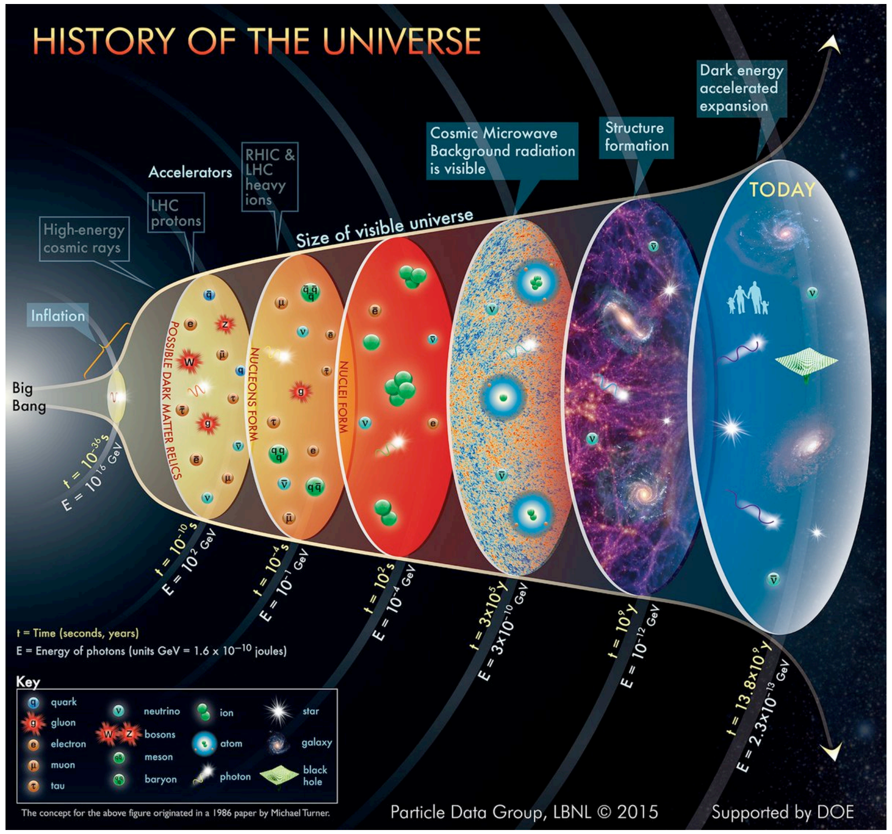
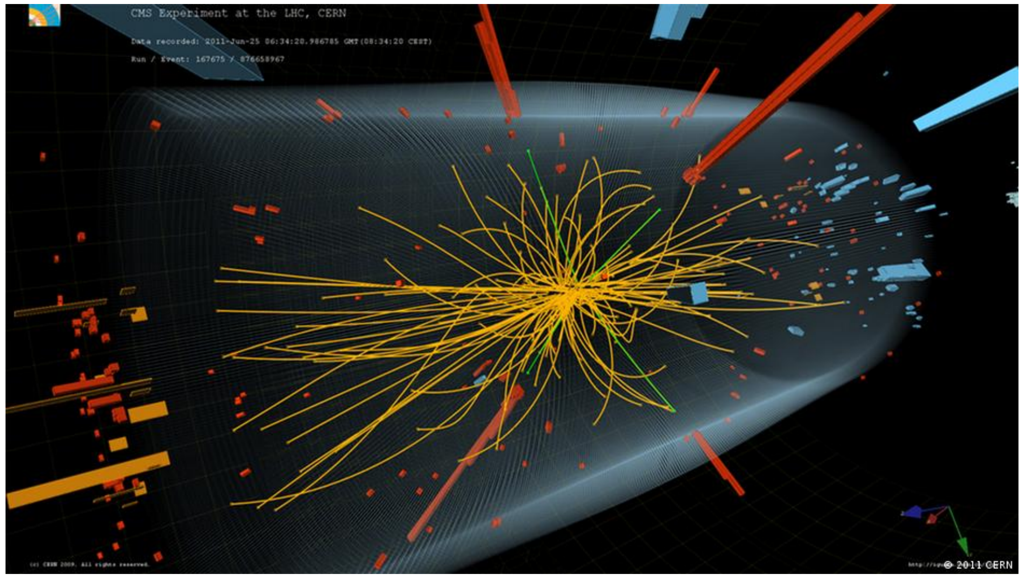
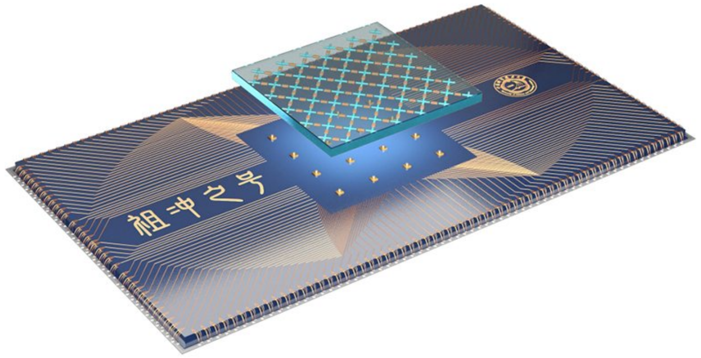






Ying-Ying Li (李英英), USTC

PRD.104,094519, PRD.106, 114504,
PRL.129, 051601, arXiv: 2402.16780
in collaboration with
Marcela Carena, Erik J. Gustafson,
Henry Lamm, Wanqiang Liu

Quantum Computing for Lattice Gauge Theories

April, 2024 @ 第六届重味物理与QCD研讨会

QUANTUM EASY

High Energy Physics

real-time dynamics

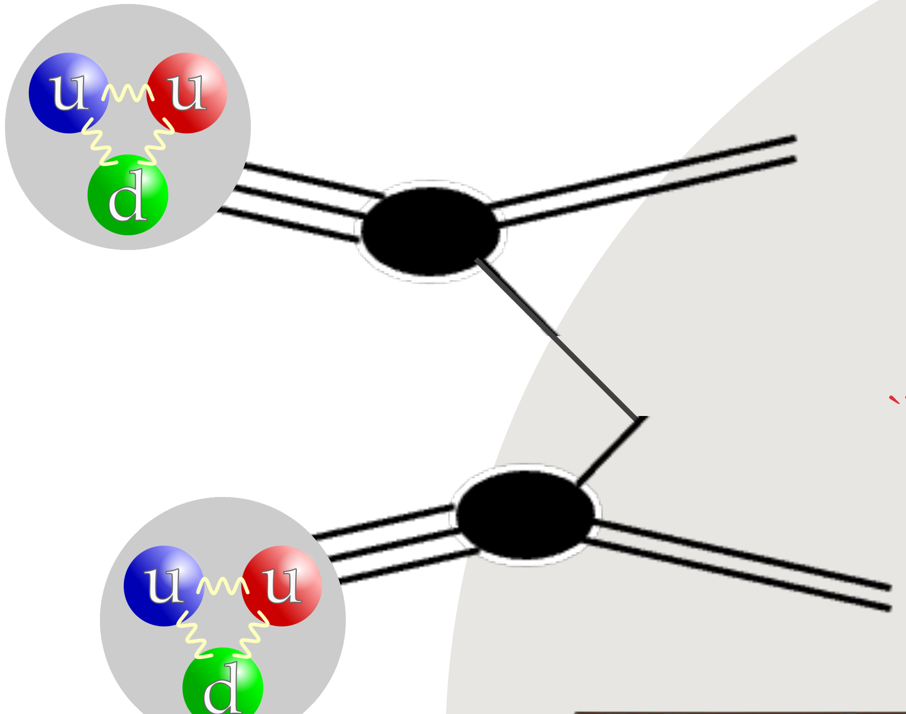
finite density

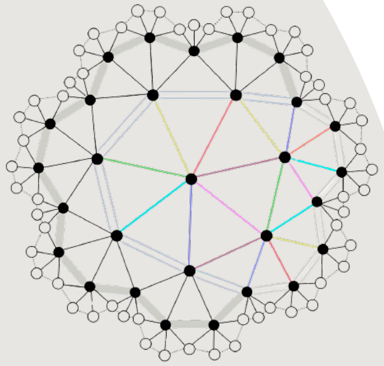
quantum interference

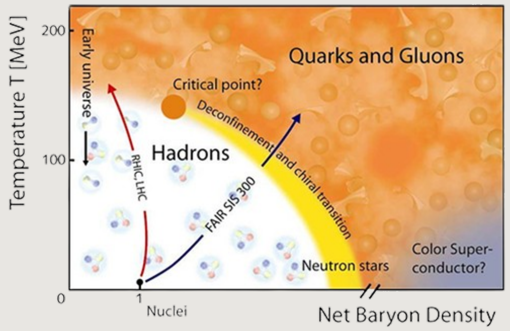
out-of equilibrium

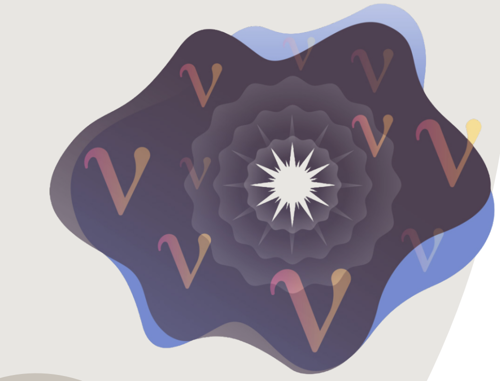
``strongly interacting many-body system''

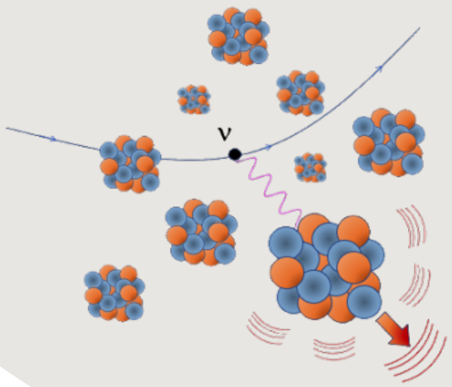
CLASSICAL EASY

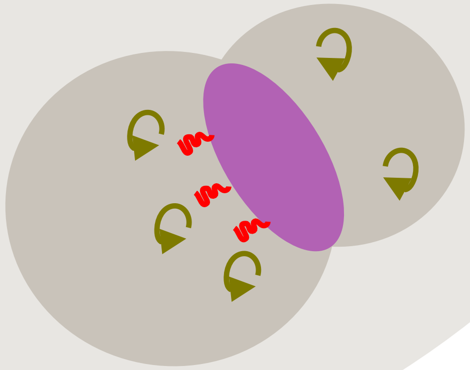


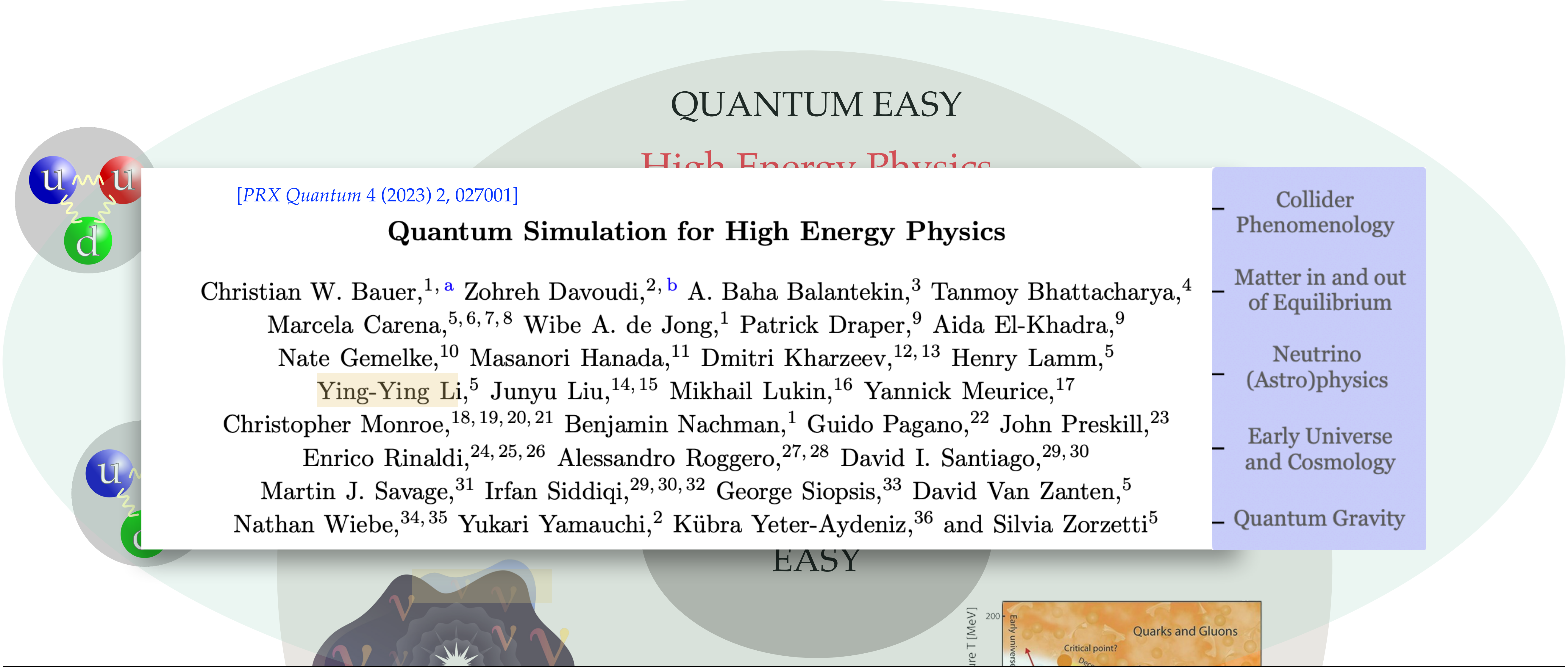












[*PRX Quantum* 4 (2023) 2, 027001]

