QCD Factorization for Hadronic B Decays

李 新 强

华 中 师 范 大 学

In collaboration with G. Bell, M. Beneke, T. Huber, and S. Krankl

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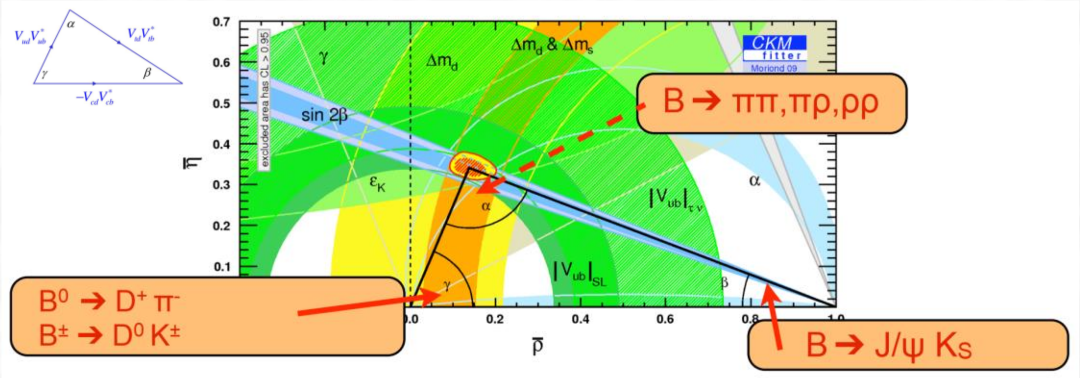
Outline

- □ **Introduction**
- □ **QCDF approach for hadronic B decays**
- □ **NNLO QCD corrections to hadronic matrix elements**
- □ **Possible higher-order power corrections motivated by data**
- □ **Summary**

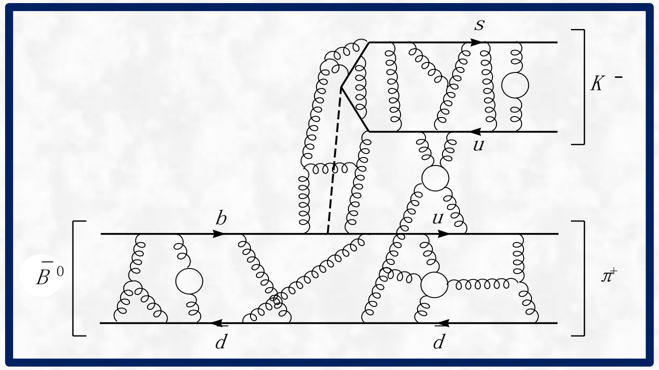
Why hadronic B decays

- direct access to the CKM parameters, especially to the **three angles of UT**



- further insight into the **strong-interaction effects** involved in hadronic weak decays

factorization? strong phase origin?...



- deepen our understanding of the **origin & mechanism of CPV**

$$\mathcal{A}(\bar{B}^0 \to \pi^+ K^-) = \frac{G_F}{\sqrt{2}} \sum_{ij} V_{\rm CKM} \left(\mathcal{C}_i^{\rm SM} + \mathcal{C}_i^{\rm NP}\right) \left[F_j^{B\to\pi}(m_K^2) \int_0^1 du\, T_{ij}^{\rm I}(u) \Phi_K(u) + (\pi \leftrightarrow K) + \int_0^1 d\xi\, du\, dv\, T_i^{\rm II}(\xi, u, v) \Phi_B(\xi) \Phi_\pi(v) \Phi_K(u) \right]$$

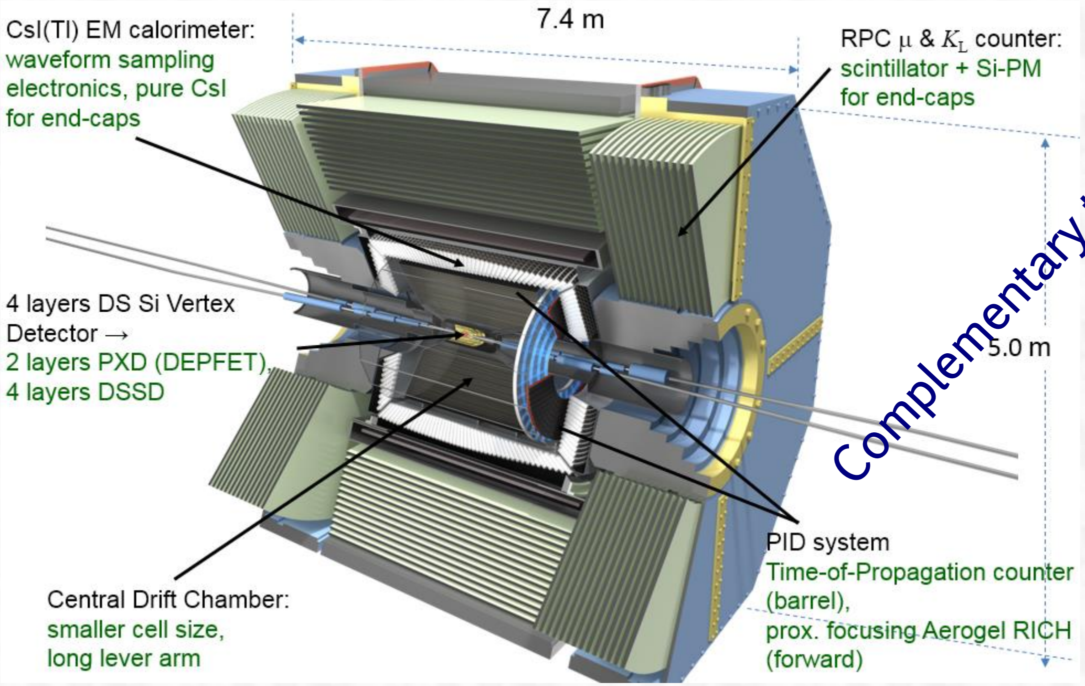
- insight into the **hadron structures**: especially **exotic hadronic states**

- Probing NP beyond the SM

→ ***although complicated but necessary!***

Exp. status of B physics

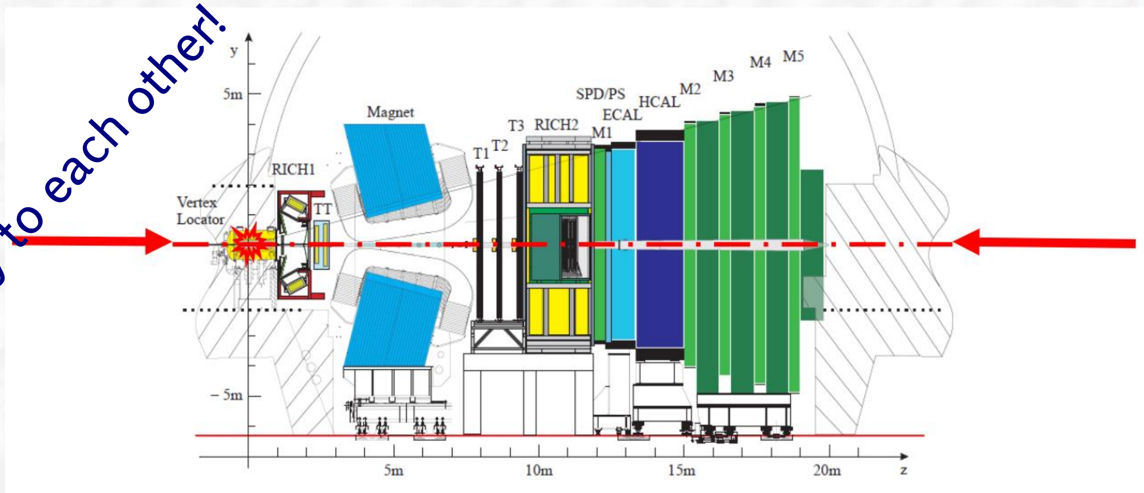
Super B-factories (e^+e^-): Belle II



[E. Kou *et al.* [Belle II], PTEP 2019 (2019) 123C01]

Hadron colliders (pp): LHCb @LHC

Complementary to each other!



[R. Aaij *et al.* [LHCb Collaboration], arXiv:1808.08865]

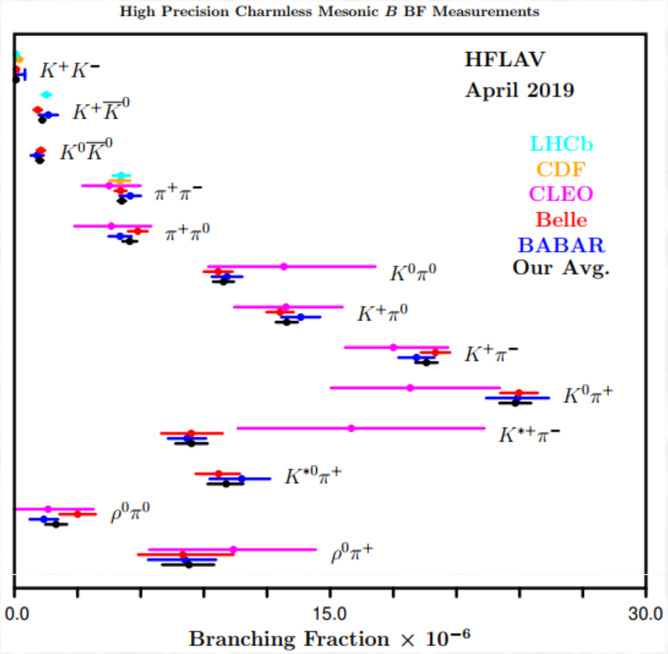
LHCb & Belle II: the two currently running experiments aimed at heavy flavor physics!

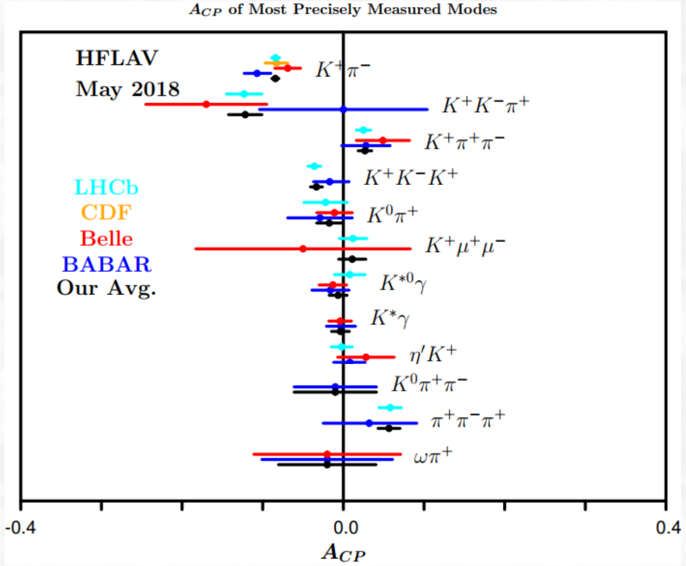
Two main goals among others:

- **Check if there are any extra new CP-violation mechanisms beyond the KM?**
- **Check if there are new particles/interactions that are sensitive to flavor structures?**

Precision era of B physics

- More precise data from these dedicated experiments





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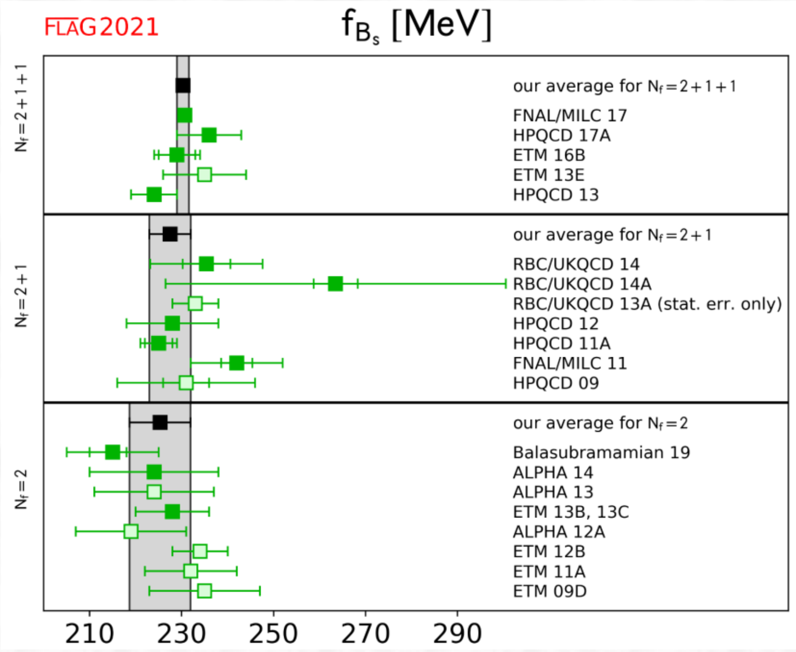

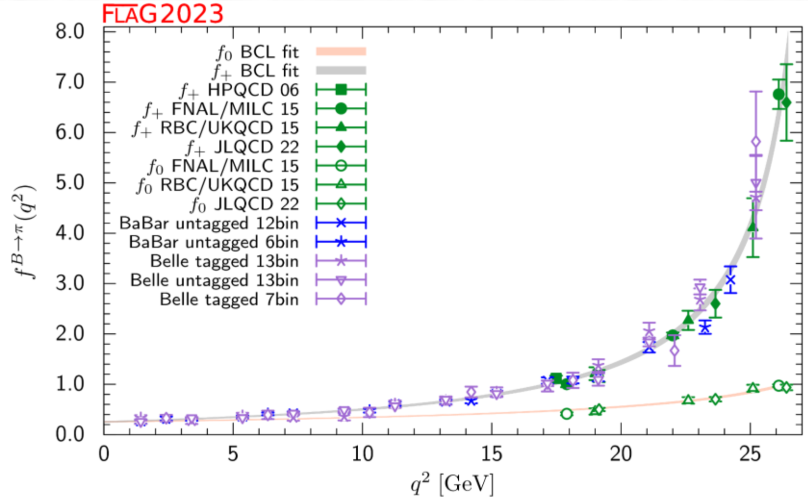

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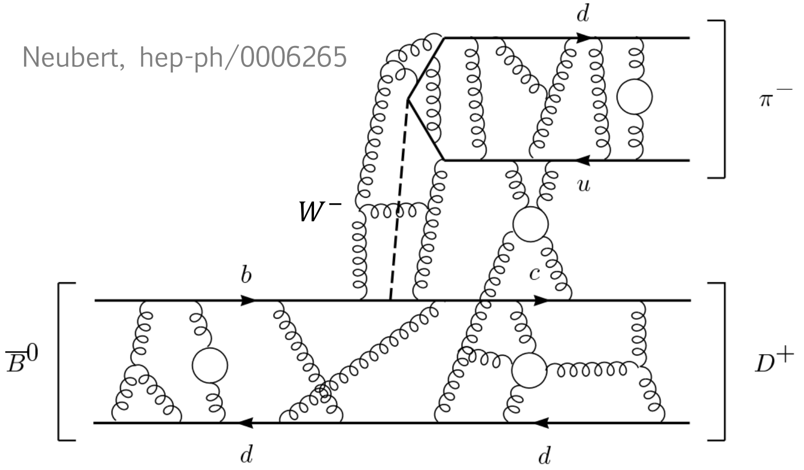
- **Lattice QCD** & **LCSR** etc. also provide more precise results for the **non-pert. hadronic parameters**

↳ we are entering an ***era of precision flavor physics*** !

Effective Hamiltonian for hadronic B decays

- **For hadronic B decays: typical multi-scale problem;** → **EFT formalism more suitable!**



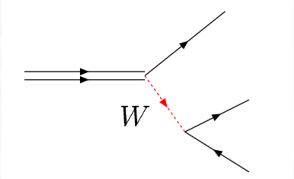
multi-scale problem with highly hierarchical scales!

