Based on arXiv: **2403.07499**

Wen-Fei Wang, Li-Fei Yang, Ai-Jun Ma, Angels Ramos

The low-mass enhancement of kaon pair in the decays $B^+ \to \overline{D}^{(*)0} K^+ \overline{K}^0$ *and* $B^0 \to D^{(*)-} K^+ \overline{K}^0$

王文飞（山西大学）

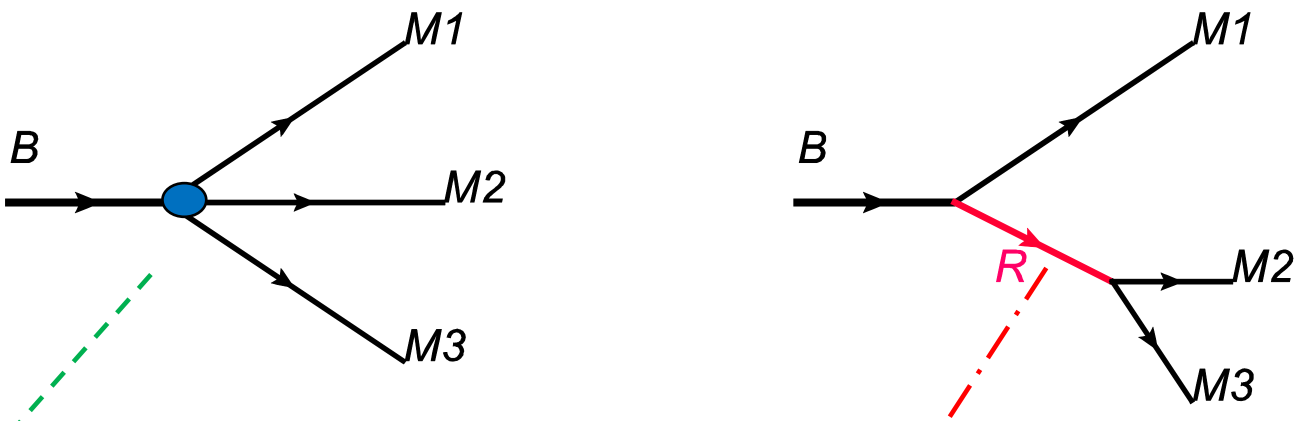
第六届重味物理与量子色动力学研讨会
中国海洋大学　2024-04-21

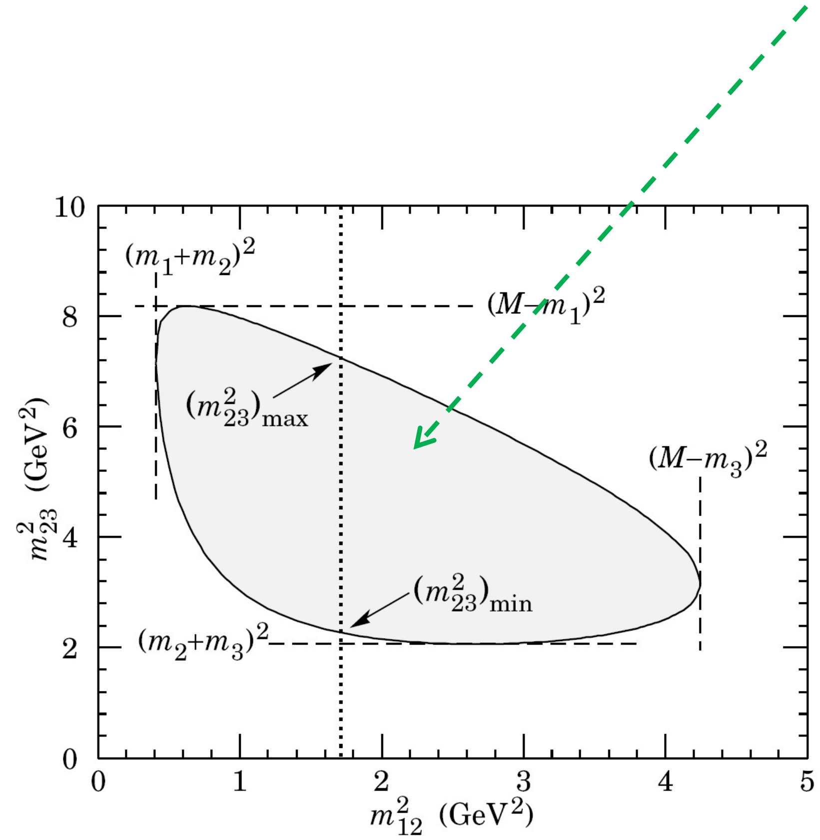
Outline

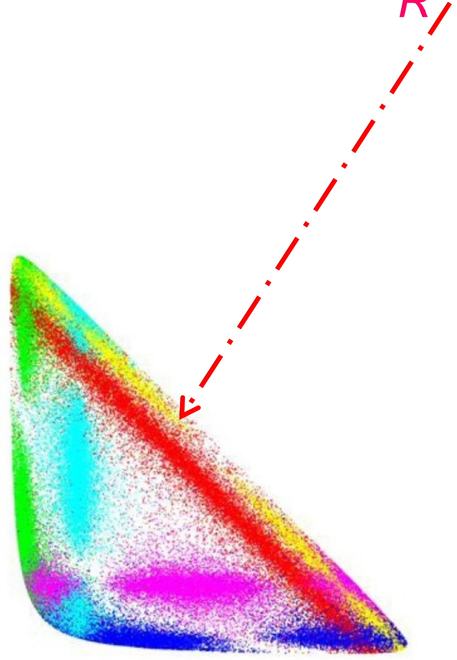
- **3-body B meson decays and the virtual contribution**
- **Kaon pair in $B^+ \to \bar{D}^{(*)0} K^+ \bar{K}^0$ and $B^0 \to D^{(*)-} K^+ \bar{K}^0$ decays**
- **Results and discussions**
- **Conclusions**

3-body B meson decays and the virtual contribution

B → M1 M2 M3

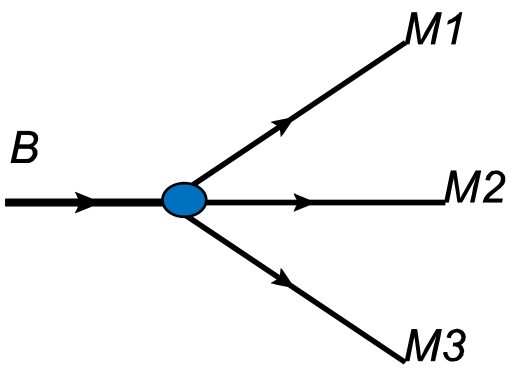


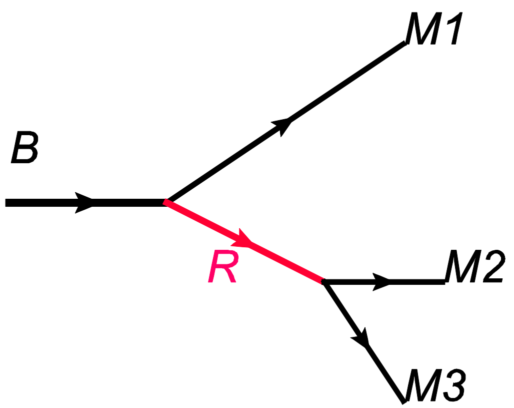




Dalitz Plot

$B \to M1\ M2\ M3$





The weak effective Hamiltonian:

1.

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\Big\{V_{ub}V_{uq}^*[C_1(\mu)Q_1^u(\mu) + C_2(\mu)Q_2^u(\mu)] - V_{tb}V_{tq}^*\Big[\sum_{i=3}^{10} C_i(\mu)Q_i(\mu)\Big]\Big\} + \text{H.c.}, \quad (2)$$

where $q = d, s$. The functions Q_i $(i = 1, \ldots, 10)$ are the local four-quark operators:

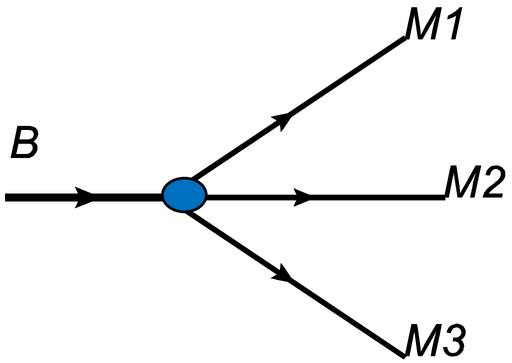
The total amplitude within isobar approach:

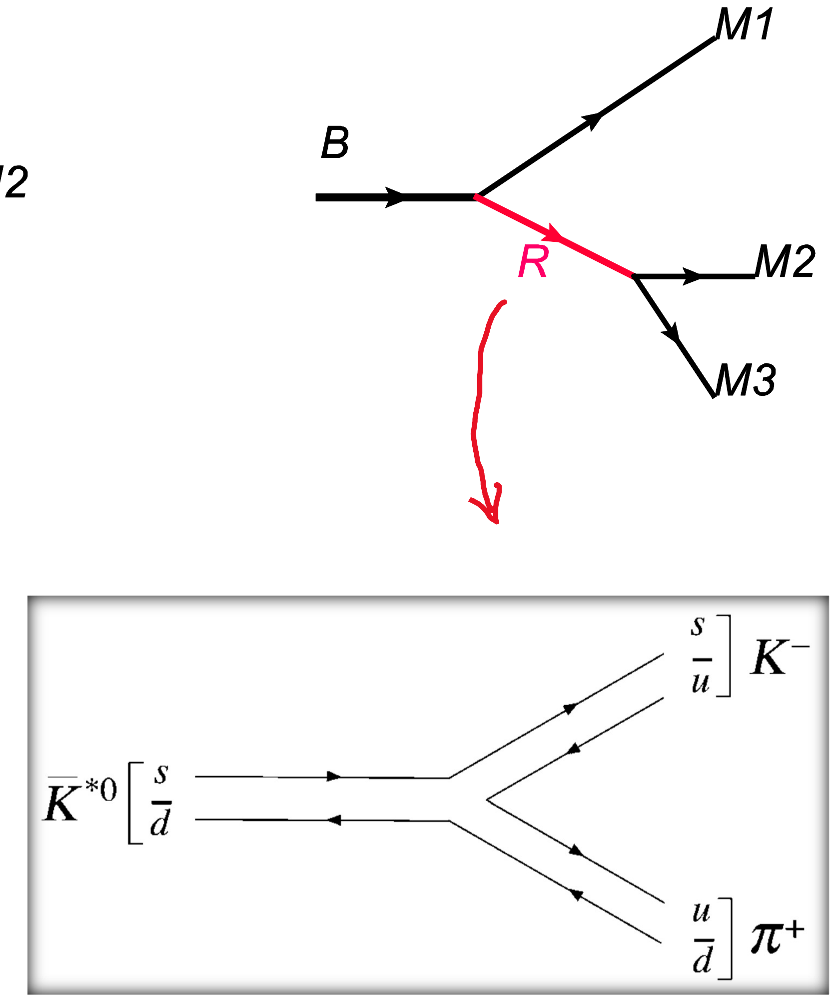
2.

