University of Chinese Academy of Sciences

Hyperon semileptonic decays in QCD sum rules

Sheng-qi Zhang (张圣奇)

University of Chinese Academy of Sciences

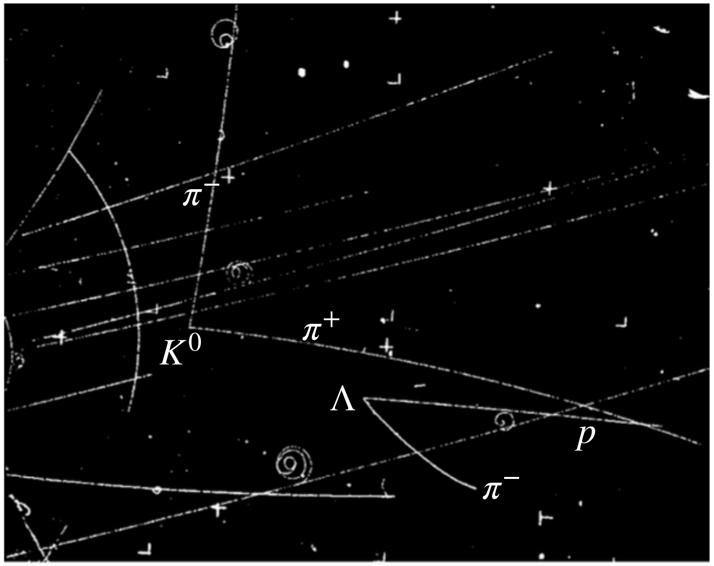
Collaborator: Xuan-Heng Zhang, Cong-Feng Qiao

arXiv:2402.15088

第六届重味物理与量子色动力学研讨会

2024.04.21 @青岛

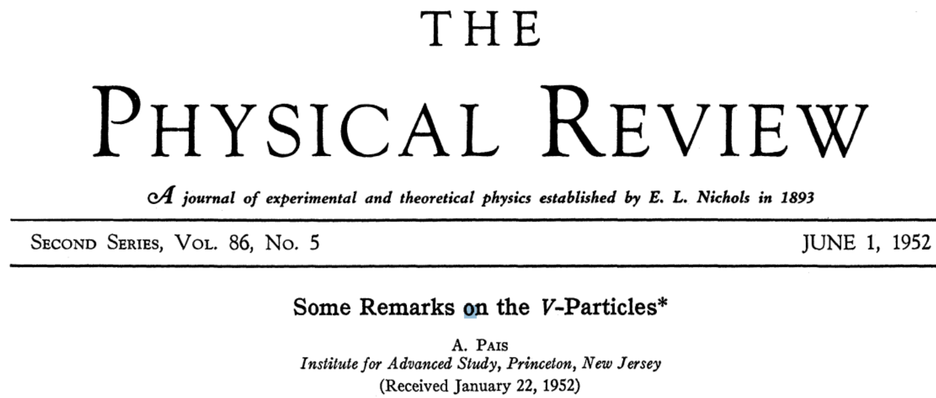
Hyperon history



"V" particles

G.D. Rochester, C. C. Butler, Nature 160, 855 (1947)

Hyperon: heavier than proton


THE
PHYSICAL REVIEW
A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 86, NO. 5 — JUNE 1, 1952

Some Remarks on the *V*-Particles*

A. PAIS
Institute for Advanced Study, Princeton, New Jersey
(Received January 22, 1952)

- Produce in pairs in strong interactions
- Decay via the weak interactions

A. Pais, Phys.Rev. 86, 663 (1952)

- Strangeness(S): conserve in strong interaction, violate in weak interaction
- Selection rule: $\Delta S = 0, \pm 1$

M. Gell-Mann, Phys.Rev. 92, 833 (1953)

T. Nakano, K. Nishijima, Prog.Theor.Phys 10, 581(1953)

Hyperon decays

	Λ	Σ^+	Σ^0	Σ^-	Ξ^0	Ξ^-
$M(\text{GeV})$	1.116	1.189	1.192	1.197	1.315	1.322
$\tau(10^{-10}s)$	2.632	0.802	7.4×10^{-10}	1.479	2.90	1.639

Hyperon decays → Semileptonic decays

- ✓ CKM matrix element V_{us}
- ✓ $SU(3)$ flavor symmetry
- ✓ Source of CP violation

- ★ Simple environment
- ★ Decay dynamics (form factors)

J. D. Richman and P. R. Burchat, Rev. Mod. Phys. 67, 893 (1995)

Experimental perspective

➡ Current status: long history, large uncertainties, not direct measurement……

Obstacles

- Dominant two-body decay backgrounds

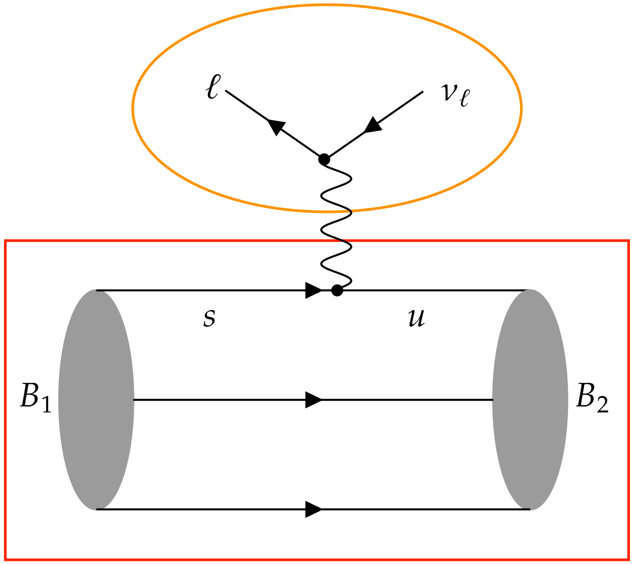
  $\Lambda \to p + \pi^-, \ \Sigma^- \to n + \pi^-, \ \Xi^- \to \Lambda + \pi^-$

