第六届重味物理与量子色动力学研讨会

NLO calculation in CGC

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Y. Shi, L. Wang, S.Y. Wei, B.W. Xiao, PRL 128, 202302 (2022)
E. Iancu, A.H. Mueller, D.N. Trantafyllopoulos, S.Y. Wei, JHEP 10, 103 (2022)
E. Iancu, A.H. Mueller, D.N. Trantafyllopoulos, S.Y. Wei, EPJC 83, 1078 (2023)
S. Hauksson, E. Iancu, A.H. Mueller, D.N. Trantafyllopoulos, S.Y. Wei, arXiv:2402.14748

Contents

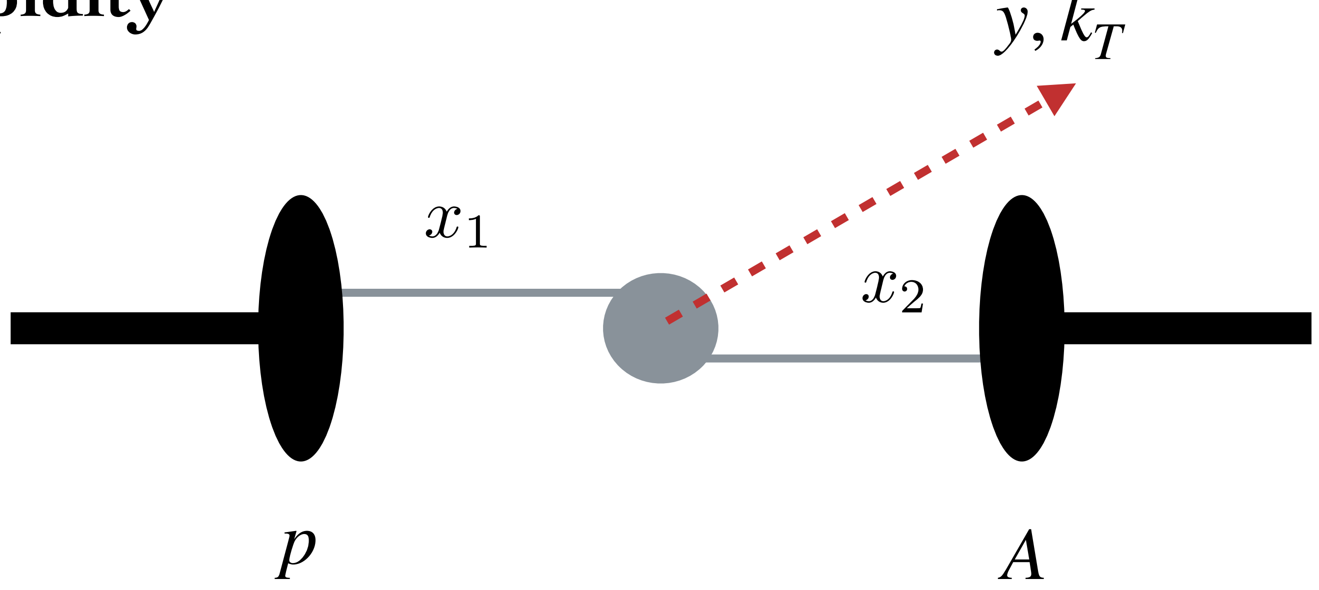
- Introduction
- Threshold Resummation
- Diffractive Dijet in UPC
- Summary