$$\mathcal{A}_{total} = \sum_i \mathcal{A}_{NR}^i + \sum_j \mathcal{A}_R^j$$

3-body B meson decays and the virtual contribution

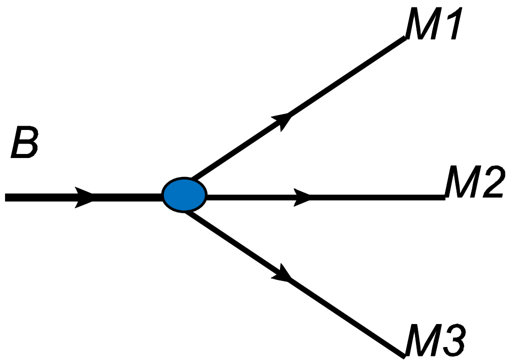
$$B^+ \to K^+ K^- \pi^+$$

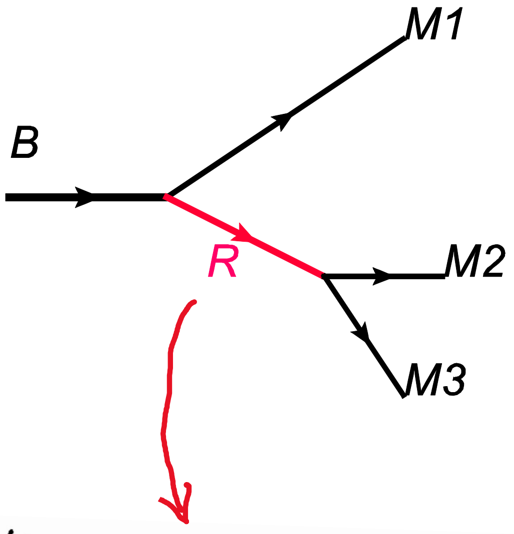




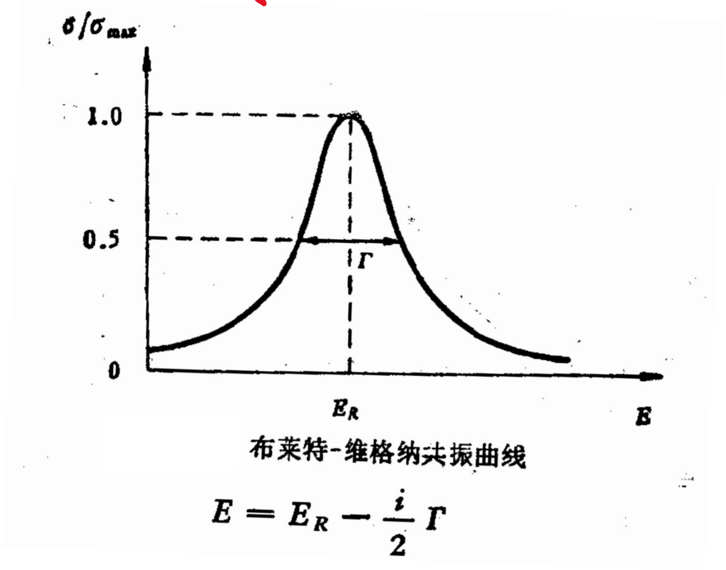
3-body B meson decays and the virtual contribution

$$B^+ \to K^+K^-\pi^+$$



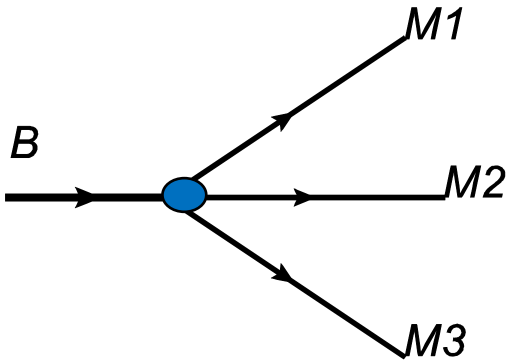


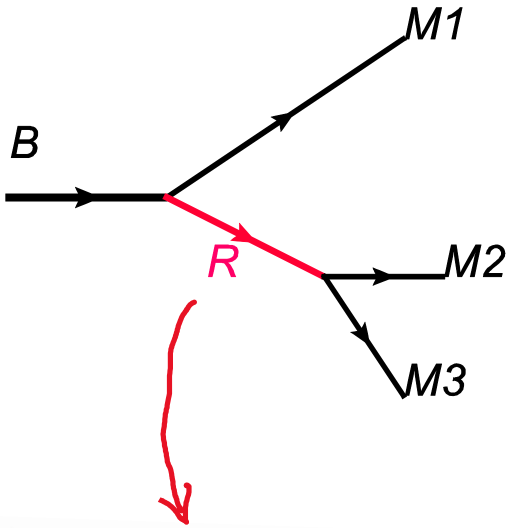
$$\mathrm{BW}_R = \frac{m_R^2}{m_R^2 - s - i m_R \Gamma_R(s)},$$



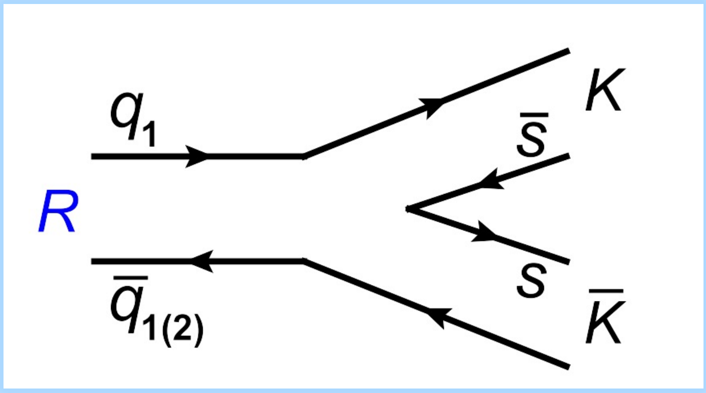
3-body B meson decays and the virtual contribution

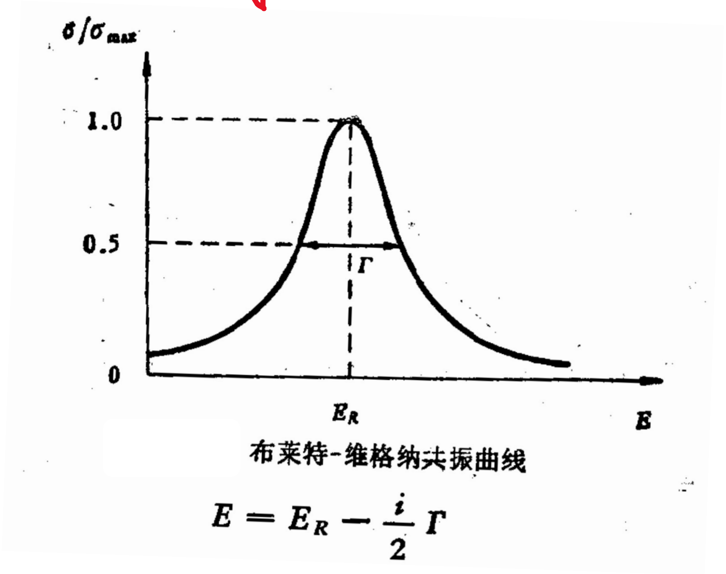
$$B^+ \to K^+ K^- \pi^+$$





$$\rho(770)^0 \xrightarrow{?} K^+ K^-$$

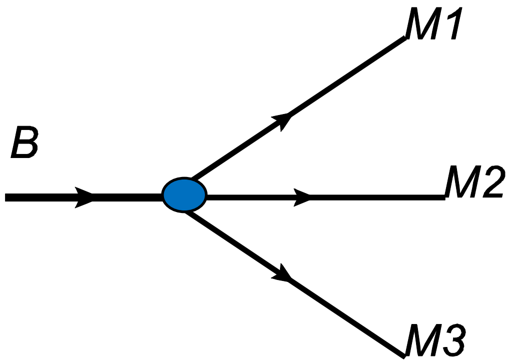


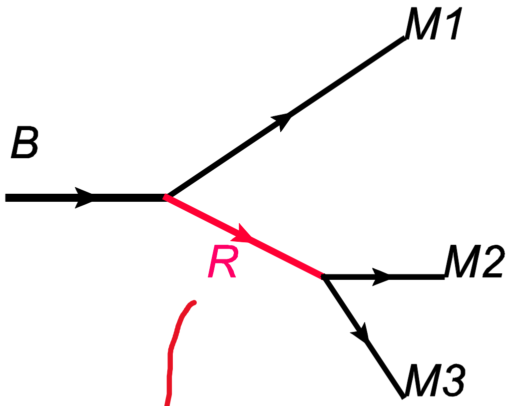


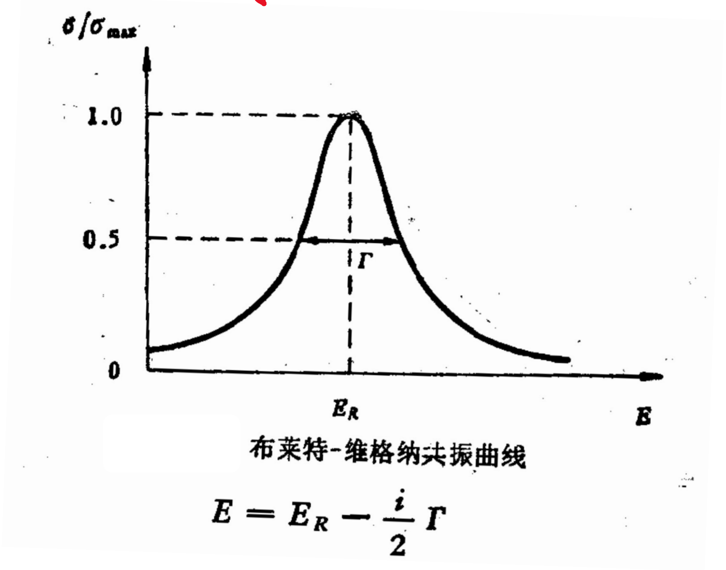
3-body B meson decays and the virtual contribution

$$B^+ \to K^+K^-\pi^+$$

$$\rho(770)^0 \xrightarrow{?} K^+K^-$$







PHYSICAL REVIEW LETTERS **123,** 231802 (2019)

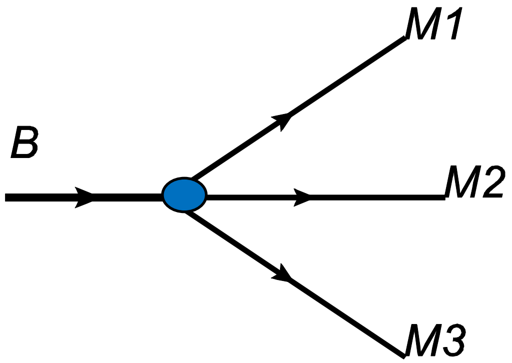
Amplitude Analysis of $B^\pm \to \pi^\pm K^+ K^-$ Decays

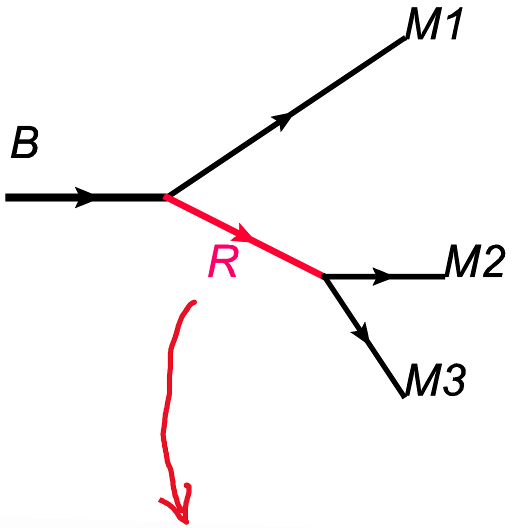
R. Aaij *et al.**
(LHCb Collaboration)

(Received 12 June 2019; revised manuscript received 15 October 2019; published 6 December 2019)

3-body B meson decays and the virtual contribution

$$B^+ \to K^+K^-\pi^+$$





$$\rho(770)^0 \xrightarrow{?} K^+K^-$$

PHYSICAL REVIEW LETTERS 123, 231802 (2019)

PHYSICAL REVIEW D 101, 111901(R) (2020)

Rapid Communications

Will the subprocesses $\rho(770, 1450)^0 \to K^+K^-$ contribute large branching fractions for $B^\pm \to \pi^\pm K^+K^-$ decays?