EW interaction scale	$\gg$	ext. mom'a in B rest frame	$\gg$	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	$\gg$	$m_b \sim 5$ GeV	$\gg$	$\Lambda_{\rm QCD} \sim 1$ GeV

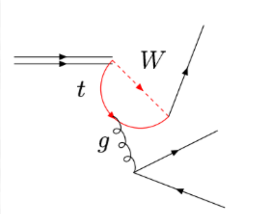
$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pD}^* \left(C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2 + \sum_{i=\rm pen} C_i \mathcal{O}_{i,\rm pen} \right)$$

- **Starting point $\mathcal{H}_{\rm eff} = -\mathcal{L}_{\rm eff}$: obtained after integrating out heavy d.o.f. ($m_{W,Z,t} \gg m_b$)**

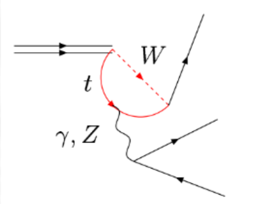
[Buras, Buchalla, Lautenbacher '96; Chetyrkin, Misiak, Munz '98]



tree



QCD penguin



EW penguin

- **Wilson coefficients C_i: all physics above m_b; perturbatively calculable & NNLL program now complete!** [Gorbahn, Haisch '04; Misiak, Steinhauser '04]

Hadronic matrix elements

- **For a typical two-body decay $\bar{B} \to M_1 M_2$:**

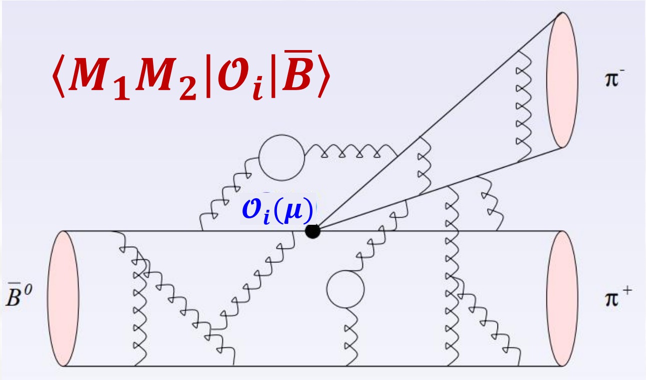
$$\mathcal{A}(\bar{B} \to M_1 M_2) = \sum_i [\lambda_{\rm CKM} \times C_i \times \langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle]$$

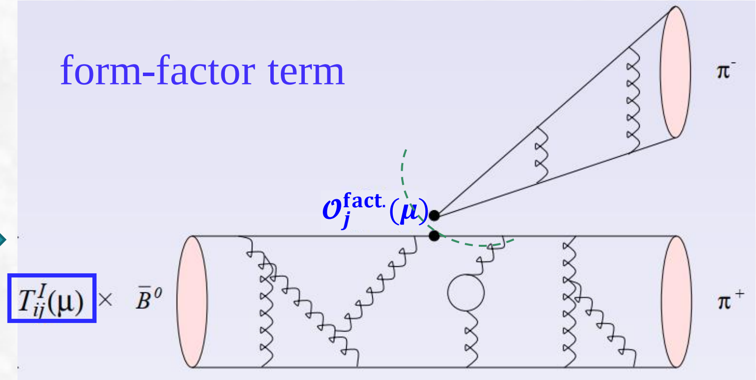
- $\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle$: depending on spin & parity of $M_{1,2}$; final-state re-scattering introduces strong phases, and hence non-zero direct CPV; ➡ *A quite difficult, multi-scale, strong-interaction problem!*

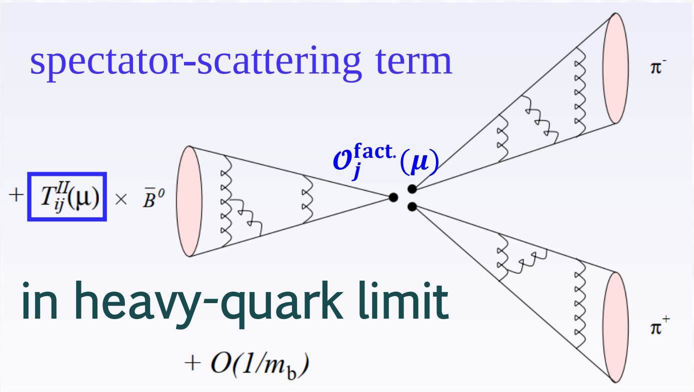
- **Different methods proposed for dealing with $\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle$:** naïve fact., generalized fact., ……

  - Dynamical approaches based on factorization theorems: PQCD, QCDF, SCET, · · · [Keum, Li, Sanda, Lü, Yang '00; Beneke, Buchalla, Neubert, Sachrajda, '00; Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02]
    - **how to include higher-order perturbative & power corrections?**
  - Symmetries of QCD: Isospin, U-Spin, V-Spin, and flavour SU(3) symmetries, · · · [Zeppenfeld, '81; London, Gronau, Rosner, He, Chiang, Cheng *et al.*]
    - **how to systematically estimate symmetry-breaking effects?**

- **QCDF/SCET:** systematic framework to all orders in α_s, limited by $\Lambda_{\rm QCD}/m_b$ corrections [BBNS '99-'03]

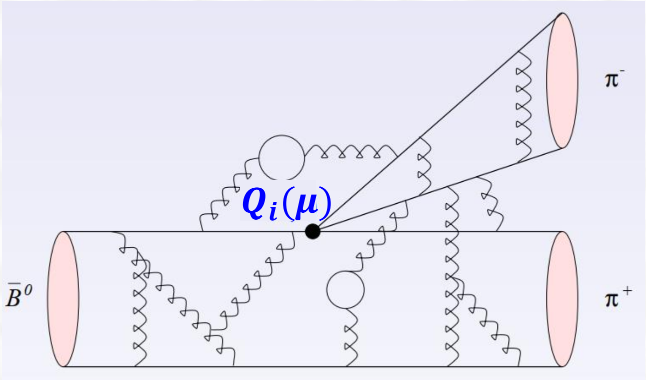






QCDF formula for charmless B decays

- **QCDF formula:** [BBNS '99-'03]



$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \sim F^{B\to M_1}(q^2=0) \int_0^1 \mathrm{d}x\, \mathbf{T}_i^{\mathrm{I}}(x)\, \phi_{M_2}(x)$$

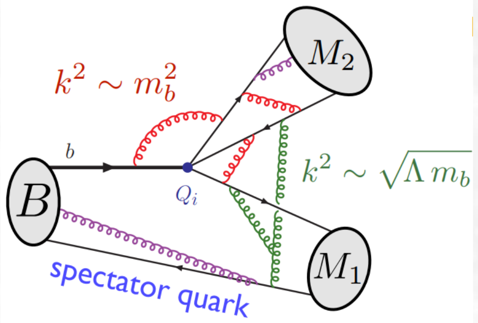
form-factor term

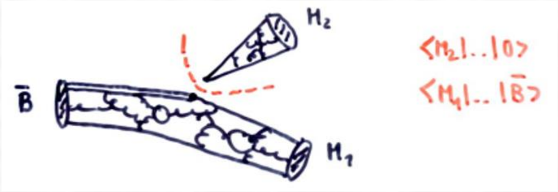
$$+ \int_0^\infty \frac{\mathrm{d}\omega}{\omega} \int_0^1 \mathrm{d}x\, \mathrm{d}y\, \mathbf{T}_i^{\mathrm{II}}(x,y,\omega)\, \phi_{M_1}(y)\, \phi_{M_2}(x)\, \phi_B^+(\omega)$$

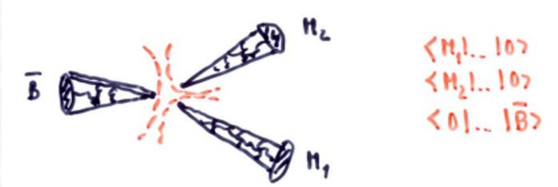
spectator-scattering term

- **How to proof QCDF formula:**

- based on diagrammatic factorization [BBNS '99-'03]
- method of expansion by regions [Beneke, Smirnov '97]
- use heavy-quark & collinear expansion for hard processes [Lepage, Brodsky '80]





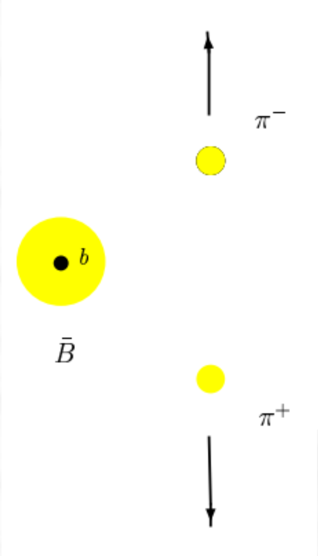


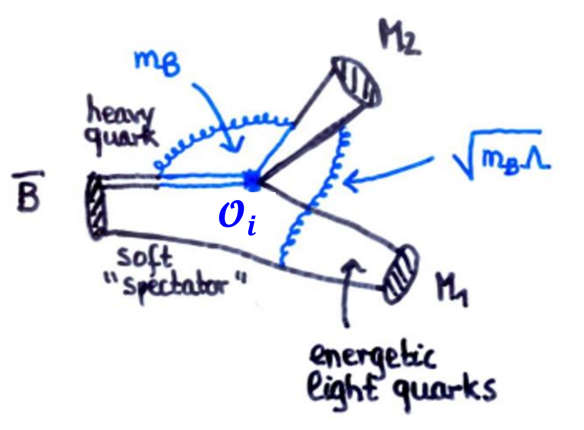
universal non-perturbative hadronic parameters

$\langle M_1 M_2 | Q_i | \bar{B} \rangle$ factorized into $\langle M | j_\mu | \bar{B} \rangle$ (transition form factors), $\langle M | j_\mu | 0 \rangle$, $\langle 0 | j_\mu | \bar{B} \rangle$ (decay constants & LCDAs)

Soft-collinear factorization from SCET

- **For a two-body decay:** simple kinematics, but complicated dynamics with several typical modes





- low-virtuality modes:
  - ⋆ HQET fields: $p - m_b v \sim$ $\mathcal{O}(\Lambda)$
  - ⋆ soft spectators in B meson: $p_s^\mu \sim \Lambda \ll m_b, \quad p_s^2 \sim$ $\mathcal{O}(\Lambda^2)$
  - ⋆ collinear quarks and gluons in pion: $E_c \sim m_b, \quad p_c^2 \sim$ $\mathcal{O}(\Lambda^2)$
- high-virtuality modes:
  - ⋆ hard modes: (heavy quark + collinear)$^2 \sim$ $\mathcal{O}(m_b^2)$
  - ⋆ hard-collinear modes: (soft + collinear)$^2 \sim$ $\mathcal{O}(m_b\Lambda)$

- **SCET:** a very suitable framework for studying factorization and re-summation for processes involving energetic & light particles/jets [Bauer *et al.* '00; Beneke *et al.* '02]

- **From SCET point of view:** introduce different fields/modes for different momentum regions, and SCET diagrams must reproduce precisely QCD diagrams in collinear & soft momentum region!

↳ achieve *soft-collinear factorization* & hence QCDF formula via QFT machinery [Beneke, 1501.07374]

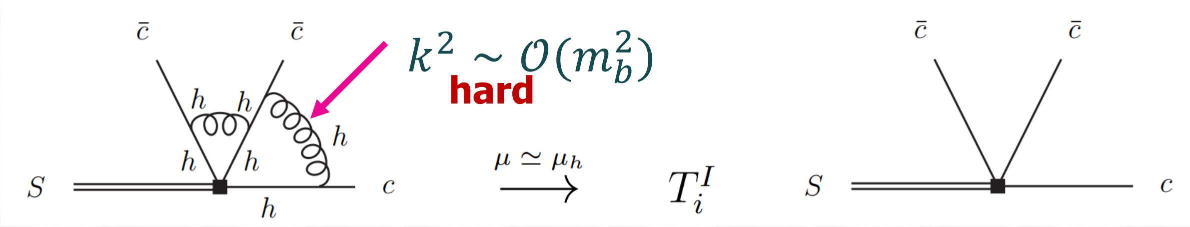
Soft-collinear factorization from SCET

- **QCDF formula from SCET:** $T^{I,II}$ = matching coefficients from QCD to SCET

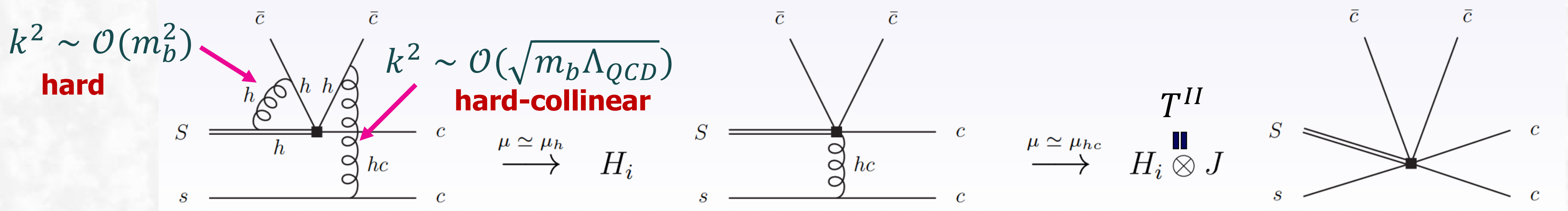
$$\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

→ **QCD - SCET = T^I & T^{II}**

- **For T^I:** only hard scale involved, one-step matching from QCD → $\mathrm{SCET_I}(\mathrm{hc}, \mathrm{c}, \mathrm{s})$!



- **For T^{II}:** two scales involved, two-step matching from QCD → $\mathrm{SCET_I}(\mathrm{hc}, \mathrm{c}, \mathrm{s})$ → $\mathrm{SCET_{II}}(\mathrm{c}, \mathrm{s})$!

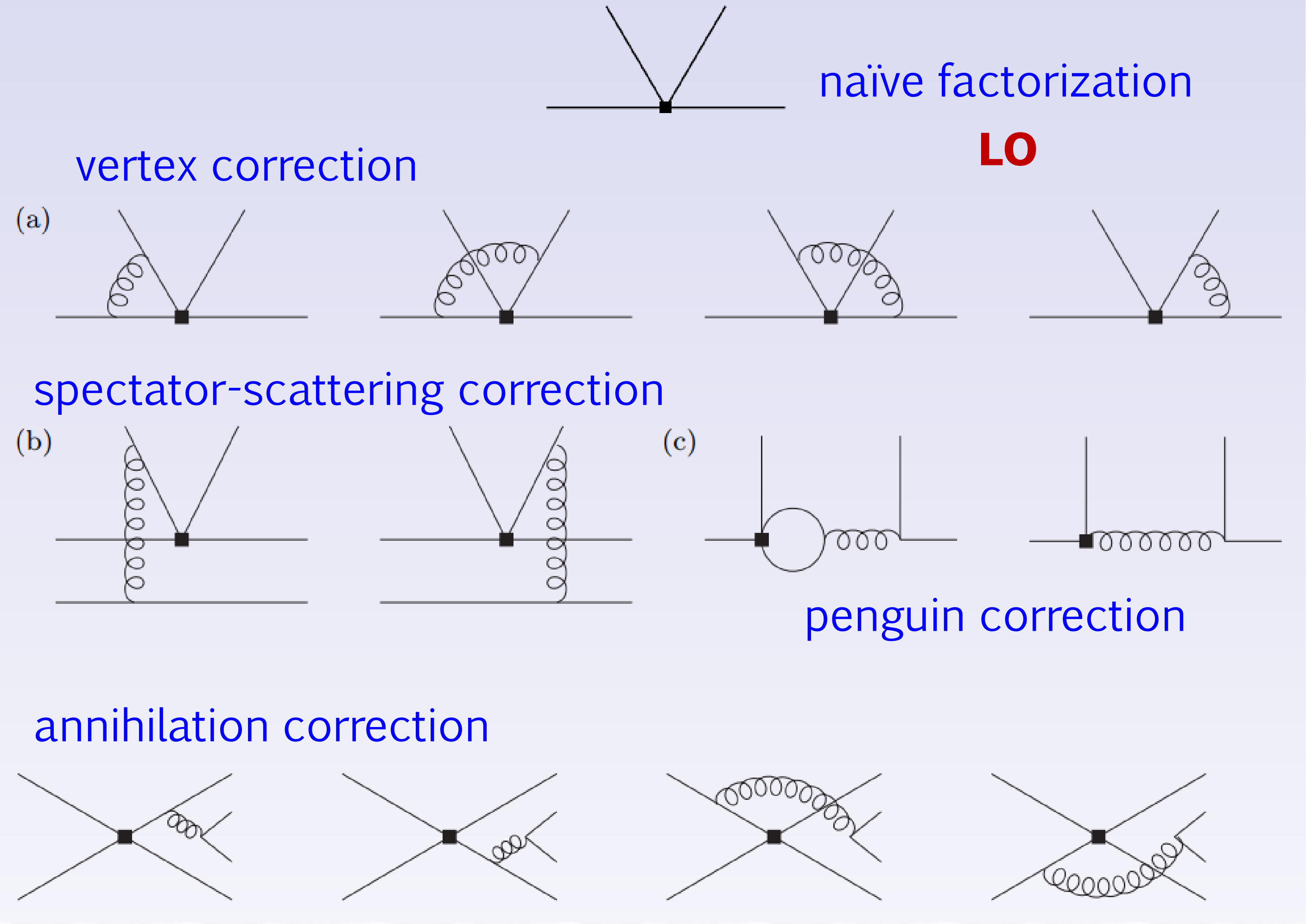


- **SCET formalism reproduces exact QCDF formula, but more apparent & efficient;** [Beneke, 1501.07374]

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = T^I(\mu_h) * \phi_{M_2}(\mu_h) f_+^{BM_1}(0) + H_i(\mu_h) * U_\parallel(\mu_h, \mu_{hc}) * J(\mu_{hc}) * \phi_{M_2}(\mu_h) * \phi_{M_1}(\mu_{hc}) * \phi_B(\mu_{hc})$$

Phenomenological analyses based on NLO

- **Various analyses based on NLO hard kernels**

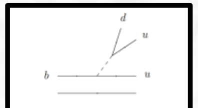


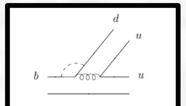
- **complete sets of final states:**
  - $B \to PP, PV$: [Beneke, Neubert, hep-ph/0308039; Cheng, Chua, 0909.5229, 0910.5237;]
  - $B \to VV$: [Beneke, Rohrer, Yang, hep-ph/0612290; Cheng, Yang, 0805.0329; Cheng, Chua, 0909.5229, 0910.5237;]
  - $B \to AP, AV, AA$: [Cheng, Yang, 0709.0137, 0805.0329;]
  - $B \to SP, SV$: [Cheng, Chua, Yang, hep-ph/0508104, 0705.3079; Cheng, Chua, Yang, Zhang, 1303.4403;]
  - $B \to TP, TV$: [Cheng, Yang, 1010.3309;]

very successful but also with some problems phenomenologically. !