- Tiny branching fractions ($10^{-4} \sim 10^{-6}$)

Super τ-charm facility

Front.Phys 19, 14701(2024)

Theoretical perspective



leptonic part:

- lepton current: $\bar{\ell}\gamma^{\mu}\left(1-\gamma_5\right)\nu_{\ell}$

hadronic part:

- $s \to u$ transition, $\bar{u}\,\gamma_{\mu}\left(1-\gamma_5\right)s$
- parametrized in terms of form factors

$$\langle B_2 | J_{\mu}^{h} | B_1 \rangle = \bar{u}_2\left[f_1\gamma_{\mu} + i f_2 \sigma_{\mu\nu}\frac{q^{\nu}}{M_1} + f_3\frac{q_{\mu}}{M_1}\right]u_1 - \bar{u}_2\left[g_1\gamma_{\mu} + i g_2 \sigma_{\mu\nu}\frac{q^{\nu}}{M_1} + g_3\frac{q_{\mu}}{M_1}\right]\gamma_5 u_1$$

Form factors

Cabibbo's predictions: flavor $SU(3)$ symmetry

	$f_1(0)$	$g_1(0)$	$g_1(0)/f_1(0)$	$f_2(0)/f_1(0)$
$\Lambda \to p$	$-\sqrt{\frac{3}{2}}$	$-\sqrt{\frac{3}{2}}(F+\frac{D}{3})$	$F+\frac{D}{3}$	$\frac{M_\Lambda}{M_p}\frac{\mu_p}{2}$
$\Sigma^0 \to p$	$-\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}(F-D)$	$F-D$	$\frac{M_{\Sigma^0}}{M_p}\frac{(\mu_p+2\mu_n)}{2}$
$\Xi^- \to \Lambda$	$\sqrt{\frac{3}{2}}$	$\sqrt{\frac{3}{2}}(F-\frac{D}{3})$	$F-\frac{D}{3}$	$-\frac{M_{\Xi^-}}{M_p}\frac{(\mu_p+\mu_n)}{2}$
$\Xi^0 \to \Sigma^+$	1	$F+D$	$F+D$	$\frac{M_{\Xi^0}}{M_p}\frac{(\mu_p-\mu_n)}{2}$
$n \to p$	1	$F+D$	$F+D$	$\frac{M_n}{M_p}\frac{(\mu_p-\mu_n)}{2}$

★ Isospin symmetry is considered.

★ g_2 vanishes in $SU(3)$ symmetry.

★ f_3 and g_3 have little contributions to the decays.

N. Cabibbo, Phys. Rev. Lett. 10, 531–533 (1963)

N. Cabibbo et al., Ann. Rev. Nucl. Part. Sci. 53, 39–75 (2003)

Potential symmetry breaking $m_s \gg m_d \sim m_u$

- Symmetry breaking of f_1: $f_1/f_1^{SU(3)}$
- In $SU(3)$ symmetry, $(g_1/f_1)^{n\to p} = (g_1 f_1)^{\Xi^0\to\Sigma^+}$

Donoghue et al., PRD(1987); Schlumpf, PRD(1995); Villadoro, PRD(2006); Faessler et al., PRD(2008) Ledwig et al., JHEP(2008); L. S. Geng et al., PRD(2009); Sasaki, PRD(2012); G.S. Yang et al., PRC(2015)

$$\text{PDG}: (g_1/f_1)^{n\to p} = 1.2754 \pm 0.013, \quad (g_1/f_1)^{\Xi^0\to\Sigma^+} = 1.22 \pm 0.05$$

Form factors in QCD sum rules

SUM RULES AND THE PION FORM FACTOR IN QCD

V.A. NESTERENKO and A.V. RADYUSHKIN
Laboratory of Theoretical Physics, JINR, Dubna, USSR

