SOUTH CHINA NORMAL UNIVERSITY

Lattice Parton Collaboration

Light hadron distribution with LQCD

Jun Hua
South China Normal University (SCNU)
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第六届重味物理与量子色动力学研讨会

Outline

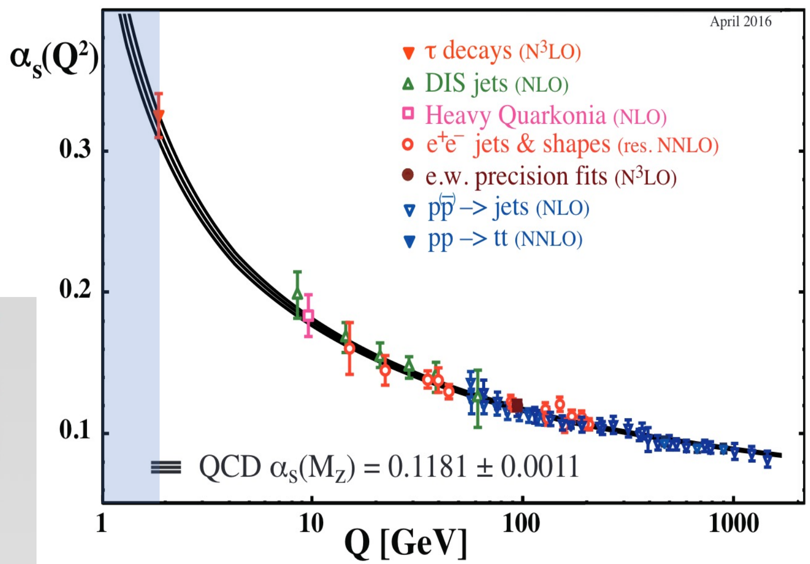
- Motivation
  - Light hadron PDF and LCDA
- Light meson LCDAs
  - LCDA research & in lattice QCD
  - Quasi distribution method by LaMET
- Light baryon (Lambda) LCDAs
- Summary

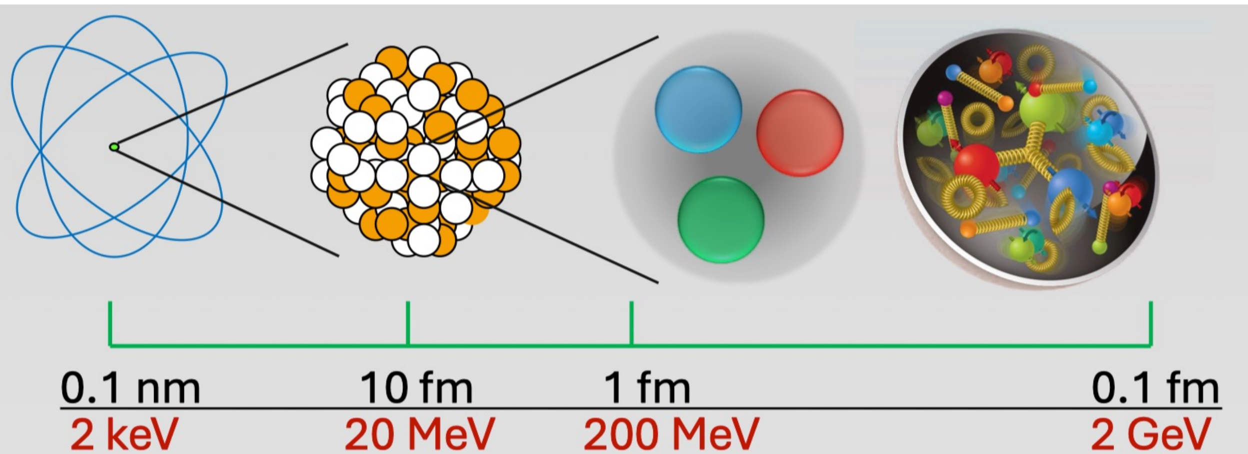
Motivation

➢ The basics of Quantum Chromodynamics:

- Yang-Mills theory, SU(3) gauge group
- Gluons themselves carry color charges and interact with each other
- Conservation of color charges and color neutrality
- Quark confinement and asymptotic freedom

α_s running coupling





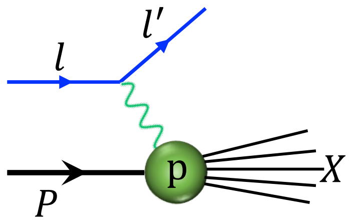
Quark confinement Asymptotic freedom

Motivation

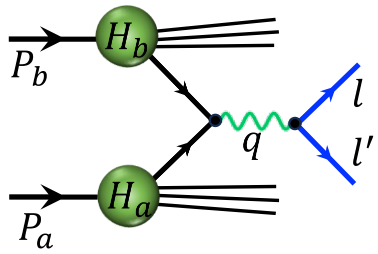
- **PDFs:** the probability distribution of partons (quarks and gluons) within a hadron

—— Inclusive process

DIS



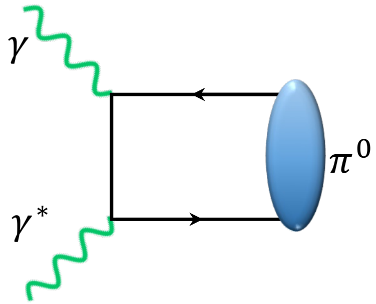
Drell-Yan



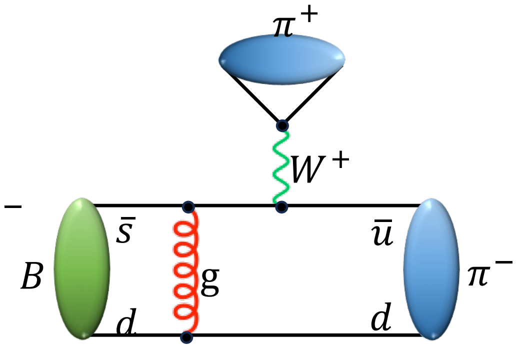
- **LCDAs:** the probability amplitude for partons within a hadron