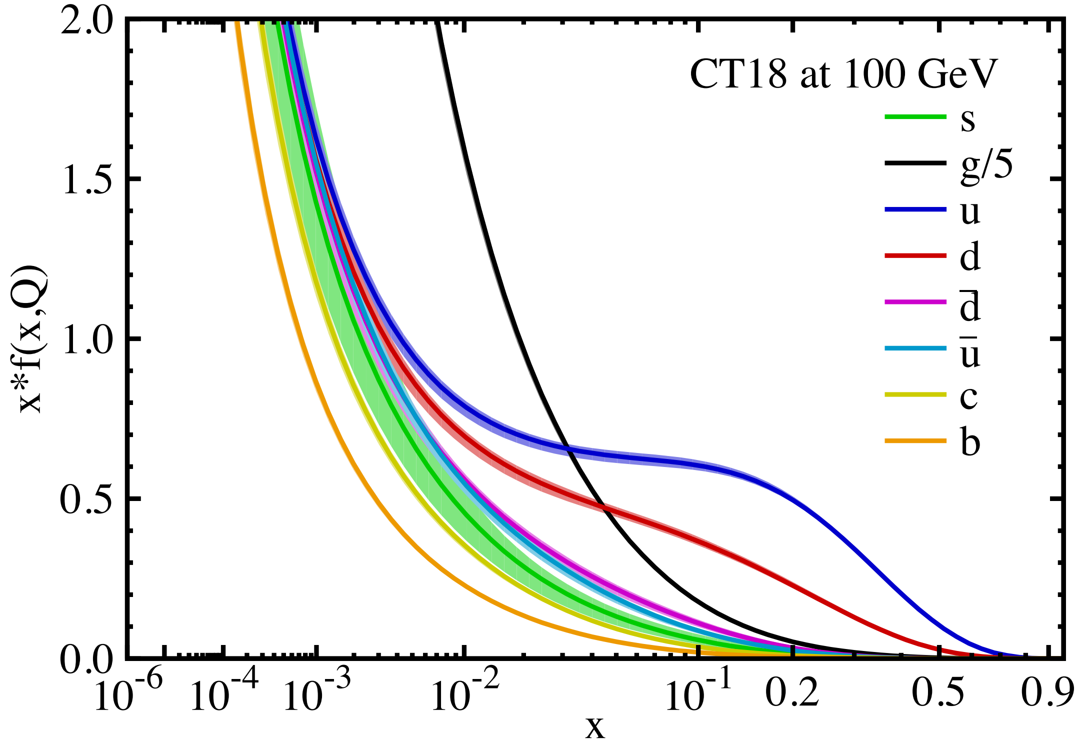
Introduction

Particles produced in the forward rapidity

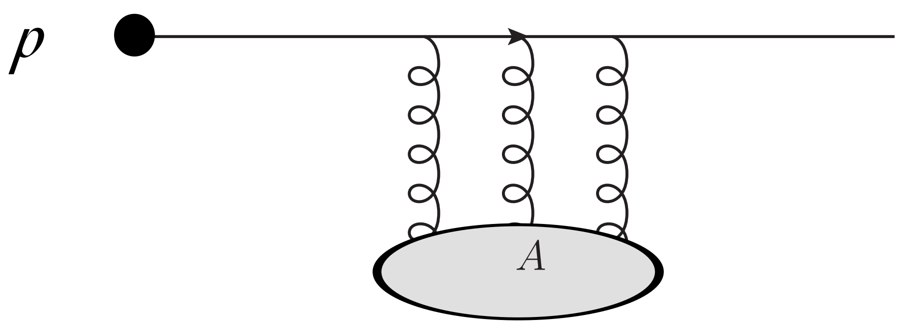
$$x_1 = k_T e^{y}/\sqrt{s} \gg 0$$

$$x_2 = k_T e^{-y}/\sqrt{s} \ll 1$$





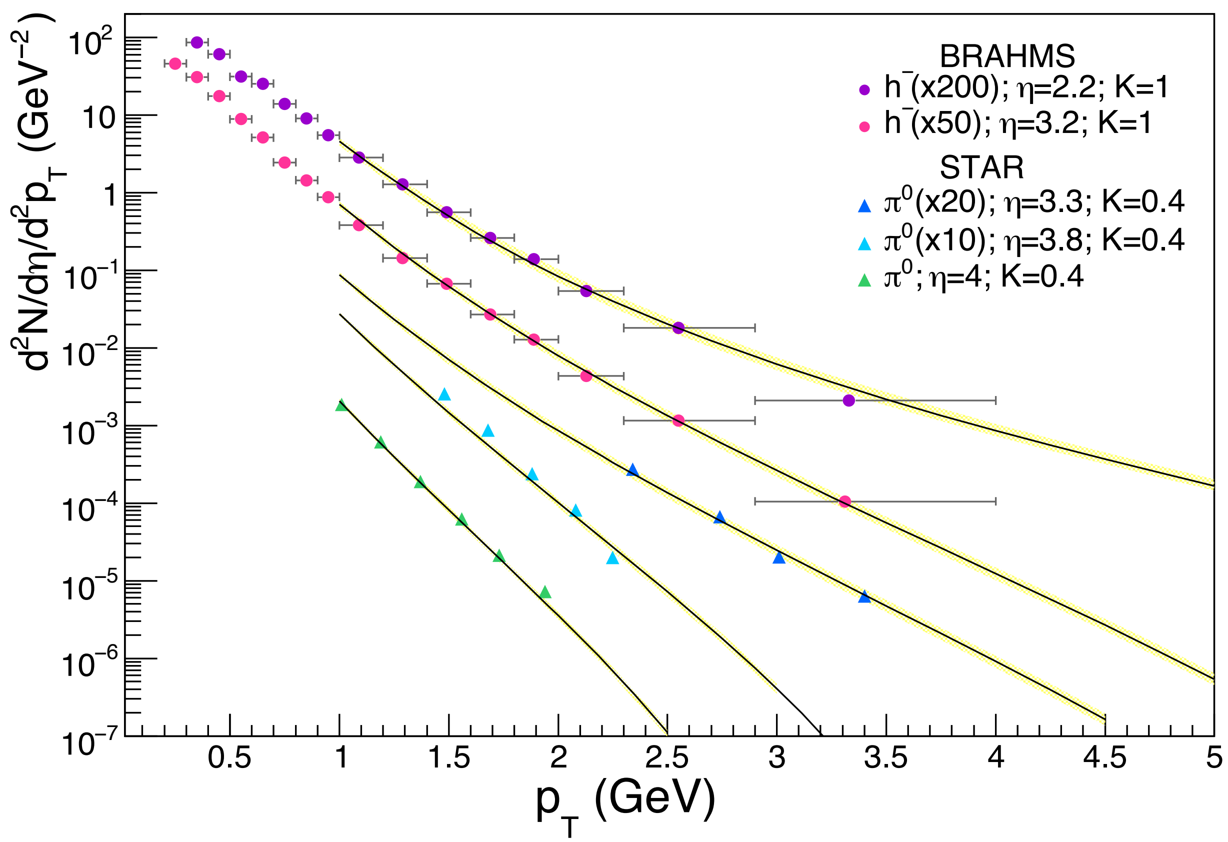
dilute-dense-system; gluon saturation; non-linear evolution; CGC

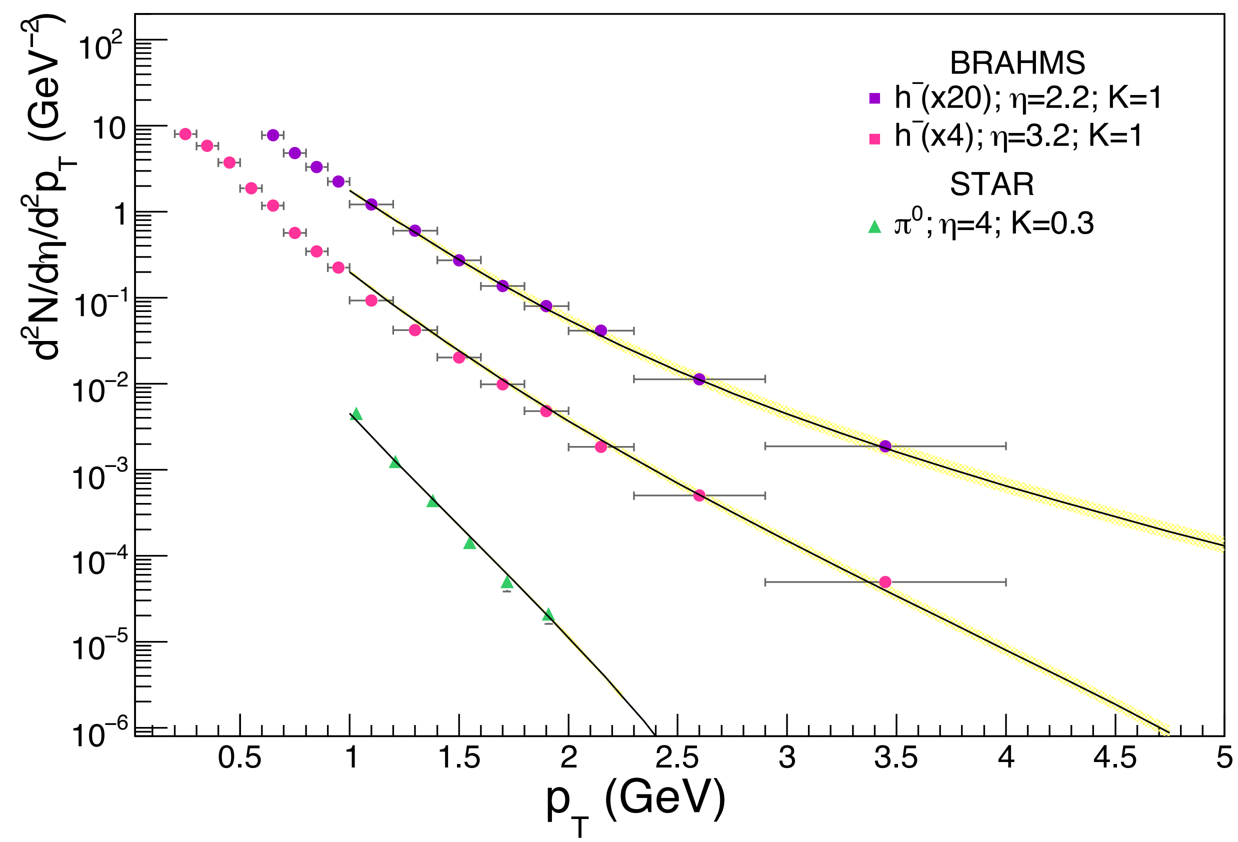


$$\mathscr{F}(k_T) = \mathrm{FT}[S(r_\perp)]$$

dipole scattering amplitude

Life is simple at LO





Albacete and Marquet, PLB 2010

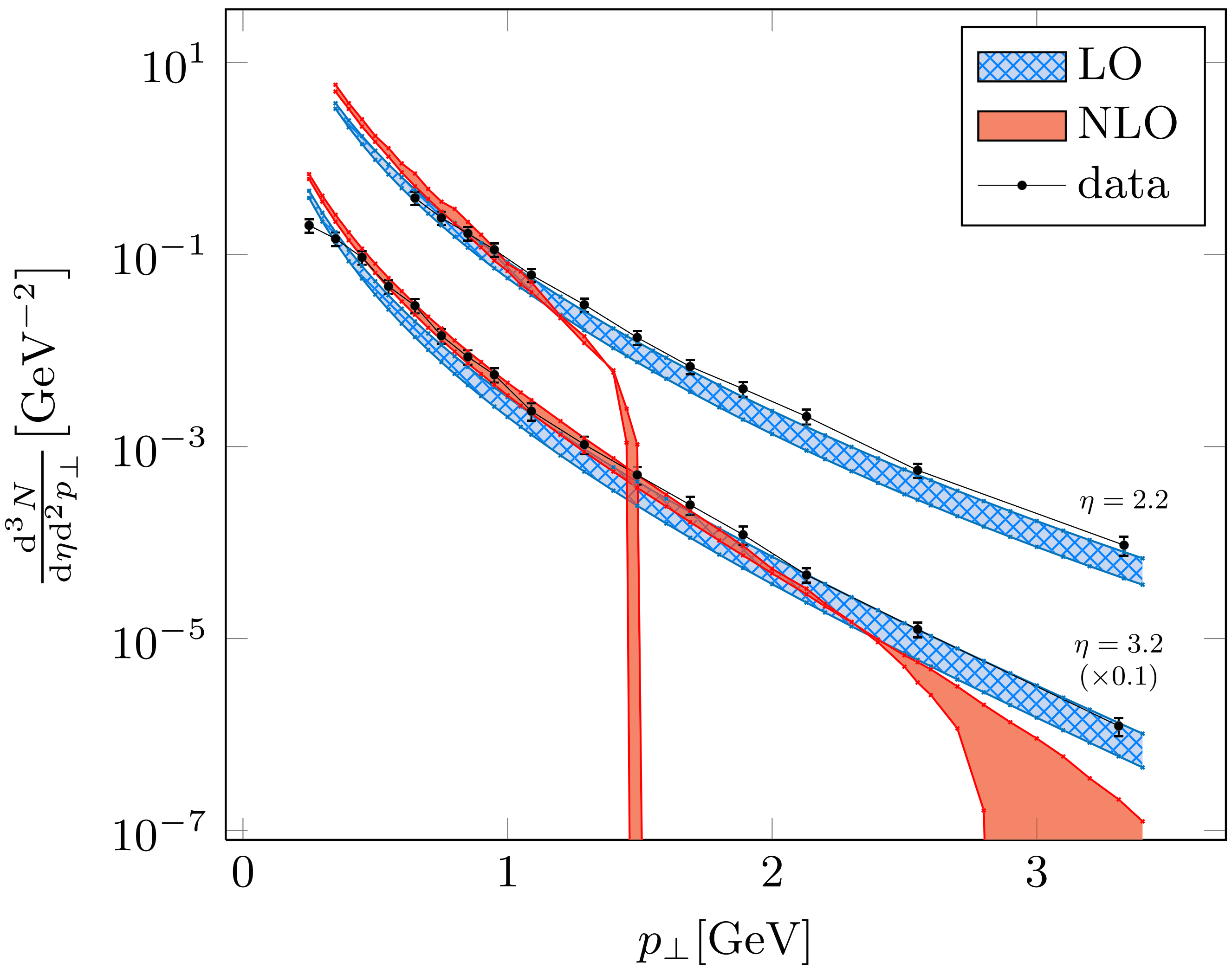
- ☑ Large theoretical uncertainties

Dumitru and Jalilian-Marian, PRL 2002
Albacete and Marquet, PLB 2010
Levin and Rezaeian, PRD 2010
……

