重介子四体半轻衰变的一些探讨

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Overview

重味半轻衰变的关键问题

Dipion 系统光锥分布振幅

重介子四体半轻衰变的一些探索

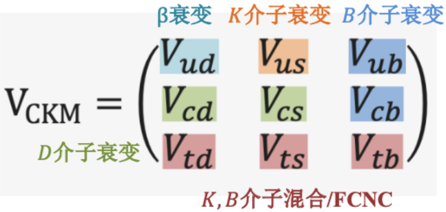
$B \to [\rho^+ \to] \, \pi^+ \pi^0 \, l \bar{\nu}$

$D_s \to [f_0 \to] \, [\pi^+ \pi^-]_{\mathrm{S}} \, e^+ \nu_e$

总结和展望

重味半轻衰变的关键问题

重味半轻衰变



- $VV^\dagger = V^\dagger V = I_3$ in the Standard Model
- $VV^\dagger \neq V^\dagger V \neq I_3$ New Physics
- $|V_{ub}|/|V_{cb}|$ contributes to *CPV* measurement in *B* decays

- CKM matrix elements are mainly measured via **the charged current processes**, i.e, $b \to u l^- \bar{\nu}, c \to s l^+ \nu$
- **Flavor changing neutral current precesses** are sensitive to new physical contributions, i.e, $b \to s l^+ l^-$

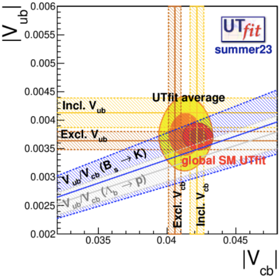
关键问题

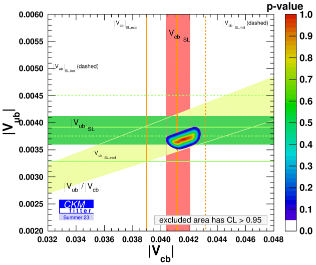
- $|V_{ub}|$ tension $\quad |V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$ [PDG 2022]

‡ $\sim 2.5\sigma$ tension between $(4.13 \pm 0.25) \times 10^{-3}$ and $(3.70 \pm 0.16) \times 10^{-3}$ measured via the $B \to X_u l^- \bar{\nu}$ and $B \to \pi l^- \bar{\nu}$ processes, respectively.

- $|V_{cb}|$ tension $\quad |V_{ub}| = (40.8 \pm 1.4) \times 10^{-3}$ [PDG 2022]

‡ $\sim 2.5\sigma$ tension between $(42.2 \pm 0.8) \times 10^{-3}$ and $(39.4 \pm 0.8) \times 10^{-3}$ measured via the $B \to X_c l^- \bar{\nu}$ and $B \to D^{(*)} l^- \bar{\nu}$ processes, respectively.

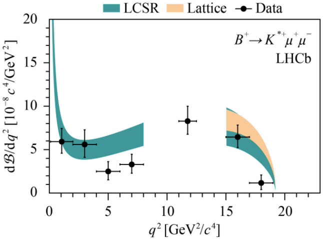


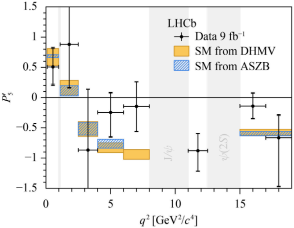


关键问题

- LFU in $b \to c l^- \bar{\nu}$ processes $\quad R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)} \tau^- \bar{\nu})}{\mathcal{B}(B \to D^{(*)} \mu^- \bar{\nu})}$
  - $R_D = 0.407 \pm 0.046,\ R_{D^*} = 0.306 \pm 0.015$ Average with [Belle PRL124, 161803 (2020)]
  - $2.1\sigma, 3.0\sigma$ derivations from the SM predictions of $R_D = 0.298 \pm 0.004,\ R_{D^*} = 0.254 \pm 0.005$ [HFLAV]
  - $R_D = 0.441 \pm 0.089,\ R_{D^*} = 0.281 \pm 0.030$ [LHCb PRL131,111802 (2023)]
  - would make the CKM measurements more complicated if confirmed

