# Decay width of the $X(3842)$ as the $\psi_{3}\left(1^{3} D_{3}\right)$ state 

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## Outline

I. Introduction
II. Model introduction
III. Relativistic wave function and transition amplitude
IV. Results and discussions
V. Summary

## I. Introduction

## 1. Background

- In 2019, a new bound state $X$ (3842) was discovered at LHCb, which is considered to spin triplet $D$ wave charmonium $\psi_{3}\left(1^{3} D_{3}\right)$ with $J^{P C}=3^{--}$.

R. Aaij et al., (LHCb Collaboration), JHEP 07 (2019) 035.


## I. Introduction

## 2. Previous research

- Other works Different models have studied the decays of $X(3842)$. They had predicted the $\psi_{3}\left(1^{3} D_{3}\right)$ state to have a natural width $0.5 \sim 4 \mathrm{MeV}$.
T. Barnes and S. Godfrey, Phys. Rev. D 69, 054008 (2004)
T. Barnes, S. Godfrey and E. S. Swanson, Phys. Rev. D 72, 054026 (2005)
E. J. Eichten, K. Lane and C. Quigg, Phys. Rev. D 73, 014014 (2006)
G.-L. Yu and Z.-G. Wang, Int. J. Mod. Phys. A 34, 1950151 (2019).
- Our works The Bethe-Salpeter (BS) equation is a relativistic dynamic equation used to describe bound state. By using the BS equation method, theoretical results were obtained that were in good agreement with experimental data.
Chang, Chen, and Wang, Commun. Theor. Phys. 46, 467 (2006)
Wang and Wang, Phys. Lett. B 697, 233 (2011)
Wang, Jiang and Wang, J. High Energy Phys. 03, 209 (2016)
Wang, Wang and Chang, J. High Energy Phys. 05, 006 (2022)


## II. Model introduction

## 1. Bethe-Salpeter equation

- Bethe-Salpeter equation

For meson consisting of quark and antiquark, the general Bethe-Salpeter equation in momentum space

$$
\begin{equation*}
\left(\not p_{1}-m_{1}\right) \chi_{P}(q)\left(\not p_{2}+m_{2}\right)=i \int \frac{d^{4} k}{(2 \pi)^{4}} V(P, k, q) \chi_{P}(k) \tag{2.1}
\end{equation*}
$$

$\chi_{P}(q)-\mathrm{BS}$ wave function, $V(P, k, q)$ - interaction kernel.

- The meson momentum $P$ and relative momentum $q$ :

$$
\begin{array}{ll}
p_{1}=\alpha_{1} P+q, & \alpha_{1}=\frac{m_{1}}{m_{1}+m_{2}}, \\
p_{2}=\alpha_{2} P-q, & \alpha_{2}=\frac{m_{2}}{m_{1}+m_{2}} .
\end{array}
$$

## II. Model introduction

- Although BS equation is the relativistic dynamic equation, it is difficult to solve analytically
such as, complete interacting kernel $V(P, k, q)$ of BS equation, it cannot be fully calculated.
- Reduced version is needed -(instantaneous version).


## II. Model introduction

## 2. Salpeter equation

- Salpeter equation

The reduced (instantaneous) Bethe-Salpeter wave function

$$
\begin{equation*}
\varphi_{p}\left(q_{\perp}^{\mu}\right) \equiv i \int \frac{d q_{p}}{(2 \pi)} \chi_{P}\left(q_{\|}^{\mu}, q_{\perp}^{\mu}\right) \tag{2.2}
\end{equation*}
$$

and new integration kernel

$$
\begin{equation*}
\eta\left(q_{\perp}^{\mu}\right) \equiv \int \frac{k_{T}^{2} d k_{T} d s}{(2 \pi)^{2}} V\left(k_{\perp}, q_{\perp}\right) \varphi_{P}\left(k_{\perp}^{\mu}\right) \tag{2.3}
\end{equation*}
$$

The useful notations

$$
\begin{gathered}
\omega_{i}=\sqrt{m_{i}^{2}-p_{i \perp}^{2}}, \quad \Lambda_{i}^{ \pm}\left(q_{\perp}\right)=\frac{1}{2 \omega_{i}}\left[\frac{p}{M} \omega_{i} \pm J_{i}\left(m_{i}+q_{\perp}\right)\right] \\
\varphi_{P}^{ \pm \pm}\left(q_{\perp}\right)=\Lambda_{1}^{ \pm}\left(q_{\perp}\right) \frac{p P}{M} \varphi_{P}\left(q_{\perp}\right) \frac{p p}{M} \Lambda_{2}^{ \pm}\left(q_{\perp}\right) .
\end{gathered}
$$