An Odyssey of NLO

Dumitru, Hayashigakia and Jalilian-Marian, NPA, 2006
Altinoluk and Kovner, PRD, 2011
Chirilli, Xiao and Yuan, PRL, 2012
Chirilli, Xiao and Yuan, PRD, 2012
Watanabe, Xiao, Yuan and Zaslavsky, PRD, 2015

BRAHMS $\eta = 2.2, 3.2$



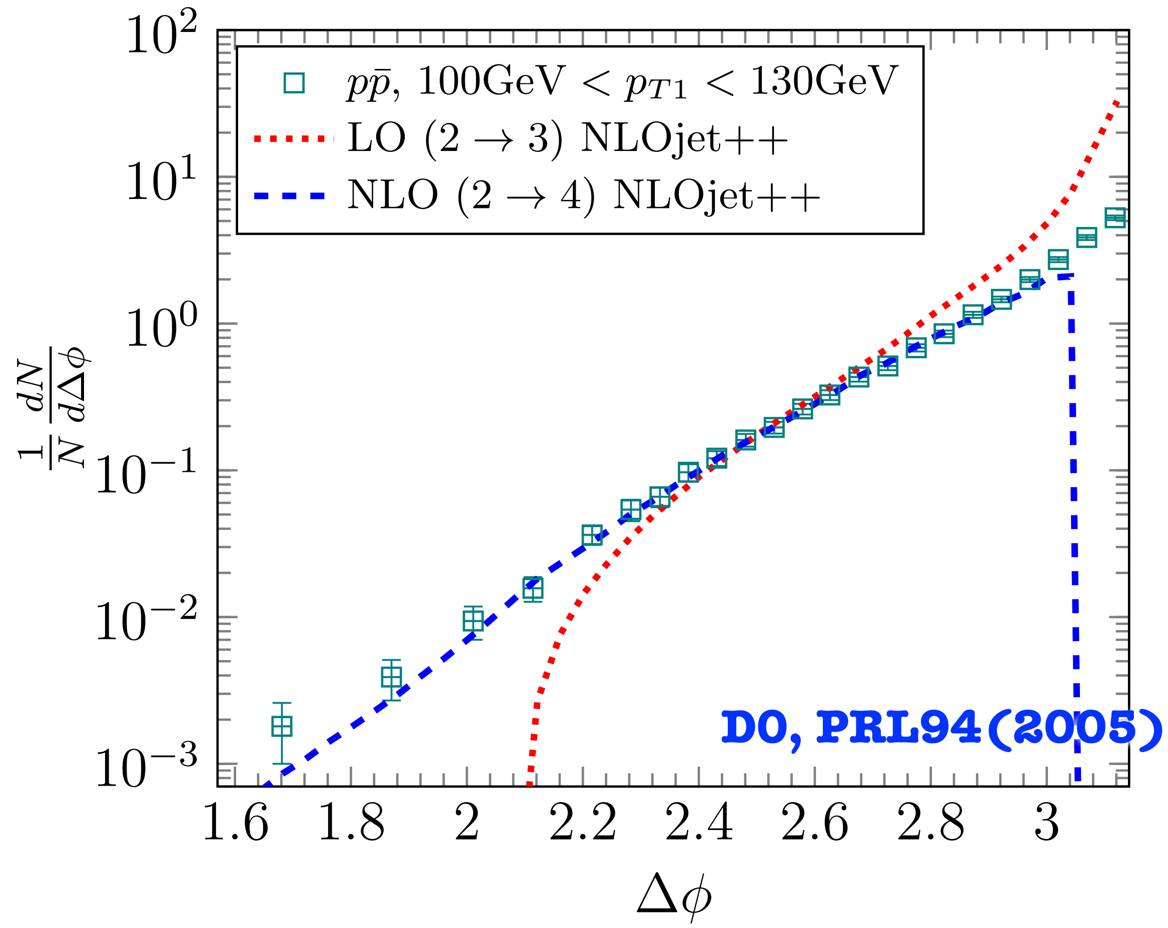
Proposals to fix this problem:

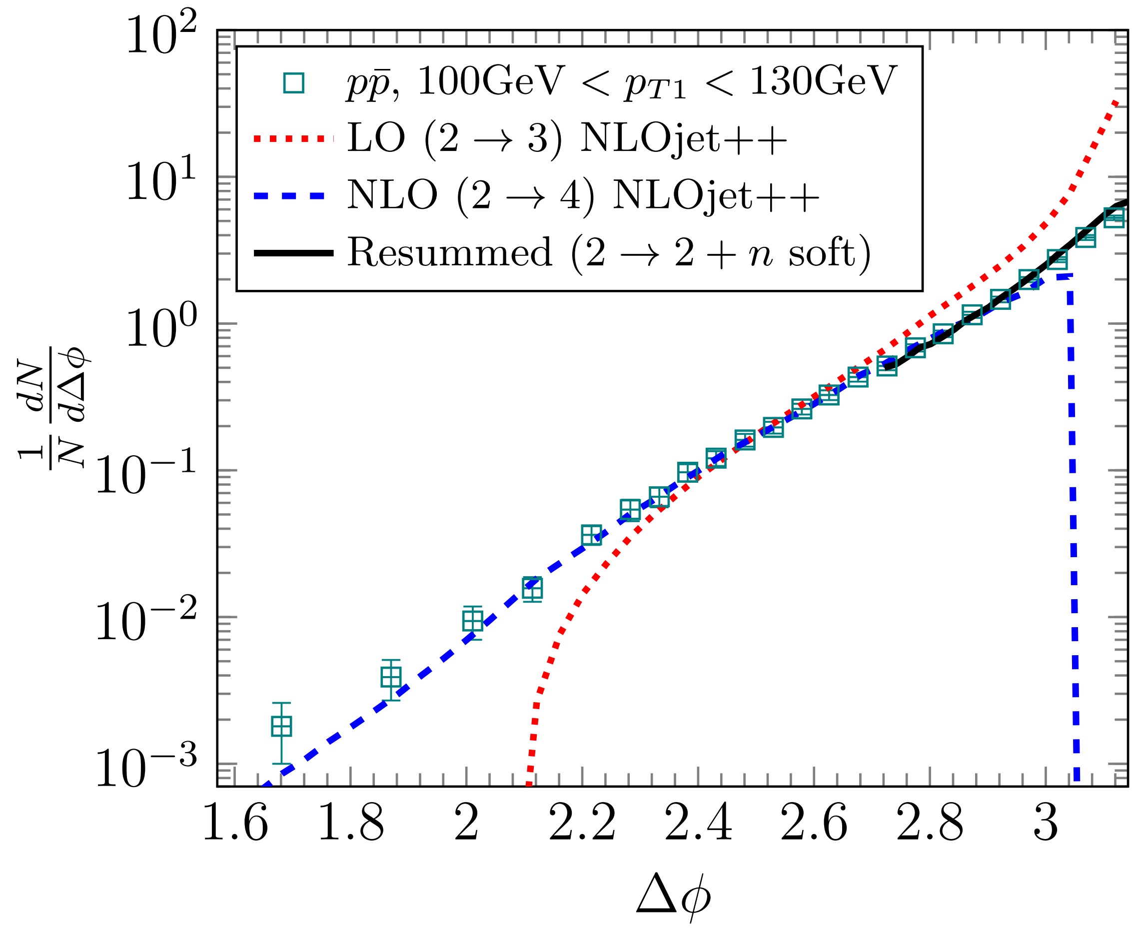
Kang, Vitev, Xing, PRL, 2014
Altinoluk, et al, PRD, 2015
Iancu, et al, JHEP, 2016
Ducloué, Lappi, Zhu, PRD, 2016, 2017
Ducloué, et al, PRD, 2018
Xiao, Yuan, PLB, 2019
Liu, Ma, Chao, PRD, 2019
Liu, Kang, Liu, PRD, 2020
Liu, Liu, Shi, Zheng, Zhou, 2022

factorisation scheme;
kinematic constraint;
running coupling effect;
resummation…

☑ NLO cross section turns negative at high p_T.