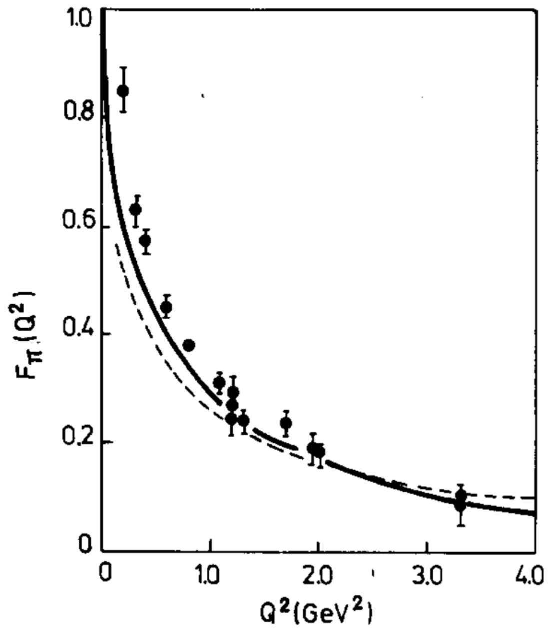
Received 12 May 1982

MESON WIDTHS AND FORM FACTORS AT INTERMEDIATE MOMENTUM TRANSFER IN NON-PERTURBATIVE QCD

B.L. IOFFE and A.V. SMILGA
ITEP, Moscow, 117259, USSR

Received 14 September 1982

pion electromagnetic form factor



light and heavy hadrons. The applications include the pion electromagnetic form factor,[100] radiative charmonium decays such as $J/\psi \to \eta_c\gamma$,[101] D and B semileptonic and flavor-changing neutral current (FCNC) transitions [102–107] and, more recently, the radiative decays $\phi \to (\eta, \eta')\gamma$.[108]

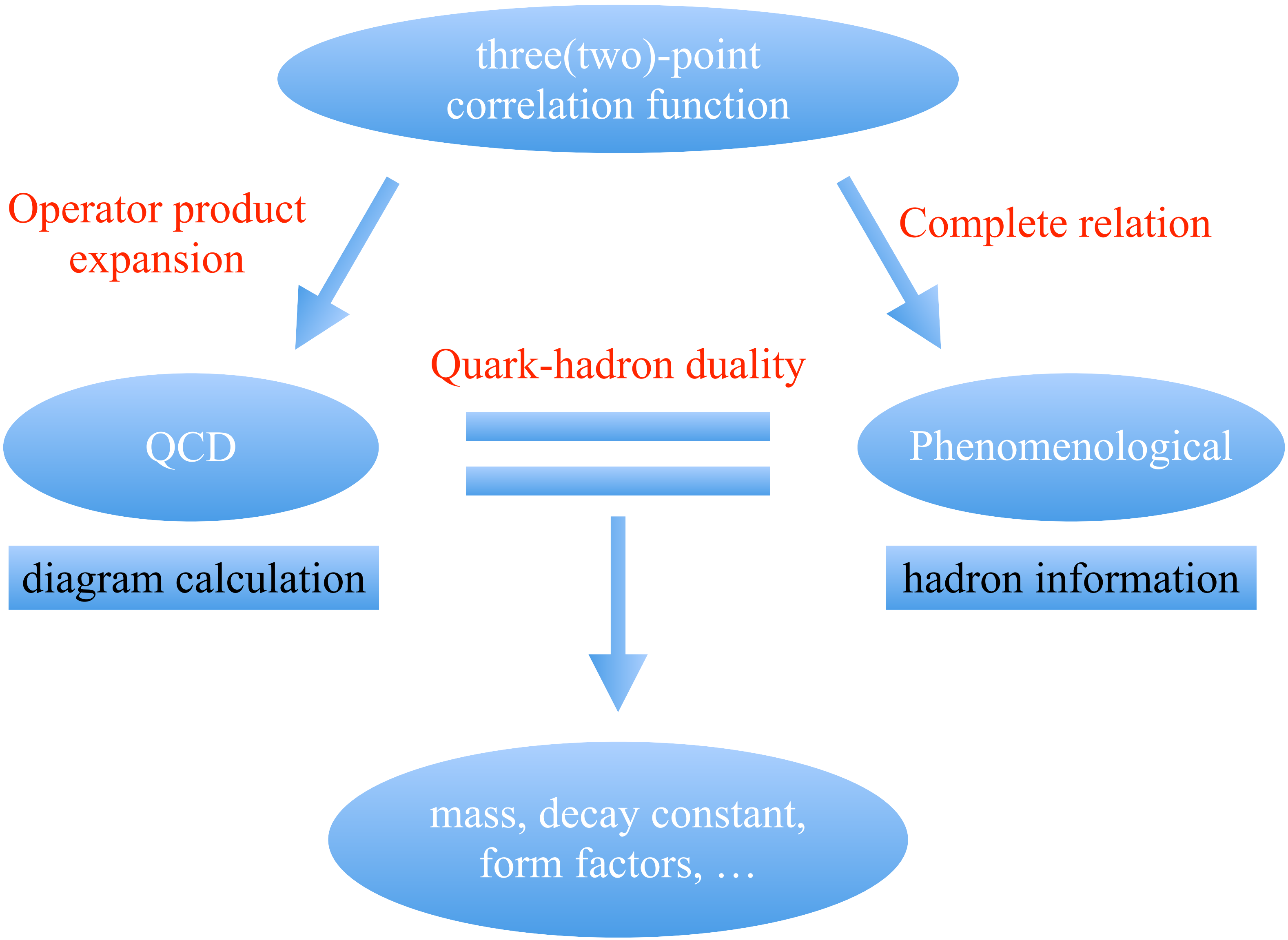
P. Colangelo et al., *QCD sum rules, a modern perspective*(2000)

