On-Shell Construction of Chiral Effective Theory

— supported by package `ABC4EFT`[1]

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I. Low, J. Shu, **MLX** and Y.-H. Zheng, *JHEP 01 (2023) 031*

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[1] https://abc4eft.hepforge.org/

Outline

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The Non-Linear Symmetry

Symmetry breaking $\mathcal{G} \to \mathcal{H}$ with Goldstone fields $\xi = e^{i\pi^a X^a/f} \in \mathcal{G}/\mathcal{H}$.

- Although spontaneously broken, $\mathcal{G}$ is realized non-linearly at low energy

$$\pi \to \pi + \epsilon$$

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where $\mathcal{T}_{ab} = f^{-2}\sum_i (T^i\pi)_a (T^i\pi)_b$, without any knowledge of the broken group $\mathcal{G}$.

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- $\mathcal{G}$-invariant ($\mathcal{H}$-invariant + Adler Zero) Chiral Lagrangian

$$\mathcal{L}^{(2)} = \frac{f^2}{2}\mathrm{Tr}(d_\mu d^\mu) \supset \pi^2\partial^2, \pi^4\partial^2, \pi^6\partial^2, \dots$$

Chiral Perturbation Theory (ChPT)

- Consider $\mathcal{G} = SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V = \mathcal{H}$ by $\langle \bar{q}q \rangle \neq 0$
- External sources in QCD: $\mathcal{L} = \mathcal{L}^0_{\rm QCD} + \bar{q}\gamma^\mu(v_\mu + a_\mu\gamma^5)q - \bar{q}(s - ip\gamma^5)q$.

  Hidden Local Symmetry: $\nabla_\mu X = \partial_\mu X - iA_\mu(X)$, $\quad A \in \{v, a, e\}$
- Two ways to parameterize the chiral Lagrangian:
  - Left-right basis: $U = \xi^2$, $\chi \sim s + ip$, $F_L^{\mu\nu}$, $F_R^{\mu\nu}$.
  - u basis: $u_\mu \sim d_\mu$, $\chi_\pm$, $f_\pm^{\mu\nu}$.
- Redundancy relations to build Lagrangian:
  - IBP: $(\nabla X_1)X_2 + X_1(\nabla X_2) \simeq 0$.
  - EOM (field redef.): $\nabla_\mu u^\mu = \frac{i}{2}\left(\chi_- - \frac{1}{N_f}\langle\chi_-\rangle\right)$.
  - CDC: $[\nabla_\mu, \nabla_\nu] = \frac{1}{4}[u_\mu, u_\nu] - \frac{i}{2}f_{+\mu\nu}$, $\nabla_{[\mu}u_{\nu]} + f_{-\mu\nu} = 0$.
  - Schouten Identity (specific for fixed $D = 4$): $\mathcal{O}_{[\mu_1\mu_2\mu_3\mu_4\mu_5]} = 0$.
- $O(p^4)$ (Gasser&Leutwyler 84'), $O(p^6)$ (Bijnens *et al.* 99'), $O(p^8)$ (Bijnens *et al.* 19').

On-Shell Construction of ChPT

- It is known for more than half a century that the non-linear constraint can be reproduced in the amplitude by Adler Zero: $\lim_{p_i=0}\mathcal{M}=0$ (Susskind&Frye 69'):

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Algebraic Aspects of Pionic Duality Diagrams

LEONARD SUSSKIND* AND GRAHAM FRYE
Belfer Graduate School of Science, Yeshiva University, New York, New York 10033
(Received 9 May 1969)

Certain algebraic aspects are abstracted from the duality principle and are incorporated in a simple model of pion n-point functions. An algorithm for constructing the n-point function in the tree-graph approximation is based on the duality assumption and the Adler condition which states that the amplitudes vanishes if any pion four-momentum vanishes, all others remaining on shell. The resulting amplitudes satisfy the constraints of current algebra and partial conservation of axial-vector current for $n=4$, 6, and 8, and (we conjecture) for all n. In addition, duality specifies a definite form for chiral symmetry breaking.

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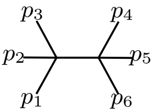
$\mathcal{M}(p_1, \dots, p_6) =$


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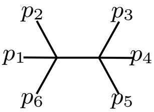

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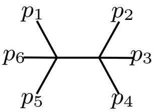

$$= \underbrace{\frac{1}{f^4}\left(\frac{s_{13}s_{46}}{s_{123}} + \frac{s_{24}s_{51}}{s_{234}} + \frac{s_{35}s_{62}}{s_{345}}\right)}_{\text{violate Adler Zero}}$$

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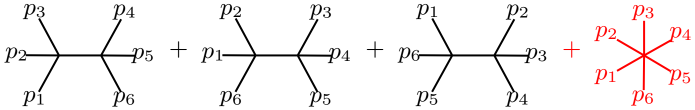

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- Extended to arbitrary n: Soft Recursion Relation (Cheung *et al.* 15')

$$p_i \to \hat{p}_i = (1-a_i z)p_i \ , \quad \mathcal{M}(\hat{p}_1,\dots,\hat{p}_n) \equiv \hat{\mathcal{M}}_n(z) \ ,$$

Poles at $\hat{s}_I(z_I^{\pm})=0$, thus the Cauchy's theorem gives

$$\mathcal{M}(p_1,\dots,p_n) = \hat{\mathcal{M}}_n(0) = \sum_{I,\pm}\frac{1}{s_I}\frac{\hat{\mathcal{M}}_L^{(I)}(z_I^{\pm})\hat{\mathcal{M}}_R^{(I)}(z_I^{\pm})}{F_n(z_I^{\pm})(1-z_I^{\pm}/z_I^{\mp})} \ , \quad F_n(z) \equiv \prod_{i=1}^{n}(1-a_i z) \, .$$

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- How about higher derivatives? Need new inputs (LEC)! (Low&Yin 19')

Single Trace: $\mathcal{S}_1^{(4)}(1,2,3,4) = \frac{c_1}{\Lambda^2 f^2} s_{13}^2\,, \quad \mathcal{S}_2^{(4)}(1,2,3,4) = \frac{c_2}{\Lambda^2 f^2} s_{13}s_{23}\,,$

Double Trace: $\mathcal{S}_1^{(4)}(1,2|3,4) = \frac{d_1}{\Lambda^2 f^2} s_{13}^2\,, \quad \mathcal{S}_2^{(4)}(1,2|3,4) = \frac{d_2}{\Lambda^2 f^2} s_{13}s_{23}\,.$

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(1) All soft blocks $\mathcal{S}$ satisfy Adler Zero.

(2) 1-to-1 correspond to Lagrangian terms $\mathcal{L}^{(4)} = \sum_{i=1}^4 \frac{L_{4,i}}{\Lambda^2 f^2}\mathcal{O}_i$.