Classical examples dijet azimuthal angle correlation

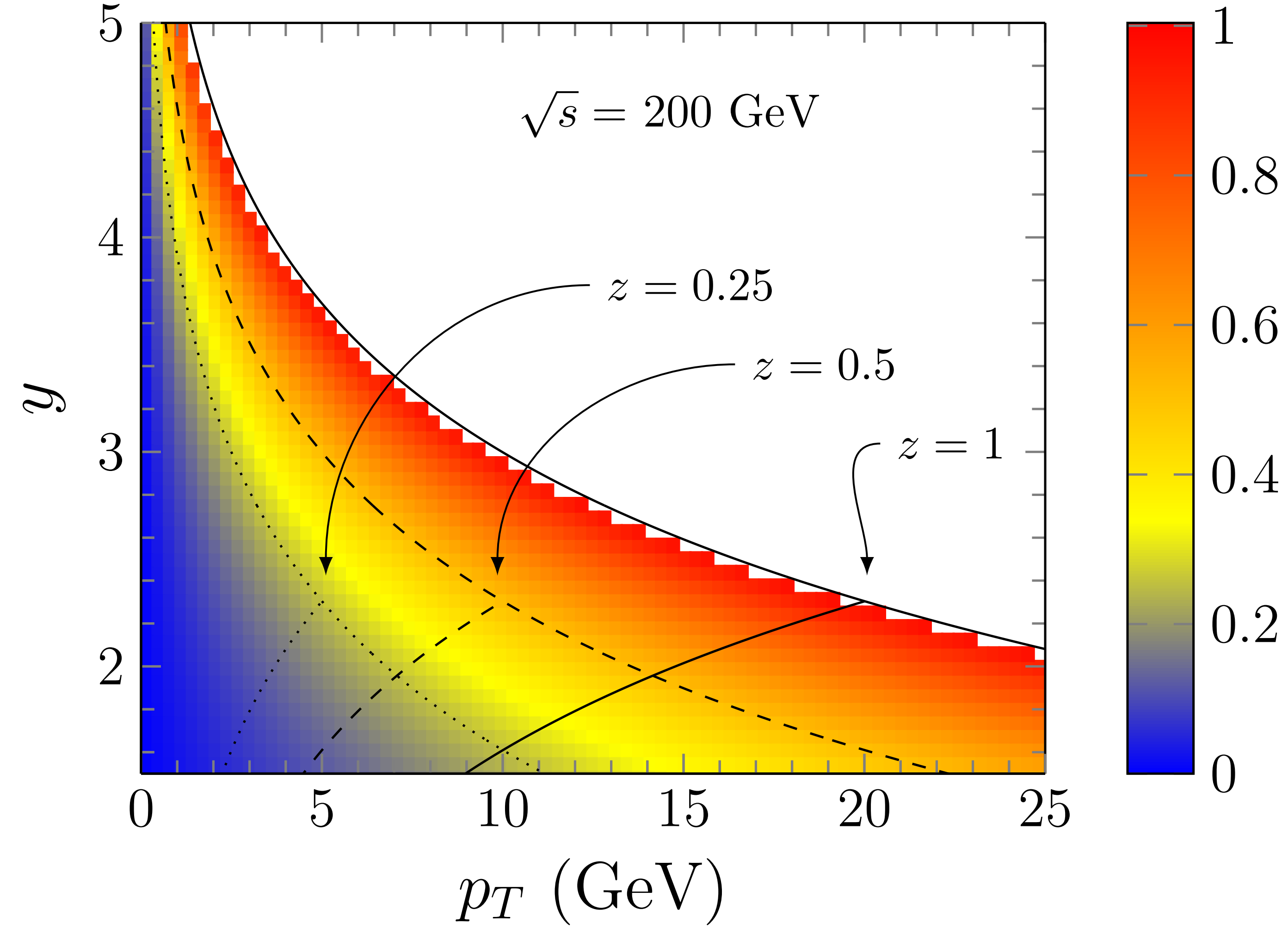




Perturbative Expansion	Resummation
$\sigma_0 \sum_{i=0}^{n} \Big((\alpha_s \text{Log})^i + \alpha_s^i C_i \Big)$	$\sigma_0 \sum_{i=0}^{n} \Big((\alpha_s \text{Log})^i \Big) + \sigma_0 \sum_{n+1}^{\infty} \Big((\alpha_s \text{Log})^i \Big)$

- ☑ Perturbative Expansion: α_s is small
- ☑ Resummation: large logs

Threshold resummation



$$\tau = xz\xi = p_T e^y/\sqrt{s}$$

$$P_{qq}(\xi) = \frac{1+\xi^2}{(1-\xi)_+} + \frac{3}{2}\delta(1-\xi)$$

$$\int_\tau^1 \frac{d\xi}{(1-\xi)_+} f(\xi) = \int_\tau^1 d\xi \frac{f(\xi)-f(1)}{1-\xi} + f(1)\ln(1-\tau)$$

☑ At higher p_T region, more contribution comes from $\xi \to 1$.

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Threshold Resummation

1. cross section in the coordinate space

$$\frac{d\sigma}{d\mathcal{P.S.}} \propto \int \frac{d^2 r_\perp}{(2\pi)^2} \exp[-i\vec{k}_T \cdot \vec{r}_\perp]$$

numerical FT becomes unstable at large k_T

$$P(\xi) \otimes \ln \frac{\mu^2}{\mu_r^2} \qquad \sigma_0 \otimes \ln \frac{k_T^2}{\mu_r^2} \qquad \sigma_0 \otimes \ln^2 \frac{k_T^2}{\mu_r^2} \qquad \mu_r \equiv c_0/r_\perp$$

$$\int \frac{\mathrm{d}^2 r_\perp}{(2\pi)^2} e^{-ik_\perp \cdot r_\perp} S^{(2)}(r_\perp) \ln \frac{c_0^2}{r_\perp^2 \mu^2} = \frac{1}{\pi} \int \frac{\mathrm{d}^2 l_\perp}{l_\perp^2} \left[F(k_\perp - l_\perp) - J_0\left(\frac{c_0}{\mu}|l_\perp|\right) F(k_\perp) \right]$$
$$= \frac{1}{\pi} \int \frac{\mathrm{d}^2 l_\perp}{l_\perp^2} \left[F(k_\perp - l_\perp) - \frac{\Lambda^2}{\Lambda^2 + l_\perp^2} F(k_\perp) \right] + F(k_\perp) \ln \frac{\Lambda^2}{\mu^2}$$

2. cross section in the momentum space

auxiliary semi-hard scale Λ

$$P(\xi) \otimes [\ln \frac{\mu^2}{\Lambda^2} + I_1(\Lambda)] \qquad \sigma_0 \otimes [\ln \frac{k_T^2}{\Lambda^2} + I_1(\Lambda)] \qquad \sigma_0 \otimes [\ln^2 \frac{k_T^2}{\Lambda^2} + I_2(\Lambda)]$$

Λ-independent

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Threshold Resummation

Resummation of the collinear logarithm

1. reverse-evolution approach

$$\begin{bmatrix} q(x_p,\mu) \\ g(x_p,\mu) \end{bmatrix} + \frac{\alpha_s}{2\pi}\ln\frac{\Lambda^2}{\mu^2}\int_{x_p}^{1}\frac{d\xi}{\xi}\begin{bmatrix} C_F\mathcal{P}_{qq}(\xi) & T_R\mathcal{P}_{qg}(\xi) \\ C_F\mathcal{P}_{gq}(\xi) & N_C\mathcal{P}_{gg}(\xi) \end{bmatrix}\begin{bmatrix} q(x_p/\xi,\mu) \\ g(x_p/\xi,\mu) \end{bmatrix} \Rightarrow \begin{bmatrix} q(x_p,\Lambda) \\ g(x_p,\Lambda) \end{bmatrix}$$

2. renormalization group equation approach

$$\mathcal{P}_{qq}(N) = -2\gamma_E - 2\psi(N) + \frac{3}{2} - \frac{1}{N} - \frac{1}{N+1} = -2\gamma_E - 2\ln N + \frac{3}{2} + \mathcal{O}(\frac{1}{N})$$

large-N limit

$$q^{\rm res}(x_p,\Lambda^2,\mu^2) = \int_{x_p}^{1}\frac{dx}{x}q(x,\mu^2)\Delta^q(\Lambda^2,\mu^2,\omega=\ln\frac{x}{x_p})$$

$$\frac{d\Delta^q(\Lambda^2,\mu^2,\omega)}{d\ln\mu^2} = -\frac{\alpha_s C_F}{\pi}\left[\ln\omega + \frac{3}{4}\right]\Delta^q(\Lambda^2,\mu^2,\omega) + \frac{\alpha_s C_F}{\pi}\int_0^{\omega}d\omega'\,\frac{\Delta^q(\Lambda^2,\mu^2,\omega) - \Delta^q(\Lambda^2,\mu^2,\omega')}{\omega-\omega'}$$

threshold jet function

Analogous to Becher, Neubert and Pecjak PRL 2006, JHEP 2007.