- Anomalies in FCNC processes $B \to K^* \mu^+ \mu^-$
  - 3.6σ derivation from SM of $d\mathcal{B}(B \to K^* \mu^+ \mu^-)/dq^2$ in $q^2 \in [1, 6]$ GeV^2
  - 1.9σ derivation from SM of $p_5' = S_5/\sqrt{F_L(1 - F_L)}$ in $q^2 \in [4, 8]$ GeV^2



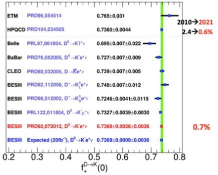


解决方案　在传统过程继续奋斗

- $|V_{cs}|$ issue　$|V_{cs}| = 0.975 \pm 0.006$ [PDG 2022]

* 0.972 ± 0.007 and 0.984 ± 0.012 measured via the $D \to K l \nu$ and $D_s \to \mu^+ \nu_\mu$ processes $\sim 3\sigma \to \sim 1.5\sigma$

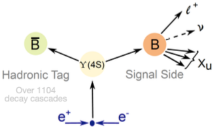
更精确的测量和格点计算



- $|V_{ub}|$ result from Belle collaboration with Simultaneous Determination in excl. and incl. processes

* $3.78 \pm 0.23 \pm 0.16 \pm 0.14$ and $3.88 \pm 0.20 \pm 0.31 \pm 0.09$ [Belle PRL131, 211801 (2023)]

更全面更系统的物理分析方法



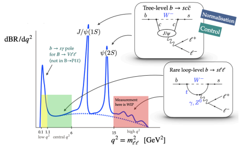
$\mathcal{B}(B \to \pi^0 \ell \nu) + \mathcal{B}(B \to \pi^+ \ell \nu) + \mathcal{B}(B \to X_u^{\text{other}} \ell \nu)$
$= \mathcal{B}(B \to X_u \ell \nu)$

- high order QCD corrections, more structures

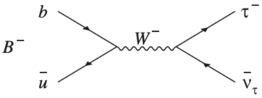
[AK, TM, YMW, JHEP 02 (2013) 010]

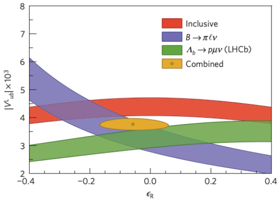
[AK, TM, AAP, YMW, JHEP 09 (2010) 089]

精细结构、丰富的 QCD 效应



解决方案　寻找新的增长极





‡ $|V_{ub}|f_B$ in **pure leptonic decay**

* 0.72 ± 0.09 MeV from Belle, 1.01 ± 0.14 MeV from BABAR, 0.77 ± 0.12 MeV average [FLAG2021]

‡ $|V_{ub}|$ in **baryon decay** [Nature Physics 11, 743 (2015)]

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} = \frac{\mathcal{B}(\Lambda_b^0 \to p\mu^-\bar{\nu})}{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+\mu^-\bar{\nu})} R_{\rm FF} = 0.68 \pm 0.07 \Downarrow$$

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.06 \xrightarrow{|V_{cb}|} |V_{ub}| = (3.27 \pm 0.23) \times 10^{-3}$$

* consistent with determinations in exclusive $B \to \pi l \bar{\nu}$ decay

confirms the existing incompatibility with the inclusive sample

‡ $|V_{cb}|$ in $B_s \to D_s \mu^+ \nu$

‡ $|V_{ub}|/|V_{cb}|$ in $\mathcal{B}(B_s \to K^- \mu^+ \nu)/\mathcal{B}(B_s \to D_s^- \mu^+ \nu)$

‡ $d\mathcal{B}(B \to K^* \mu^+ \mu^-)/dq^2$ and p_5' in baryon decay $\Lambda_b^0 \to \Lambda \mu^+ \mu^-$

解决方案　解放思想

- 以上过程都只涉及到基态粒子
- 含有激发态粒子的过程也可以提供独立的测量

- $|V_{ub}|$ in the $B \to \rho l \nu$ process

∗ ρ is reconstructed by the $\pi\pi$ invariant mass spectral, the underlying consideration is $B \to [\rho^+ \to] \, \pi^+ \pi^0 l^- \bar{\nu}_l \, (B_{l4})$ [Faller 2014]

- $|V_{cs}|$ in the $D_s \to [f_0 \to] \, [\pi\pi]_{\mathrm{S}} \, l\nu$ process

∗ f_0 large uncertainty due to the width and complicate structure

- $|V_{cb}|$ in the $B \to D^* l\nu$ processes
- B anomalies in $B \to K^* l^+ l^-$ processes

∗ How to calculate the width effect/nonresonant contribution ?