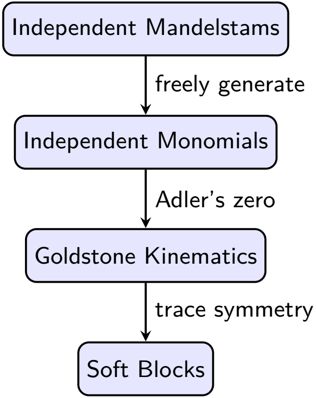
Outline

Soft Block Construction: First Attempt

Systematic construction of soft blocks (Dai *et al.* 20'):

- Only P even Building block s_{ij} (no $\epsilon^{\mu\nu\rho\sigma}$).
- Mandelstams constrained by Gram determinant
  - $D = 4$ at $O(p^{10})$: $\det s_{ij} = 0$
- The resulting soft blocks are polynomials.
  - Do not correspond to monomial operator
- Assuming independent traces (large N_f).
  - Cayley-Hamilton when $N_f < N$.

$$e.g.\quad \langle A^4\rangle = \frac{1}{2}\langle A^2\rangle^2\,,\quad \forall A\in\mathfrak{su}(3)$$



On-Shell Operator Construction in a Broader Sense

Amplitude/Operator basis correspondence (Shu, Ma, **MLX** 19')

- Operators $\mathcal{O}$ are isomorphic to **local gauge invariant on-shell** amplitudes $\mathcal{B} = \mathcal{T}\mathcal{M}$
  - IBP $\simeq$ momentum conservation $\sum_i p_i = \sum_i |i\rangle[i| = 0$.
  - EOM $\simeq$ on-shell condition $\langle ii\rangle = [ii] = 0$.
  - $D = 4$ Schouten Identity / Gram Determinants $\simeq$ spinor Schouten Identities.

$$\langle ij\rangle\langle kl\rangle + \langle ik\rangle\langle lj\rangle + \langle il\rangle\langle jk\rangle = 0\ , \quad [ij][kl] + [ik][lj] + [il][jk] = 0\ .$$

An independent operator basis $\simeq$ an independent amplitude basis.

- Young Tensor Method (Li, Ren, Shu, **MLX**, Yu, Zheng 20').

$$\mathcal{M} \xrightarrow{\text{reduce}} \sum_i c_i \mathcal{M}_i^{(y)}\ , \qquad \mathcal{M}^{(y)} \sim \langle ij\rangle \dots [kl] \dots$$

- Mathematica Package `ABC4EFT` (Li, Ren, **MLX**, Yu, Zheng 22')

Soft Block Construction: Second Attempt

Young Tensor approach (Low, Shu, **MLX**, Zheng 22'):

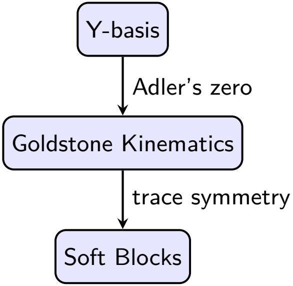
✓ Both parities are included (but mixed).

$$\langle ij\rangle[jk]\langle kl\rangle[li] \pm [ij]\langle jk\rangle[kl]\langle li\rangle = \begin{cases} 4(s_{ij}s_{kl} - s_{ik}s_{jl} + s_{il}s_{jk}) \\ 4i\epsilon_{\mu\nu\rho\sigma}p_i^\mu p_j^\nu p_k^\rho p_l^\sigma \end{cases}$$

✓ Free from Gram Det. and Schouten ($D = 4$)

	ϕ^6p^8	ϕ^6p^{10}	ϕ^6p^{12}	ϕ^7p^8	ϕ^7p^{10}	ϕ^7p^{12}	ϕ^8p^{10}	ϕ^8p^{12}
$f^{\text{even}}(s_{ij})$	–	1287	3003	–	8568	27132	42504	177100
y-basis	–	1286	2994	–	8547	26873	42308	173915
$f^{\text{odd}}(s_{ij},\epsilon)$	225	825	2475	1575	8400	35700	53900	309925
y-basis	180	600	1650	1106	5019	18305	28196	132335

- The resulting soft blocks are still polynomials.
- Assuming independent traces (large N_f).

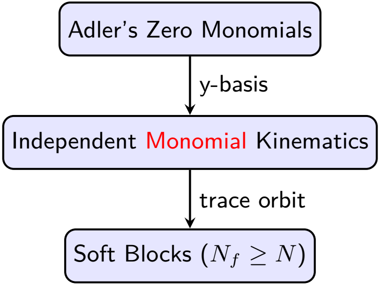


Soft Block Construction: Final Algorithm

Young Tensor approach (Low, Shu, **MLX**, Zheng 22'):

✓ Both parities are included separately.

$s_{12}s_{34}s_{56} = \sum_{i=1}^{205} c_i^{(\text{even})}\mathcal{M}_i^{\text{y}}, \ \epsilon_{1234}s_{56} = \sum_{i=1}^{205} c_i^{(\text{odd})}\mathcal{M}_i^{\text{y}}$



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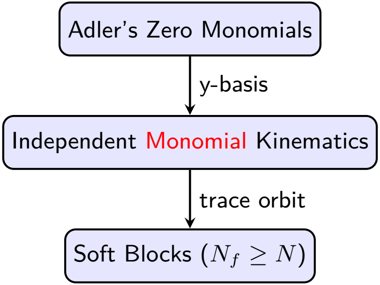

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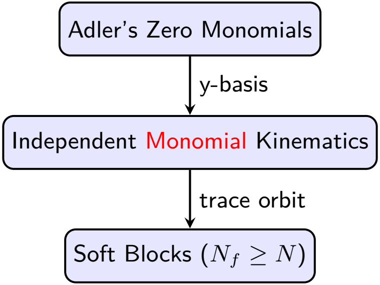
✓ Free from Gram Det. and Schouten $(D=4)$

- Orbits of residual group $H^{(\mathcal{T})}$ (*e.g.* $H^{(6)} = Z_6$):

$$\mathcal{Y}(\mathcal{T}\mathcal{M}) = \mathcal{Y}(\mathcal{T}\mathcal{M}'), \quad \text{if } \mathcal{M}' \in H^{(\mathcal{T})}(\mathcal{M})$$
$$\mathcal{Y}(\mathcal{B}) = \mathcal{Y}(\mathcal{B}') \Leftrightarrow \mathcal{O} \simeq \mathcal{O}'$$



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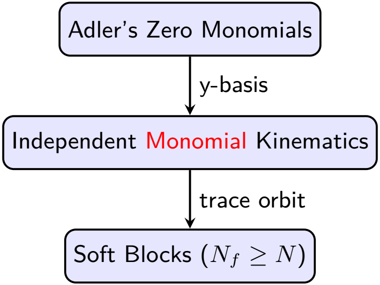
e.g. $\mathcal{Y}(\text{tr}[123456]s_{12}s_{34}s_{56}) = \mathcal{Y}(\text{tr}[123456]s_{23}s_{45}s_{61})$