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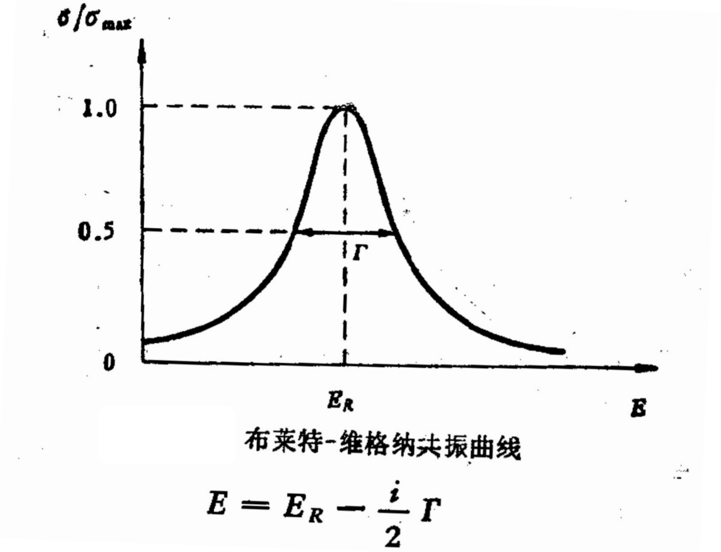

3-body B meson decays and the virtual contribution

PHYSICAL REVIEW D **94,** 072001 (2016)

Amplitude analysis of $B^- \to D^+\pi^-\pi^-$ decays

R. Aaij *et al.**
(LHCb Collaboration)

(Received 4 August 2016; published 5 October 2016)

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Contribution	Fit fraction (%)
$D_2^*(2460)^0$	35.7 ± 0.6
$D_1^*(2680)^0$	8.3 ± 0.6
$D_3^*(2760)^0$	1.0 ± 0.1
$D_2^*(3000)^0$	0.23 ± 0.07
$D_v^*(2007)^0$	10.8 ± 0.7
B_v^{*0}	2.7 ± 1.0
Total S wave	57.0 ± 0.8
Total fit fraction	115.7

PHYSICAL REVIEW D VOLUME 15, NUMBER 11 1 JUNE 1977

High-statistics study of the reactions $\pi^- p \to K^- K^+ n$ and $\pi^+ n \to K^- K^+ p$ at 6 GeV/c*

A. J. Pawlicki, D. S. Ayres, D. Cohen, R. Diebold, S. L. Kramer, and A. B. Wicklund
Argonne National Laboratory, Argonne, Illinois 60439
(Received 23 December 1976; revised manuscript received 15 March 1977)

ambiguities. As discussed above, the P wave is consistent with the tail of the ρ^0 decaying into K^-K^+, with a ρKK coupling that agrees with SU(3), including the sign. Only one of the ambiguous sol-

3-body B meson decays and the virtual contribution

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BW-Tail

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Volume 25, Issue 4, 4 September 1967, Pages 294-297

Further study of the $I = 1 K\bar{K}$ structure near threshold

A. Astier, J. Cohen-Ganouna, M. Della Negra, B. Maréchal, L. Montanet, M. Tomas †, M. Baubillier, J. Duboc

3) The $I = 1$ $K\bar{K}$ channel is dominated, at threshold, by a virtual bound state resonance

Page-4

Kaon pair in $B^+ \to \overline{D}^{(*)0} K^+ \overline{K}^0$ and $B^0 \to D^{(*)-} K^+ \overline{K}^0$ decays

MOTIVATION:

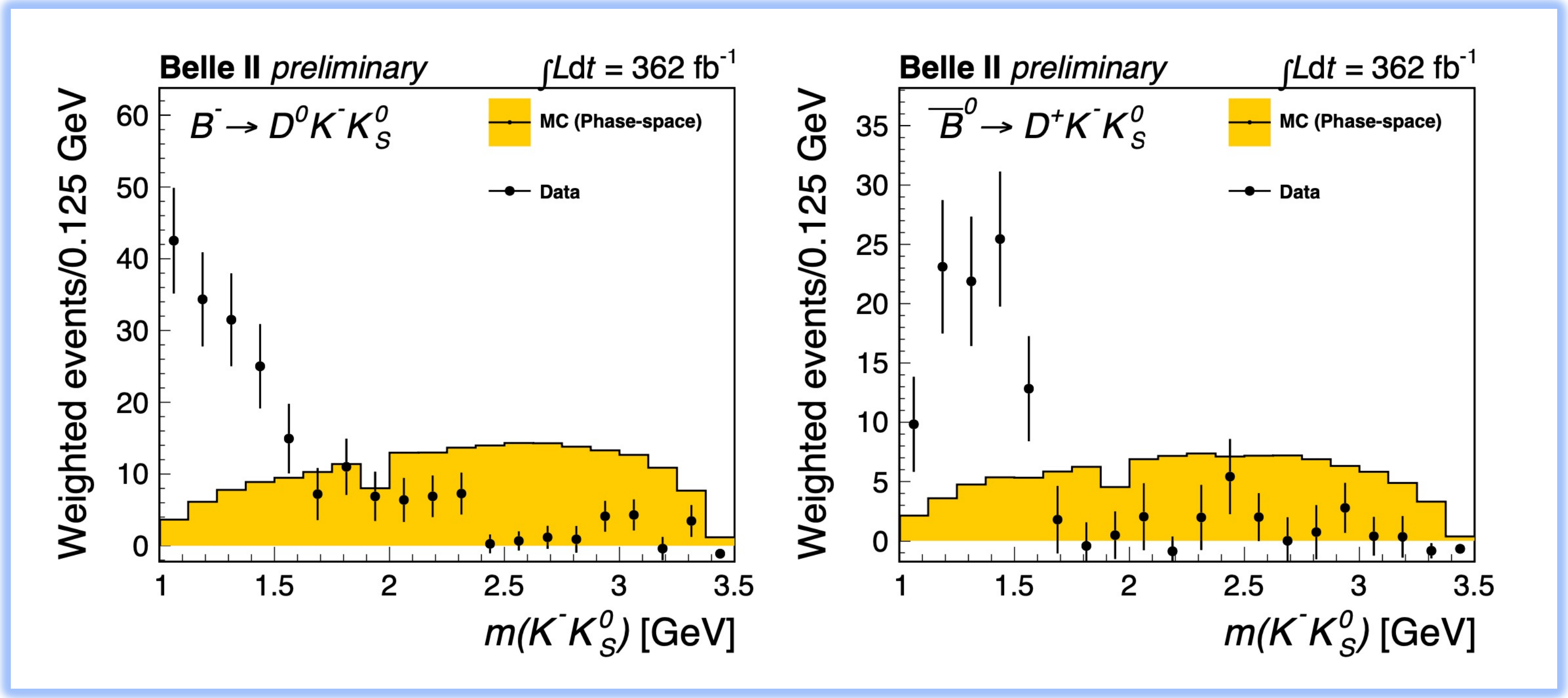
Belle-II arXiv:2305.01321

$$
\begin{aligned}
\mathcal{B}(B^- \to D^0 K^- K_S^0) &= (1.89 \pm 0.16 \pm 0.10) \times 10^{-4}, \\
\mathcal{B}(\overline{B}^0 \to D^+ K^- K_S^0) &= (0.85 \pm 0.11 \pm 0.05) \times 10^{-4}, \\
\mathcal{B}(B^- \to D^{*0} K^- K_S^0) &= (1.57 \pm 0.27 \pm 0.12) \times 10^{-4}, \\
\mathcal{B}(\overline{B}^0 \to D^{*+} K^- K_S^0) &= (0.96 \pm 0.18 \pm 0.06) \times 10^{-4},
\end{aligned}
$$

Kaon pair in $B^+ \to \overline{D}^{(*)0} K^+ \overline{K}^0$ and $B^0 \to D^{(*)-} K^+ \overline{K}^0$ decays

Motivation:

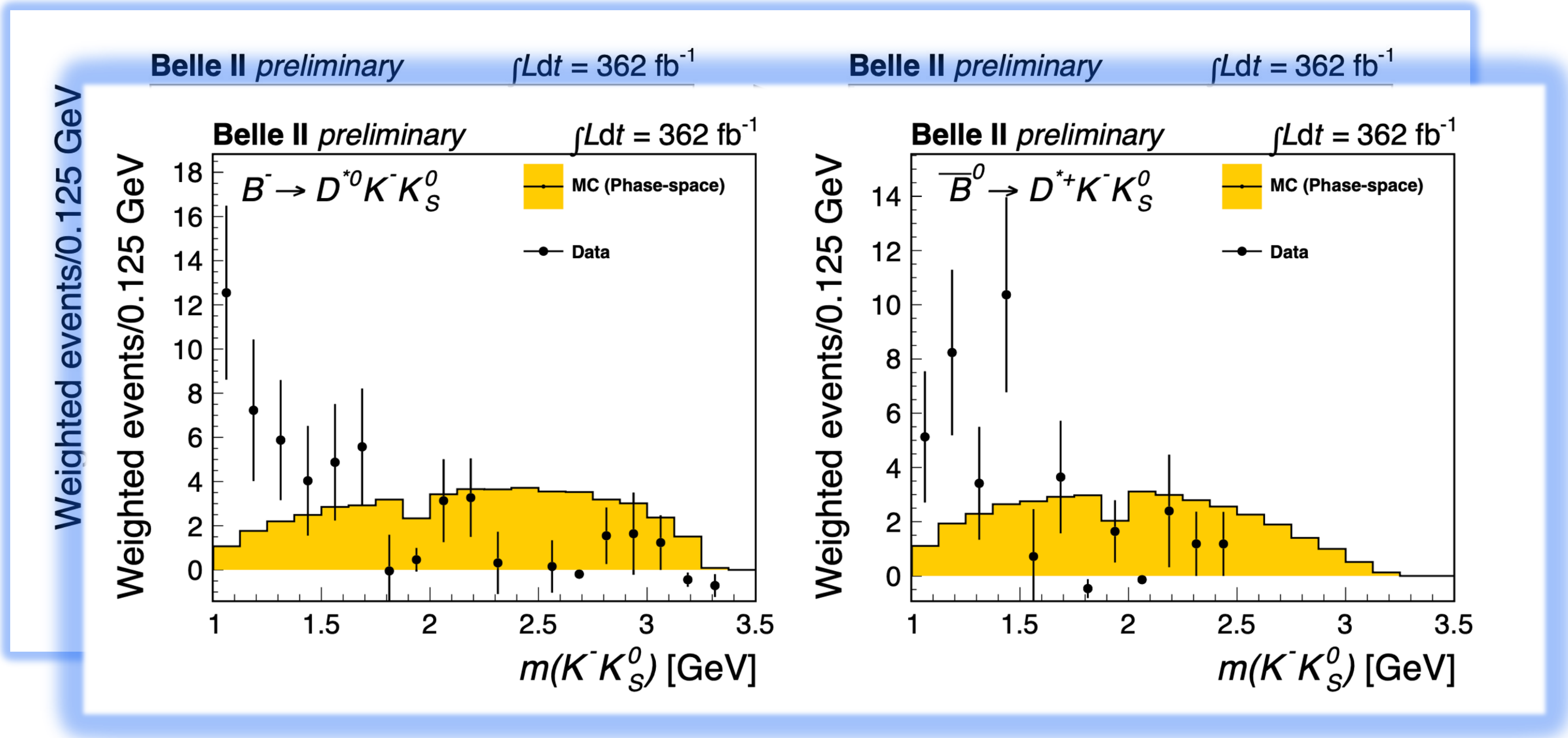
Belle-II arXiv:2305.01321



Kaon pair in $B^+ \to \overline{D}^{(*)0} K^+ \overline{K}^0$ and $B^0 \to D^{(*)-} K^+ \overline{K}^0$ decays