——Exclusive process

$\gamma\gamma^* \to \pi$



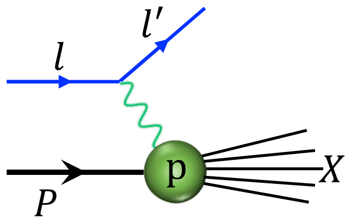
$B \to \pi^+\pi^-$



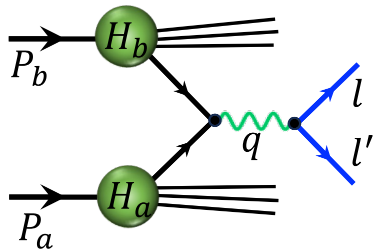
Motivation

- PDF

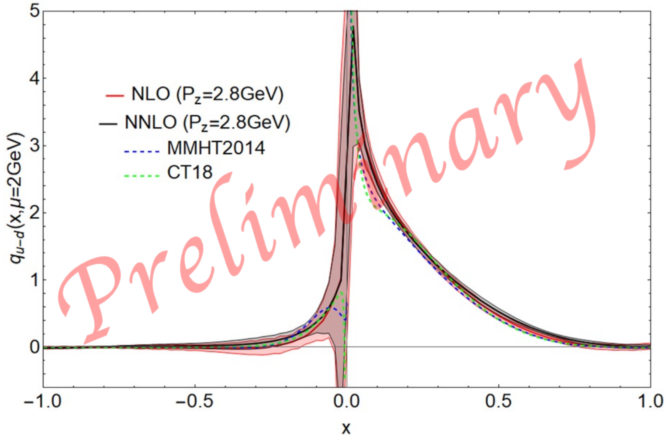
DIS



Drell-Yan

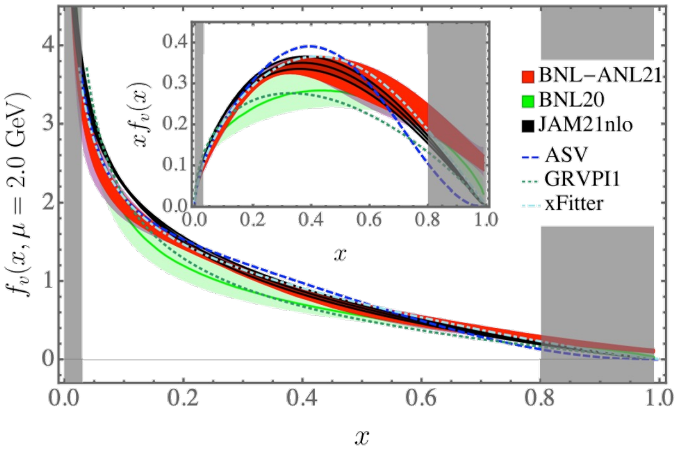


Proton Unpolarized PDF



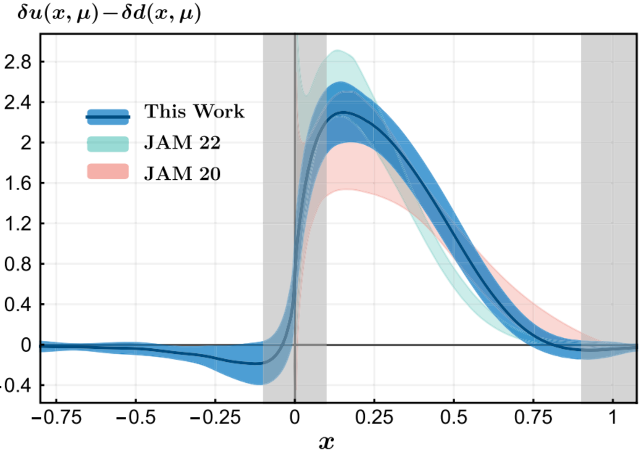
Preliminary from LPC

Pion valancePDF



X.Gao, et.al. PRL.128.142003 (2022)

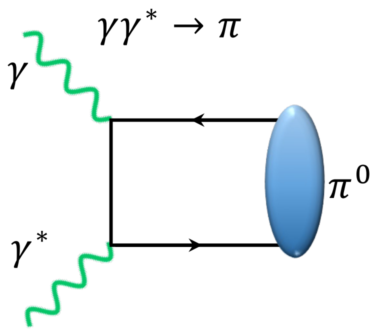
Pion valancePDF

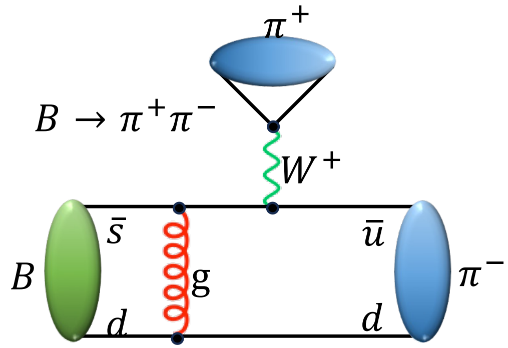


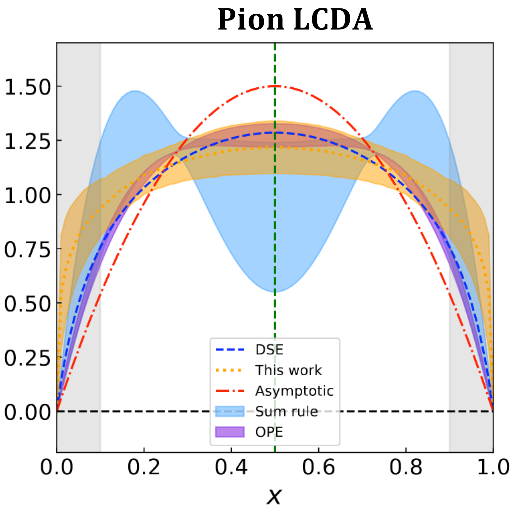
(LPC) PRL.131,261901 (2023)

Motivation

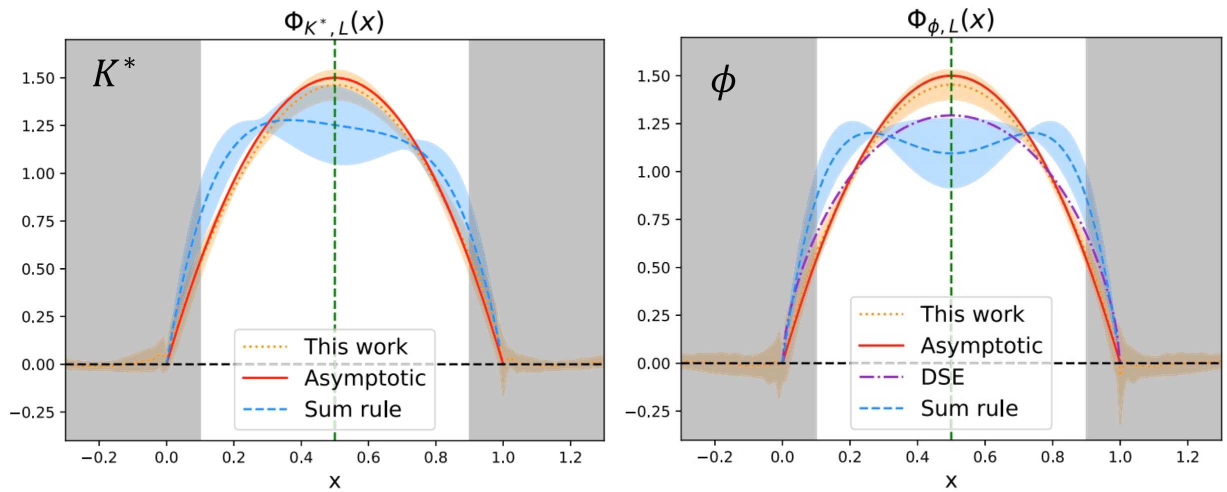
➢ LCDA







(LPC) J.Hua et.al. PRL.129,132001 (2022)