Quantum Simulation for High Energy Physics

Christian W. Bauer,[1, a] Zohreh Davoudi,[2, b] A. Baha Balantekin,[3] Tanmoy Bhattacharya,[4]
Marcela Carena,[5, 6, 7, 8] Wibe A. de Jong,[1] Patrick Draper,[9] Aida El-Khadra,[9]
Nate Gemelke,[10] Masanori Hanada,[11] Dmitri Kharzeev,[12, 13] Henry Lamm,[5]
Ying-Ying Li,[5] Junyu Liu,[14, 15] Mikhail Lukin,[16] Yannick Meurice,[17]
Christopher Monroe,[18, 19, 20, 21] Benjamin Nachman,[1] Guido Pagano,[22] John Preskill,[23]
Enrico Rinaldi,[24, 25, 26] Alessandro Roggero,[27, 28] David I. Santiago,[29, 30]
Martin J. Savage,[31] Irfan Siddiqi,[29, 30, 32] George Siopsis,[33] David Van Zanten,[5]
Nathan Wiebe,[34, 35] Yukari Yamauchi,[2] Kübra Yeter-Aydeniz,[36] and Silvia Zorzetti[5]

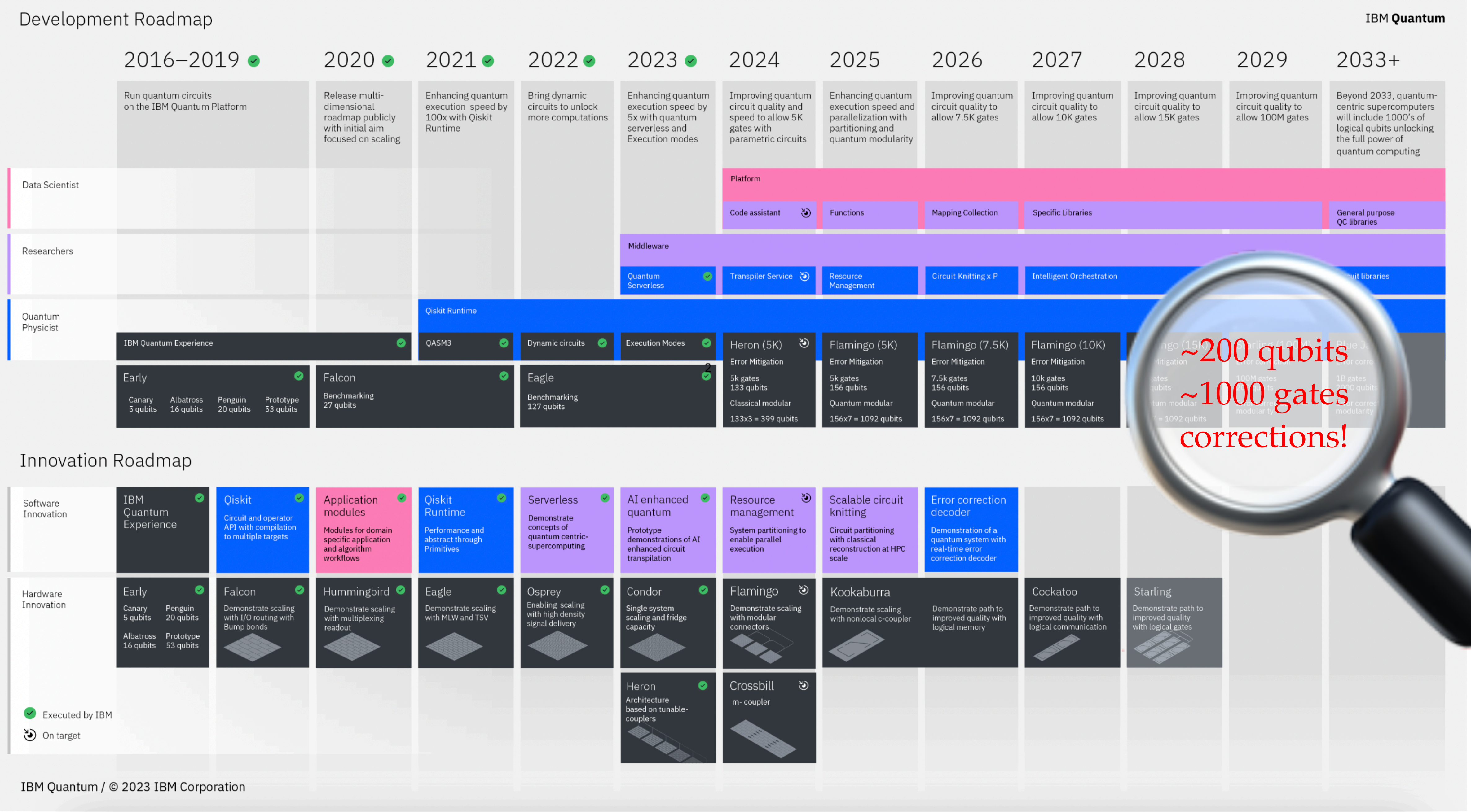
- Collider Phenomenology
- Matter in and out of Equilibrium
- Neutrino (Astro)physics
- Early Universe and Cosmology
- Quantum Gravity

International Conference on Quantum Technology for High-Energy Physics (QT4HEP), CERN, 2022
Quantum Computing Methods For High Energy Physics, Munich, 2023
Quantum Computing and Machine Learning Workshop, 青岛, 2023
人工智能、量子信息、量子计算在粒子物理、核物理和宇宙学等学科前沿的应用，吉林，2024
Quantum Technologies and Computation for High Energy Physics, Munich, 2024

Quantum Computing

- Now - Noisy Intermediate Scale Quantum (NISQ) era
  more than 50 well controlled qubits, not error-corrected yet

- Next decades



Development Roadmap

IBM Quantum

2016–2019 ✓	2020 ✓	2021 ✓	2022 ✓	2023 ✓	2024	2025	2026	2027	2028	2029	2033+
Run quantum circuits on the IBM Quantum Platform	Release multi-dimensional roadmap publicly with initial aim focused on scaling	Enhancing quantum execution speed by 100x with Qiskit Runtime	Bring dynamic circuits to unlock more computations	Enhancing quantum execution speed by 5x with quantum serverless and Execution modes	Improving quantum circuit quality and speed to allow 5K gates with parametric circuits	Enhancing quantum execution speed and parallelization with partitioning and quantum modularity	Improving quantum circuit quality to allow 7.5K gates	Improving quantum circuit quality to allow 10K gates	Improving quantum circuit quality to allow 15K gates	Improving quantum circuit quality to allow 100M gates	Beyond 2033, quantum-centric supercomputers will include 1000's of logical qubits unlocking the full power of quantum computing

Data Scientist

- Platform
- Code assistant ↻
- Functions
- Mapping Collection
- Specific Libraries
- General purpose QC libraries

Researchers

- Middleware
- Quantum Serverless ✓
- Transpiler Service ↻
- Resource Management
- Circuit Knitting x P
- Intelligent Orchestration
- uit libraries

Quantum Physicist

- Qiskit Runtime
- IBM Quantum Experience ✓
- QASM3 ✓
- Dynamic circuits ✓
- Execution Modes ✓
- Heron (5K) ↻ — Error Mitigation — 5k gates, 133 qubits — Classical modular — 133x3 = 399 qubits
- Flamingo (5K) — Error Mitigation — 5k gates, 156 qubits — Quantum modular — 156x7 = 1092 qubits
- Flamingo (7.5K) — Error Mitigation — 7.5k gates, 156 qubits — Quantum modular — 156x7 = 1092 qubits
- Flamingo (10K) — Error Mitigation — 10k gates, 156 qubits — Quantum modular — 156x7 = 1092 qubits
- Starling (100M) — 100M gates
- Blue J — 1B gates, 2000 qubits

Hardware

- Early ✓ — Canary 5 qubits, Albatross 16 qubits, Penguin 20 qubits, Prototype 53 qubits
- Falcon ✓ — Benchmarking 27 qubits
- Eagle ✓ — Benchmarking 127 qubits

~200 qubits
~1000 gates
corrections!

Innovation Roadmap

Software Innovation

- IBM Quantum Experience ✓
- Qiskit ✓ — Circuit and operator API with compilation to multiple targets
- Application modules ✓ — Modules for domain specific application and algorithm workflows
- Qiskit Runtime ✓ — Performance and abstract through Primitives
- Serverless ✓ — Demonstrate concepts of quantum centric-supercomputing
- AI enhanced quantum ✓ — Prototype demonstrations of AI enhanced circuit transpilation
- Resource management ↻ — System partitioning to enable parallel execution
- Scalable circuit knitting — Circuit partitioning with classical reconstruction at HPC scale
- Error correction decoder — Demonstration of a quantum system with real-time error correction decoder

Hardware Innovation

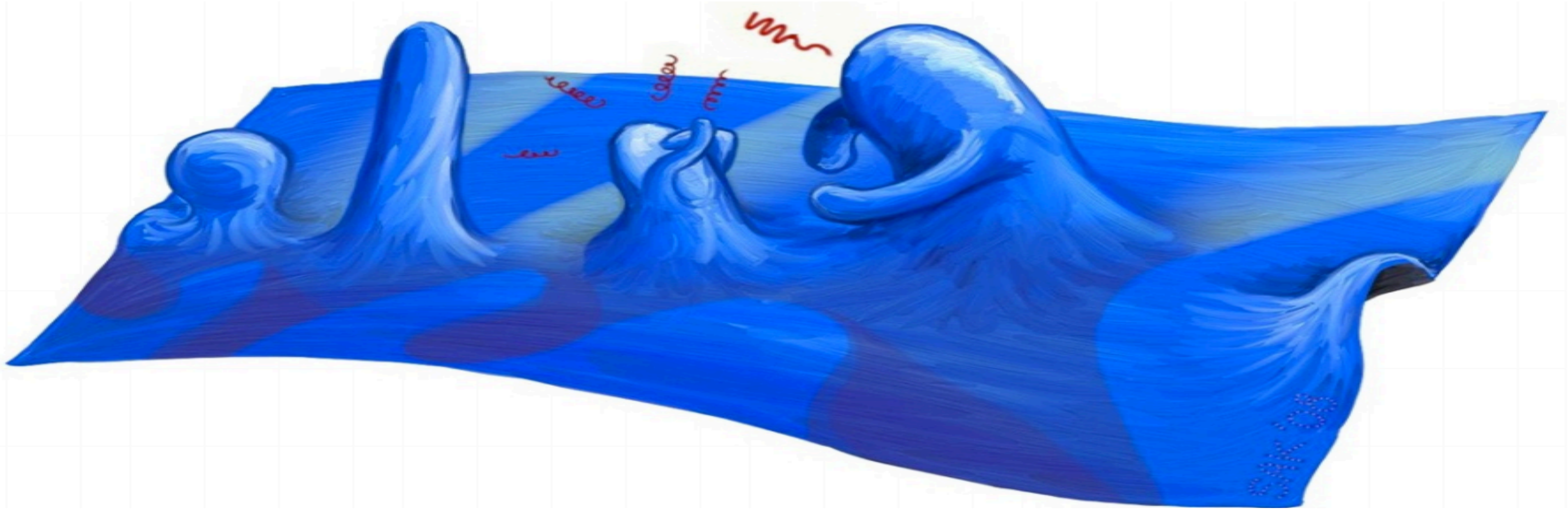
- Early ✓ — Canary 5 qubits, Penguin 20 qubits, Albatross 16 qubits, Prototype 53 qubits
- Falcon ✓ — Demonstrate scaling with I/O routing with Bump bonds
- Hummingbird ✓ — Demonstrate scaling with multiplexing readout
- Eagle ✓ — Demonstrate scaling with MLW and TSV
- Osprey ✓ — Enabling scaling with high density signal delivery
- Condor ✓ — Single system scaling and fridge capacity
- Heron ✓ — Architecture based on tunable-couplers
- Flamingo ↻ — Demonstrate scaling with modular connectors
- Crossbill ↻ — m- coupler
- Kookaburra — Demonstrate scaling with nonlocal c-coupler — Demonstrate path to improved quality with logical memory
- Cockatoo — Demonstrate path to improved quality with logical communication
- Starling — Demonstrate path to improved quality with logical gates

✓ Executed by IBM

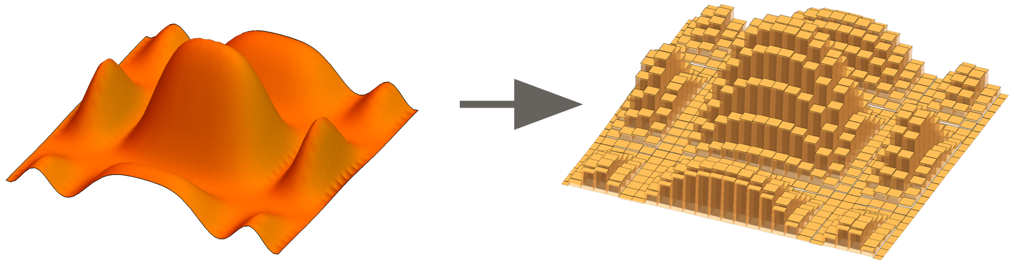
↻ On target

Quantum Computing for QFT

$$\int \mathcal{D}\phi e^{iS} = \langle x| \, e^{-iHt} \, |y\rangle$$



Discretization



infinities in space

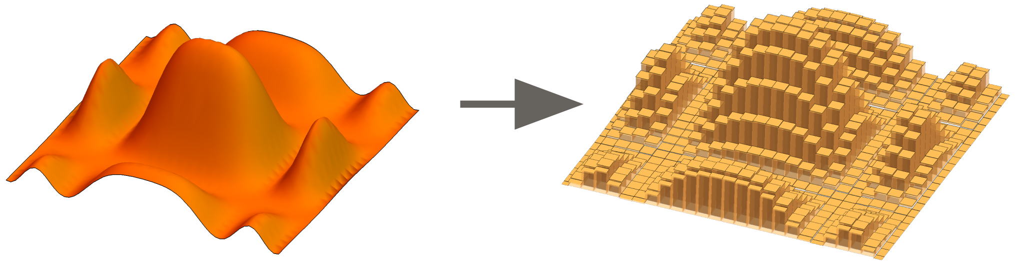
Digitization

$|q\rangle^N \to |G\rangle$

infinities in field variables

Quantum Computing for QFT

General Framework — **How Costly?**

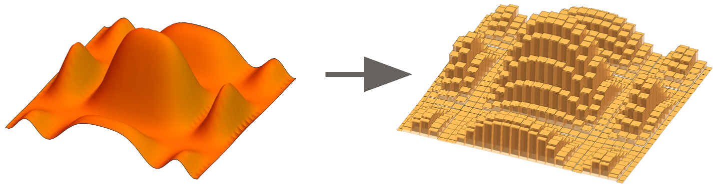
Discretization		*infinities in space*
Digitization	$\|q\rangle^N \rightarrow \|G\rangle$	*infinities in field variables*
Initialization	$\mathcal{U}\,\|G\rangle^L \rightarrow \|\psi_0\rangle$	*ground/thermal/bound state prep*
Propagation	$\mathcal{U}\,\|\psi_0\rangle \rightarrow \|\psi(t)\rangle$	*efficiency of time evolutions*
Evaluation	$\langle\mathcal{O}\rangle$	*parton distribution function,* *particle decay, …*
Error mitigation/corrections		

check references in
[M. Carena, H. Lamm, **YYL**, W. Liu, PRD. 104, 094519]