$\Rightarrow \langle u_\mu u^\mu u_\nu u^\nu u_\rho u^\rho \rangle = \langle u_\rho u^\mu u_\mu u^\nu u_\nu u^\rho \rangle$



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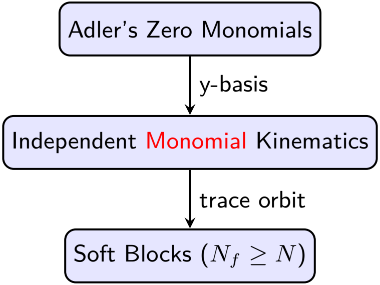
- ✓ Free from Gram Det. and Schouten $(D = 4)$
- ✓ Linearly independent orbits $\{H^{(\mathcal{T})}(\mathcal{M}_a)\} \Leftrightarrow$ Monomial soft blocks $\mathcal{O}_a^{(\mathcal{T})} \simeq \mathcal{Y}(\mathcal{T}\mathcal{M}_a)$.
- They are independent at $N_f \geq N$.



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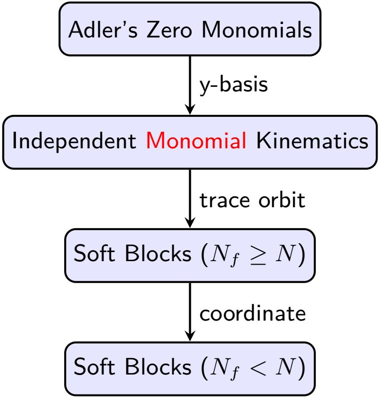
- ✓ Linearly independent orbits $\{H^{(\mathcal{T})}(\mathcal{M}_a)\} \Leftrightarrow$ Monomial soft blocks $\mathcal{O}_a^{(\mathcal{T})} \simeq \mathcal{Y}(\mathcal{T}\mathcal{M}_a)$.
- ✓ Reduce: $\mathcal{Y}(\mathcal{T}\mathcal{M}_a) = \sum_{i,j} c_{a,ij}\mathcal{T}_i\mathcal{M}_j^{\mathrm{y}}$

  Find independent coordinates $c_{a,ij}$!



Independent Trace Structures

Group $SU(N_f)$	$SU(2)$	$SU(3)$	$SU(4)$	$SU(5)$	$SU(6)$	$SU(7)$	Trace
$\mathcal{T}^{a_1a_2a_3}$	1	2	2	2	2	2	2
$\mathcal{T}^{a_1a_2a_3a_4}$	3	8	9	9	9	9	9
$\mathcal{T}^{a_1a_2a_3a_4a_5}$	6	32	43	44	44	44	44
$\mathcal{T}^{a_1a_2a_3a_4a_5a_6}$	15	145	245	264	265	265	265

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$\mathcal{T}^{a_1a_2a_3a_4a_5}$	6	32	43	44	44	44	44
$\mathcal{T}^{a_1a_2a_3a_4a_5a_6}$	15	145	245	264	265	265	265

Inner Product Treatment

Define $g_{ij} \equiv \langle \mathcal{T}_i, \mathcal{T}_j \rangle = (\mathcal{T}_i^\dagger)_{a_1 \ldots a_N} (\mathcal{T}_j)^{a_1 \cdots a_N}$, we have

$$\det g(N_f) \neq 0 \quad \Leftrightarrow \quad \{\mathcal{T}_i\} \text{ is independent}$$

$$\text{where} \qquad T^a_{ij} T^a_{kl} = \delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl}/N_f$$

Tensor reduction for $\mathcal{T} = \sum_i c^i \mathcal{T}_i$: $\quad c^i = \left\langle (g^{-1})^{ij} \mathcal{T}_j, \mathcal{T} \right\rangle$.

Outline

The Operator Basis

1. Obtain independent orbits $\mathcal{Y}(\mathcal{T}\mathcal{M}_a)$ (independent soft blocks for large N_f);
2. Evaluate $\mathcal{Y}$ and find the unique coordinate on the $\mathcal{T}_i\mathcal{M}_j$;
3. Select independent monomials via linear algebra;
4. Translate to operators

$$p_{i\mu_1}p_{i\mu_2}\cdots p_{i\mu_n} \quad \Leftrightarrow \quad \nabla_{(\mu_1}\nabla_{\mu_2}\cdots\nabla_{\mu_{n-1}}u_{\mu_n)}$$

$O(p^6)$ and $O(p^8)$ bases for 6-point soft blocks (agree with Graf *et al.* 20'):

6-pt $O(p^6)$	$SU(2)$	$SU(3)$	$SU(4)$	$SU(5)$	$SU(6)$
P-even	3	8	13	14	15
P-odd	0	3	4	4	4

6-pt $O(p^8)$	$SU(2)$	$SU(3)$	$SU(4)$	$SU(5)$	$SU(6)$
P-even	9	40	68	74	76
P-odd	2	20	33	35	35

The Operator Basis

$O(p^6)$ operators without external sources:

$SU(N_f)$	Operator Basis	Amplitude Basis
$SU(2)$	$\mathcal{O}_1 = \langle u^\mu u^\nu u^\rho u_\mu u_\nu u_\rho \rangle$	$\mathcal{B}_1 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{14} s_{25} s_{36}$
	$\mathcal{O}_2 = \langle u^\mu u^\nu u^\rho u_\mu u_\rho u_\nu \rangle$	$\mathcal{B}_2 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{14} s_{26} s_{35}$
	$\mathcal{O}_3 = \langle u^\mu u^\nu u^\rho u_\rho u_\mu u_\nu \rangle$	$\mathcal{B}_3 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{15} s_{26} s_{34}$
$SU(3)$	$\mathcal{O}_4 = \langle u^\mu u^\nu u^\rho u_\rho u_\nu u_\mu \rangle$	$\mathcal{B}_4 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{16} s_{25} s_{34}$
	$\mathcal{O}_5 = \langle u^\mu u^\nu u_\nu u^\rho u_\rho u_\mu \rangle$	$\mathcal{B}_5 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{16} s_{23} s_{45}$
	$\mathcal{O}_6 = \langle u^\mu u^\nu u^\rho u_\mu \rangle \langle u_\nu u_\rho \rangle$	$\mathcal{B}_6 = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{14} s_{25} s_{36}$
	$\mathcal{O}_7 = \langle u^\mu u^\nu u^\rho u_\nu \rangle \langle u_\mu u_\rho \rangle$	$\mathcal{B}_7 = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{15} s_{24} s_{36}$
	$\mathcal{O}_8 = \langle u^\mu u^\nu u_\mu u^\nu \rangle \langle u_\rho u_\rho \rangle$	$\mathcal{B}_8 = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{13} s_{24} s_{56}$
$SU(4)$	$\mathcal{O}_9 = \langle u^\mu u^\nu u_\nu u_\mu \rangle \langle u^\rho u_\rho \rangle$	$\mathcal{B}_9 = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{14} s_{23} s_{56}$
	$\mathcal{O}_{10} = \langle u^\mu u^\nu u^\rho \rangle \langle u_\mu u_\nu u_\rho \rangle$	$\mathcal{B}_{10} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{14} s_{25} s_{36}$
	$\mathcal{O}_{11} = \langle u^\mu u^\nu u^\rho \rangle \langle u_\mu u_\rho u_\nu \rangle$	$\mathcal{B}_{11} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{14} s_{26} s_{35}$
	$\mathcal{O}_{12} = \langle u^\mu u^\nu u_\mu \rangle \langle u^\rho u_\nu u_\rho \rangle$	$\mathcal{B}_{12} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{13} s_{25} s_{46}$
	$\mathcal{O}_{13} = \langle u^\mu u^\rho \rangle \langle u^\nu u_\mu \rangle \langle u_\rho u_\nu \rangle$	$\mathcal{B}_{13} = \mathcal{Y} \circ \mathrm{tr}[12\|34\|56] s_{14} s_{25} s_{36}$
$SU(5)$	$\mathcal{O}_{14} = \langle u^\mu u^\nu \rangle \langle u^\rho u_\rho \rangle \langle u_\mu u_\nu \rangle$	$\mathcal{B}_{14} = \mathcal{Y} \circ \mathrm{tr}[12\|34\|56] s_{15} s_{26} s_{34}$
$SU(N_f \geq 6)$	$\mathcal{O}_{15} = \langle u^\mu u_\mu \rangle \langle u^\nu u_\nu \rangle \langle u^\rho u_\rho \rangle$	$\mathcal{B}_{15} = \mathcal{Y} \circ \mathrm{tr}[12\|34\|56] s_{12} s_{34} s_{56}$

Induced Linear Relations

For example $N_f = 2$, choose $\{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3\}$ as the basis:

$$\begin{pmatrix} \mathcal{B}_4 \\ \mathcal{B}_5 \\ \mathcal{B}_6 \\ \mathcal{B}_7 \\ \mathcal{B}_8 \\ \mathcal{B}_9 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \\ 2 & 2 & -2 \end{pmatrix} \times \begin{pmatrix} \mathcal{B}_1 \\ \mathcal{B}_2 \\ \mathcal{B}_3 \end{pmatrix}, \quad \begin{pmatrix} \mathcal{B}_{10} \\ \mathcal{B}_{11} \\ \mathcal{B}_{12} \\ \mathcal{B}_{13} \\ \mathcal{B}_{14} \\ \mathcal{B}_{15} \end{pmatrix} = \begin{pmatrix} 0 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \\ 2 & 2 & 0 \\ 4 & 4 & -4 \end{pmatrix} \times \begin{pmatrix} \mathcal{B}_1 \\ \mathcal{B}_2 \\ \mathcal{B}_3 \end{pmatrix}$$