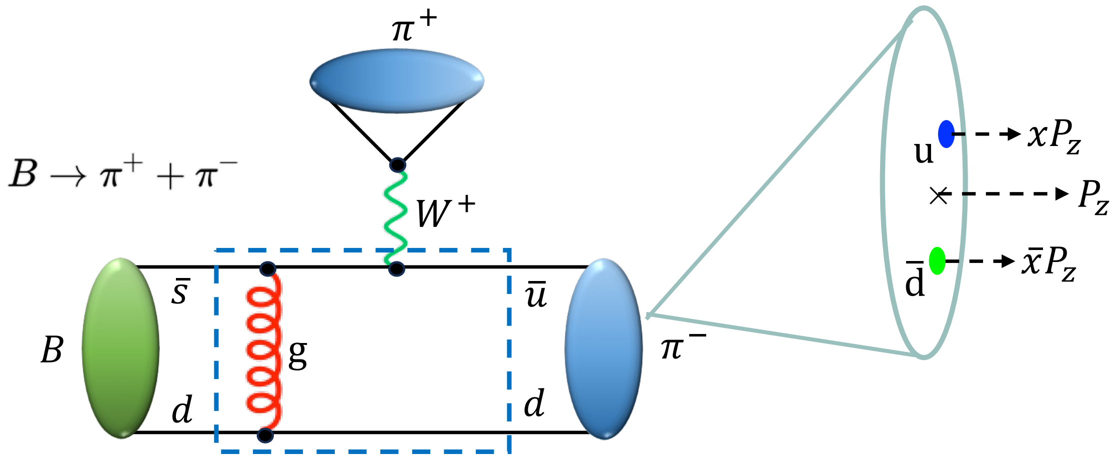


(LPC) J.Hua et.al. PRL.129,132001 (2022)

Motivation

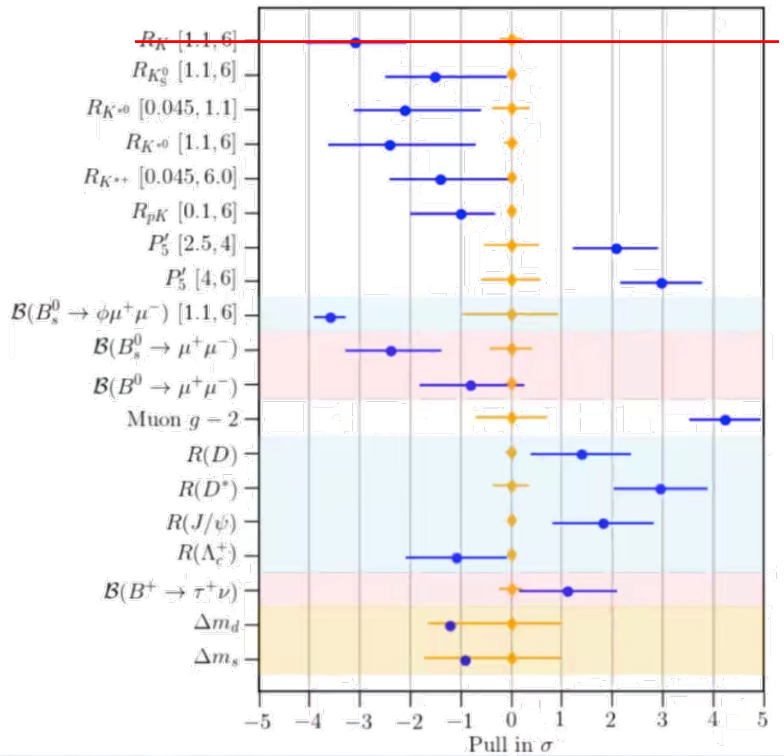
- LCDA as most important input in flavor physics：
  - $B \to \pi l \nu_l, B \to \pi\pi, \ldots$
  - $\gamma^* \to \gamma\pi, \gamma\gamma \to \pi\pi$
  - $eN \to eN\pi$
  - $B \to K^* l^+ l^-$
  - $B \to \phi l^+ l^-$
  - $\ldots$

- **CKM matrix**
- **CP violation**
- New physics ⋯



$$\langle \pi(p')\,\pi(q)\,|Q_i|\,\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx T_i^{\mathrm{I}}(x)\phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{\mathrm{II}}(\xi,x,y)\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$

Anomalous:



LCDA research & in lattice QCD

LCDA research & in lattice QCD

➢ Light meson LCDAs have been extensively pursued: (1970s - now)

- **Asymptotic LCDAs**
  Chernyak, Zhitnitsky, 1977; Lepage, Brodsky, 1979; Efremov, Radyushkin, 1980
- **Dyson-Schwinger Equation**
  Chang, Cloet, Cobos-Martinez, Roberts, Schmidt, 2013; Gao, Chang, Liu, Roberts, Schmidt, 2014; Roberts, Richards, Chang, 2021
- **Sum rules**
  Chernyak, Zhitnitsky, 1982; Braun, Filyanov, 1989; Ball, Braun, Koike, Tanaka, 1998; Ball, Braun, 1998; Khodjamirian, Mannel, Melcher, 2004; Ball,, Lenz, 2007
- **Inverse Problem**
  Li, 2022
- **Models**
  Arriola, Broniowski, 2002, 2006; Zhong, Zhu, Fu, Wu, Huang, 2021;
- **Global Fits**
  Stefanis, 2020; Cheng, Khodjamirian, Rusov, 2020; Hua, Li, Lu, Wang, Xing, 2021
- **Lattice with current-current correlation**
  Bali, Braun, Gläßle, Göckeler, Gruber, 2017, 2018;
- **Lattice with OPE**
  Martinelli, Sachrajda, 1987; Braun, Bruns, et al., 2016; RQCD collaboration, 2019, 2020
- **Lattice with LaMET**
  Zhang, Chen, Ji, Jin, Lin, 2017; LP3 Collaboration, 2019; Zhang, Honkala, Lin, Chen, 2020; Lin, Chen, Fan, Zhang², 2021; LPC Collaboration, 2021, 2022
- **Quantum Computing**
  QuNu Collaboration, 2023, 2024