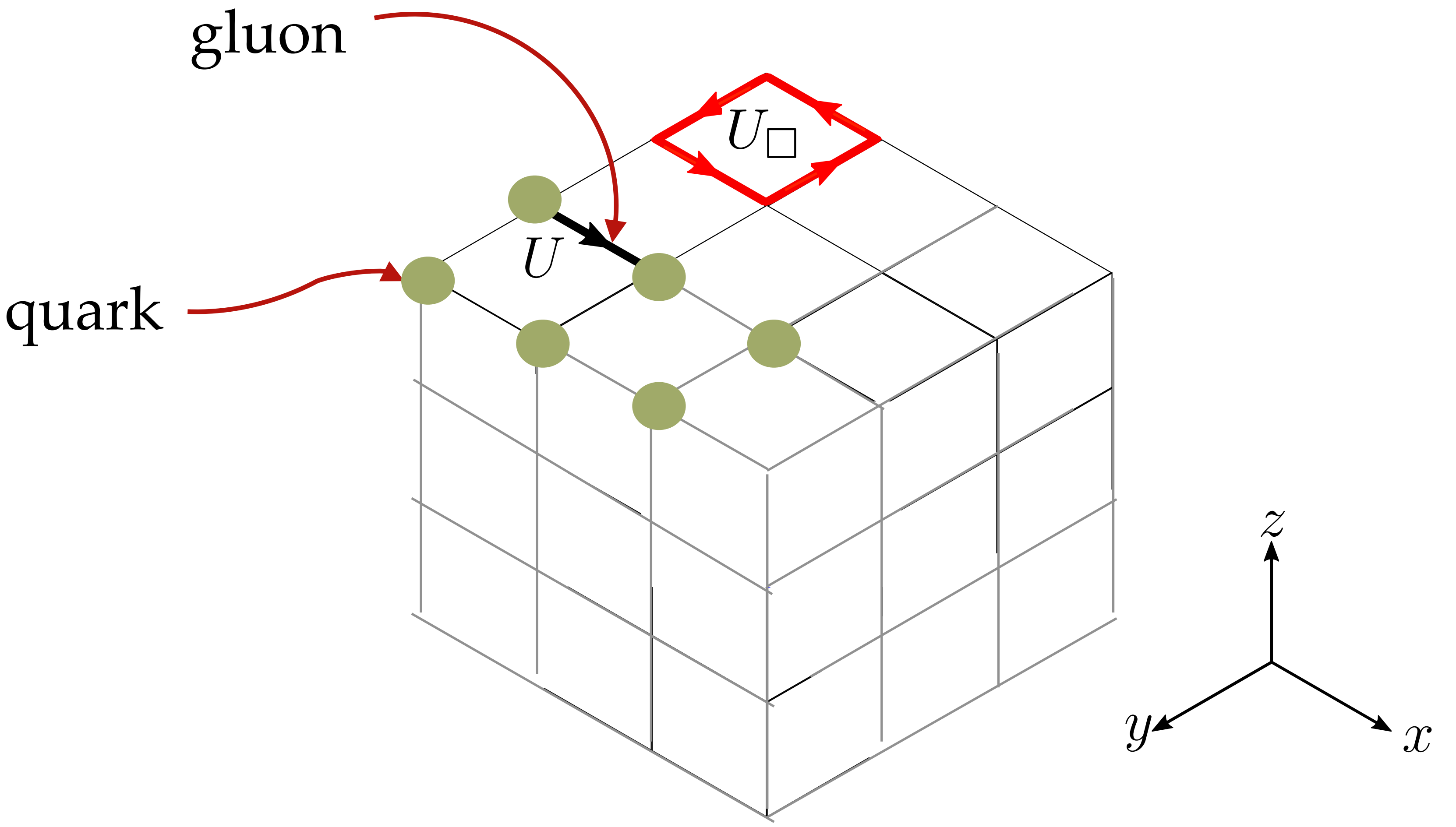
Discretization

infinities in space

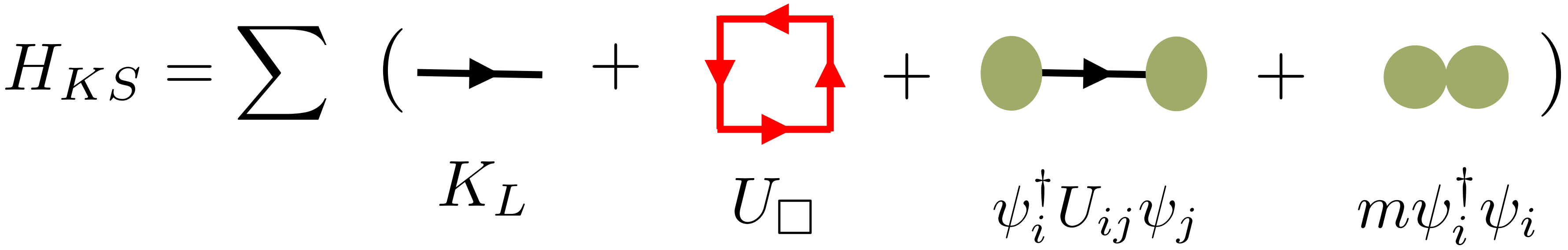
Discretization



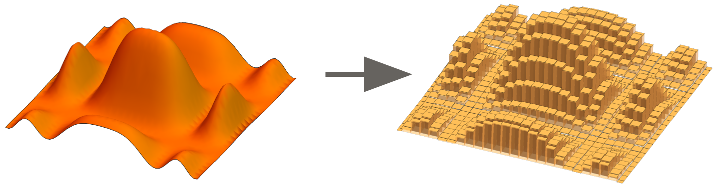
infinities in QFT

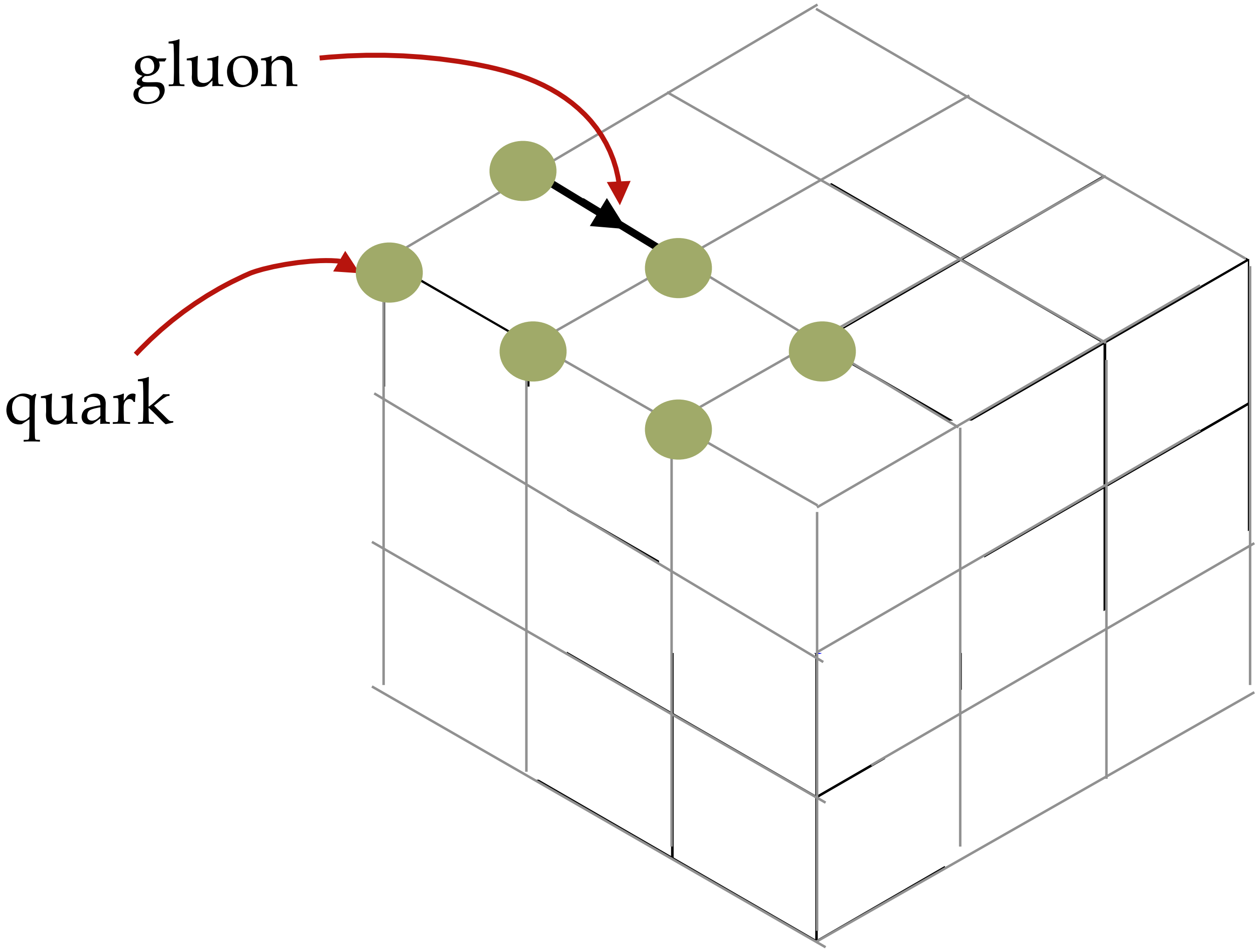


KS Hamiltonian Phys. Rev. D 11, 395 (1975)



Discretization





spatial dimension d

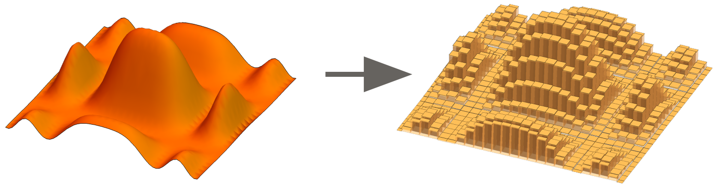
lattice spacing a

$$(na)^{-1} \lesssim E \lesssim a^{-1}$$

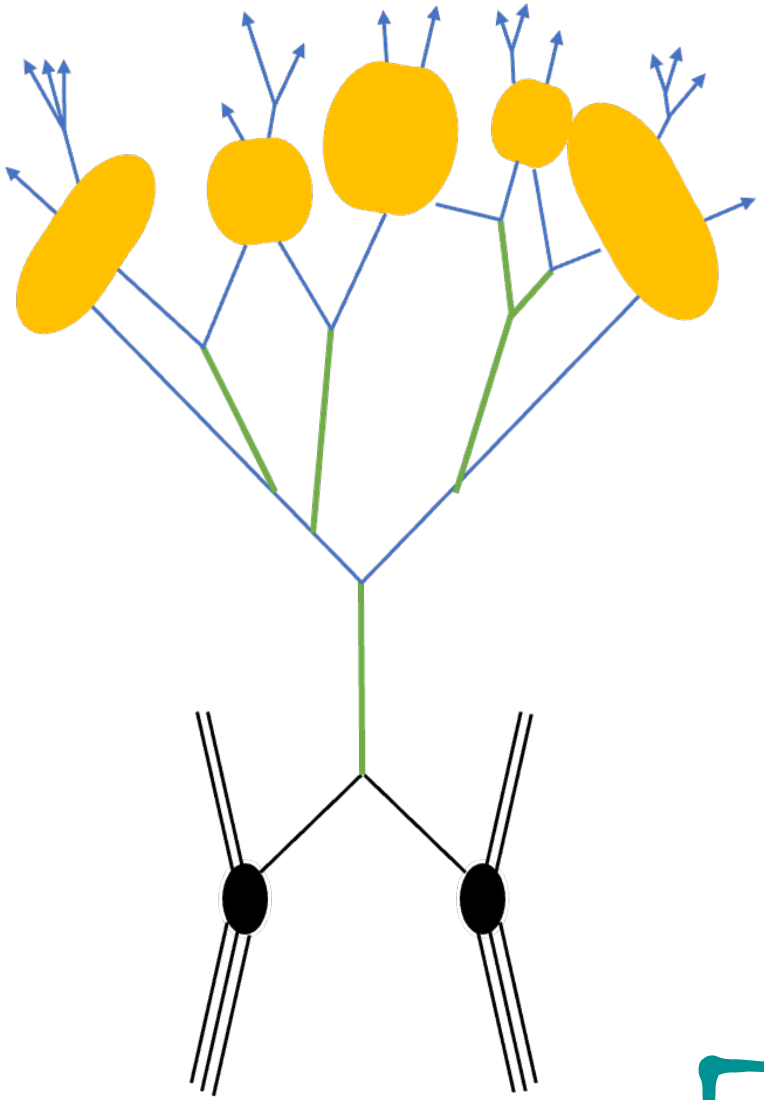
$$n_{\text{qubits}} \sim (n_{\text{gluon}} + n_{\text{quark}}) \times n^d$$