Dipion 系统光锥分布振幅

DiPion 系统光锥分布振幅

- Chiral-even LC expansion with gauge factor $[x, 0]$ [Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1)\pi^b(k_2)|\bar{q}_f(zn)\gamma_\mu \tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

† Three independent kinematic variables

† Normalization conditions $\int_0^1 \Phi_{\parallel}^{I=1}(u,\zeta,k^2) = (2\zeta-1)F_\pi(k^2)$

$$\int_0^1 dz\,(2z-1)\Phi_{\parallel}^{I=0}(z,\zeta,k^2) = -2M_2^{(\pi)}\zeta(1-\zeta)F_\pi^{\rm EMT}(k^2)$$

△ $M_2^{(\pi)}$ is the momentum fraction carried by quarks in the pion associated to the usual quark distribution

- 2πDAs is decomposed in terms of $C_n^{3/2}(2z-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=0,\mathrm{even}}^{\infty}\sum_{l=1,\mathrm{odd}}^{n+1} B_{n\ell}^{I=1}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=1,\mathrm{odd}}^{\infty}\sum_{l=0,\mathrm{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

- $B_{n\ell}(k^2,\mu)$ have similar scale dependence as the a_n of π, ρ, f_0 mesons

DiPion 系统光锥分布振幅

- Soft pion theorem relates the chirarlly even coefficients with a_n^π

  △ relation between the a_n^π and the coefficients $B_{n\ell}$

- 2πDAs relate to the skewed parton distributions (SPDs) by crossing

  △ express the moments M_N^π of SPDs in terms of $B_{nl}(k^2)$ in the forward limit as $k^2 = 0$

- In the vicinity of the resonance, 2πDAs reduce to the DAs of ρ/f_0

  △ relation between the a_n^ρ and the coefficients $B_{n\ell}$ △ f_ρ relates to the imaginary part of $B_{nl}(m_\rho^2)$

- How to describe **the evolution from $4m_\pi^2$ to large invariant mass** $k^2 \sim \mathcal{O}(m_c^2)$? furtherly to $\mathcal{O}(m_b\lambda_{\rm QCD})$

‡ Watson theorem of π-π scattering amplitudes

  △ implies an intuitive way to express the imaginary part of 2πDAs

  △ leads to the Omnés solution of N-subtracted DR for the coefficients

$$B_{n\ell}^I(k^2) = B_{n\ell}^I(0)\,{\rm Exp}\left[\sum_{m=1}^{N-1}\frac{k^{2m}}{m!}\frac{d^m}{dk^{2m}}\ln B_{n\ell}^I(0) + \frac{k^{2N}}{\pi}\int_{4m_\pi^2}^{\infty} ds\,\frac{\delta_\ell^I(s)}{s^N(s-k^2-i0)}\right]$$

- **2πDAs in a wide range of energies is given by δ_ℓ^I and a few subtraction constants**

DiPion 系统光锥分布振幅

- The subtraction constants of $B_{n\ell}(s)$ at low s (around the threshold)

(nl)	$B^{\parallel}_{n\ell}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2}\ln B^{\parallel}_{n\ell}(0)$	$B^{\perp}_{n\ell}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2}\ln B^{\perp}_{n\ell}(0)$
(01)	1	0	1.46 $\to$ 1.80	1	0	0.68 $\to$ 0.60
(21)	-0.113 $\to$ 0.218	-0.340	0.481	0.113 $\to$ 0.185	-0.538	-0.153
(23)	0.147 $\to$ -0.038	0	0.368	0.113 $\to$ 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

△ firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

△ updated with the kinematical constraints and the new a_2^{π}, a_2^{ρ} [**SC** 2019, 2023]