LCDA research & in lattice QCD

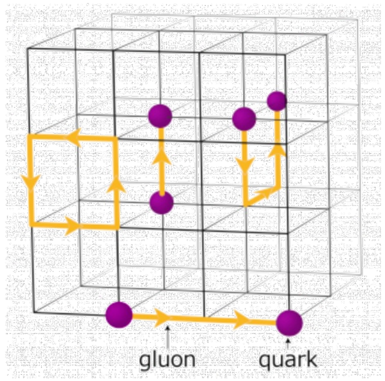
LQCD is formulated as a Feynman path integral on a discrete 4D Euclidean grid.

Numerical simulations based on a QCD Lagrangian with discrete from:

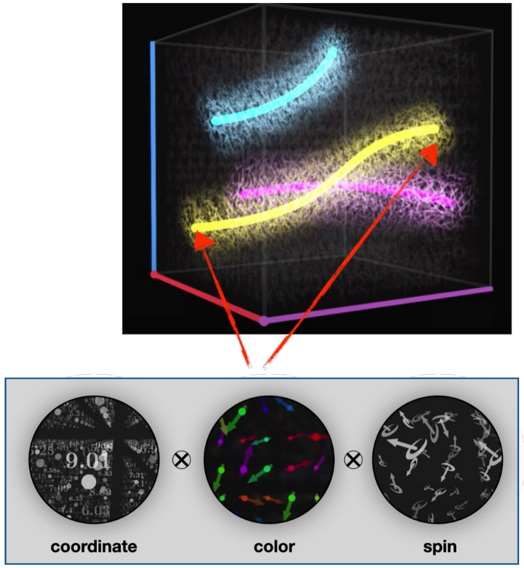
$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu} D_{\mu} - m)\psi - \frac{1}{4} G^{a}_{\mu\nu} G^{a,\mu\nu}$$

$$S_E^{\text{latt}} = -\sum_{\Box} \frac{6}{g^2} \mathrm{Re}\, \mathrm{tr}_N \left(U_{\Box,\mu\nu} \right) - \sum_q \bar{q} \left(D_{\mu}^{\text{lat}} \gamma_{\mu} + a m_q \right) q$$

- Gluon fields on links
- Quark fields on sites



- lattice spacing $a \rightarrow$ UV regulator;
- box length $L \rightarrow$ IR regulator;
- Chiral extrapolation ($M_{\pi} \rightarrow 135$MeV);
- Numerical sampling of path integral with highly dimension $n_s^3 \times n_t \times N_{\text{color}} \times N_{\text{spin}}$



LCDA research & in lattice QCD

◆ Pion LCDA:

$$\int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \langle 0 | \bar{\psi}_1(0) n \cdot \gamma\gamma_5 U(0, \xi^-) \psi_2(\xi^-) | \pi(p) \rangle = i f_\pi \Phi_\pi(x)$$

$$\phi_\pi(x) = 6x(1-x) \sum_{n=1,2,\cdots} a^\pi_{2n-2} C^{(3/2)}_{2n-2}(2x-1)$$ Gegenbauer expansion

$$\langle \xi^n \rangle = \int_0^1 dx (2x-1)^n \phi_\pi(x)$$

- **Lattice based OPE to local moments**

  V.M.Braun et.al. PRD 92.014504 (2015) , V.M.Braun et.al. JHEP 04082 (2017), (RQCD) G.S.Bali et.al. JHEP 08065 (2019)

- **Quasi-correlation (LaMET) to entire x range**

  J.H.Zhang PRD95. 094514(2017), R.Zhang H.W.Lin et.al. PRD102. 094519(2020), (LPC)J.Hua et.al. PRL127. 062002(2021), (LPC)J.Hua et.al. PRL129. 132001(2022)

Recent progresses with lattice QCD

- **Light-like correlators cannot be simulated on Euclidean lattice directly**

  ⇒ OPE to local correlators

OPE moments ⇒ Gegenbauer moments

- Lattice with OPE:

$$\langle \xi^n \rangle = \int_0^1 dx (2x-1)^n \phi_\pi(x)$$

- The nonlocal operator can be defined as a generating function for renormalized local operators:

$$\bar{d}(z_2 n) \not{n}\gamma_5 [z_2 n, z_1 n] u(z_1 n) = \sum_{k,l=0}^{\infty} \frac{z_2^k z_1^l}{k! l!} n^\rho n^{\mu_1} \ldots n^{\mu_{k+l}} \mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}}$$

$$\mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}} = \bar{d}(0) \overleftarrow{D}_{(\mu_1} \ldots \overleftarrow{D}_{\mu_k} \vec{D}_{\mu_{k+1}} \ldots \vec{D}_{\mu_{k+l}} \gamma_{\rho)} \gamma_5 u(0)$$

- Moments of the pion DA are given by matrix elements of local operators:

$$i^{k+l} \left\langle 0 \left| \mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}} \right| \pi(p) \right\rangle = i f_\pi p_{(\rho} p_{\mu_1} \ldots p_{\mu_{k+l}} \left\langle x^l (1-x)^k \right\rangle$$

$l + k = 2$
Second moment

Recent progresses with lattice QCD

- **Results by OPE moments:**

- **Precise at low order moments ✓**
- **Hard to get high moments ?**

$$\langle \xi^n \rangle = \int_0^1 dx (2x-1)^n \phi_\pi(x)$$

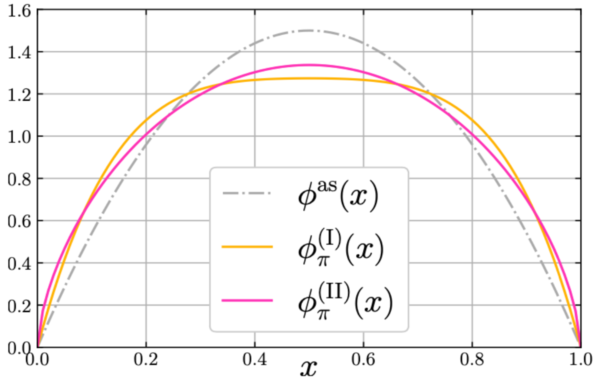
$$\phi_\pi(x) = 6x(1-x) \sum_{n=1,2,\cdots} a^\pi_{2n-2} C^{(3/2)}_{2n-2}(2x-1)$$