$O(p^8)$ 6-pt P-even operators

$SU(N_f)$	Operator Basis	Amplitude Basis
$SU(2)$	$\mathcal{O}_1 = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma u_\mu u_\nu u_\rho u_\sigma \rangle$	$\mathcal{B}_1 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13} s_{14} s_{15} s_{26}$
	$\mathcal{O}_2 = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma u_\mu u_\nu u_\sigma u_\rho \rangle$	$\mathcal{B}_2 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13} s_{14} s_{16} s_{25}$
	$\mathcal{O}_3 = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u^\sigma u_\nu u_\sigma u_\rho \rangle$	$\mathcal{B}_3 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{14} s_{16} s_{35}$
	$\mathcal{O}_4 = \langle \nabla^\mu u^\nu \nabla^\rho u^\sigma u_\mu u_\nu u_\rho u_\sigma \rangle$	$\mathcal{B}_4 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13} s_{14} s_{25} s_{26}$
	$\mathcal{O}_5 = \langle \nabla^\mu u^\nu \nabla^\rho u^\sigma u_\mu u_\rho u_\nu u_\sigma \rangle$	$\mathcal{B}_5 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13} s_{15} s_{24} s_{26}$
	$\mathcal{O}_6 = \langle \nabla^\mu u^\nu \nabla^\rho u^\sigma u_\rho u_\mu u_\sigma u_\nu \rangle$	$\mathcal{B}_6 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{14} s_{16} s_{23} s_{25}$
	$\mathcal{O}_7 = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u^\sigma u_\nu u_\rho u_\sigma \rangle$	$\mathcal{B}_7 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{14} s_{25} s_{36}$
	$\mathcal{O}_8 = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u^\sigma u_\nu u_\sigma u_\rho \rangle$	$\mathcal{B}_8 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{14} s_{26} s_{35}$
	$\mathcal{O}_9 = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u^\sigma u_\rho u_\nu u_\sigma \rangle$	$\mathcal{B}_9 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{15} s_{24} s_{36}$
$SU(3)$	$\mathcal{O}_{10} = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma u_\sigma u_\mu u_\nu u_\rho \rangle$	$\mathcal{B}_{10} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{14} s_{15} s_{16} s_{23}$
	$\mathcal{O}_{11} = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma u_\mu u_\sigma u_\nu u_\rho \rangle$	$\mathcal{B}_{11} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13} s_{15} s_{16} s_{24}$
	$\mathcal{O}_{12} = \langle \nabla^\mu u^\nu \nabla^\rho u^\sigma u_\rho u_\sigma u_\mu u_\nu \rangle$	$\mathcal{B}_{12} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{15} s_{16} s_{23} s_{24}$
	$\mathcal{O}_{13} = \langle \nabla^\mu u^\nu \nabla^\rho u^\sigma u_\mu u_\rho u_\sigma u_\nu \rangle$	$\mathcal{B}_{13} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13} s_{16} s_{24} s_{25}$
	$\mathcal{O}_{14} = \langle \nabla^\mu u^\nu \nabla^\rho u^\sigma u_\rho u_\mu u_\nu u_\sigma \rangle$	$\mathcal{B}_{14} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{14} s_{15} s_{23} s_{26}$
	$\mathcal{O}_{15} = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u^\sigma u_\sigma u_\nu u_\rho \rangle$	$\mathcal{B}_{15} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{15} s_{16} s_{34}$
	$\mathcal{O}_{16} = \langle \nabla^\mu u^\nu u^\rho u^\sigma \nabla_\rho u_\sigma u_\mu u_\nu \rangle$	$\mathcal{B}_{16} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{15} s_{16} s_{24} s_{34}$
	$\mathcal{O}_{17} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u^\sigma u_\sigma u_\rho u_\nu \rangle$	$\mathcal{B}_{17} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{16} s_{25} s_{34}$
	$\mathcal{O}_{18} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u^\sigma u_\sigma u_\nu u_\rho \rangle$	$\mathcal{B}_{18} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{15} s_{26} s_{34}$
	$\mathcal{O}_{19} = \langle \nabla^\mu u^\nu u^\rho \nabla_\mu u^\sigma u_\sigma u_\nu u_\rho \rangle$	$\mathcal{B}_{19} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13} s_{15} s_{26} s_{34}$
	$\mathcal{O}_{20} = \langle \nabla^\mu u^\nu u^\rho u^\sigma u_\sigma \nabla_\mu u_\nu u_\rho \rangle$	$\mathcal{B}_{20} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{15}^2 s_{26} s_{34}$
	$\mathcal{O}_{21} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u^\sigma u_\rho u_\sigma u_\nu \rangle$	$\mathcal{B}_{21} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{16} s_{24} s_{35}$
	$\mathcal{O}_{22} = \langle \nabla^\mu u^\nu u^\rho \nabla_\mu u^\sigma u_\nu u_\sigma u_\rho \rangle$	$\mathcal{B}_{22} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13} s_{14} s_{26} s_{35}$
	$\mathcal{O}_{23} = \langle \nabla^\mu u^\nu u^\rho u^\sigma \nabla_\mu u_\nu u_\sigma u_\rho \rangle$	$\mathcal{B}_{23} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{14}^2 s_{26} s_{35}$
	$\mathcal{O}_{24} = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u^\sigma u_\nu u_\rho u_\sigma \rangle$	$\mathcal{B}_{24} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{14} s_{15} s_{36}$
	$\mathcal{O}_{25} = \langle \nabla^\mu u^\nu u^\rho \nabla_\mu u^\sigma u_\rho u_\nu u_\sigma \rangle$	$\mathcal{B}_{25} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13} s_{15} s_{24} s_{36}$
	$\mathcal{O}_{26} = \langle \nabla^\mu u^\nu u^\rho \nabla_\mu u^\sigma u_\nu u_\rho u_\sigma \rangle$	$\mathcal{B}_{26} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13} s_{14} s_{25} s_{36}$
	$\mathcal{O}_{27} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u_\rho u^\sigma u_\sigma u_\nu \rangle$	$\mathcal{B}_{27} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{16} s_{23} s_{45}$
	$\mathcal{O}_{28} = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u_\nu u^\sigma u_\rho u_\sigma \rangle$	$\mathcal{B}_{28} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{13} s_{15} s_{46}$
	$\mathcal{O}_{29} = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u_\nu u_\rho u^\sigma u_\sigma \rangle$	$\mathcal{B}_{29} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{13} s_{14} s_{56}$
	$\mathcal{O}_{30} = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma u_\mu u_\sigma \rangle \langle u_\nu u_\rho \rangle$	$\mathcal{B}_{30} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{13} s_{15} s_{16} s_{24}$
	$\mathcal{O}_{31} = \langle \nabla^\mu u^\nu \nabla^\rho u^\sigma u_\rho u_\sigma \rangle \langle u_\mu u_\nu \rangle$	$\mathcal{B}_{31} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{15} s_{16} s_{23} s_{24}$
	$\mathcal{O}_{32} = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma u_\mu u_\nu \rangle \langle u_\rho u_\sigma \rangle$	$\mathcal{B}_{32} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{13} s_{14} s_{15} s_{26}$
	$\mathcal{O}_{33} = \langle \nabla^\mu u^\nu \nabla^\rho u^\sigma u_\mu u_\rho \rangle \langle u_\nu u_\sigma \rangle$	$\mathcal{B}_{33} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{13} s_{15} s_{24} s_{26}$
	$\mathcal{O}_{34} = \langle \nabla^\mu u^\nu \nabla^\rho u^\sigma u_\mu u_\nu \rangle \langle u_\rho u_\sigma \rangle$	$\mathcal{B}_{34} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{13} s_{14} s_{25} s_{26}$
	$\mathcal{O}_{35} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u^\sigma u_\sigma \rangle \langle u_\nu u_\rho \rangle$	$\mathcal{B}_{35} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{12} s_{15} s_{26} s_{34}$
	$\mathcal{O}_{36} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u^\sigma u_\rho \rangle \langle u_\nu u_\sigma \rangle$	$\mathcal{B}_{36} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{12} s_{15} s_{24} s_{36}$
	$\mathcal{O}_{37} = \langle \nabla^\mu u^\nu u^\rho \nabla_\mu u^\sigma u_\rho \rangle \langle u_\nu u_\sigma \rangle$	$\mathcal{B}_{37} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{13} s_{15} s_{24} s_{36}$
	$\mathcal{O}_{38} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u^\sigma u_\nu \rangle \langle u_\rho u_\sigma \rangle$	$\mathcal{B}_{38} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{12} s_{14} s_{25} s_{36}$
	$\mathcal{O}_{39} = \langle \nabla^\mu u^\nu u^\rho \nabla_\mu u^\sigma u_\nu \rangle \langle u_\rho u_\sigma \rangle$	$\mathcal{B}_{39} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{13} s_{14} s_{25} s_{36}$
	$\mathcal{O}_{40} = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u^\nu u^\rho \rangle \langle u^\sigma u_\sigma \rangle$	$\mathcal{B}_{40} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{12} s_{13} s_{14} s_{56}$