- All discussions are at leading twist, subleading twist LCDAs are not known yet

DiMeson LCDAs widely used in the three-body B decays studied from pQCD and QCDF is the asymptotic formula [J Chai, **SC**, A-J Ma 2109.00664]

normalized to unit as $\Gamma^{I=1}_{M_1M_2}(0) = 1$. When the invariant mass of dimeson system is small, the higher $\mathcal{O}(s)$ terms in the expansion of coefficient $B_{nl}(s,\mu)$ around the resonance pole can be safely neglected due to the large suppression $\mathcal{O}(s/m_b^2)$ in contrast to the energetic dimeson system in B decay, so the relation $B_{n1}(s,\mu) \to a_n(\mu)\Gamma^{I=1}_{M_1M_2}(s)$ can be obtained in the lowest partial wave approximation. This argument induces the basic assumption in PQCD that the energetic dimeson DAs can be deduced from the DAs of resonant meson by replacing the decay constant by the timelike form factor.

重介子四体半轻衰变的一些探索

$$B \to [\rho^+ \to]\, \pi^+\pi^0 l\bar{\nu}$$

$$D_s \to [f_0 \to]\, [\pi\pi]_{\rm S}\, e^+\nu_e$$

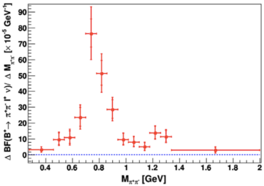
$B \to [\rho^+ \to] \pi^+ \pi^0 l \bar{\nu}$ 衰变（B_{l4} 过程）

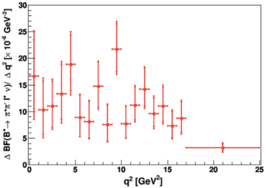
- B_{l4} decays have rich observables, nontrivial tests of SM [Faller 2014]
- Different exclusive $b \to u$ processes in the $|V_{ub}|$ determination [Faller 2014] [Gao, Lü, Shen, Wang, Wei 1902.11092]

$$|V_{ub}| = \left(3.05^{+0.67}_{-0.52}\Big|_{\rm theo} \; {}^{+0.19}_{-0.20}\Big|_{\rm exp}\right) \times 10^{-3}, \quad \text{from } B \to \rho l \nu$$

$$|V_{ub}|_{\rm PDG} = (3.70 \pm 0.12|_{\rm theo} \; \pm 0.10|_{\rm exp}) \times 10^{-3} \quad \text{from } B \to \pi l \nu$$

- $B \to \pi\pi l \bar{\nu}_l$ has already been measured, mainly its resonant part $B \to \rho l \bar{\nu}_l$ $(1.58 \pm 0.11) \times 10^{-4}$ [CLEO 2000, BABAR 2011, Belle 2013]
- We propose to measure the $B \to \pi^+ \pi^0 l^- \bar{\nu}$ decay with $B \to \pi^+ \pi^0$ form factor calculation from B meson LCSRs [**SC**, Khodjamirian, Virto 1701,01633]
- First measurement of the branching fraction of $B^+ \to \pi^+ \pi^- l^+ \bar{\nu}_l$ $(2.3 \pm 0.4) \times 10^{-4}$ [Belle 2005.07766] More data (Belle II) is on the way





$B \to \pi\pi$ 形状因子

- Dynamics of B_{l4} is governed by the $B \to \pi\pi$ form factors
- A big task for the practitioners of QCD-based methods
- First Lattice QCD study of the $B \to \pi\pi l \bar{\nu}$ transition amplitude in the region of large q^2 and $\pi\pi$ invariant mass near the ρ resonance [Leskovec et.al. 2212.08833[hep-lat]]

$B \to \pi\pi$ form factors [Hambrock, Khodjamirian, 1511.02509]

$$
\begin{aligned}
i\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle = {} & F_\perp(q^2,k^2,\zeta)\,\frac{2}{\sqrt{k^2}\sqrt{\lambda_B}}\,i\epsilon_{\nu\alpha\beta\gamma}\,q^\alpha\,k^\beta\,\bar{k}^\gamma \\
& + F_t(q^2,k^2,\zeta)\,\frac{q_\nu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k_\nu - \frac{k\cdot q}{q^2}q_\nu\right) \\
& + F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}_\nu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_B}\,k_\nu + \frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q_\nu\right)
\end{aligned}
$$