$$\sim f \times n^d$$

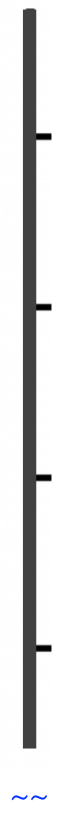
Discretization



infinities in QFT



Energy Scales



$$n \sim 10^3$$

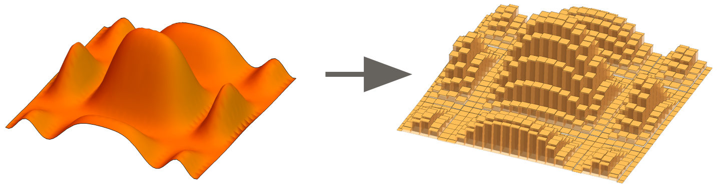
$$n_{\text{qubits}} \sim f \times n^d$$

non-perturbative regime

$$100\text{MeV} \lesssim E \lesssim \text{GeV}$$

$$n_q \sim f \times 1000$$

Discretization

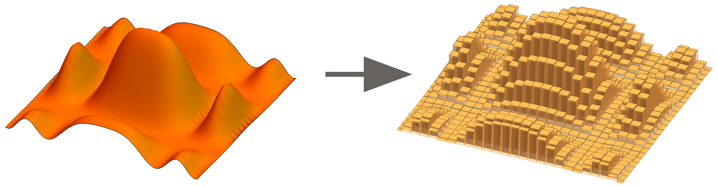


infinities in QFT

$$\begin{tikzcd}
 & H_2 \arrow[ddr, "a \to 0"] & \cdots & H_n \arrow[ddl, "a \to 0"] \\
H_1 \arrow[drr, "a \to 0"] & & & \\
H_{KS} \arrow[rr, "a \to 0"] & & H = \int dx(E^2 + B^2) & \\
H_I \arrow[urr, "a \to 0"] & & &
\end{tikzcd}$$

Hamiltonian in continuous space

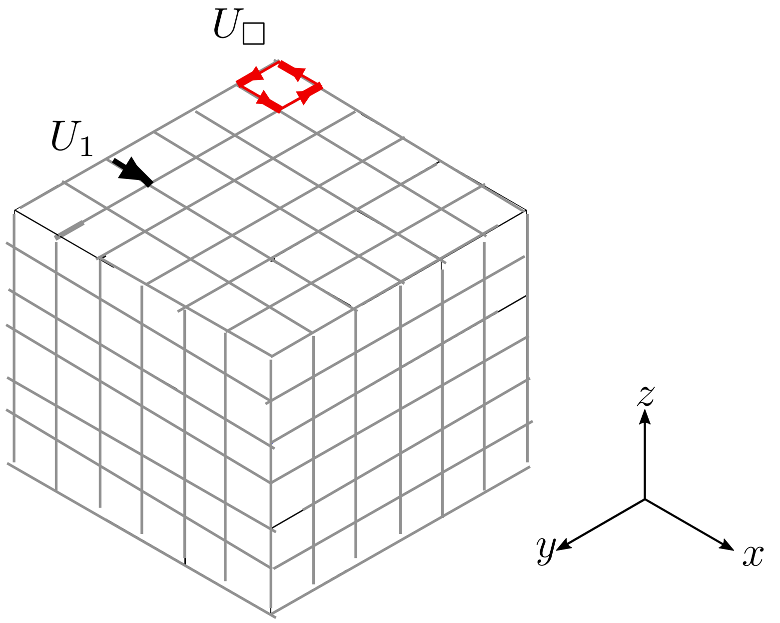
Discretization

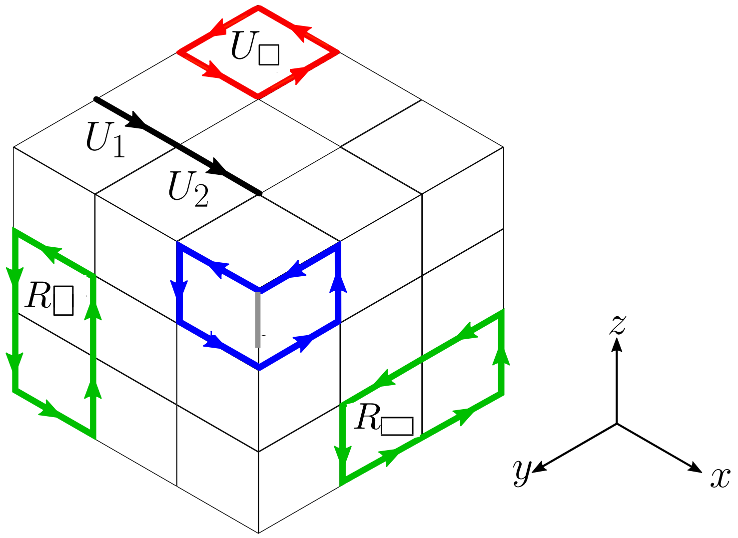


infinities in QFT

[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. 129, 051601]

$$|\langle H_{KS}(a)-H\rangle| \sim |\langle H_I(2a)-H\rangle|?$$





improved Hamiltonian

non-perturbative regime

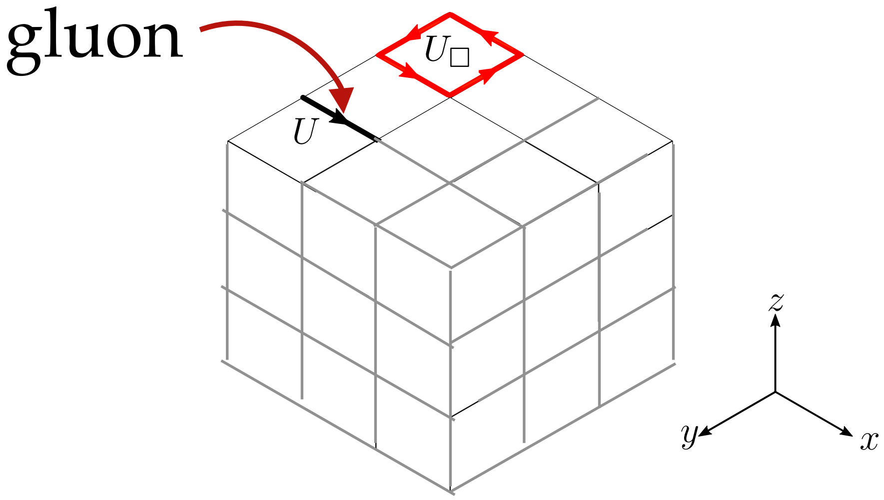
$100\text{MeV} \lesssim E \lesssim \text{GeV}$

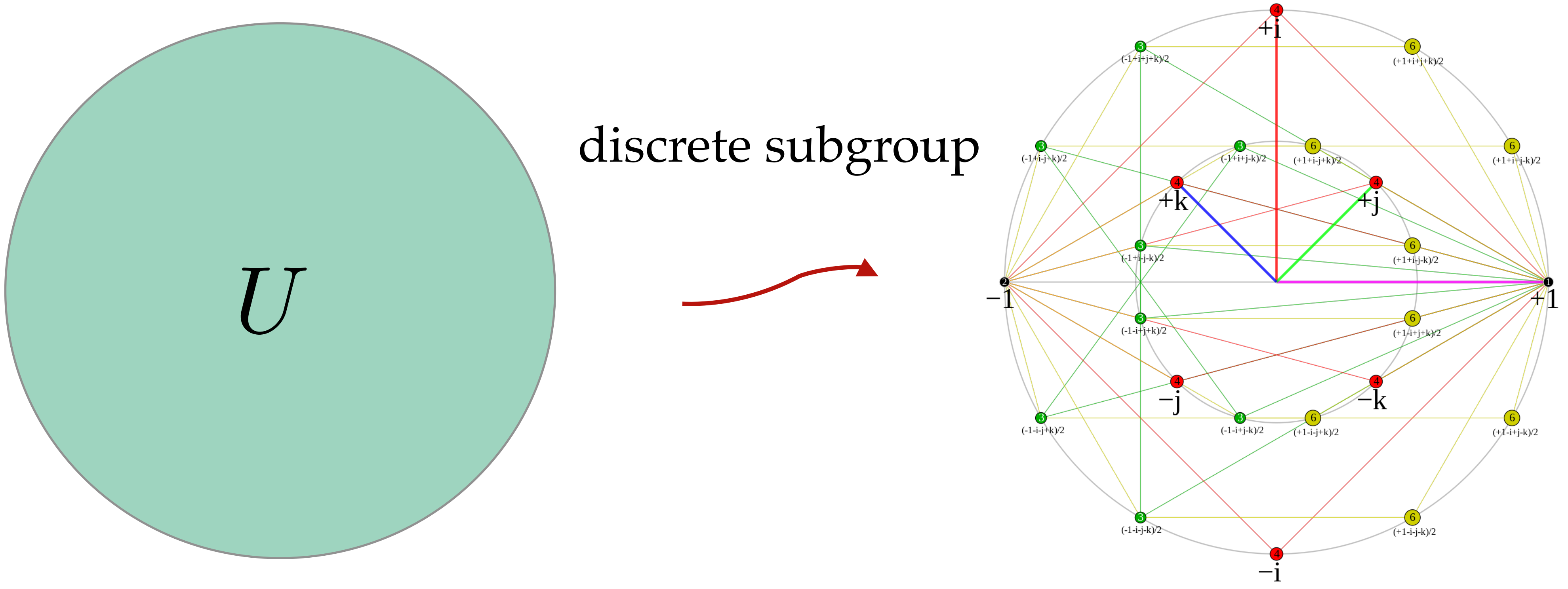
$n_q \sim f \times 125$

Digitization

infinities in field variables

Digitization $|q\rangle^N \to |G\rangle$



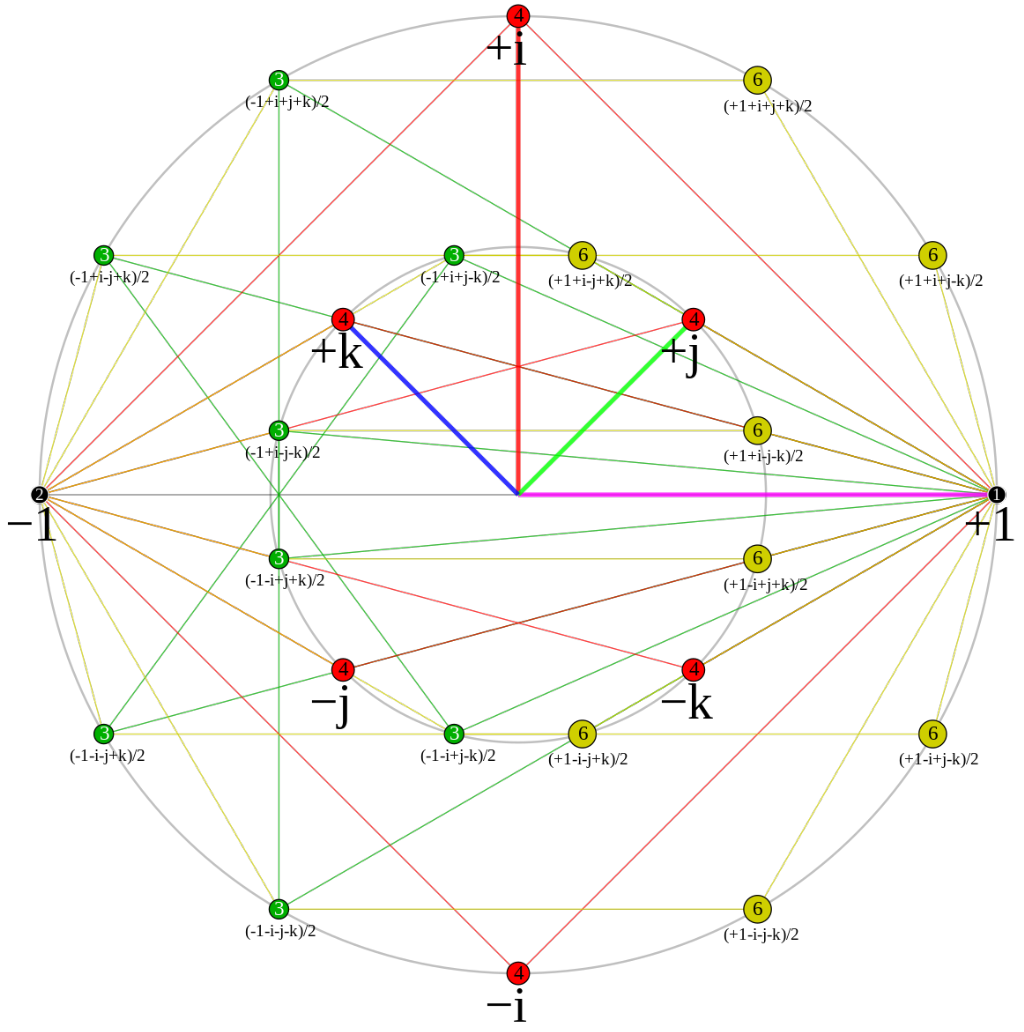


discrete subgroup

continuous field variables

$G-$register : $|U\rangle$

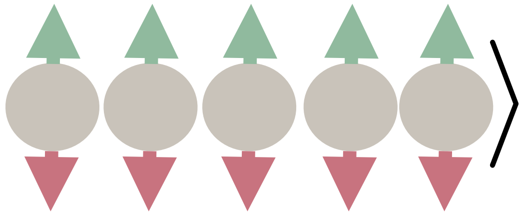
Digitization $|q\rangle^N \rightarrow |G\rangle$



