北京航空航天大學
BEIHANG UNIVERSITY

Exploring the heavy meson distribution amplitudes from first principles

Based on arXiv:2403.17492

In collaboration with:

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Outline

- **Motivation**
- **Heavy meson LCDAs:**
  - **Challenges and recent progresses**
  - **Two-step factorization for heavy meson LCDAs**
  - **LCDAs in QCD and HQET**
- **Summary and outlook**

Motivation

➢ Weak decays of B meson are critical for:

- **Precise tests of SM**
- **Searching for NP**
- **Understanding the origins of CPV**
- **……**

- $B \to \pi\pi$: ***Beneke, Buchalla, Neubert, Sachrajda, 1999; 1422 citations***
  Lu, Ukai, Yang, 2001; 568 citations
- $B \to \pi K$: ***Beneke, Buchalla, Neubert, Sachrajda, 2001; 1177 citations***
- $B \to \pi \ell \nu$: ***Becher, Hill, 2005; 215 citations***
  Khodjamirian, Mannel, Offen, Wang, 2011; 192 citations
- $B \to K^{(*)} \ell\ell$: ***Khodjamirian, Mannel, Pivavorov, Wang, 2010; 486 citations***
- $B \to D \ell \nu$: ***HPQCD Collaboration, 2015; 387 citations***

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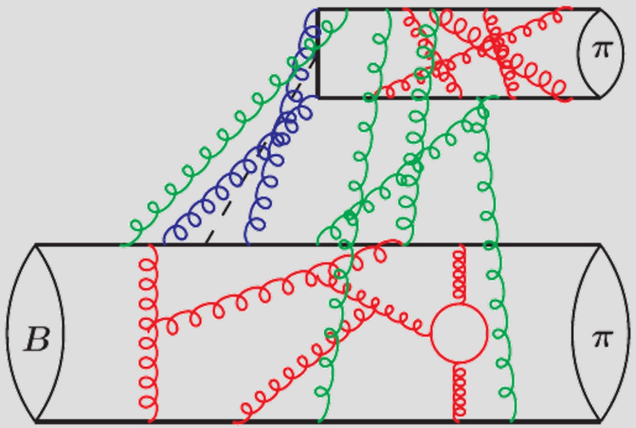
- Factorization: categories by different characteristic scales

$$\langle \pi(p')\pi(q)|Q_i|\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx T_i^{\mathrm{I}}(x)\phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{\mathrm{II}}(\xi,x,y)\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$

Form factor =
Hard kernel + LCDAs

Hard kernel
(Perturbative)

Meson LCDAs
(Nonperturbative)



Light Meson LCDAs: Research Progresses and Challenges

- Light meson LCDAs have been extensively pursued: (1970s - now)

- **Asymptotic LCDAs**
  Chernyak, Zhitnitsky, 1977; Lepage, Brodsky, 1979; Efremov, Radyushkin, 1980
- **Dyson-Schwinger Equation**
  Chang, Cloet, Cobos-Martinez, Roberts, Schmidt, 2013; Gao, Chang, Liu, Roberts, Schmidt, 2014; Roberts, Richards, Chang, 2021
- **Sum rules**
  Chernyak, Zhitnitsky, 1982; Braun, Filyanov, 1989; Ball, Braun, Koike, Tanaka, 1998; Ball, Braun, 1998; Khodjamirian, Mannel, Melcher, 2004; Ball,, Lenz, 2007
- **Inverse Problem**
  Li, 2022
- **Models**
  Arriola, Broniowski, 2002, 2006; Zhong, Zhu, Fu, Wu, Huang, 2021;
- **Global Fits**
  Stefanis, 2020; Cheng, Khodjamirian, Rusov, 2020; Hua, Li, Lu, Wang, Xing, 2021
- **Lattice with current-current correlation**
  Bali, Braun, Gläßle, Göckeler, Gruber, 2017, 2018;
- **Lattice with OPE**
  Martinelli, Sachrajda, 1987; Braun, Bruns, et al., 2016; RQCD collaboration, 2019, 2020
- **Lattice with LaMET**
  Zhang, Chen, Ji, Jin, Lin, 2017; LP3 Collaboration, 2019; Zhang, Honkala, Lin, Chen, 2020; Lin, Chen, Fan, Zhang², 2021; LPC Collaboration, 2021, 2022
- **Quantum Computing**
  QuNu Collaboration, 2023, 2024

Heavy Meson LCDAs: Research Progresses and Challenges

- **The HQET matrix element of heavy meson** [Grozin, Neubert, 1997; Beneke, Feldmann, 2000]

$$\langle 0\,|\bar{q}_\beta(\xi)[\xi,0]h_{v\alpha}(0)|\,\bar{B}(v)\rangle = -\frac{i\tilde{f}_B m_B}{8}\left\{\left[\tilde{\phi}_B^+(t,\mu)v_+\gamma_- + \tilde{\phi}_B^-(t,\mu)v_-\gamma_+\right]\gamma_5\right\}_{\alpha\beta}$$