Phenomenological successes based on NLO

□ Successes at NLO:


colour-allowed tree α_1


QCD penguins α_4

- For color-allowed tree- & penguin-dominated decay modes, branching ratios usually quantitatively OK
- Dynamical explanation of intricate patterns of penguin interference seen in PP, PV, VP and VV modes

$$PP \sim a_4 + r_\chi a_6, \quad PV \sim a_4 \approx \frac{PP}{3}$$
$$VP \sim a_4 - r_\chi a_6 \sim -PV$$
$$VV \sim a_4 \sim PV$$

$$r_\chi = \frac{2m_L^2}{m_b\,(m_q + m_s)}$$

⇒ $\mathrm{Br}\left(B^{\pm,0} \to \eta^{(\prime)} K^{(*)\pm,0}\right)$

- Qualitative explanation of polarization puzzle in $B \to VV$ decays, due to the large weak annihilation
- Strong phases start at $\mathcal{O}(\alpha_s)$, dynamical explanation of smallness of direct CP asymmetries

□ Some problems encountered at NLO:

- Factorization of power corrections generally broken, due to endpoint divergence
- Could not account for some data, such as $\mathrm{Br}(B^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K)$
- How important the higher-order pert. corr.? Fact. theorem is still established for them?
- As strong phases start at $\mathcal{O}(\alpha_s)$, NNLO is only NLO to them; quite relevant for A_{CP}?

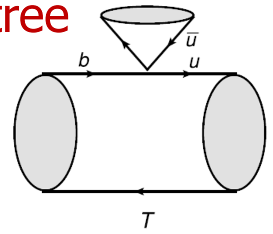
we need go beyond the LO in pert. and power corrections!

Status of NNLO calculation of T^{I} & T^{II}

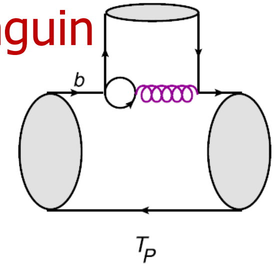
- For each Q_i insertion, both **tree** & **penguin** topologies relevant for **charmless decays**

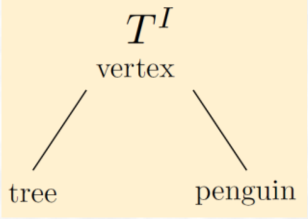
$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq F^{B\to M_1}\, T_i^{I} \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

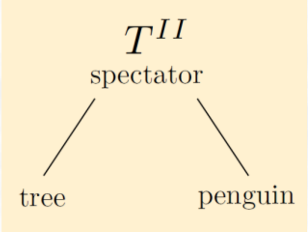
tree



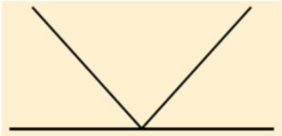
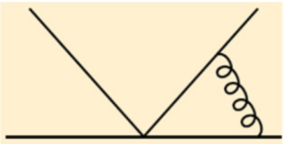
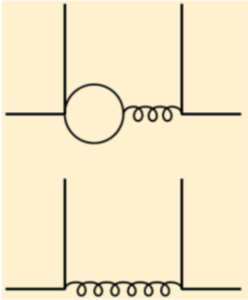
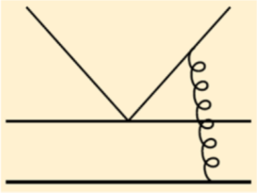
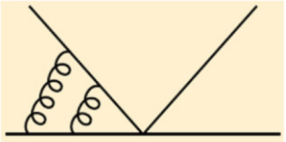
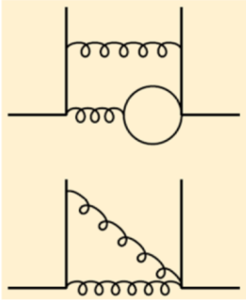
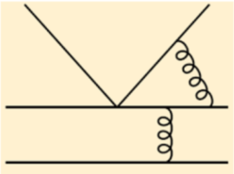
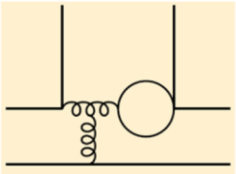
penguin







- For **tree** & **penguin** topologies, both contribute to T^{I} & T^{II}

	T_i^{I}, tree	T_i^{I}, penguin	T_i^{II}, tree	T_i^{II}, penguin
LO: $\mathcal{O}(1)$		$T^{I} = 1 + \mathcal{O}(\alpha_s) + \cdots$		
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'03				$T^{II} = \mathcal{O}(\alpha_s) + \cdots$
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07, '09; Beneke, Huber, Li '09; Huber, Krankl, Li '16	 Kim, Yoon '11; Bell, Beneke, Huber, Li '15, '20	 Beneke, Jager '05; Kivel '06, Pilipp '07	 Beneke, Jager '06; Jain, Rothstein, Stewart '07

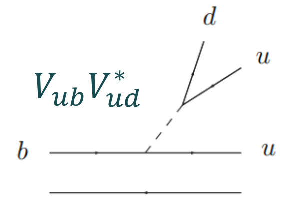
Tree-dominated B decays

- $B \to \pi\pi$ **decay amplitudes in QCDF:**

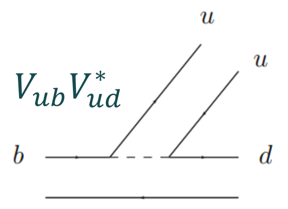
$$\sqrt{2}\,\langle \pi^-\pi^0 | \mathcal{H}_{eff} | B^- \rangle = \lambda_u \left[\alpha_1(\pi\pi) + \alpha_2(\pi\pi)\right] A_{\pi\pi}$$

$$\langle \pi^+\pi^- | \mathcal{H}_{eff} | \bar{B}^0 \rangle = \left\{\lambda_u \left[\alpha_1(\pi\pi) + \alpha_4^u(\pi\pi)\right] + \lambda_c\, \alpha_4^c(\pi\pi)\right\} A_{\pi\pi}$$

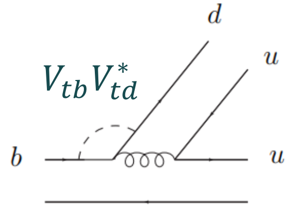
$$-\langle \pi^0\pi^0 | \mathcal{H}_{eff} | \bar{B}^0 \rangle = \left\{\lambda_u \left[\alpha_2(\pi\pi) - \alpha_4^u(\pi\pi)\right] - \lambda_c\, \alpha_4^c(\pi\pi)\right\} A_{\pi\pi}$$



colour-allowed tree α_1



colour-suppressed tree α_2



QCD penguins α_4

$b \to u\bar{u}d$: $\lambda_u = V_{ub}V_{ud}^* \sim \mathcal{O}(\lambda^3) \sim \lambda_c = V_{cb}V_{cd}^* \sim \mathcal{O}(\lambda^3)$ → α_4 loop-suppressed vs $\alpha_{1,2}$ → Tree-dominated!

- α_2 **at NLO:**

$$\alpha_2(\pi\pi) = 0.220 - [0.179 + 0.077i]_{\text{NLO}} + \left[\frac{r_{\text{sp}}}{0.485}\right]\{[0.123]_{\text{LOsp}} + [0.072]_{\text{tw3}}\}$$

↳ large cancellation between 1-loop vertex correction & LO result

$$r_{\text{sp}} = \frac{9 f_{M_1} \hat{f}_B}{m_b f_+^{B\pi}(0)\,\lambda_B}$$

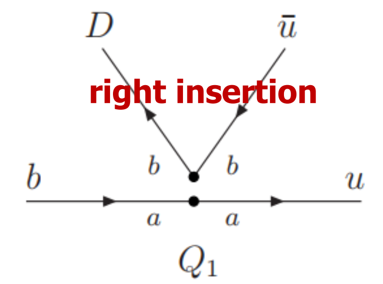
↳ **making α_2 sensitive to NNLO corrections, and large effect possible?**

Hard kernel T^I at NNLO

□ QCD → SCETI matching calculation:

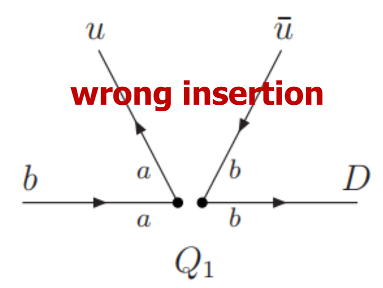
- For "right insertion":

$$\langle Q_i \rangle = T_i \langle O_{\rm QCD} \rangle + \sum_{a>1} H_{ia} \langle O_a \rangle$$



- For "wrong insertion":

$$\langle Q_i \rangle = \tilde{T}_i \langle O_{\rm QCD} \rangle + \tilde{H}_{i1} \langle \tilde{O}_1 - O_1 \rangle + \sum_{a>1} \tilde{H}_{ia} \langle \tilde{O}_a \rangle$$



□ On-shell matrix elements at NNLO: full QCD side

$$\langle Q_i \rangle = \Big\{ A_{ia}^{(0)} + \frac{\alpha_s}{4\pi} \Big[A_{ia}^{(1)} + Z_{ext}^{(1)} A_{ia}^{(0)} + Z_{ij}^{(1)} A_{ja}^{(0)} \Big] + \left(\frac{\alpha_s}{4\pi}\right)^2 \Big[A_{ia}^{(2)} + Z_{ij}^{(1)} A_{ja}^{(1)} + Z_{ij}^{(2)} A_{ja}^{(0)} + Z_{ext}^{(1)} A_{ia}^{(1)} + Z_{ext}^{(2)} A_{ia}^{(0)} + Z_{ext}^{(1)} Z_{ij}^{(1)} A_{ja}^{(0)} + Z_{\alpha}^{(1)} A_{ia}^{(1)} + (-i)\, \delta m^{(1)} A_{ia}^{\prime(1)} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \langle O_a \rangle^{(0)}$$

□ On-shell matrix elements at NNLO: SCET side

$$\langle O_a \rangle = \Big\{ \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi} \Big[M_{ab}^{(1)} + Y_{ext}^{(1)} \delta_{ab} + Y_{ab}^{(1)} \Big] + \left(\frac{\hat{\alpha}_s}{4\pi}\right)^2 \Big[M_{ab}^{(2)} + Y_{ac}^{(1)} M_{cb}^{(1)} + Y_{ab}^{(2)} + Y_{ext}^{(1)} M_{ab}^{(1)} + Y_{ext}^{(2)} \delta_{ab} + Y_{ext}^{(1)} Y_{ab}^{(1)} + \hat{Z}_{\alpha}^{(1)} M_{ab}^{(1)} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \langle O_b \rangle^{(0)}$$

□ Master formula for T^I: right insertion

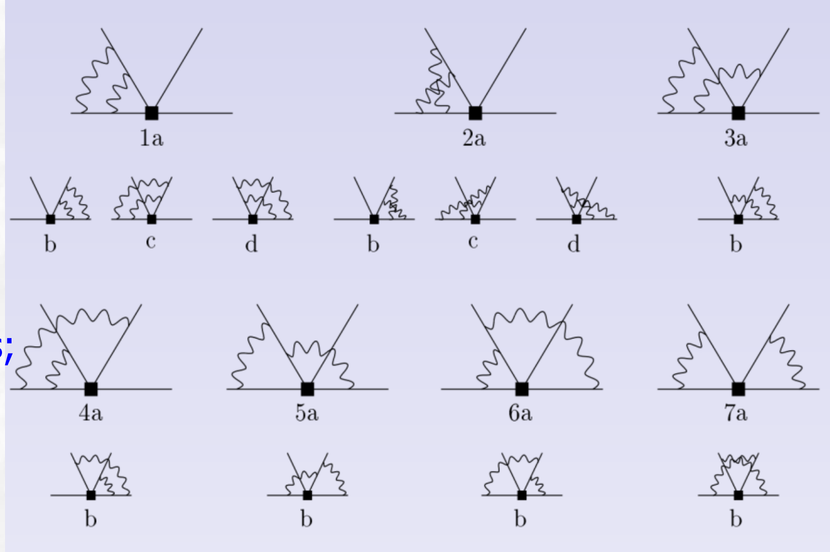
$$T_i^{(0)} = A_{i1}^{(0)} ,$$
$$T_i^{(1)} = A_{i1}^{(1)\rm nf} + Z_{ij}^{(1)} A_{j1}^{(0)} ,$$
$$T_i^{(2)} = \boxed{A_{i1}^{(2)\rm nf}} + Z_{ij}^{(1)} A_{j1}^{(1)} + Z_{ij}^{(2)} A_{j1}^{(0)} + Z_{\alpha}^{(1)} A_{i1}^{(1)\rm nf} + (-i)\, \delta m^{(1)} A_{i1}^{\prime(1)\rm nf} - T_i^{(1)} [C_{FF}^{(1)} + Y_{11}^{(1)} - Z_{ext}^{(1)}] - \sum_{b>1} H_{ib}^{(1)} Y_{b1}^{(1)} .$$

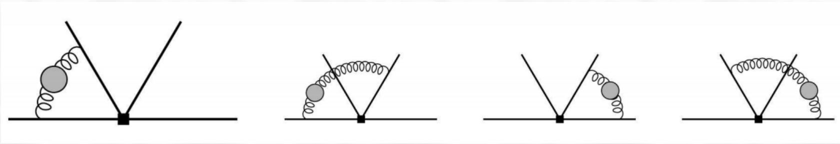
□ Master formula for T^I: wrong insertion