$$U = (-1)^m \mathbf{i}^n \mathbf{j}^o \mathbf{l}^{p+2q}$$

binary variables : m, n, o, p, q

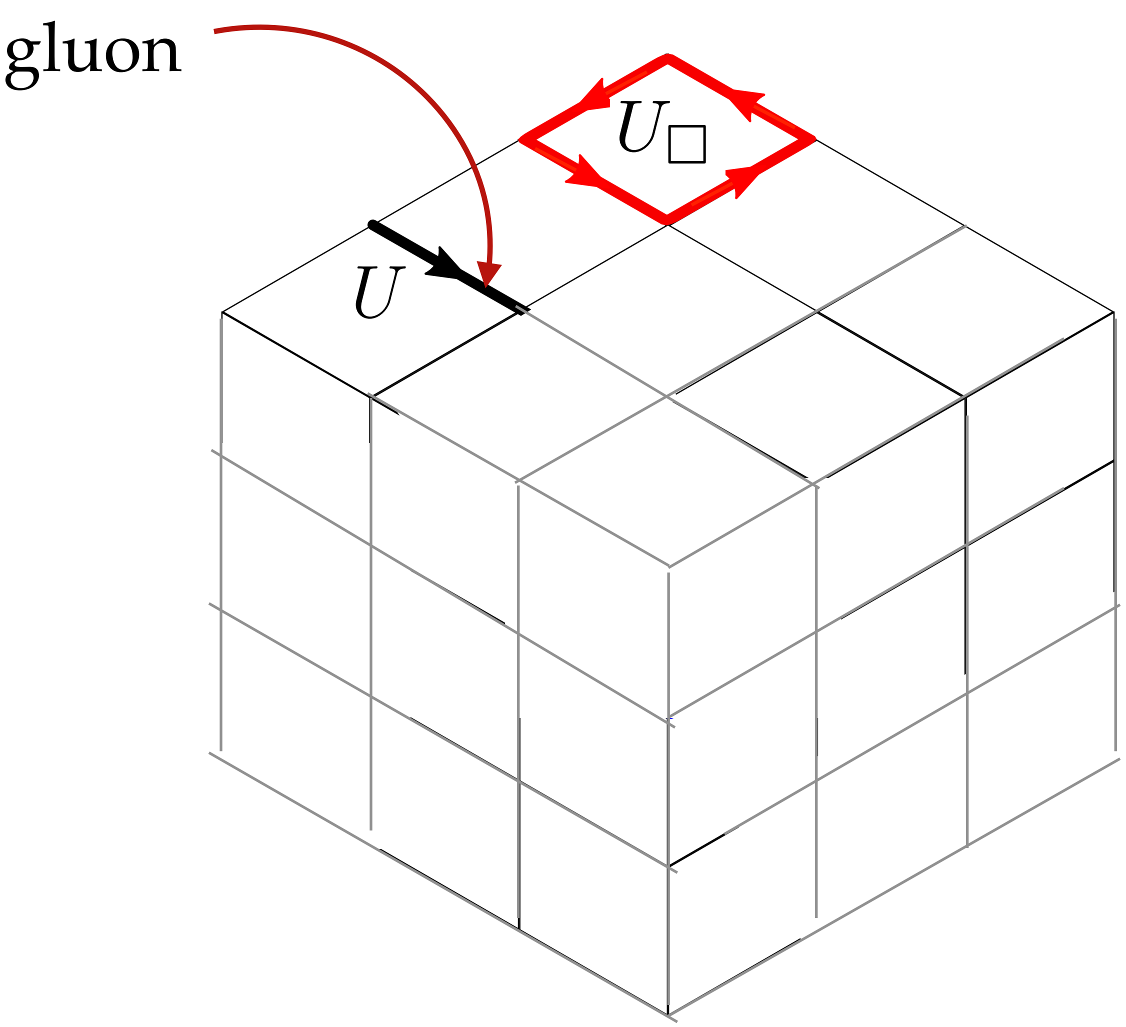
$|U\rangle = |$

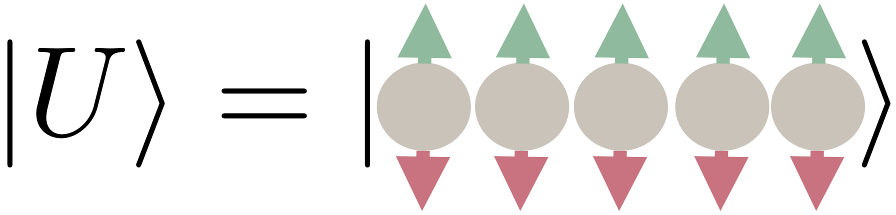


$\rangle$

Digitization $|q\rangle^N \to |G\rangle$

[M. Carena, H. Lamm, **YYL**, W. Liu, PRL. 129, 051601]





$$n_q \sim 5 \times n^d$$

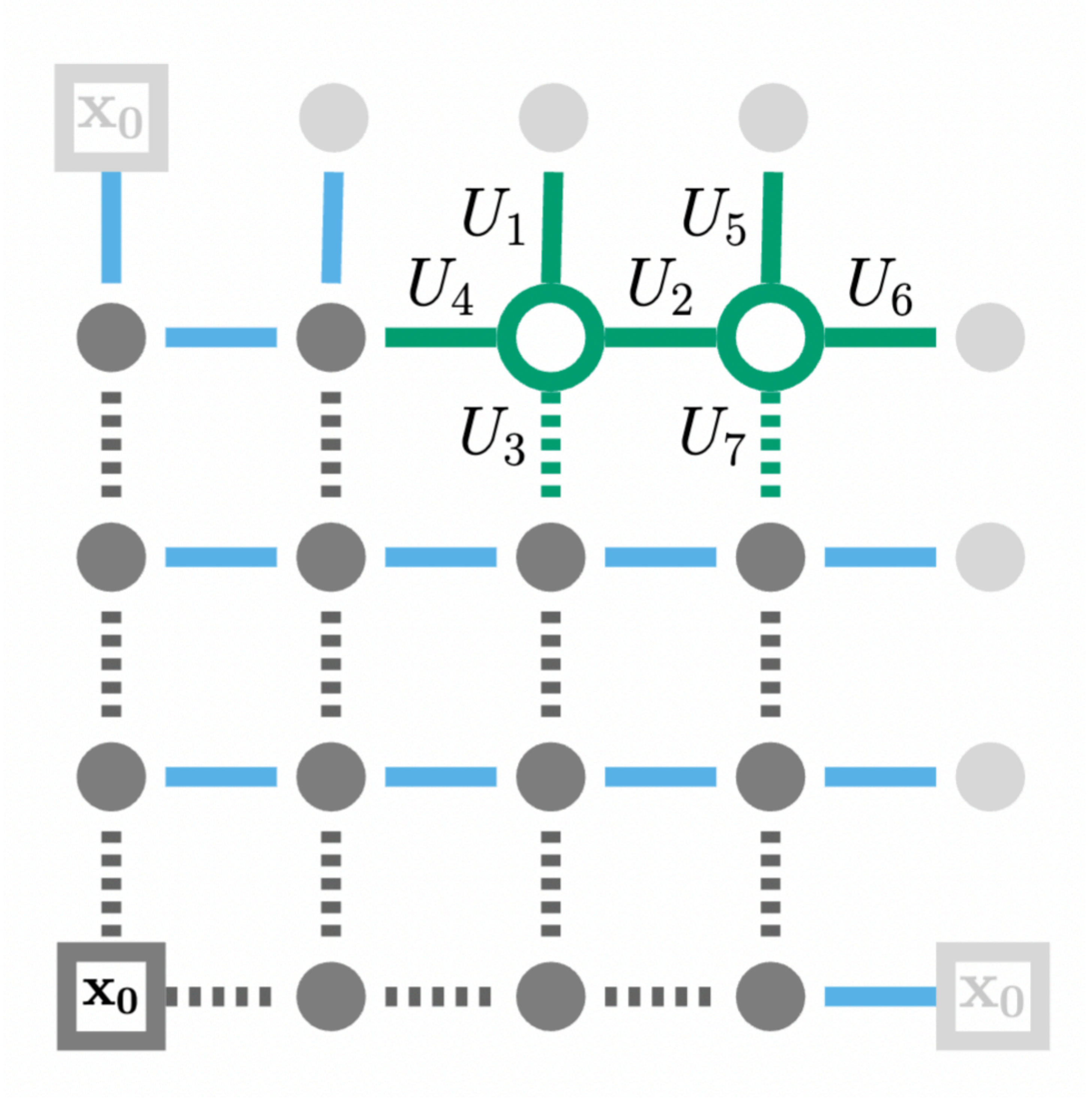
non-perturbative regime

$$100\text{MeV} \lesssim E \lesssim \text{GeV}$$

$$H_I \qquad n_q \sim 625$$

To correct quantum errors - keep gauge redundancies

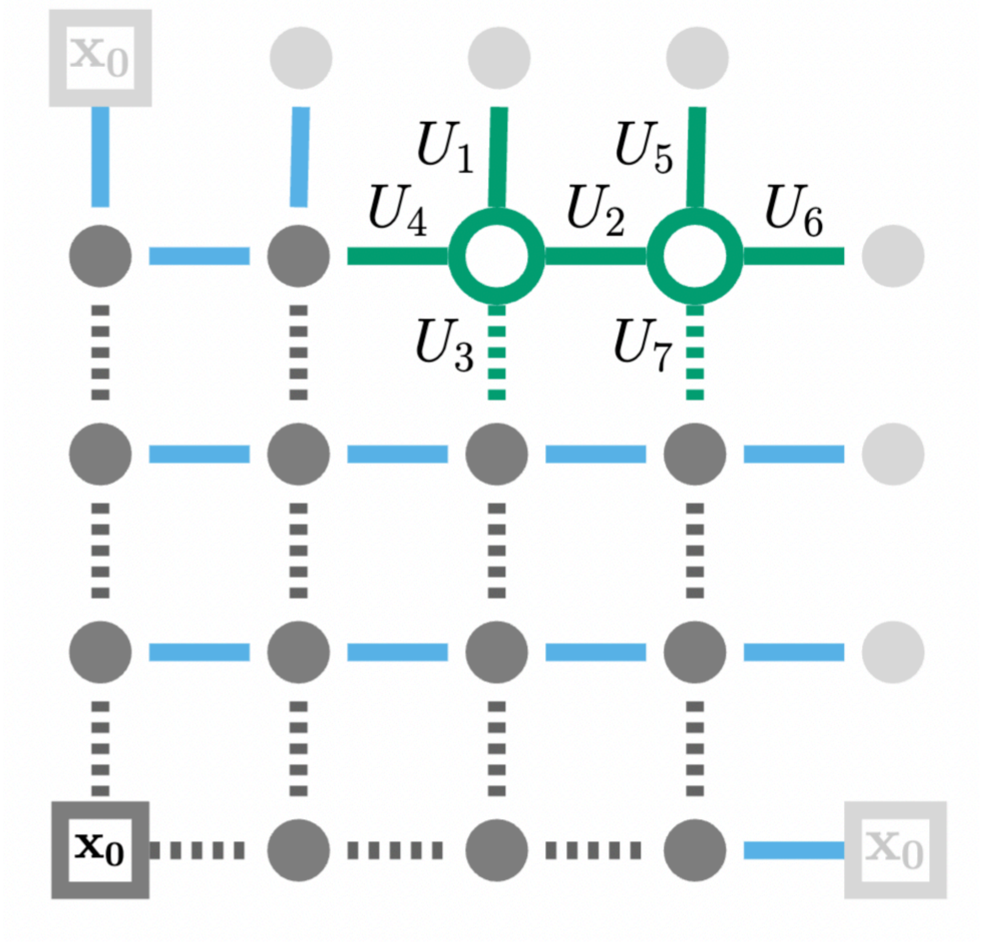
[M. Carena, H. Lamm, **YYL**, W. Liu, arXiv:2402.16780]



