Power Correction of SIDIS at Large Transverse Momentum

第六届重味物理与量子色动力学研讨会
Apr. 19th-23rd @ Qingdao, Shandong

Tianbo Liu (刘天博)

Key Laboratory of Particle Physics and Particle Irradiation (MOE)
Institute of Frontier and Interdisciplinary Science, Shandong University
Southern Center for Nuclear-Science Theory, IMP, CAS

In collaboration with W. Melnitchouk, J.W. Qiu, N. Sato

SHANDONG UNIVERSITY

SCNT
Southern Center for Nuclear-Science Theory

Lepton-hadron Deep Inelastic Scattering

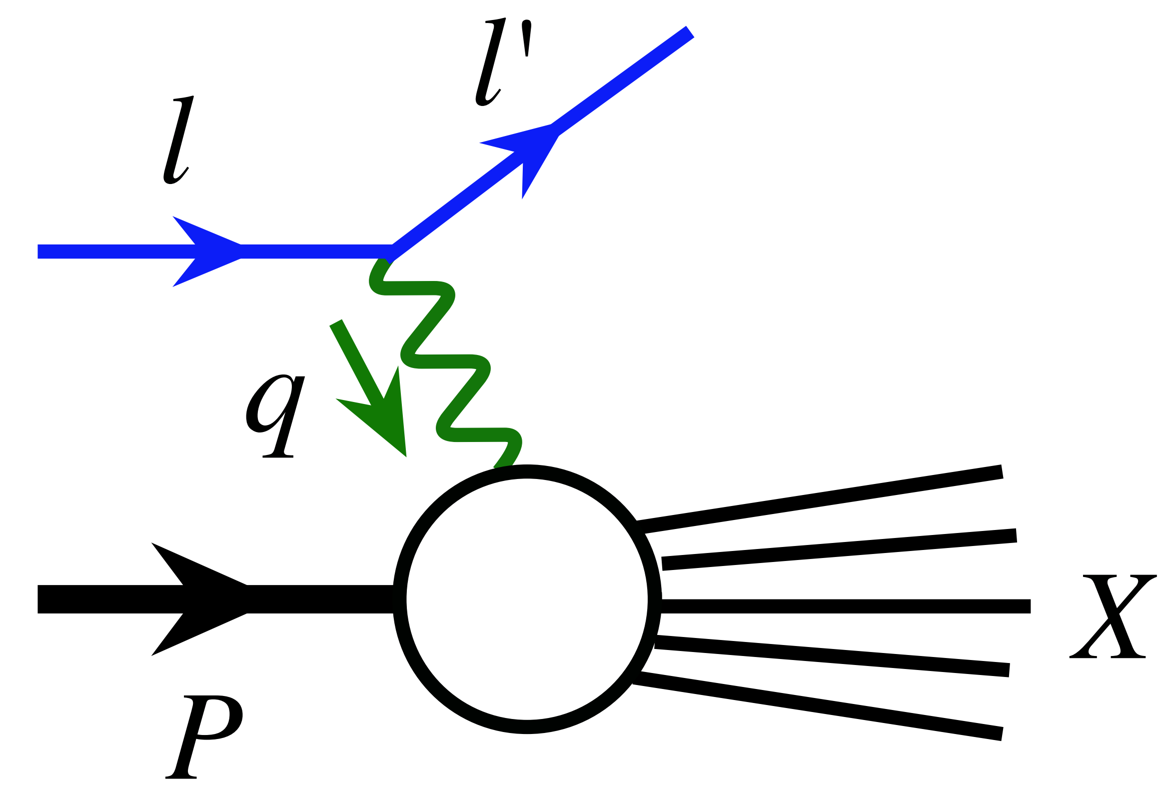
Inclusive DIS at a large momentum transfer $Q \gg \Lambda_{\mathrm{QCD}}$

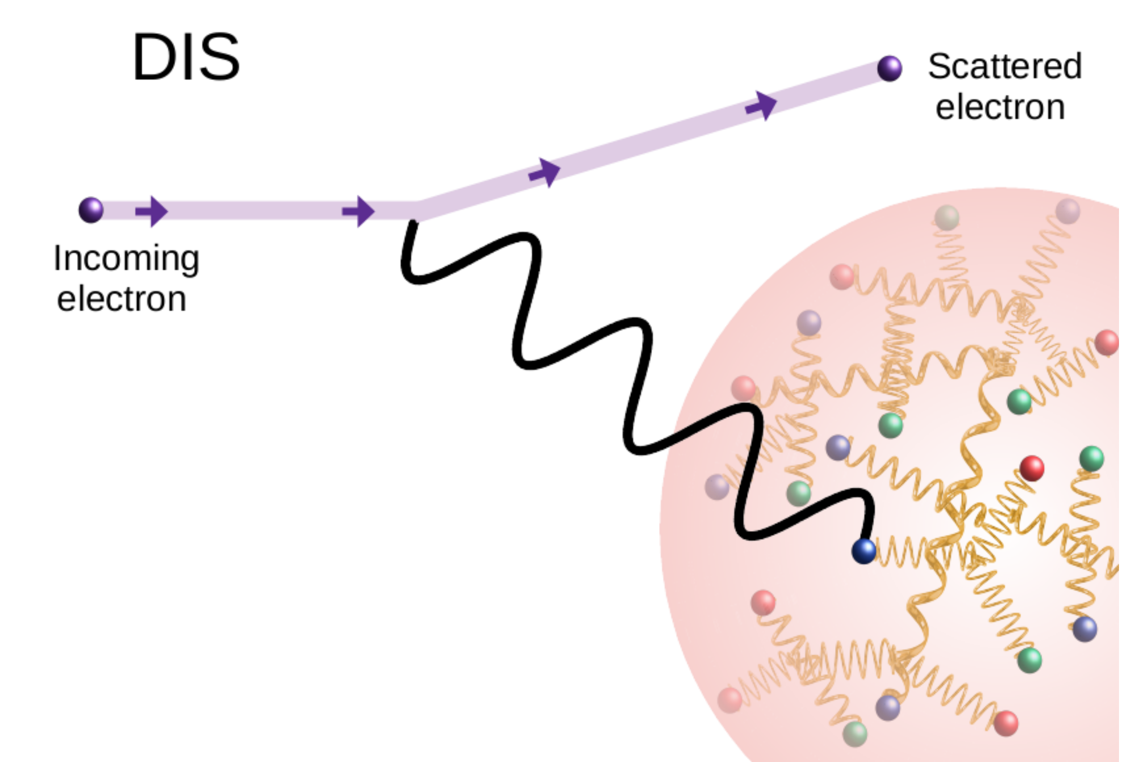
- dominated by the scattering of the lepton off an active quark/parton
- not sensitive to the dynamics at a hadronic scale ~ 1/fm
- factorized $\sigma \propto H(Q) \otimes \phi_{a/P}(x, \mu^2)$
- overall corrections suppressed by $1/Q^n$

QCD factorization

- provides the probe to "see" quarks, gluons and their dynamics indirectly
- predictive power relies on
  - precision of the probe
  - universality of $\phi_{a/P}(x, \mu^2)$

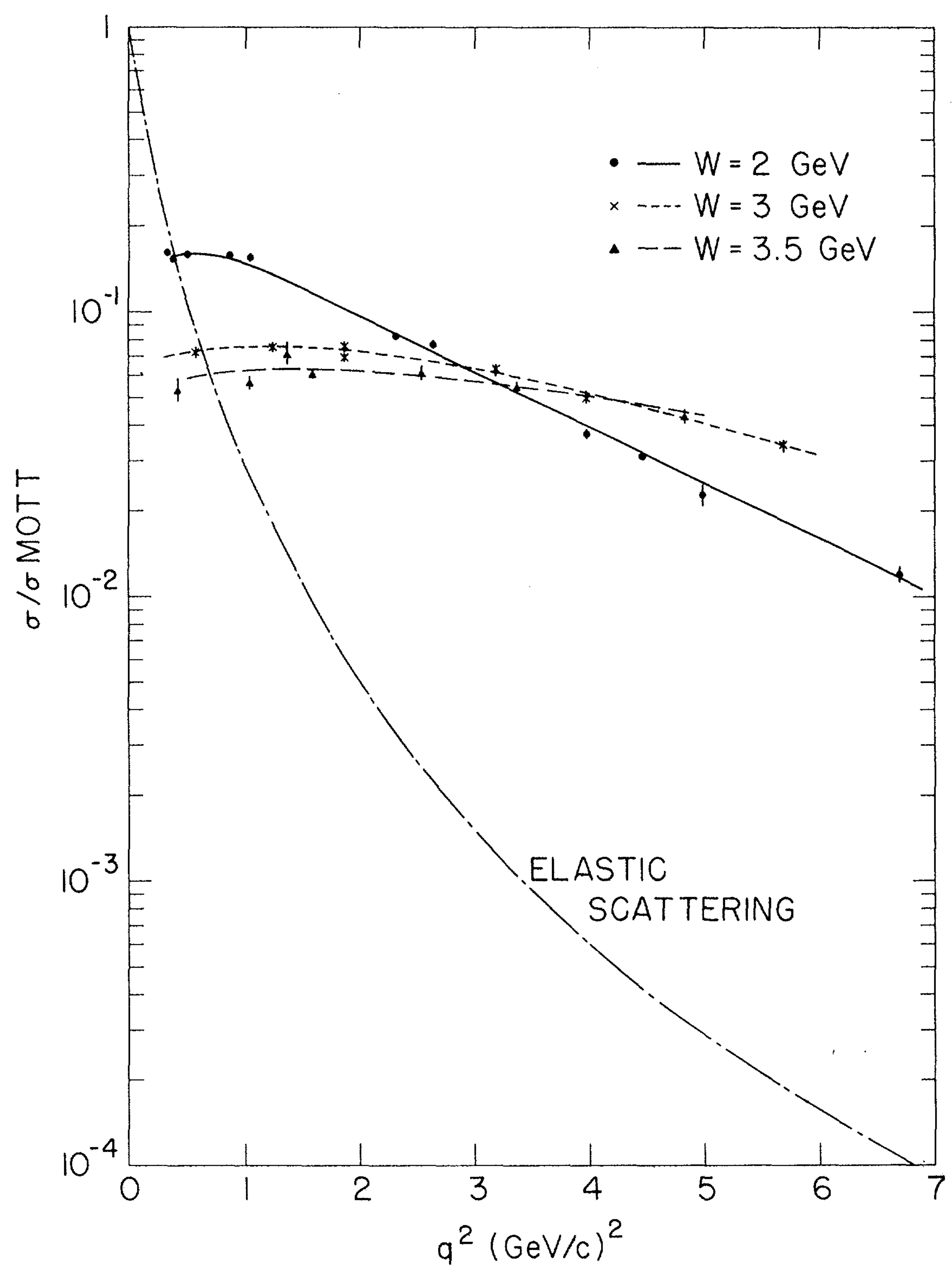
Modern "Rutherford" experiment.



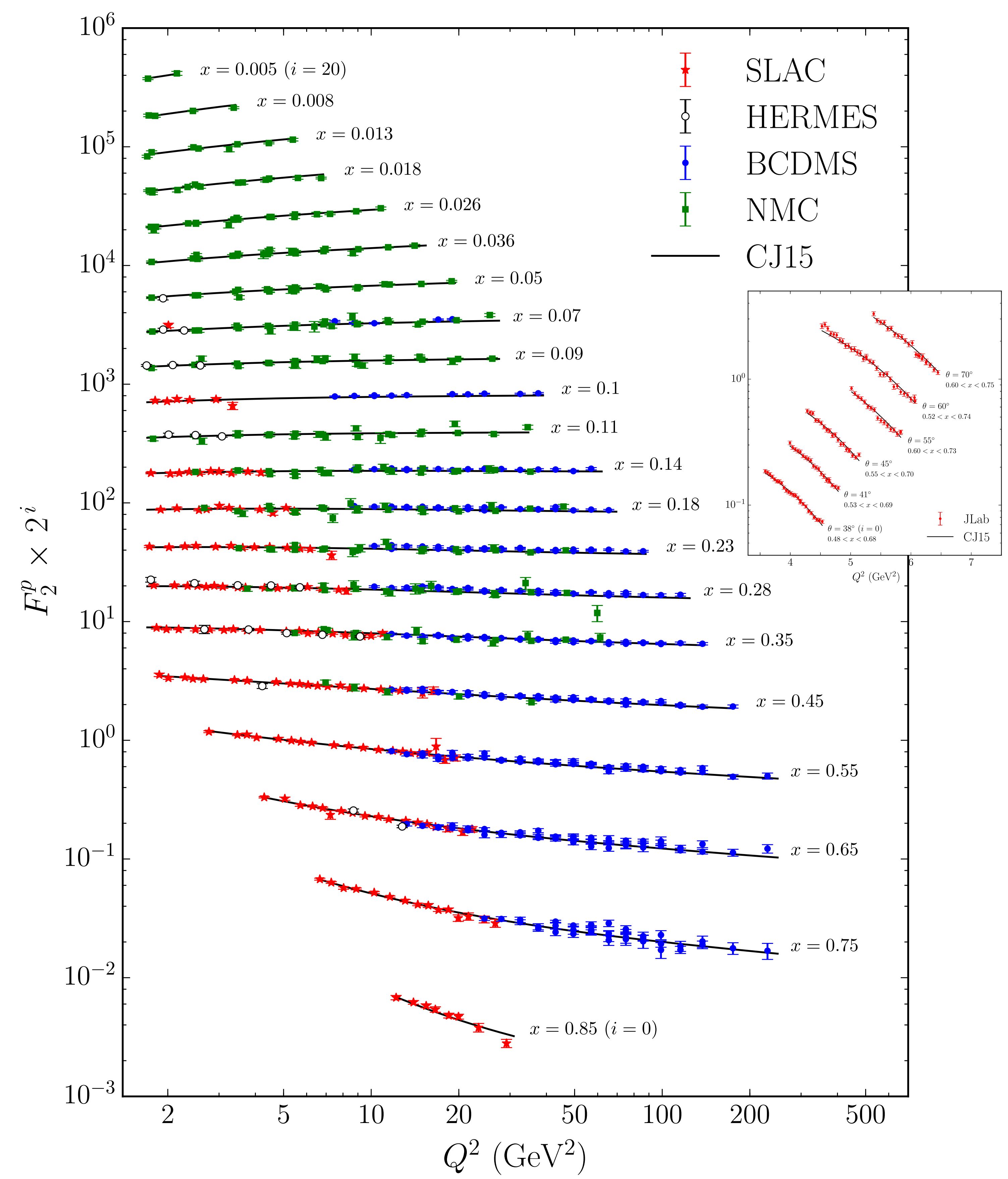


[Figure from DESY-21-099]