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Threshold Resummation

Resummation of the soft logarithms

Sudakov resummation / Sudakov factor

$$\frac{d\sigma_{\text{resummed}}}{dy d^2 p_T} = S_\perp \int_\tau^1 \frac{dz}{z^2} x_p q(x_p, \Lambda^2) D_{h/q}(z, \Lambda^2) F(k_\perp) e^{-S_{\text{Sud}}^{qq}}$$

$$S_{\text{Sud}}^{qq} = C_F \int_{\Lambda^2}^{k_\perp^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu^2)}{\pi} \ln \frac{k_\perp^2}{\mu^2} - 3C_F \int_{\Lambda^2}^{k_\perp^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu^2)}{2\pi}$$

Final formula

$$\frac{d\sigma}{dy d^2 p_T} = \frac{d\sigma_{\text{resummed}}}{dy d^2 p_T} + \frac{d\sigma_{\text{NLO matching}}}{dy d^2 p_T} + \frac{d\sigma_{\text{Sud matching}}}{dy d^2 p_T}$$

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Determining the semi-hard scale Λ: saddle point approximation

$$\frac{d\sigma^{qq}_{\rm resummed}}{dy d^2p_T} = S_\perp \int_\tau^1 \frac{dz}{z^2} \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-ik_\perp \cdot r_\perp} S^{(2)}(r_\perp) e^{-S^{qq}_{\rm Sud}} \int_{x_p}^1 \frac{dx}{x} q(x,\mu) \frac{e^{(3/4-\gamma_E)\gamma^q_{\mu_r,\mu}}}{\Gamma(\gamma^q_{\mu_r,\mu})} \left[\ln \frac{x}{x_p}\right]_*^{\gamma^q_{\mu_r,\mu}-1}$$
$$\times \int_z^1 \frac{dz'}{z'} D_{h/q}(z') \frac{e^{(3/4-\gamma_E)\gamma^q_{\mu_r,\mu}}}{\Gamma(\gamma^q_{\mu_r,\mu})} \left[\ln \frac{z'}{z}\right]_*^{\gamma^q_{\mu_r,\mu}-1},$$

$$P(\xi) \otimes \ln \frac{\mu^2}{\mu_r^2}$$

$$P(\xi) \otimes [\ln \frac{\mu^2}{\Lambda^2} + I_1(\Lambda)]$$

saddle point approximation

$$\Lambda \sim \mu_r = \frac{c_0}{r_\perp}$$

$$\Lambda^2 \approx \max\left\{\Lambda^2_{\rm QCD}\left[\frac{k_\perp^2(1-\xi)}{\Lambda^2_{\rm QCD}}\right]^{\frac{C_F}{C_F+N_c\beta_0}}, Q_s^2\right\}$$

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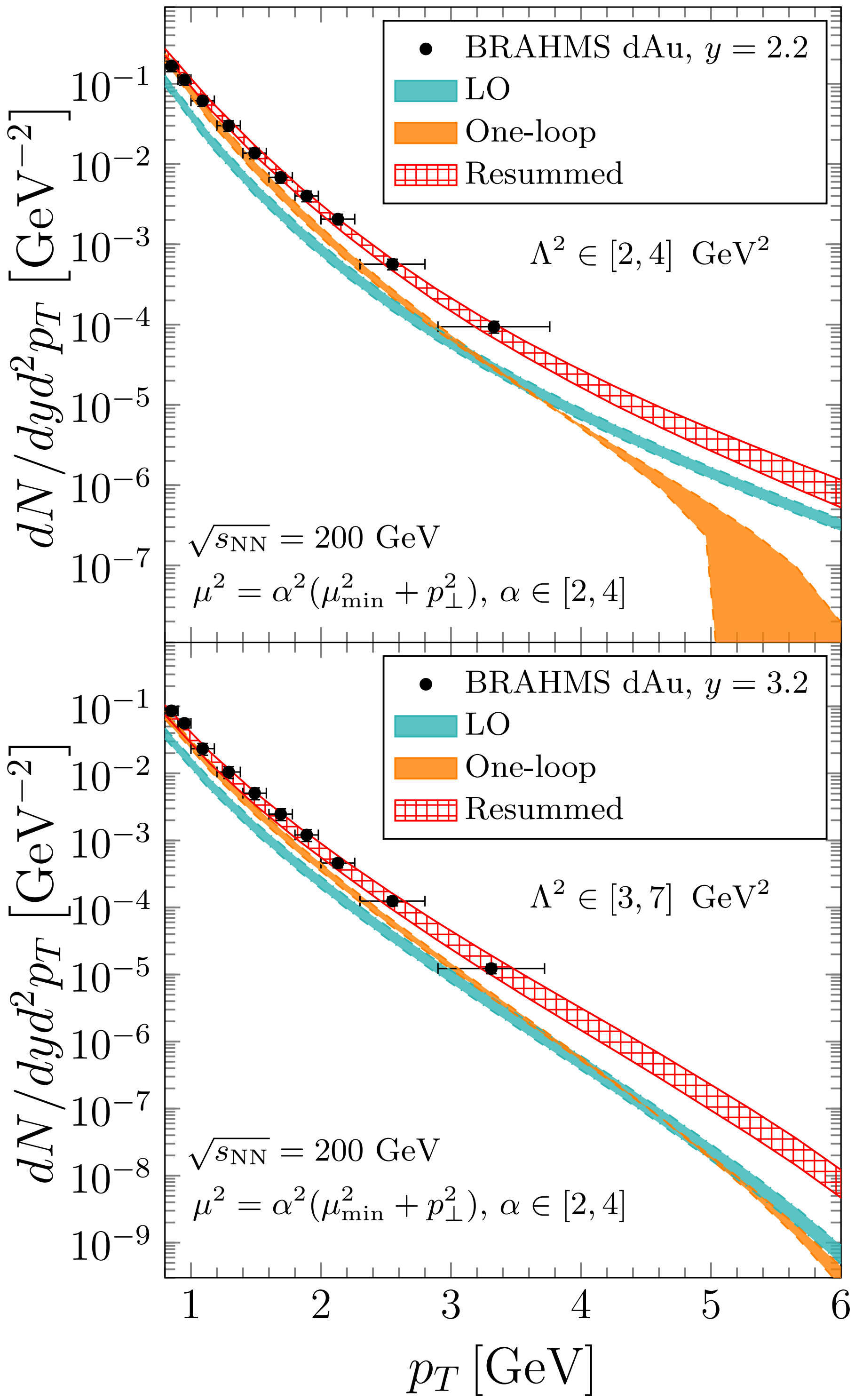
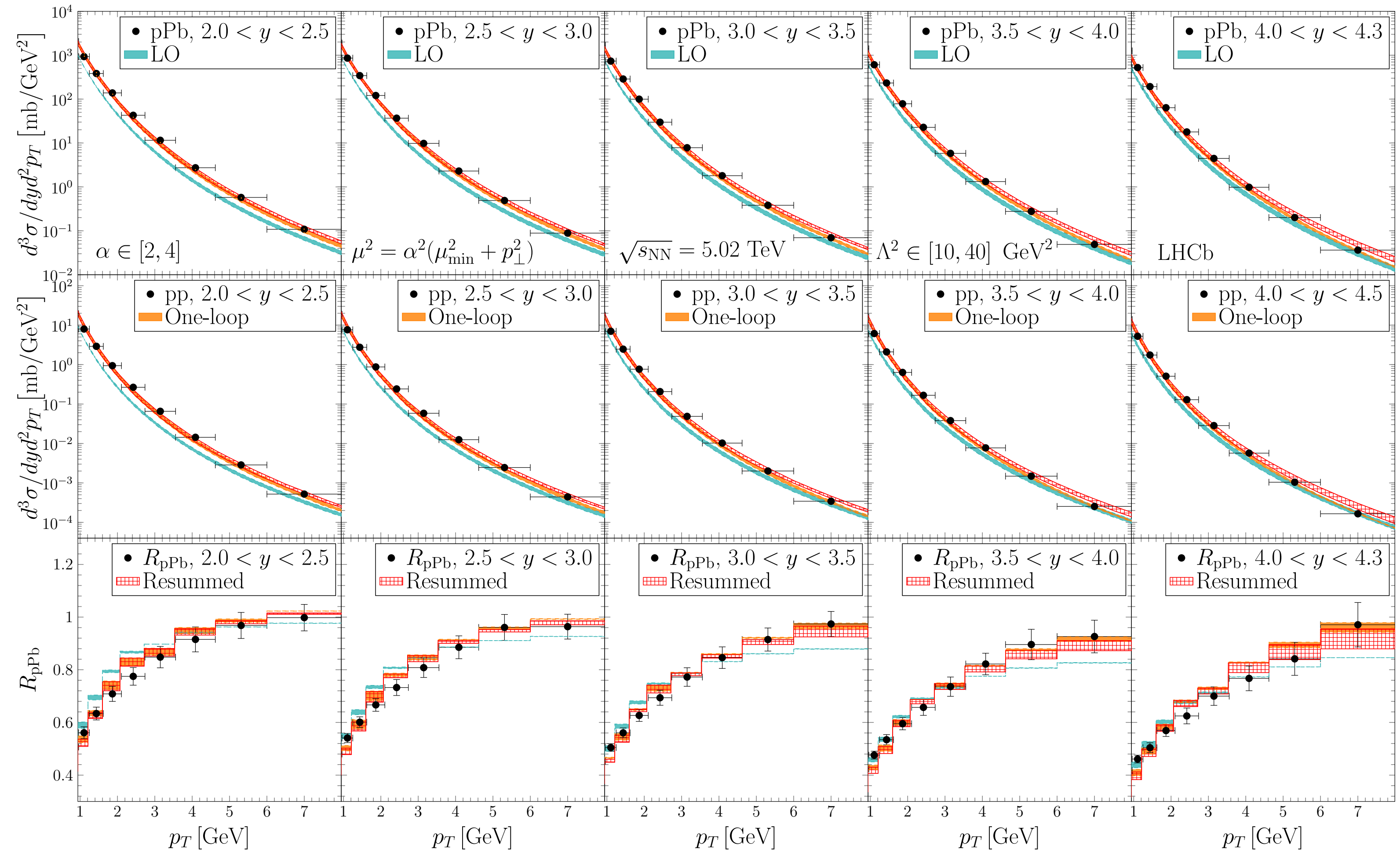
Threshold Resummation