- ☑ Radiative decay
- ☑ FCNC transition
- ☑ Strong decay
- ☑ Semileptonic decay

Semileptonic decays in QCD sum rules

Channel	work
Meson	
$D \to \pi, \rho, K^{(*)}$	Dosch et al.(1991); Ball et al.(1993); 杨茂志 (2006);
$D_s \to \phi$	杜东升 & 李竞武 & 杨茂志 (2006);
$J/\psi \to D_s^{(*)}$	王玉明 & Hao Zou & 魏正涛 & 李学潜 & 吕才典 (2006);
$B \to \pi, \rho, K$	Narison et al.(1992); Khodjamirian et al.(1993);
$B_c \to D, J/\psi, \eta_c, \chi_{c0}, B, B_s$	Colangelo et al.(1993); Kiselev et al.(2000); Azizi et al.(2009); 王志刚 (2014); Leljak et al.(2020);
Baryon	
$\Lambda_b \to \Lambda_c$	戴元本 & 黄朝商 & 黄明球 & 刘纯 (1996); Nielsen et al.(1999); 赵振兴 & 李润辉 & 沈月龙 & 施瑀基 & Yan-Sheng Yang(2020);
$\Lambda_b \to p$	黄朝商 & 乔从丰 & Hua-Gang Yan(1998);
$\Lambda_c \to \Lambda$	Dosch et al.(1999);
$\Lambda_c \to n$	ZSQ & 乔从丰 (2023);
$\{\Xi_{QQ}, \Omega_{QQ}\} \to \{\Lambda_Q, \Sigma_Q, \Xi_Q, \Omega_Q\}$	施瑀基 & 王伟 & 赵振兴 (2020)

Basic idea


three(two)-point
correlation function
Operator product
expansion
Complete relation
Quark-hadron duality
QCD
Phenomenological
diagram calculation
hadron information
mass, decay constant,
form factors, …

Three-point correlation function

$$\Pi_\mu(q_1^2, q_2^2, q^2) = i^2 \int d^4x\; d^4y\; e^{i(-q_1 x + q_2 y)} < 0 \,|\, T\{j_2(y) j_\mu(0) j_1^\dagger(x)\} \,|\, 0 >$$

$j_{1,2}$: interpolating currents for the initial and final state baryons

j_μ: weak transition current

$$j = \epsilon^{abc} \left(q_i^{aT} C \sigma^{\mu\nu} q_j^b \right) \sigma_{\mu\nu} \gamma_5 q_k^c$$

$$j_\mu = \bar{u} \gamma_\mu (1 - \gamma_5) s$$

dispersion relations

$$\Pi_\mu(q_1^2, q_2^2, q^2) = \int ds_1 \int ds_2 \frac{\rho_\mu(s_1, s_2, q^2)}{(s_1 - q_1^2)(s_2 - q_2^2)}$$

QCD side

$$\Pi_\mu(q_1^2, q_2^2, q^2) = i^2 \int d^4x \; d^4y \; e^{i(-q_1 x + q_2 y)} < 0 \,|\, T\{j_2(y) j_\mu(0) j_1^\dagger(x)\} \,|\, 0 >$$

dispersion relations

operator product expansion:

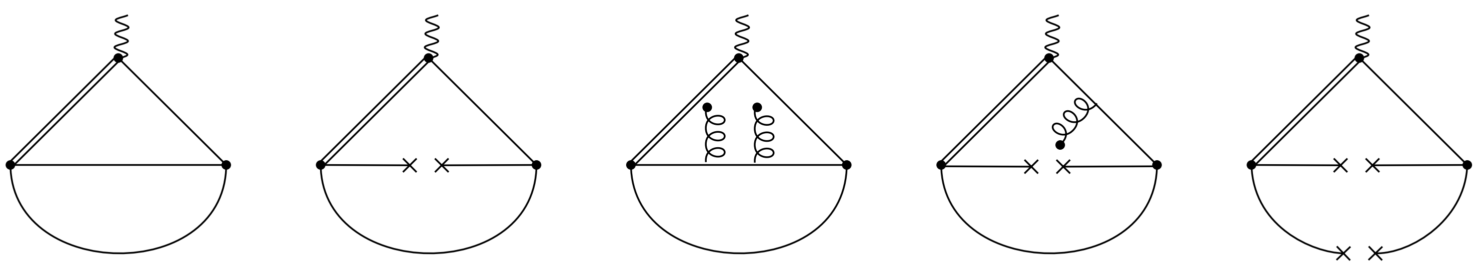
$$\Pi_\mu(q_1^2, q_2^2, q^2) = \sum_d C_{d,\mu}(q_1^2, q_2^2) < 0 \,|\, O_d(0) \,|\, 0 >$$

vacuum condensate

$$\Pi_\mu^{\rm QCD}(q_1^2, q_2^2, q^2) = \int_{s_1^{\rm min}}^{\infty} ds_1 \int_{s_2^{\rm min}}^{\infty} ds_2 \frac{\rho_\mu^{\rm QCD}(s_1, s_2, q^2)}{(s_1 - q_1^2)(s_2 - q_2^2)}$$