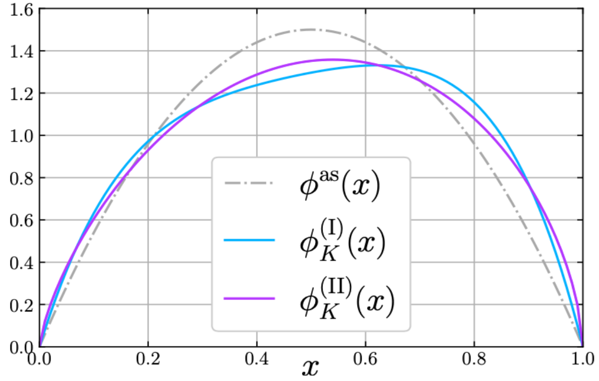
$$a_0^\pi = \langle \xi^0 \rangle,$$

$$a_2^\pi = \frac{7}{12}\left(5\langle \xi^2 \rangle - \langle \xi^0 \rangle\right),$$

$$a_4^\pi = \frac{11}{24}\left(21\langle \xi^4 \rangle - 14\langle \xi^2 \rangle + \langle \xi^0 \rangle\right),$$

(RQCD) JHEP 08065 (2019)





$\langle \xi^2 \rangle = 0.235^{\pm 0.008}$

$\langle \xi^4 \rangle = 0.109^{\pm 0.005}$

➡ $a_2^\pi = 0.101^{+24}_{-24}$ $a_4^\pi = 0.002^{+71}_{-71}$

$a_1^K = 0.0533^{+34}_{-35}$ $a_2^K = 0.090^{+19}_{-20}$

- Cancellation of large numbers breaks the convergence of moments

Reflections on current issues

- **Consider the form of Gegenbauer expansion:**

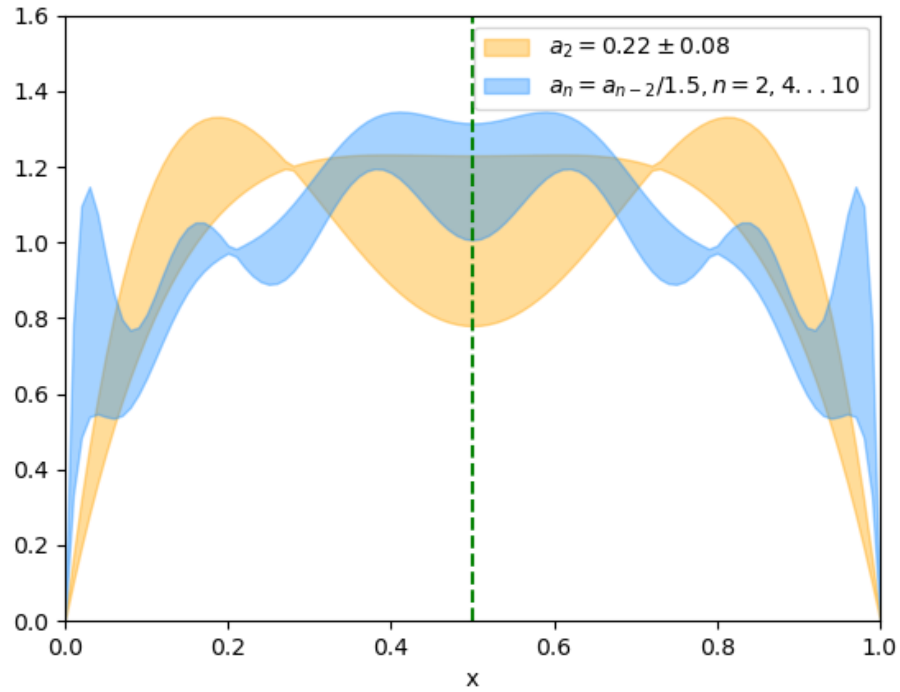
$$\phi_\pi(x) = 6x(1-x) \sum_{n=1,2,\cdots} a^{\pi}_{2n-2} C^{(3/2)}_{2n-2}(2x-1)$$

$$C_2 = \frac{3}{2}(5 * (2x-1)^2 - 1)$$

$$C_4 = \frac{15}{8}(1 - 14 * (2x-1)^2 + 21 * (2x-1)^4)$$

$$C_6 = \cdots$$

The endpoint region of LCDA is more sensitive to the high order moments.

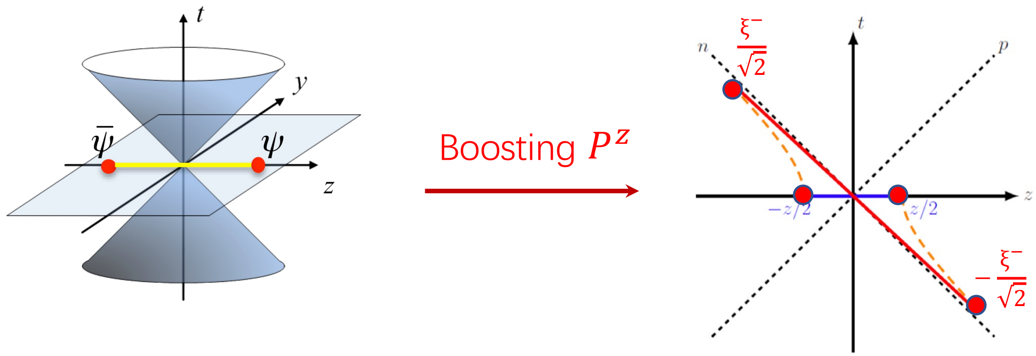


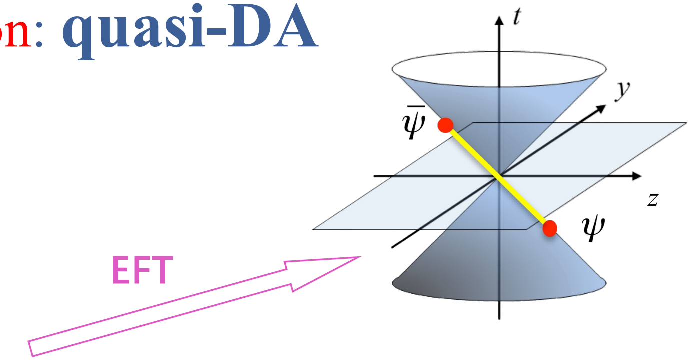
- The convergence of Gegenbauer expansion itself is not so good

