

Deciphering the Long-distance Penguin contribution to $B \rightarrow \gamma\gamma$ decays

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QQ, Y.-L. Shen, C. Wang, Y.-M. Wang, *Phys.Rev.Lett.* 131(2023)091902



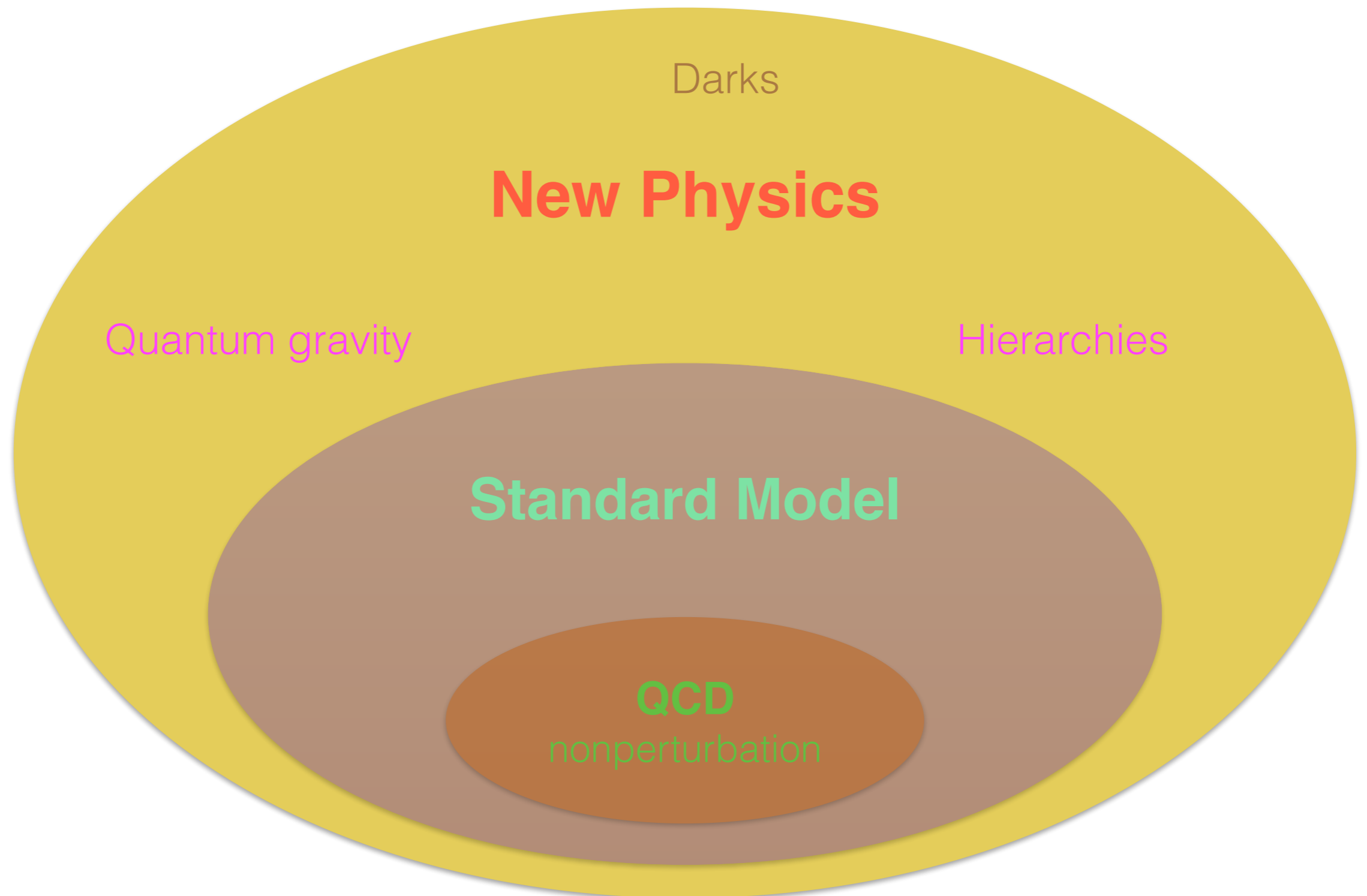
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Contents

- Why $\bar{B} \rightarrow \gamma\gamma$?
- History of (1) $\bar{B} \rightarrow \gamma\gamma$ and (2) the long-distance penguin contribution
- Factorization of the long-distance penguin contribution
 - — a novel B-meson distribution amplitude
- Numerics
- Summary and Prospects

B decays are important!



Why $\bar{B} \rightarrow \gamma\gamma$?

- **FCNC:** Sensitive to dynamics beyond the SM, e.g. CP violation
- **Simplest** decay (as $B \rightarrow \mu^+ \mu^-$): to address the intricate strong interaction mechanism of the heavy-meson systems

— — structure of the B meson

Belle II

Physics Book

| Process | Observable | Theory | Sys. limit (Discovery) [ab ⁻¹] | vs LHCb | vs Belle | Anomaly | NP |
|--|---------------|--------|--|---------|----------|---------|-----|
| ● $B \rightarrow X_s l^+ l^-$ | R_{X_s} | *** | >50 | *** | *** | ** | *** |
| ● $B \rightarrow K^{(*)} e^+ e^-$ | $R(K^{(*)})$ | *** | >50 | ** | *** | *** | *** |
| ● $B \rightarrow X_s \gamma$ | $Br.$ | ** | 1-5 | *** | * | * | ** |
| ● $B_{d,(s)} \rightarrow \gamma\gamma$ | $Br., A_{CP}$ | ** | > 50(5) | ** | ** | - | ** |
| ● $B \rightarrow K^* e^+ e^-$ | P'_5 | ** | >50 | *** | ** | *** | *** |
| ● $B \rightarrow K \tau l$ | $Br.$ | *** | >50 | ** | *** | ** | *** |

Why $\bar{B} \rightarrow \gamma\gamma$?