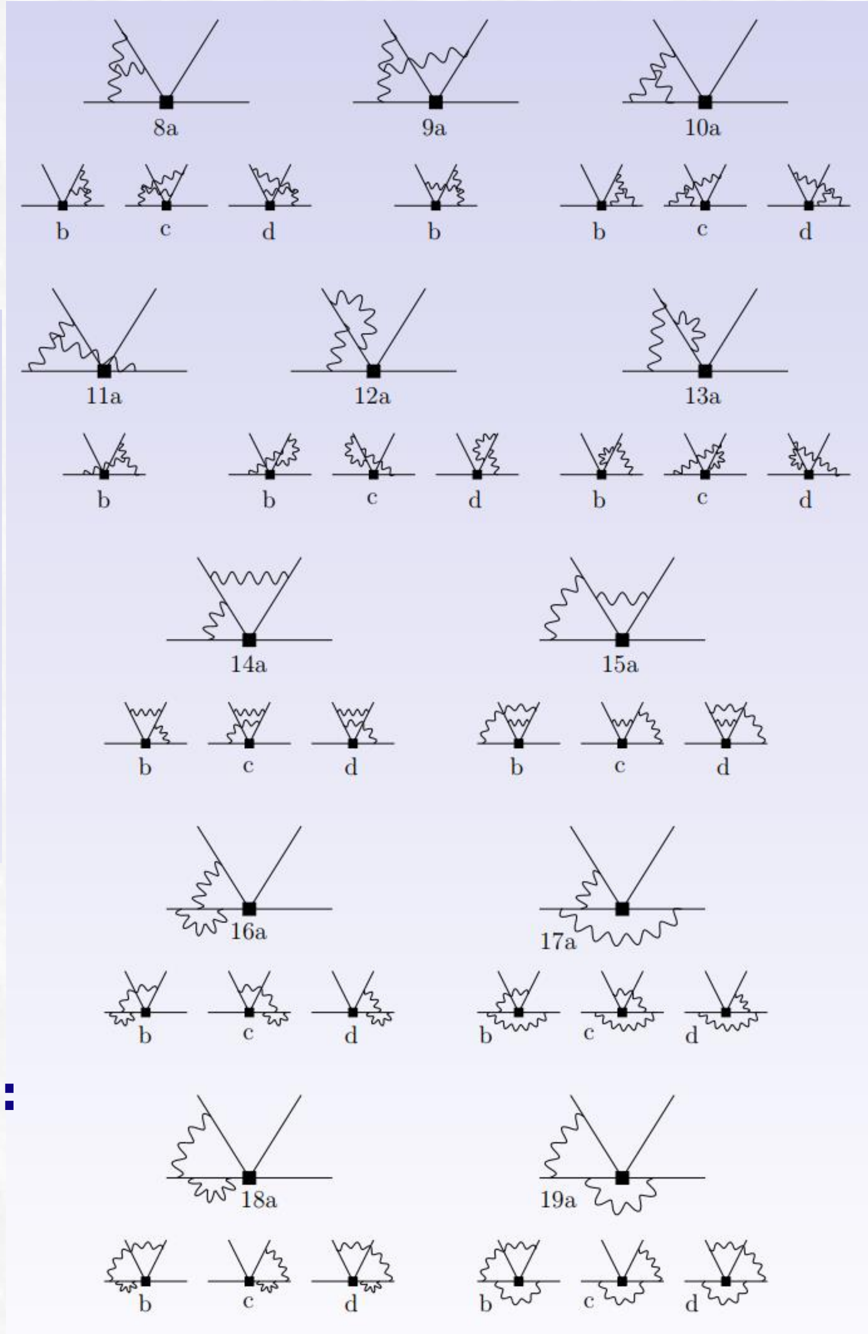
$$\widetilde{T}_i^{(0)} = \widetilde{A}_{i1}^{(0)} ,$$
$$\widetilde{T}_i^{(1)} = \widetilde{A}_{i1}^{(1)\rm nf} + Z_{ij}^{(1)} \widetilde{A}_{j1}^{(0)} + \underbrace{\widetilde{A}_{i1}^{(1)\rm f} - A_{21}^{(1)\rm f} \widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] \widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} ,$$
$$\widetilde{T}_i^{(2)} = \boxed{\widetilde{A}_{i1}^{(2)\rm nf}} + Z_{ij}^{(1)} \widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)} \widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)} \widetilde{A}_{i1}^{(1)\rm nf} + (-i)\, \delta m^{(1)} \widetilde{A}_{i1}^{\prime(1)\rm nf} + Z_{ext}^{(1)} [\widetilde{A}_{i1}^{(1)\rm nf} + Z_{ij}^{(1)} \widetilde{A}_{j1}^{(0)}] - \widetilde{T}_i^{(1)} [C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}] - \sum_{b>1} \widetilde{H}_{ib}^{(1)} \widetilde{Y}_{b1}^{(1)} + [\widetilde{A}_{i1}^{(2)\rm f} - A_{21}^{(2)\rm f} \widetilde{A}_{i1}^{(0)}] + (-i)\, \delta m^{(1)} [\widetilde{A}_{i1}^{\prime(1)\rm f} - A_{21}^{\prime(1)\rm f} \widetilde{A}_{i1}^{(0)}] + (Z_{\alpha}^{(1)} + Z_{ext}^{(1)}) [\widetilde{A}_{i1}^{(1)\rm f} - A_{21}^{(1)\rm f} \widetilde{A}_{i1}^{(0)}] - [\widetilde{M}_{11}^{(2)} - M_{11}^{(2)}] \widetilde{A}_{i1}^{(0)} - (C_{FF}^{(1)} - \xi_{45}^{(1)}) [\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] \widetilde{A}_{i1}^{(0)} - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}] \widetilde{A}_{i1}^{(0)} .$$

Two-loop QCD diagrams

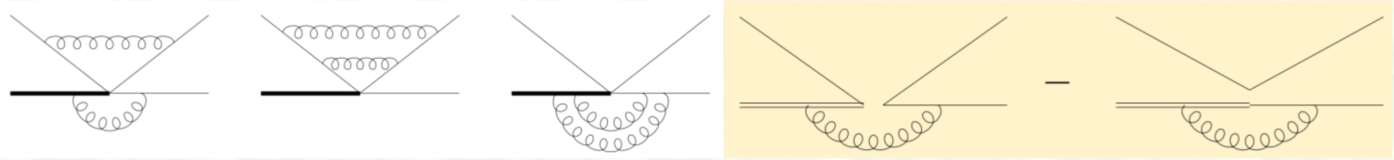
- $\widetilde{A}_{i1}^{(2)\mathrm{nf}}$: **relevant two-loop non-factorizable Feynman diagrams in full QCD:**
  - totally ~ 70 diagrams;
  - needs modern multi-loop Feynman diagrams techniques;
  - IBP reduction, Mellin-Barnes representation, Differential equations, …







- **Complicated counter-terms from QCD & SCET operators:**



Final results for $\alpha_{1,2}$

$$V_{1j}^{(0)} = \int_0^1 du\, T_j^{(0)} \phi_M(u), \qquad \frac{C_F}{2N_c} V_{1j}^{(l)} = \int_0^1 du\, T_j^{(l)}(u) \phi_M(u),$$

$$V_{2j}^{(0)} = \int_0^1 du\, \widetilde{T}_j^{(0)} \phi_M(u), \qquad \frac{C_F}{2N_c} V_{2j}^{(l)} = \int_0^1 du\, \widetilde{T}_j^{(l)}(u) \phi_M(u).$$

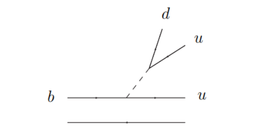
- **Tree amplitudes $\alpha_{1,2}$, after convolution with LCDAs:**

$$\alpha_i(M_1 M_2) = \sum_j C_j V_{ij}^{(0)} + \sum_{l \geqslant 1} \left(\frac{\alpha_s}{4\pi}\right)^l \left[\frac{C_F}{2N_c} \sum_j C_j V_{ij}^{(l)} + P_i^{(l)}\right] + \cdots$$

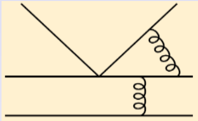
free from endpoint divergence

- **Numerical results including the NNLO corrections:**

$$\alpha_1(\pi\pi) = 1.009 + [0.023 + 0.010\, i]_{\rm NLO} + [0.026 + 0.028\, i]_{\rm NNLO}$$
$$- \left[\frac{r_{\rm sp}}{0.445}\right] \left\{[0.014]_{\rm LOsp} + [0.034 + 0.027 i\,]_{\rm NLOsp} + [0.008]_{\rm tw3}\right\}$$
$$= 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050}) i$$

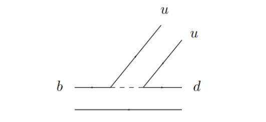


colour-allowed tree α_1

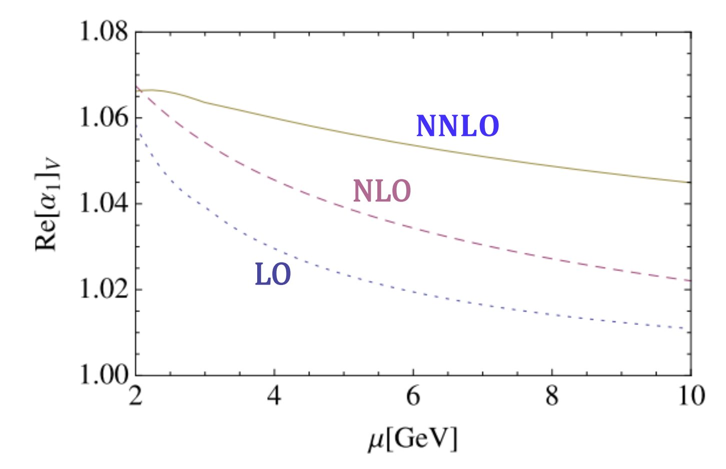


Beneke, Jager '05
Kivel '06, Pilipp '07

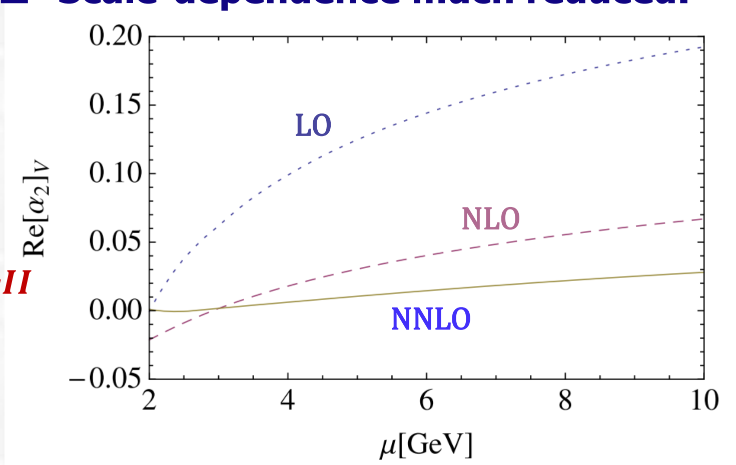
$$\alpha_2(\pi\pi) = 0.220 - [0.179 + 0.077\, i]_{\rm NLO} - [0.031 + 0.050\, i]_{\rm NNLO}$$
$$+ \left[\frac{r_{\rm sp}}{0.445}\right] \left\{[0.114]_{\rm LOsp} + [0.049 + 0.051 i\,]_{\rm NLOsp} + [0.067]_{\rm tw3}\right\}$$
$$= 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078}) i$$



colour-suppressed tree α_2



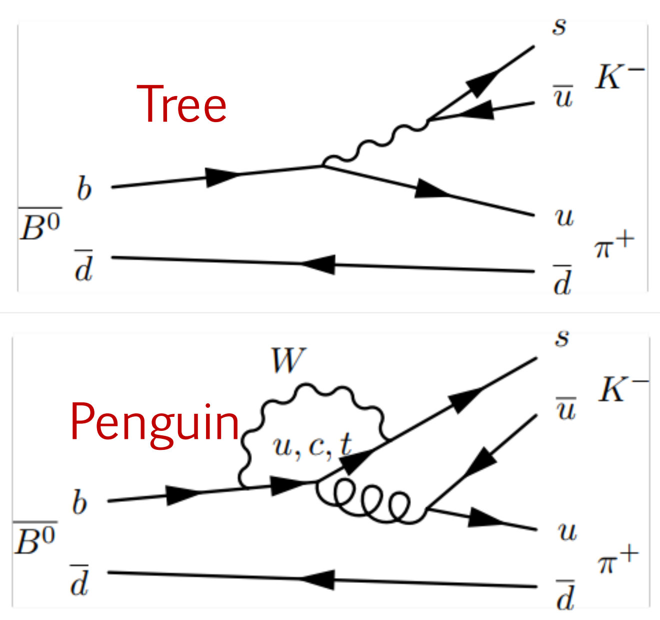
- **Scale-dependence much reduced!**



- **NNLO corrections both large, but cancelled between T^I & T^{II}**

$$\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

Penguin-dominated B decays



- $B \to \pi K$ **decay amplitudes:** mediated by $b \to s q\bar{q}$ transitions

$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} = A_{\pi\bar{K}}\big[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\big] + A_{\bar{K}\pi}\big[\delta_{pu}\alpha_2 + \delta_{pc}\tfrac{3}{2}\alpha_{3,\rm EW}^c\big],$$
$$\mathcal{A}_{\bar{B}^0\to\pi^+K^-} = A_{\pi\bar{K}}\big[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\big],$$

$\lambda_u = V_{ub}V_{us}^* \sim \mathcal{O}(\lambda^4) \ll \lambda_c = V_{cb}V_{cs}^* \sim \mathcal{O}(\lambda^2)$ → Penguin-dominated!

- **In QCDF, strong phases generated firstly at NLO in α_s**

$$A_{\rm CP} = [c \times \alpha_s]_{\rm NLO} + \mathcal{O}(\alpha_s^2, \Lambda/m_b)$$

→ ***NNLO is only NLO for A_{CP}, large effects still possible?***

- **To predict accurately direct CPV, we must calculate both tree & penguin up to NNLO!**

- **Driven by the current exp. data on $B \to \pi K$:**

$$\Delta A_{CP}(\pi K) = A_{CP}(B^- \to \pi^0 K^-) - A_{CP}(\bar{B}^0 \to \pi^+ K^-)$$
$$= (11.3 \pm 1.2)\%$$

differs from 0 by ~9σ

ΔA_{CP} puzzle

→ ***How about the situation @ NNLO?***

Penguin topologies with various insertions

☐ Effective Hamiltonian including penguin operators: [BBL '96; CMM '98]

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} V_{pD}^* V_{pb} \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$$

CMM operator basis

$$Q_1^p = (\bar{p}_L \gamma^\mu T^A b_L)\, (\bar{D}_L \gamma_\mu T^A p_L),$$
$$Q_2^p = (\bar{p}_L \gamma^\mu b_L)\, (\bar{D}_L \gamma_\mu p_L),$$

current-current operators

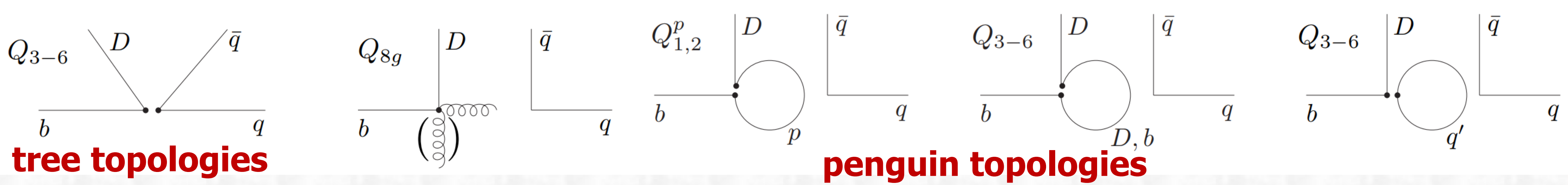
$$Q_3 = (\bar{D}_L \gamma^\mu b_L) \textstyle\sum_q (\bar{q} \gamma_\mu q),$$
$$Q_4 = (\bar{D}_L \gamma^\mu T^A b_L) \textstyle\sum_q (\bar{q} \gamma_\mu T^A q),$$
$$Q_5 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho q),$$
$$Q_6 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho T^A b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho T^A q)\,.$$

QCD penguin operators

$$Q_{8g} = \frac{-g_s}{32\pi^2}\, \overline{m}_b\, \bar{D} \sigma_{\mu\nu} (1+\gamma_5) G^{\mu\nu} b,$$

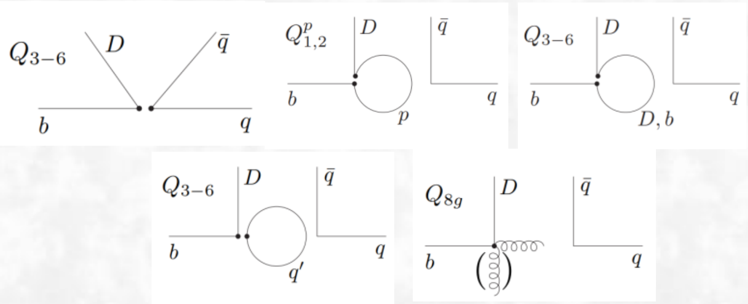
chromo-magnetic dipole operators

☐ Various operator insertions:



(i) Dirac structure of Q_i, (ii) color structure of Q_i, (iii) types of contraction, and (iv) quark masses in the fermion loop

Hard kernel T^I at NNLO



- **QCD → SCETI matching calculation:**

$$\langle Q_i \rangle = \sum_a \widetilde{H}_{ia} \langle \widetilde{O}_a \rangle$$

- **Note: always wrong insertion!**

- **Complete SCET operator basis:**

$$Q_3 = (\bar{D}_L \gamma^\mu b_L) \textstyle\sum_q (\bar{q}\gamma_\mu q),$$
$$Q_4 = (\bar{D}_L \gamma^\mu T^A b_L) \textstyle\sum_q (\bar{q}\gamma_\mu T^A q),$$
$$Q_5 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \textstyle\sum_q (\bar{q}\gamma_\mu \gamma_\nu \gamma_\rho q),$$
$$Q_6 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho T^A b_L) \textstyle\sum_q (\bar{q}\gamma_\mu \gamma_\nu \gamma_\rho T^A q).$$

$$O_1 = \sum_{q=u,d,s} \left[\bar{\chi}_D \frac{\not{n}_-}{2} (1-\gamma_5) \chi_q \right] \left[\bar{\xi}_q \not{n}_+ (1-\gamma_5) h_v \right],$$

the only physical operator and factorizes into FF*LCDA.