Leading twist — Sub-leading twist

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Leading twist　　Sub-leading twist

- Evolution of φ_B^+ and φ_B^-: [Lange, Neubert, 2003; Bell, Feldmann, 2008]

$$\frac{d}{d\ln\mu}\varphi_B^+(\omega,\mu) = -\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\gamma_+^{(1)}(\omega,\omega',\mu)\,\varphi_B^+(\omega',\mu) + \mathcal{O}\left(\alpha_s^2\right)$$

$$\frac{d}{d\ln\mu}\varphi_B^-(\omega,\mu) = -\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\gamma_-^{(1)}(\omega,\omega',\mu)\,\varphi_B^-(\omega',\mu) + \mathcal{O}\left(\alpha_s^2\right)$$

with

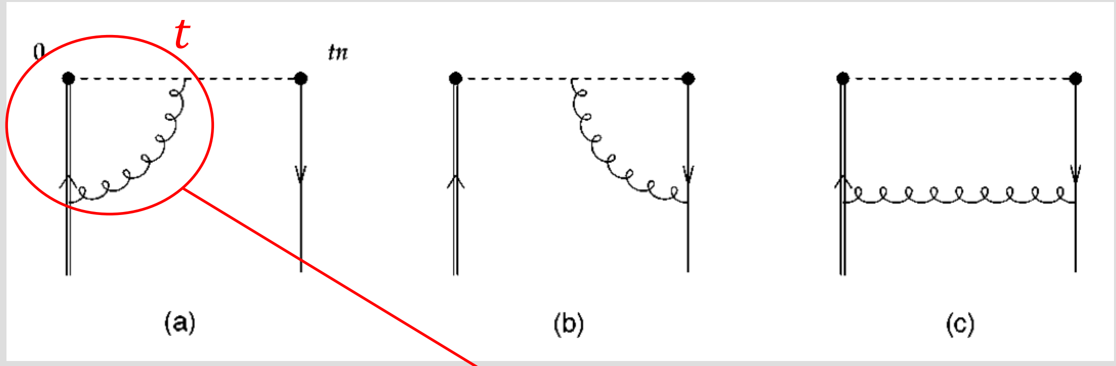
$$\gamma_+^{(1)}(\omega,\omega',\mu) = \left(\Gamma_{\rm cusp}^{(1)}\ \ln\frac{\mu}{\omega} - 2\right)\delta(\omega-\omega') - \Gamma_{\rm cusp}^{(1)}\,\omega\left[\frac{\theta(\omega'-\omega)}{\omega'(\omega'-\omega)} + \frac{\theta(\omega-\omega')}{\omega(\omega-\omega')}\right]_+,$$

$$\gamma_-^{(1)}(\omega,\omega';\mu) = \gamma_+^{(1)}(\omega,\omega';\mu) - \Gamma_{\rm cusp}^{(1)}\frac{\theta(\omega'-\omega)}{\omega'}.$$

- Solution of evolution equations: [Bell, Feldmann, Wang and Yip, 2013; Braun, Manashov, 2014]
- RG equations of $\varphi_B^+(\omega,\mu)$ at two-loops: [Braun, Ji, Manashov, 2019; Liu, Neubert, 2020]
- RG equations of the higher-twist B-meson distribution amplitudes: [Braun, Ji, Manashov, 2017]
- NNLO QCD correction to relevant hadronic B-meson decays: [Bell, Beneke, Huber, Li, 2020]

Heavy Meson LCDAs: Research Progresses and Challenges

➢ **Difficulties in calculating the moments of heavy meson LCDA:**



Diverge at $t \to 0$!

$$O_{+}^{\rm ren}(t,\mu) = O_{+}^{\rm bare}(t) + \frac{\alpha_s C_F}{4\pi}\left\{\left(\frac{4}{\hat{\epsilon}^2} + \frac{4}{\hat{\epsilon}}\ln(it\mu)\right) O_{+}^{\rm bare}(t) - \frac{4}{\hat{\epsilon}}\int_0^1 du \frac{u}{1-u}\left[O_{+}^{\rm bare}(ut) - O_{+}^{\rm bare}(t)\right]\right.$$

[Braun, Ivanov, Korchemsky, 2004]

- Diverge at $t \to 0 \Leftrightarrow$ No local limit
- Non-negative moments $\int dk\, k^n \varphi_+(k)$ for $n=0,1,2,\ldots$ are not related to OPE, and actually they diverge
- Cannot obtain φ_B from lattice QCD through their moments.