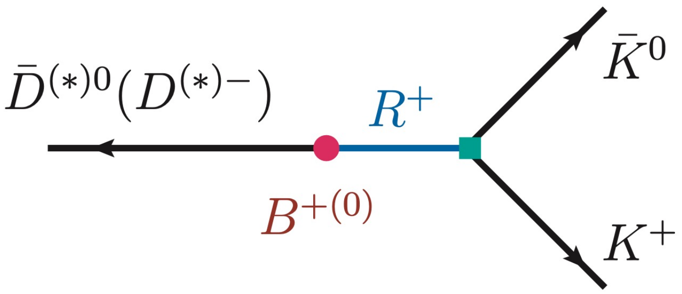
MOTIVATION:

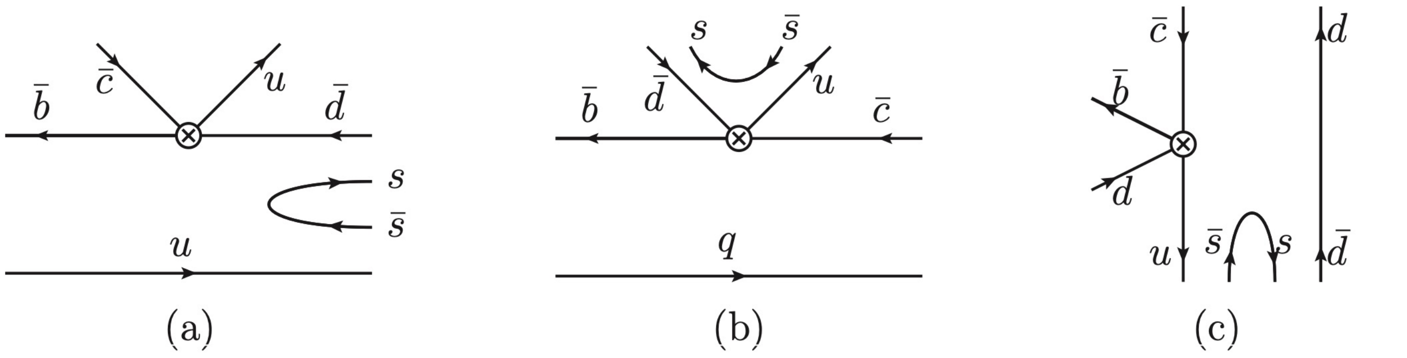
Belle-II arXiv:2305.01321



Kaon pair in $B^+ \to \overline{D}^{(*)0} K^+ \overline{K}^0$ and $B^0 \to D^{(*)-} K^+ \overline{K}^0$ decays

APPROACH:



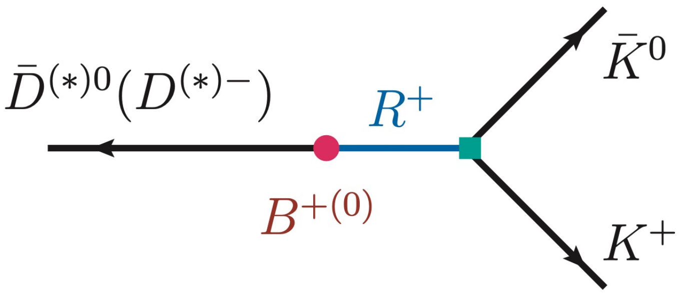


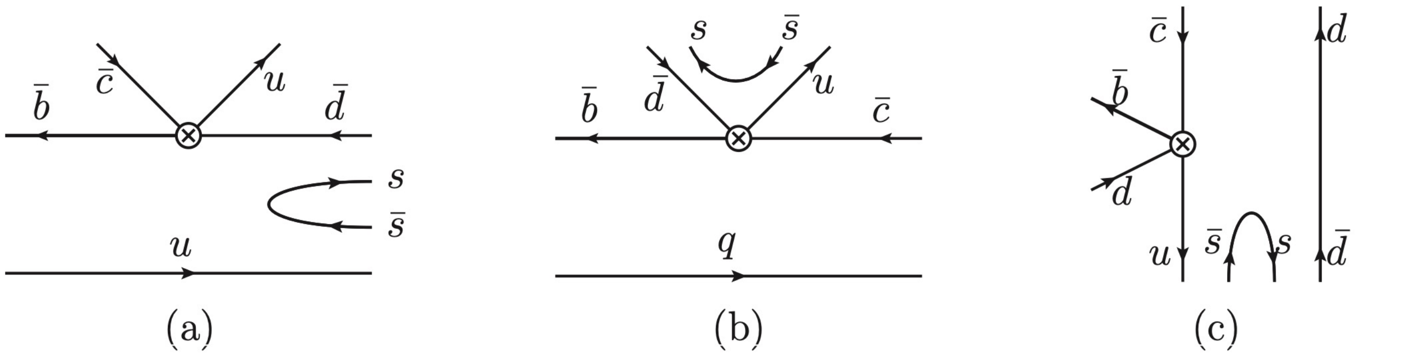
$$\mathcal{A}_V = M_{2B} \cdot \frac{\langle hh'|\rho^+\rangle}{\mathcal{D}_{\rho^+}(s)}$$

$$F^R_{\pi,K}(s) = c_R^{\pi,K}\mathrm{BW}_R(s) \equiv c_R^{\pi,K}\frac{m_R^2}{\mathcal{D}_R(s)},$$

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud}\left[C_1(\mu)O_1^c(\mu) + C_2(\mu)O_2^c(\mu)\right],$$

Approach:



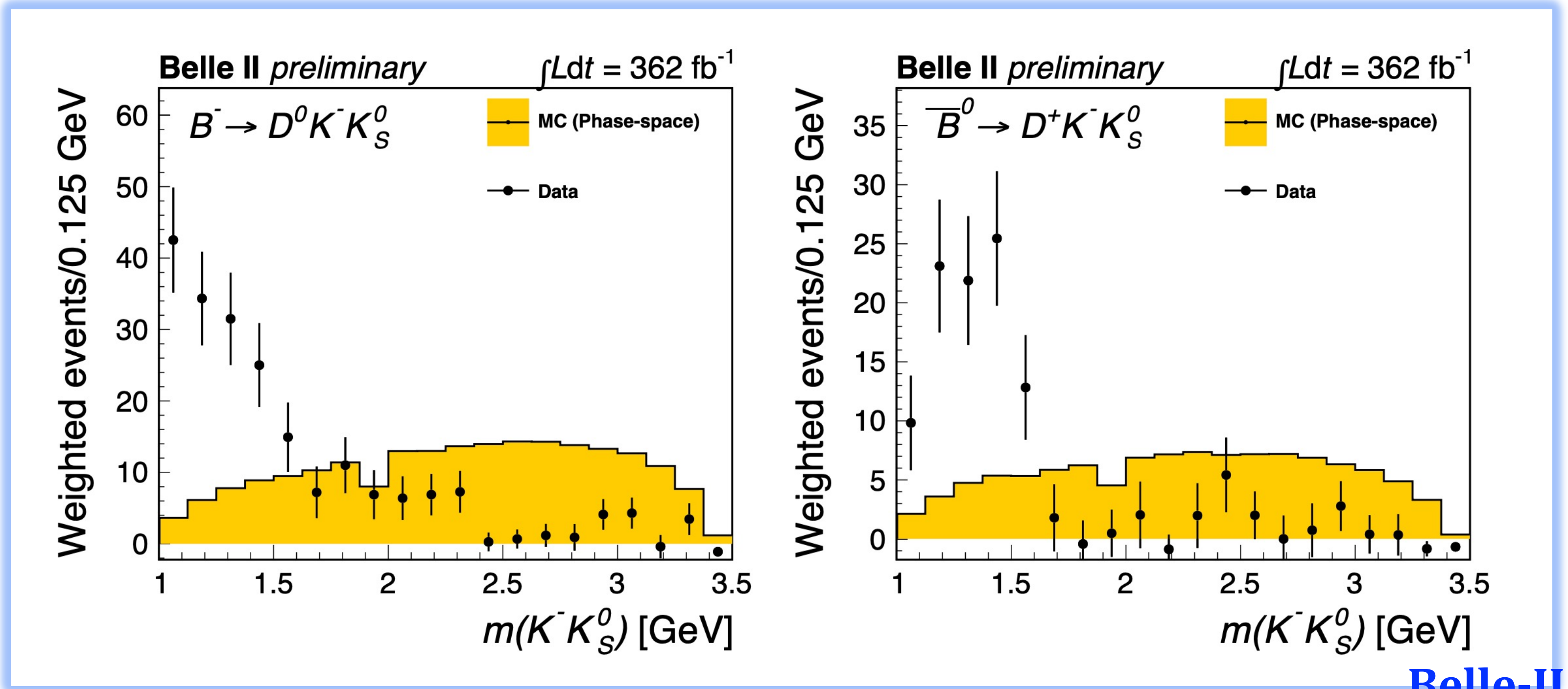


Isospin $I = 1$ $K^+\overline{K}^0$

- $\rho(770)^+$ & $\rho(1450)^+$ & $\rho(1700)^+$
- $a_0(980)^+$ & $a_0(1450)^+$
- $a_2(1320)^+$ & $a_2(1700)^+$

Kaon pair in $B^+ \to \overline{D}^{(*)0} K^+ \overline{K}^0$ and $B^0 \to D^{(*)-} K^+ \overline{K}^0$ decays

APPROACH:



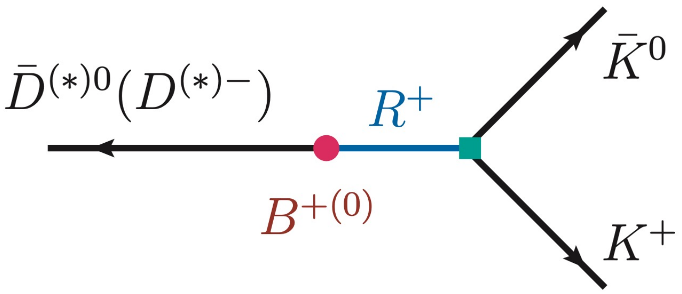
Belle-II arXiv:2305.01321

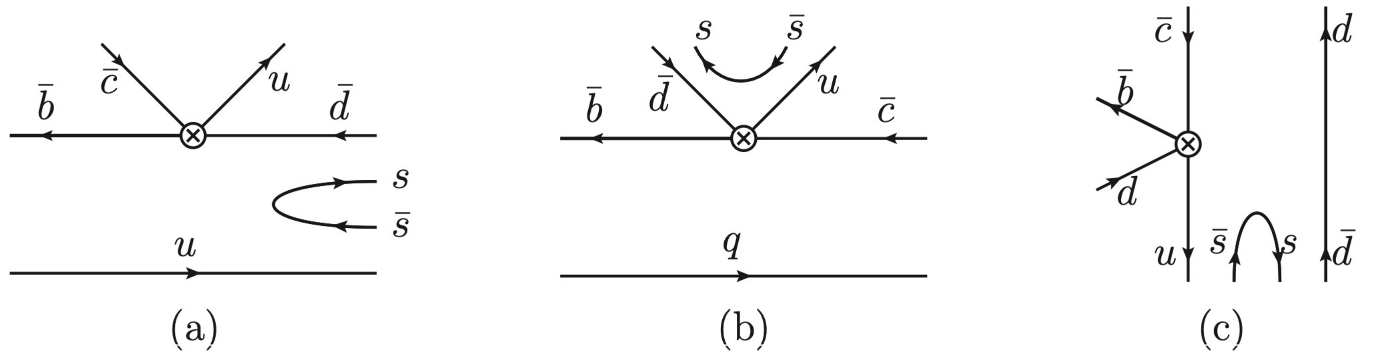
Isospin $I = 1$ $K^+\overline{K}^0$

- $\rho(770)^+$ & $\rho(1450)^+$ ~~& $\rho(1700)^+$~~
- $a_0(980)^+$ & $a_0(1450)^+$
- $a_2(1320)^+$ ~~& $a_2(1700)^+$~~

Kaon pair in $B^+ \to \overline{D}^{(*)0} K^+ \overline{K}^0$ and $B^0 \to D^{(*)-} K^+ \overline{K}^0$ decays

APPROACH:



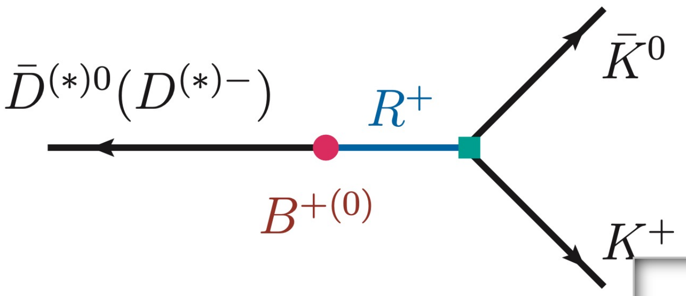


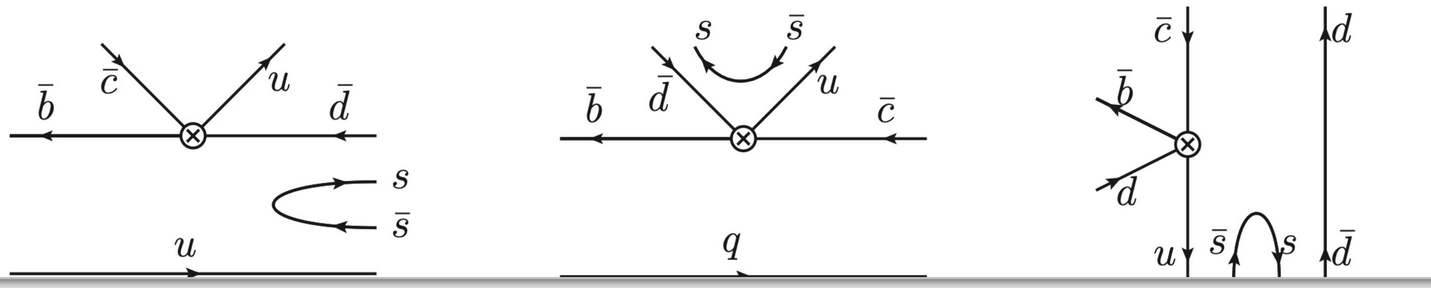
With PQCD approach

$$\mathcal{A} = \langle (\omega\pi)_{P\text{-wave}} D^{(*)} | \mathcal{H}_{\text{eff}} | B \rangle$$
$$= \phi_B \otimes \mathcal{H} \otimes \phi_{\omega\pi}^{P\text{-wave}} \otimes \phi_{D^{(*)}}$$

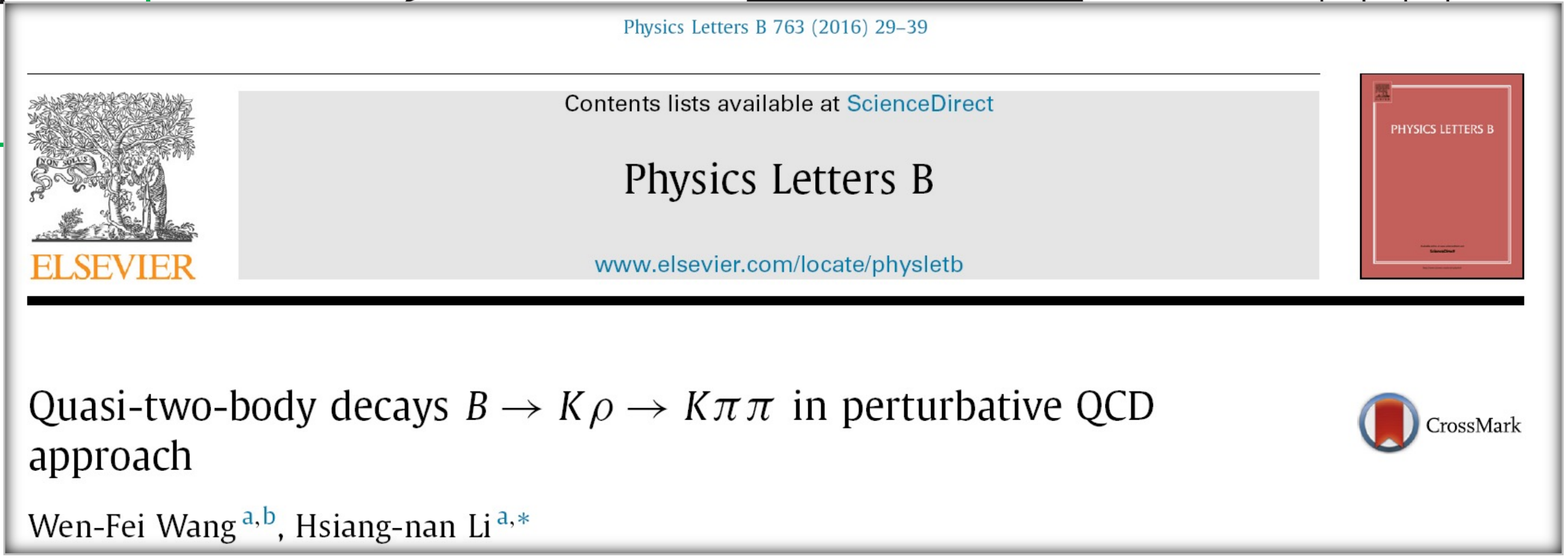
Kaon pair in $B^+ \to \bar{D}^{(*)0} K^+ \bar{K}^0$ and $B^0 \to D^{(*)-} K^+ \bar{K}^0$ decays

Approach:





With PQCD approach


Physics Letters B 763 (2016) 29–39

Contents lists available at ScienceDirect

Physics Letters B

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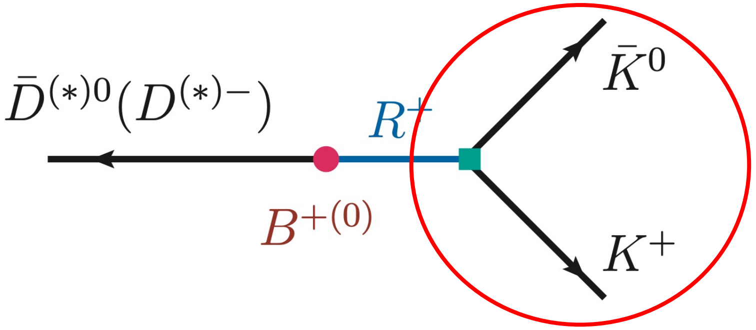
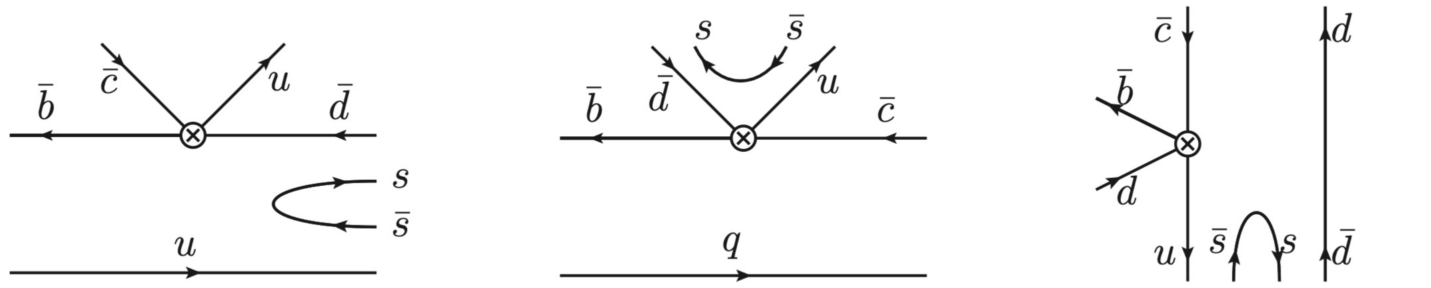
Quasi-two-body decays $B \to K\rho \to K\pi\pi$ in perturbative QCD approach

Wen-Fei Wang [a,b], Hsiang-nan Li [a,*]

Kaon pair in $B^+ \to \bar{D}^{(*)0} K^+ \bar{K}^0$ and $B^0 \to D^{(*)-} K^+ \bar{K}^0$ decays

Approach:





(a) (b) (c)

With PQCD approach

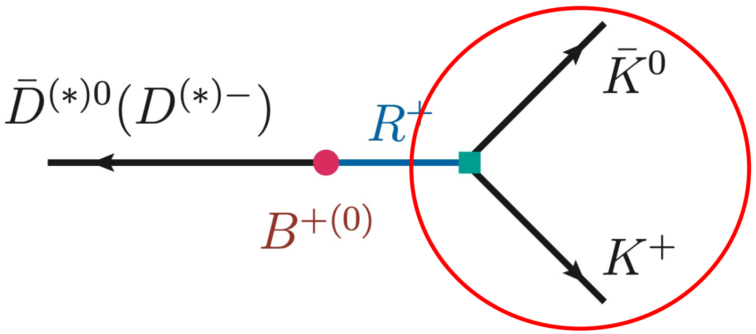
$$F^R_{\pi,K}(s) = c_R^{\pi,K}\,\mathrm{BW}_R(s) \equiv c_R^{\pi,K}\frac{m_R^2}{\mathcal{D}_R(s)};$$