Numerical Results





- ☑ Threshold resummation solves the "negativity" problem.
- ☑ Numerical results can universally describe the experimental data from RHIC and the LHC.

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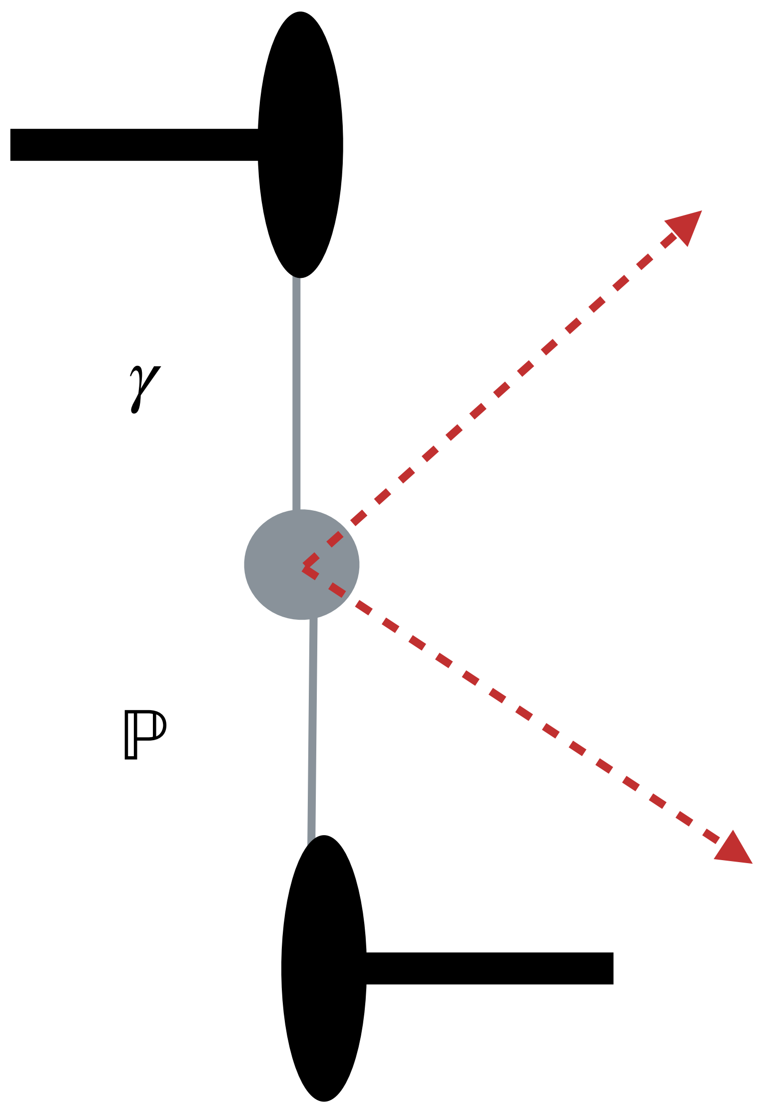
Diffractive Dijet in UPC

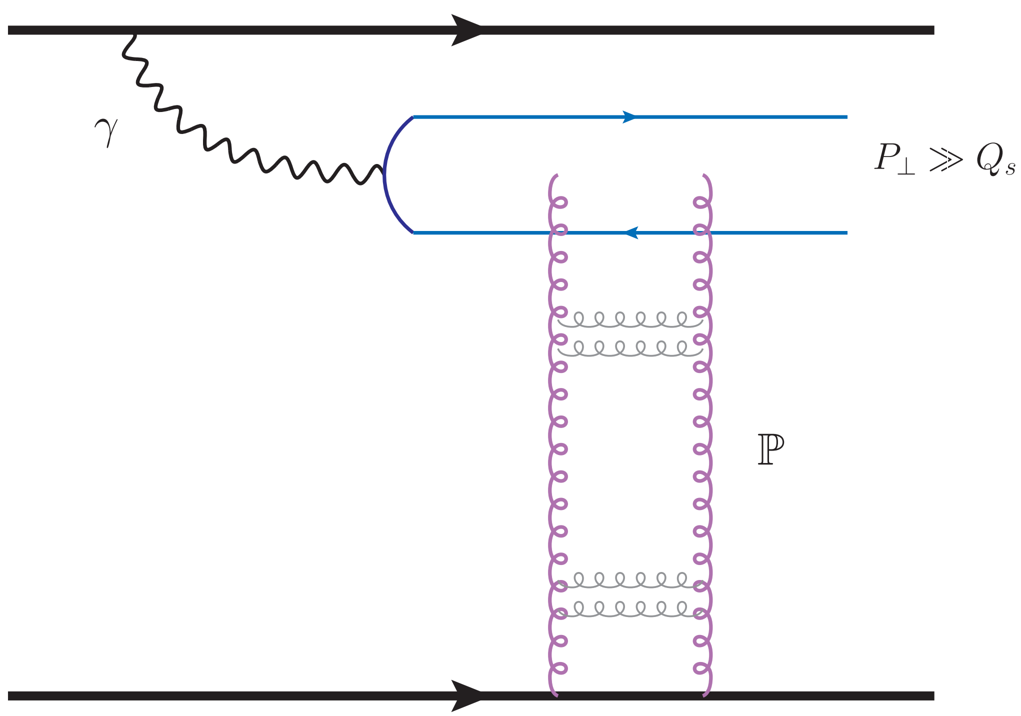
Diffractive Process in UPC



Golden channel to study gluon saturation

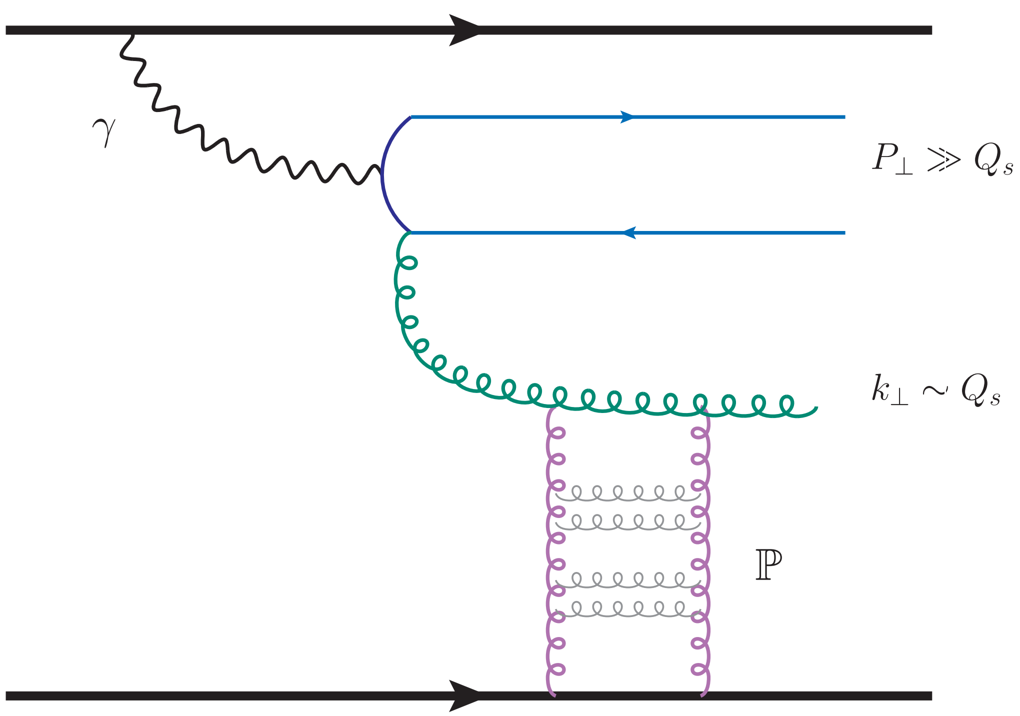
Iancu, Mueller, Triantafyllopoulos, SYW, JHEP 2022, EPJC 2023
Hauksson, Iancu, Mueller, Triantafyllopoulos, SYW, arXiv:2402.14748