## II. Model introduction

- Then, BS equations can be written as

$$
\begin{equation*}
\chi(q)=S\left(p_{1}\right) \eta\left(q_{\perp}\right) S\left(-p_{2}\right) \tag{2.4}
\end{equation*}
$$

$S\left(p_{1}\right)$ and $S\left(-p_{2}\right)$ represent fermion and antifermion propagators, respectively.

$$
\begin{aligned}
-i S\left(p_{1}\right) & =\frac{i \Lambda_{1}^{+}}{q_{P}+\alpha_{1} M-\omega_{1}+i \epsilon}+\frac{i \Lambda_{1}^{-}}{q_{P}+\alpha_{1} M+\omega_{1}-i \epsilon} \\
i S\left(-p_{2}\right) & =\frac{i \Lambda_{2}^{+}}{q_{P}-\alpha_{2} M+\omega_{2}-i \epsilon}+\frac{i \Lambda_{2}^{-}}{q_{P}-\alpha_{2} M-\omega_{2}+i \epsilon}
\end{aligned}
$$

- Further carry out a contour integration for the time-component $q_{P}$ on both sides of equation (2.4), then obtain Salpeter wave function

$$
\begin{equation*}
\varphi\left(q_{\perp}\right)=\frac{\Lambda_{1}^{+}\left(q_{\perp}\right) \eta\left(q_{\perp}\right) \Lambda_{2}^{+}\left(q_{\perp}\right)}{M-\omega_{1}-\omega_{2}}-\frac{\Lambda_{1}^{-}\left(q_{\perp}\right) \eta\left(q_{\perp}\right) \Lambda_{2}^{-}\left(q_{\perp}\right)}{M+\omega_{1}+\omega_{2}} \tag{2.5}
\end{equation*}
$$

## II. Model introduction

- Salpeter equation

To apply the complete set of the projection operators $\Lambda_{i}^{ \pm}\left(q_{\perp}\right)$, then obtain the four equations

$$
\begin{gathered}
\left(M-\omega_{1}-\omega_{2}\right) \varphi_{P}^{++}\left(q_{\perp}\right)=\Lambda_{1}^{+}\left(q_{\perp}\right) \eta_{P}\left(q_{\perp}\right) \Lambda_{2}^{+}\left(q_{\perp}\right), \\
\left(M+\omega_{1}+\omega_{2}\right) \varphi_{P}^{--}\left(q_{\perp}\right)=\Lambda_{1}^{-}\left(q_{\perp}\right) \eta_{P}\left(q_{\perp}\right) \Lambda_{2}^{-}\left(q_{\perp}\right), \\
\varphi_{P}^{+-}\left(q_{\perp}\right)=\varphi_{P}^{-+}\left(q_{\perp}\right)=0 .
\end{gathered}
$$

E. E. Salpeter, PRD 87 (1952) 328.
C.-H. Chang, J.-K. Chen, and G.-L.Wang, Commun. Theor. Phys. 46, 467 (2006).

## II. Model introduction

3. ${ }^{3} P_{0}$ model

- The ${ }^{3} P_{0}$ model (quark pair creation model) is a nonrelativistic model. This model is widely used in the Okubo-Zweig-lizuka (OZI) allowed strong decays of a meson.
- In previous work, we have extended the ${ }^{3} P_{0}$ model to the relativistic case

$$
H_{I}=\left.g \int d^{3} \vec{x} \bar{\psi} \psi\right|_{\text {nonrel }} \longrightarrow H_{I}=-i g \int d^{4} x \bar{\psi} \psi,
$$

where, the input relativistic wave function $\psi$ comes from a strict solution of the Salpeter equation.
$g=2 m_{q} \gamma$ is phenomenological parameters of the model, $\gamma$ is dimensionless interaction strength.

Simonov Y A, Phys. Atom. Nucl., 2008, 71(6).
Danilkin I V, Simonov Y A, Phys. Rev. Lett., 2010, 105(10).
H.-F. Fu, G.-L. Wang, T-H. Wang, Int. J. Mod. Phys. A 27, 1250027 (2012).
T. Wang, G.-L. Wang, H.-F. Fu, , J. High Energy Phys. 07 (2013) 120.

## III. Transition amplitude and relativistic wave function

1. Amplitude and form factors

- For the two-body strong decay $A \rightarrow B+C$, the Feynman diagram as follows


Figure: The Feynman diagram for the two-body strong decay process with a ${ }^{3} P_{0}$ vertex.