Heavy Meson LCDAs: Research Progresses and Challenges

- A theoretical attempt to extract the heavy meson LCDAs in the framework of LaMET:

$$\tilde{\varphi}_B^+(\xi,\mu) \propto \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} e^{in_z\cdot v\xi\tau} \langle 0 \left| \boxed{(\bar{q}W_c)(\tau n_z)\,\eta_z\gamma_5\,(W_c^\dagger h_v)(0)} \right| \bar{B}(v)\rangle \quad \Rightarrow \quad \varphi_B^+(\omega,\mu)$$

Connecting the between **equal-time HQET correlator** and **light-cone one**.

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Connecting the between **equal-time HQET correlator** and **light-cone one**.

- Leading twist matching @ 1-loop: *[Wang[2], Xu, Zhao, 2020; Xu, Zhang, 2022]*
- Sub-leading twist matching: *[Hu, Wang, Xu, Zhao, 2024]*
- Inverse moment and log moments: *[Xu, Zhang, Zhao, 2022; Hu, Xu, Zhao, 2024]*
- Ioffe-time distributions: *[Zhao, Radyushkin, 2021]*

🤔 Difficult to realize the boosted HQET field (time-like Wilson link) on lattice QCD.

Two-step factorization to access heavy meson LCDA

- Focus on the **Quasi DA with QCD fields**, which is easy to implement in LQCD:
  - Light meson (2 characteristic scales: $P^z, \Lambda_{\rm QCD}$) → Heavy meson (3 scales: $P^z, m_H, \Lambda_{\rm QCD}$)

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$(P^z, m_H, \Lambda_{\rm QCD})$ **Quasi DA** —**LaMET**→ $(m_H, \Lambda_{\rm QCD})$ **LCDA in QCD** —**bHQET**→ $(\Lambda_{\rm QCD})$ **LCDA in HQET**

LaMET: $\mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$

bHQET: $\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_H}\right)$

- A multi-scale processes:

1. LaMET requires $\Lambda_{\rm QCD}, m_H \ll P^z$ and finally integrate out P^z;
2. bHQET requires $\Lambda_{\rm QCD} \ll m_H$ and integrate out m_H;

⇒ **Hierarchy $\boldsymbol{\Lambda_{\rm QCD} \ll m_H \ll P^z}$.**

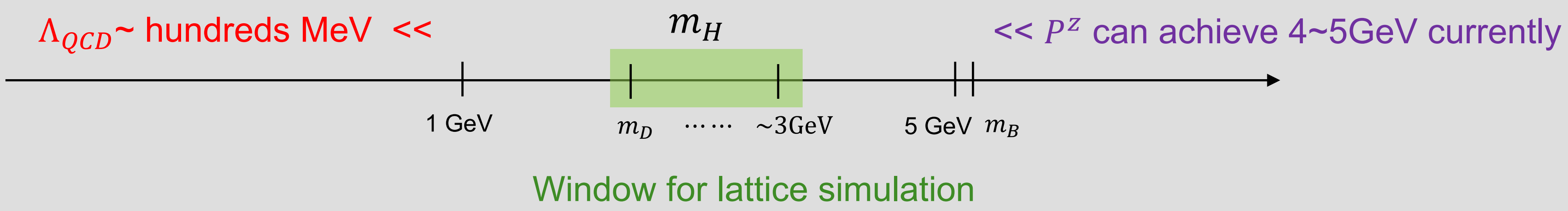
Two-step factorization to access heavy meson LCDA

$(P^z, m_H, \Lambda_{\rm QCD})$ **Quasi DA** —**LaMET**→ $(m_H, \Lambda_{\rm QCD})$ **LCDA in QCD** —**bHQET**→ $(\Lambda_{\rm QCD})$ **LCDA in HQET**

LaMET: $\mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$

bHQET: $\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_H}\right)$

⇒ **Hierarchy $\boldsymbol{\Lambda_{\rm QCD} \ll m_H \ll P^z}$:** A big challenge for lattice simulation



<u>At this stage, the heavy meson could be *D*, but hard to directly implement *B*!</u>

Matching I: from quasi DAs to LCDAs in QCD

- D meson quasi DA $\tilde{\phi}(x, P^z)$, include the scales $\Lambda_{\rm QCD} \ll m_D \ll P^z$