Gauss's Law

To correct quantum errors - keep gauge redundancies

[M. Carena, H. Lamm, **YYL**, W. Liu, arXiv:2402.16780]



$$\mathcal{H}_{\rm full} = {\rm span}(\{|U\rangle\,, U \in G\})^{\otimes N_L}$$

$$\hat{\Theta}_\Omega(x)\,|\psi\rangle = |\psi'\rangle$$

gauge redundant

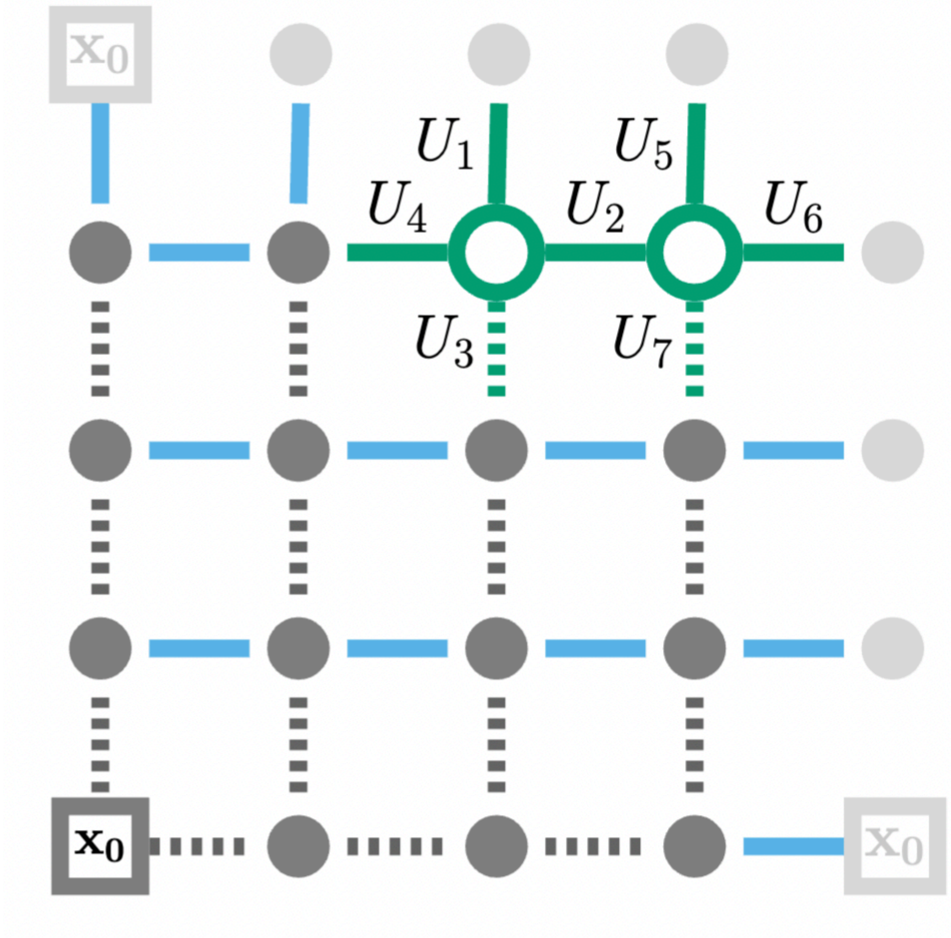
$$\hat{G}^a(x)\,|\psi\rangle = 0$$

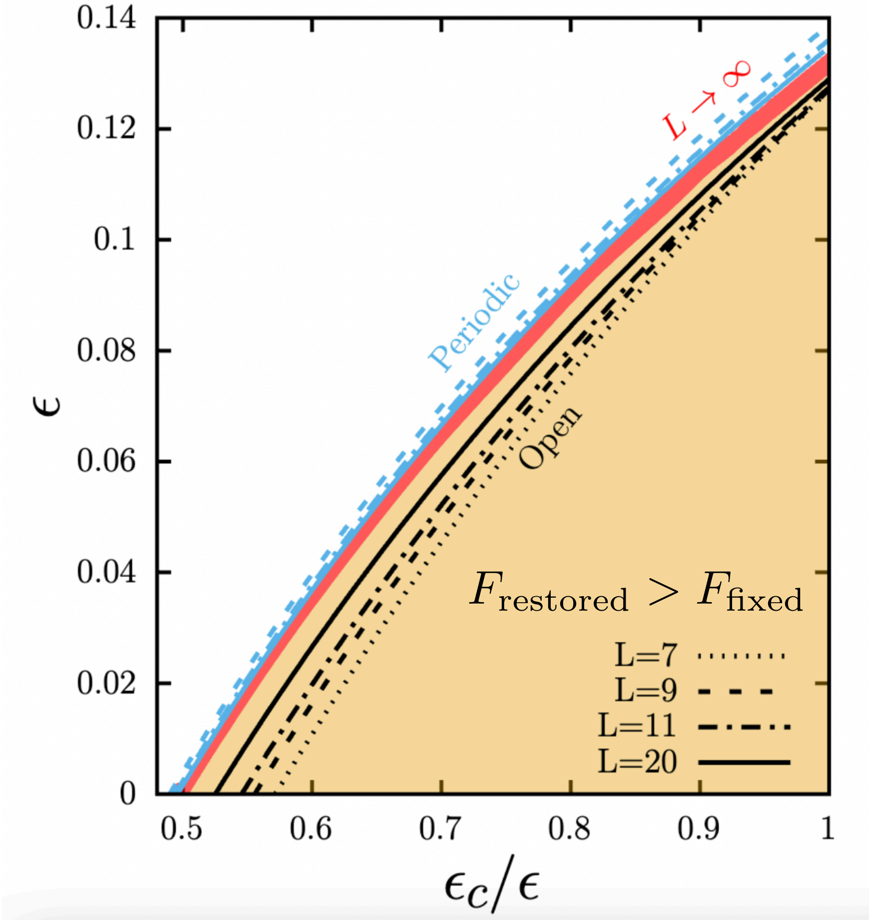
$$\mathcal{H}_{\rm inv} = {\rm span}(\{|U\rangle\,, U \in G\})^{\otimes N_L - N_V + 1}$$

$$\hat{\Theta}_\Omega(x)\,|\psi_{\rm phys}\rangle = |\psi_{\rm phys}\rangle$$

To correct quantum errors - keep gauge redundancies

[M. Carena, H. Lamm, **YYL**, W. Liu, arXiv:2402.16780]





$$\mathcal{H}_{\text{full}} = \text{span}(\{|U\rangle\,, U \in G\})^{\otimes N_L}$$

$$\hat{\Theta}_\Omega(x)\,|\psi\rangle = |\psi'\rangle$$

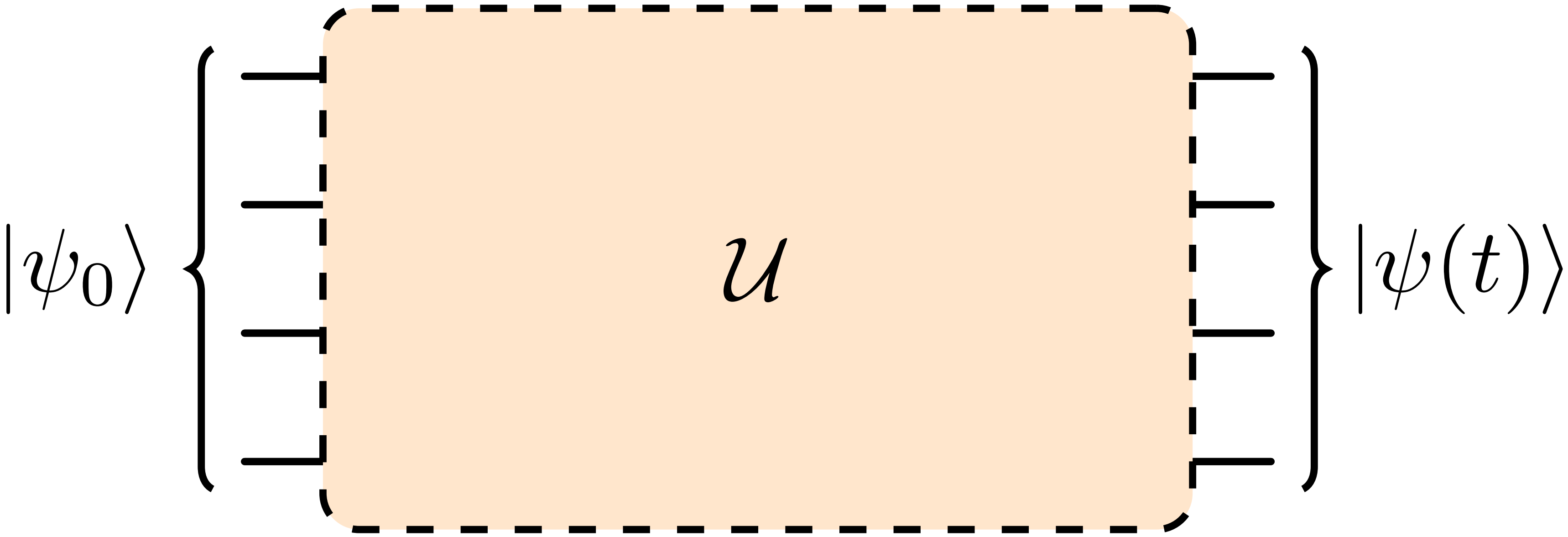
gauge redundant

$$\hat{G}^a(x)\,|\psi\rangle = 0$$

$$\mathcal{H}_{\text{inv}} = \text{span}(\{|U\rangle\,, U \in G\})^{\otimes N_L - N_V + 1}$$

$$\hat{\Theta}_\Omega(x)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle$$

Propagation



$|\psi_0\rangle$

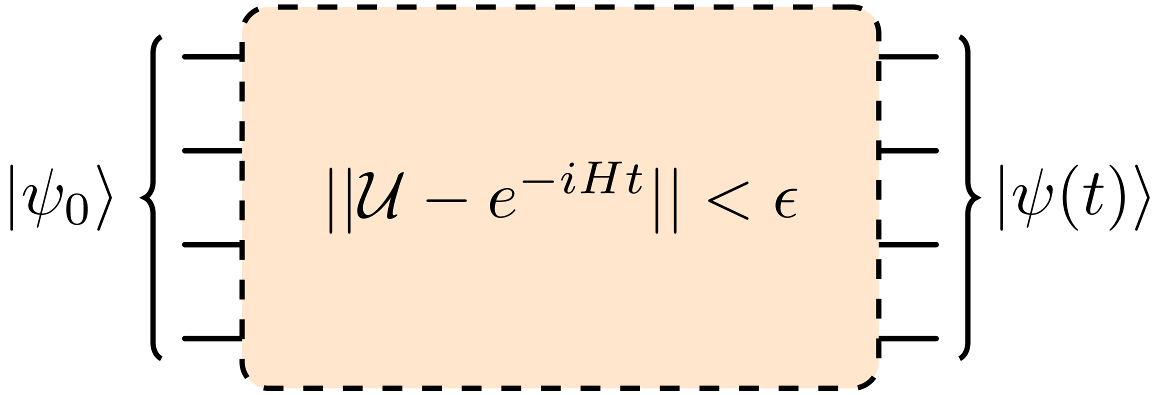
$\mathcal{U}$

$|\psi(t)\rangle$

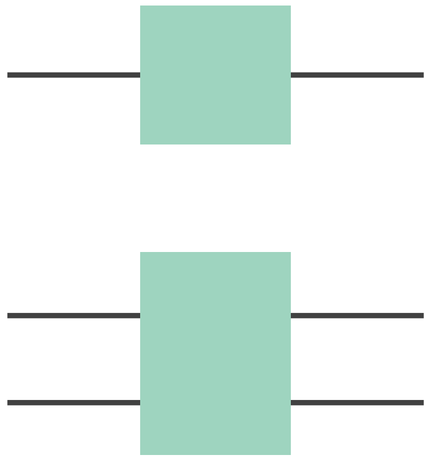
DIGITAL

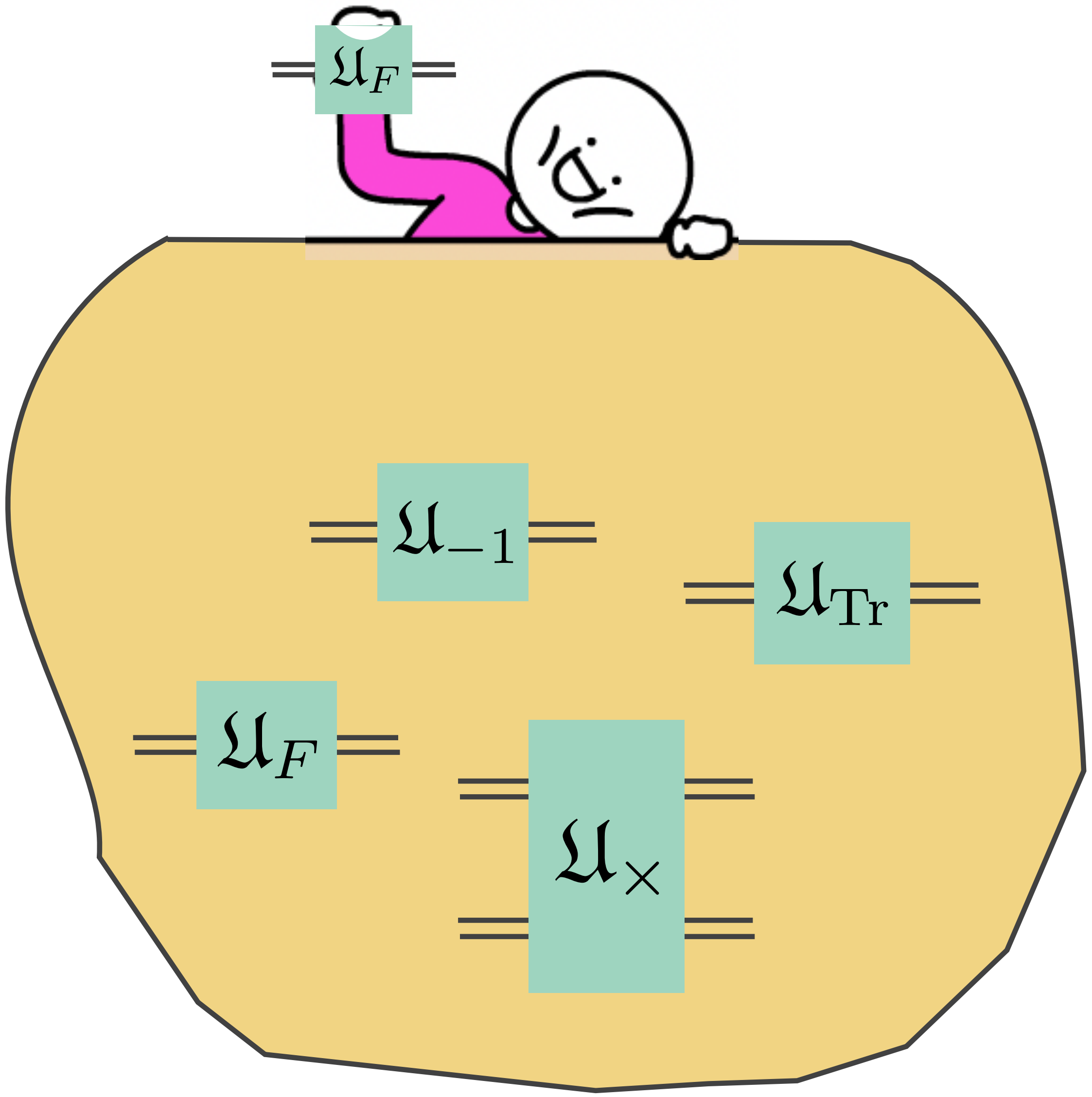
Propagation $\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$