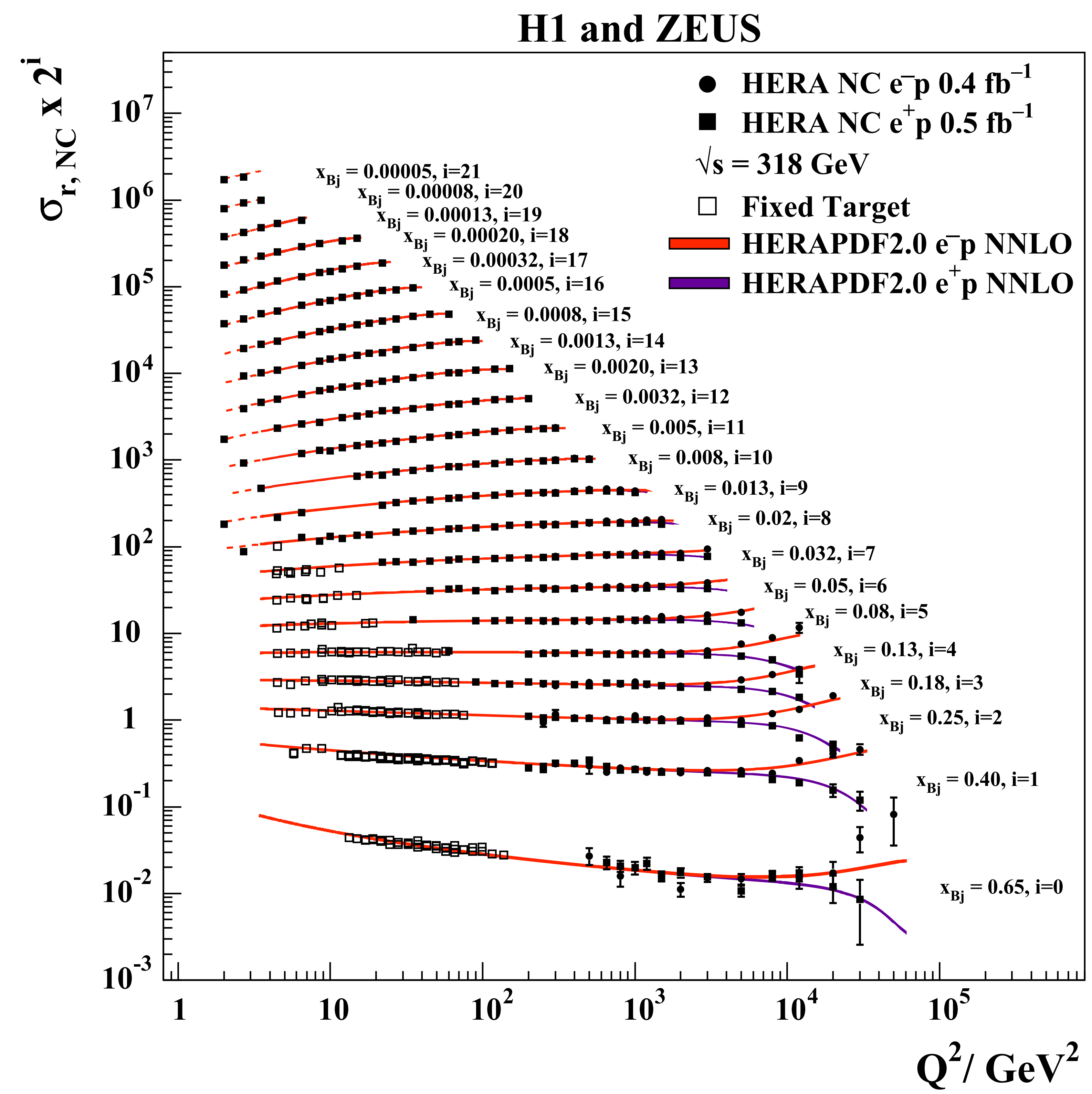
Lepton-hadron Deep Inelastic Scattering



M. Breidenbach *et al.*, PRL 23, 935 (1969).



A. Accardi *et al.*, PRD 93, 114017 (2016).

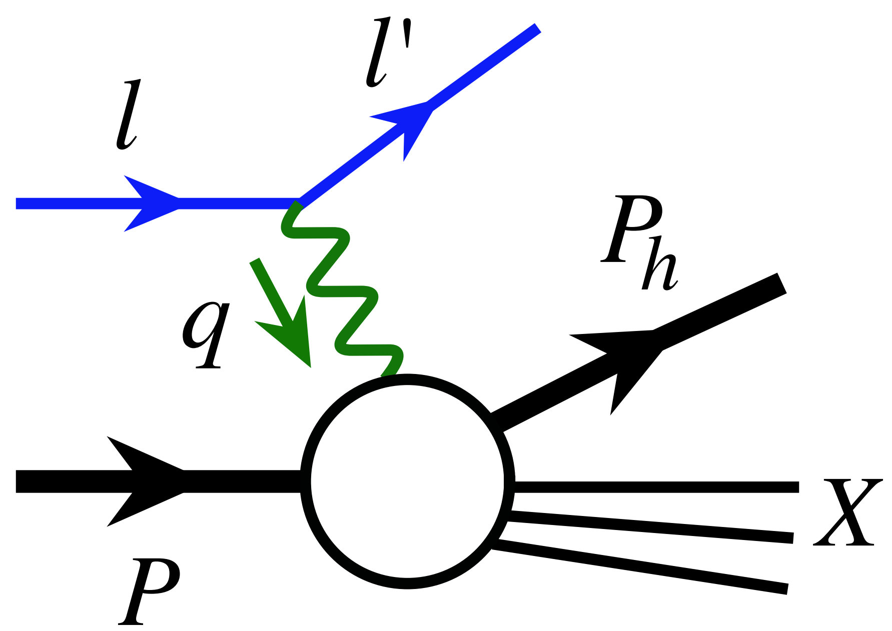


H. Abramowicz *et al.*, EPJC 78, 580 (2015).

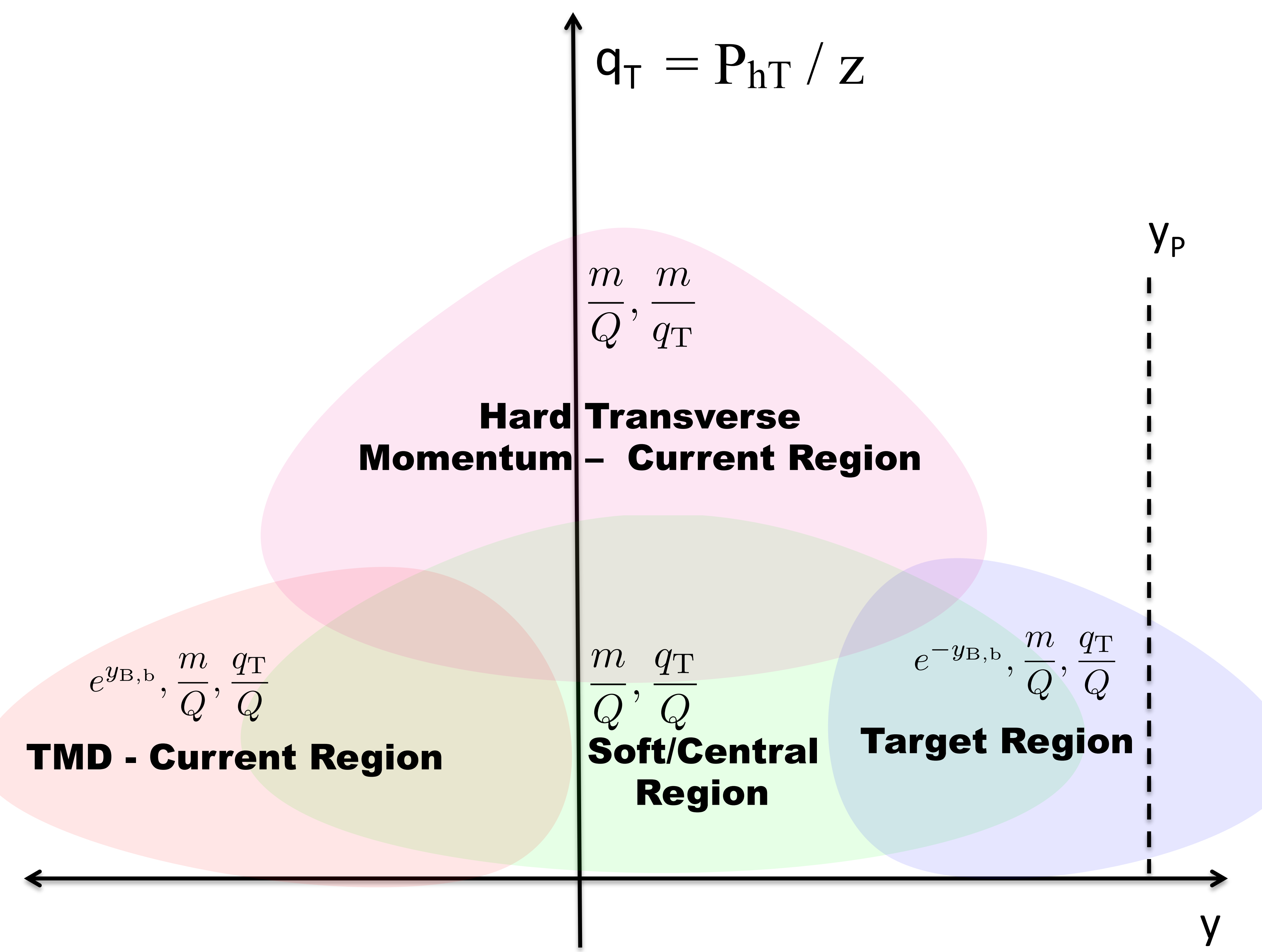
A successful story of QCD, factorization and evolution!

Semi-inclusive Deep Inelastic Scattering

SIDIS: identify a hadron h in the final state



- enable us to explore the emergence of color neutral hadrons from colored quarks/gluons
- flavor dependence by selecting different types of observed hadrons: pions, kaons, …
- a large momentum transfer Q provides a short-distance probe
- an additional and adjustable momentum scale



[Figure from JHEP10(2019)122]

W + Y Formalism

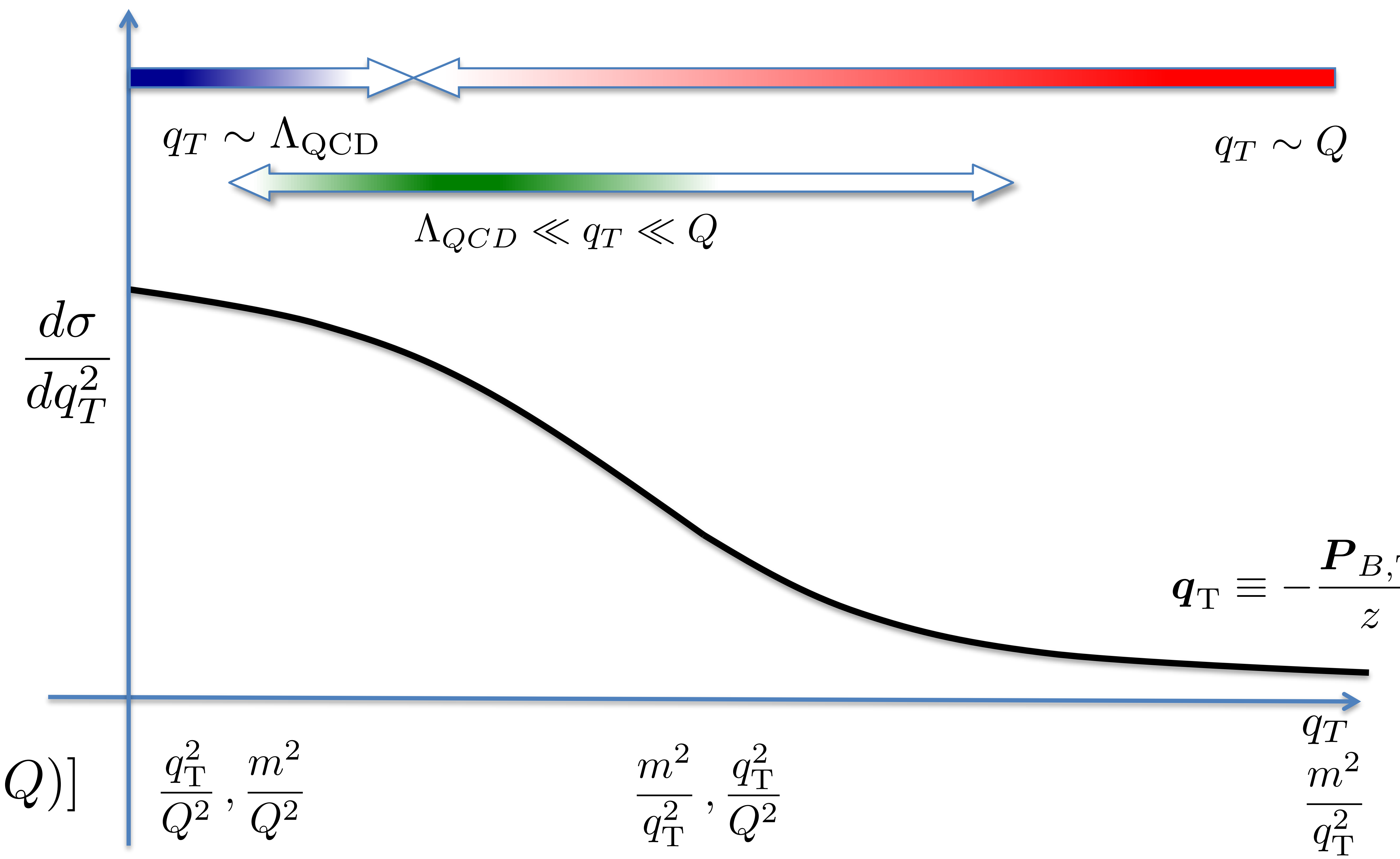
The $W + Y$ formalism

$$\Gamma(q_{\mathrm{T}}, Q) = \frac{d\sigma}{d^2 q_{\mathrm{T}} dQ \cdots}$$

$$\Gamma(q_{\mathrm{T}}, Q) = W(q_{\mathrm{T}}, Q) + Y(q_{\mathrm{T}}, Q) + \mathcal{O}\left(\frac{m}{Q}\right)^c \Gamma(q_{\mathrm{T}}, Q)$$

$$W(q_{\mathrm{T}}, Q) = \mathrm{T}_{\mathrm{TMD}} \Gamma(q_{\mathrm{T}}, Q)$$

$$\begin{aligned} Y(q_{\mathrm{T}}, Q) &= X(q_{\mathrm{T}}/\lambda)[\mathrm{T}_{\mathrm{coll}} \Gamma(q_{\mathrm{T}}, Q) \\ &\quad - \mathrm{T}_{\mathrm{coll}} \mathrm{T}_{\mathrm{TMD}} \Gamma(q_{\mathrm{T}}, Q)] \\ &= X(q_{\mathrm{T}}/\lambda)[\mathrm{FO}(q_{\mathrm{T}}, Q) - \mathrm{ASY}(q_{\mathrm{T}}, Q)] \end{aligned}$$



[Figure by Ted Rogers]

J. Collins, L. Gamberg, A. Prokudin, T.C. Rogers,
N. Sato, B. Wang, Phys. Rev. D 94, 034014 (2016).

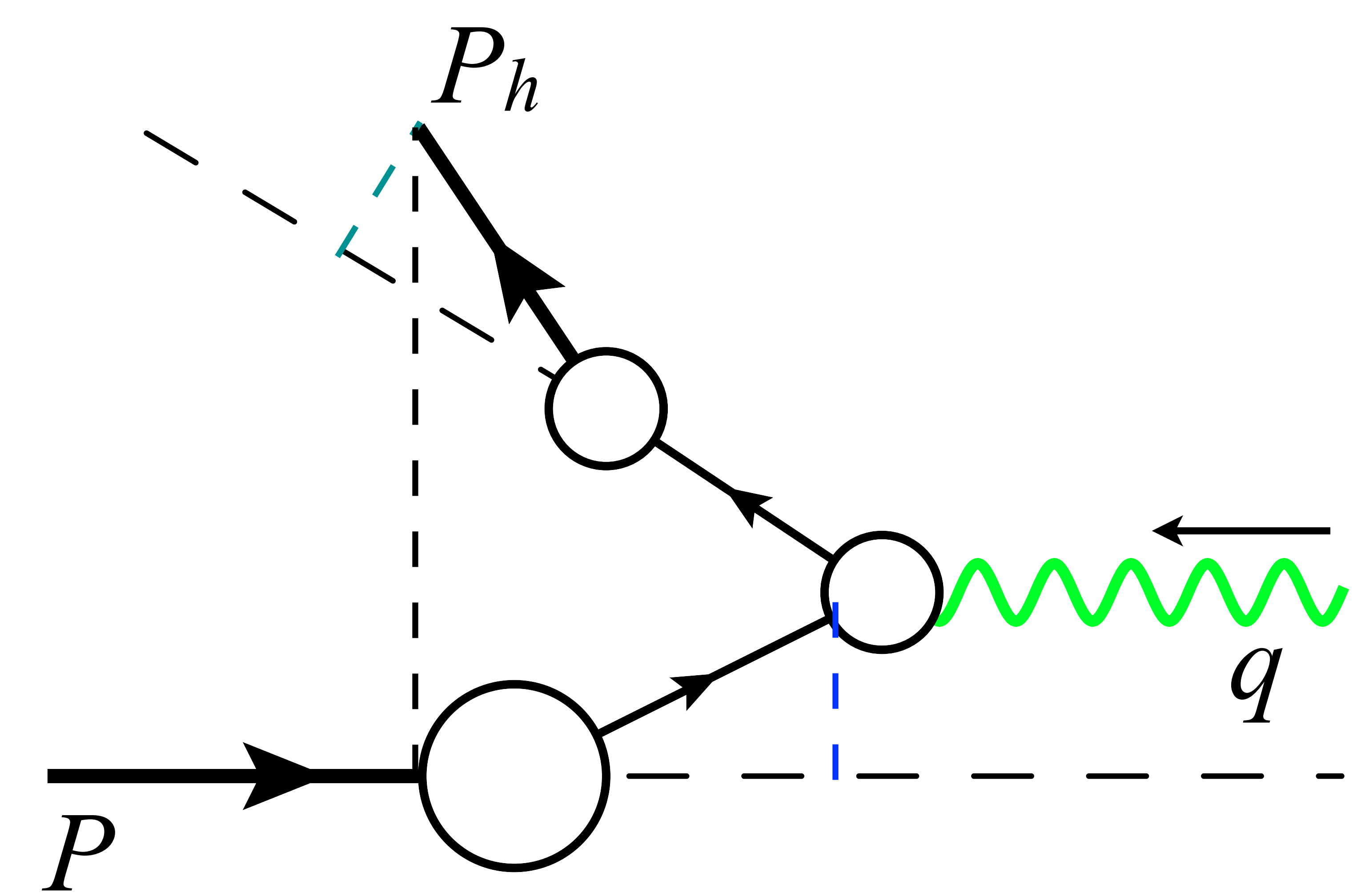
Small Transverse Momentum Region

Small transverse momentum $P_{h_T} \ll Q$

- the hard scale Q localizes the probe to "see" quarks and gluons
- the soft scale P_{h_T} is sensitive to the confined motion of quarks and gluons
- TMD factorization

$$\sigma \propto H(Q) \otimes \phi_{a/P}(x, k_T, \mu^2) \otimes D_{f \to h}(z, p_T, \mu^2)$$

- corrections suppressed by powers of P_{h_T}/Q
- dominated by the W-term in the "W+Y" prescription