$SU(N_f)$	Operator Basis	Amplitude Basis
$SU(4)$	$\mathcal{O}_{41} = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma u_\sigma u_\mu \rangle \langle u_\nu u_\rho \rangle$	$\mathcal{B}_{41} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{14} s_{15} s_{16} s_{23}$
	$\mathcal{O}_{42} = \langle \nabla^\mu u^\nu \nabla^\rho u^\sigma u_\rho u_\mu \rangle \langle u_\nu u_\sigma \rangle$	$\mathcal{B}_{42} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{14} s_{15} s_{23} s_{26}$
	$\mathcal{O}_{43} = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u^\sigma u_\sigma \rangle \langle u_\nu u_\rho \rangle$	$\mathcal{B}_{43} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{12} s_{15} s_{16} s_{34}$
	$\mathcal{O}_{44} = \langle \nabla^\mu u^\nu u^\rho \nabla_\mu u^\sigma u_\sigma \rangle \langle u_\nu u_\rho \rangle$	$\mathcal{B}_{44} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{13} s_{15} s_{26} s_{34}$
	$\mathcal{O}_{45} = \langle \nabla^\mu u^\nu u^\rho u^\sigma u_\sigma \rangle \langle \nabla_\mu u_\nu u_\rho \rangle$	$\mathcal{B}_{45} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{15}^2 s_{26} s_{34}$
	$\mathcal{O}_{46} = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u^\sigma u_\nu \rangle \langle u_\rho u_\sigma \rangle$	$\mathcal{B}_{46} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{12} s_{14} s_{15} s_{36}$
	$\mathcal{O}_{47} = \langle \nabla^\mu u^\nu u^\rho \nabla_\rho u^\sigma u_\mu \rangle \langle u_\nu u_\sigma \rangle$	$\mathcal{B}_{47} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{14} s_{15} s_{23} s_{36}$
	$\mathcal{O}_{48} = \langle \nabla^\mu u^\nu u^\rho u^\sigma \nabla_\rho u_\mu \rangle \langle u_\nu u_\sigma \rangle$	$\mathcal{B}_{48} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{14} s_{15} s_{24} s_{36}$
	$\mathcal{O}_{49} = \langle \nabla^\mu u^\nu u^\rho u^\sigma \nabla_\mu u_\nu \rangle \langle u_\rho u_\sigma \rangle$	$\mathcal{B}_{49} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{14}^2 s_{25} s_{36}$
	$\mathcal{O}_{50} = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u_\nu u^\sigma \rangle \langle u_\rho u_\sigma \rangle$	$\mathcal{B}_{50} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{12} s_{13} s_{15} s_{46}$
	$\mathcal{O}_{51} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u_\rho u^\sigma \rangle \langle u^\nu u_\sigma \rangle$	$\mathcal{B}_{51} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{12} s_{15} s_{23} s_{46}$
	$\mathcal{O}_{52} = \langle \nabla^\mu u^\nu u^\rho \nabla_\mu u_\nu u^\sigma \rangle \langle u_\rho u_\sigma \rangle$	$\mathcal{B}_{52} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{13}^2 s_{25} s_{46}$
	$\mathcal{O}_{53} = \langle u^\mu u^\nu u^\rho u^\sigma \rangle \langle \nabla_\mu \nabla_\nu u_\rho u_\sigma \rangle$	$\mathcal{B}_{53} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{15} s_{25} s_{35} s_{46}$
	$\mathcal{O}_{54} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u_\nu u_\rho \rangle \langle u^\sigma u_\sigma \rangle$	$\mathcal{B}_{54} = \mathcal{Y} \circ \mathrm{tr}[1234\|56] s_{12} s_{13} s_{24} s_{56}$
	$\mathcal{O}_{55} = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma u_\sigma \rangle \langle u_\mu u_\nu u_\rho \rangle$	$\mathcal{B}_{55} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{14} s_{15} s_{16} s_{23}$
	$\mathcal{O}_{56} = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma u_\mu \rangle \langle u_\nu u_\rho u_\sigma \rangle$	$\mathcal{B}_{56} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{13} s_{14} s_{15} s_{26}$
	$\mathcal{O}_{57} = \langle \nabla^\mu u^\nu \nabla^\rho u^\sigma u_\rho \rangle \langle u_\mu u_\nu u_\sigma \rangle$	$\mathcal{B}_{57} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{14} s_{15} s_{23} s_{26}$
	$\mathcal{O}_{58} = \langle \nabla^\mu u^\nu \nabla^\rho u^\sigma u_\mu \rangle \langle u_\nu u_\rho u_\sigma \rangle$	$\mathcal{B}_{58} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{13} s_{14} s_{25} s_{26}$
	$\mathcal{O}_{59} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u^\sigma \rangle \langle u_\nu u_\sigma u_\rho \rangle$	$\mathcal{B}_{59} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{12} s_{14} s_{26} s_{35}$
	$\mathcal{O}_{60} = \langle \nabla^\mu u^\nu u^\rho u^\sigma \rangle \langle \nabla_\mu u_\nu u_\sigma u_\rho \rangle$	$\mathcal{B}_{60} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{14}^2 s_{26} s_{35}$
	$\mathcal{O}_{61} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u^\sigma \rangle \langle u_\nu u_\rho u_\sigma \rangle$	$\mathcal{B}_{61} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{12} s_{14} s_{25} s_{36}$
	$\mathcal{O}_{62} = \langle \nabla^\mu u^\nu u^\rho u^\sigma \rangle \langle \nabla_\mu u_\nu u_\rho u_\sigma \rangle$	$\mathcal{B}_{62} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{14}^2 s_{25} s_{36}$
	$\mathcal{O}_{63} = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u_\nu \rangle \langle u^\sigma u_\rho u_\sigma \rangle$	$\mathcal{B}_{63} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{12} s_{13} s_{15} s_{46}$
	$\mathcal{O}_{64} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u_\rho \rangle \langle u^\sigma u_\nu u_\sigma \rangle$	$\mathcal{B}_{64} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{12} s_{15} s_{23} s_{46}$
	$\mathcal{O}_{65} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u_\nu \rangle \langle u^\sigma u_\rho u_\sigma \rangle$	$\mathcal{B}_{65} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{12} s_{13} s_{25} s_{46}$
	$\mathcal{O}_{66} = \langle \nabla^\mu u^\nu u^\rho u_\mu \rangle \langle \nabla_\nu u^\sigma u_\rho u_\sigma \rangle$	$\mathcal{B}_{66} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{13} s_{14} s_{25} s_{46}$
	$\mathcal{O}_{67} = \langle \nabla^\mu u^\nu \nabla^\rho u^\sigma \rangle \langle u_\mu u_\nu \rangle \langle u_\rho u_\sigma \rangle$	$\mathcal{B}_{67} = \mathcal{Y} \circ \mathrm{tr}[12\|34\|56] s_{13} s_{14} s_{25} s_{26}$
	$\mathcal{O}_{68} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho \rangle \langle u^\sigma u_\nu \rangle \langle u_\rho u_\sigma \rangle$	$\mathcal{B}_{68} = \mathcal{Y} \circ \mathrm{tr}[12\|34\|56] s_{12} s_{14} s_{25} s_{36}$
$SU(5)$	$\mathcal{O}_{69} = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u^\sigma \rangle \langle u_\nu u_\rho u_\sigma \rangle$	$\mathcal{B}_{69} = \mathcal{Y} \circ \mathrm{tr}[123\|456] s_{12} s_{14} s_{15} s_{36}$
	$\mathcal{O}_{70} = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma \rangle \langle u_\mu u_\nu \rangle \langle u_\rho u_\sigma \rangle$	$\mathcal{B}_{70} = \mathcal{Y} \circ \mathrm{tr}[12\|34\|56] s_{13} s_{14} s_{15} s_{26}$
	$\mathcal{O}_{71} = \langle \nabla^\mu u^\nu \nabla^\rho u^\sigma \rangle \langle u_\mu u_\rho \rangle \langle u_\nu u_\sigma \rangle$	$\mathcal{B}_{71} = \mathcal{Y} \circ \mathrm{tr}[12\|34\|56] s_{13} s_{15} s_{24} s_{26}$
	$\mathcal{O}_{72} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho \rangle \langle u^\sigma u_\sigma \rangle \langle u_\nu u_\rho \rangle$	$\mathcal{B}_{72} = \mathcal{Y} \circ \mathrm{tr}[12\|34\|56] s_{12} s_{15} s_{26} s_{34}$
	$\mathcal{O}_{73} = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu \rangle \langle u^\sigma u_\nu \rangle \langle u_\rho u_\sigma \rangle$	$\mathcal{B}_{73} = \mathcal{Y} \circ \mathrm{tr}[12\|34\|56] s_{12} s_{14} s_{15} s_{36}$
	$\mathcal{O}_{74} = \langle \nabla^\mu u^\nu u^\rho \rangle \langle \nabla_\mu u^\sigma u_\nu \rangle \langle u_\rho u_\sigma \rangle$	$\mathcal{B}_{74} = \mathcal{Y} \circ \mathrm{tr}[12\|34\|56] s_{13} s_{14} s_{25} s_{36}$
$SU(N_f \geq 6)$	$\mathcal{O}_{75} = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu \rangle \langle u^\sigma u_\sigma \rangle \langle u_\nu u_\rho \rangle$	$\mathcal{B}_{75} = \mathcal{Y} \circ \mathrm{tr}[12\|34\|56] s_{12} s_{15} s_{16} s_{34}$
	$\mathcal{O}_{76} = \langle \nabla^\mu u^\nu u^\rho \rangle \langle \nabla_\mu u^\sigma u_\rho \rangle \langle u_\nu u_\sigma \rangle$	$\mathcal{B}_{76} = \mathcal{Y} \circ \mathrm{tr}[12\|34\|56] s_{13} s_{15} s_{24} s_{36}$