Quasi distribution method by LaMET

Quasi distribution method by LaMET

- Define a lattice calculable, equal-time correlation: **quasi-DA**





- Effective field theory:
  - Instead of taking $P^z \to \infty$ calcuation, one can perform an expansion for large but finite P^z:

LCDA, Quasi-DA

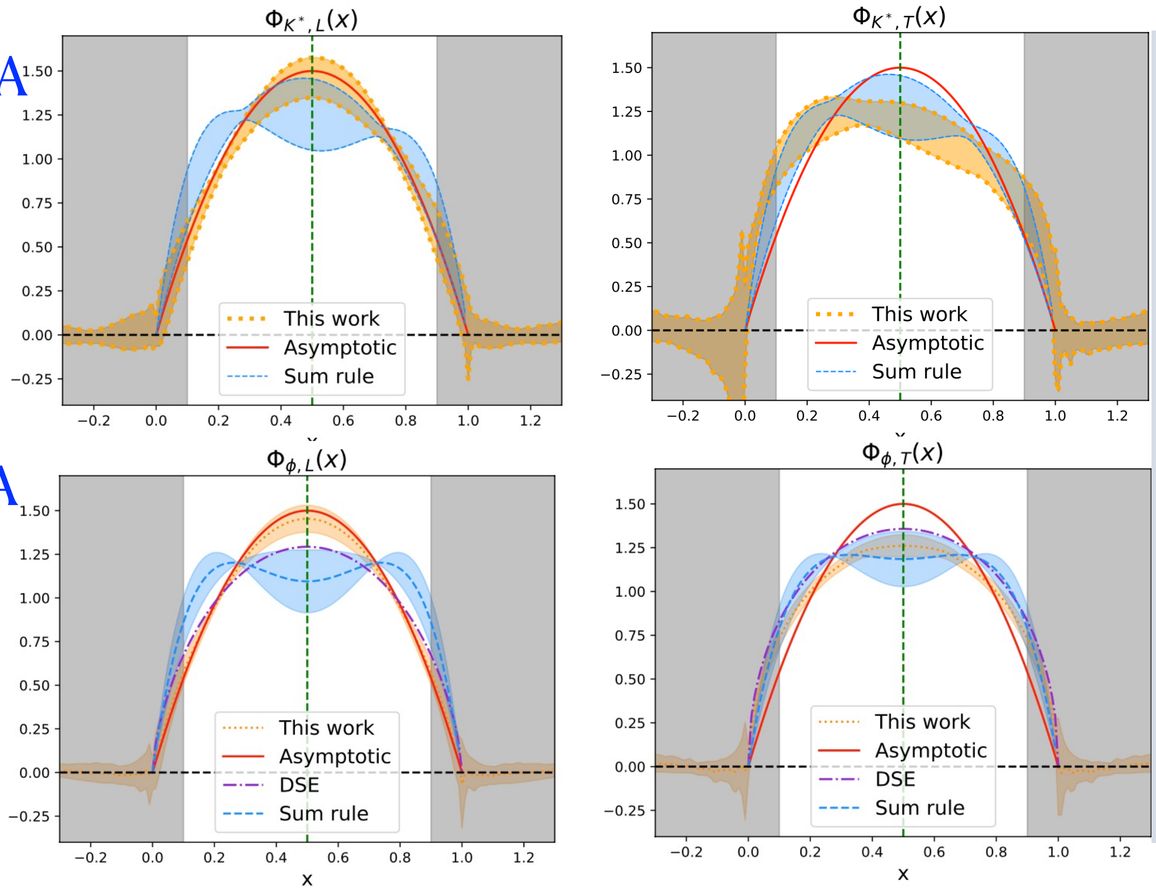
$$q(y, P^z, \mu) = \int dx C^{-1}(x, y, P^z, \mu) \tilde{q}(x, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{(yP^z)^2}, \frac{\Lambda_{QCD}^2}{((1-y)P^z)^2}\right)$$

Matching kernel; High power correction

Quasi distribution method by LaMET

K^* LCDA

ϕ LCDA

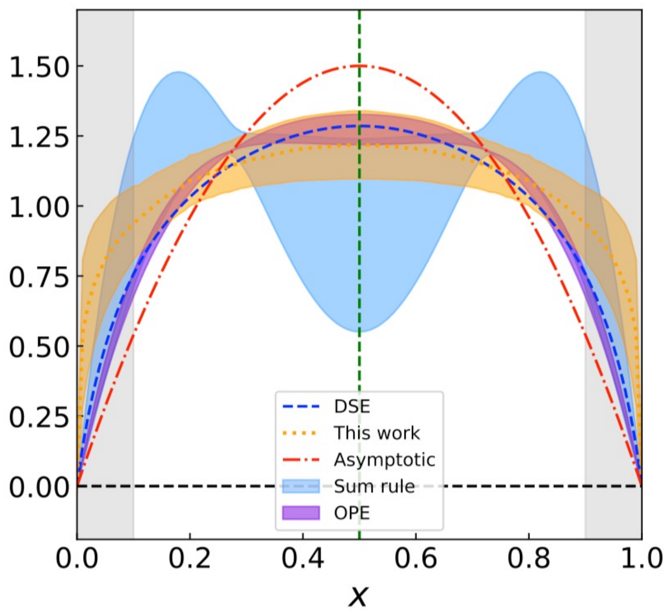


(LPC)J.Hua et.al. PRL127. 062002(2021)

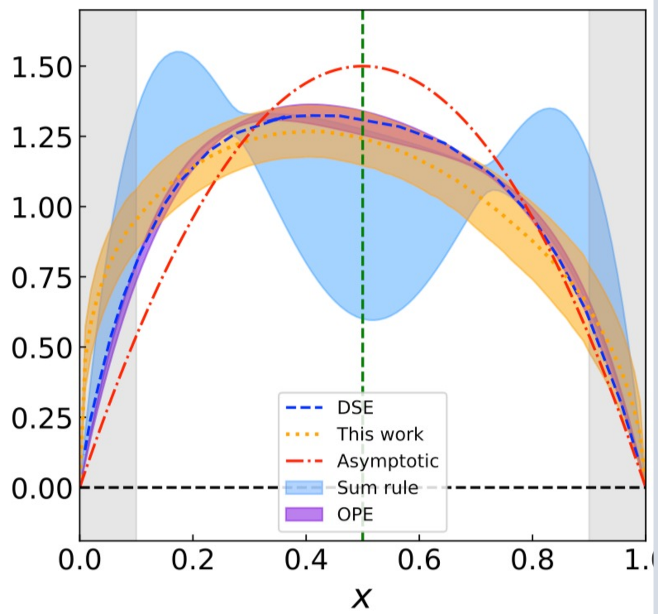
- 3 lattice spacings: (0.12, 0.09, 0.06) fm, largest volume ($96^3 \times 192$)
- 3 momentum: (1.29, 1.72, 2.15) GeV
- mass: K^*: 0.89GeV, ϕ: 1.02GeV
- Hybrid scheme (based on RI/MOM)