- Sensitive to dynamics beyond the SM (FCNC), e.g. CP violation
- **Simplest** decay (as $B \rightarrow \mu^+ \mu^-$) to address the intricate strong interaction mechanism of the heavy-meson systems

— — structure of the B meson

Belle II Physics Book

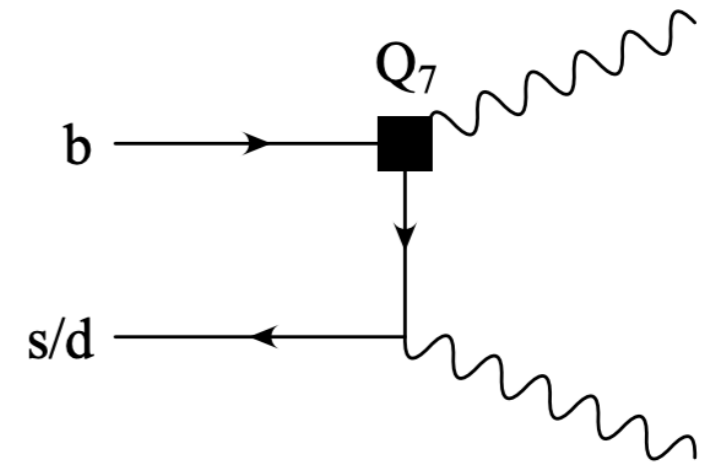
| Observables | Belle 0.71 ab ⁻¹ (0.12 ab ⁻¹) | Belle II 5 ab ⁻¹ | Belle II 50 ab ⁻¹ |
|--|--|-----------------------------|------------------------------|
| Br($B_d \rightarrow \gamma\gamma$) | < 740% | 30% | 9.6% |
| $A_{CP}(B_d \rightarrow \gamma\gamma)$ | — | 78% | 25% |
| Br($B_s \rightarrow \gamma\gamma$) | < 250% | 23% | — |

$$\mathcal{BR}(B_d \rightarrow \gamma\gamma) = (1.352_{-0.745}^{+1.242}) \times 10^{-8}, \quad \mathcal{BR}(B_s \rightarrow \gamma\gamma) = (2.964_{-1.614}^{+1.800}) \times 10^{-7}$$

[Y.-L. Shen, Y.-M. Wang, Y.-B. Wei, 2009.02723]

History of $\bar{B} \rightarrow \gamma\gamma$

- LO + NLO



$$\bar{A}(\bar{B}_q \rightarrow \gamma\gamma) = -\frac{4 G_F}{\sqrt{2}} \frac{\alpha_{\text{em}}}{4\pi} \epsilon^{*\alpha}(p) \epsilon^{*\beta}(q) \times \sum_{p=u,c} V_{pb} V_{pq}^* \sum_{i=1}^8 C_i T_{i,\alpha\beta}^{(p)},$$

Leading power

$$T_{i,\alpha\beta}^{(p)} = i m_{B_q}^3 \left[\left(g_{\alpha\beta}^\perp - i \epsilon_{\alpha\beta}^\perp \right) F_{i,L}^{(p)} - \left(g_{\alpha\beta}^\perp + i \epsilon_{\alpha\beta}^\perp \right) F_{i,R}^{(p)} \right],$$

Two polarizations

$F_L^{\text{LP}} \propto m_b \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega, \mu) \propto \frac{m_b}{\lambda_b}$

[Bosch, Buchalla, hep-ph/0208202; Descotes-Genon, Sacharajda, hep-ph/0212162]

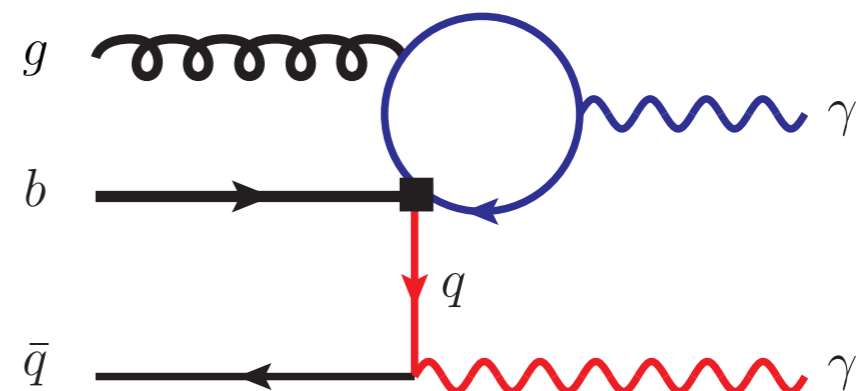
- NLL corrections + Systematic power corrections

both $\sim \mathcal{O}(10\%)$

[Y.-L. Shen, Y.-M. Wang, Y.-B. Wei, 2009.02723]

- One important but tough piece missing — — **long-distance**

penguin contribution



History of Long-distance penguin contribution

In inclusive $b \rightarrow s$ decays

- Realized in $\bar{B} \rightarrow X_s \gamma$, expansion of $\frac{\Lambda_{\text{QCD}}^2}{m_c^2} \left(\frac{m_b \Lambda_{\text{QCD}}}{m_c^2} \right)^n$, beyond $\frac{\Lambda_{\text{QCD}}^n}{m_b^n}$

[Voloshin, '96; Ligeti, Randall, Wise, '97; Buchalla, Isidori, Rey, '97]

- Factorization in $\bar{B} \rightarrow X_s \gamma$ using SCET, $m_c^2 \sim m_b \Lambda_{\text{QCD}}$