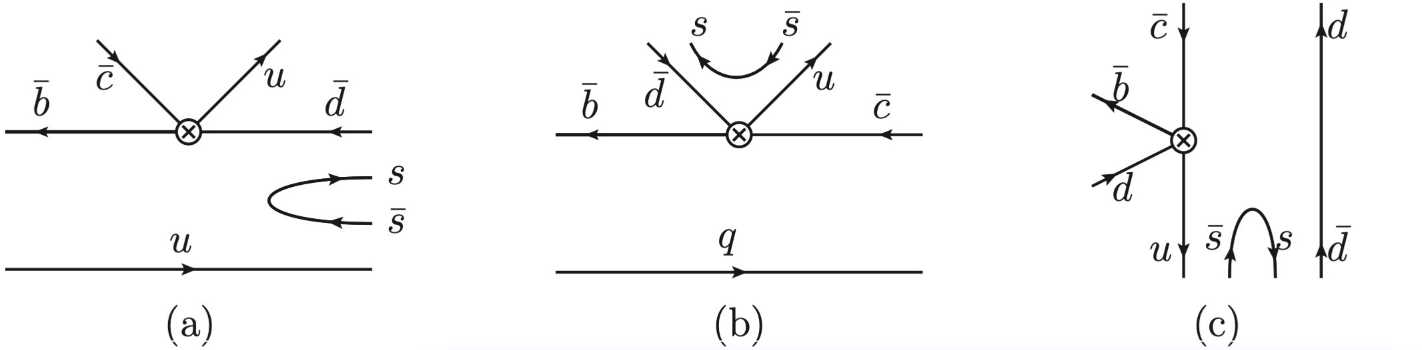
$$\phi^{P\text{-wave}}_{K\bar{K},L}(x,s) = \frac{-1}{\sqrt{2N_c}}\left[\sqrt{s}\,\not\epsilon_L \phi^0(x,s) + \not\epsilon_L \not p \phi^t(x,s) + \sqrt{s}\phi^s(x,s)\right]$$

$$\phi^{P\text{-wave}}_{K\bar{K},T}(x,s) = \frac{-1}{\sqrt{2N_c}}\left[\sqrt{s}\,\not\epsilon_T \phi^v(x,s) + \not\epsilon_T \not p \phi^T(x,s) + \sqrt{s}\, i\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu \epsilon_T^{*\nu} n^\rho v^\sigma \phi^a(x)\right]$$

Kaon pair in $B^+ \to \bar{D}^{(*)0} K^+ \bar{K}^0$ and $B^0 \to D^{(*)-} K^+ \bar{K}^0$ decays

APPROACH:





With PQCD approach

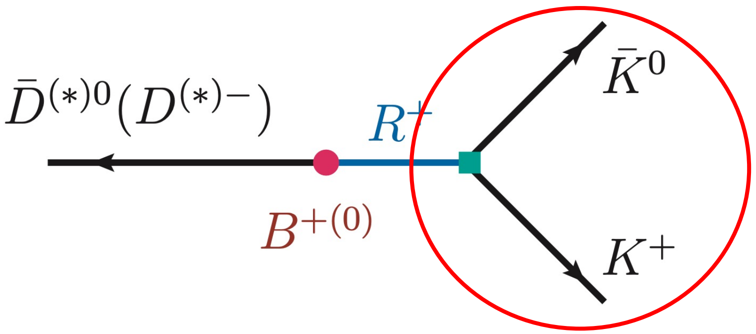
$$F^R_{\pi,K}(s) = c^{\pi,K}_R \mathrm{BW}_R(s) \equiv c^{\pi,K}_R \frac{m_R^2}{\mathcal{D}_R(s)}$$

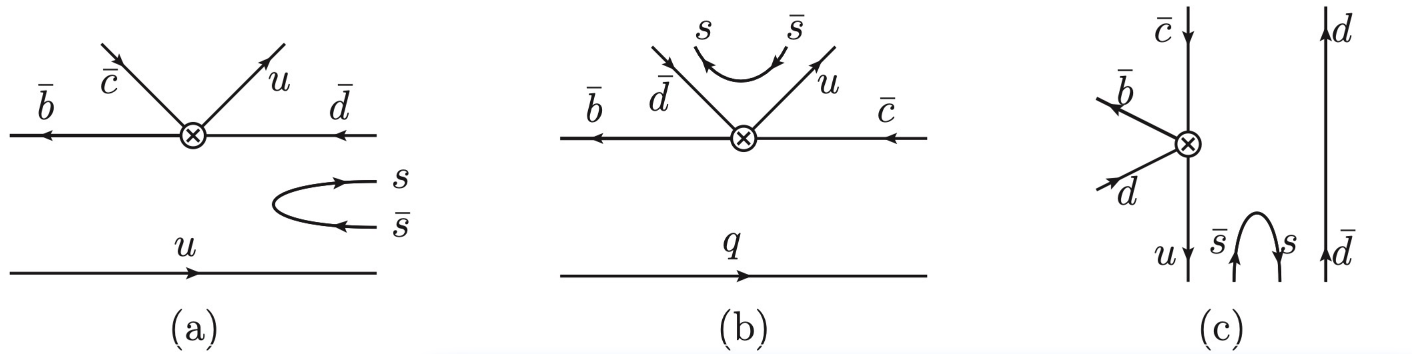
$$f_R g_{R\pi\pi}/(\sqrt{2} m_R)$$

$$g_{\rho(770)^0 K^+ K^-} = g_{\rho(770)^0 \pi^+ \pi^-}/2$$

Kaon pair in $B^+ \to \bar{D}^{(*)0} K^+ \bar{K}^0$ and $B^0 \to D^{(*)-} K^+ \bar{K}^0$ decays

Approach:





With PQCD approach

$$F^R_{\pi,K}(s) = c_R^{\pi,K}\,\mathrm{BW}_R(s) \equiv c_R^{\pi,K}\frac{m_R^2}{\mathcal{D}_R(s)}$$

PHYSICAL REVIEW D **103,** 056021 (2021)

Contributions for the kaon pair from $\rho(770)$, $\omega(782)$ and their excited states in the $B \to K\bar{K}h$ decays

Wen-Fei Wang[1,2,*]

$$c^K_{\rho(770)} = 1.247 \pm 0.019$$
$$c^K_{\rho(1450)} = -0.156 \pm 0.015$$

TABLE II: PQCD results for the branching fractions of the quasi-two-body decays $B^+ \to \bar{D}^{(*)0}[\rho(770)^+ \to]\pi^+\pi^0$ and $B^0 \to D^{(*)-}[\rho(770)^+ \to]\pi^+\pi^0$.

Decay modes	Units	PQCD
$B^+ \to \bar{D}^0[\rho(770)^+ \to]\pi^+\pi^0$	%	$1.21^{+0.16+0.10+0.05}_{-0.16-0.12-0.06}$
$B^0 \to D^-[\rho(770)^+ \to]\pi^+\pi^0$	10^{-3}	$7.63^{+0.58+0.97+0.34}_{-0.58-0.73-0.21}$
$B^+ \to \bar{D}^{*0}[\rho(770)^+ \to]\pi^+\pi^0$	10^{-3}	$9.03^{+1.55+0.73+0.51}_{-1.55-0.64-0.46}$
$B^0 \to D^{*-}[\rho(770)^+ \to]\pi^+\pi^0$	10^{-3}	$8.15^{+1.31+0.64+0.03}_{-1.31-0.62-0.07}$

$$
\begin{aligned}
\mathcal{B}(B^+ \to \bar{D}^0\rho(770)^+) &= (1.34 \pm 0.18)\%\,, \\
\mathcal{B}(B^0 \to D^-\rho(770)^+) &= (7.6 \pm 1.2) \times 10^{-3}\,, \\
\mathcal{B}(B^+ \to \bar{D}^{*0}\rho(770)^+) &= (9.8 \pm 1.7) \times 10^{-3}\,, \\
\mathcal{B}(B^0 \to D^{*-}\rho(770)^+) &= (6.8 \pm 0.9) \times 10^{-3}\,,
\end{aligned}
$$

PDG-2022

TABLE III: PQCD predictions for the branching fractions of the concerned quasi-two-body decays with the subprocess $\rho^+ \to K^+\bar{K}^0$, here $\rho^+ = \rho(770)^+ + \rho(1450)^+$.

Mode	Unit	$\mathcal{B}$
$B^+ \to \bar{D}^0[\rho^+ \to]K^+\bar{K}^0$	10^{-4}	$1.68^{+0.20+0.17+0.12}_{-0.20-0.15-0.12}$
$B^0 \to D^-[\rho^+ \to]K^+\bar{K}^0$	10^{-4}	$0.98^{+0.06+0.13+0.06}_{-0.06-0.12-0.06}$
$B^+ \to \bar{D}^{*0}[\rho^+ \to]K^+\bar{K}^0$	10^{-4}	$1.33^{+0.21+0.11+0.05}_{-0.21-0.11-0.07}$
$B^0 \to D^{*-}[\rho^+ \to]K^+\bar{K}^0$	10^{-4}	$1.16^{+0.19+0.08+0.02}_{-0.19-0.09-0.02}$