$\rho_\mu^{\rm QCD}$: spectral density

$$\rho_\mu^{\rm QCD}(s_1, s_2, q^2) = \rho_\mu^{\rm pert}(s_1, s_2, q^2) + \rho_\mu^{\langle \bar{q}q \rangle}(s_1, s_2, q^2) + \rho_\mu^{\langle g_s^2 G^2 \rangle}(s_1, s_2, q^2) + \rho_\mu^{\langle g_s \bar{q} \sigma \cdot G q \rangle}(s_1, s_2, q^2) + \rho_\mu^{\langle \bar{q}q \rangle^2}(s_1, s_2, q^2)$$



Phenomenological side

$$\Pi_\mu(q_1^2, q_2^2, q^2) = i^2 \int d^4x\, d^4y\, e^{i(-q_1x+q_2y)} < 0 \,|\, T\{j_2(y) j_\mu(0) j_1^\dagger(x)\} \,|\, 0 >$$

dispersion relations

$$\sum_H |H\rangle \langle H| = 1$$

$$\Pi_\mu^{\rm phe}(q_1^2, q_2^2, q^2) = \sum_{\rm spins} \frac{< 0 \,|\, j_2 \,|\, B_2(q_2) > \;\; < B_2(q_2) \,|\, j_\mu \,|\, B_1(q_1) > \;\; < B_1(q_1) \,|\, j_1 \,|\, 0 >}{(q_1^2 - M_1^2)(q_2^2 - M_2^2)}$$

$$+\text{higher resonances and continuum states},$$

$$< 0 \,|\, j_i \,|\, B_i(p) > \; = \lambda_i u_i(p)$$

$$\begin{aligned} &< B_2(q_2) \,|\, \bar{u}\gamma_\mu \left(1 - \gamma_5\right) s \,|\, B_1(q_1) > \\ &= \bar{u}_2(q_2) \left[f_1(q^2)\gamma_\mu + i f_2(q^2)\sigma_{\mu\nu}\frac{q^\nu}{M_1} + f_3(q^2)\frac{q_\mu}{M_1} \right] u_1(q_1) \\ &- \bar{u}_2(q_2) \left[g_1(q^2)\gamma_\mu + i g_2(q^2)\sigma_{\mu\nu}\frac{q^\nu}{M_1} + g_3(q^2)\frac{q_\mu}{M_1} \right] \gamma_5 u_1(q_1) \,. \end{aligned}$$

$$\begin{aligned} \Pi_\mu^{\rm phe}(q_1^2, q_2^2, q^2) = & \frac{\lambda_2 \left(\not{q}_2 + M_2\right) \left[f_1(q^2)\gamma_\mu + i f_2(q^2)\sigma_{\mu\nu}\frac{q^\nu}{M_1} + f_3(q^2)\frac{q_\mu}{M_1} \right] \lambda_1 \left(\not{q}_1 + M_1\right)}{(q_1^2 - M_1^2)(q_2^2 - M_2^2)} \\ & - \frac{\lambda_2 \left(\not{q}_2 + M_2\right) \left[g_1(q^2)\gamma_\mu + i g_2(q^2)\sigma_{\mu\nu}\frac{q^\nu}{M_1} + g_3(q^2)\frac{q_\mu}{M_1} \right] \gamma_5 \lambda_1 \left(\not{q}_1 + M_1\right)}{(q_1^2 - M_1^2)(q_2^2 - M_2^2)} \end{aligned}$$

Borel transformation

Suppress the higher excited states and continuum states contributions

$$\mathscr{B}[g(Q^2)] \equiv g(M_B^2) = \lim_{\substack{Q^2, n \to \infty \\ n/Q^2 = 1/M_B^2}} \frac{(-1)^n (Q^2)^{n+1}}{n!} \left(\frac{\partial}{\partial Q^2} \right)^n g(Q^2)$$

Quark-hadron duality

Establish the equivalence between two sides

$$\Pi_\mu^{\text{phe}}(q_1^2, q_2^2, q^2) \simeq \int_{s_1^{\min}}^{s_1^0} ds_1 \int_{s_2^{\min}}^{s_2^0} ds_2 \frac{\rho_\mu^{\text{QCD}}(s_1, s_2, q^2)}{(s_1 - q_1^2)(s_2 - q_2^2)}$$

$s_{1(2)}^0$: threshold parameters

Symmetry breaking of vector form factor $f_1(0)$

negative corrections

	$\Lambda \to N$	$\Sigma \to N$	$\Xi \to \Lambda$	$\Xi \to \Sigma$	Refs
$f_1(0)/f_1^{SU(3)}$					
QCDSR	0.963 ± 0.061	0.993 ± 0.059	0.993 ± 0.061	0.956 ± 0.049	This work
Quark model	0.987	0.987	0.987	0.987	J. F. Donoghue et al., PRD(1987)
Quark model	0.976	0.975	0.976	0.976	F. Schlumpf, PRD(1995)
lattice QCD		0.975 ± 0.01		0.973 ± 0.007	S. Sasaki, PRD(2012)
$1/N_c$ expansion	1.02 ± 0.02	1.04 ± 0.02	1.10 ± 0.04	1.12 ± 0.05	R. Flores-Mendiet et al., PRD(1998)
χPT	1.027	1.041	1.043	1.009	G. Villadoro, PRD(2006)
χPT	$1.001^{+0.013}_{-0.010}$	$1.087^{+0.042}_{-0.031}$	$1.040^{+0.028}_{-0.021}$	$1.017^{+0.022}_{-0.016}$	L. S. Geng et al., PRD(2009)