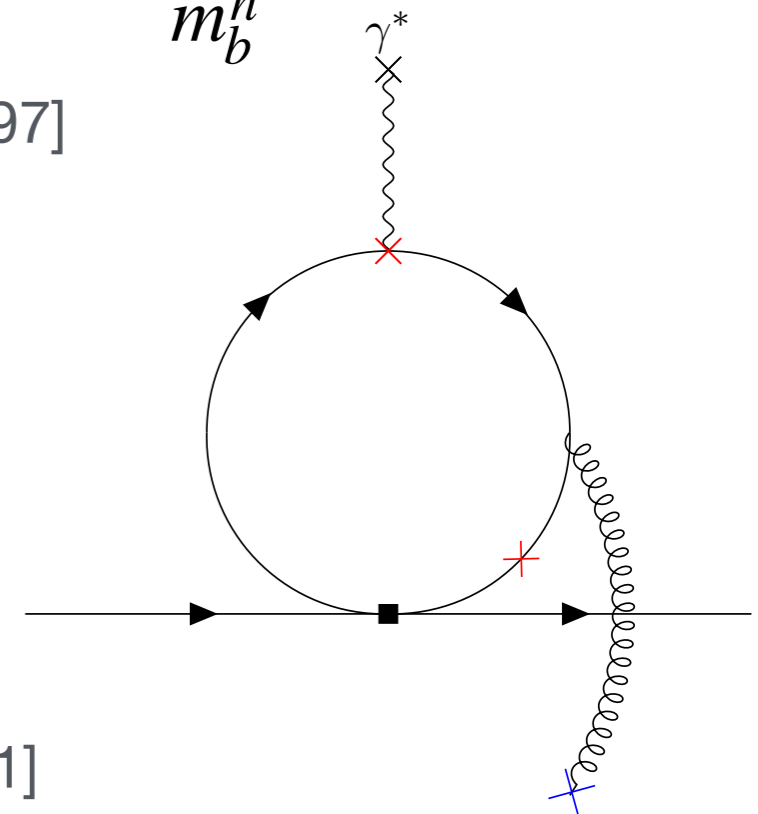
[Benzke, Lee, Neubert, Paz, 1003.5012]

- Factorization in $\bar{B} \rightarrow X_s \ell \ell$

[Benzke, Hurth, Turczyk, 1705.10366]

- Phenomenological Application in $\bar{B} \rightarrow X_{d,s} \ell \ell$

[Huber, Hurth, Enrico, Jenkins, **QQ**, Vos, 1908.07507, 2007.04191]



Soft gluon from charm-loop

In exclusive $b \rightarrow s$ decays

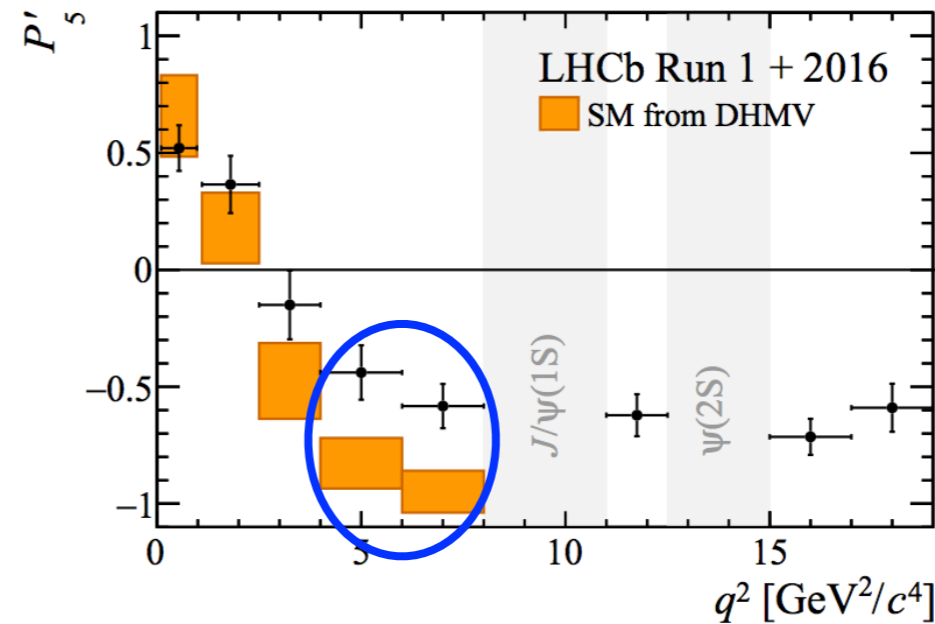
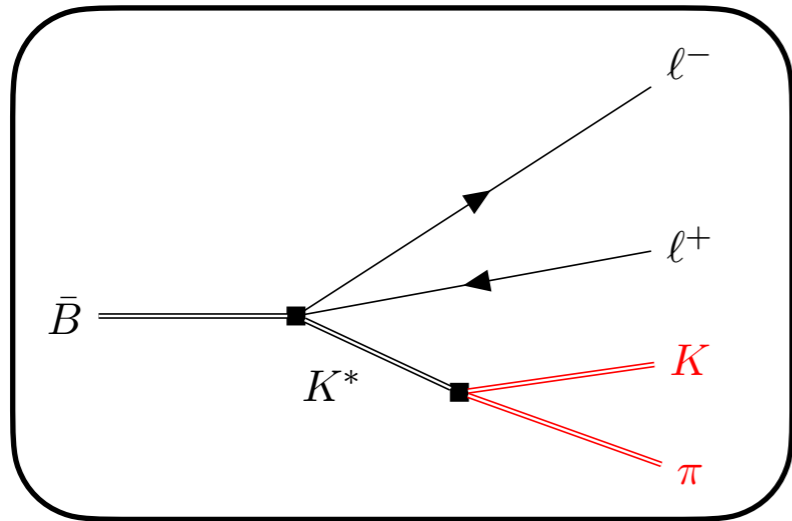
- Initiated in $B \rightarrow K^* \gamma$

[Khodjamirian, Ruckl, Stoll, Wyler, '97]

- Developed in $B \rightarrow K^* \ell \ell$

[Khodjamirian, Mannel, Pivovarov, Wang, 1006.4945]

History of Long-distance penguin contribution



P'_5 : an angular-distribution observable

[LHCb, 2003.04831]

Charm-loop effect in $B \rightarrow K^{(*)} \ell^+ \ell^-$ and $B \rightarrow K^* \gamma$