$$\tilde{O}_n = \sum_{q=u,d,s} \left[\bar{\xi}_q \gamma_\perp^\alpha \gamma_\perp^{\mu_1} \gamma_\perp^{\mu_2} \cdots \gamma_\perp^{\mu_{2n-2}} \chi_q \right] \left[\bar{\chi}_q (1+\gamma_5) \gamma_{\perp\alpha} \gamma_{\perp\mu_{2n-2}} \gamma_{\perp\mu_{2n-3}} \cdots \gamma_{\perp\mu_1} h_v \right],$$

n now up to 4, with 7 gamma matrices

$\tilde{O}_1 - O_1/2$ is another evanescent operator

- **On-shell matrix elements at NNLO: on the full QCD side**

$$\langle Q_i \rangle = \Big\{ \widetilde{A}^{(0)}_{ia} + \frac{\alpha_s}{4\pi} \left[\widetilde{A}^{(1)}_{ia} + Z^{(1)}_{\rm ext} \widetilde{A}^{(0)}_{ia} + Z^{(1)}_{ij} \widetilde{A}^{(0)}_{ja} \right] + \left(\frac{\alpha_s}{4\pi} \right)^2 \left[\widetilde{A}^{(2)}_{ia} + Z^{(1)}_{ij} \widetilde{A}^{(1)}_{ja} + Z^{(2)}_{ij} \widetilde{A}^{(0)}_{ja} + Z^{(1)}_{\rm ext} \widetilde{A}^{(1)}_{ia} + Z^{(2)}_{\rm ext} \widetilde{A}^{(0)}_{ia} + Z^{(1)}_{\rm ext} Z^{(1)}_{ij} \widetilde{A}^{(0)}_{ja} + Z^{(1)}_{\alpha} \widetilde{A}^{(1)}_{ia} + (-i)\, \delta m^{(1)} \widetilde{A}'^{(1)}_{ia} \right] + \mathcal{O}(\alpha_s^3) \Big\} \langle \widetilde{O}_a \rangle^{(0)}$$

- **On-shell matrix elements at NNLO: SCET side**

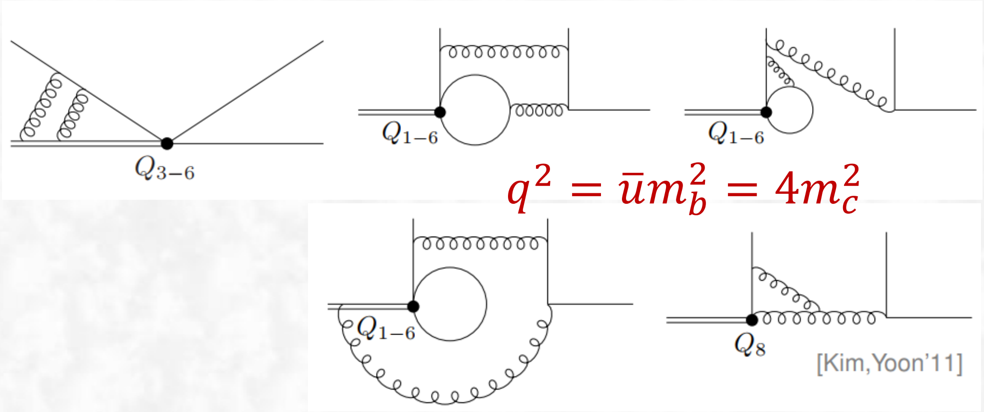
$$\langle O_a \rangle = \Big\{ \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi} \left[M^{(1)}_{ab} + Y^{(1)}_{\rm ext} \delta_{ab} + Y^{(1)}_{ab} \right] + \left(\frac{\hat{\alpha}_s}{4\pi} \right)^2 \left[M^{(2)}_{ab} + Y^{(1)}_{ac} M^{(1)}_{cb} + Y^{(2)}_{ab} + Y^{(1)}_{\rm ext} M^{(1)}_{ab} + Y^{(2)}_{\rm ext} \delta_{ab} + Y^{(1)}_{\rm ext} Y^{(1)}_{ab} + \hat{Z}^{(1)}_{\alpha} M^{(1)}_{ab} \right] + \mathcal{O}(\alpha_s^3) \Big\} \langle O_b \rangle^{(0)}$$

T^I up to NNLO

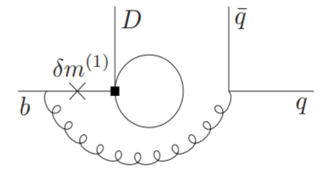
◻ Master formulae for T^I:

$$
\begin{aligned}
\frac{1}{2}\widetilde{T}_i^{(2)} = {} & \widetilde{A}_{i1}^{(2)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\,\widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)}\,\widetilde{A}_{i1}^{(1)\mathrm{nf}} \\
& + (-i)\,\delta m^{(1)}\,\widetilde{A}_{i1}^{\prime(1)\mathrm{nf}} + Z_{\mathrm{ext}}^{(1)}\,[\widetilde{A}_{i1}^{(1)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)}] \\
& - \frac{1}{2}\widetilde{T}_i^{(1)}\,[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}] - \sum_{b>1}\widetilde{H}_{ib}^{(1)}\,\widetilde{Y}_{b1}^{(1)} \\
& + [\widetilde{A}_{i1}^{(2)\mathrm{f}} - A_{31}^{(2)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}] + (-i)\,\delta m^{(1)}\,[\widetilde{A}_{i1}^{\prime(1)\mathrm{f}} - A_{31}^{\prime(1)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}] \\
& + (Z_{\alpha}^{(1)} + Z_{\mathrm{ext}}^{(1)})\,[\widetilde{A}_{i1}^{(1)\mathrm{f}} - A_{31}^{(1)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}] \\
& - [\widetilde{M}_{11}^{(2)} - M_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)} \\
& - (C_{FF}^{(1)} - \xi_{45}^{(1)})\,[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\,\widetilde{A}_{i1}^{(0)} - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)} \\
& - \sum_{b>1}\widetilde{A}_{ib}^{(0)}\,\widetilde{M}_{b1}^{(2)} - \sum_{b>1}\widetilde{A}_{ib}^{(0)}\,\widetilde{Y}_{b1}^{(2)}.
\end{aligned}
$$

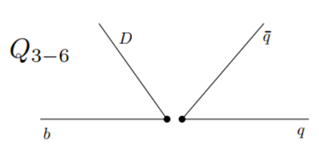
~ 100 two-loop Feynman diagrams



$q^2 = \bar{u}m_b^2 = 4m_c^2$



non-vanishing fermion-tadpole contraction of QCD penguin operators



tree-level matching of Q_i involves already evanescent SCET operators

◻ Complication during calculations:

(i) fermion loop with either $m = 0, m = m_c$ or $m = m_b$

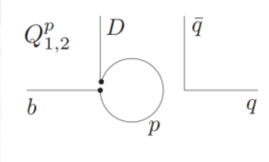
(ii) genuine 2-loop two-scale problem: $\bar{u},\ z_c = m_c^2/m_b^2$

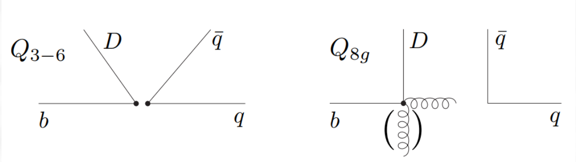
(iii) threshold at $\bar{u} = 4z_c$ introduces strong phase

Final results for a_4^p

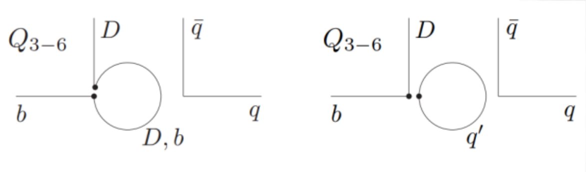
- **Final numerical results:**

$$\langle M_1 M_2|\mathcal{O}_i|\bar{B}\rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$





$$\begin{aligned}
a_4^u(\pi\bar{K})/10^{-2} &= -2.87 - [0.09+0.09i]_{\mathrm{V}_1} + [0.49-1.32i]_{\mathrm{P}_1} - [0.32+0.71i]_{\mathrm{P}_2,\,Q_{1,2}} + [0.33+0.38i]_{\mathrm{P}_2,\,Q_{3-6,8}} \\
&\quad + \left[\frac{r_{\mathrm{sp}}}{0.434}\right]\left\{[0.13]_{\mathrm{LO}} + [0.14+0.12i]_{\mathrm{HV}} - [0.01-0.05i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}}\right\} \\
&= (-2.12^{+0.48}_{-0.29}) + (-1.56^{+0.29}_{-0.15})i\,,
\end{aligned}$$

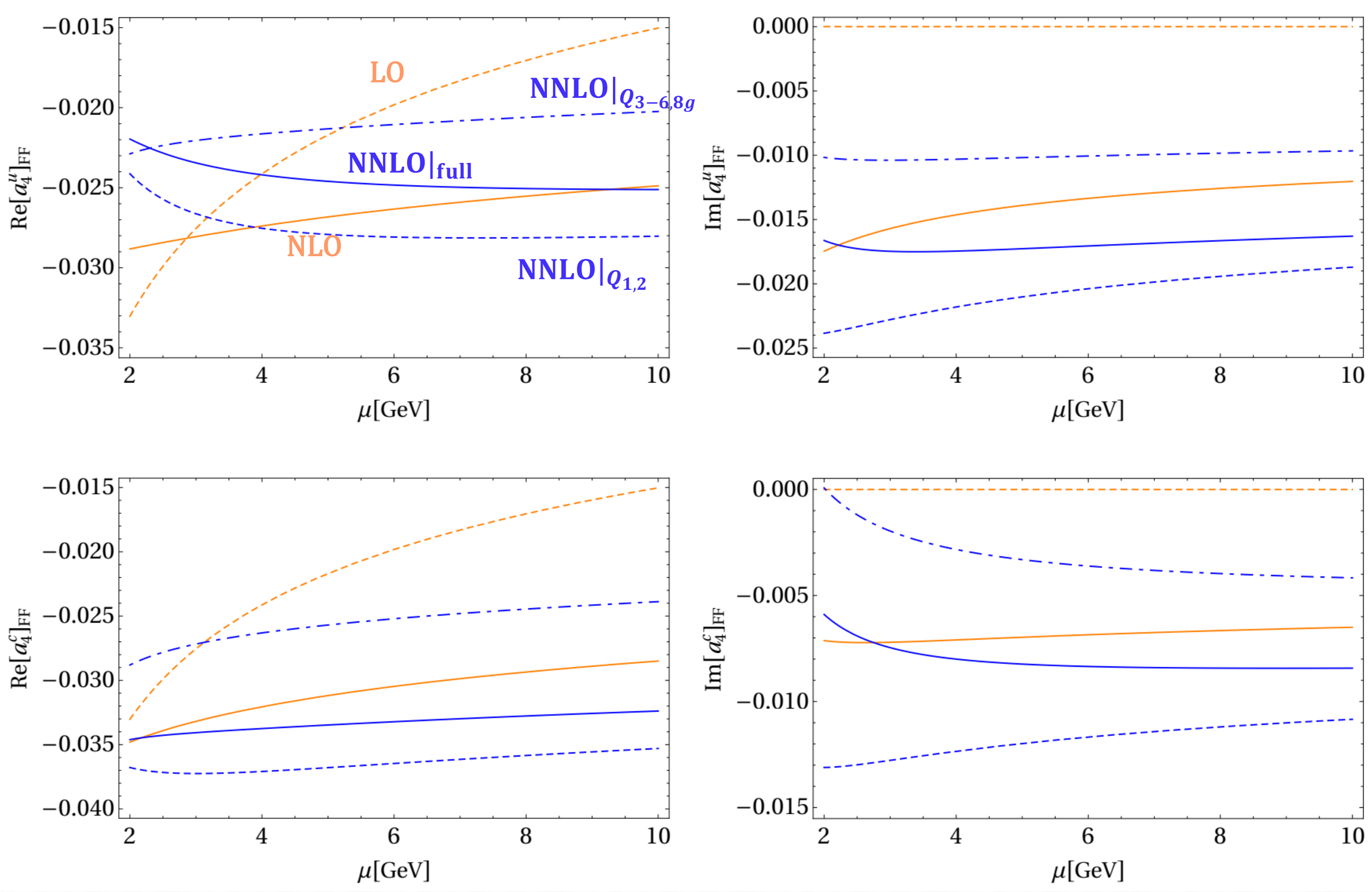


$$\begin{aligned}
a_4^c(\pi\bar{K})/10^{-2} &= -2.87 - [0.09+0.09i]_{\mathrm{V}_1} + [0.05-0.62i]_{\mathrm{P}_1} - [0.77+0.50i]_{\mathrm{P}_2,\,Q_{1,2}} + [0.33+0.38i]_{\mathrm{P}_2,\,Q_{3-6,8}} \\
&\quad + \left[\frac{r_{\mathrm{sp}}}{0.434}\right]\left\{[0.13]_{\mathrm{LO}} + [0.14+0.12i]_{\mathrm{HV}} + [0.01+0.03i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}}\right\} \\
&= (-3.00^{+0.45}_{-0.32}) + (-0.67^{+0.50}_{-0.39})i\,.
\end{aligned}$$

- **individual NNLO contributions from $Q_{1,2}^p$ and $Q_{3-6,8g}$ significant**
- **strong cancellation between NNLO corrections from $Q_{1,2}^p$ and $Q_{3-6,8g}$**

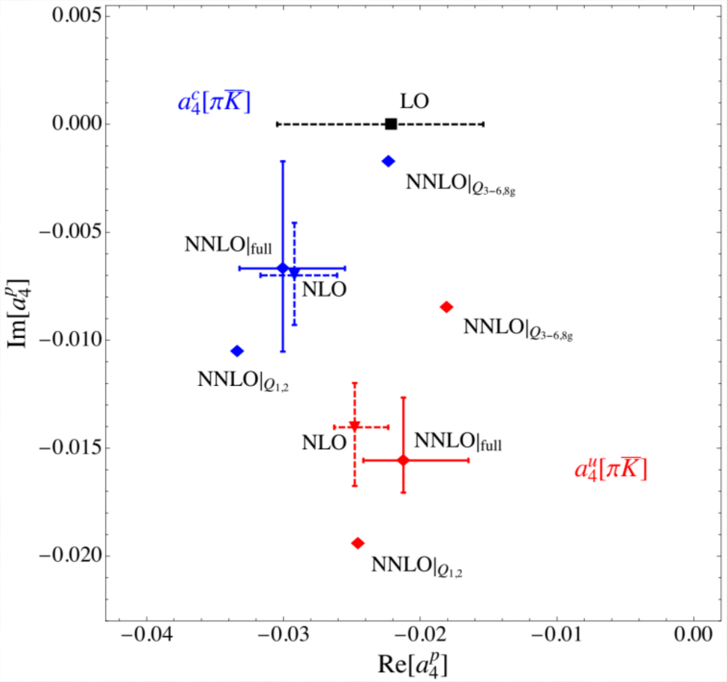
Scale dependence of a_4^p

☐ Scale dependence of a_4^p: only form-factor term



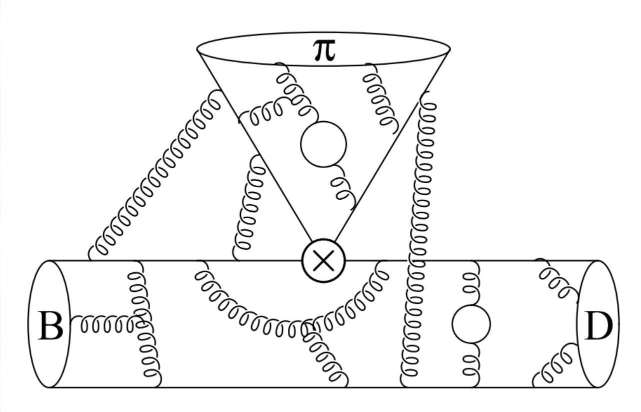
- Scale dependence negligible, especially for μ > 4 GeV.