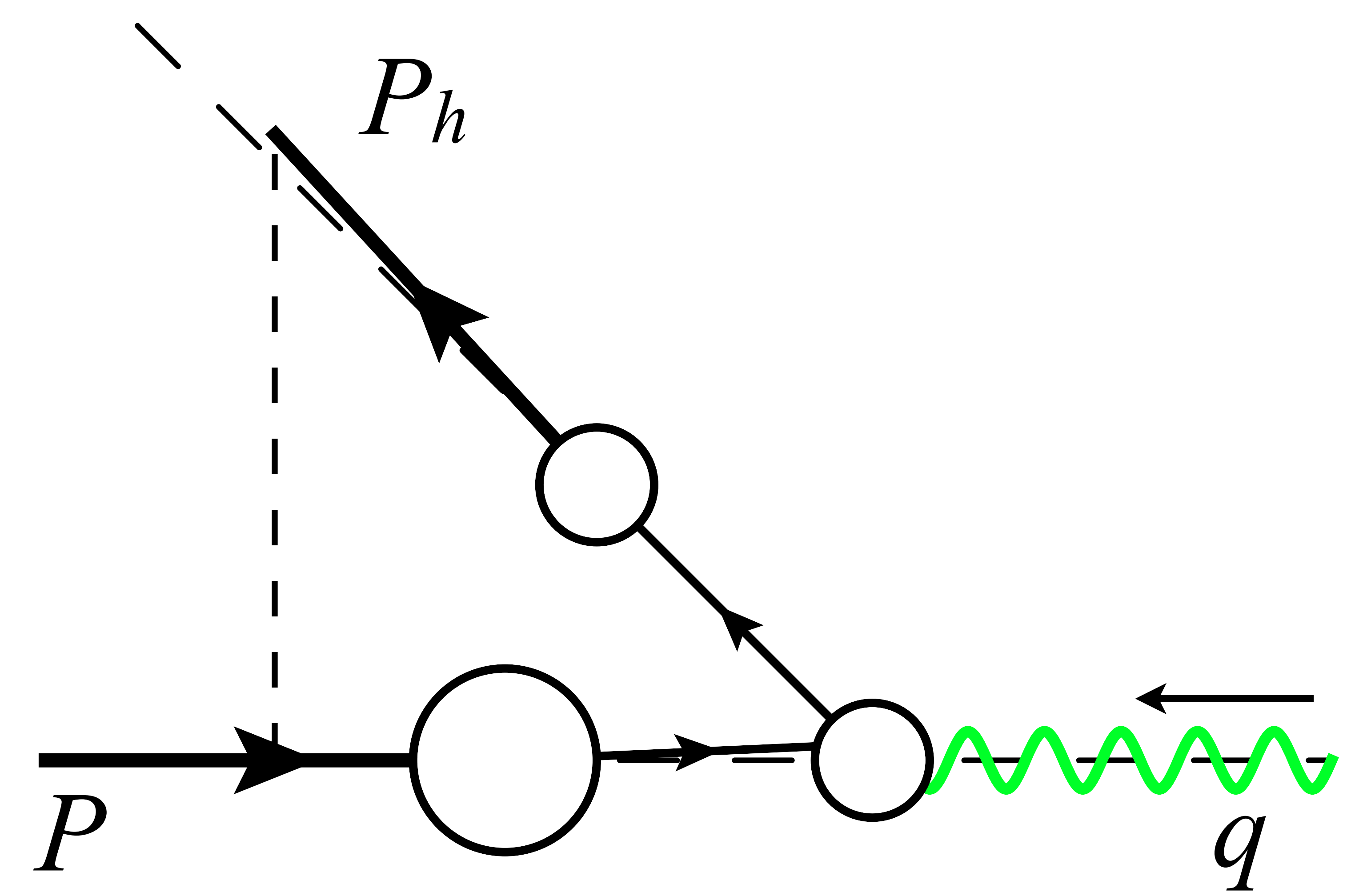


Large Transverse Momentum Region

Large transverse momentum $P_{h_T} \sim Q$

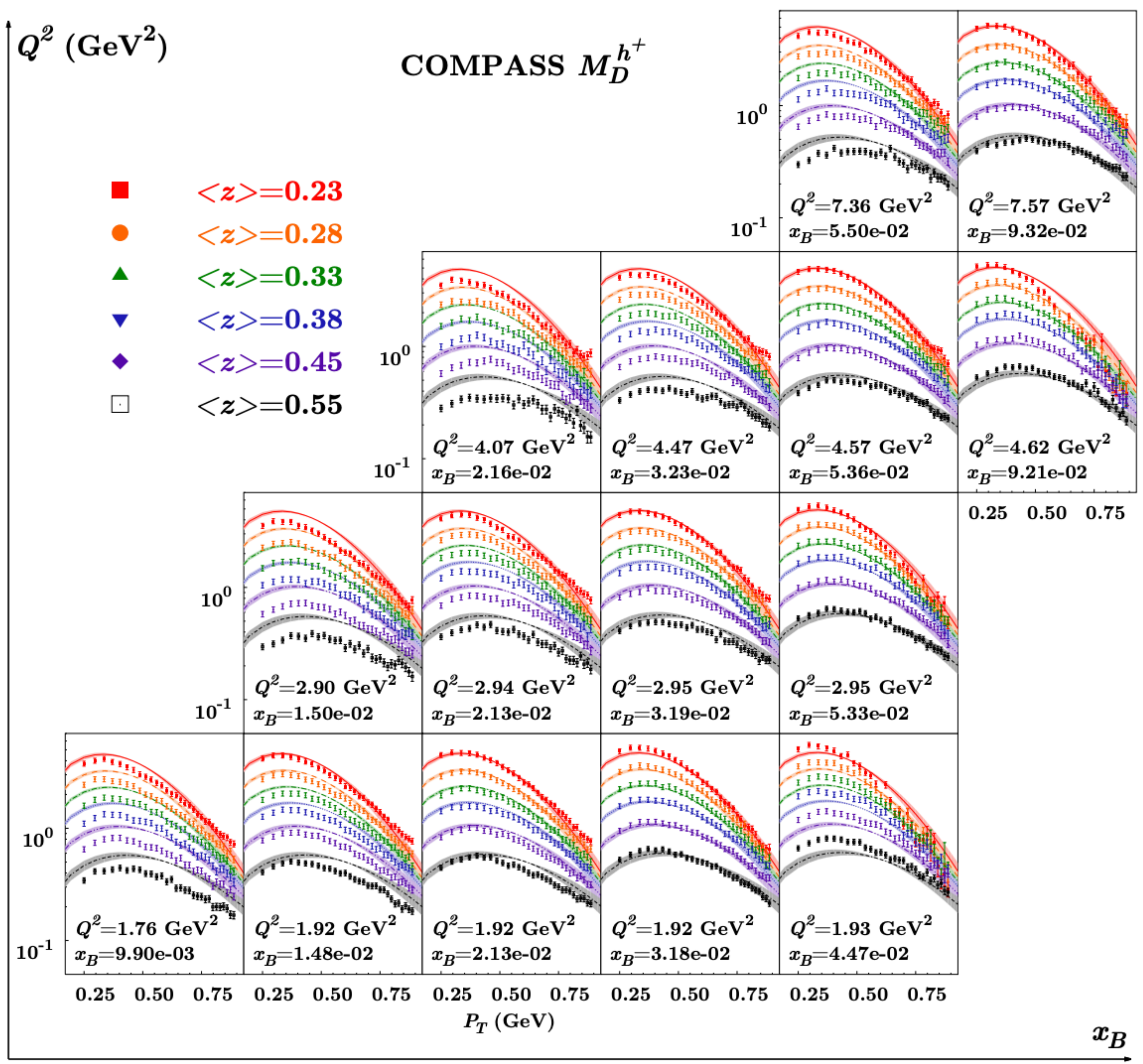
- dominated by a single hard scale
- not sensitive to the active parton's transverse momentum k_T or p_T
- described by collinear factorization

  $\sigma \propto H(Q, P_{h_T}) \otimes \phi_{a/P}(x, \mu^2) \otimes D_{f\to h}(z, \mu^2)$
- corrections suppressed by $1/P_{h_T}^2$ or $1/Q^2$
- dominated by the fixed-order (FO) term in the "W+Y" prescription

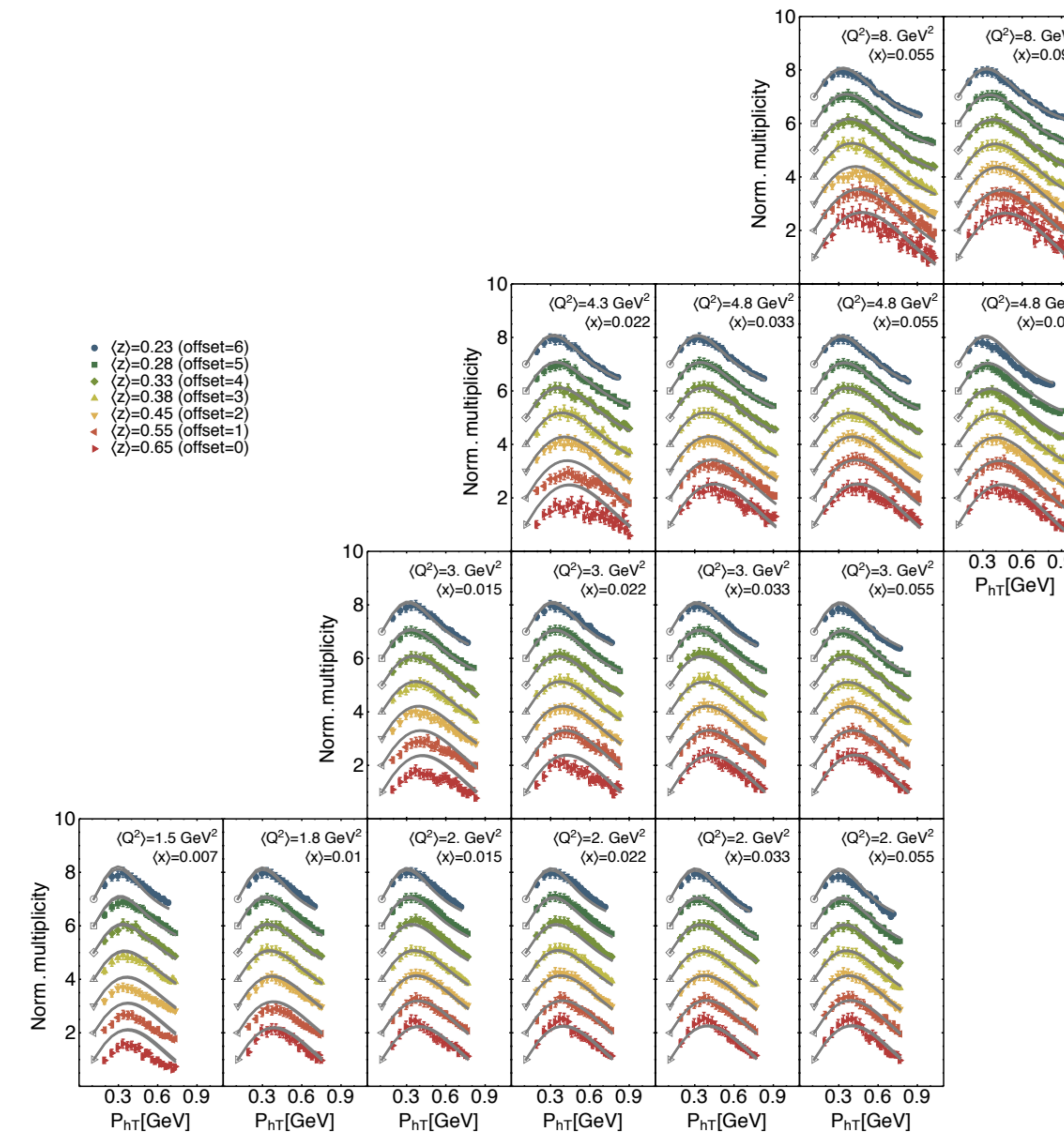


Phenomenology Fits with TMDs

Recent global analyses using *W-term* only



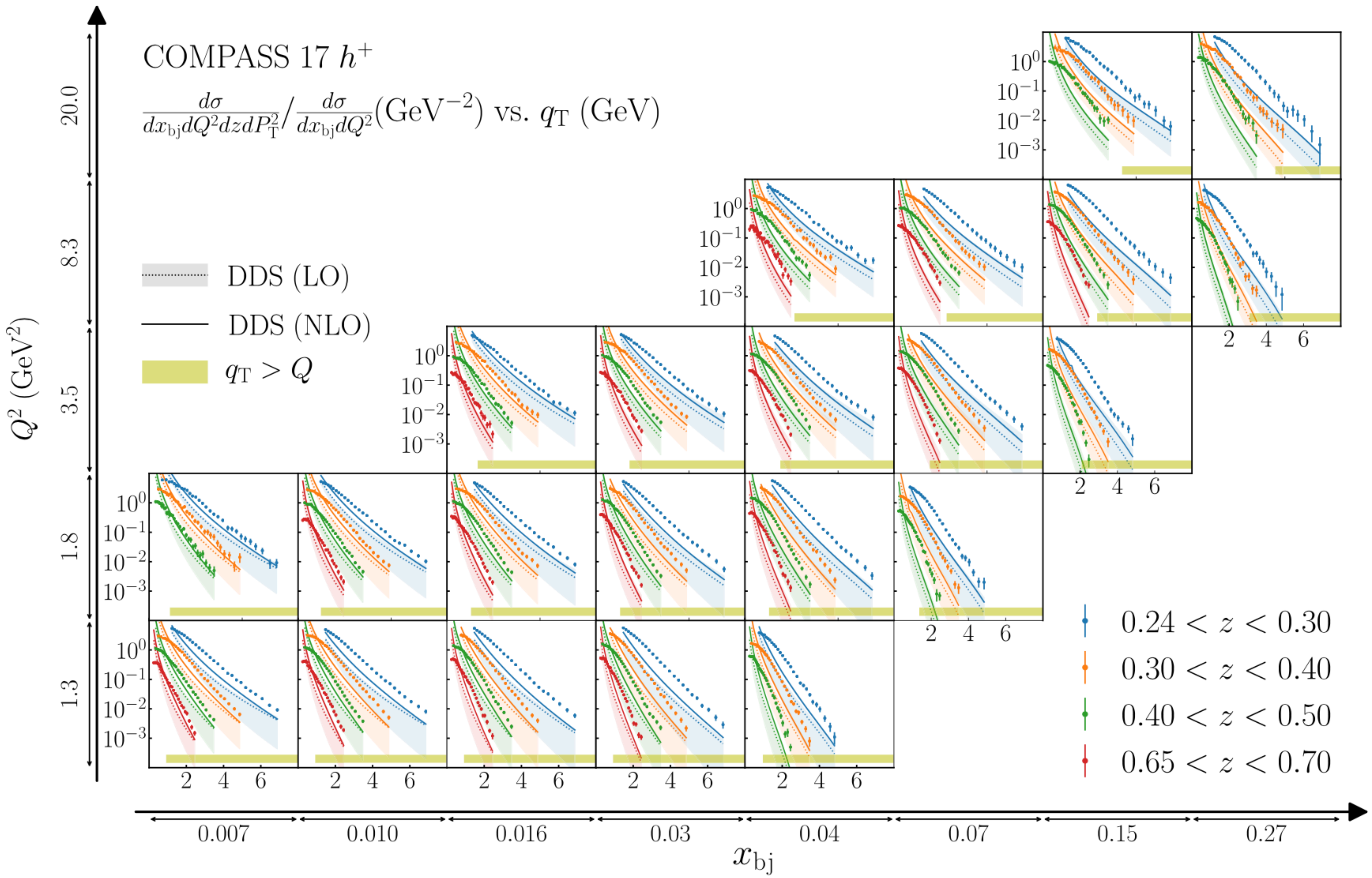
M. Anselmino, M. Boglione, J.O. Gonzalez-Hernandez, S. Melis, A. Prokudin, JHEP 04 (2014) 005.



A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori, JHEP 06 (2017) 081.

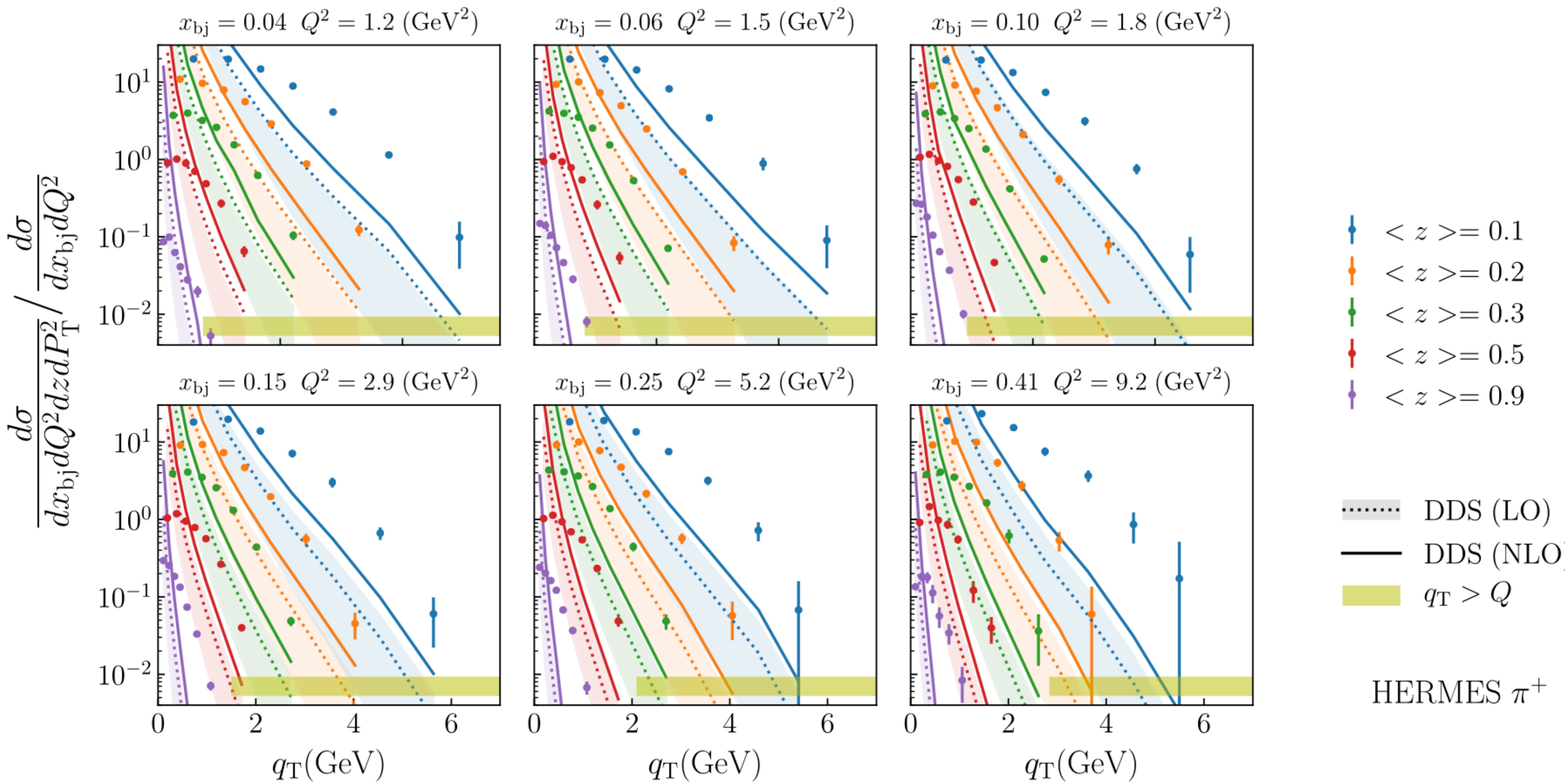
Challenge at Large Transverse Momentum

About an order of magnitude discrepancy between data and theory



M. Aghasyan *et al.* (COMPASS Collaboration), Phys. Rev. D 97, 032006 (2018).
J.O. Gonzalez-Hernandez, T.C. Rogers, N. Sato, B. Wang, Phys. Rev. D 98 114005 (2018).