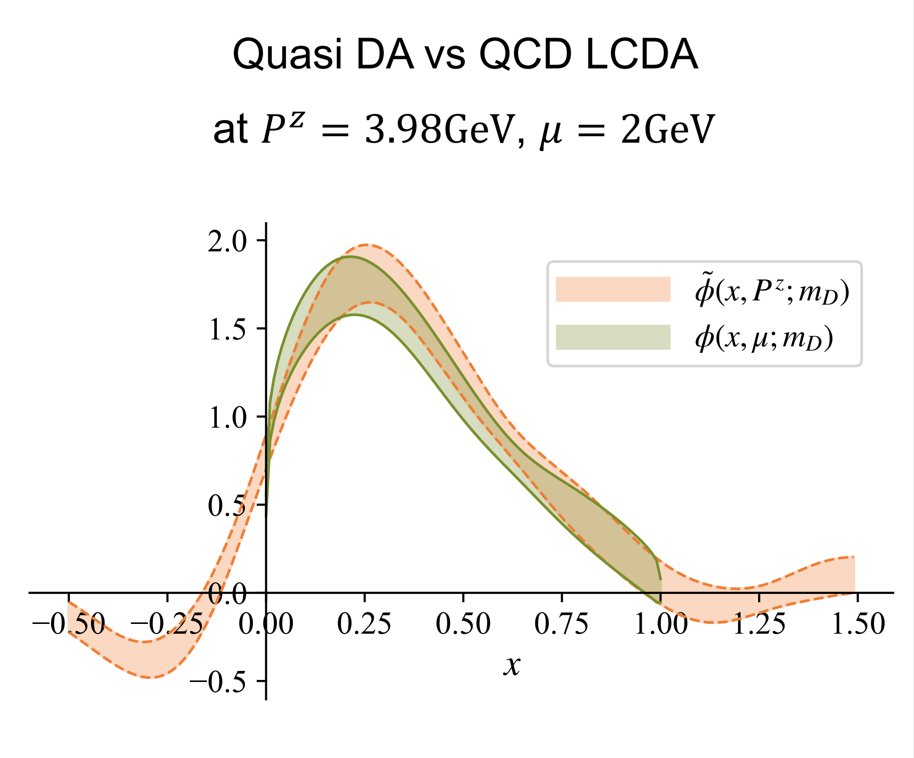
$$\tilde{\phi}(x, P^z) = \int \frac{dz}{2\pi} e^{-ixP^z z} \tilde{M}(z, P^z)$$

- Matching formula in LaMET:

$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

Liu, Wang, Xu, QAZ, Zhao, 2019;
Han, Hua, Ji, Lu, Wang, Xu, QAZ, Zhao, 2024

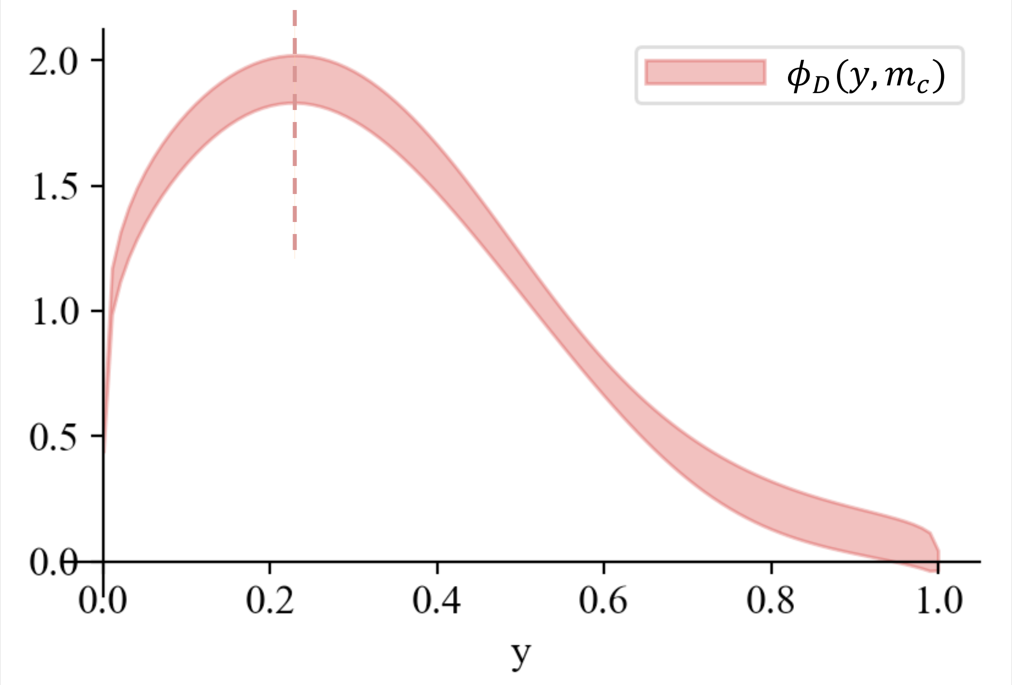
This matching integrate out P^z, obtain the LCDAs in QCD.