"LO" Exclusive Process



$$\frac{d\sigma}{d^2P_\perp} \propto \frac{1}{P_\perp^6}$$

"NLO" 2+1 Process

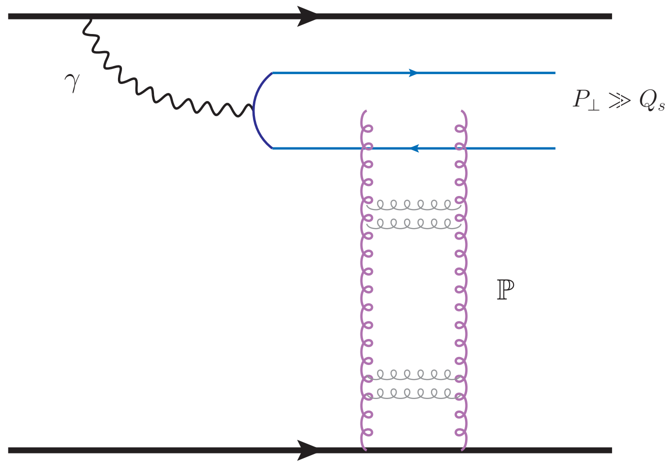


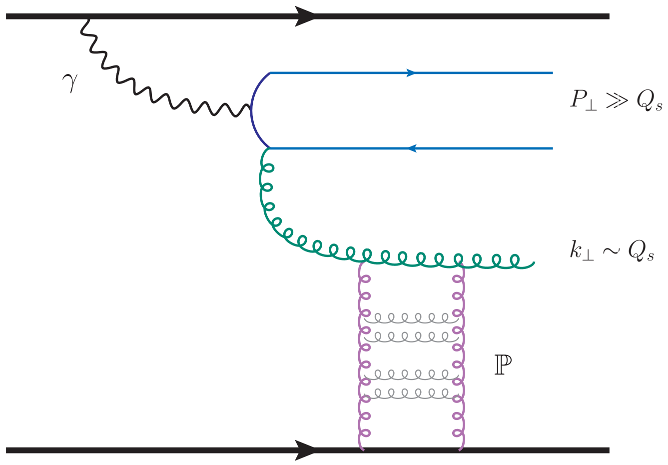
$$\frac{d\sigma}{d^2P_\perp} \propto \frac{1}{P_\perp^4}$$

Iancu, Mueller, Triantafyllopoulos, SYW, JHEP 2022, EPJC 2023
Hauksson, Iancu, Mueller, Triantafyllopoulos, SYW, arXiv:2402.14748

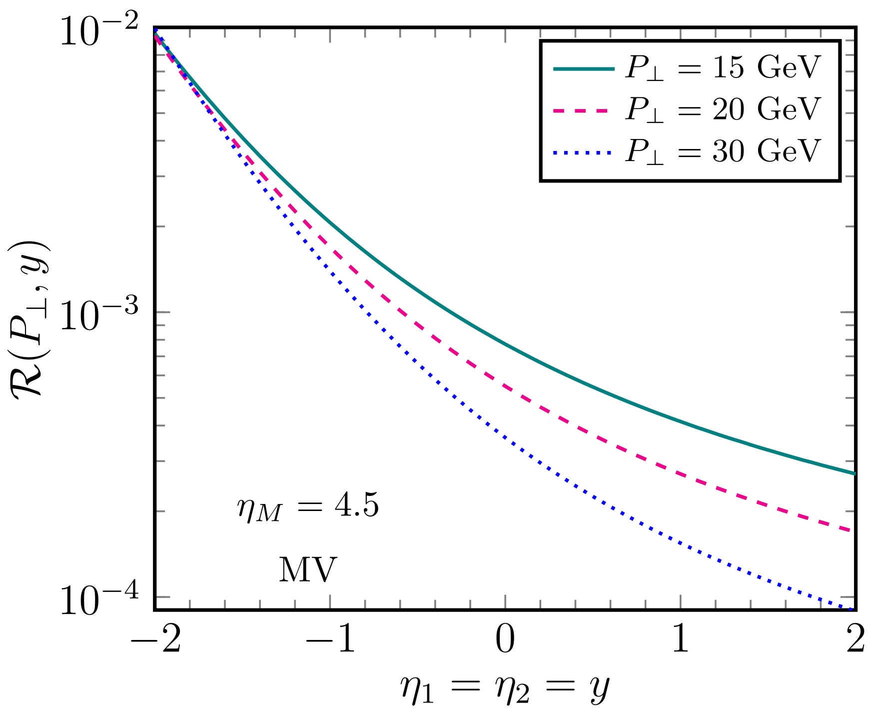
Diffractive Dijet in UPC

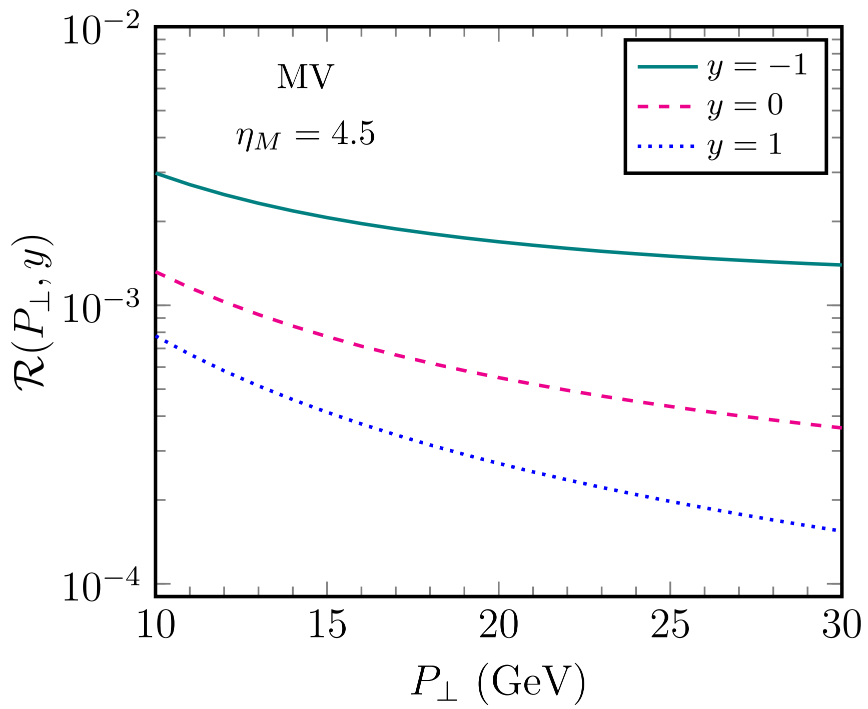
"LO" Exclusive Process ≪ **"NLO" 2+1 Process**