QCDSR and $1/N_c$ expansion suggest relatively larger breaking in $\Xi \to \Sigma$.

positive corrections

Axial-vector form factor $g_1(0)$

	$\Lambda \to N$	$\Sigma \to N$	$\Xi \to \Lambda$	$\Xi \to \Sigma$	Refs
$g_1(0)/f_1(0)$					
QCDSR	0.708 ± 0.047	-0.327 ± 0.046	0.271 ± 0.045	1.293 ± 0.100	This work
Cabibbo model	0.731	−0.341	0.195	1.267	N. Cabibbo et al., Annu. Rev(2003)
Quark model	0.724	−0.260	0.265	1.20	A. Faessler et al., PRD(2008)
Soliton model	0.68	−0.27	0.21	1.16	T. Ledwig et al., JHEP(2008)
Soliton model	0.718 ± 0.003	-0.340 ± 0.003	0.250 ± 0.002	1.210 ± 0.005	G.S. Yang et al., PRC(2015)
$1/N_c$ expansion	0.73	−0.34	0.22	1.03	R. Flores-Mendiet et al., PRD(1998)
lattice QCD		-0.287 ± 0.052		1.248 ± 0.029	D. Guadagnol et al., NPB(2007) S. Sasaki et al., PRD(2009)
Exp	0.718 ± 0.015	-0.340 ± 0.017	0.25 ± 0.05	1.22 ± 0.05	PDG(2022)

$$\left(g_1/f_1\right)^{n\to p}_{\mathrm{Exp}} = 1.2754 \pm 0.013$$

- The results are generally consistent.
- $\left(g_1/f_1\right)^{n\to p}$ slightly deviates from the $\left(g_1/f_1\right)^{\Xi\to\Sigma}$

⋆ In $SU(3)$ symmetry:

$$\left(g_1/f_1\right)^{n\to p} = \left(g_1 f_1\right)^{\Xi^0\to\Sigma^+}$$

Weak magnetism form factor $f_2(0)$

	$\Lambda \to N$	$\Sigma \to N$	$\Xi \to \Lambda$	$\Xi \to \Sigma$	Refs
$f_2(0)/f_1(0)$					
QCDSR	0.752 ± 0.074	-1.042 ± 0.090	0.118 ± 0.032	1.957 ± 0.255	This work
Cabibbo model	1.066	-1.292	0.085	2.609	N. Cabibbo et al., Annu. Rev(2003)
Quark model	1	-0.962	0.129	2.402	A. Faessler et al., PRD(2008)
Soliton model	0.637 ± 0.041	-0.709 ± 0.036	-0.069 ± 0.027	1.143 ± 0.061	G.S. Yang et al., PRC(2015)
Soliton model	0.71	-0.96	-0.02	2.02	T. Ledwig et al., JHEP(2008)
$1/N_c$ expansion	0.90	-1.02	-0.06	1.85	R. Flores-Mendiet et al., PRD(1998)
lattice QCD		-1.52 ± 0.81			S. Sasaki et al., PRD(2009)
Exp	1.32 ± 0.81	-0.97 ± 0.14	-0.24 ± 0.25	2.0 ± 0.9	M. Bourquin et al., Z. Phys. C(1983) PDG(2022)