Challenge at Large Transverse Momentum

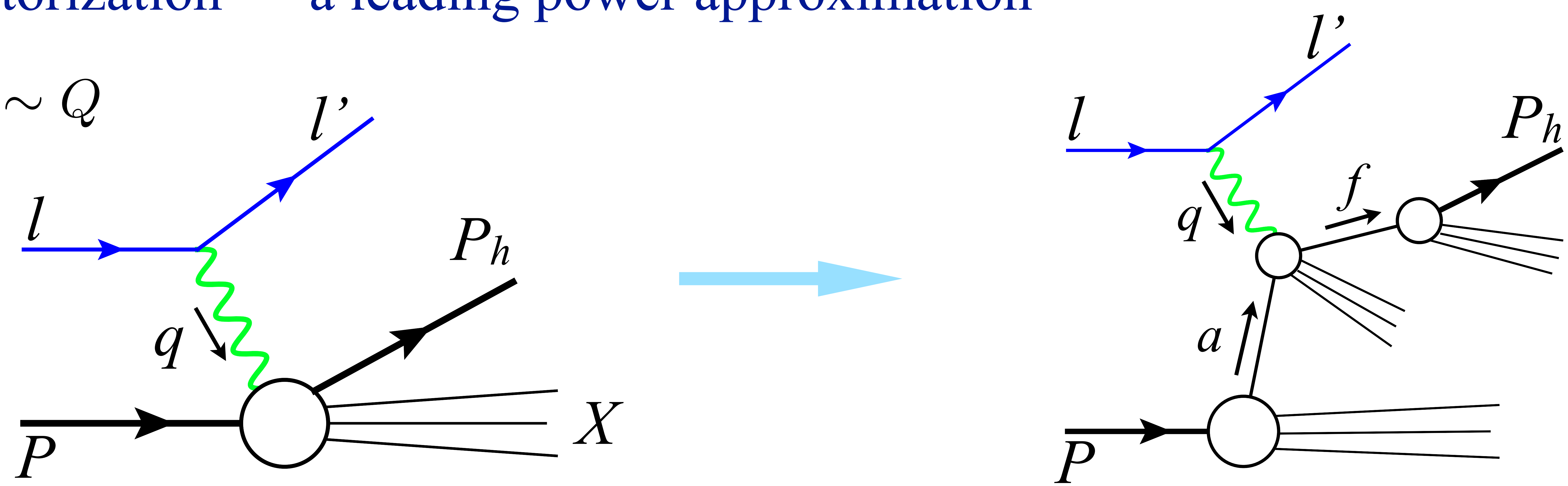


J.O. Gonzalez-Hernandez, T.C. Rogers, N. Sato, B. Wang, Phys. Rev. D 98 114005 (2018).

Leading Power Approximation

QCD factorization — a leading power approximation

$P_{h_T} \sim Q$



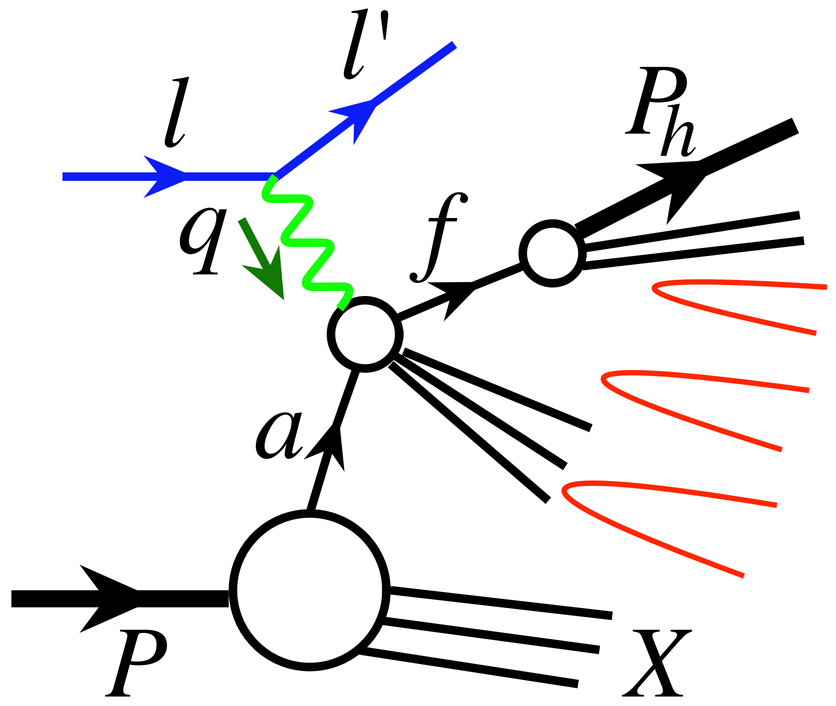
$$\frac{d\sigma_{l+P\to l'+P_h+X}}{d^3\mathbf{l}'/(2E')\; d^3\mathbf{P}_h/(2E_h)} \approx \sum_{a,f}\int_{x_B}^{1}\frac{dx}{x}\int_{z_h}^{1}\frac{dz}{z^2}\phi_{a/P}(x)\; D_{f\to h}(z)\frac{d\hat{\sigma}_{l+a\to l'+f+X}}{d^3\mathbf{l}'/(2E')d^3\mathbf{p}_f/(2E_f)}$$

$$+\;\; \mathcal{O}\left(\frac{1}{Q^n},\frac{1}{P_{h_T}^n}\right)$$

corrections are formally suppressed by inverse powers of large momentum scale

Leading Power Approximation

Color neutralization



soft pions …

Need large enough phase space to shower

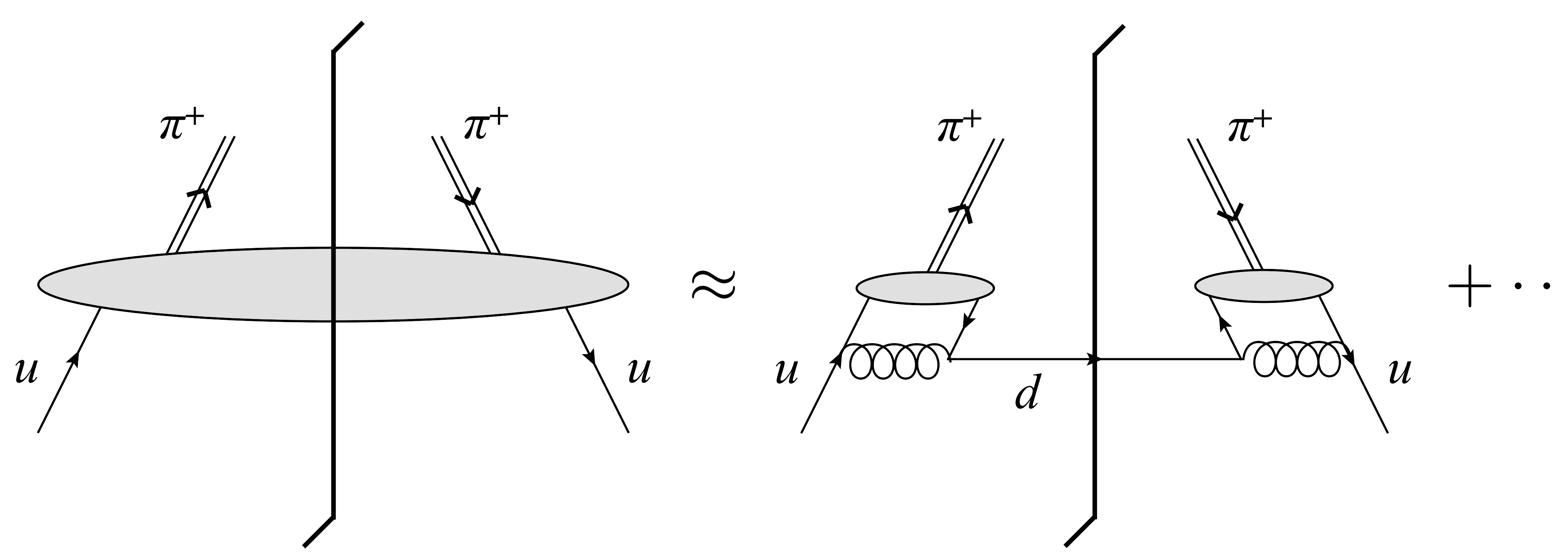
Sufficiently high multiplicity

Near the edge of phase space — large P_{hT}, large z_h?

Low multiplicity?

LP fragmentation functions:

$D_{u\to\pi^+}(z)$



At the edge of phase space

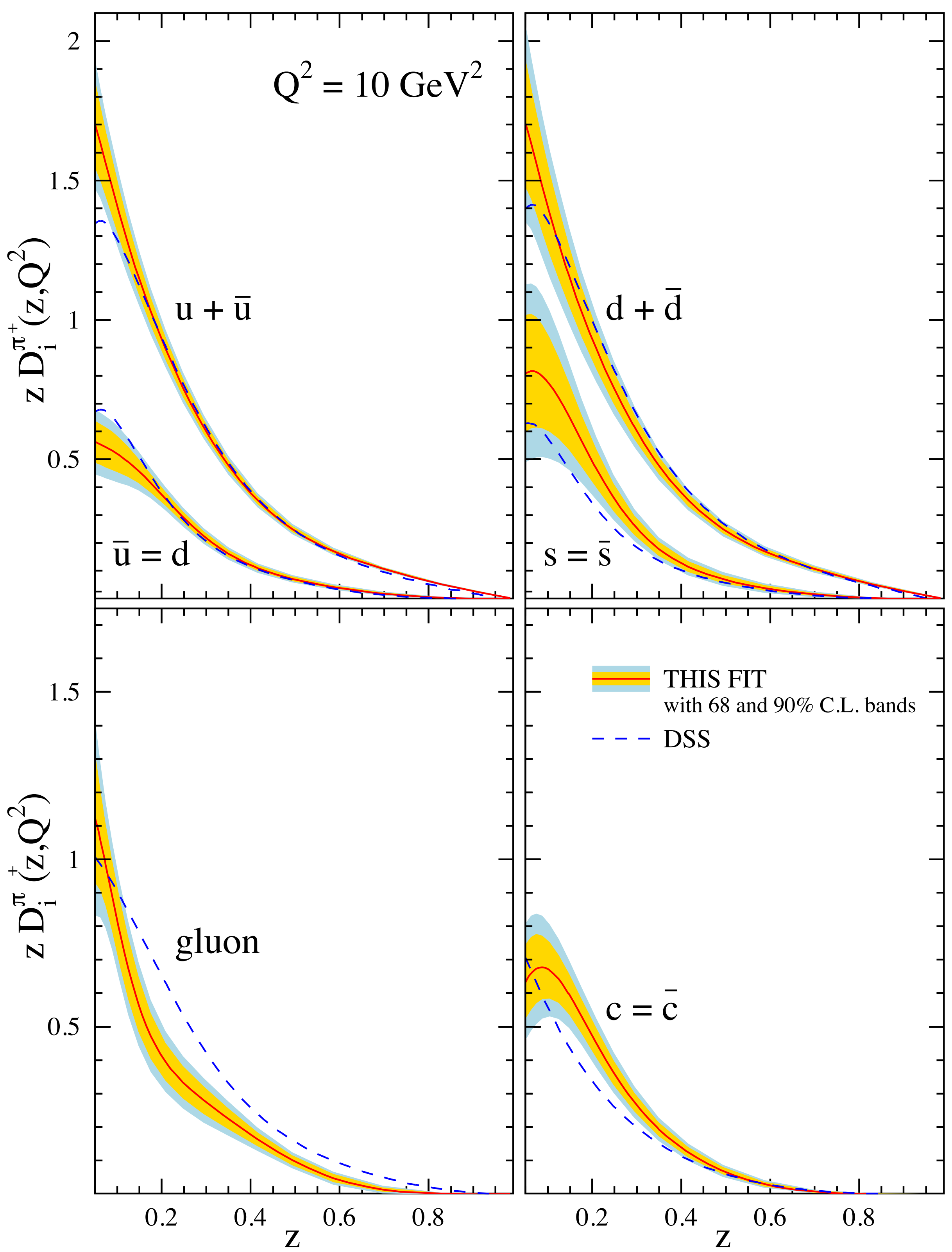
— large P_{hT}, large z_h

$$\sigma \propto D_{f\to h}(z) \propto (1-z)^n$$

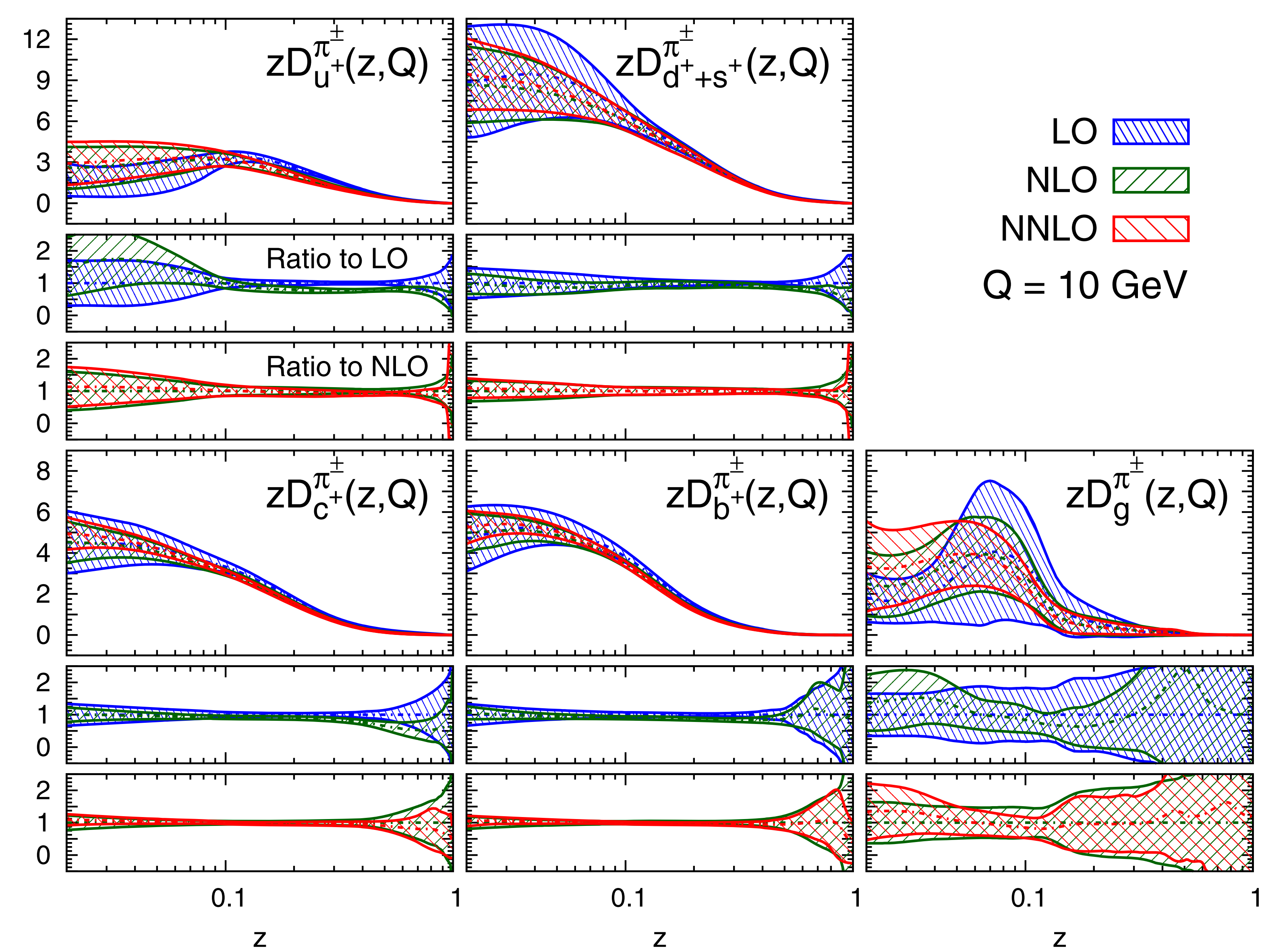
Some Extractions of Fragmentation Functions