A. Khodjamirian (Siegen U.), Th. Mannel (Siegen U.), A.A. Pivovarov (Siegen U.), Y.-M. Wang (Siegen U.)

Jun, 2010

35 pages

Published in: *JHEP* 09 (2010) 089

e-Print: [1006.4945](https://arxiv.org/abs/1006.4945) [hep-ph]

pdf

cite

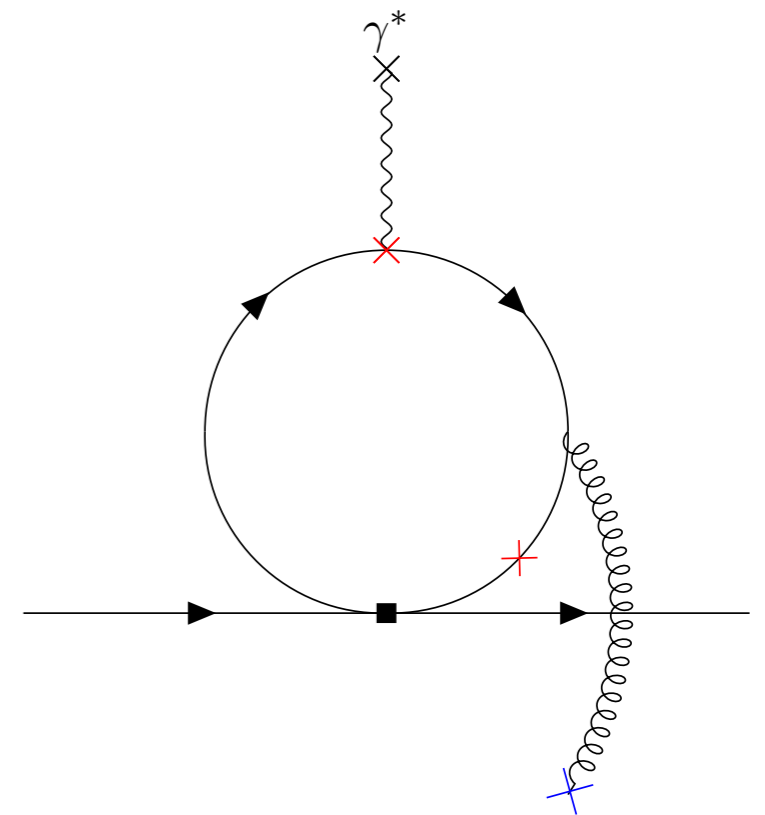
claim

reference search

472 citations

History of Long-distance penguin contribution

- Long-distance penguin contribution exist in basically all $b \rightarrow s$ decays
- Factorization completed in inclusive decays, but not yet in exclusive decays
- Attempt in the simplest decay $\bar{B} \rightarrow \gamma\gamma$



Soft gluon from charm-loop

Factorization

Factorization

$$\mathcal{H}_{\text{eff}} = \frac{4 G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pq}^* \left[C_1(\nu) P_1^p(\nu) + C_2(\nu) P_2^p(\nu) + \sum_{i=3}^8 C_i(\nu) P_i(\nu) + \sum_{i=3}^6 C_i(\nu) P_i^Q(\nu) \right] + \text{h.c.},$$

$$P_1^p = (\bar{q}_L \gamma_\mu T^a p_L) (\bar{p}_L \gamma^\mu T^a b_L),$$

$$P_2^p = (\bar{q}_L \gamma_\mu p_L) (\bar{p}_L \gamma^\mu b_L),$$

$$P_3 = (\bar{q}_L \gamma_\mu b_L) \sum_{q'} (\bar{q}' \gamma^\mu q'),$$

$$P_4 = (\bar{q}_L \gamma_\mu T^a b_L) \sum_{q'} (\bar{q}' \gamma^\mu T^a q'),$$

$$P_5 = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \sum_{q'} (\bar{q}' \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q'),$$

$$P_6 = (\bar{q}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L) \sum_{q'} (\bar{q}' \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q'),$$

Factorization

Integrate out the hard and hard-collinear d.o.f.

$$M = H * J * S \quad (m_b \gg m_c \sim \mathcal{O}(\sqrt{\Lambda m_b}) \gg \Lambda_{\text{QCD}})$$

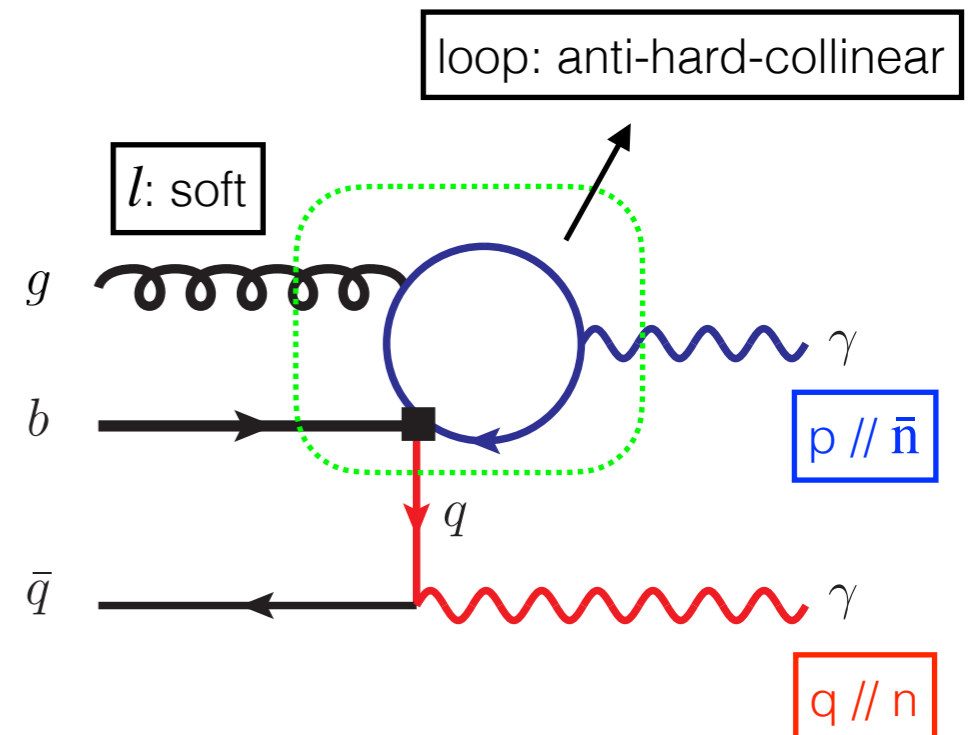
First-step match:

$$M \ni \left(C_2 - \frac{C_1}{2N_c} \right) Q_p \left[F\left(\frac{m_p^2 - i0^+}{(p-l)^2}\right) - 1 \right] \frac{p^\alpha}{(p-l)^2} \left[\bar{q}(\tilde{q}) \gamma_\beta P_L G_{\mu\alpha} \tilde{F}^{\mu\beta} b(v) \right]$$

$$F(x) = 4x \arctan^2\left(\frac{1}{\sqrt{4x-1}}\right)$$

$$(p-l)^2 = -2p \cdot l = -m_b \bar{n} \cdot l$$

➔ Non-local operator!



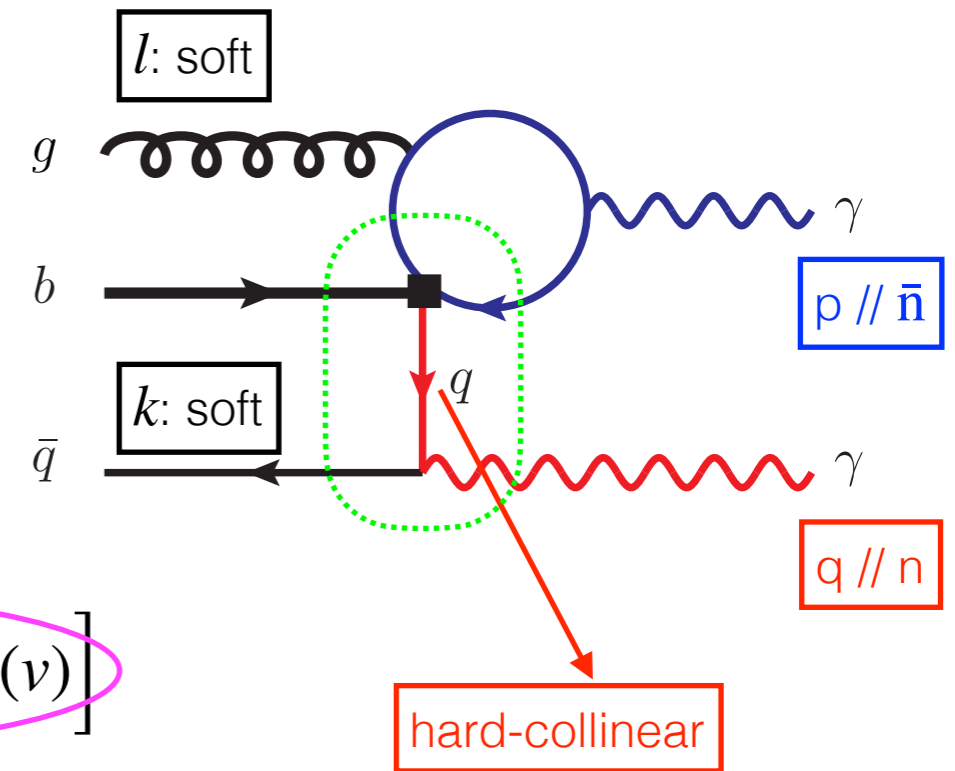
Factorization

Second-step match:

$$\langle \gamma(p) \gamma(q) | \bar{q} \gamma_\beta P_L G_{\mu\alpha} \tilde{F}^{\mu\beta} b | g(l) b(v) \bar{q}(k) \rangle$$

$$\Rightarrow \frac{i g_{\text{em}} e_q}{(q-k)^2} \epsilon^{\mu\beta\lambda\tau} p_\lambda \epsilon_\tau^*(p) \epsilon_\rho^*(q) \times \left[\bar{q}(k) \gamma_\perp^\rho \not{q} \gamma_\beta P_L G_{\mu\alpha}(\ell) b(v) \right]$$

$$(q-k)^2 = -2q \cdot k = -m_b n \cdot k$$



- The hard-kernel (jet functions) depends on 2 different light-cone components of the gluon and light quark momenta.
- It becomes evident to introduce the 3-particle B-meson distribution amplitude with 2 light-cone directions.

$$H \star J \star \bar{J} \star \Phi_G$$

Factorization

The explicit factorization formula:

$$\sum_{i=1}^8 C_i F_{i,L}^{(p), \text{soft } 4q} = -\frac{Q_q f_{B_q}}{m_{B_q}} \int_{-\infty}^{+\infty} \frac{d\omega_1}{\omega_1} \int_{-\infty}^{+\infty} \frac{d\omega_2}{\omega_2} \left(C_2 - \frac{C_1}{2N_c} \right) Q_p \left[F\left(-\frac{m_p^2}{m_b \omega_2}\right) - 1 \right] \times \Phi_G(\omega_1, \omega_2, \mu)$$

The light quark momentum component $\omega_1 = n \cdot k$;

The soft gluon momentum component $\omega_2 = \bar{n} \cdot l$.

The novel B-meson DA:

$$\begin{aligned} & \langle 0 | \bar{q}_s(\tau_1 n) (g_s G_{\mu\nu})(\tau_2 \bar{n}) \bar{n}^\nu \not{n} \gamma_\perp^\mu \gamma_5 h_\nu(0) | \bar{B}_\nu \rangle \\ & = 2 \tilde{f}_B(\mu) m_B \int_{-\infty}^{+\infty} d\omega_1 \int_{-\infty}^{+\infty} d\omega_2 \exp[-i(\omega_1 \tau_1 + \omega_2 \tau_2)] \Phi_G(\omega_1, \omega_2, \mu) \end{aligned}$$

- The quark and gluon fields are localized on 2 distinct light-cone directions.