Theory

significant disparities

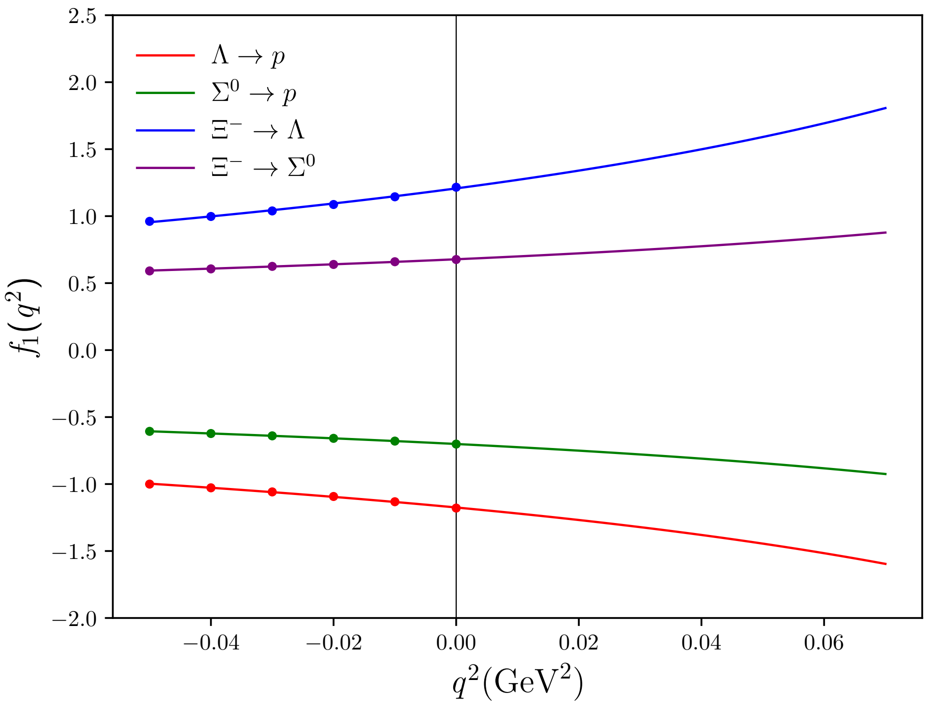
Experiment

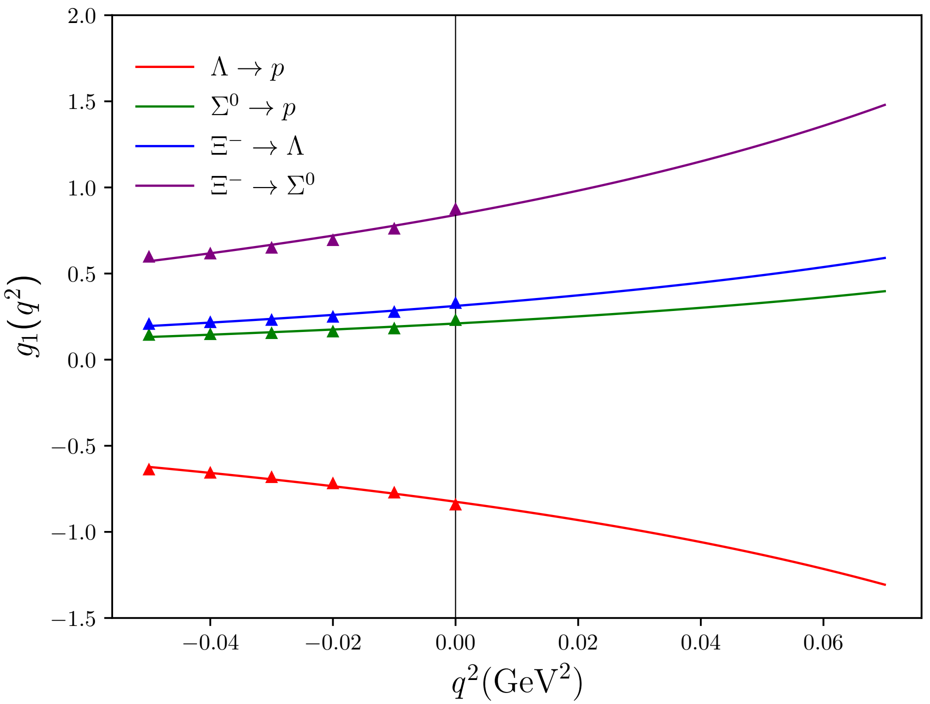
large uncertainties

q^2 dependence of the form factors

☆ Employ BCL parametrization to extrapolate the form factors:

$$f_i(q^2) = \frac{f_i(0)}{1 - q^2/(m_{\text{pole}})^2}\{1 + a_1(z(q^2, t_0) - z(0,t_0))\}$$