Parton to pion FFs

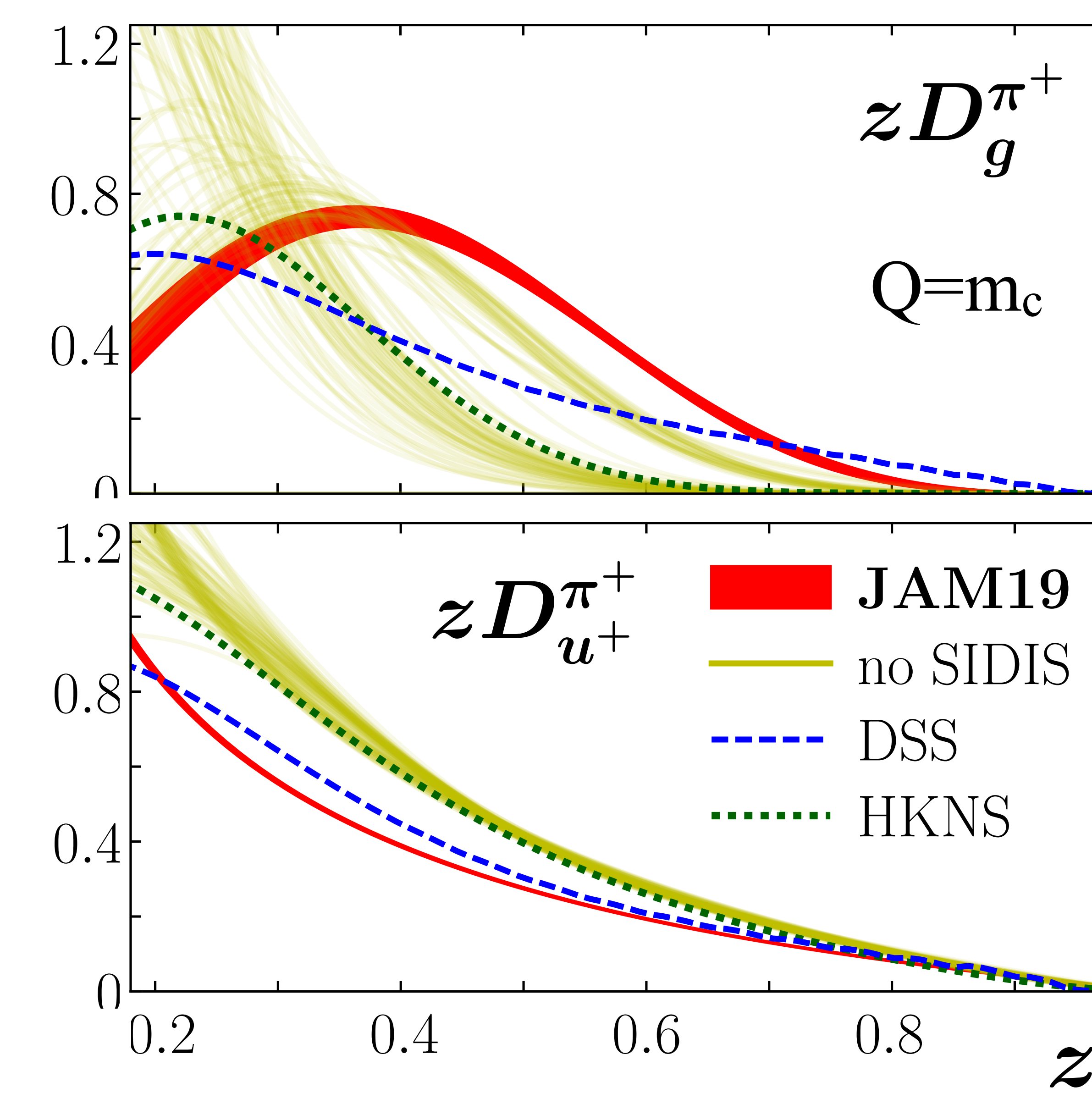
DSS14



NNFF1.0



JAM19



D. de Florian, R. Sassot, M. Epele, R.J. Hernández-Pinto, M. Stratmann, PRD 91, 014035 (2015).

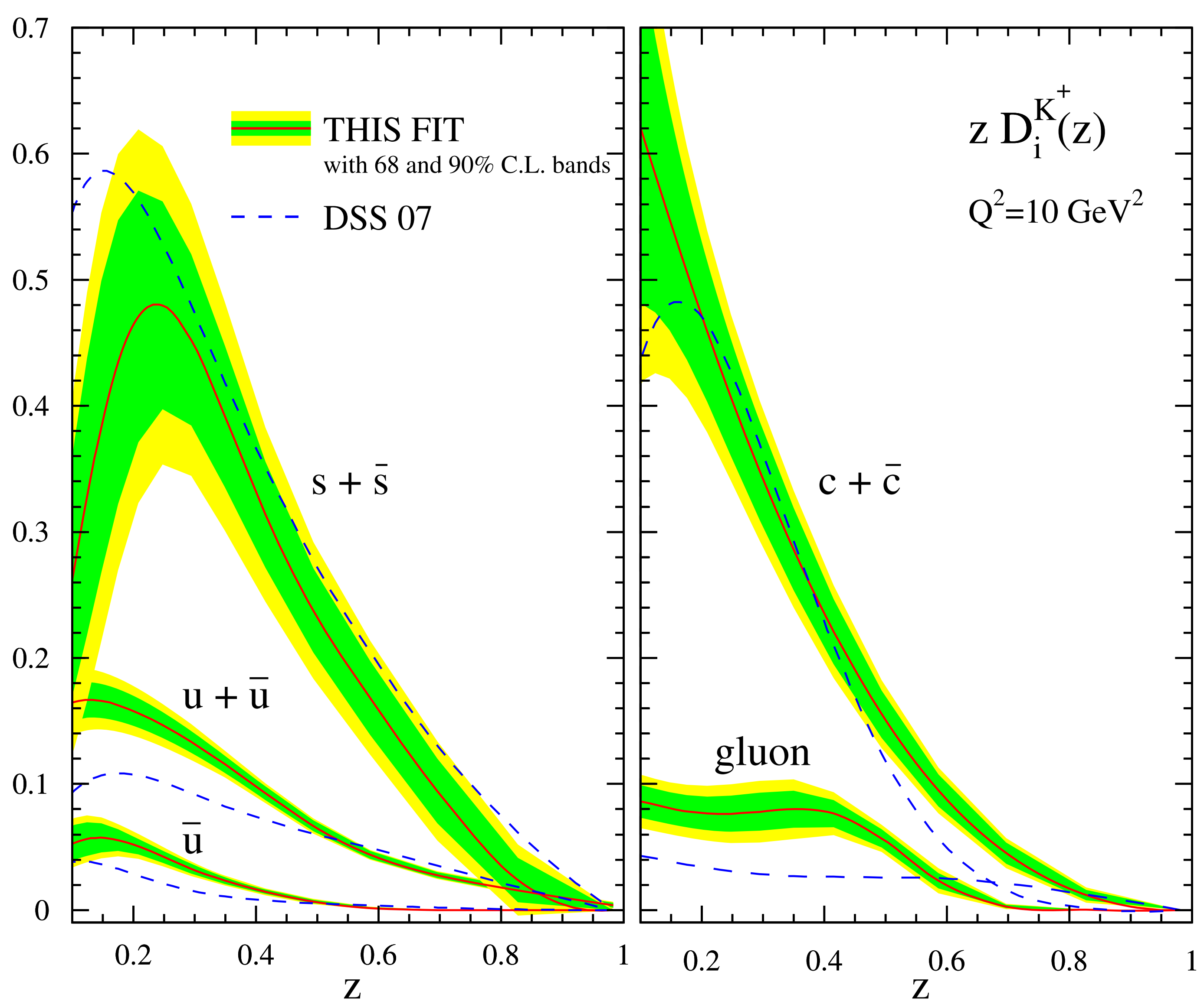
V. Bertone, S. Carrazza, N.P. Hartland, E.R. Nocera, J. Rojo, EPJC 77, 516 (2017).

N. Sato, C. Andres, J.J. Ethier, W. Melnitchouk, PRD 101, 074020 (2020).

Some Extractions of Fragmentation Functions

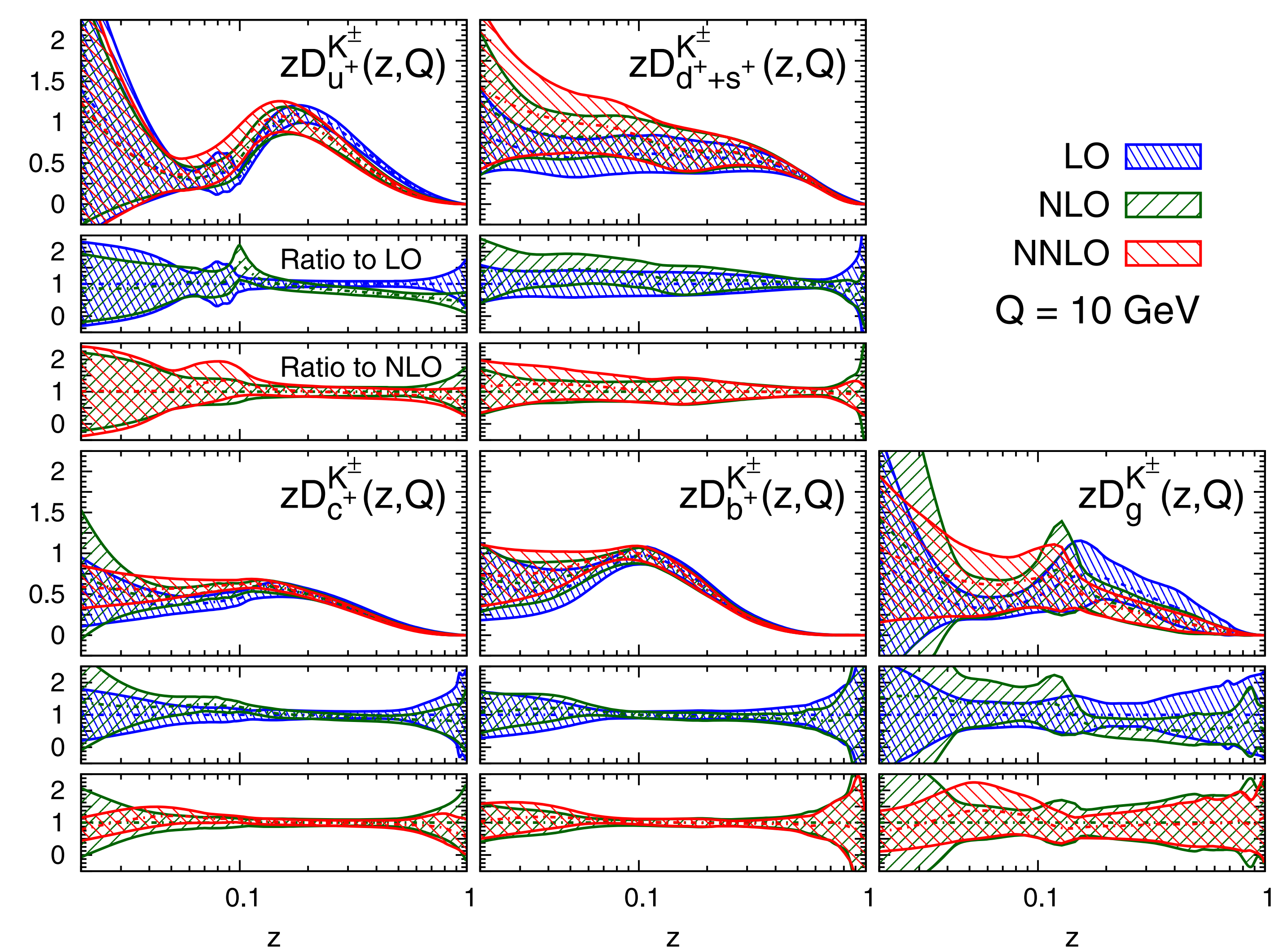
Parton to kaon FFs

DSS17



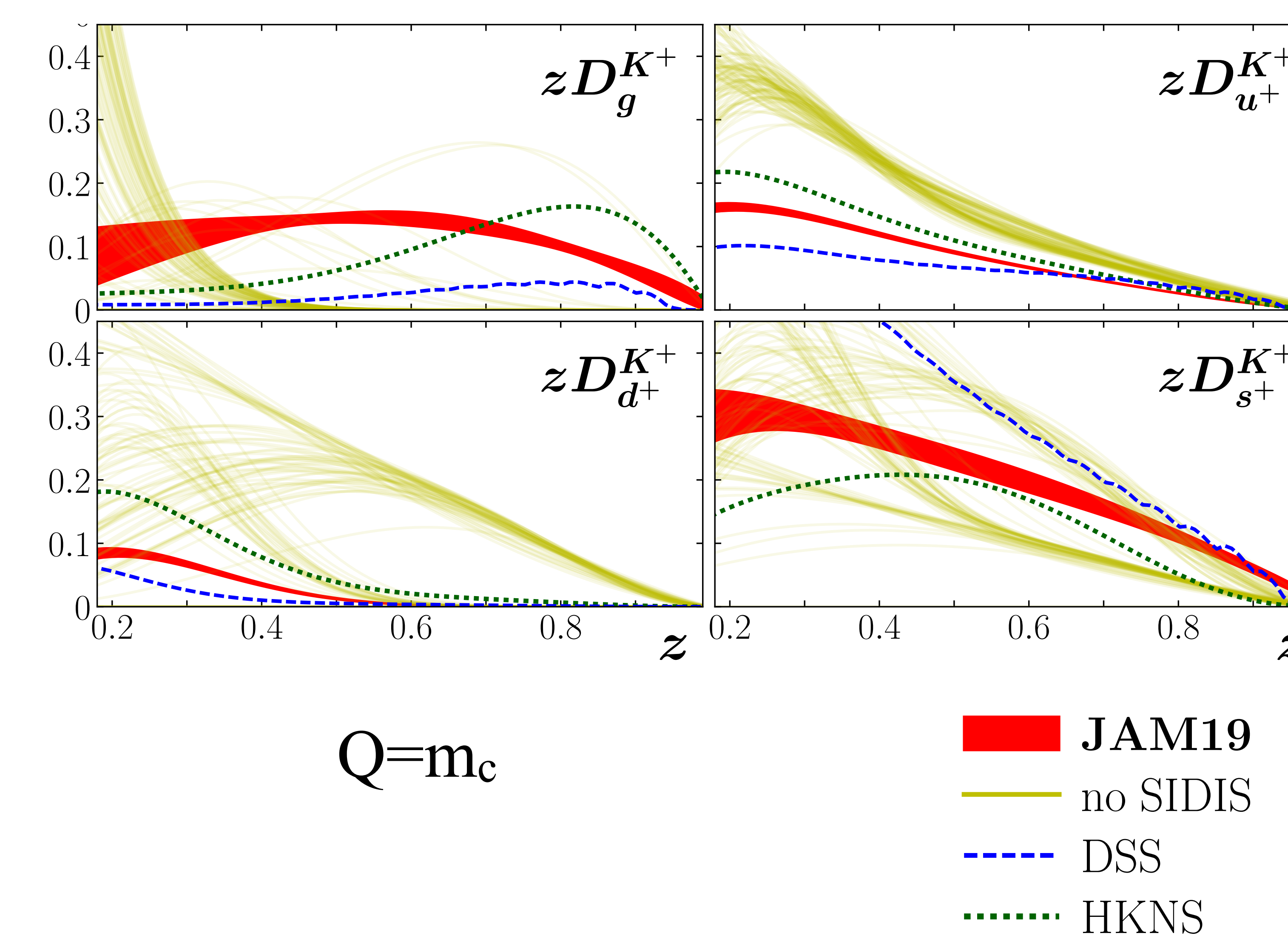
D. de Florian, M. Epele, R.J. Hernández-Pinto, R. Sassot, M. Stratmann, PRD 95, 094019 (2017).

NNFF1.0



V. Bertone, S. Carrazza, N.P. Hartland, E.R. Nocera, J. Rojo, EPJC 77, 516 (2017).

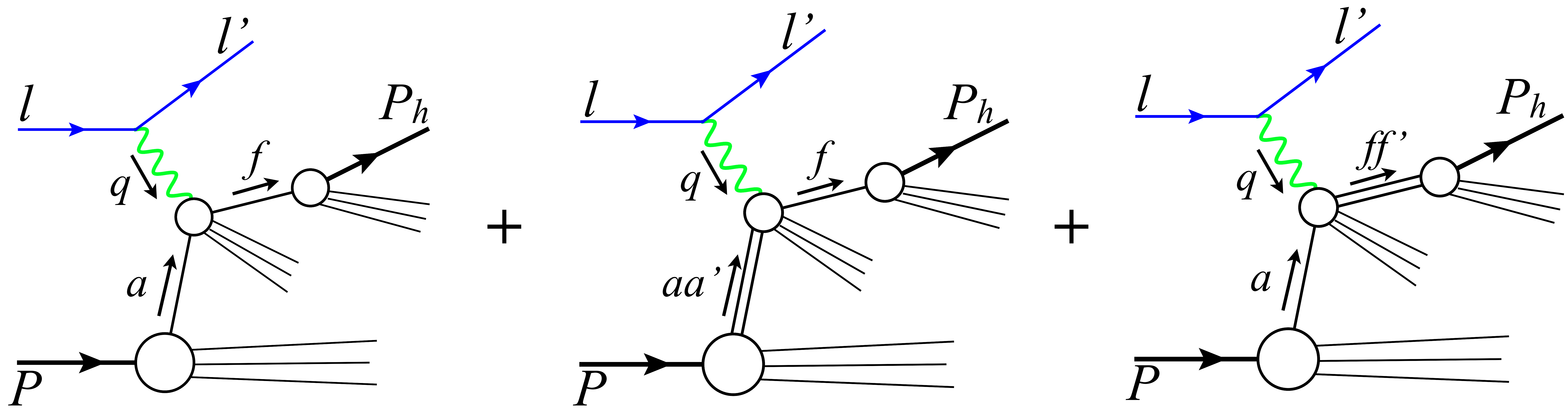
JAM19



N. Sato, C. Andres, J.J. Ethier, W. Melnitchouk, PRD 101, 074020 (2020).

Next-to-Leading Power Correction

LP + NLP:



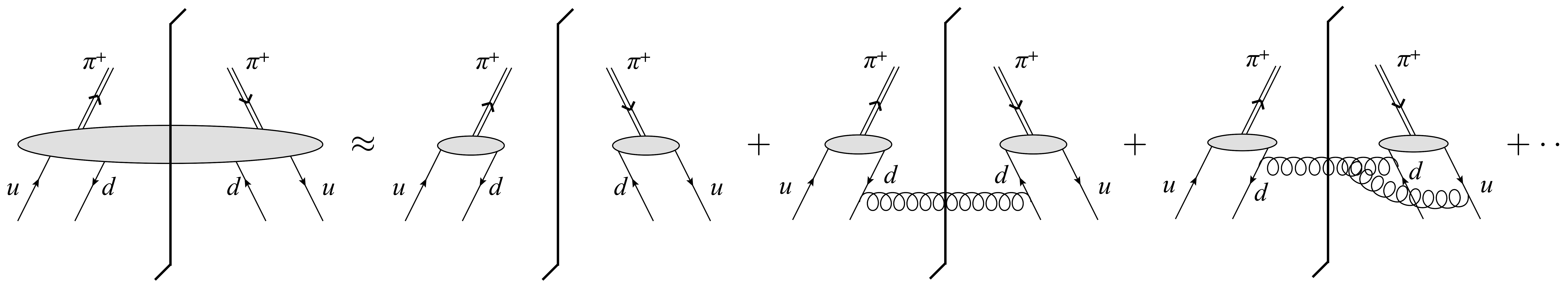
(LP) (NLP) $\mathcal{O}\left(\frac{1}{Q^2}\right)$ (NLP) $\mathcal{O}\left(\frac{1}{P_{h_T}^2}\right)$