$O(p^8)$ 6-pt P-odd operators

$SU(N_f)$	Operator Basis	Amplitude Basis
$SU(2)$	$\mathcal{O}_1 = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u_\nu u^\sigma u^\eta u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_1 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{13} \epsilon(2,4,5,6)$
	$\mathcal{O}_2 = \langle \nabla^\mu u^\rho \nabla^\sigma u^\nu u_\nu u_\mu u^\eta u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_2 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{14} s_{23} \epsilon(1,2,5,6)$
$SU(3)$	$\mathcal{O}_3 = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma u_\mu u^\eta u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_3 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13} s_{15} \epsilon(1,2,4,6)$
	$\mathcal{O}_4 = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma u_\mu u_\nu u^\eta u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_4 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13} s_{14} \epsilon(1,2,5,6)$
	$\mathcal{O}_5 = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u^\sigma u^\eta u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_5 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{15} \epsilon(1,3,4,6)$
	$\mathcal{O}_6 = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u^\sigma u_\nu u^\eta u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_6 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{14} \epsilon(1,3,5,6)$
	$\mathcal{O}_7 = \langle \nabla^\mu u^\rho u^\nu \nabla_\mu u^\sigma u_\nu u^\eta u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_7 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13} s_{24} \epsilon(1,3,5,6)$
	$\mathcal{O}_8 = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u_\nu u^\sigma u^\eta u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_8 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{13} \epsilon(1,4,5,6)$
	$\mathcal{O}_9 = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u^\sigma u^\eta u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_9 = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{15} \epsilon(2,3,4,6)$
	$\mathcal{O}_{10} = \langle \nabla^\mu u^\nu u^\rho \nabla_\mu u^\sigma u^\eta u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{10} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13} s_{15} \epsilon(2,3,4,6)$
	$\mathcal{O}_{11} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u^\sigma u_\nu u^\eta u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{11} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12} s_{14} \epsilon(2,3,5,6)$
	$\mathcal{O}_{12} = \langle \nabla^\mu u^\nu u^\rho \nabla_\mu u^\sigma u_\nu u^\eta u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{12} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13} s_{14} \epsilon(2,3,5,6)$
	$\mathcal{O}_{13} = \langle \nabla^\mu u^\nu u^\rho u^\sigma \nabla_\mu u_\nu u^\eta u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{13} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{14}^2 \epsilon(2,3,5,6)$
	$\mathcal{O}_{14} = \langle \nabla^\mu u^\nu u^\rho \nabla_\mu u_\nu u^\sigma u^\eta u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{14} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{13}^2 \epsilon(2,4,5,6)$
	$\mathcal{O}_{15} = \langle \nabla^\mu u^\nu \nabla_\mu u_\nu u^\rho u^\sigma u^\eta u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{15} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{12}^2 \epsilon(3,4,5,6)$
	$\mathcal{O}_{16} = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma u^\eta u_\mu u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{16} = \mathcal{Y} \circ \mathrm{tr}[123456] s_{14} s_{15} \epsilon(1,2,3,6)$
	$\mathcal{O}_{17} = \langle \nabla^\mu u^\rho \nabla^\nu u^\sigma u_\mu u^\eta \rangle \langle u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{17} = \mathcal{Y} \circ \mathrm{tr}[1234\vert 56] s_{13} s_{25} \epsilon(1,2,4,6)$
	$\mathcal{O}_{18} = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u^\sigma u^\eta \rangle \langle u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{18} = \mathcal{Y} \circ \mathrm{tr}[1234\vert 56] s_{12} s_{15} \epsilon(1,3,4,6)$
	$\mathcal{O}_{19} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u^\sigma u^\eta \rangle \langle u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{19} = \mathcal{Y} \circ \mathrm{tr}[1234\vert 56] s_{12} s_{15} \epsilon(2,3,4,6)$
	$\mathcal{O}_{20} = \langle \nabla^\mu u^\nu u^\rho \nabla_\mu u^\sigma u^\eta \rangle \langle u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{20} = \mathcal{Y} \circ \mathrm{tr}[1234\vert 56] s_{13} s_{15} \epsilon(2,3,4,6)$
$SU(4)$	$\mathcal{O}_{21} = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma u^\eta u_\mu \rangle \langle u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{21} = \mathcal{Y} \circ \mathrm{tr}[1234\vert 56] s_{14} s_{15} \epsilon(1,2,3,6)$
	$\mathcal{O}_{22} = \langle \nabla^\mu u^\rho \nabla^\nu u^\sigma u^\eta u_\mu \rangle \langle u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{22} = \mathcal{Y} \circ \mathrm{tr}[1234\vert 56] s_{14} s_{25} \epsilon(1,2,3,6)$
	$\mathcal{O}_{23} = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma u_\mu u^\eta \rangle \langle u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{23} = \mathcal{Y} \circ \mathrm{tr}[1234\vert 56] s_{13} s_{15} \epsilon(1,2,4,6)$
	$\mathcal{O}_{24} = \langle \nabla^\mu u^\rho u^\nu \nabla_\mu u^\sigma u^\eta \rangle \langle u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{24} = \mathcal{Y} \circ \mathrm{tr}[1234\vert 56] s_{13} s_{25} \epsilon(1,3,4,6)$
	$\mathcal{O}_{25} = \langle \nabla^\mu u^\rho u^\nu u^\sigma \nabla_\mu u^\eta \rangle \langle u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{25} = \mathcal{Y} \circ \mathrm{tr}[1234\vert 56] s_{14} s_{25} \epsilon(1,3,4,6)$
	$\mathcal{O}_{26} = \langle \nabla^\mu u^\nu u^\rho u^\sigma \nabla_\mu u^\eta \rangle \langle u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{26} = \mathcal{Y} \circ \mathrm{tr}[1234\vert 56] s_{14} s_{15} \epsilon(2,3,4,6)$
	$\mathcal{O}_{27} = \langle u^\mu \nabla^\nu u^\rho u^\sigma \nabla_\mu u^\eta \rangle \langle u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{27} = \mathcal{Y} \circ \mathrm{tr}[1234\vert 56] s_{14} s_{25} \epsilon(2,3,4,6)$
	$\mathcal{O}_{28} = \langle \nabla^\mu u^\rho u^\nu u^\sigma \nabla_\mu \rangle \langle u^\eta u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{28} = \mathcal{Y} \circ \mathrm{tr}[123\vert 456] s_{14} s_{25} \epsilon(1,3,4,6)$
	$\mathcal{O}_{29} = \langle \nabla^\mu \nabla^\nu u^\rho u_\mu u_\nu \rangle \langle u^\sigma u^\eta u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{29} = \mathcal{Y} \circ \mathrm{tr}[123\vert 456] s_{12} s_{13} \epsilon(1,4,5,6)$
	$\mathcal{O}_{30} = \langle \nabla^\mu u^\nu u^\rho u^\sigma \nabla_\mu \rangle \langle u_\nu u^\eta u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{30} = \mathcal{Y} \circ \mathrm{tr}[123\vert 456] s_{14}^2 \epsilon(2,3,5,6)$
	$\mathcal{O}_{31} = \langle \nabla^\mu u^\nu \nabla_\mu u^\rho u_\nu \rangle \langle u^\sigma u^\eta u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{31} = \mathcal{Y} \circ \mathrm{tr}[123\vert 456] s_{12} s_{13} \epsilon(2,4,5,6)$
	$\mathcal{O}_{32} = \langle \nabla^\mu u^\nu \nabla_\mu u_\nu u^\rho \rangle \langle u^\sigma u^\eta u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{32} = \mathcal{Y} \circ \mathrm{tr}[123\vert 456] s_{12}^2 \epsilon(3,4,5,6)$
	$\mathcal{O}_{33} = \langle \nabla^\mu u^\nu u^\rho \nabla_\mu \rangle \langle u^\sigma u^\eta \rangle \langle u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{33} = \mathcal{Y} \circ \mathrm{tr}[12\vert 34\vert 56] s_{13} s_{15} \epsilon(2,3,4,6)$
$SU(N_f \geq 5)$	$\mathcal{O}_{34} = \langle \nabla^\mu \nabla^\nu u^\rho u^\sigma u^\eta \rangle \langle u_\mu u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{34} = \mathcal{Y} \circ \mathrm{tr}[123\vert 456] s_{14} s_{15} \epsilon(1,2,3,6)$
	$\mathcal{O}_{35} = \langle \nabla^\mu u^\rho \nabla^\nu u^\sigma \rangle \langle u_\mu u^\eta \rangle \langle u_\nu u^\lambda \rangle \epsilon_{\rho\sigma\eta\lambda}$	$\mathcal{B}_{35} = \mathcal{Y} \circ \mathrm{tr}[12\vert 34\vert 56] s_{13} s_{25} \epsilon(1,2,4,6)$