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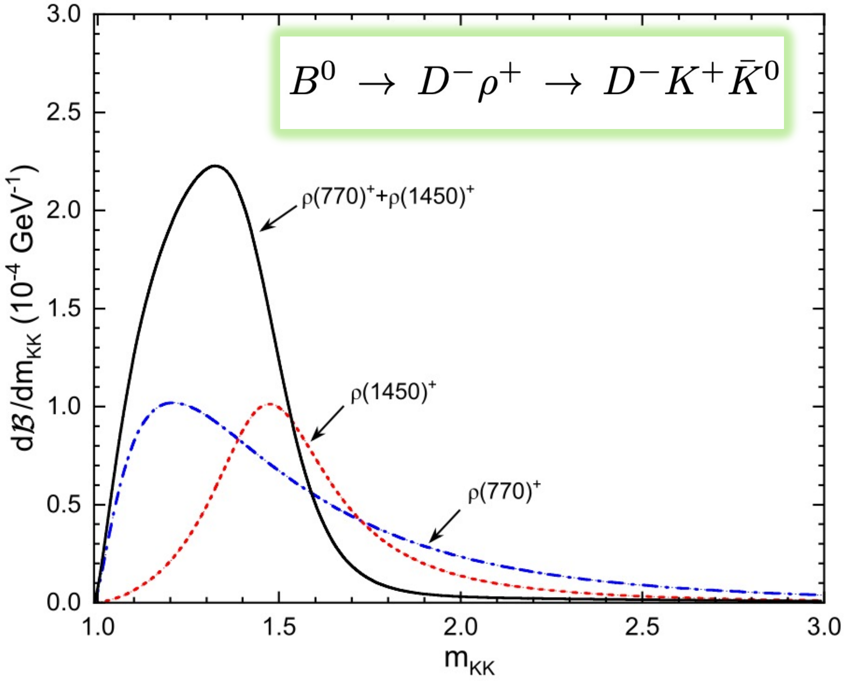
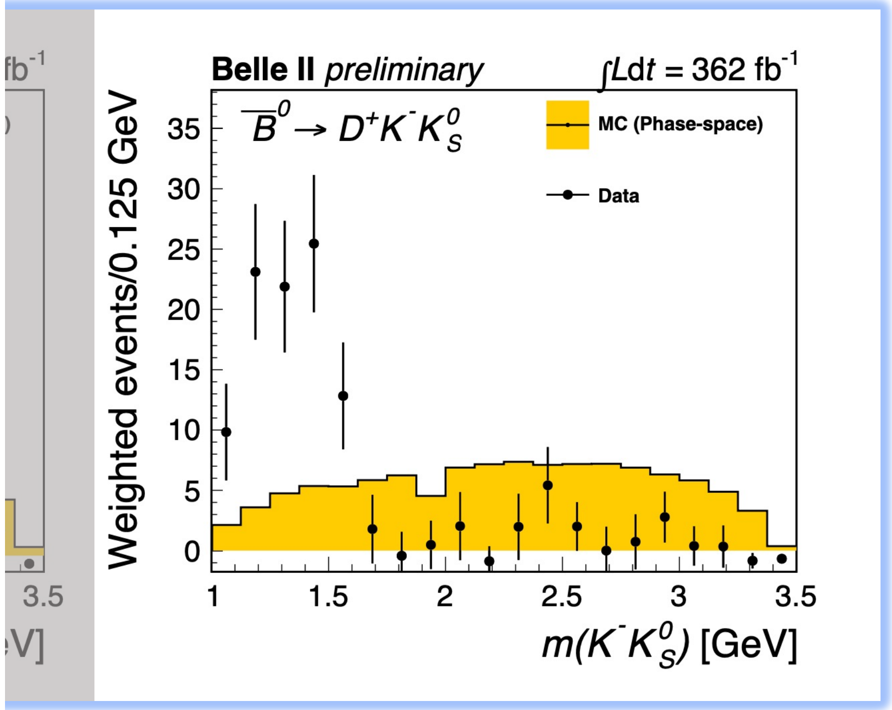
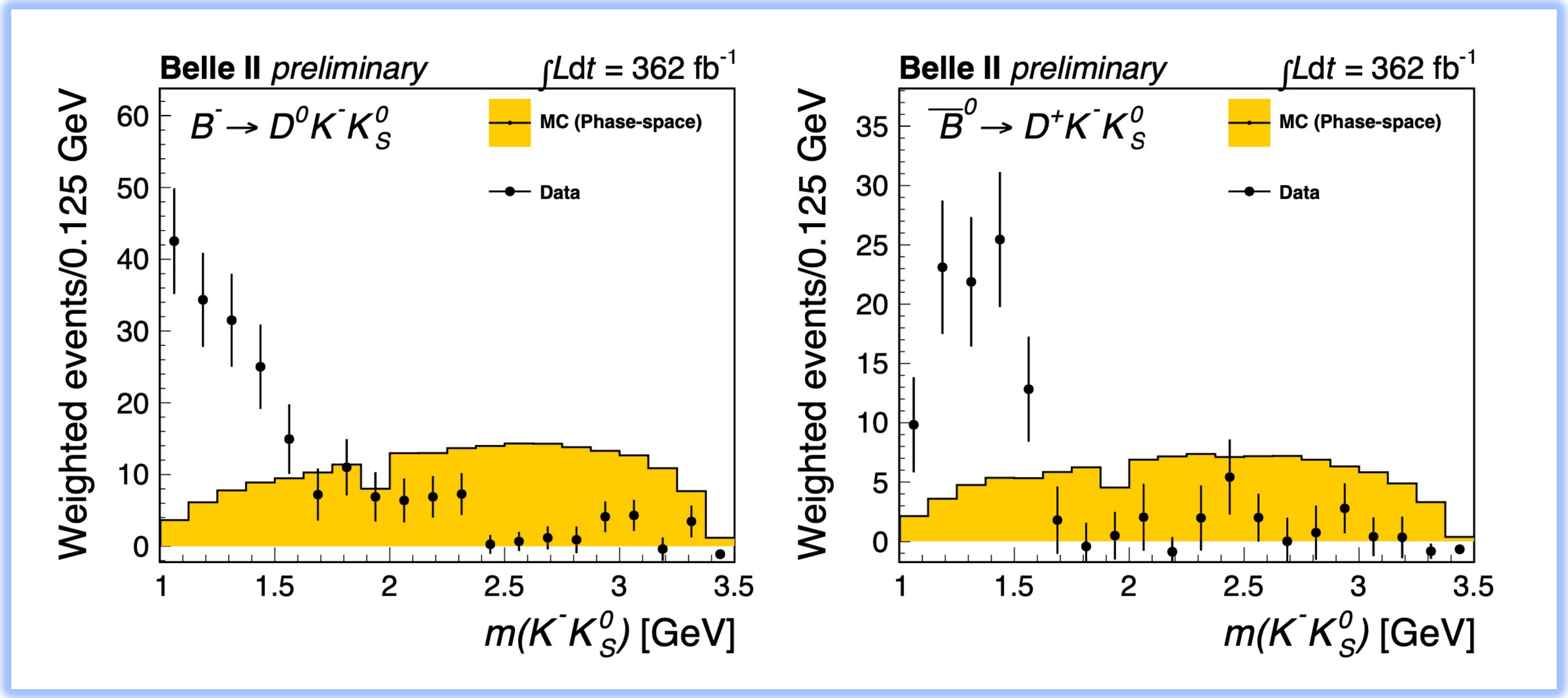
$$\mathcal{B}(B^- \to D^0 K^- K_S^0) = (1.89 \pm 0.16 \pm 0.10) \times 10^{-4},$$
$$\mathcal{B}(\overline{B}^0 \to D^+ K^- K_S^0) = (0.85 \pm 0.11 \pm 0.05) \times 10^{-4},$$
$$\mathcal{B}(B^- \to D^{*0} K^- K_S^0) = (1.57 \pm 0.27 \pm 0.12) \times 10^{-4},$$
$$\mathcal{B}(\overline{B}^0 \to D^{*+} K^- K_S^0) = (0.96 \pm 0.18 \pm 0.06) \times 10^{-4},$$

Results and discussions





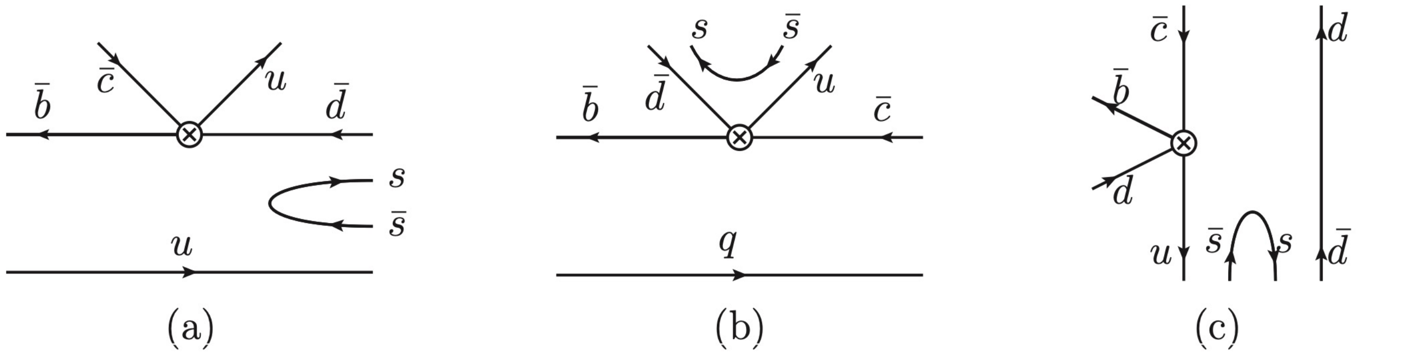
Results and discussions



Results and discussions

Contributions:

$a_0(980)^+$ & $a_0(1450)^+$



$$\langle K^+ \bar{K}^0 | (\bar{u}d)_{V-A} | 0 \rangle \Rightarrow f_{a_0(980)} \approx 1.1\text{MeV}$$

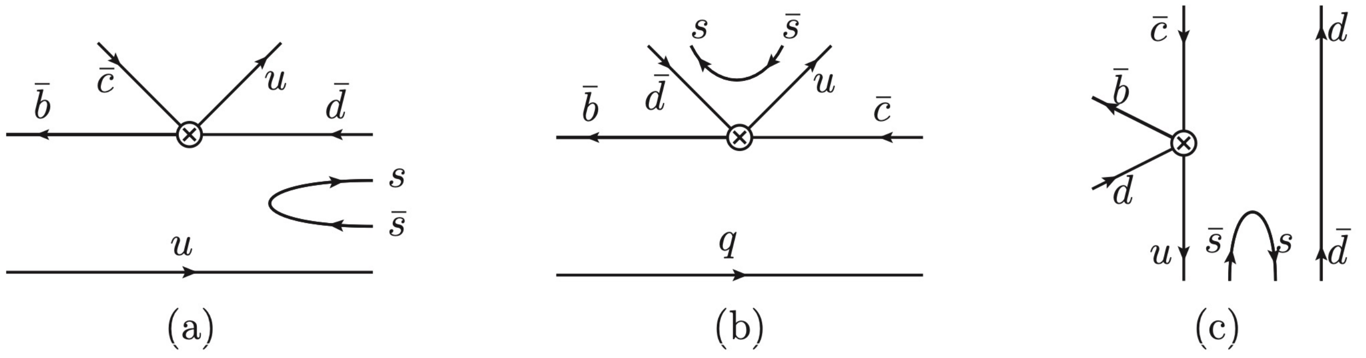
$$\mu_S f_S = \bar{f}_S, \quad \text{with} \quad \mu_S = \frac{m_S}{m_2(\mu) - m_1(\mu)}$$

$$\bar{f}_{a_0}(1\ \text{GeV}) \approx 385\ \text{MeV}$$

PRD73-014017

CONTRIBUTIONS:

$a_0(980)^+$ & $a_0(1450)^+$



$$\mathcal{B}(B^0 \to D_s^+ a_0(980)^-) = 1.93 \times 10^{-5}$$

upper limit 1.9×10^{-5} at 90% C.L. presented by the $BABAR$ ⇨ 2.24×10^{-5}

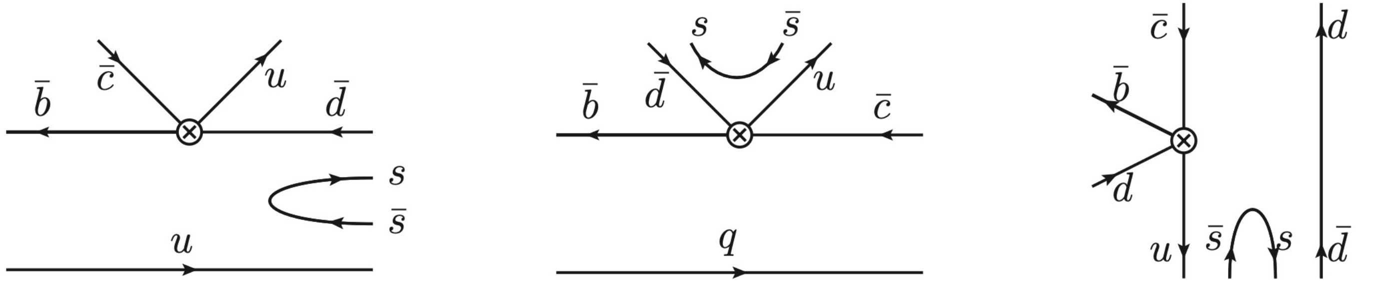
PRD73-071103

$$\mathcal{B} = 0.72 \times 10^{-5} \text{ for } B^+ \to \bar{D}^0 a_0(1450)^+ \to \bar{D}^0 K^+ \bar{K}^0$$

$$\mathcal{B} = 1.56 \times 10^{-5} \quad B^+ \to \bar{D}^0 a_0(980)^+ \to \bar{D}^0 K^+ \bar{K}^0$$