LCDAs in QCD

➢ Heavy meson LCDAs in QCD

$$\phi(y,\mu) = \frac{1}{if_H}\int_{-\infty}^{+\infty}\frac{d\tau}{2\pi}e^{iyP_H\tau n_+} \times \langle 0\,|\bar{q}(\tau n_+)\not{n}_+\gamma_5 W_c(\tau n_+,0)Q(0)|\,H(P_H)\rangle$$



- The peak position dominated by m_H and μ;
- At very large scale $\mu \gg m_H$, asymptotic form;

LCDAs in QCD

- Heavy meson LCDAs in QCD

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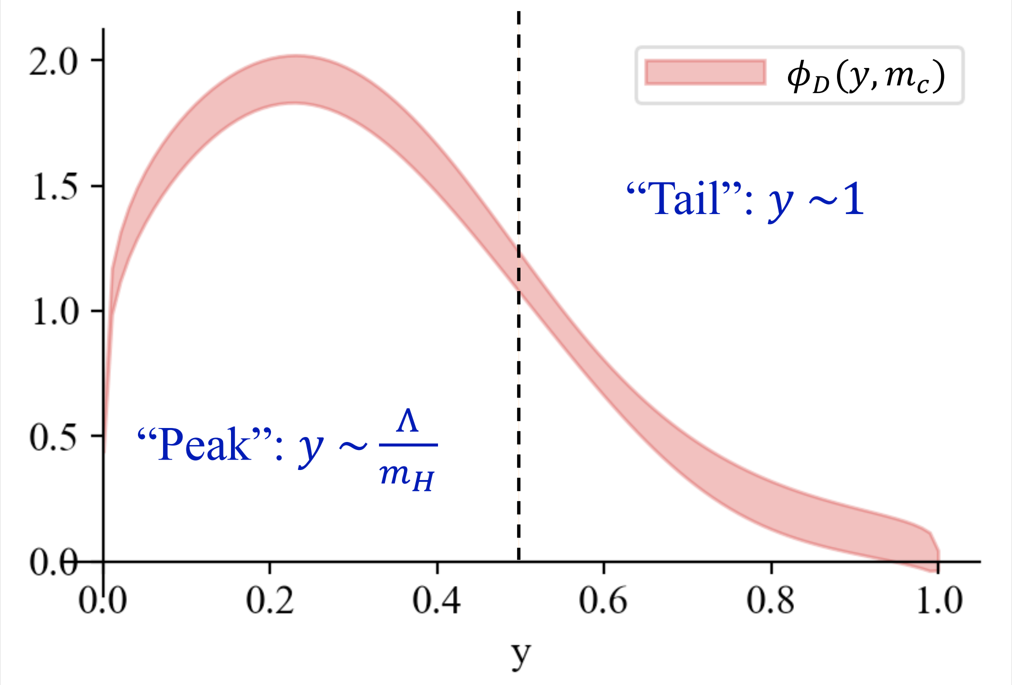


- The peak position dominated by m_H and μ;
- At very large scale $\mu \gg m_H$, asymptotic form;
- For the scale $\mu \lesssim m_Q$,
  - Light quark carries small momentum fraction $y \sim \Lambda/m_H$ peak region, related to the HQET LCDA;

    [Ishaq, Jia, Xiong, Yang, 2020; Beneke, Finauri, Vos, Wei, 2023]
  - $y \sim O(1)$ region be suppressed in LCDA:

    Only hard-collinear physics (perturbative), and starts at the one-loop level.

Matching II: connecting LCDAs in QCD and HQET

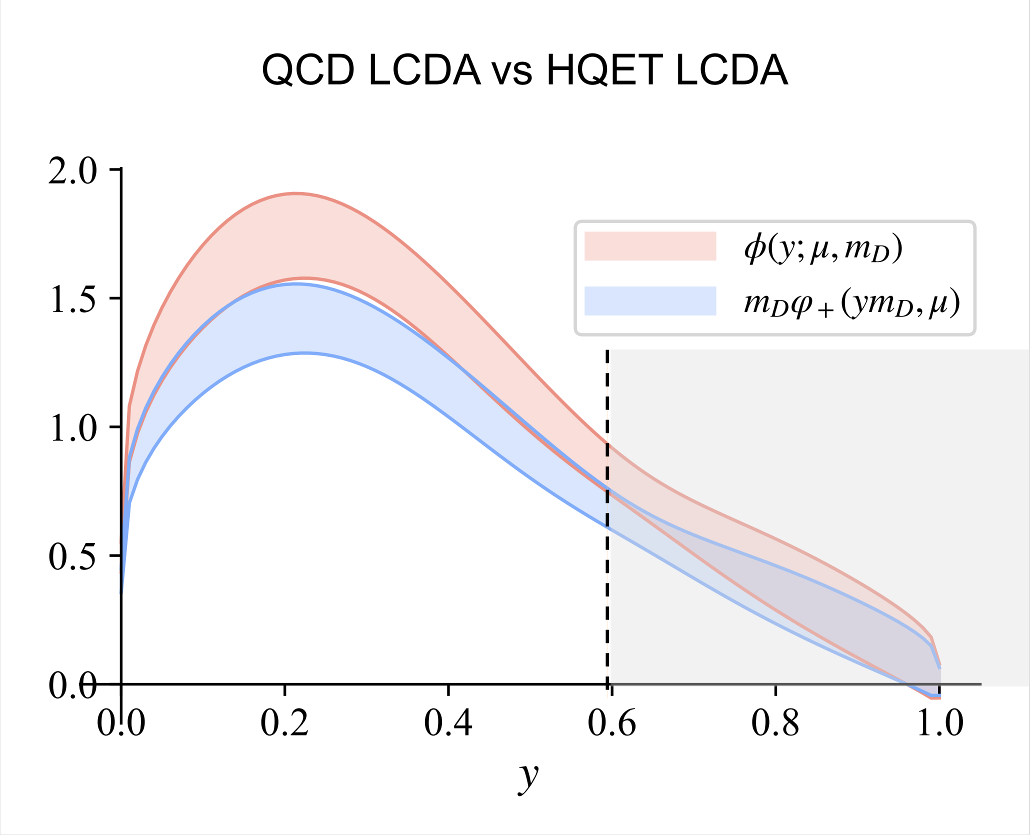
- Leading twist heavy meson LCDA in HQET