Conclusion

- Features of our algorithm:
  1. Monomial operators basis (consistent with Hilbert Series counting);
  2. Parity-odd amplitude/operator basis;
  3. Gram determinant considered;
  4. Cayley-Hamilton theorem considered (finite N_f);
  5. Capable of deriving linear relations among the operators.
- Easy to add external sources: EWChEFT (NLO, NNLO) (Sun, **MLX**, Yu, 22'&23').

Classes	$\mathcal{N}_{\text{type}}$	$\mathcal{N}_{\text{term}}$	$\mathcal{N}_{\text{operator}}$
UhD^4	$3+6+0+0$	15	15
X^2Uh	$6+4+0+0$	10	10
$XUhD^2$	$2+6+0+0$	8	8
X^3	$4+2+0+0$	6	6
ψ^2UhD	$4+8+0+0$	13(16)	${n_f}^2$ $(16{n_f}^2)$
ψ^2UhD^2	$6+10+0+0$	60(80)	$60{n_f}^2$ $(80{n_f}^2)$
ψ^2UhX	$7+7+0+0$	22(28)	$22{n_f}^2$ $(28{n_f}^2)$
ψ^4	$12+24+4+8$	117(160)	$\frac{1}{4}{n_f}^2(31-6n_f+335{n_f}^2)$ $({n_f}^2(9-2n_f+125{n_f}^2))$
Total	123	261(313)	$\frac{335{n_f}^4}{4}-\frac{3{n_f}^3}{2}+\frac{411{n_f}^2}{4}+39$ $(39+133{n_f}^2-2{n_f}^2-2{n_f}^3+125{n_f}^4)$ $\mathcal{N}_{\text{operatrs}}(n_f=1)=224(295)$, $\mathcal{N}_{\text{operatrs}}(n_f=3)=7704(11307)$

Thank you for your attention!

Gram Det Makes a Difference

ONLY at $O(p^{10})$ or beyond (# independent trace orbits):

Trace Class	(6)	$(4\|2)$	$(3\|3)$	$(2\|2\|2)$
General D	112	91	43	25
$D=4$	111	90	42	24

	(8)	$(6\|2)$	$(5\|3)$	$(4\|4)$	$(4\|2\|2)$	$(3\|3\|2)$	$(2\|2\|2\|2)$
General D	435	320	226	129	149	117	26
$D=4$	427	314	222	126	146	115	25

	(10)	$(8\|2)$	$(7\|3)$	$(6\|4)$	$(5\|5)$	$(6\|2\|2)$
General D	105	74	45	50	29	37
$D=4$	99	71	43	47	27	35

	$(5\|3\|2)$	$(4\|4\|2)$	$(4\|3\|3)$	$(4\|2\|2\|2)$	$(3\|3\|2\|2)$	$(2\|2\|2\|2\|2)$
General D	35	30	21	18	18	7
$D=4$	35	28	20	17	17	6