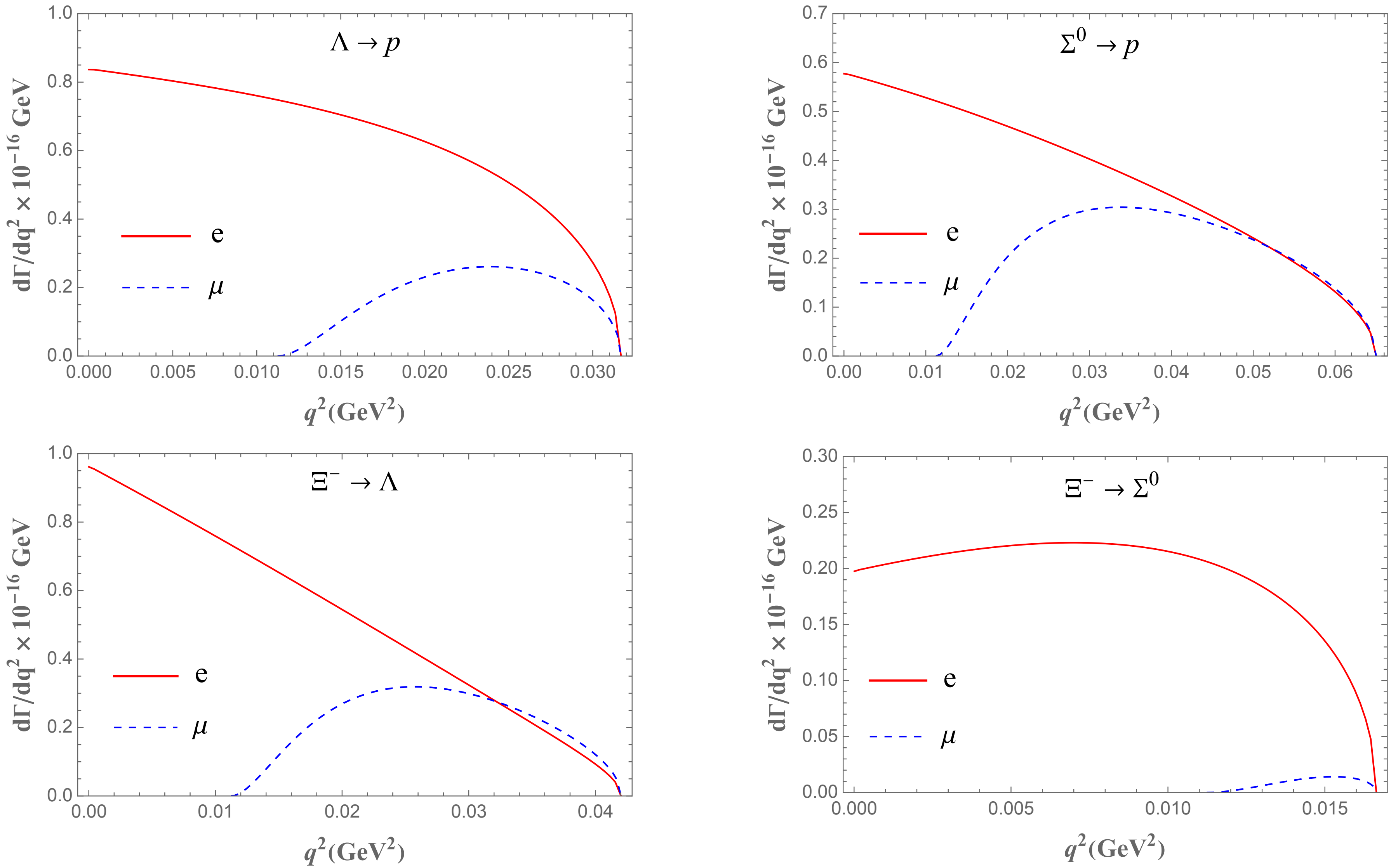




q^2 dependence of the differential decay width

helicity amplitude
(form factors)

Differential decay width: $$\frac{d\Gamma\left(B_1 \to B_2 \ell \nu_\ell\right)}{dq^2} = \frac{G_F^2 \left|V_{us}\right|^2 q^2 \sqrt{Q_+ Q_-}}{384\pi^3 M_1^3}\left(1-\frac{m_\ell^2}{q^2}\right)^2 H_{\rm tot}$$



Branching fractions

not include q^2 dependence of the form factors

	QCDSR	QCDSR^0	Exp
$\mathcal{B}(\Lambda \to p e^- \bar{\nu}_e) \times 10^{-4}$	8.09 ± 0.73	7.12 ± 0.65	8.34 ± 0.14
$\mathcal{B}(\Lambda \to p \mu^- \bar{\nu}_\mu) \times 10^{-4}$	1.44 ± 0.13	1.15 ± 0.11	1.51 ± 0.19
$\mathcal{B}(\Sigma^- \to n e^- \bar{\nu}_e) \times 10^{-3}$	1.06 ± 0.20	0.87 ± 0.14	1.017 ± 0.034
$\mathcal{B}(\Sigma^- \to n \mu^- \bar{\nu}_\mu) \times 10^{-4}$	5.17 ± 1.03	3.74 ± 0.59	4.5 ± 0.4
$\mathcal{B}(\Sigma^0 \to p e^- \bar{\nu}_e) \times 10^{-13}$	2.67 ± 0.49	2.18 ± 0.34	$2.46 \pm 0.32^*$
$\mathcal{B}(\Sigma^0 \to p \mu^- \bar{\nu}_\mu) \times 10^{-13}$	1.30 ± 0.25	0.94 ± 0.15	$1.13 \pm 0.15^*$
$\mathcal{B}(\Xi^- \to \Lambda e^- \bar{\nu}_e) \times 10^{-4}$	5.41 ± 0.79	4.67 ± 0.63	5.63 ± 0.31
$\mathcal{B}(\Xi^- \to \Lambda \mu^- \bar{\nu}_\mu) \times 10^{-4}$	1.66 ± 0.26	1.24 ± 0.17	$3.5^{+3.5}_{-2.2}$
$\mathcal{B}(\Xi^0 \to \Sigma^+ e^- \bar{\nu}_e) \times 10^{-4}$	2.48 ± 0.24	2.39 ± 0.25	2.52 ± 0.08
$\mathcal{B}(\Xi^0 \to \Sigma^+ \mu^- \bar{\nu}_\mu) \times 10^{-6}$	2.18 ± 0.24	1.91 ± 0.20	2.33 ± 0.35
$\mathcal{B}(\Xi^- \to \Sigma^0 e^- \bar{\nu}_e) \times 10^{-5}$	8.07 ± 0.77	7.72 ± 0.81	8.7 ± 1.7
$\mathcal{B}(\Xi^- \to \Sigma^0 \mu^- \bar{\nu}_\mu) \times 10^{-6}$	1.12 ± 0.12	0.97 ± 0.10	< 800

* R.M. Wang, M.Z. Yang, H.B. Li, and X.D. Cheng, PRD(2019)

- Contributions from the q^2 dependence of the form factors: $10\,\% \sim 30\,\%$.
- Much more impact to μ mode.

Summary

- We calculated the transition form factors of hyperon semileptonic decays in the frame work of QCD sum rules.
- We analyze the flavor $SU(3)$ symmetry breaking effects based on the relevant form factors.
- We derive the branching fractions of hyperon semileptonic decays and the tensor form factor related to the new physics beyond the standard model.
- Potential improvement: inverse problem, NLO corrections……

Thanks