Quasi distribution method by LaMET

π LCDA:



K LCDA:



(LPC)J.Hua et.al. PRL129. 132001(2022)

- 3 lattice spacings: (0.12, 0.09, 0.06) fm, largest volume ($96^3 \times 192$)
- 3 momentum: (1.29, 1.72, 2.15) GeV
- mass: π: 0.13GeV, K: 0.49GeV
- Hybrid scheme (Self renormalization)

- There are uncontrolled systematic uncertainty in the endpoint region due to high power correction

Quasi distribution method by LaMET

- Research based on our results:

Next-to-next-to-leading-order QCD corrections to pion electromagnetic form factors

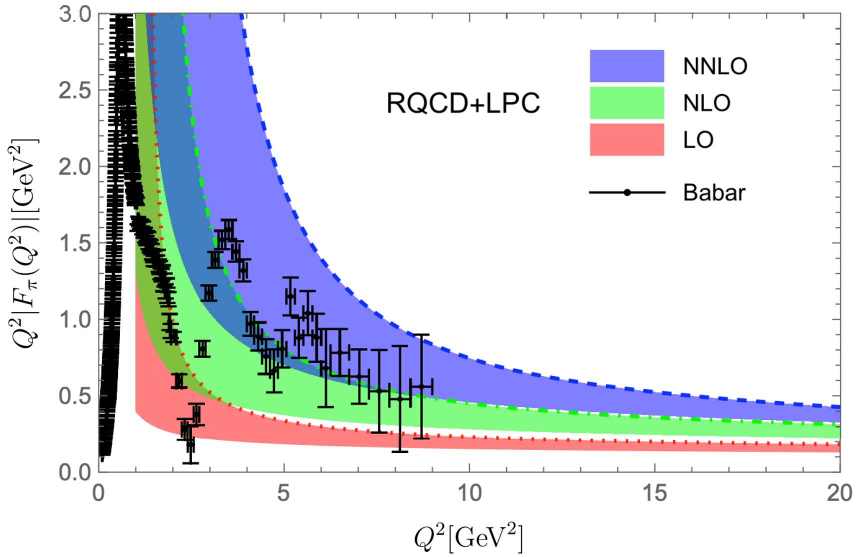
Long-Bin Chen *,[1] Wen Chen †,[2,3] Feng Feng ‡,[4,5] and Yu Jia §[5,6]

New determination of $|V_{ub}/V_{cb}|$ from $B_s^0 \to \{K^-, D_s^-\}\mu^+\nu$

Carolina Bolognani,[a,b] Danny van Dyk,[c] K. Keri Vos[a,b]

$$\left.\frac{V_{ub}}{V_{cb}}\right|_{q^2<7\ \mathrm{GeV}^2} = 0.0681 \pm 0.0040 \quad \text{and} \quad \left|\frac{V_{ub}}{V_{cb}}\right|_{q^2>7\ \mathrm{GeV}^2} = 0.0801 \pm 0.0047\ ,$$

C.Bolognani et.al. JHEP 11 ,082 (2023)



L.B.Chen arXiv:2312.17228 (2023)
Accepted by PRL

Quasi distribution method by LaMET

Problems we still face:

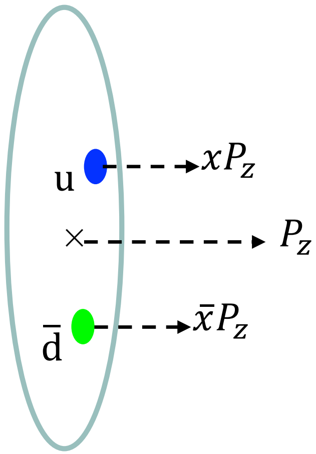
- **OPE: Cancellation of large numbers breaks the convergence of moments**
  - The convergence of Gegenbauer expansion itself
- **LaMET: There are uncontrolled systematic uncertainty in the endpoint region due to high power correction**
  - A gap between moments by lattice OPE and LaMET

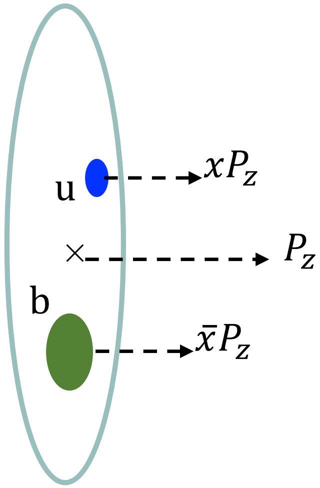
How to combine and give a reliable accurate results ?

Light baryon (Lambda) LCDAs

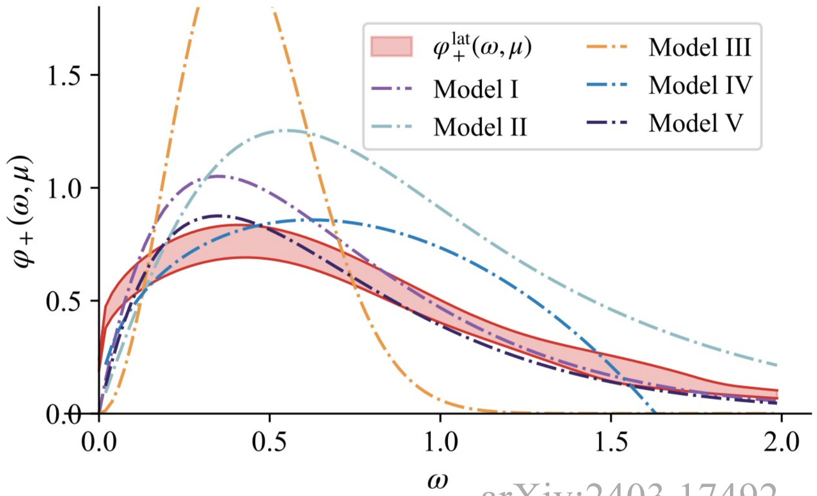
Light baryon (Lambda) LCDAs

Light meson π/K

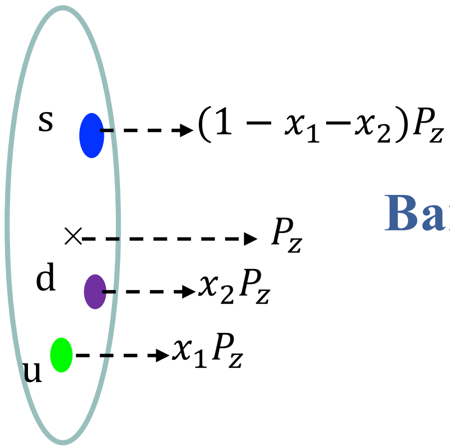




Heavy meson D/B



arXiv:2403.17492



Baryon Λ, proton

Light baryon (Lambda) LCDAs

- Light cone distribution of baryons:

$$\int \frac{d\xi_1^-}{2\pi}\frac{d\xi_2^-}{2\pi} e^{ixp_1^+\xi_1^-} e^{ixp_2^+\xi_2^-} \epsilon^{ijk} \left\langle 0 \left| W^{ii\prime}(\infty,\xi_1^-)\psi_\alpha^{i\prime}(\xi_1^-)\Gamma_{\alpha\beta}W^{jj\prime}(\infty,\xi_2^-)\,\psi_\beta^{j\prime}(\xi_2^-)\,\psi_\gamma^j(0)\right| M(P)\right\rangle$$

$$= if_M(p_1\cdot n)(p_2\cdot n)\phi_M(x_1,x_2).$$

- Quasi distribution of baryons:

$$\tilde{\Phi}^0(x_1,x_2) = \int \frac{p_{z1}dz_1}{2\pi}\frac{p_{z2}dz_2}{2\pi} e^{-i(x_1p_{z1}z_1+x_2p_{z2}z_2)} \left\langle 0\left|\tilde{O}\left(z_1,z_2,\tilde{\Gamma}\right)\right|P^z\right\rangle$$

$$\tilde{O}^\Lambda\left(z_1,z_2,\tilde{\Gamma}\right) = \varepsilon^{ijk}U^i(z_1)\tilde{\Gamma}D^j(z_2)S^k(0)$$
$$= \varepsilon^{ijk}W^{ii'}(\lambda,z_1)u_\alpha^{i'}(z_1)\Gamma_{\alpha\beta}W^{jj'}(\lambda,z_2)d_\beta^{j'}(z_2)s_\gamma^k(0)$$

Similar with a three dimension problem

- Matching kernel

$$\tilde{\Phi}(x_1,x_2,\mu) = \int dy_1dy_2\mathcal{C}(x_1,x_2,y_1,y_2,\mu)\Phi(y_1,y_2,\mu)+\mathcal{O}\left(\frac{1}{x_1p^z},\frac{1}{x_2p^z},\frac{1}{(1-x_1-x_2)p^z}\right)$$

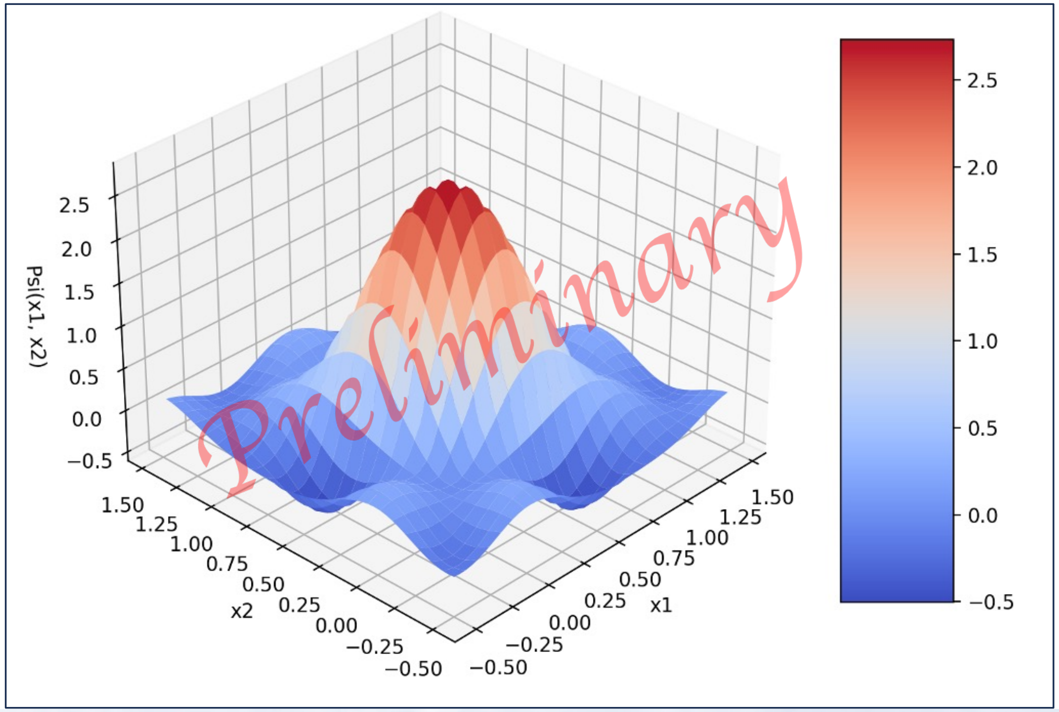
Z.F.Deng et.al. JHEP 07. 191(2023)

Light baryon (Lambda) LCDAs

$$\tilde{\Phi}^0(x_1,x_2)=\int\frac{p_{z1}dz_1}{2\pi}\frac{p_{z2}dz_2}{2\pi}e^{-i(x_1p_{z1}z_1+x_2p_{z2}z_2)}\left\langle 0\left|\tilde{O}\left(z_1,z_2,\tilde{\Gamma}\right)\right|P^z\right\rangle$$

$$\begin{aligned}\tilde{O}^{\Lambda}\left(z_1,z_2,\tilde{\Gamma}\right)&=\varepsilon^{ijk}U^i(z_1)\tilde{\Gamma}D^j(z_2)S^k(0)\\&=\varepsilon^{ijk}W^{ii'}(\lambda,z_1)u_\alpha^{i'}(z_1)\Gamma_{\alpha\beta}W^{jj'}(\lambda,z_2)d_\beta^{j'}(z_2)s_\gamma^k(0)\end{aligned}$$

- Two dimensional quasi distribution



Summary

Light meson LCDAs:

- ➢ The disagree between lattice OPE and LaMET lattice calcuation

LaMET:

- Large P^z to suppress the power corrections;
- Some resummation skills ……

OPE:

- High order moment

- ➢ Still working towards a more accurate LCDA

More hadrons as heavy mesons, baryons …

More generalized distribution, TMD-WFs …

Thanks!