NLP hard part is formally suppressed by $1/Q^2$ or $1/P_{h_T}^2$

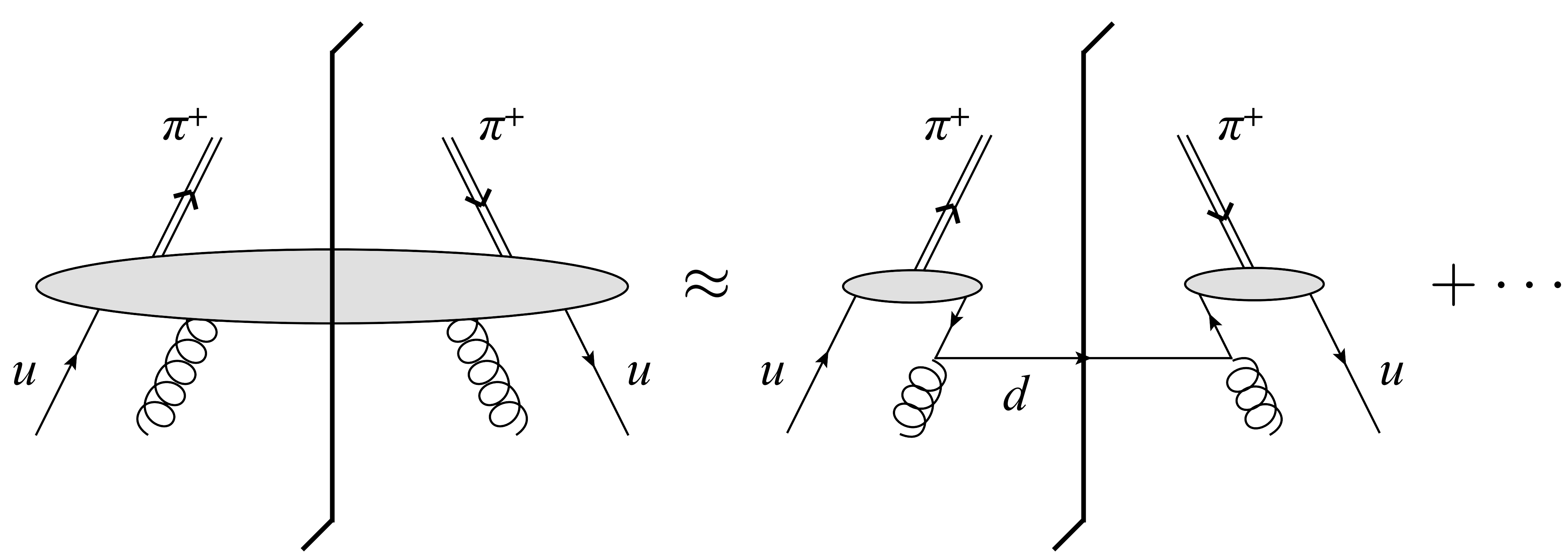
NLP contribution to the cross section is not necessarily small if get enhancement from hadronization

Enhancement of Hadronization at NLP

NLP fragmentation functions:



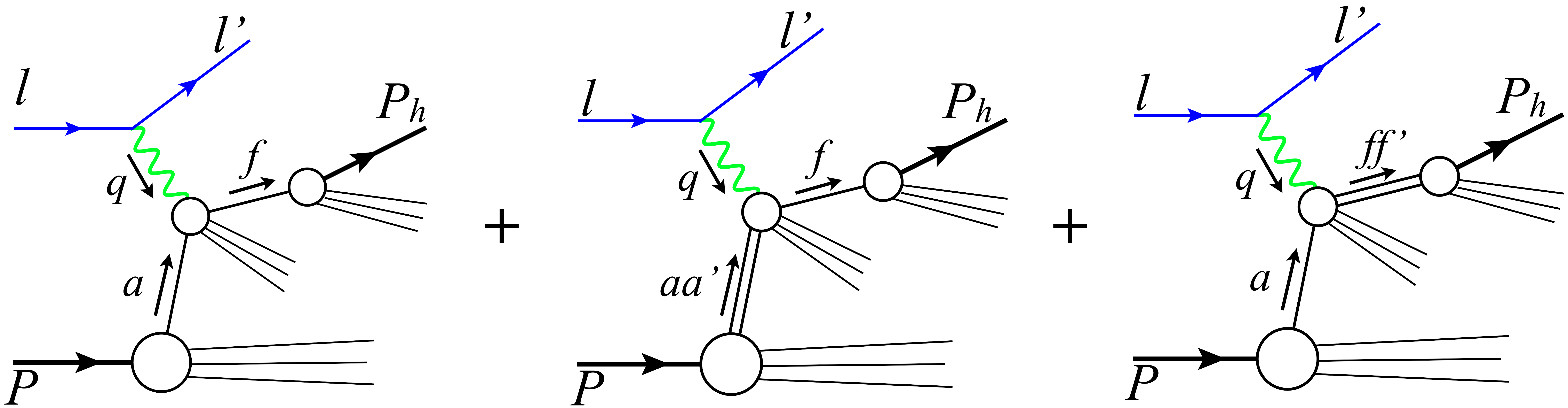
$\propto \delta(1-z)$ $\qquad$ $\propto (1-z)^m$



$\propto (1-z)^m$

Parton pair with the right quantum number has better chance to form the hadron.

Leading Power vs. Next-to-Leading Power



$\mathcal{O}\big(D_{f\to h}(z)\big)$ ✔ $\quad$ $\mathcal{O}\big(\frac{1}{Q^2} \times D_{f\to h}(z)\big)$ ✘ $\quad$ $\mathcal{O}\big(\frac{1}{P_{h_T}^2} \times D_{ff'\to h}(z)\big)$ ✔

At the edge of phase space — large P_{hT}, large z_h

trade off between $(1-z)^n$ suppression in the FF at LP
and $1/P_{h_T}^2$ suppression in the hard part at NLP

NLP Theoretical Calculation

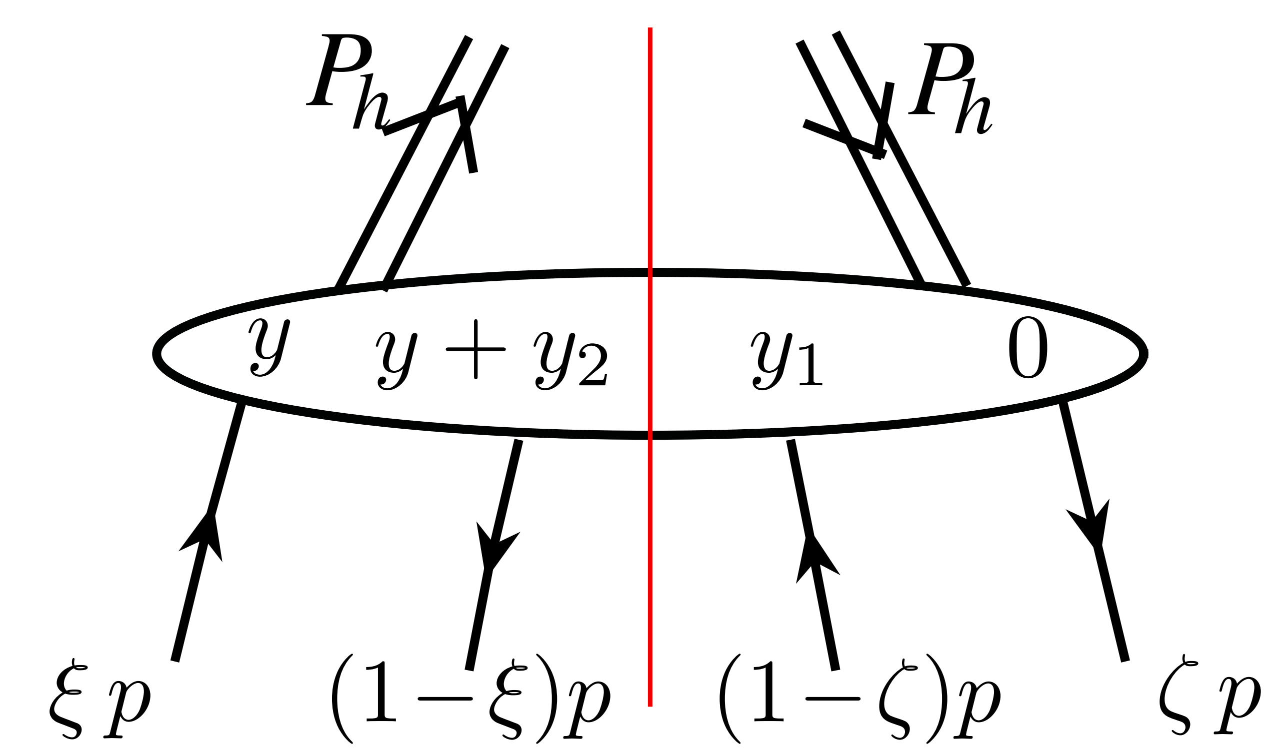
QCD factorization

$$\frac{d\sigma_{\gamma^*+A\to h+X}}{d^3\mathbf{P}_h/(2E_h)} \approx \sum_{a,f}\int_{x_B}^{1}\frac{dx}{x}\int_{z_h}^{1}\frac{dz}{z^2}\,\phi_{a/P}(x)\,D_{f\to h}(z)\,\frac{d\hat{\sigma}_{\gamma^*+a(l)\to f(p)+X}}{d^3\mathbf{p}/(2E_p)} \qquad \text{(LP)}$$

$$+\sum_{a,[ff'(\kappa)]}\int_{x_B}^{1}\frac{dx}{x}\int_{z_h}^{1}\frac{dz}{z^2}\int_{0}^{1}d\xi d\zeta\,\phi_{a/P}(x)\,D_{[ff'(\kappa)]\to h}(z,\xi,\zeta)\,\frac{d\hat{\sigma}_{\gamma^*+a(l)\to[ff'(\kappa)](p,\xi,\zeta)+X}}{d^3\mathbf{p}/(2E_p)} \qquad \text{(NLP)}$$

Two-parton (quark-antiquark) fragmentation functions

$$D_{[q\bar{q}'(\kappa)]\to h}(z,\xi,\zeta) = \sum_X \int\frac{P_h^+ dy^-}{2\pi}\int\frac{P_h^+ dy_1^-}{2\pi}\int\frac{P_h^+ dy_2^-}{2\pi}$$
$$\times\, e^{i(1-\zeta)\frac{P_h^+}{z}y_1^-}\,e^{-i\frac{P_h^+}{z}y^-}\,e^{-i(1-\xi)\frac{P_h^+}{z}y_2^-}$$
$$\times\, \mathcal{C}\,\mathcal{P}\,\langle 0|\bar{q}'(y_1^-)[\Phi_n(y_1^-)]^\dagger[\Phi_n(0)]q(0)|h(P_h)X\rangle$$
$$\times\,\langle h(P_h)X|\bar{q}(y^-)[\Phi_n(y^-)]^\dagger[\Phi_n(y^-+y_2^-)]q'(y^-+y_2^-)|0\rangle$$



$$\Phi_n(y^-) = P\exp\left[-ig_s\int_{y^-}^{\infty} d\lambda n\cdot G^A(\lambda n)t^A\right]$$

Color and Spin States

Color projection

for hard part:

$$\tilde{\mathcal{C}}^{[1]}_{ba,dc} = \delta_{ba}\delta_{dc}\,,$$

$$\tilde{\mathcal{C}}^{[8]}_{ba,dc} = \sum_A \sqrt{2}\, t^A_{ba} \sqrt{2} t^A_{dc}$$

for fragmentation function:

$$\mathcal{C}^{[1]}_{ab,cd} = \frac{1}{N_c^2}\delta_{ab}\delta_{cd}\,,$$

$$\mathcal{C}^{[8]}_{ab,cd} = \frac{1}{N_c^2 - 1}\sum_A \sqrt{2}\, t^A_{ab} \sqrt{2} t^A_{cd}$$

$$\sum_{abcd} \tilde{\mathcal{C}}^I_{ba,dc} \mathcal{C}^J_{ab,cd} = \delta^{IJ} \qquad I, J = [1], [8]$$

Spin projection

for hard part:

$$\tilde{\mathcal{P}}^{(v)}(p)_{ji,lk} = (\gamma \cdot p)_{ji}(\gamma \cdot p)_{lk}\,,$$

$$\tilde{\mathcal{P}}^{(a)}(p)_{ji,lk} = (\gamma \cdot p\gamma_5)_{ji}(\gamma \cdot p\gamma_5)_{lk}\,,$$

$$\tilde{\mathcal{P}}^{(t)}(p)_{ji,lk} = \sum_{\alpha=1,2}(\gamma \cdot p\gamma^\alpha_\perp)_{ji}(\gamma \cdot p\gamma^\alpha_\perp)_{lk}\,,$$

for fragmentation function:

$$\mathcal{P}^{(v)}(p)_{ij,kl} = \frac{1}{4p \cdot n}(\gamma \cdot n)_{ij}\frac{1}{4p \cdot n}(\gamma \cdot n)_{kl}\,,$$

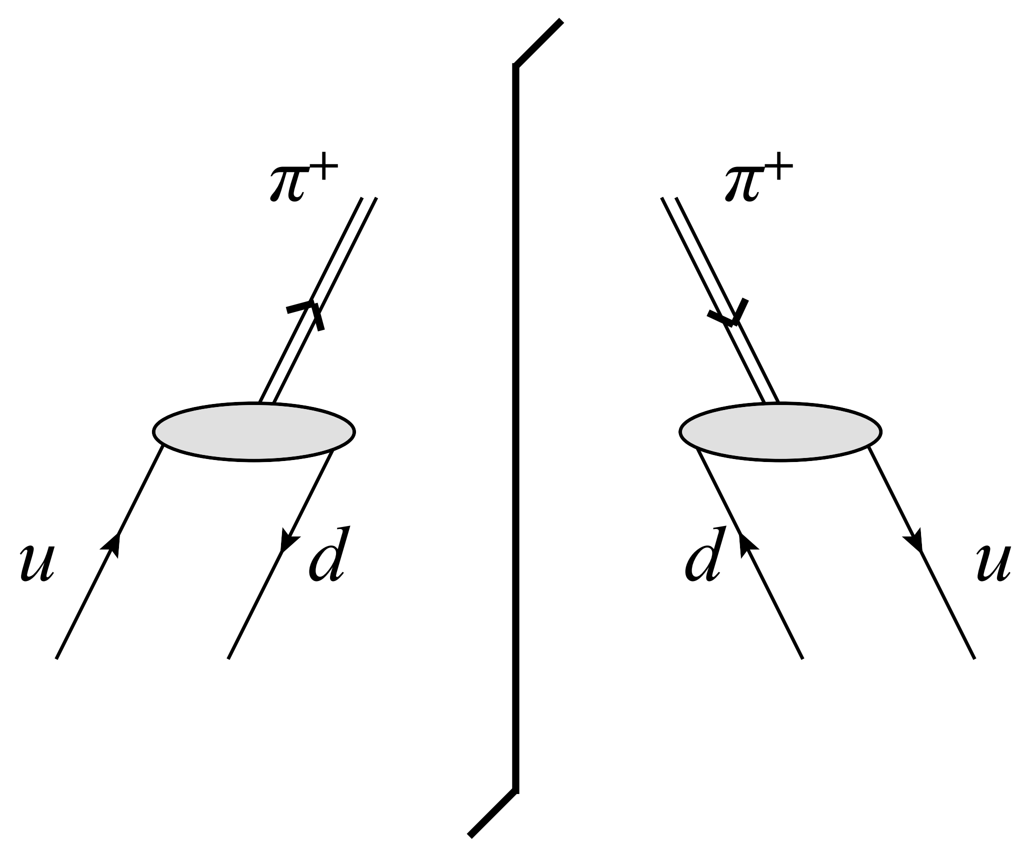
$$\mathcal{P}^{(a)}(p)_{ij,kl} = \frac{1}{4p \cdot n}(\gamma \cdot n\gamma_5)_{ij}\frac{1}{4p \cdot n}(\gamma \cdot n\gamma_5)_{kl}\,,$$

$$\mathcal{P}^{(t)}(p)_{ij,kl} = \frac{1}{2}\sum_{\alpha=1,2}\frac{1}{4p \cdot n}(\gamma \cdot n\gamma^\alpha_\perp)_{ij}\frac{1}{4p \cdot n}(\gamma \cdot n\gamma^\alpha_\perp)_{kl}$$

$$\sum_{ijkl} \tilde{\mathcal{P}}^{(s)}_{ji,lk} \mathcal{P}^{(s')}_{ij,kl} = \delta^{ss'} \qquad s, s' = v,\ a,\ t$$

An Estimation of Two-Parton FFs

Lowest order quark-antiquark fragmentation function



$$
\begin{aligned}
D_{[q\bar{q}'(1a)]}(z,\xi,\zeta,\mu_0) \approx & \int \frac{P_h^+ dy^-}{2\pi} \int \frac{P_h^+ dy_1^-}{2\pi} \int \frac{P_h^+ dy_2^-}{2\pi} e^{i(1-\zeta)\frac{P_h^+}{z}y_1^-} e^{-i\frac{P_h^+}{z}y^-} e^{-i(1-\xi)\frac{P_h^+}{z}y_2^-} \\
& \times \frac{1}{4N_c P_h^+} \langle 0|\bar{q}'_{c',k}(y_1^-)(\gamma\cdot n\gamma_5)_{kl} U_{c'd'}(y_1^-,0) q_{d',l}(0)|h(P_h)\rangle \\
& \times \frac{1}{4N_c P_h^+} \langle h(P_h)|\bar{q}_{a',i}(y^-)(\gamma\cdot n\gamma_5)_{ij} U_{a'b'}(y^-,y^-+y_2^-) q'_{b',j}(y^-+y_2^-)|0\rangle \\
= & \frac{1}{16N_c^2} \int \frac{P_h^+ dy^-}{2\pi} \int \frac{P_h^+ dy_1^-}{2\pi} \int \frac{P_h^+ dy_2^-}{2\pi} e^{i(1-\zeta)\frac{P_h^+}{z}y_1^-} e^{-i\frac{P_h^+}{z}y^-} e^{-i(1-\xi)\frac{P_h^+}{z}y_2^-} \\
& \times f_h^2\, e^{iP_h^+ y^-} \int_0^1 d\zeta' e^{-i(1-\zeta')P_h^+ y_1^-} \phi_h(\zeta',\mu_0) \int_0^1 d\xi' e^{i(1-\xi')P_h^+ y_2^-} \phi_h(\xi',\mu_0) \\
= & \frac{f_h^2}{16N_c^2}\, z\,\delta(1-z)\phi_h(\zeta,\mu_0)\phi_h(\xi,\mu_0).
\end{aligned}
$$