$$\varphi^+(\omega,\mu) = \frac{1}{i\tilde{f}_H(\mu)m_H}\int_{-\infty}^{+\infty}\frac{d\eta}{2\pi}\,e^{i\omega n_+\cdot v\eta} \times \langle 0\,|\bar{q}(\eta n_+)\not{n}_+\gamma_5 W_c(\eta n_+,0)h_v(0)|\,H(v)\rangle$$

connected with the QCD LCDA through a multiplicative factorization in the peak region:

[Beneke, Finauri, Vos, Wei, 2023]

$$\phi\,(y,\mu;m_H) = \frac{\tilde{f}_H}{f_H} J_{\text{peak}} m_H \varphi^+\,(\omega,\mu) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_H}\right)$$



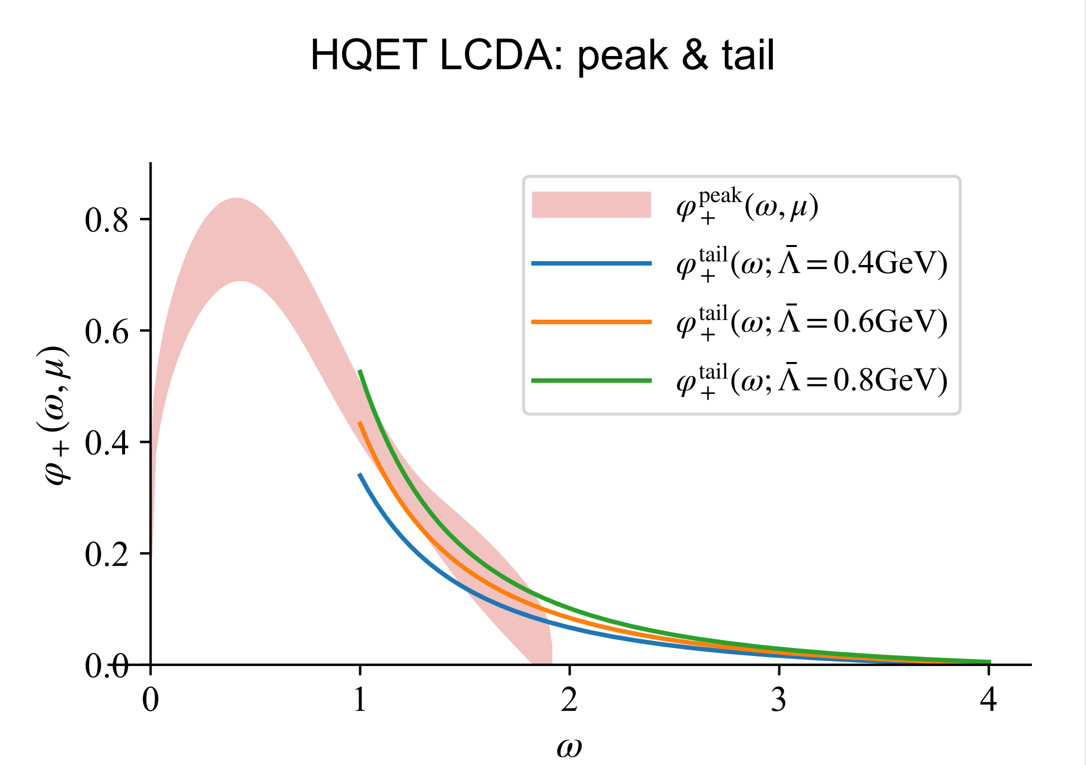
Tails of HQET LCDA

➢ The tail region of HQET LCDA is perturbative: *[Lee, Neubert, 2005]*

$$\varphi^{+}_{\mathrm{tail}}(\omega,\mu) = \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right]$$

where $\bar{\Lambda} \equiv m_H - m_Q^{\mathrm{pole}}$ reflect the power correction, and usually be chosen as 400~600MeV.

The final results of HQET LCDA will merge the peak (from LQCD) and tail region (from 1-loop calculation).