$$\mathcal{R} \equiv \frac{\sigma_{\text{exclusive}}}{\sigma_{2+1}}$$



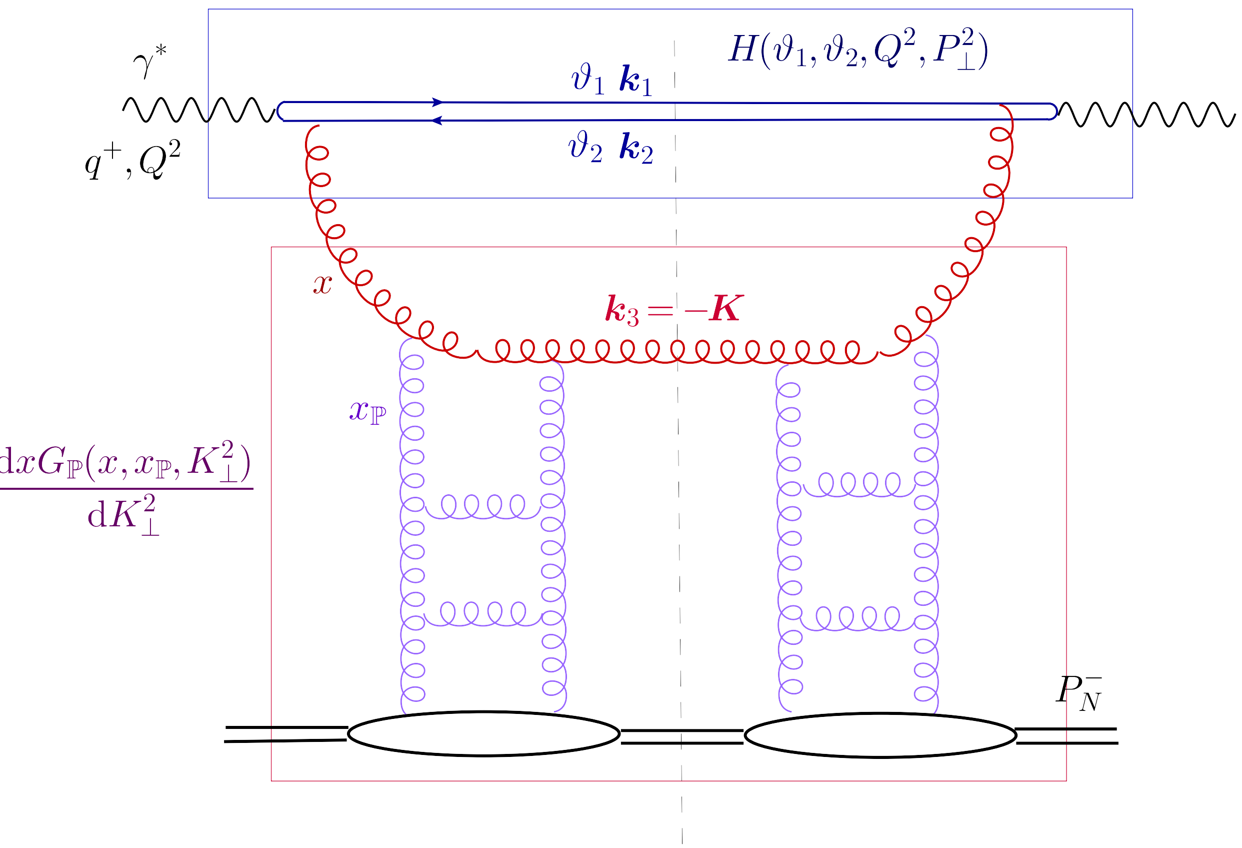


Iancu, Mueller, Triantafyllopoulos, SYW, JHEP 2022, EPJC 2023
Hauksson, Iancu, Mueller, Triantafyllopoulos, SYW, arXiv:2402.14748

Resummation: Shao, Shi, Zhang, Zhou, Zhou; arXiv:2402.05465

TMD factorisation of 2+1 Process

$$\frac{d\sigma}{d\eta_1 d\eta_2 d^2\boldsymbol{P} d^2\boldsymbol{K} dY_{\mathbb{P}}} = x_\gamma f_\gamma(x_\gamma)\,\frac{1}{\pi}\frac{d\hat{\sigma}^{\gamma g\to q\bar{q}}}{d\hat{t}}\;\frac{dxG_{\mathbb{P}}(x, Y_{\mathbb{P}}, K_\perp^2)}{d^2\boldsymbol{K}}$$



Gluon distribution of Pomeron

$$\frac{dxG_{\mathbb{P}}(x, Y_{\mathbb{P}}, K_\perp^2)}{d^2\boldsymbol{K}} = \frac{S_\perp(N_c^2-1)}{4\pi^3}\frac{1}{2\pi(1-x)} \times \left[\mathcal{M}^2\int dR\,R\,J_2(K_\perp R)K_2(\mathcal{M}R)\mathcal{T}_g(R, Y_{\mathbb{P}})\right]^2$$

Probability of finding gluon with momentum fraction x inside a pomeron with momentum fractuon $x_{\mathbb{P}}$

$$\mathcal{M}^2 = \frac{x}{1-x}K_\perp^2$$

Iancu, Mueller, Triantafyllopoulos, SYW, JHEP 2022, EPJC 2023
Hauksson, Iancu, Mueller, Triantafyllopoulos, SYW, arXiv:2402.14748

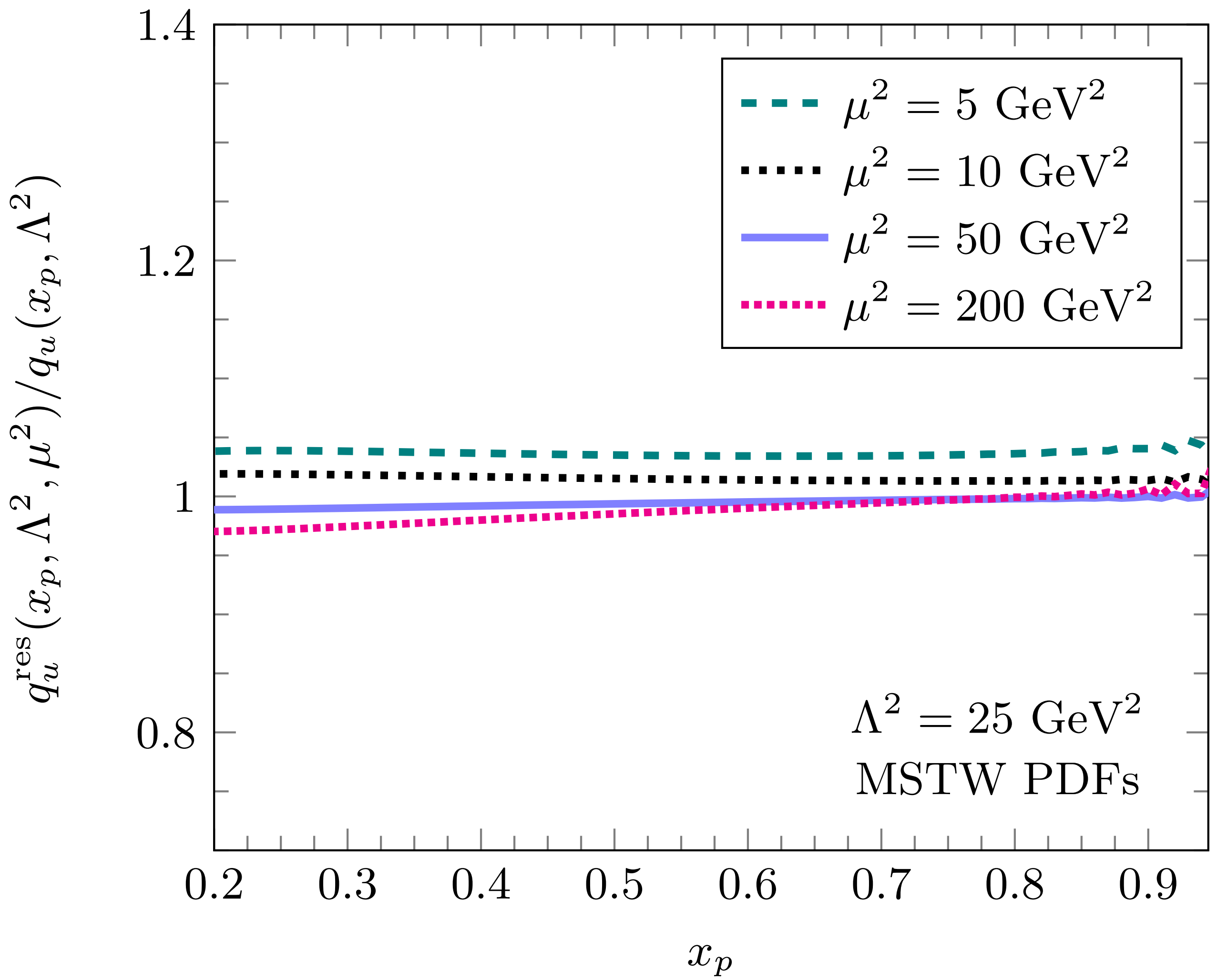
Summary

- ☑ Threshold resummation solves the "negativity" problem in the single inclusive hadron cross section at the NLO.
- ☑ The 2+1 process dominates in the diffractive dijet production in UPC and the diffractive TMD factorisation naturally arise in the CGC effective theory.

Thanks for your attention!

The End

Resummation of the collinear logarithm



☑ Two approaches are numerically equivalent.

Determining the semi-hard scale Λ: saddle point approximation