Non-trivial RG evolution of this soft function, mixing positive into negative support of $\omega_{1,2}$. See an upcoming paper [Huang, Ji, Shen, Wang, Wang, Zhao, 2312.15439]. See also Yong-Kang's talk.

Factorization

The (tree-level) normalization conditions of Φ_G :

Matching the conventional 3-particle B meson DAs as τ_1 or $\tau_2 \rightarrow 0$.

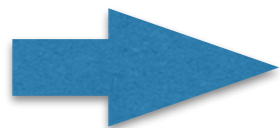
$$\langle 0 | \bar{q}(z_1)(g_s G_{\mu\nu})(z_2) \bar{n}^\nu \not{n} \gamma_\perp^\mu \gamma_5 h_\nu(0) | \bar{B}_\nu \rangle = 2 \tilde{f}_B(\mu) \Phi_4(z_1, z_2, \mu)$$

Twist 4

$$\langle 0 | \bar{q}(z_1)(g_s G_{\mu\nu})(z_2) n^\nu \not{n} \gamma_\perp^\mu \gamma_5 h_\nu(0) | \bar{B}_\nu \rangle = 2 \tilde{f}_B(\mu) \Phi_5(z_1, z_2, \mu)$$

Twist 5

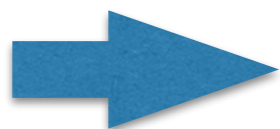
[Braun, Ji, Manashov, 1703.02446]



$$\int_0^\infty d\omega_1 \Phi_G(\omega_1, \omega_2, \mu) = \int_0^\infty d\omega_1 \Phi_4(\omega_1, \omega_2, \mu),$$

$$\int_0^\infty d\omega_2 \Phi_G(\omega_1, \omega_2, \mu) = \int_0^\infty d\omega_2 \Phi_5(\omega_1, \omega_2, \mu),$$

$$\int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \Phi_G(\omega_1, \omega_2, \mu) = \frac{\lambda_E^2 + \lambda_H^2}{3},$$



The power counting: $F_L^{\text{soft}, 4q} / F_L^{\text{LP}} \sim \lambda_B / m_b$

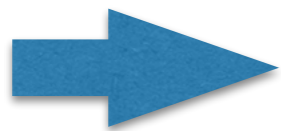
Factorization

The asymptotic behaviors of Φ_G :

$$\Phi_G(\omega_1, \omega_2, \mu) \sim \omega_1 \omega_2^2 \text{ at } \omega_1, \omega_2 \rightarrow 0$$

The explicit factorization formula:

$$\sum_{i=1}^8 C_i F_{i,L}^{(p), \text{soft } 4q} = -\frac{Q_q f_{B_q}}{m_{B_q}} \int_0^{+\infty} \frac{d\omega_1}{\omega_1} \int_0^{+\infty} \frac{d\omega_2}{\omega_2} \left(C_2 - \frac{C_1}{2N_c} \right) Q_p \left[F\left(-\frac{m_p^2}{m_b \omega_2}\right) - 1 \right] \times \Phi_G(\omega_1, \omega_2, \mu)$$