DIGITAL



building blocks

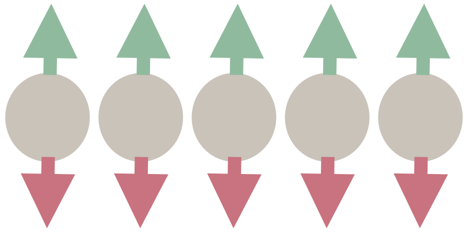




Propagation $\mathcal{U}\,|\psi_0\rangle \rightarrow |\psi(t)\rangle$

discrete subgroup

$$U = (-1)^m \mathbf{i}^n \mathbf{j}^o \mathbf{l}^{p+2q}$$

$|U\rangle = |$  $\rangle$

Qiskit v0.37.2

:~1000 CNOT gates

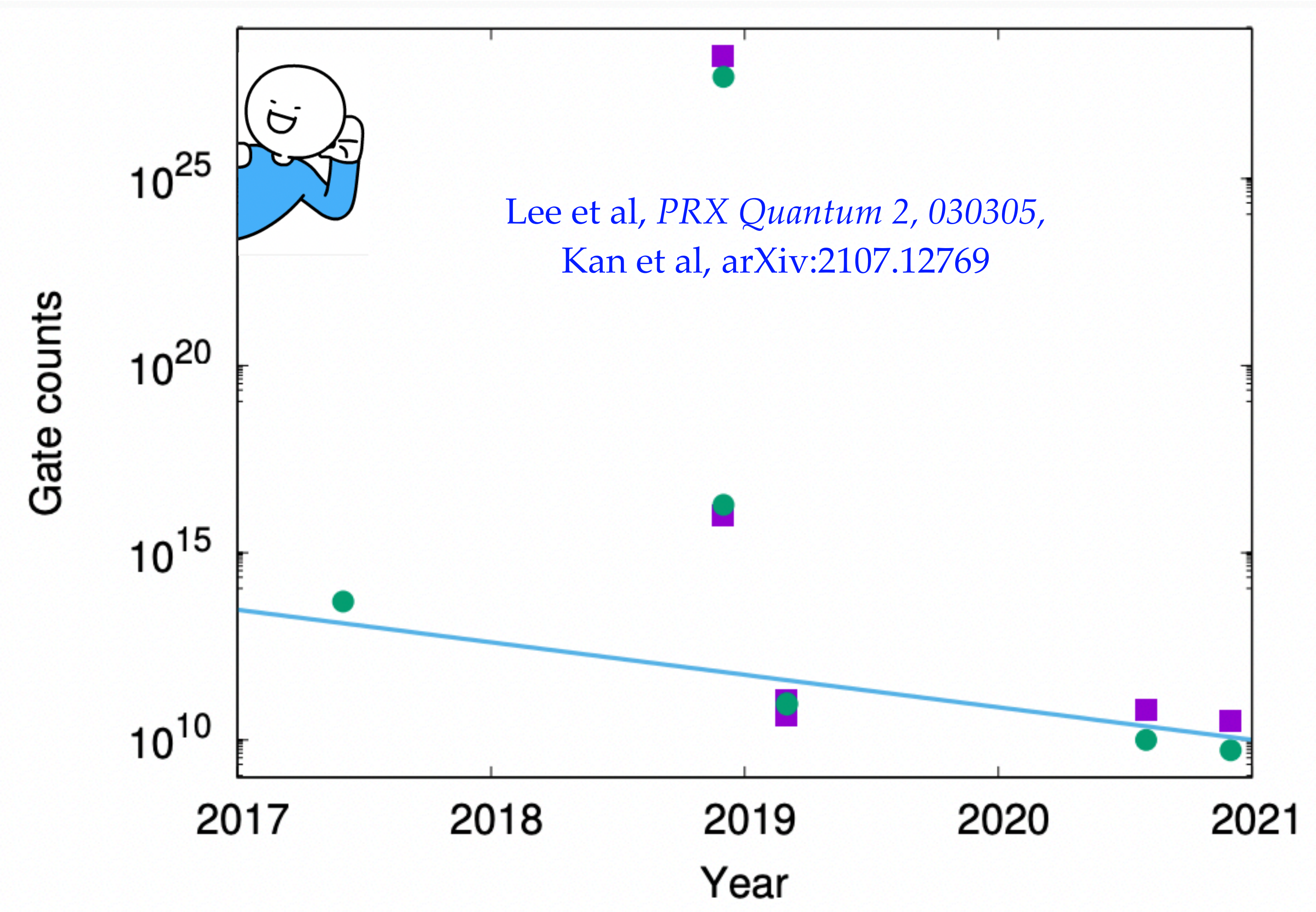
Qiskit v0.43.1

:~400 CNOT gates

Propagation $\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$

$$\mathcal{U} = \left[\prod_{l=1}^{\Gamma} e^{-itH^{(l)}/r}\right]^r$$

RESOURCE ESTIMATION AND CIRCUITS CONSTRUCTION IMPROVEMENT

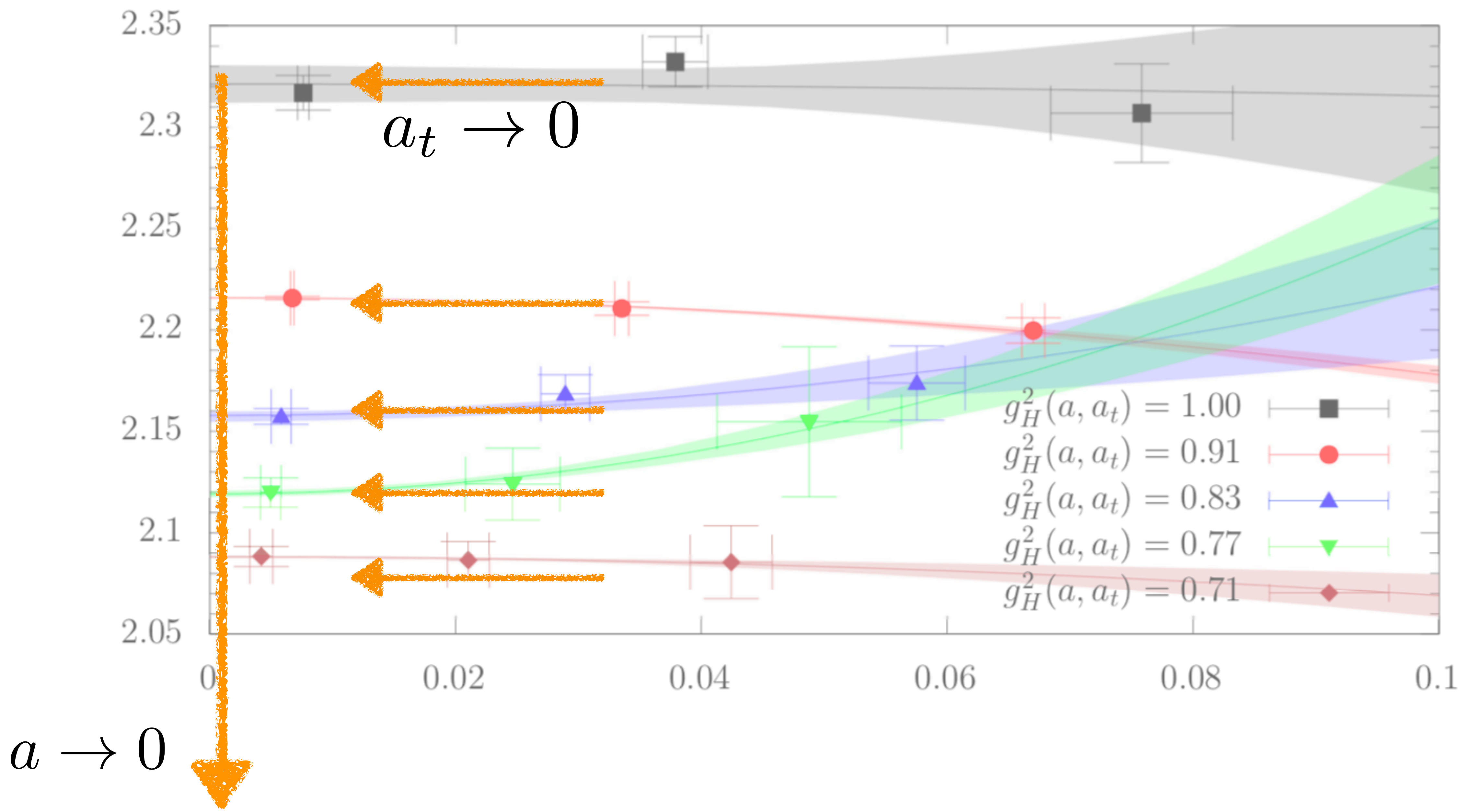


Lee et al, *PRX Quantum 2, 030305,*
Kan et al, arXiv:2107.12769

To reach observables in the continuum limit

[M. Carena, H. Lamm,YYL, W. Liu, PRD. 104, 094519]

TRAJECTORY TO THE CONTINUUM LIMIT

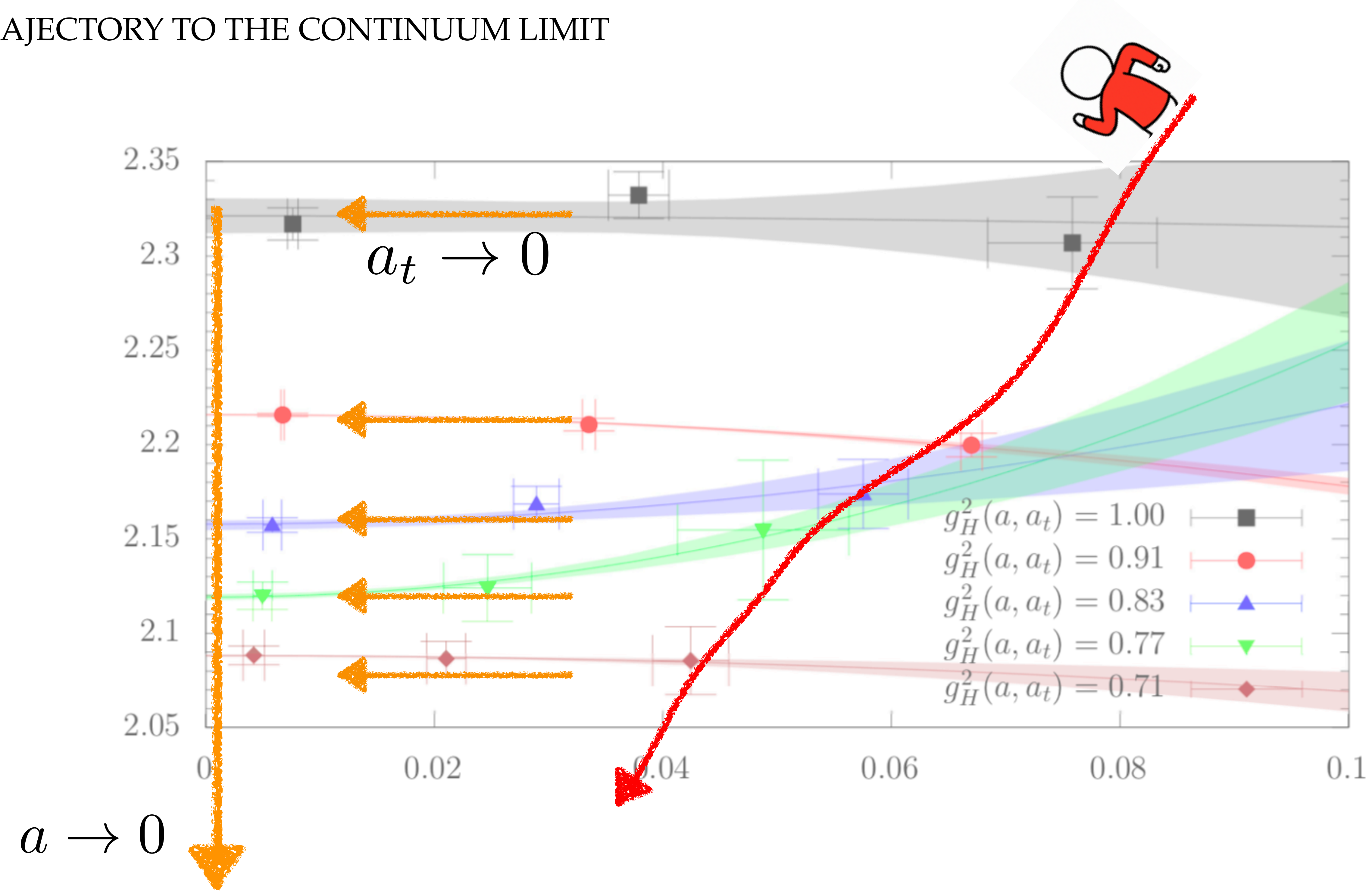


continuous
space-time limit

To reach observables in the continuum limit

[M. Carena, H. Lamm,YYL, W. Liu, PRD. 104, 094519]

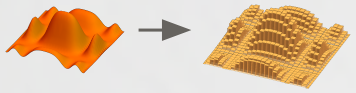
TRAJECTORY TO THE CONTINUUM LIMIT



continuous
space-time limit

“It’s time to unleash the quantum potential!”

(2010s) galactic algorithms



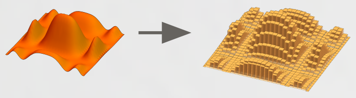
$$\begin{tikzcd}
{|q\rangle^N \to |G\rangle} \arrow[d] \\
{\mathcal{U}\,|G\rangle^L \to |\psi_0\rangle} \arrow[d] \\
{\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle} \arrow[d] \\
{\langle\mathcal{O}\rangle}
\end{tikzcd}$$

S. P. Jordan,
K. S. M. Lee,
J. Preskill

2011-

“It’s time to unleash the quantum potential!”