☐ Results at different orders:



- total NNLO effects small
- uncertainty at NNLO larger than at NLO, due to non-trivial charm mass

$B_q^0 \to D_q^{(*)-} L^+$ class-I decays



- **At quark-level, these decays mediated by $b \to c\bar{u}d(s)$**

  all four flavors different from each other,

  no penguin operators & no penguin topologies!

$$\mathcal{Q}_2 = \bar{d}\gamma_\mu(1-\gamma_5)u \;\; \bar{c}\gamma^\mu(1-\gamma_5)b$$
$$\mathcal{Q}_1 = \bar{d}\gamma_\mu(1-\gamma_5)T^A u \;\; \bar{c}\gamma^\mu(1-\gamma_5)T^A b$$

- **For class-I decays:** QCDF formula much simpler;

  only the form-factor term at leading power

  [Beneke, Buchalla, Neubert, Sachrajda '99-'03; Bauer, Pirjol, Stewart '01]

$$\langle D_q^{(*)+} L^- | \mathcal{Q}_i | \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u) \phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

i) only color-allowed tree topology a_1

ii) spectator & annihilation power-suppressed

iii) annihilation absent in $B_{d(s)}^0 \to D_{d(s)}^- K(\pi)^+$ etc.

iv) they are theoretically simpler and cleaner

→ these decays used to test factorization theorems

- **Hard kernel T:** both NLO and NNLO results known;

  [Beneke, Buchalla, Neubert, Sachrajda '01; Huber, Kränkl, Li '16]

$$T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)$$

Calculation of T^I

❑ Matching QCD onto $\mathrm{SCET_I}$: [Huber, Kränkl, Li '16]

m_c also heavy, must keep m_c/m_b fixed as $m_b \to \infty$,

thus needing two sets of SCET operator basis.

$$\langle \mathcal{Q}_i \rangle = \hat{T}_i \langle \mathcal{Q}^{\rm QCD} \rangle + \hat{T}'_i \langle \mathcal{Q}'^{\rm QCD} \rangle + \sum_{a>1} \left[H_{ia} \langle \mathcal{O}_a \rangle + H'_{ia} \langle \mathcal{O}'_a \rangle \right]$$

physical operators and factorizes into FF*LCDA.

$$\mathcal{O}_1 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \chi \;\; \bar{h}_{v'} \not{n}_+ (1-\gamma_5) h_v \,,$$

$$\mathcal{O}_2 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \chi \;\; \bar{h}_{v'} \not{n}_+ (1-\gamma_5) \gamma_{\perp,\beta} \gamma_{\perp,\alpha} h_v \,,$$

$$\mathcal{O}_3 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma \gamma_\perp^\delta \chi \;\; \bar{h}_{v'} \not{n}_+ (1-\gamma_5) \gamma_{\perp,\delta} \gamma_{\perp,\gamma} \gamma_{\perp,\beta} \gamma_{\perp,\alpha} h_v$$

$$\mathcal{O}'_1 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \chi \;\; \bar{h}_{v'} \not{n}_+ (1+\gamma_5) h_v \,,$$

$$\mathcal{O}'_2 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \chi \;\; \bar{h}_{v'} \not{n}_+ (1+\gamma_5) \gamma_{\perp,\alpha} \gamma_{\perp,\beta} h_v \,,$$

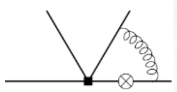
$$\mathcal{O}'_3 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma \gamma_\perp^\delta \chi \;\; \bar{h}_{v'} \not{n}_+ (1+\gamma_5) \gamma_{\perp,\alpha} \gamma_{\perp,\beta} \gamma_{\perp,\gamma} \gamma_{\perp,\delta} h_v$$

evanescent operators and must be renormalized to zero.

❑ Renormalized on-shell QCD amplitudes:

$$\begin{aligned}\langle \mathcal{Q}_i \rangle = \Big\{ & A_{ia}^{(0)} + \frac{\alpha_s}{4\pi} \left[A_{ia}^{(1)} + Z_{ext}^{(1)} A_{ia}^{(0)} + Z_{ij}^{(1)} A_{ja}^{(0)} \right] \\ & + \left(\frac{\alpha_s}{4\pi} \right)^2 \left[A_{ia}^{(2)} + Z_{ij}^{(1)} A_{ja}^{(1)} + Z_{ij}^{(2)} A_{ja}^{(0)} + Z_{ext}^{(1)} A_{ia}^{(1)} + Z_{ext}^{(2)} A_{ia}^{(0)} + Z_{ext}^{(1)} Z_{ij}^{(1)} A_{ja}^{(0)} \right. \\ & \left. + (-i)\delta m_b^{(1)} A_{ia}^{*(1)} + (-i)\delta m_c^{(1)} A_{ia}^{**(1)} + Z_\alpha^{(1)} A_{ia}^{(1)} \right] + \mathcal{O}(\alpha_s^3) \Big\} \langle \mathcal{O}_a \rangle^{(0)} \\ & + (A \leftrightarrow A') \langle \mathcal{O}'_a \rangle^{(0)} \,. \end{aligned}$$

on QCD side



❑ Renormalized on-shell SCET amplitudes:

$$\begin{aligned}\langle \mathcal{O}_a \rangle = \Big\{ & \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi} \left[M_{ab}^{(1)} + Y_{ext}^{(1)} \delta_{ab} + Y_{ab}^{(1)} \right] \\ & + \left(\frac{\hat{\alpha}_s}{4\pi} \right)^2 \left[M_{ab}^{(2)} + Y_{ext}^{(1)} M_{ab}^{(1)} + Y_{ac}^{(1)} M_{cb}^{(1)} + \hat{Z}_\alpha^{(1)} M_{ab}^{(1)} + Y_{ext}^{(2)} \delta_{ab} \right. \\ & \left. + Y_{ext}^{(1)} Y_{ab}^{(1)} + Y_{ab}^{(2)} \right] + \mathcal{O}(\hat{\alpha}_s^3) \Big\} \langle \mathcal{O}_b \rangle^{(0)} \,, \end{aligned}$$

on SCET side

❑ Master formulas for hard kernels:

$$T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)$$

$$\hat{T}_i^{(0)} = A_{i1}^{(0)}$$

$$\hat{T}_i^{(1)} = A_{i1}^{(1)nf} + Z_{ij}^{(1)} A_{j1}^{(0)}$$

$$\begin{aligned}\hat{T}_i^{(2)} = & A_{i1}^{(2)nf} + Z_{ij}^{(1)} A_{j1}^{(1)} + Z_{ij}^{(2)} A_{j1}^{(0)} + Z_\alpha^{(1)} A_{i1}^{(1)nf} - \hat{T}_i^{(1)} \left[C_{FF}^{\rm D(1)} + Y_{11}^{(1)} - Z_{\rm ext}^{(1)} \right] \\ & - C_{FF}^{\rm ND(1)} \hat{T}_i'^{(1)} + (-i)\delta m_b^{(1)} A_{i1}^{*(1)nf} + (-i)\delta m_c^{(1)} A_{i1}^{**(1)nf} - \sum_{b\neq 1} H_{ib}^{(1)} Y_{b1}^{(1)} \,. \end{aligned}$$

Decay amplitudes for $B_q^0 \to D_q^- L^+$

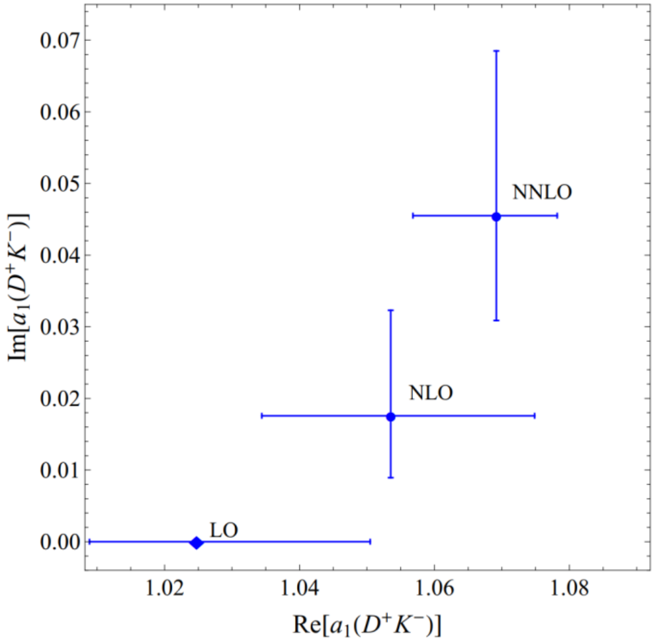
- **Color-allowed tree amplitude a_1: collinear factorization established @ NNLO!**

$$a_1(D^+L^-) = \sum_{i=1}^{2} C_i(\mu) \int_0^1 du \left[\hat{T}_i(u,\mu) + \hat{T}_i'(u,\mu)\right] \Phi_L(u,\mu),$$
$$a_1(D^{*+}L^-) = \sum_{i=1}^{2} C_i(\mu) \int_0^1 du \left[\hat{T}_i(u,\mu) - \hat{T}_i'(u,\mu)\right] \Phi_L(u,\mu),$$

free from the endpoint divergence ⇓ collinear factorization established

- **Numerical result:**

$$a_1(D^+K^-) = 1.025 + [0.029 + 0.018i]_{\rm NLO} + [0.016 + 0.028i]_{\rm NNLO} = (1.069^{+0.009}_{-0.012}) + (0.046^{+0.023}_{-0.015})i\,,$$



- both NLO and NNLO add always constructively to LO result!
- NNLO corrections to real part quite small (2%), but rather large to imaginary part (60%).

- **For different decay modes:** *quasi-universal*, with small process dependence from *different LCDA of light mesons*.

$$a_1(D^+K^-) = (1.069^{+0.009}_{-0.012}) + (0.046^{+0.023}_{-0.015})i\,,$$
$$a_1(D^+\pi^-) = (1.072^{+0.011}_{-0.013}) + (0.043^{+0.022}_{-0.014})i\,,$$
$$a_1(D^{*+}K^-) = (1.068^{+0.010}_{-0.012}) + (0.034^{+0.017}_{-0.011})i\,,$$
$$a_1(D^{*+}\pi^-) = (1.071^{+0.012}_{-0.013}) + (0.032^{+0.016}_{-0.010})i\,.$$

Non-leptonic/semi-leptonic ratios

◻ Non-leptonic/semi-leptonic ratios : [Bjorken '89; Neubert, Stech '97; Beneke, Buchalla, Neubert, Sachrajda '01]

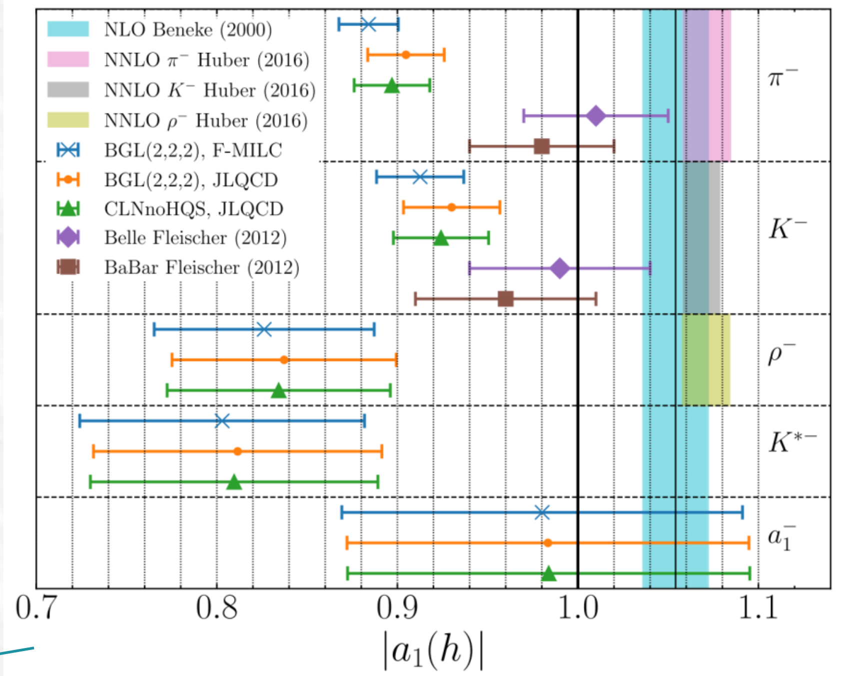
$$R^{(*)}_{(s)L} \equiv \frac{\Gamma(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} L^-)}{d\Gamma(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} \ell^- \bar{\nu}_\ell)/dq^2 \,|_{q^2=m_L^2}} = 6\pi^2 \,|V_{uq}|^2 f_L^2 \,|a_1(D^{(*)+}_{(s)} L^-)|^2 X_L^{(*)}$$

free from uncertainties from V_{cb} & $B_{d,s} \to D^{(*)}_{d,s}$ form factors

◻ Updated predictions vs data: [Huber, Kränkl, Li '16; Cai, Deng, Li, Yang '21]

$R^{(*)}_{(s)L}$	LO	NLO	NNLO	Exp.	Deviation (σ)
R_π	1.01	$1.07^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.74 ± 0.06	5.4
R^*_π	1.00	$1.06^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.80 ± 0.06	4.5
R_ρ	2.77	$2.94^{+0.19}_{-0.19}$	$3.02^{+0.17}_{-0.18}$	2.23 ± 0.37	1.9
R_K	0.78	$0.83^{+0.03}_{-0.03}$	$0.85^{+0.01}_{-0.02}$	0.62 ± 0.05	4.4
R^*_K	0.72	$0.76^{+0.03}_{-0.03}$	$0.79^{+0.01}_{-0.02}$	0.60 ± 0.14	1.3
R_{K^*}	1.41	$1.50^{+0.11}_{-0.11}$	$1.53^{+0.10}_{-0.10}$	1.38 ± 0.25	0.6
$R_{s\pi}$	1.01	$1.07^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.72 ± 0.08	4.4
R_{sK}	0.78	$0.83^{+0.03}_{-0.03}$	$0.85^{+0.01}_{-0.02}$	0.46 ± 0.06	6.3

◻ Latest Belle data: 2207.00134



$|a_1(\bar{B} \to D^{*+}\pi^-)| = 0.884 \pm 0.004 \pm 0.003 \pm 0.016\ [1.071^{+0.020}_{-0.016}]$;

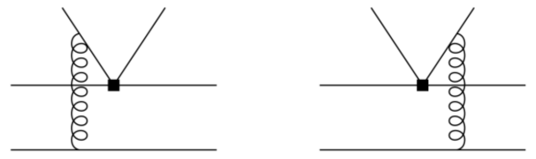
15% lower than SM

$|a_1(\bar{B} \to D^{*+}K^-)| = 0.913 \pm 0.019 \pm 0.008 \pm 0.013\ [1.069^{+0.020}_{-0.016}]$;

Power corrections

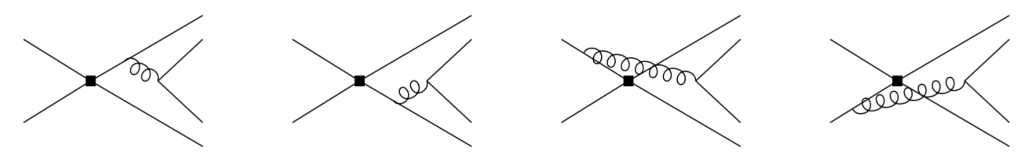
$$\langle D_q^{(*)+} L^- | \mathcal{Q}_i | \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u) \phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

- **Sources of sub-leading power corrections:** [Beneke, Buchalla, Neubert, Sachrajda '01; Bordone, Gubernari, Huber, Jung, van Dyk '20]
  - non-factorizable spectator interactions

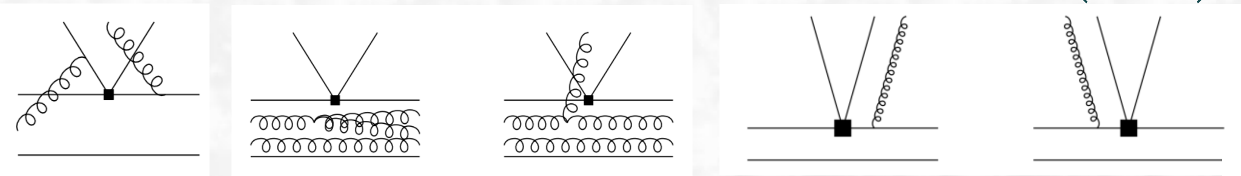


$\frac{\Lambda_{\rm QCD}}{m_b}$

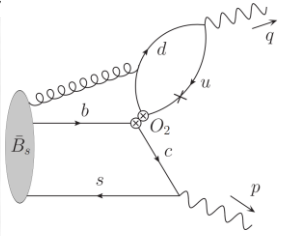
  - annihilation topologies



  - non-leading higher Fock-state contributions $\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2$



  - non-factorizable soft-gluon contributions in LCSR with B-meson LCDA: [Maria Laura Piscopo, Aleksey V. Rusov '23]



$$\frac{C_2 \langle O_2^d \rangle}{C_1 \langle O_1^d \rangle} = 0.051^{+0.059}_{-0.052}\,, \qquad \bar{B}_s^0 \to D_s^+ \pi^-\,,$$

$$\frac{C_2 \langle O_2^s \rangle}{C_1 \langle O_1^s \rangle} = 0.039^{+0.042}_{-0.034}\,, \qquad \bar{B}^0 \to D^+ K^-\,.$$

- **Scaling of the leading-power contribution:** [BBNS '01]