Pseudoscalar meson distribution amplitude ϕ_h

$$
\begin{aligned}
& \langle 0|\bar{q}_{a,i}(y^-+y_1^-)(\gamma\cdot n\gamma_5)_{ij} U_{ab}(y^-+y_1^-,y^-) q_{b,j}(y^-)|h(P_h)\rangle \\
& = iP_h^+ f_h \int_0^1 dx\, e^{-ixP_h^+ y^- - i(1-x)P_h^+(y^-+y_1^-)} \phi_h(x,\mu) \\
& = iP_h^+ f_h\, e^{-iP_h^+ y^-} \int_0^1 dx\, e^{-i(1-x)P_h^+ y_1^-} \phi_h(x,\mu)
\end{aligned}
$$

$$
U_{ab}(y_2^-,y_1^-) = [\Phi_n(y_2^-)]^\dagger_{ac}[\Phi_n(y_1^-)]_{cb}
$$

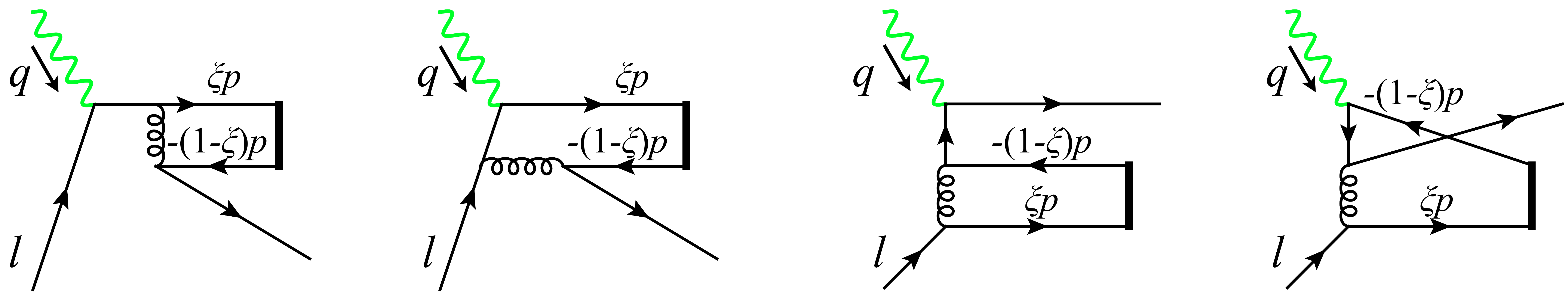
Calculation of the Partonic Hard Part

Hard part

$$\frac{E_p d\hat{\sigma}_{\gamma^*+a(l)\to[ff'](p)+X}}{d^3\mathbf{p}} = \frac{|\overline{\mathcal{M}}_{\gamma^*+a(l)\to[ff'(\kappa)](p)+X}|^2}{2(\hat{s}+Q^2)} \frac{1}{2(2\pi)^2}\delta(\hat{s}+\hat{t}+\hat{u}+Q^2)$$

$$\hat{s} = (q+l)^2, \quad \hat{t} = (q-p)^2, \quad \hat{u} = (l-p)^2$$

LO Feynman diagrams



two possible channels

Calculation of the Partonic Hard Part

Color factor

same color factor for LO diagrams

$$C^{[1]} = \frac{1}{N_c}\sum_{AB}\mathrm{Tr}[t^A t^A t^B t^B] = \frac{(N_c^2-1)^2}{4N_c^2}$$

$$C^{[8]} = \frac{1}{N_c}\sum_{ABC}2\mathrm{Tr}[t^A t^C t^A t^B t^C t^B] = \frac{N_c^2-1}{4N_c^3}$$

Virtual photon spin states

transverse: $\sum_{\lambda=\pm}\epsilon_\lambda^{*\mu}\epsilon_\lambda^\nu = -g^{\mu\nu} + v^\mu \bar{v}^\nu + \bar{v}^\mu v^\nu$

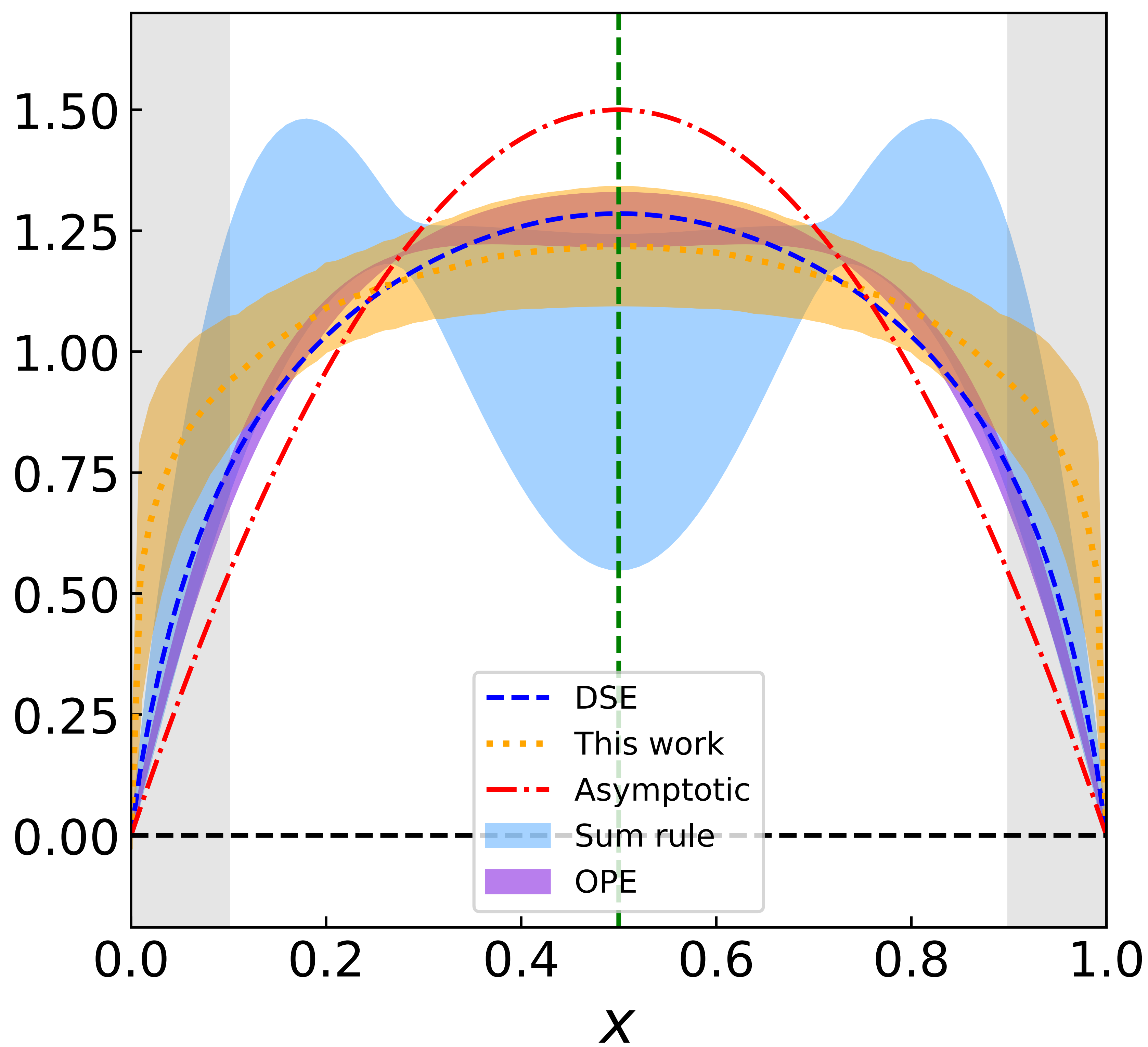
$$q^\mu = (q\cdot v)\bar{v}^\mu + (q\cdot\bar{v})v^\mu$$

longitudinal: $\epsilon_L^{*\mu}\epsilon_L^\nu = \frac{1}{-q^2}[(q\cdot\bar{v})^2 v^\mu v^\nu + (q\cdot v)^2\bar{v}^\mu\bar{v}^\nu] + \frac{1}{2}(v^\mu\bar{v}^\nu + \bar{v}^\mu v^\nu),$

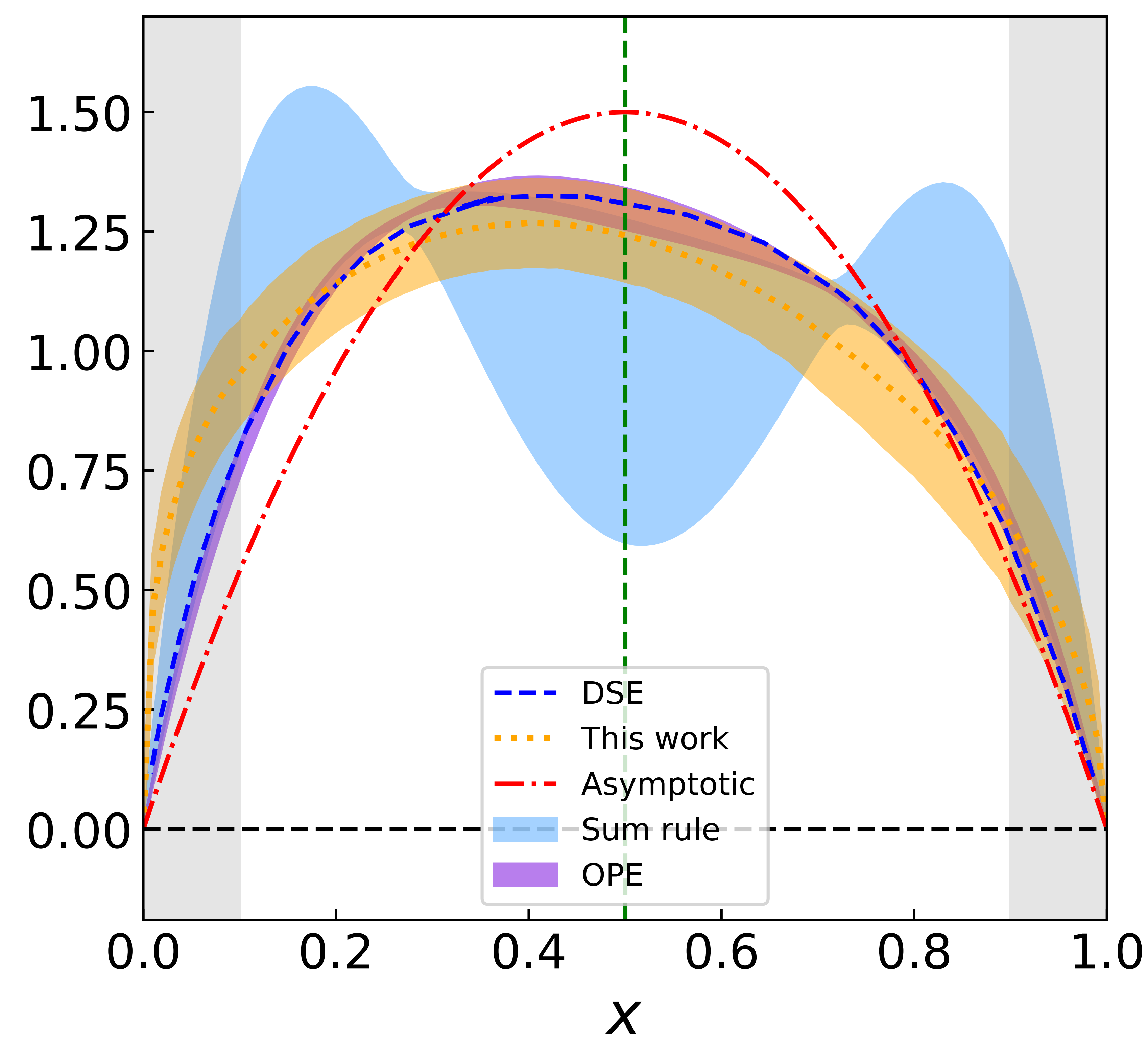
$$v^2 = \bar{v}^2 = 0, \quad v\cdot\bar{v} = 1$$

Pion and Kaon Distribution Amplitudes

pion:



kaon:

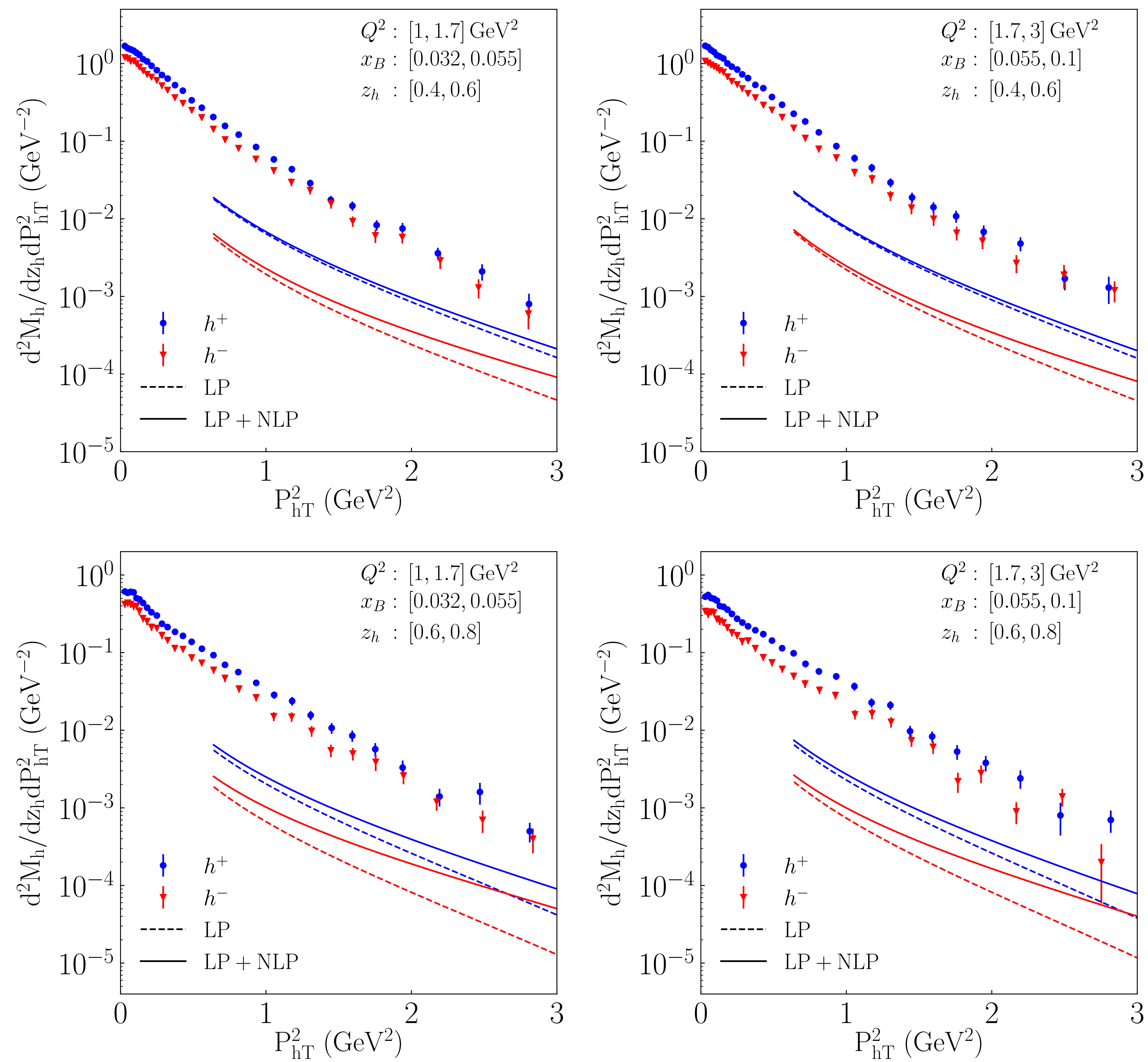


[Figure from PRL129 (2022) 132001]

Numerical Estimate: COMPASS Kinematics

Differential multiplicity

$$\frac{d^2 M_h}{dz_h dP_{h_T}^2} = \left(\frac{d^4\sigma_h^{\mathrm{SIDIS}}}{dx_B dQ^2 dz_h dP_{h_T}^2}\right) \Big/ \left(\frac{d^2\sigma^{\mathrm{DIS}}}{dx_B dQ^2}\right)$$



Only use the leading term of
two-parton fragmentation functions.

Lower limit for power corrections.