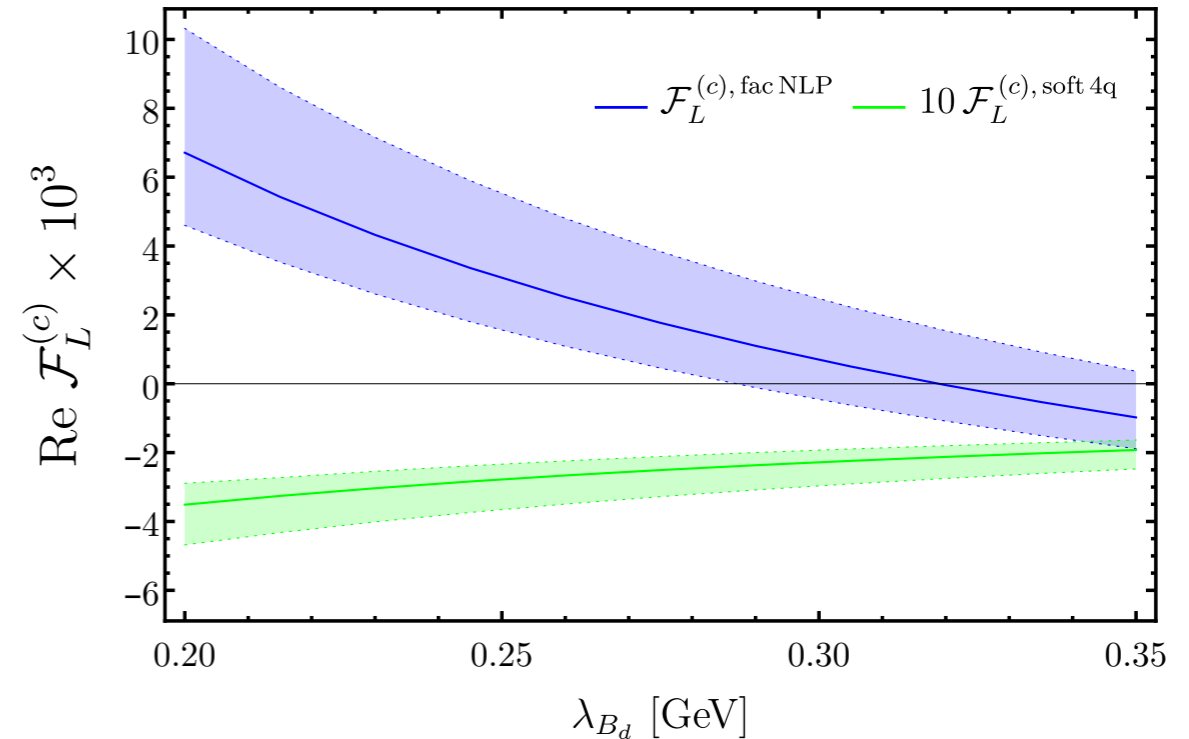
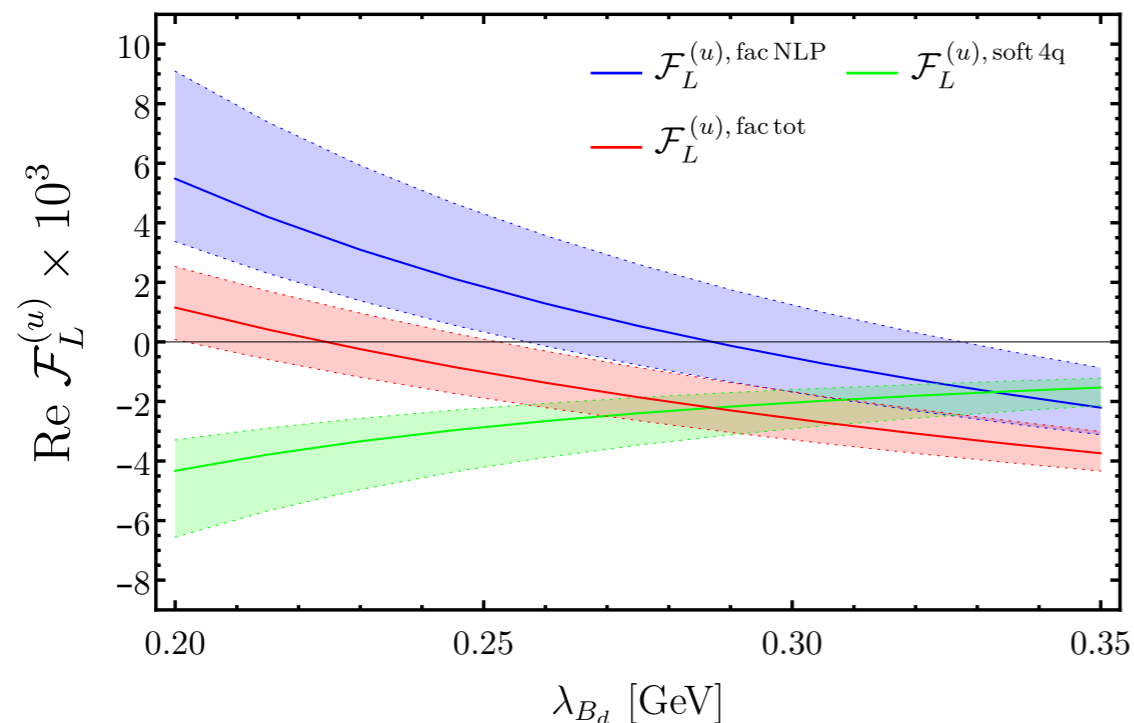


The convolution integral converges.

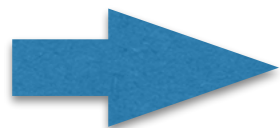
Numerics

The Φ_G parametrization:

$$\Phi_G(\omega_1, \omega_2, \mu_0) = \frac{\lambda_E^2 + \lambda_H^2}{6} \frac{\omega_1 \omega_2^2}{\omega_0^5} \exp\left(-\frac{\omega_1 + \omega_2}{\omega_0}\right) \frac{\Gamma(\beta + 2)}{\Gamma(\alpha + 2)} U\left(\beta - \alpha, 4 - \alpha, \frac{\omega_1 + \omega_2}{\omega_0}\right)$$



- The up-loop contribution dominates; the charm-loop is 1-order smaller.
- The new power correction accidentally cancels the previous ones.



Clean channel to determine λ_B and to probe new physics.

Numerics

The B_d results:

| | Central Value | Total Error | λ_{B_d} | $\{\hat{\sigma}_{B_d}^{(1)}, \hat{\sigma}_{B_d}^{(2)}\}$ | μ | ν | μ_h | $\bar{\Lambda}$ | m_c^{PS} |
|---|-----------------|------------------|------------------|--|------------------|------------------|------------------|------------------|-------------------|
| $10^8 \times \mathcal{BR}$ | 1.929 [1.900] | +1.096 -1.012 | +0.680 -0.439 | +0.736 -0.779 | +0.083 -0.299 | +0.278 -0.287 | +0.246 -0.066 | +0.212 -0.200 | +0.043 -0.043 |
| f_{\parallel} | 0.408 [0.407] | +0.044 -0.046 | +0.015 -0.015 | +0.016 -0.033 | +0.002 -0.009 | +0.037 -0.026 | +0.007 -0.002 | +0.005 -0.006 | +0.002 -0.002 |
| f_{\perp} | 0.592 [0.593] | +0.046 -0.044 | +0.015 -0.015 | +0.033 -0.016 | +0.009 -0.002 | +0.026 -0.037 | +0.002 -0.007 | +0.006 -0.005 | +0.002 -0.002 |
| $\mathcal{A}_{\text{CP}}^{\text{dir}, \parallel}$ | 0.126 [0.129] | +0.043 -0.027 | +0.007 -0.004 | +0.017 -0.010 | +0.013 -0.008 | +0.027 -0.018 | +0.024 -0.012 | +0.007 -0.007 | +0.004 -0.004 |
| $\mathcal{A}_{\text{CP}}^{\text{mix}, \parallel}$ | -0.197 [-0.154] | +0.053 -0.084 | +0.019 -0.036 | +0.001 -0.002 | +0.021 -0.047 | +0.026 -0.040 | +0.015 -0.029 | +0.011 -0.013 | +0.008 -0.009 |
| $\mathcal{A}_{\Delta\Gamma}^{\parallel}$ | -0.972 [-0.980] | +0.024 -0.013 | +0.009 -0.004 | +0.003 -0.002 | +0.013 -0.005 | +0.013 -0.007 | +0.010 -0.004 | +0.004 -0.003 | +0.002 -0.002 |
| $\mathcal{A}_{\text{CP}}^{\text{dir}, \perp}$ | 0.330 [0.326] | +0.078 -0.053 | +0.015 -0.012 | +0.060 -0.035 | +0.035 -0.014 | +0.012 -0.024 | +0.014 -0.010 | +0.018 -0.016 | +0.018 -0.017 |
| $\mathcal{A}_{\text{CP}}^{\text{mix}, \perp}$ | 0.136 [0.101] | +0.087 -0.066 | +0.043 -0.028 | +0.015 -0.035 | +0.025 -0.014 | +0.060 -0.038 | +0.026 -0.012 | +0.003 -0.003 | +0.009 -0.008 |
| $\mathcal{A}_{\Delta\Gamma}^{\perp}$ | 0.934 [0.940] | +0.017 -0.030 | +0.000 -0.003 | +0.009 -0.019 | +0.007 -0.017 | +0.001 -0.002 | +0.005 -0.009 | +0.006 -0.007 | +0.007 -0.008 |