Comparison with phenomenological models

➢ Several commonly used models: [Wang, Shen, 2015; Beneke, Braun, Ji, Wei, 2018; Gao, Huber, Ji, Wang², 2022]

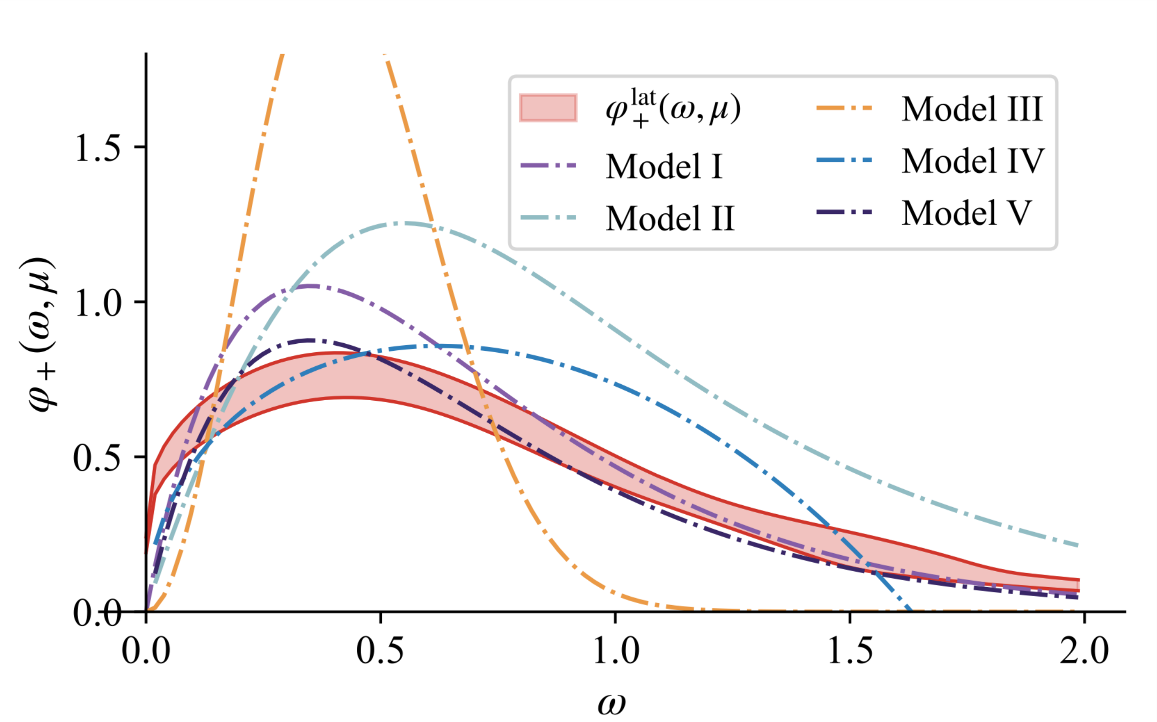
$$\varphi_{\mathrm{I}}^{+}(\omega,\mu_0)=\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0}\,,$$

$$\varphi_{\mathrm{II}}^{+}(\omega,\mu_0)=\frac{4}{\pi\omega_0}\frac{k}{k^2+1}\left[\frac{1}{k^2+1}-\frac{2\left(\sigma_B^{(1)}-1\right)}{\pi^2}\ln k\right]\,,$$

$$\varphi_{\mathrm{III}}^{+}(\omega,\mu_0)=\frac{2\omega^2}{\omega_0\omega_1^2}e^{-(\omega/\omega_1)^2}\,,$$

$$\varphi_{\mathrm{IV}}^{+}(\omega,\mu_0)=\frac{\omega}{\omega_0\omega_2}\frac{\omega_2-\omega}{\sqrt{\omega\left(2\omega_2-\omega\right)}}\theta\left(\omega_2-\omega\right)\,,$$

$$\varphi_{\mathrm{V}}^{+}(\omega,\mu_0)=\frac{\Gamma(\beta)}{\Gamma(\alpha)}\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0}U(\beta-\alpha,3-\alpha,\omega/\omega_0)\,,$$