Numerical Estimate: JLab Kinematics

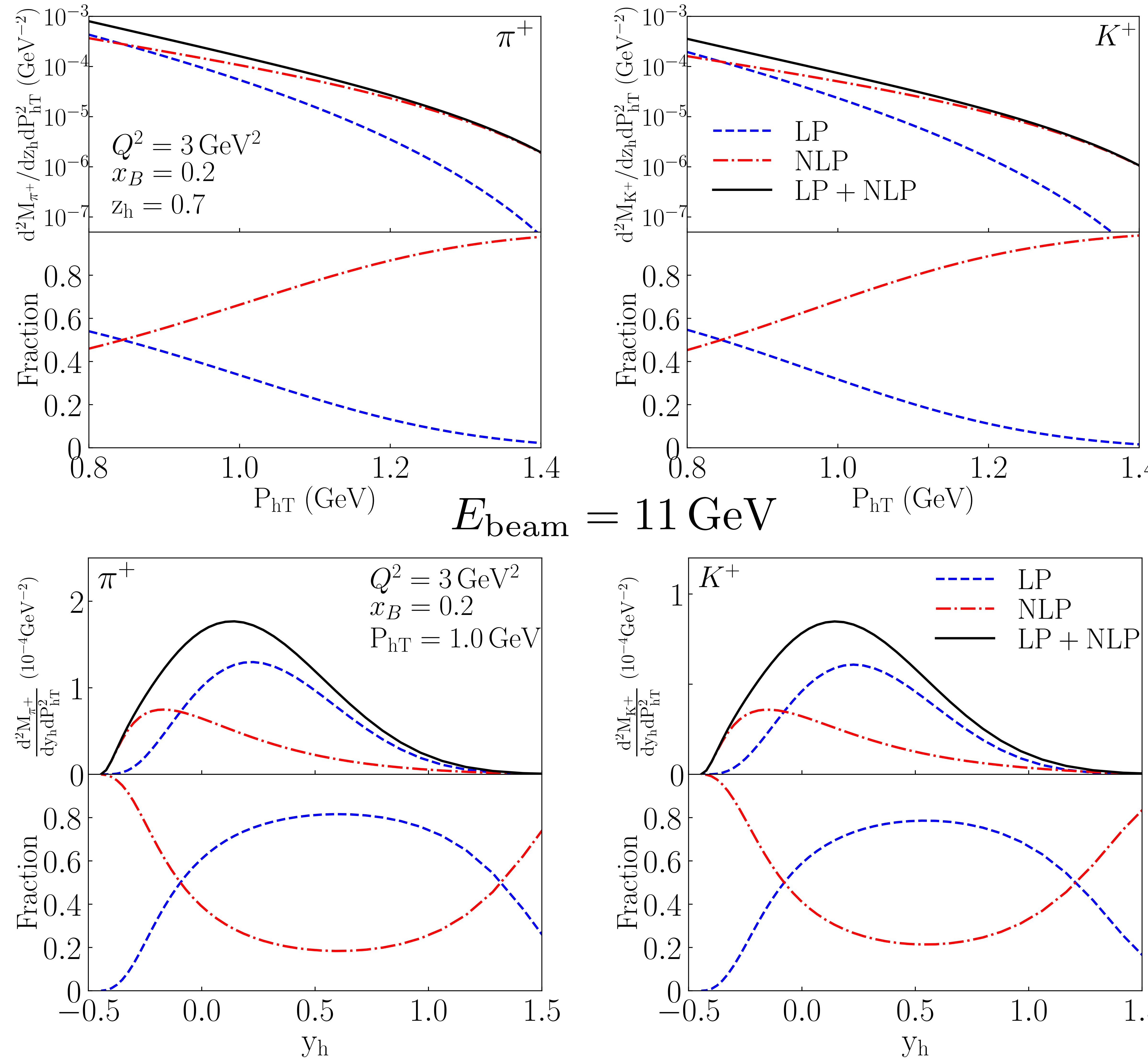
Differential multiplicity

$$\frac{d^2 M_h}{dz_h dP_{h_T}^2} = \left(\frac{d^4\sigma_h^{\mathrm{SIDIS}}}{dx_B dQ^2 dz_h dP_{h_T}^2}\right) \Big/ \left(\frac{d^2\sigma^{\mathrm{DIS}}}{dx_B dQ^2}\right)$$

compare to COMPASS:

- lower collision energy
- less high multiplicity events
- NLP contribution is more significant

y_h is defined in photon-target frame with $q = (-Q/\sqrt{2}, Q/\sqrt{2}, \mathbf{0}_\perp)$



NLP Correction to the Evolution Equation

Physical observable is independent of the choice of factorization scale

$$\frac{d}{d\ln\mu^2}\Big(D_{f\to h}\otimes d\hat{\sigma}_{\gamma^{(*)}+A\to f+X}$$
$$+D_{[ff'(\kappa)]\to h}\otimes d\hat{\sigma}_{\gamma^{(*)}+A\to[ff'(\kappa)]+X}\Big)=0$$

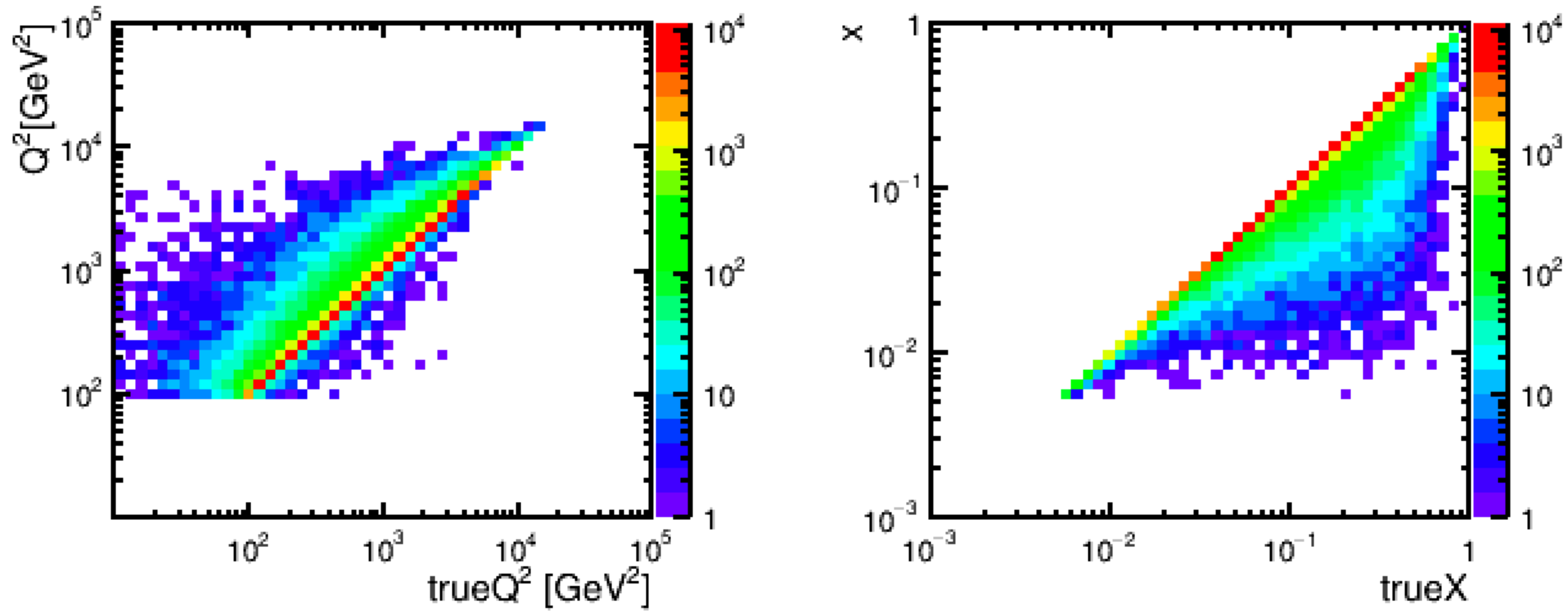
A closed set of evolution equations

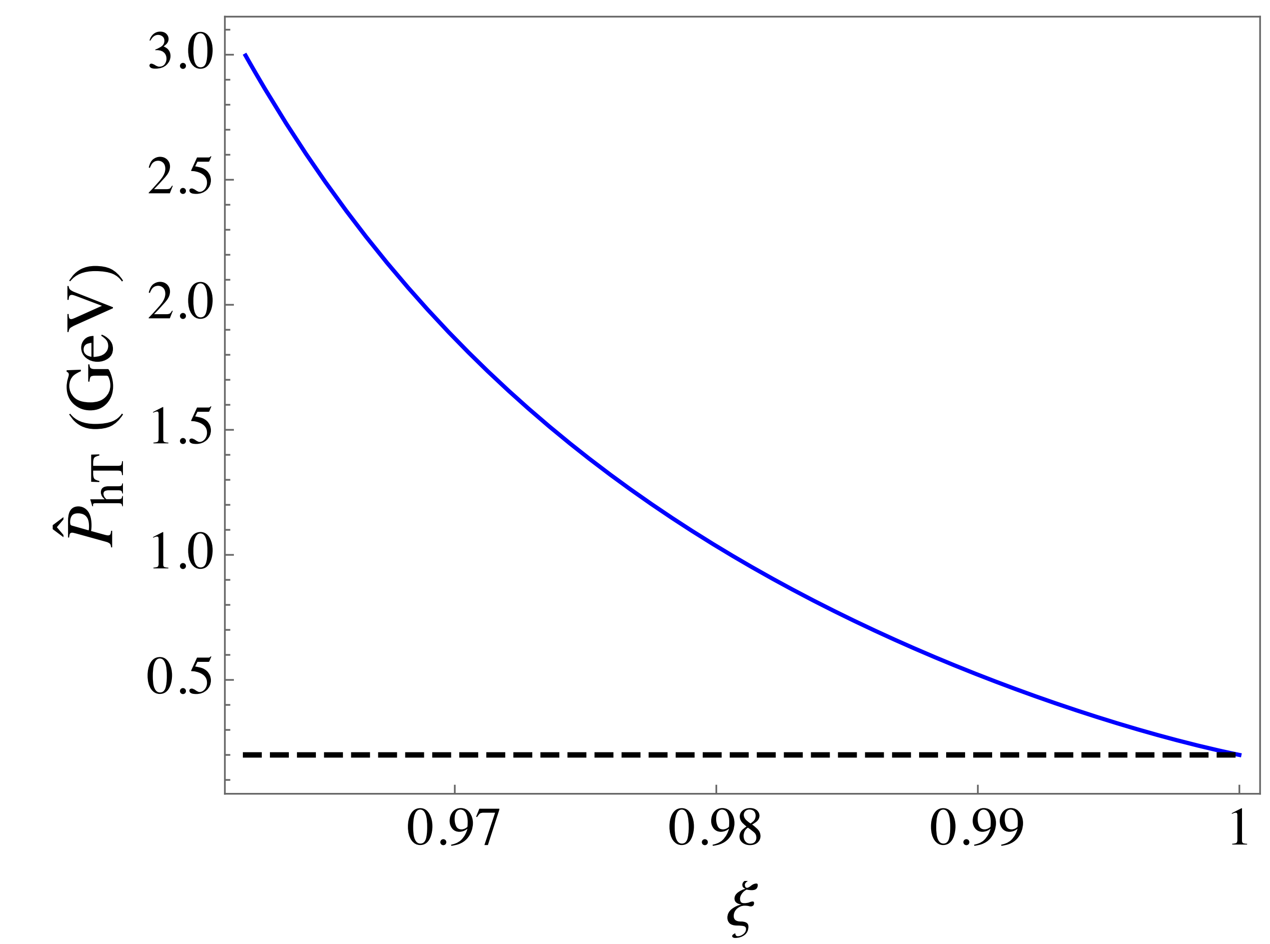
$$\frac{\partial}{\partial\ln\mu^2}D_{[ff'(\kappa)]\to h}$$
$$=\sum_{[\tilde{f}\tilde{f}'(\kappa)']}D_{[\tilde{f}\tilde{f}'(\kappa')]\to h]}\otimes\Gamma_{[ff'(\kappa)]\to[\tilde{f}\tilde{f}'(\kappa')]}$$

$$\frac{\partial}{\partial\ln\mu^2}D_{f\to h}=\sum_{f'}D_{f'\to h}\otimes\gamma_{f\to f'}$$
$$+\frac{1}{\mu^2}\sum_{[ff'(\kappa)]}D_{[ff'(\kappa)]\to h}\otimes\tilde{\gamma}_{f\to[ff'(\kappa)]}$$

QED Radiative Effects

Kinematic shifted by QED radiation





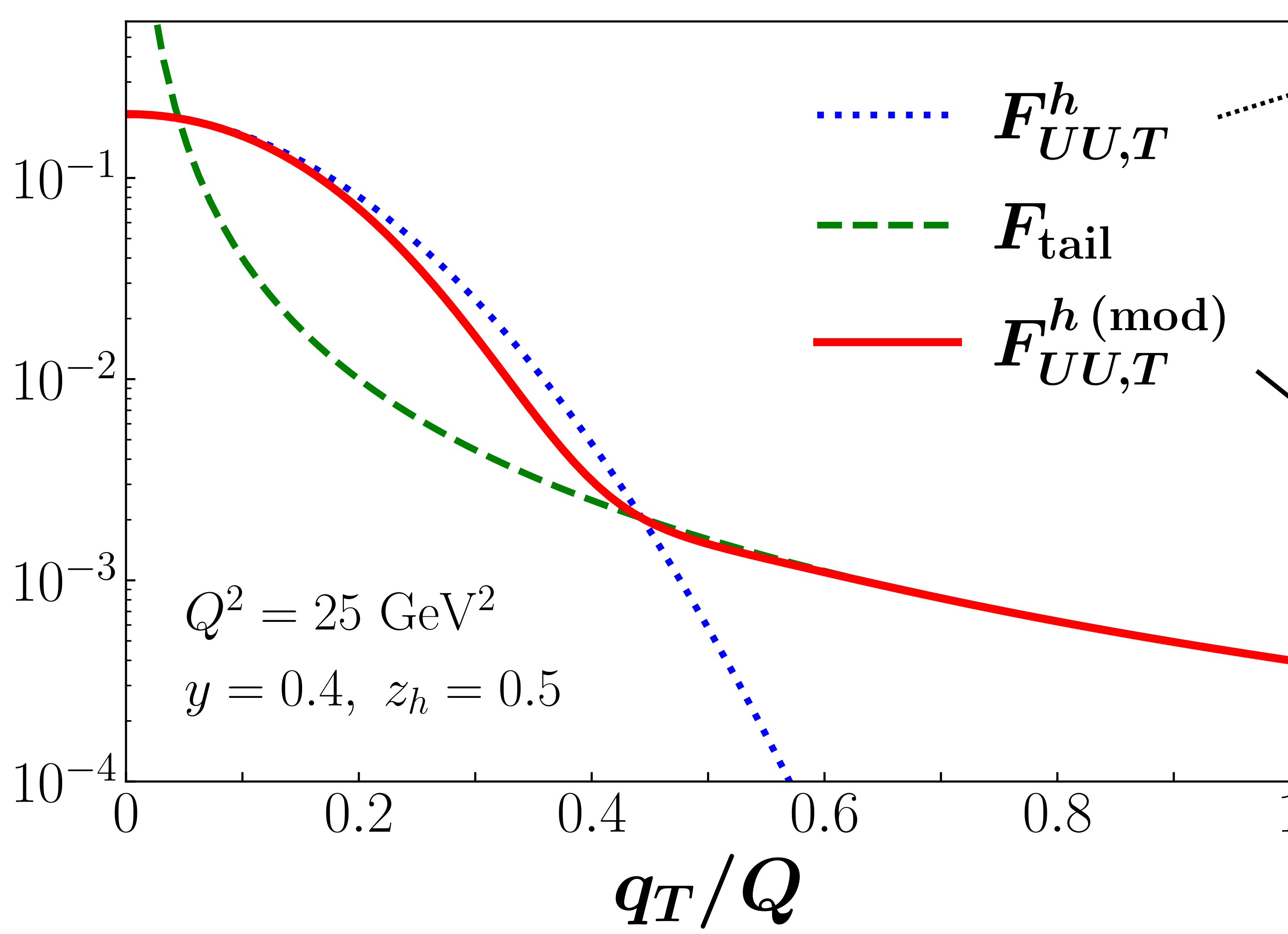
[Figure from X. Chu at 2nd EIC YR workshop]

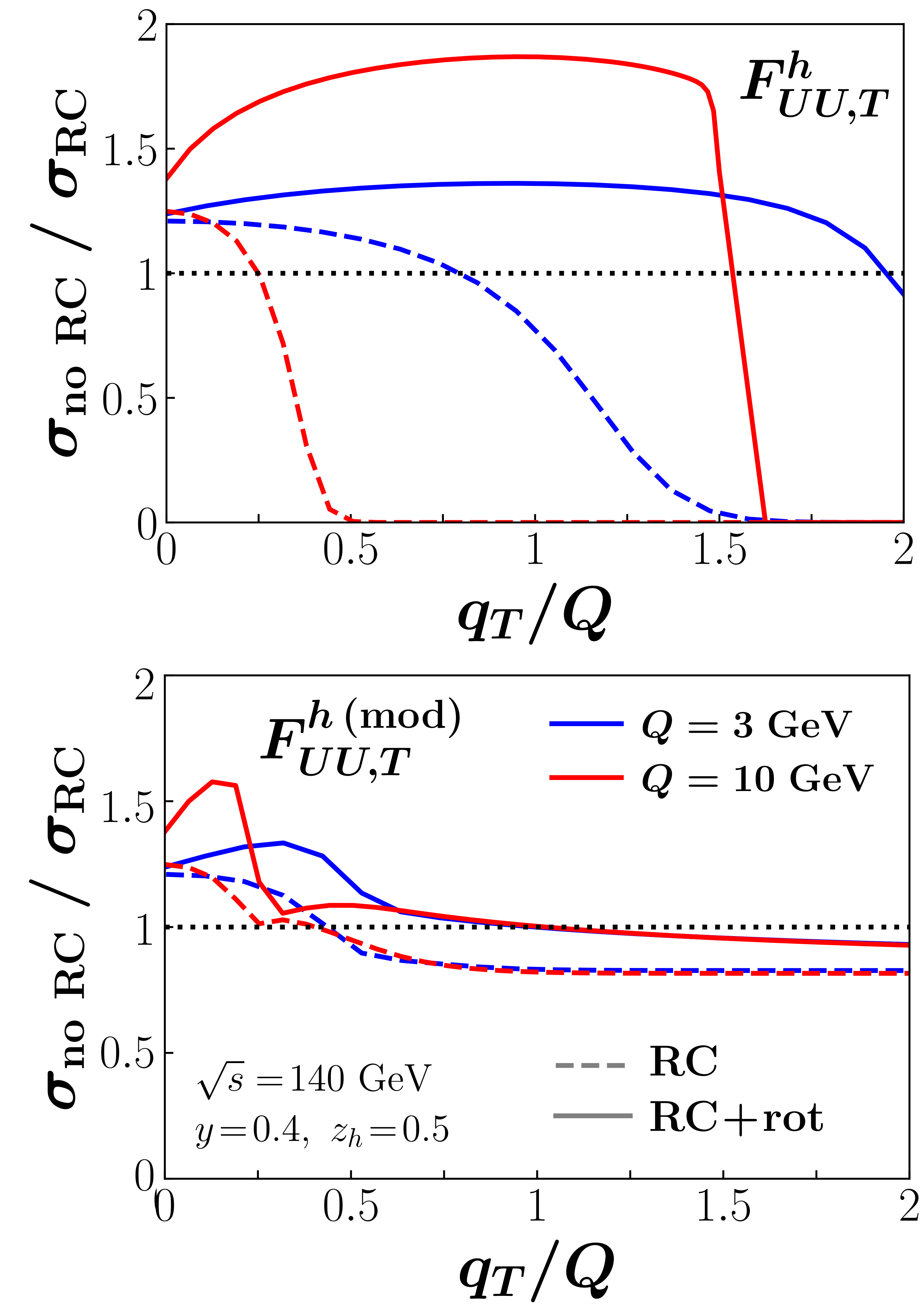
kinematic experienced by the parton ≠ kinematic reconstructed from observed momenta

"In many nuclear physics experiments, radiative corrections quickly become a dominant source of systematics. In fact, the uncertainty on the corrections might be the dominant source for high-statistics experiment"

—— EIC Yellow Report

QED Radiative Effects





Radiative correction factor depends on the hadronic physics we want to extract.

Summary and Outlook

- Formally suppressed NLP contribution to SIDIS cross section is not necessarily smaller than the formal LP contribution.
- Produced parton pair with the right quantum number has better chance to turn to the measured meson.
- Power corrections are very important for events near the edge of phase space where the multiplicity is low.
- Evolution equation should be modified consistently to NLP.
- Other effects, such as QED radiations, may also be important.
- A simultaneous fit of FFs and PDFs including power corrections is desired.
- Opportunities from experiments at JLab and the future EicC/EIC/STCF.

Thank you!