Numerics

Electroweak and radiative penguin
 B meson decays at Belle and Belle II



Giulio Dujany, Moriond 2024

| | $\mathcal{B}(B^0 \rightarrow \gamma\gamma)$ | $\mathcal{B}(B^0 \rightarrow \gamma\gamma)$ (at 90% CL) |
|----------|--|--|
| Belle | $(5.4_{-2.6}^{+3.3} \pm 0.5) \times 10^{-8}$ | $< 9.9 \times 10^{-8}$ |
| Belle II | $(1.7_{-2.4}^{+3.7} \pm 0.3) \times 10^{-8}$ | $< 7.4 \times 10^{-8}$ |
| Combined | $(3.7_{-1.8}^{+2.2} \pm 0.7) \times 10^{-8}$ | $< 6.4 \times 10^{-8}$ |

Summary and prospects

- We have factorized the long-distance penguin contribution to $\bar{B} \rightarrow \gamma\gamma$ decay, for the first time in an exclusive decay.
- A novel B-meson DA is introduced, with quark and gluon fields localized on two different light-cone directions. It provides a new window to probe the inner structure of the B meson.
- The new contribution cancels the known factorizable power corrections, making $\bar{B} \rightarrow \gamma\gamma$ a clean channel to determine λ_B and to probe the non-standard dynamics.
- The developed formalism has a broad field of applications to the entire spectrum of the exclusive FCNC B-meson decays, including flagship modes, e.g. $B \rightarrow K^*\gamma$, $B \rightarrow K^*\mu\mu$.

Thank you!

Backup

| | B_d | B_s |
|--|---|---|
| $\mathcal{A}^{\text{LP,NLL}} [10^{-4}]$ | $3.4 + 1.9i$ | $-20 - 0.37i$ |
| $\mathcal{A}^{\text{fac,NLP}} [10^{-4}]$ | $-0.15 - 0.53i$ | $0.92 + 2.6i$ |
| $\mathcal{A}_R^{\text{fac,NLP}} [10^{-4}]$ | $0.25 - 0.36i$ | $-1.6 + 2.6i$ |
| $\mathcal{A}^{\text{had},\gamma} [10^{-4}]$ | $-0.30 - 0.17i$ | $1.4 - 0.0021i$ |
| $\mathcal{A}^{\text{soft},4q} [10^{-4}]$ | $(-0.0079 + 0.078i)$ | $-0.11 + 0.016i$ |
| $(F_u^{\text{LP,NLL}}, F_c^{\text{LP,NLL}})$ | $(-0.056 - 0.0092i, -0.048 - 0.0019i)$ | $(-0.057 - 0.0094i, -0.049 - 0.0020i)$ |
| $(F_u^{\text{had},\gamma}, F_c^{\text{had},\gamma})$ | $(0.0051 + 0.00092i, 0.0043 + 0.00019i)$ | $(0.0094 + 0.0016i, 0.0034 + 0.00016i)$ |
| $(F_u^{\text{soft},4q}, F_c^{\text{soft},4q})$ | $(-0.0024, -0.00025)$ | $(-0.0021, -0.00025)$ |
| $(F_u^{\text{HC}}, F_c^{\text{HC}})$ | $(0.0055, 0.0055)$ | $(0.0067, 0.0067)$ |
| $(F_u^{\text{mq}}, F_c^{\text{mq}})$ | $(0.000049, 0.000049)$ | $(0.00078, 0.00078) [0.00079]$ |
| $(F_u^{\text{A}_2}, F_c^{\text{A}_2})$ | $(-0.0010, -0.0010)$ | $(-0.0011, -0.0011)$ |
| $(F_u^{\text{HT}}, F_c^{\text{HT}})$ | $(0.0046, 0.0046) [0.0047]$ | $(0.0048, 0.0048) [0.0050]$ |
| $(F_u^{\text{Q}_b}, F_c^{\text{Q}_b})$ | $(-0.0036, -0.0036)$ | $(-0.0043, -0.0043)$ |
| $(F_u^{\text{WA}}, F_c^{\text{WA}})$ | $(-0.0049 + 0.000092i, -0.0037 + 0.0056i)$ | $(-0.0059 + 0.00011i, -0.0045 + 0.0065i)$ |
| $(F_u^{\text{fac,NLP}}, F_c^{\text{fac,NLP}})$ | $(0.00054 + 0.000092i, 0.0018 + 0.0056i)$ | $(0.00098 + 0.00011i, 0.0023 + 0.0065i)$ |
| | $[(0.00063 + 0.000092i, 0.0019 + 0.0056i)]$ | $(0.0011 + 0.00011i, 0.0024 + 0.0065i)$ |
| $(F_{R,u}^{\text{fac,NLP}}, F_{R,c}^{\text{fac,NLP}})$ | $(-0.0046 + 0.000092i, -0.0033 + 0.0056i)$ | $(-0.0054 + 0.00011i, -0.0041 + 0.0065i)$ |

$$A = V_{uq}^* V_{ub} F_u + V_{cq}^* V_{cb} F_c \quad (q = d, s)$$