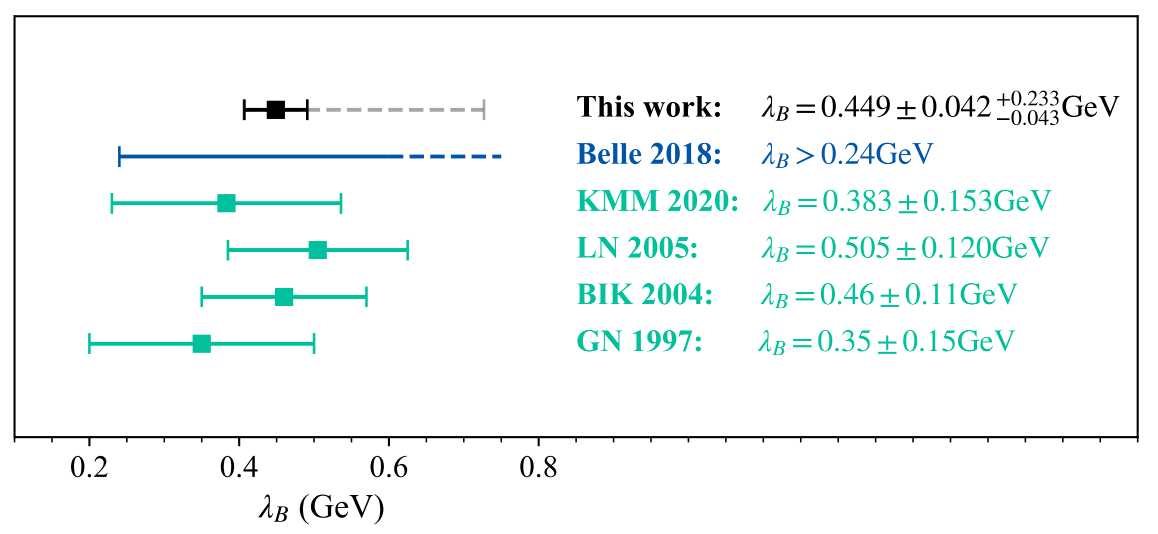
First inverse moment

➢ The first inverse moment

$$\lambda_B^{-1}(\mu) = \int_{-\infty}^{\infty} d\omega \frac{\varphi^+(\omega,\mu)}{\omega}$$

Models	I	II	III	IV	V
Parameters	$\omega_0 = 0.433(23)\mathrm{GeV}$	$\omega_0 = 0.682(45)\mathrm{GeV}$ $\sigma_B^{(1)} = 2.78(48)$	——	$\omega_0 = 0.427(21)\mathrm{GeV}$	$\omega_0 = 0.449(42)\mathrm{GeV}$
fit range	$\omega \in [0.2, 1.4]\mathrm{GeV}$	$\omega \in [0.2, 1.4]\mathrm{GeV}$		$\omega \in [0.4, 0.8]\mathrm{GeV}$	$\omega \in [0.2, 1.4]\mathrm{GeV}$
χ^2/d.o.f	1.4	1.2		2.1	1.0

- The current numerical results are unable to accomplish the integration over full-ω range;
- We determine the λ_B^{-1} by fitting the parameterization forms of different model.



Belle collaboration, 2018
Khodjamirian, Mandal, Mannel, 2020 Lee, Neubert, 2005
Braun, Ivanov, Korchemsky, 2004
Grozin, Neubert, 1997

Summary and outlook

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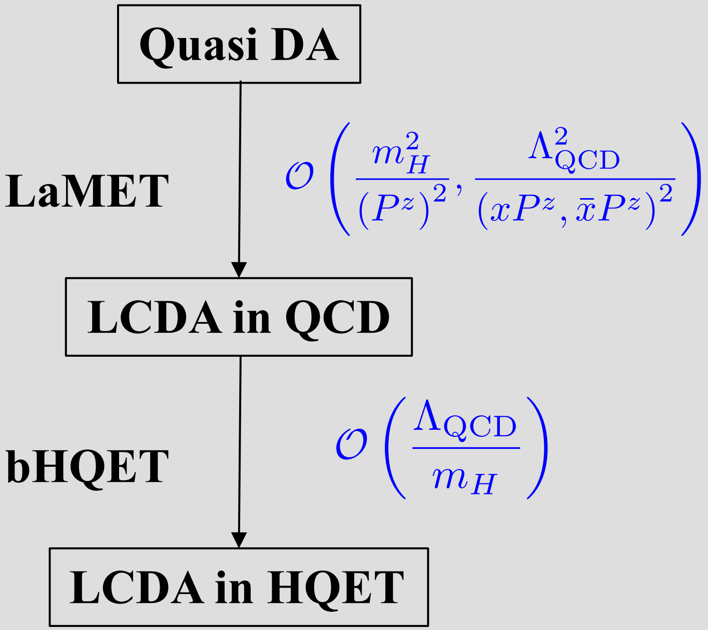
Summary and outlook

✓ **Heavy meson LCDAs**: the first lattice-implementable scheme, which can continually be **improved**.

- ☐ The most urgent is to properly control the power corrections within two step factorization:
  - Larger P^z to increase the window for lattice calculations;
  - $m_D \to m_B$ is hard, LQCD can simulate some unphysical m_H which satisfy the hierarchy better;
- ☐ More systematic lattice QCD calculations:
  - Nonperturbative renormalization, continuum and physical mass extrapolation, operator mixing effects, ……
  - More reliable method to merge the peak and tail regions
- ☐ Realize the HQET quasi DA on lattice QCD directly?



Thanks