(2020s) pocket of methods for every steps,
theoretical developments - improved H, continuous limit
benchmarks,
error corrections



$$|q\rangle^N \to |G\rangle$$

$$\downarrow$$

$$\mathcal{U}\,|G\rangle^L \to |\psi_0\rangle$$

$$\downarrow$$

$$\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$$

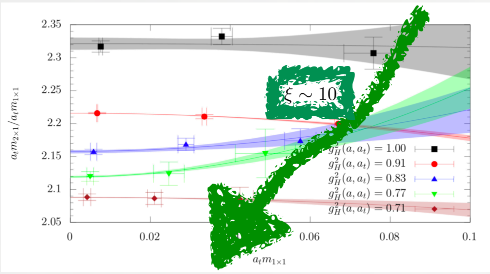
$$\downarrow$$

$$\langle\mathcal{O}\rangle$$

S. P. Jordan,
K. S. M. Lee,
J. Preskill

2011-

various methods

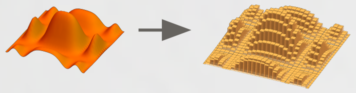


2020 -

“It’s time to unleash the quantum potential!”

(2030s) narrow down the framework with

improving algorithms,
theoretical studies of uncertainties,
hardware co-design
benchmark studies
…
HEP case calculations



$|q\rangle^N \to |G\rangle$

$\downarrow$

$\mathcal{U}\,|G\rangle^L \to |\psi_0\rangle$

$\downarrow$

$\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$

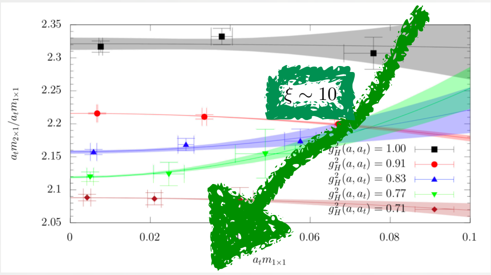
$\downarrow$

$\langle\mathcal{O}\rangle$

S. P. Jordan,
K. S. M. Lee,
J. Preskill

2011-

various
methods



2020 -

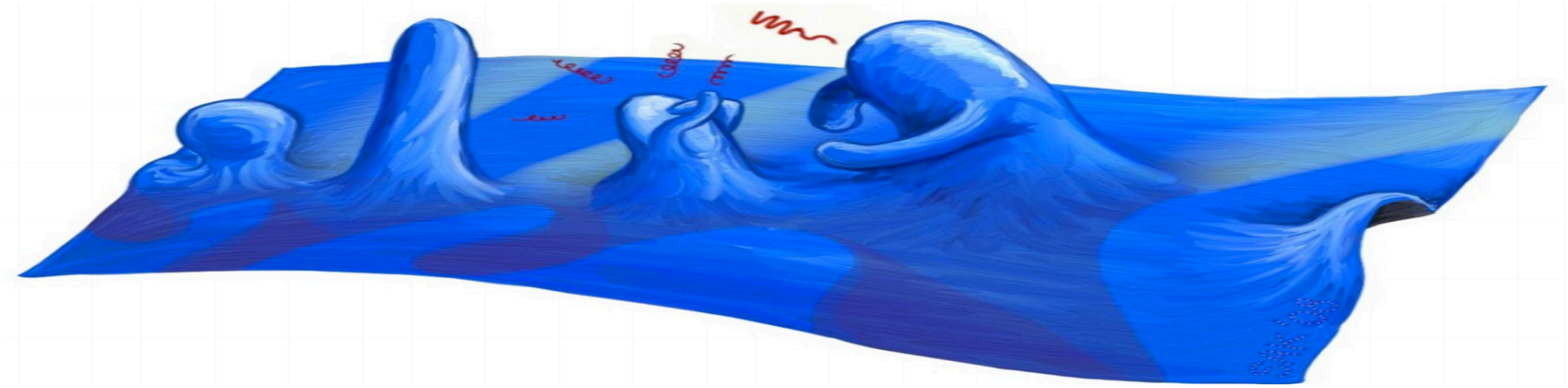
2030s -

Thank you

BACK UP

Lattice QCD - Real Time

$$\int \mathcal{D}\phi e^{iS} = \langle x| e^{-iHt} |y\rangle$$



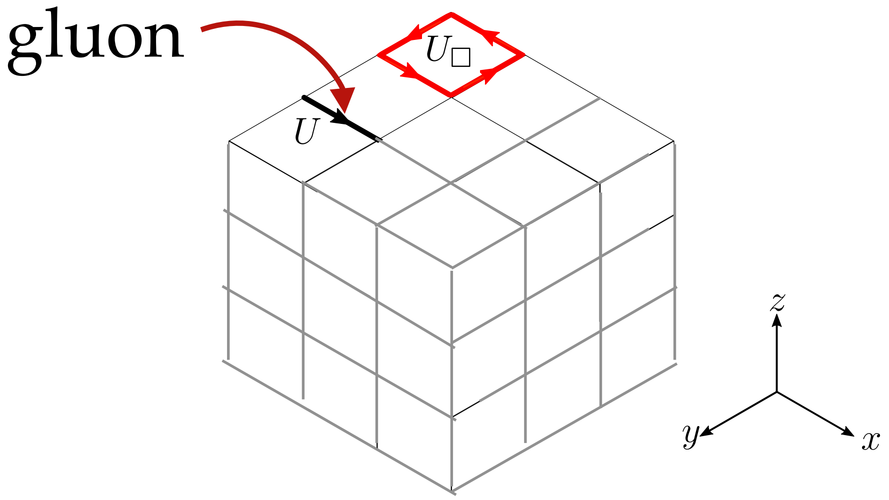
$$\mathrm{dim}H \propto |G|^{N_V}$$

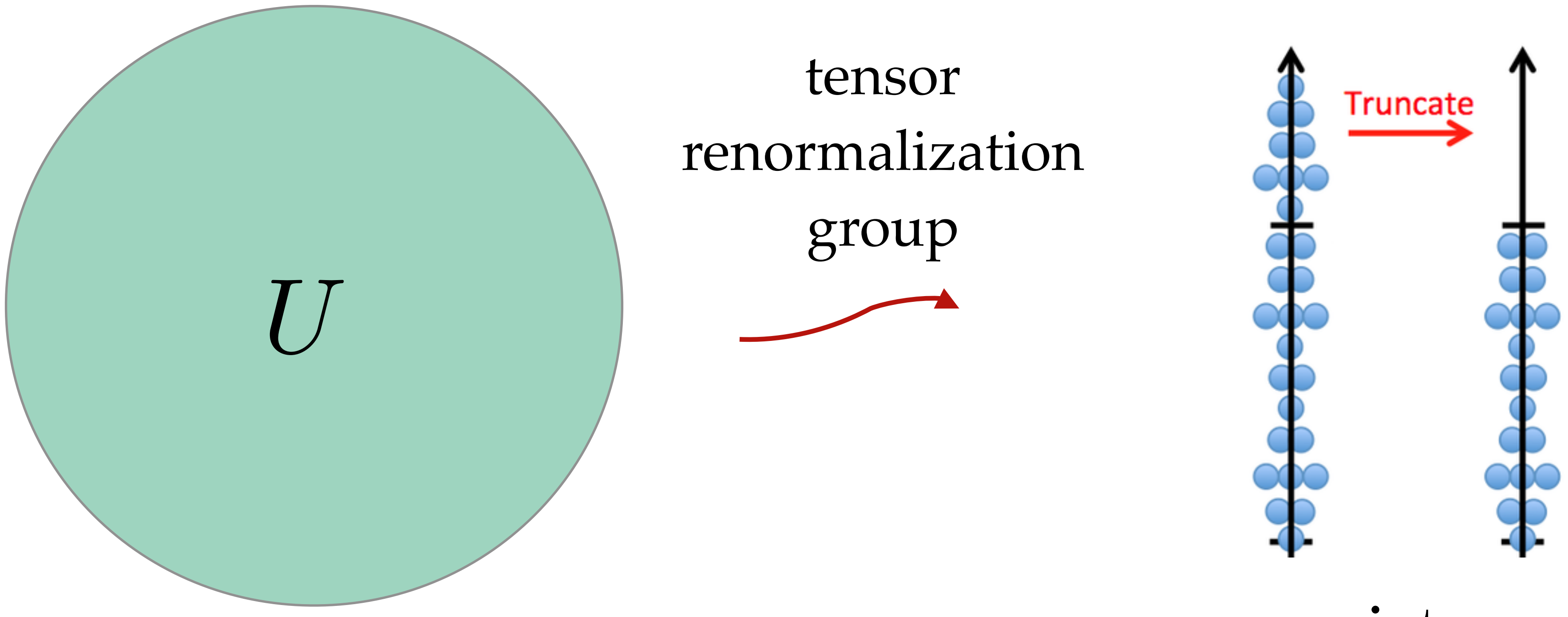
system size N_V : number of lattice sites

exponentially large number of classical bits in system size

Digitization $|q\rangle^N \to |G\rangle$

infinities in QFT





tensor renormalization group

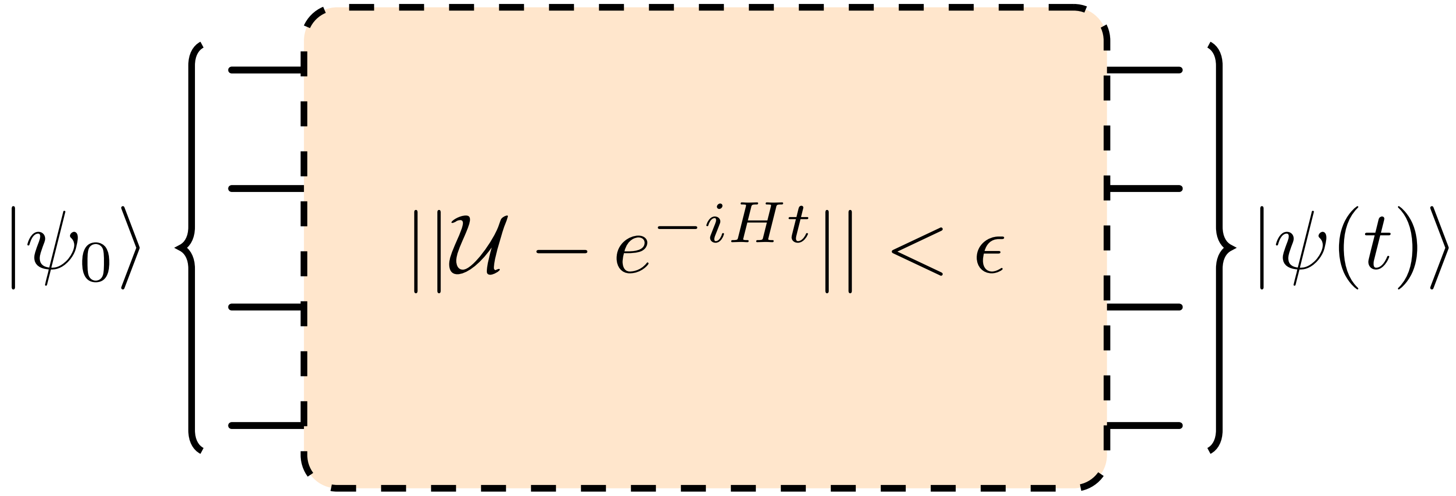
$\sigma-$register : $|\sigma\rangle$

continuous field variables

D. B. Kaplan, J. F. Haase, Y. Meurice, et al.

Propagation $\mathcal{U}\,|\psi_0\rangle \to |\psi(t)\rangle$

DIGITAL



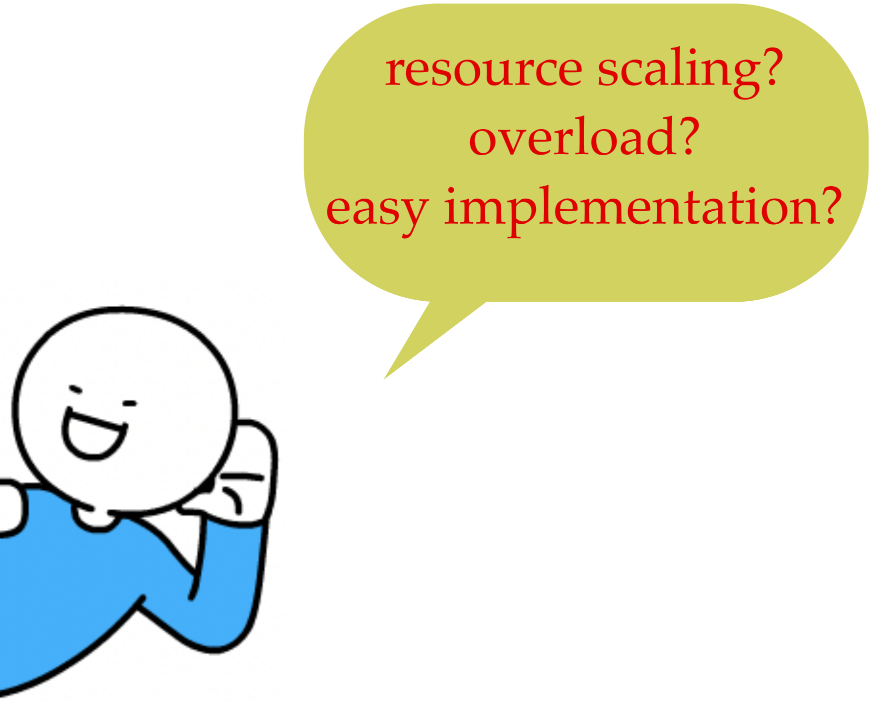
Trotter-Suzuki decomposition

logarithmic
no
usually yes

Taylor series expansion (LCU)

specify your problem..

yeah, not much
maybe not



Casanova et al (2011), Davoudi et al (2021)
trapped ion

Harmalkar et al (2022)
classical preprocessing
Zohar et al (2017), Bender et al (2018)
effective interactions
Klco et al (2018), Kokail et al (2019), Atas et al (2021) state preparations
Peruzzo et al (2014), Farhi et al (2014)
optimization methods