Contributions:

$a_2(1320)^+$ & $a_2(1700)^+$



(a) (b) (c)

$a_2(1700)$ $I^G(J^{PC}) = 1^-(2^{++})$

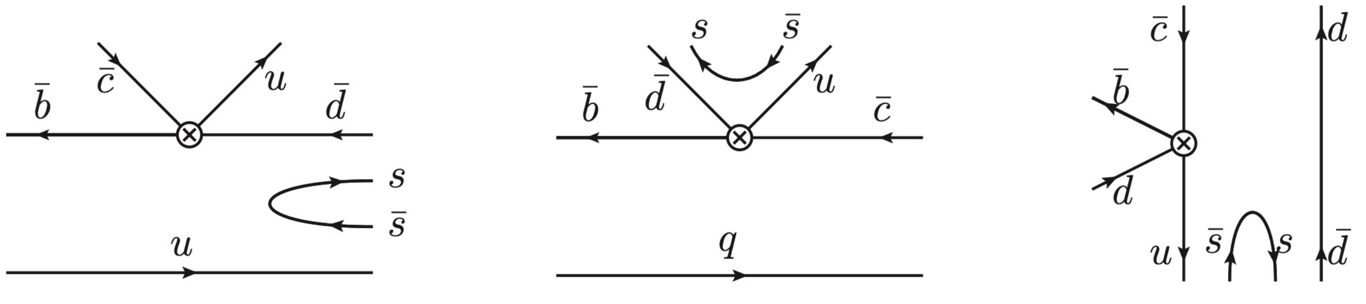
Mass $m = 1698 \pm 40$ MeV

Full width $\Gamma = 265 \pm 60$ MeV

$a_2(1700)$ DECAY MODES	Fraction (Γ_i/Γ)
$\eta\pi$	(3.6 ± 1.1) %
$\gamma\gamma$	$(1.13 \pm 0.30) \times 10^{-6}$
$K\overline{K}$	(1.9 ± 1.2) %

Contributions:

$a_2(1320)^+$ ~~& $a_2(1700)^+$~~



(a) (b) (c)

$a_2(1320)$ $I^G(J^{PC}) = 1^-(2^{++})$

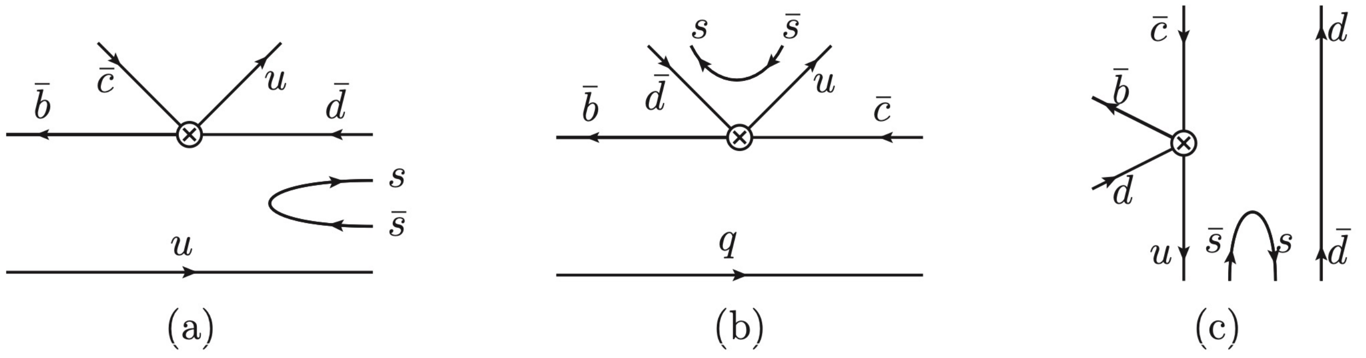
Mass $m = 1318.2 \pm 0.6$ MeV (S = 1.2)

Full width $\Gamma = 107 \pm 5$ MeV [i]

$a_2(1320)$ DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
3π	(70.1 ± 2.7) %	S=1.2	624
$\eta\pi$	(14.5 ± 1.2) %		535
$\omega\pi\pi$	(10.6 ± 3.2) %	S=1.3	366
$K\overline{K}$	(4.9 ± 0.8) %		437

Contributions:

$a_2(1320)^+$ ~~& $a_2(1700)^+$~~



$$B_s^0 \to \bar{D}^0[\bar{K}_2^*(1430)^0 \to]K^-\pi^+ \quad \mathcal{B} = (3.7 \pm 1.4) \times 10^{-5}$$

PRD90-072003 LHCb

$$s \to u$$

$$B^+ \to \bar{D}^0 a_2(1320)^+ \qquad \mathcal{B} = (0.99 \pm 0.37) \times 10^{-4}$$

For the Kaon pair in $B^+ \to \bar{D}^{(*)0}K^+\bar{K}^0$ and $B^0 \to D^{(*)-}K^+\bar{K}^0$ decays

(i). The intermediate states $\rho(770)^+$ & $\rho(1450)^+$ dominate the branching fractions for the four corresponding decay channels;

(ii). The role of $a_2(1320)^+$ in these four decay channels is negligible;

(ii). The state $a_0(980)^+$ turned out to be less important than expected for the kaon pair near the threshold;

For the Kaon pair in $B^+ \to \bar{D}^{(*)0} K^+ \bar{K}^0$ and $B^0 \to D^{(*)-} K^+ \bar{K}^0$ decays

(i). The intermediate states $\rho(770)^+$ & $\rho(1450)^+$ dominate the branching fractions for the four corresponding decay channels;

(ii). The role of $a_2(1320)^+$ in these four decay channels is negligible;

(ii). The state $a_0(980)^+$ turned out to be less important than expected for the kaon pair near the threshold;

Thank You !

Appendix

Appendix

Belle-II arXiv:2305.01321

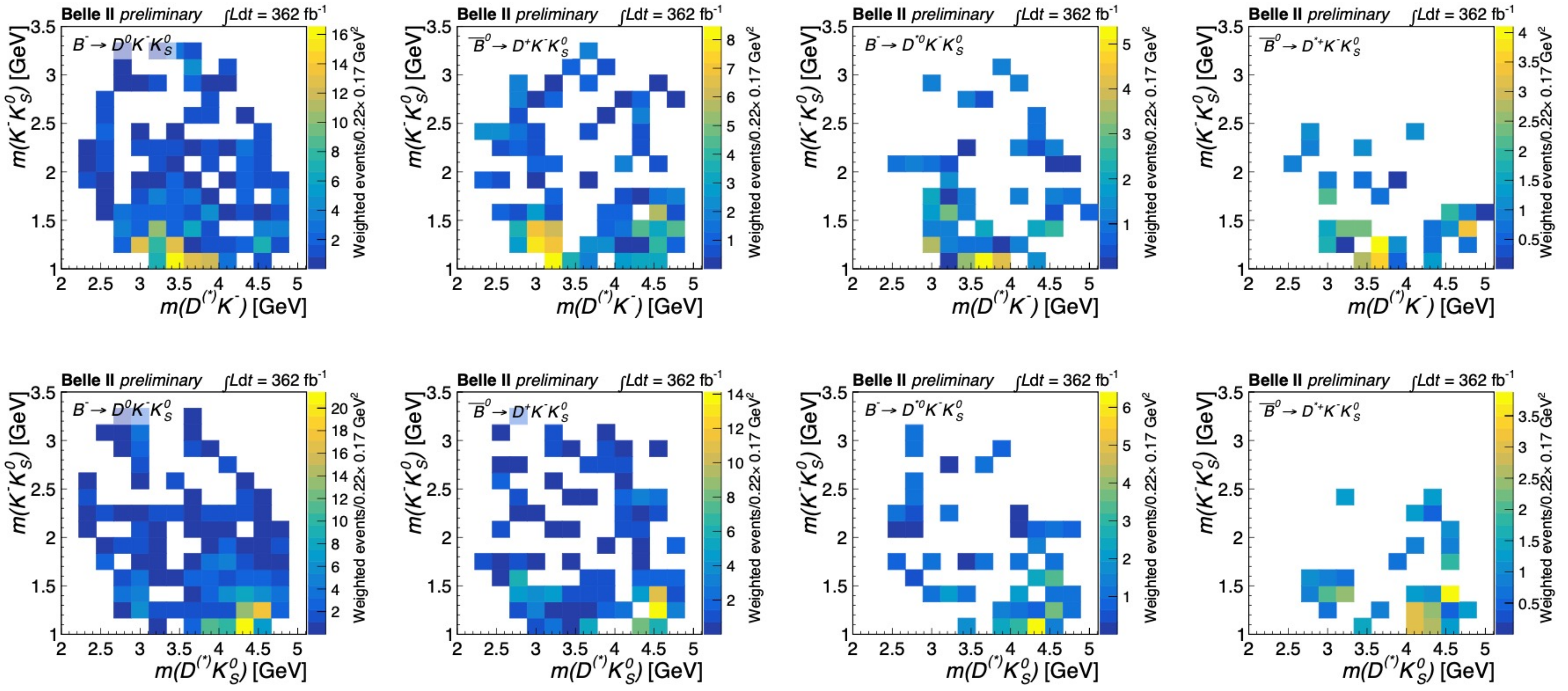


Figure 4. Dalitz distributions of $(m(D^*K^-), m(K^-K^0_S))$ (upper panels) and $(m(D^*K^0_S), m(K^-K^0_S))$ (lower panels) for (left to right) $B^- \to D^0K^-K^0_S$, $\overline{B}^0 \to D^+K^-K^0_S$, $B^- \to D^{*0}K^-K^0_S$, and $\overline{B}^0 \to D^{*+}K^-K^0_S$ channels. The background is subtracted by applying the signal sWeight.

Belle-II *preliminary*

Channel	Yield (K_S^0 / K^{*0})	Average ε (K_S^0 / K^{*0})	$\mathcal{B}$ [10^{-4}]
$B^- \to D^0 K^- K_S^0$	209 ± 17	0.098	$1.82 \pm 0.16 \pm 0.08$
$\overline{B}^0 \to D^+ K^- K_S^0$	105 ± 14	0.048	$0.82 \pm 0.12 \pm 0.05$
$B^- \to D^{*0} K^- K_S^0$	51 ± 9	0.044	$1.47 \pm 0.27 \pm 0.10$
$\overline{B}^0 \to D^{*+} K^- K_S^0$	36 ± 7	0.046	$0.91 \pm 0.19 \pm 0.05$

first observation

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Belle-II *preliminary*