† $\lambda = \lambda(m_B^2, k^2, q^2)$ is the Källén function

† $q \cdot k = (m_B^2 - q^2 - k^2)/2$ and $q \cdot \bar{k} = \sqrt{\lambda}\beta_\pi(k^2)\cos\theta_\pi/2 = \sqrt{\lambda}\,(2\zeta - 1)$

† $\beta_\pi(k^2) = \sqrt{1 - 4m_\pi^2/k^2}$, θ_π is the angle between the 3-momenta of the neutral pion and the B-meson in the dipion rest frame

$B \to \pi\pi$ 形状因子

- Starting with the correlation function

$$
\begin{aligned}
F_\mu(k_1, k_2, q) &= i \int d^4x e^{iq\cdot x} \langle \pi^+(k_1)\pi^0(k_2)|\mathrm{T}\{j_\mu^{V-A}(x), j_5(0)\}|0\rangle \\
&\equiv \varepsilon_{\mu\nu\rho\sigma} q^\nu k_1^\rho k_1^\sigma F^V + q_\mu F^{(A,q)} + k_\mu F^{(A,k)} + \bar{k}_\mu F^{(A,\bar{k})}
\end{aligned}
$$

- Take $F_\perp(q^2, k^2, \zeta)$ as an example

$$
\frac{F_\perp(q^2, k^2, \zeta)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{m_b}{\sqrt{2} f_B m_B^2 f_{2\pi}^\perp\,(2\zeta - 1)} \int_{u_0}^1 \frac{du}{u}\, \Phi_\perp^I(u, \zeta, k^2)\, e^{-\frac{s(u)+m_B^2}{M^2}}
$$

† Partial wave expansion $F_\perp(k^2, q^2, \zeta) = \sum_\ell \sqrt{2\ell+1}\, F_{\perp,\parallel}^{(\ell)}(k^2, q^2)\, \frac{P_\ell^{(1)}(\cos\theta_\pi)}{\sin\theta_\pi}$

$$
F_\perp^{(\ell)}(k^2, q^2) = \frac{\sqrt{k^2}}{\sqrt{2} f_{2\pi}^\perp} \frac{\sqrt{\lambda_B} m_b}{m_B^2 f_B}\, e^{\frac{m_B^2}{M^2}} \sum_{n=0,2,\cdots} \sum_{\ell'=1,3}^{n+1} I_{\ell\ell'} B_{n\ell'}^\perp(k^2, \mu) J_n^\perp(q^2, k^2, M^2, s_0^B).
$$

$$
I_{\ell\ell'} \propto \int_{-1}^1 \frac{dz}{z} \sqrt{1-z^2}\, P_\ell^{(1)}(z) P_{\ell'}^{(0)}(z), \quad J_n^\perp = \int_{u_0}^1 du\, e^{\frac{-s}{M^2}}\, 6(1-u) C_n^{3/2}(2u-1).
$$

† $I_{\ell\ell'} = 0$ when $\ell > \ell'$, $I_{11} = 1/\sqrt{3}$, $I_{13} = -1/\sqrt{3}$, $I_{15} = 4/(5\sqrt{3})$, $\ell' = 1$, asymptotic DAs

$B \to \pi\pi$ 形状因子

- How large of $P-$wave contribution to $B \to \pi\pi$ FFs $(\ell = 1)$?

$$R_\ell \equiv F_\perp^{(\ell>1)}(k^2, q^2)/F_\perp^{(\ell=1)}(k^2, q^2)$$

- How much ρ contained in $P-$wave $B \to \pi\pi$ FFs $(\ell = 1, \ell' = 1)$?

- Hadronic dispersion relation for the P-wave $B \to \pi\pi$ form factors

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle = \langle \pi^+(k_1)\pi^0(k_2)|\rho\rangle\langle\rho|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle + \cdots$$

$$\frac{\sqrt{3}F_\perp^{(\ell=1)}}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - im_\rho\Gamma_\rho(k^2)} \frac{V^{B\to\rho}(q^2)}{m_B + m_\rho} + \cdots$$