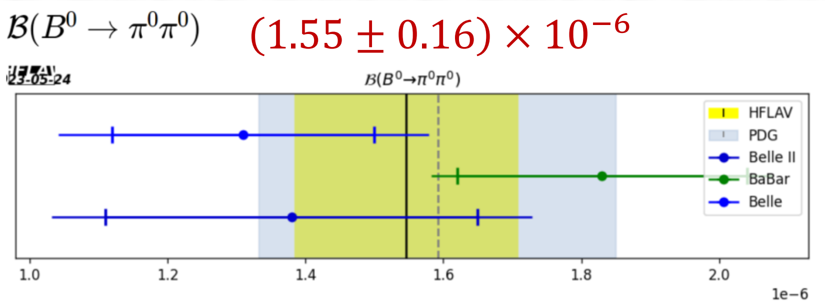
$$\mathcal{A}(\bar{B}_d \to D^+ \pi^-) \sim G_F m_b^2\, F^{B \to D}(0)\, f_\pi \sim G_F m_b^2\, \Lambda_{\rm QCD}$$

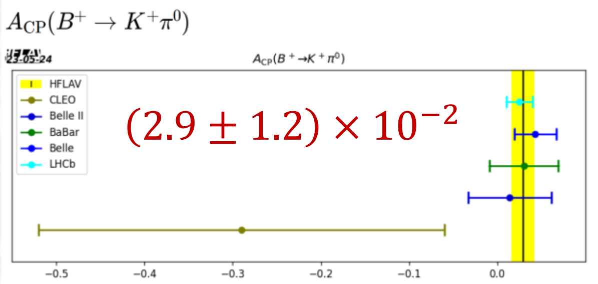
- All ESTIMATED to be power-suppressed; not even chirality-enhanced due to (V-A)(V-A)
- Difficult to explain why measured values of $|a_1(h)|$ several σ smaller than SM?
- *Must consider possible sub-leading power corrections carefully!*

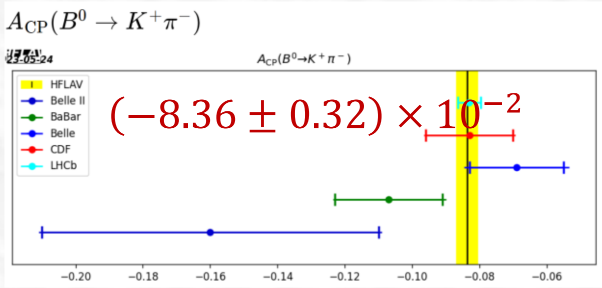
Charmless two-body hadronic B decays

- Long-standing puzzles in $\mathrm{Br}(\bar{B}^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K) = A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-)$: [HFLAV '23]

$\mathrm{Br}(B^0 \to \pi^0\pi^0) = (0.3 - 0.9) \times 10^{-6}$ $\Delta A_{CP}(\pi K) = (11.3 \pm 1.2)\%$ **differs from 0 by ~9σ**







- Decay amplitudes in QCDF:

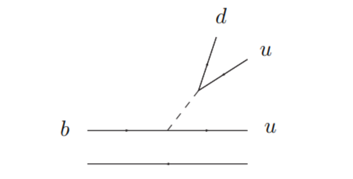
$$-\mathcal{A}_{\bar{B}^0\to\pi^0\pi^0} = A_{\pi\pi}\left[\delta_{pu}(\alpha_2 - \beta_1) - \hat{\alpha}_4^p - 2\beta_4^p\right]$$

$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} = A_{\pi\bar{K}}\left[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\right] + A_{\bar{K}\pi}\left[\delta_{pu}\alpha_2 + \delta_{pc}\tfrac{3}{2}\alpha_{3,\mathrm{EW}}^c\right],$$
$$\mathcal{A}_{\bar{B}^0\to\pi^+K^-} = A_{\pi\bar{K}}\left[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\right],$$

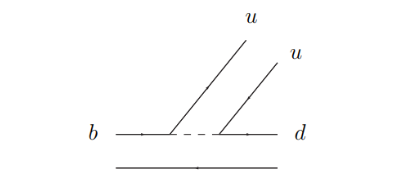
$$A_{\mathrm{CP}}(\pi^0K^\pm) - A_{\mathrm{CP}}(\pi^\mp K^\pm) = -2\sin\gamma\left(\mathrm{Im}(r_C) - \mathrm{Im}(r_T\, r_{\mathrm{EW}})\right) + \ldots$$

α_2 **always plays a key role here!**

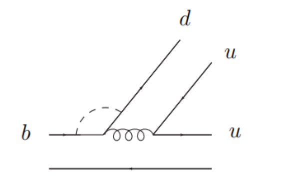
- Dominant topologies: **LP NNLO known**



colour-allowed tree α_1



colour-suppressed tree α_2



QCD penguins α_4

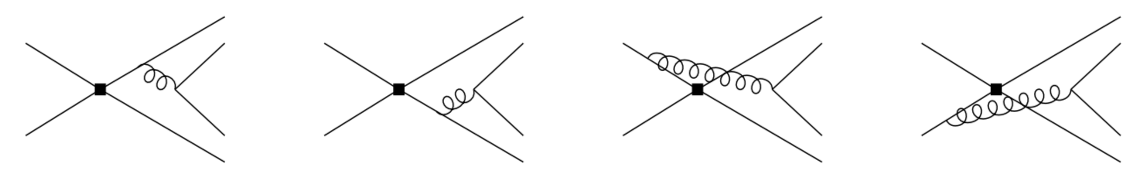
Find some mechanism (sub-leading power corrections) to enhance α_2, and hence explain both puzzles!

Pure annihilation charmless decays

- **Pure annihilation modes:** [HFLAV '23]

$$\mathrm{Br}(\bar{B}_s^0 \to \pi^+\pi^-) = (7.2 \pm 1.1) \times 10^{-7}$$

$$\mathrm{Br}(\bar{B}_d^0 \to K^+K^-) = (8.0 \pm 1.5) \times 10^{-8}$$



- **With universal X_A and different scenarios, we have:** [BBNS '03]

$$X_A = (1 + \varrho_A e^{i\varphi_A}) \ln(m_B / \Lambda_h)$$

Mode	Theory	S1 (large γ)	S2 (large a_2)	S3 ($\varphi_A = -45^\circ$)	S4 ($\varphi_A = -55^\circ$)	Exp.
$\bar{B}_s^0 \to \pi^+\pi^-$	$0.024^{+0.003+0.025+0.000+0.163}_{-0.003-0.012-0.000-0.021}$	0.027	0.032	0.149	0.155	0.72 ± 0.11
$\bar{B}^0 \to K^-K^+$	$0.013^{+0.005+0.008+0.000+0.087}_{-0.005-0.005-0.000-0.011}$	0.007	0.014	0.079	0.070	0.080 ± 0.015

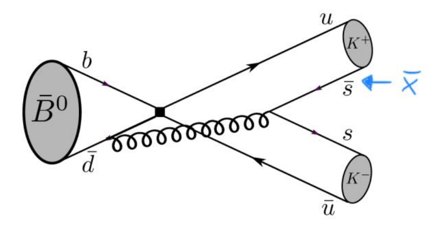
large SU(3)-flavor symmetry breaking or flavor-dependent $A_{1,2}^i$?

- **How to improve the situation?**
  - including higher Gegenbauer moments to include SU(3)-breaking effects;

$$\Phi_M(x,\mu) = 6x\bar{x}\left[1 + \sum_{n=1}^{\infty} a_n^M(\mu)\, C_n^{(3/2)}(2x-1)\right]$$

  - making the parameter X_A to be flavour dependent & depending on its origins;

    [Wang, Zhu '03; Bobeth *et al.* '14; Chang, Sun *et al.* '14-15]

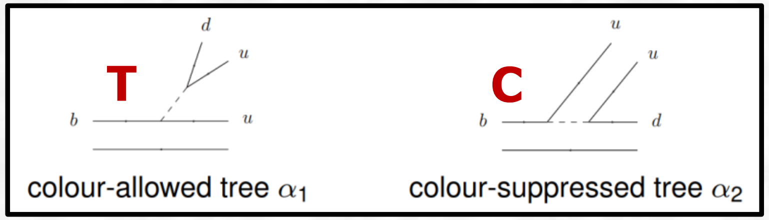


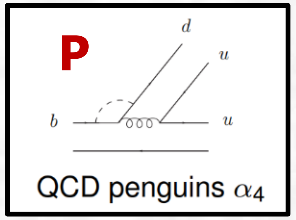
  - other interesting progress;

    Lu, Shen, Wang, Wang, Wang 2202.08073; Boer talk @ SCET2023 and CERN; Neubert talk @ Neutrinos, Flavour and Beyond 2022

Summary

- **With exp. and theor. progress, we are entering a precision era for flavour physics!**





- **Within QCDF/SCET framework, NNLO QCD corrections to color-allowed, color-suppressed tree & leading-power penguin amplitudes complete, factorization at 2-loop established**

- **Due to delicate cancellation, NNLO corrections found small; some puzzles still remain:**

  - long-standing $\mathrm{Br}(\bar{B}^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K) = A_{CP}(B^- \to \pi^0 K^-) - A_{CP}(\bar{B}^0 \to \pi^+ K^-)$;
  - for class-I $B_q^0 \to D_q^{(*)-} L^+$ decays, $\mathcal{O}(4\text{–}5\sigma)$ discrepancies observed in branching ratios;

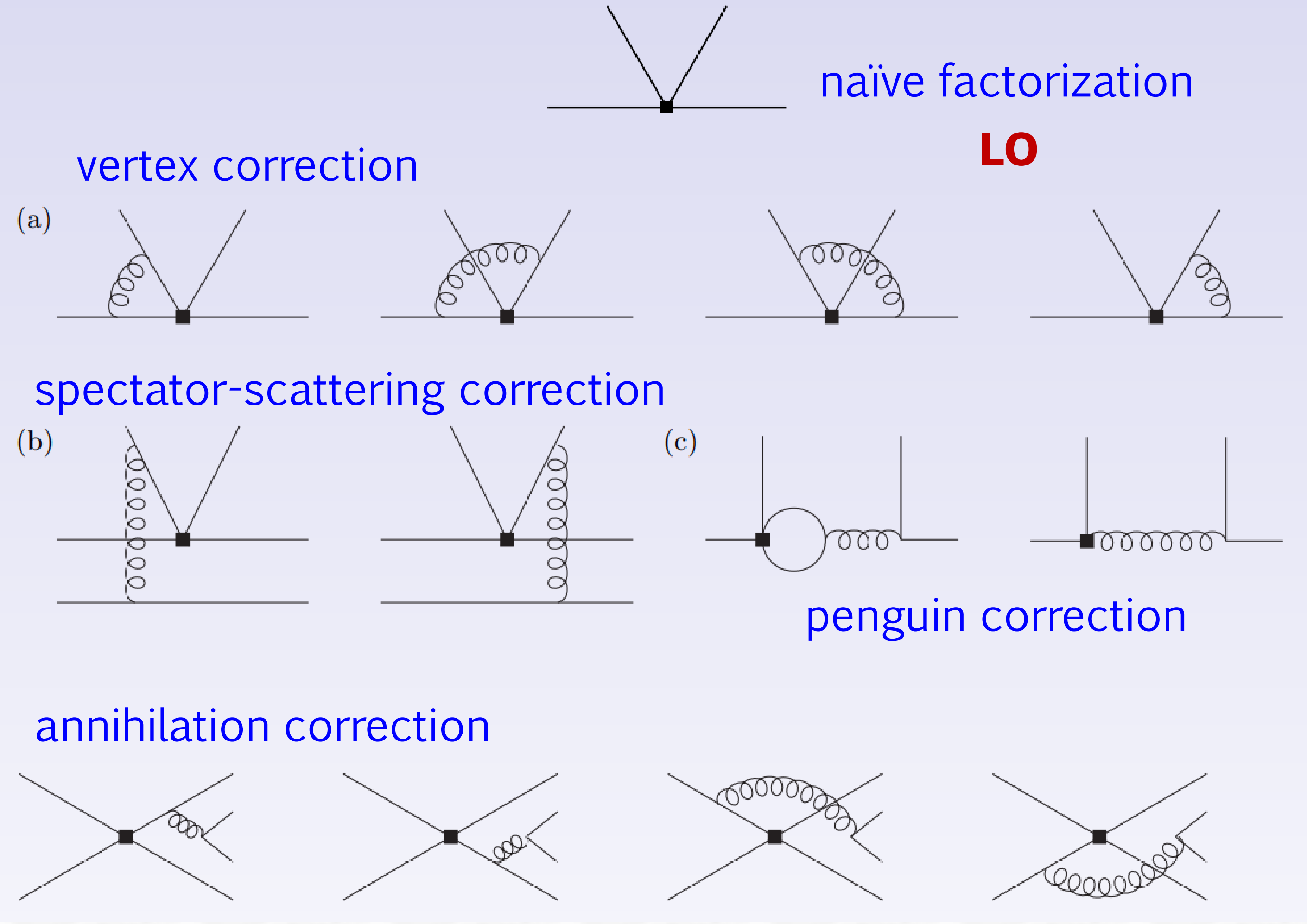
➡ **sub-leading power corrections in QCDF/SCET need to be considered!**

  - sub-leading color-octet matrix elements $\langle M_1 M_2 | [\bar{u}_c T^A h_v]_{\Gamma_1} [\bar{s}_{\bar{c}} T^A u_{\bar{c}}]_{\Gamma_2}(u) | \bar{B} \rangle$ [w.i.p]
  - improved treatments of annihilation amplitudes: SU(3)-breaking effects [w.i.p]

backup

Phenomenological analyses based on NLO

- **Various analyses based on NLO hard kernels**



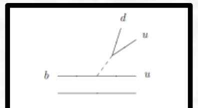
- **complete sets of final states:**

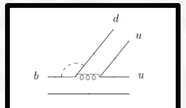
  - $B \to PP, PV$: [Beneke, Neubert, hep-ph/0308039; Cheng, Chua, 0909.5229, 0910.5237;]
  - $B \to VV$: [Beneke, Rohrer, Yang, hep-ph/0612290; Cheng, Yang, 0805.0329; Cheng, Chua, 0909.5229, 0910.5237;]
  - $B \to AP, AV, AA$: [Cheng, Yang, 0709.0137, 0805.0329;]
  - $B \to SP, SV$: [Cheng, Chua, Yang, hep-ph/0508104, 0705.3079; Cheng, Chua, Yang, Zhang, 1303.4403;]
  - $B \to TP, TV$: [Cheng, Yang, 1010.3309;]

very successful but also with some problems phenomenologically. ！

Phenomenological successes based on NLO

◻ Successes at NLO:


colour-allowed tree α_1


QCD penguins α_4

- For color-allowed tree- & penguin-dominated decay modes, branching ratios usually quantitatively OK
- Dynamical explanation of intricate patterns of penguin interference seen in PP, PV, VP and VV modes

$$PP \sim a_4 + r_\chi a_6, \quad PV \sim a_4 \approx \frac{PP}{3}$$
$$VP \sim a_4 - r_\chi a_6 \sim -PV$$
$$VV \sim a_4 \sim PV$$

$$r_\chi = \frac{2m_L^2}{m_b\,(m_q + m_s)}$$

⇒ $\mathrm{Br}\left(B^{\pm,0} \to \eta^{(\prime)} K^{(*)\pm,0}\right)$

- Qualitative explanation of polarization puzzle in $B \to VV$ decays, due to the large weak annihilation
- Strong phases start at $\mathcal{O}(\alpha_s)$, dynamical explanation of smallness of direct CP asymmetries

◻ Some problems encountered at NLO:

- Factorization of power corrections generally broken, due to endpoint divergence
- Could not account for some data, such as $\mathrm{Br}(B^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K)$
- How important the higher-order pert. corr.? Fact. theorem is still established for them?
- As strong phases start at $\mathcal{O}(\alpha_s)$, NNLO is only NLO to them; quite relevant for A_{CP}?

we need go beyond the LO in pert. and power corrections!