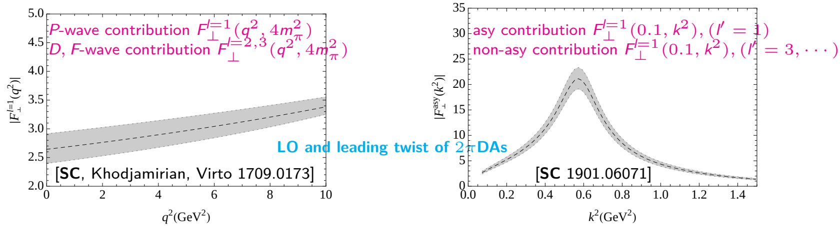
△ $\rho \to 2\pi$ strong coupling $g_{\rho\pi\pi} = 5.96 \pm 0.04 \Leftarrow$ the energy dependent total width of ρ

- $B \to \rho$ form factor obtained from leading twist ρ-LCDAs

$$V^{B\to\rho}(q^2) = \frac{(m_B + m_\rho)m_b}{2m_B^2 f_B} f_\rho^\perp \int_{u_0}^1 \frac{du}{u} \phi_\perp^{(\rho)}(u)\, e^{m_B^2/M^2}\, e^{(-q^2\bar{u}+m_\rho^2 u\bar{u})/(uM^2)}$$

$$i\langle\rho^+(k)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle = i\varepsilon_{\nu\alpha\beta\gamma}\epsilon^{*\alpha}q^\beta k^\gamma \frac{V(q^2)}{m_B + m_\rho} + \cdots$$

$B \to \pi\pi$ 形状因子

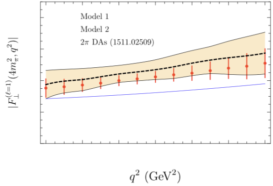


- High partial waves give few percent contributions to $B \to \pi\pi$ form factors
- ρ', ρ'' and NR background contribute $\sim 20\% - 30\%$ to P-wave

- 30% smaller than it obtained from B-meson LCSRs [SC, Khodjamirian and Virto 1701.01663]

† high twist contributions ?

† Uncertainty of B-meson LCDAs ?



$D_s \to [f_0 \to] [\pi^+\pi^-]_S e^+\nu_e$ 衰变（D_{l4} 过程）

轻标量介子结构

- $f_0(1370), f_0(1500), a_0(1450), K_0^*(1430)$ form a $SU(3)$ flavor nonet
  $q\bar{q}$ replenished with some possible gluon content
  i.e., $f_0(1370) \to 2\rho \to 4\pi$, $|n\bar{n}\rangle$, $f_0(1500) \to 4\pi, 2\pi$, gluon content

- $f_0(500)/\sigma, f_0(980), a_0(980), K_0^*(700)/\kappa$ form another nonet
  compact tetraquark and $K\bar{K}$ bound state

- the spectral analysis $q\bar{q}$ has one unit of orbital argular momentum which increases the masses, but f_0 and a_0 are mass degeneracy

- in B_s decays $q\bar{q}$ is dominated in the energetic $f_0(980)$
  $q^2\bar{q}^2$ is power suppressed, FSI is also weak [SC, J-M Shen 1907.08401]

- in D_s decays how about the energetic $q\bar{q}$ picture $f_0(980)$?

$D_s \to [f_0 \to] [\pi^+\pi^-]_S e^+\nu_e$ 衰变（D_{l4} 过程）

- $D_{(s)} \to Sl\nu$ decays provide clean environment to study the scalar meson
  $\triangle$ $D_{(s)} \to a_0 e^+\nu$[BESIII 18, 21], $D^+ \to f_0/\sigma e^+\nu$[BESIII 19], $D_s \to f_0(\to \pi^+\pi^-)e^+\nu$[CLEO 09]
- $D_s \to f_0(\to \pi^0\pi^0, K_sK_s)e^+\nu$ [BESIII 22], $D_s \to f_0(\to \pi^+\pi^-)e^+\nu$ [BESIII 23]

$$\mathcal{B}(D_s \to f_0(\to \pi^0\pi^0)e^+\nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$
$$\mathcal{B}(D_s \to f_0(\to \pi^+\pi^-)e^+\nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$
$$f_+^{f_0}(0)|V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

- Theoretical consideration

$$\frac{d\Gamma(D_s^+ \to f_0 l^+\nu)}{dq^2} = \frac{G_F^2|V_{cs}|^2\lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3}|f_+(q^2)|^2$$

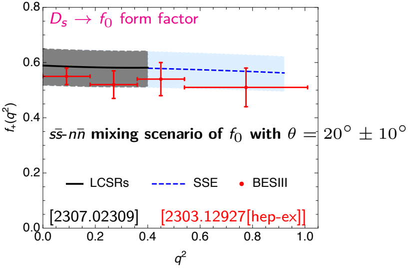
- Improvement with the width effect ($\pi\pi$ invariant mass spectral)

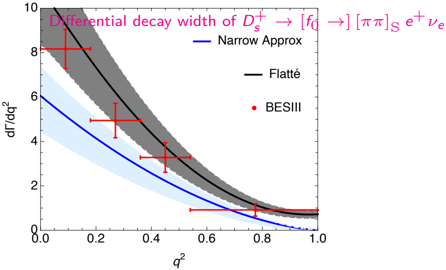
$$\frac{d\Gamma(D_s^+ \to [\pi\pi]_S l^+\nu)}{dsdq^2} = \frac{1}{\pi}\frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s}^3}|f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2)\, g_1^2\beta_\pi(s)}{|m_S^2 - s + i\left(g_1^2\beta_\pi(s)) + g_2^2\beta_K(s)\right)|^2}$$

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_S l^+\nu)}{dk^2 dq^2} = \frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s}^3}\frac{\beta_{\pi\pi}(k^2)\sqrt{\lambda_{D_s}}q^2}{16\pi}\sum_{\ell=0}^{\infty} 2|F_0^{(\ell)}(q^2, k^2)|^2$$

$D_s \to [f_0 \to][\pi^+\pi^-]_{\rm S}\, e^+\nu_e$ 衰变（D_{l4} 过程）

$D_s \to f_0$ form factors $\langle f_0(p_1)|\bar{s}\gamma_\mu\gamma_5 c|D_s^+(p)\rangle = -i\left[f_+(q^2)(p+p_1)_\mu + f_-(q^2)q_\mu\right]$





- Twist-3 LCDAs give dominate contribution in $D_s \to f_0, [\pi\pi]_{\rm S}$ transitions

† does not indicate a breakdown of the twist expansion

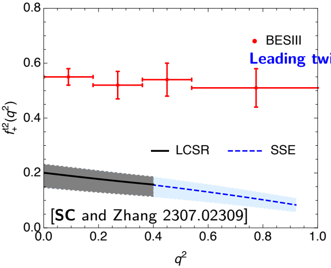
† the asymptotic term in the leading twist LCDAs is zero ($a_0 = 0$) due to the charge conjugate invariance

$D_s \to [f_0 \to] [\pi^+\pi^-]_S \, e^+\nu_e$ 衰变（D_{l4} 过程）

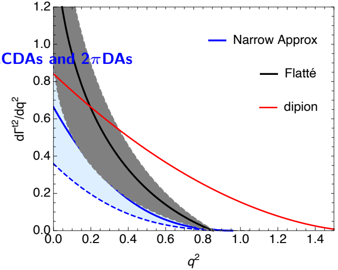
- QCD description in terms of $\pi\pi$ LCDAs

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_S \, l^+\nu)}{dk^2\, dq^2} = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} \frac{\beta_{\pi\pi}(k^2)\sqrt{\lambda_{D_s}} q^2}{16\pi} \sum_{\ell=0}^{\infty} 2|F_0^{(\ell)}(q^2, k^2)|^2$$

- $D_s \to [\pi\pi]_S$ form factors $\langle [\pi(k_1)\pi(k_2)]_S | \bar{s}\gamma_\mu\gamma_5 c | D_s^+(p)\rangle = -iF_0(q^2, s, \zeta) k_\mu^0 + \cdots$



Leading twist of f_0 LCDAs and 2πDAs



- further measurements would help us to understand DiPion system (f_0)

总结和展望

总结和展望

- 两介子系统光锥分布振幅是研究（重介子半轻衰变过程中）不稳定粒子的宽度效应和结构的重要途径
- 相关过程的研究为开展 CKM 矩阵元的独立测量和理解味物理反常提供了一个新思路
- 目前对它的理解还停留在领头扭度，任重道远
- 高精度味物理研究的新增长点（高精确度 +高准确度）

Thank you for your patience.