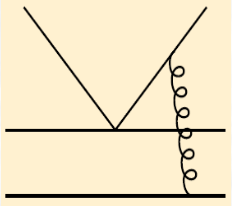
Power-suppressed color-octet contribution

- **Sub-leading power corrections to a_2: spectator scattering or final-state re-scatterings**
- **Every four-quark operator in $H_{\rm eff}$ has a color-octet piece in QCD:**

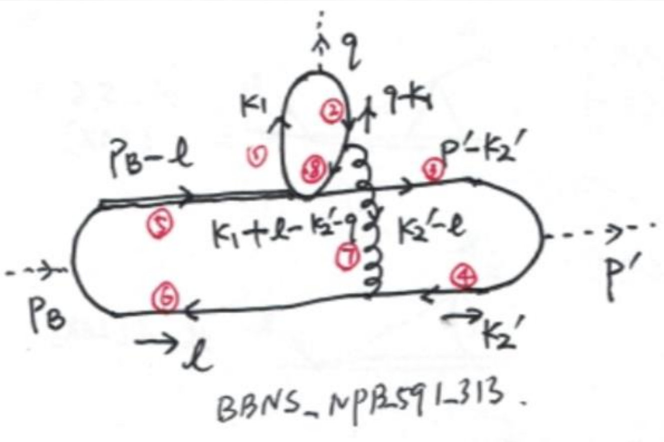
$$t^a_{ik}t^a_{jl} = \frac{1}{2}\delta_{il}\delta_{jk} - \frac{1}{2N_c}\delta_{ik}\delta_{jl},$$

$$Q_1 = (\bar{u}_i b_i)_{V-A} \otimes (\bar{s}_j u_j)_{V-A} = \frac{1}{N_c}(\bar{s}_i b_i)_{V-A} \otimes (\bar{u}_j u_j)_{V-A} + 2(\bar{s}T^A b)_{V-A} \otimes (\bar{u}T^A u)_{V-A}$$

$$Q_2 = (\bar{u}_i b_j)_{V-A} \otimes (\bar{s}_j u_i)_{V-A} = \frac{1}{N_c}(\bar{u}_i b_i)_{V-A} \otimes (\bar{s}_j u_j)_{V-A} + 2(\bar{u}T^A b)_{V-A} \otimes (\bar{s}T^A u)_{V-A}$$



- **Soft-gluon contributions with color-octet operator insertions:**



method of regions: 6 regions

- The gluon propagator can be in the hard-collinear region
  - ➡ hard-spectator scattering contribution
- Can also be in the soft region; expected to be $\mathcal{O}(1/m_b)$
  - ➡ can be non-zero at sub-leading power, numerically relevant
- Other four regions suppressed by more powers of $1/m_b$

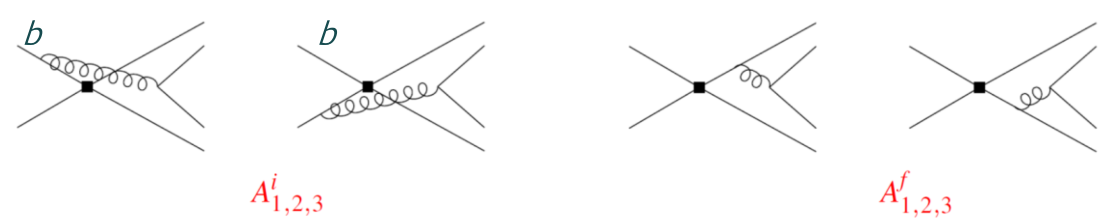
Pure annihilation B decays

- **Two typical pure annihilation decay modes:** $\bar{B}_s^0 \to \pi^+\pi^-$ vs $\bar{B}_d^0 \to K^+K^-$ related by SU(3)

$$\mathcal{A}(\bar{B}_s \to \pi^+\pi^-) = B_{\pi\pi}\left[\delta_{pu} b_1 + 2b_4^p + \frac{1}{2} b_{4,\mathrm{EW}}^p\right]$$

$$\mathcal{A}(\bar{B}_d \to K^+K^-) = A_{\bar{K}K}\left[\delta_{pu}\beta_1 + \beta_4^p + b_{4,\mathrm{EW}}^p\right] + B_{K\bar{K}}\left[b_4^p - \frac{1}{2} b_{4,\mathrm{EW}}^p\right]$$

$$= A_{\bar{K}K}\left[\delta_{pu}\beta_1 + \beta_4^p\right] + B_{K\bar{K}}\left[b_4^p\right]$$



- **Both involve $b_1 = \frac{C_F}{N_c^2} C_1 A_1^i$ & $b_4^p = \frac{C_F}{N_c^2}[C_4 A_1^i + C_6 A_2^i]$ and kernels A_1^i & A_2^i :**

$A_1^i: (V-A)\otimes(V-A)$

$A_2^i: (V-A)\otimes(V+A)$

$$A_1^i(M_1M_2) = \pi\alpha_s \int_0^1 dxdy \left\{\Phi_{M_2}(x)\,\Phi_{M_1}(y)\left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y}\right] + r_\chi^{M_1} r_\chi^{M_2}\,\Phi_{m_2}(x)\,\Phi_{m_1}(y)\,\frac{2}{\bar{x}y}\right\},$$

$$A_2^i(M_1M_2) = \pi\alpha_s \int_0^1 dxdy \left\{\Phi_{M_2}(x)\,\Phi_{M_1}(y)\left[\frac{1}{\bar{x}(1-x\bar{y})} + \frac{1}{\bar{x}y^2}\right] + r_\chi^{M_1} r_\chi^{M_2}\,\Phi_{m_2}(x)\,\Phi_{m_1}(y)\,\frac{2}{\bar{x}y}\right\},$$

- **With the asymptotic LCDAs $\Phi_M(x) = 6x\bar{x}$, we have $A_1^i = A_2^i$:**

[BBNS '99-'03]

$$A_1^i(M_1M_2) = \pi\alpha_s\left\{18X_A - 18 - 6(9-\pi^2) + r_\chi^{M_1} r_\chi^{M_2}\left(2X_A^2\right)\right\},$$

$$A_2^i(M_1M_2) = \pi\alpha_s\left\{18X_A - 18 - 6(9-\pi^2) + r_\chi^{M_1} r_\chi^{M_2}\left(2X_A^2\right)\right\},$$

$$X_A = \left(1 + \varrho_A e^{i\varphi_A}\right)\ln\left(m_B/\Lambda_h\right),$$

$\Lambda_h = 0.5\mathrm{GeV}$, $\varrho_A \le 1$ and an arbitrary phase φ_A

Ways to improve the modelling of annihilations

- **With universal X_A and different scenarios, we have:** [BBNS '03]

Mode	Theory	S1 (large γ)	S2 (large a_2)	S3 ($\varphi_A=-45^\circ$)	S4 ($\varphi_A=-55^\circ$)	Exp.
$\bar{B}_s^0 \to \pi^+\pi^-$	$0.024^{+0.003+0.025+0.000+0.163}_{-0.003-0.012-0.000-0.021}$	0.027	0.032	0.149	0.155	0.72 ± 0.11
$\bar{B}^0 \to K^-K^+$	$0.013^{+0.005+0.008+0.000+0.087}_{-0.005-0.005-0.000-0.011}$	0.007	0.014	0.079	0.070	0.080 ± 0.015

Large SU(3)-flavor symmetry breaking or flavor-dependent $A_{1,2}^i$? [Wang, Zhu '03; Bobeth *et al.* '14; Chang, Sun *et al.* '14-15]

- **How to improve the situation:**

  - including higher Gegenbauer moments to include SU(3)-breaking effects;

$$\Phi_M(x,\mu) = 6x\bar{x}\left[1+\sum_{n=1}^{\infty} a_n^M(\mu)\, C_n^{(3/2)}(2x-1)\right]$$

due to G-parity, $a_{odd}^{\pi}=0$, but $a_{odd}^{K}\neq 0$

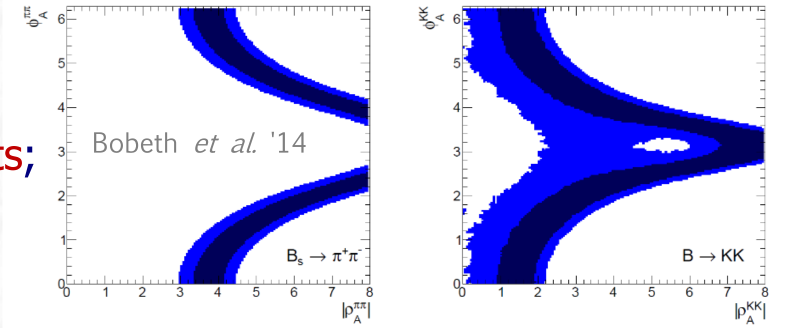


FIGURE 5.8: 68% and 95% CRs for the complex parameter $\rho_A^{\pi^+\pi^-}$ and $\rho_A^{K^+K^-}$ obtained from a branching-ratio fit assuming the SM.

$$X_A = \left(1+\varrho_A e^{i\varphi_A}\right)\ln\left(m_B/\Lambda_h\right)$$

  - including the difference between the chirality factors to include SU(3)-breaking effects;

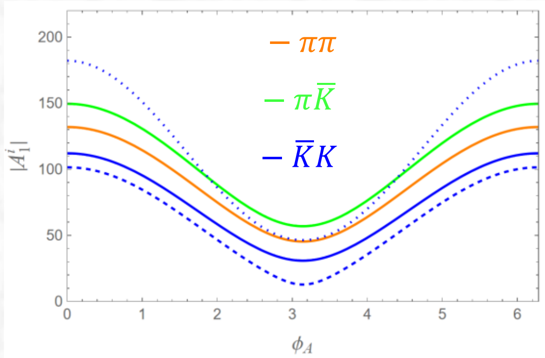
$$r_\chi^\pi(1.5\text{GeV}) = \frac{2m_\pi^2}{m_b(\mu)\left(m_u(\mu)+m_d(\mu)\right)} \simeq 0.86, \qquad r_\chi^K(1.5\text{GeV}) = \frac{2m_K^2}{m_b(\mu)\left(m_u(\mu)+m_s(\mu)\right)} \simeq 0.91$$

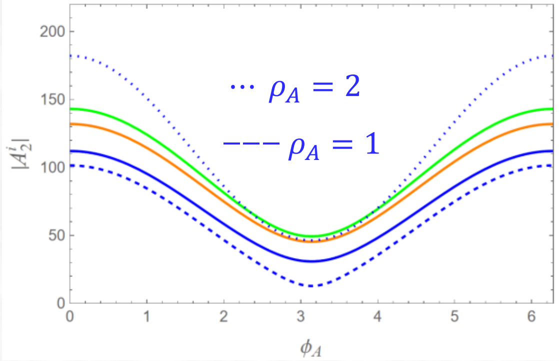
Ways to improve the modelling of annihilations

□ SU(3)-breaking effects in $A_{1,2}^i$: due to higher Gengengauber moments and quark masses

$$A_1^i(M_1M_2) = \pi\alpha_s \Big\{18(1 - a_1^{M_1} + a_2^{M_1})\Big[(1+3a_1^{M_2}+6a_2^{M_2})X_A - (1+6a_1^{M_2}+16a_2^{M_2})\Big] + 54(69-7\pi^2)a_1^{M_1}a_1^{M_2} - 36(385-39\pi^2)(a_1^{M_1}a_2^{M_2} - 2a_2^{M_1}a_1^{M_2}) - 6(9-\pi^2) - 18(10-\pi^2)(3a_1^{M_1} - a_1^{M_2}) - 6(59-6\pi^2)(6a_2^{M_1}+a_2^{M_2}) - 18(9593-972\pi^2)a_2^{M_1}a_2^{M_2} + r_\chi^{M_1}r_\chi^{M_2}\left(2X_A^2\right)\Big\},$$

$$A_2^i(M_1M_2) = \pi\alpha_s \Big\{18(1 + a_1^{M_2} + a_2^{M_2})\Big[(1-3a_1^{M_1}+6a_2^{M_1})X_A - (1-6a_1^{M_1}+16a_2^{M_1})\Big] - 6(9-\pi^2) - 18(10-\pi^2)(a_1^{M_1} - 3a_1^{M_2}) - 6(59-6\pi^2)(a_2^{M_1}+6a_2^{M_2}) + 54(69-7\pi^2)a_1^{M_1}a_1^{M_2} - 36(385-39\pi^2)(2a_1^{M_1}a_2^{M_2} - a_2^{M_1}a_1^{M_2}) - 18(9593-972\pi^2)a_2^{M_1}a_2^{M_2} + r_\chi^{M_1}r_\chi^{M_2}\left(2X_A^2\right)\Big\},$$





	$\pi\pi$	$\pi\bar{K}$	$\bar{K}K$
A_1^i	$31.7X_A - 51.5{+}6.2 + 1.5X_A^2$	$37.6X_A - 63.4{+}6.5 + 1.6X_A^2$	$23.4X_A - 36.0{+}5.2 + 1.7X_A^2$
	$[18X_A - 18{+}5.2 + 1.5X_A^2]$	$[18X_A - 18{+}5.2 + 1.6X_A^2]$	$[18X_A - 18{+}5.2 + 1.7X_A^2]$
A_2^i	$31.7X_A - 51.5{+}6.2 + 1.5X_A^2$	$34.6X_A - 56.2{+}6.9 + 1.6X_A^2$	$23.4X_A - 36.0{+}5.2 + 1.7X_A^2$
	$[18X_A - 18{+}5.2 + 1.5X_A^2]$	$[18X_A - 18{+}5.2 + 1.6X_A^2]$	$[18X_A - 18{+}5.2 + 1.7X_A^2]$

$$X_A = \ln\left(\frac{m_B}{\Lambda_h}\right)(1+\rho_A e^{i\phi_A})$$

$Br(\bar{B}_s^0 \to \pi^+\pi^-)$: $(0.72\pm0.11)\times10^{-6}$
$Br(\bar{B}^0 \to K^-K^+)$: $(0.080\pm0.015)\times10^{-6}$

- $|A_{1,2}^i|$ can differ by more than **20%** in the **BBNS+ model**!
- The amplitude ratios $A_{1,2}^i(\pi\pi)/A_{1,2}^i(KK)$ get **enhanced** in the **BBNS+ model**! ➡ what we need!

Ways to improve the modelling of annihilations

◻ How to improve:

➢ Making the parameter X_A to be flavour dependent & depending on its origins;

$$\int_0^1 dy\,\frac{\Phi_{M_1}(y)}{y^2} = \Phi'_{M_1}(0)\int_0^1 dy\,\frac{1}{y} + \int_0^1 dy\,\frac{\Phi_{M_1}(y) - y\,\Phi'_{M_1}(0)}{y^2} \longrightarrow 6X_0^{M_1} - 6\,,$$

$$\int_0^1 dx\,\frac{\Phi_{M_2}(x)}{\bar{x}^2} = \Phi'_{M_2}(1)\int_0^1 dx\,\frac{1}{\bar{x}} + \int_0^1 dx\,\frac{\Phi_{M_2}(x) - \bar{x}\,\Phi'_{M_2}(1)}{\bar{x}^2} \longrightarrow 6X_1^{M_2} - 6\,,$$

$$\int_0^1 dy\,\frac{\Phi_{m_1}(y)}{y} = \Phi_{m_1}(0)\int_0^1 dy\,\frac{1}{y} + \int_0^1 dy\,\frac{\Phi_{m_1}(y) - \Phi_{m_1}(0)}{y} \longrightarrow X_0^{m_1}\,,$$

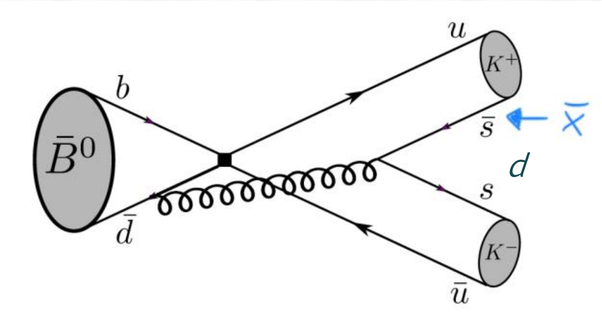
$$\int_0^1 dx\,\frac{\Phi_{m_2}(x)}{\bar{x}} = \Phi_{m_2}(1)\int_0^1 dx\,\frac{1}{\bar{x}} + \int_0^1 dx\,\frac{\Phi_{m_2}(x) - \Phi_{m_2}(1)}{\bar{x}} \longrightarrow X_1^{m_2}\,,$$

$$A_1^i(M_1M_2) = \pi\alpha_s\left\{18X_1^{M_2} - 18 - 6(9-\pi^2) + r_\chi^{M_1} r_\chi^{M_2}\left(2X_0^{m_1}X_1^{m_2}\right)\right\},$$

$$A_2^i(M_1M_2) = \pi\alpha_s\left\{18X_0^{M_1} - 18 - 6(9-\pi^2) + r_\chi^{M_1} r_\chi^{M_2}\left(2X_0^{m_1}X_1^{m_2}\right)\right\},$$

$$\Rightarrow \boxed{A_1^i(M_1M_2) \neq A_2^i(M_1M_2)}$$

➢ To make it predictive, distinguish whether the endpoint configuration mediated by a soft strange quark (X_A^s) or a soft up or down quark (X_A^{ud}) .



◻ Advantages compared to original BBNS: two free parameters!

➢ For $\pi\pi$ final states, only X_A^{ud} involved;

↳ **easily to reproduce the data!**

➢ For KK final states, both X_A^{ud} (for $M_1M_2 = K^+K^-$) and X_A^s (for $M_1M_2 = K^-K^+$) involved;

◻ Other interesting progress:

Lu, Shen, Wang, Wang, Wang 2202.08073; Boer talk @ SCET2023 and CERN;

Neubert talk @ Neutrinos, Flavour and Beyond 2022