- The transition matrix can be written as

$$
\begin{align*}
& \left\langle P_{1 f} P_{2 f}\right| S|P\rangle_{3_{P_{0}}}=(2 \pi)^{4} \delta^{4}\left(P-P_{1 f}-P_{2 f}\right) \mathcal{M}_{3_{P_{0}}} \\
& =-i g(2 \pi)^{4} \delta^{4}\left(P-P_{1 f}-P_{2 f}\right) \int \frac{d^{4} q}{(2 \pi)^{4}} \operatorname{Tr}\left[\chi_{P}(q) S_{2}^{-1}\left(-p_{2}\right) \bar{\chi}_{P_{2 f}}\left(q_{2 f}\right) \bar{\chi}_{P_{1 f}}\left(q_{1 f}\right) S_{1}^{-1}\left(p_{1}\right)\right] . \tag{3.1}
\end{align*}
$$

## III. Transition amplitude and relativistic wave function

- Through the Feynman rule and related calculations, the decay amplitude can be written as

$$
\begin{equation*}
\mathcal{M}_{3_{p_{0}}}=g \int \frac{d^{3} q_{\perp}}{(2 \pi)^{3}} T r\left[\frac{\phi}{M} \varphi_{p}^{++}\left(q_{\perp}\right) \frac{\mathscr{P}}{M} \bar{\varphi}_{p_{2}}^{++}\left(q_{z_{\perp}}\right) \bar{\varphi}_{p_{1,}}^{++}\left(q_{y_{1} \perp}\right)\right] . \tag{3.2}
\end{equation*}
$$

## III. Transition amplitude and relativistic wave function

- For the EM decay of $A \rightarrow B \gamma$, the Feynman diagram as follows


Figure: Feynman diagram for the transition $\chi_{P} \rightarrow \chi_{c l} \gamma$. The diagram on the left and right show photons come from the quark and the antiquark, respectively.

- For the EM decay of $A \rightarrow B \gamma$, the transition matrix can be written as

$$
\begin{equation*}
\left\langle\chi_{c J}\left(P_{f}, \epsilon_{2}\right) \gamma\left(k, \epsilon_{0}\right) \mid X\left(P, \epsilon_{1}\right)\right\rangle=(2 \pi)^{4} \delta^{4}\left(P-P_{f}-k\right) \epsilon_{0 \xi} \mathcal{M}^{\xi} \tag{3.3}
\end{equation*}
$$

## III. Transition amplitude and relativistic wave function

- Therefore, the decay amplitude can be written as

$$
\begin{align*}
\mathcal{M}^{\xi}=\int & \frac{d^{3} q_{\perp}}{(2 \pi)^{3}} \operatorname{Tr}\left[Q_{1} e \frac{\not P}{M} \bar{\varphi}_{f}^{++}\left(q_{\perp}+\alpha_{2} P_{f_{\perp}}\right) \gamma^{\xi} \varphi_{i}^{++}\left(q_{\perp}\right)\right. \\
& \left.+Q_{2} e \bar{\varphi}_{f}^{++}\left(q_{\perp}-\alpha_{1} P_{f_{\perp}}\right) \frac{p}{M} \varphi_{i}^{++}\left(q_{\perp}\right) \gamma^{\xi}\right] . \tag{3.4}
\end{align*}
$$

- Considering nonrelativistic limitations, we only calculate the decay channel $X(3842) \rightarrow \chi_{c 2} \gamma$ by $E_{1}$.


## III. Transition amplitude and relativistic wave function

2. Relativistic wave function

- For $J^{P C}=3^{--}$state, the relativistic wave function

$$
\begin{gather*}
\varphi_{3--}\left(q_{\perp}\right)=\epsilon_{\mu \nu \alpha} q_{\perp}^{\mu} q_{\perp}^{\nu}\left[q_{\perp}^{\alpha}\left(f_{1}+\frac{p}{M} f_{2}+\frac{\not q_{\perp}}{M} f_{3}+\frac{P \phi q_{\perp}}{M^{2}} f_{4}\right)\right. \\
\left.+M \gamma^{\alpha}\left(f_{5}+\frac{p}{M} f_{6}+\frac{q q_{\perp}}{M} f_{7}+\frac{P p q_{\perp}}{M^{2}} f_{8}\right)\right], \tag{3.5}
\end{gather*}
$$

where, radial wave function $f_{i}(i=1,2, \ldots 8)$ is function of $-q_{\perp}^{2}$. The relationships between them as follows

$$
\begin{array}{ll}
f_{1}=\frac{q_{\perp}^{2} f_{3}\left(\omega_{1}+\omega_{2}\right)+2 M^{2} f_{5} \omega_{2}}{M\left(m_{1} \omega_{2}+m_{2} \omega_{1}\right)}, f_{2}=\frac{q_{\perp}^{2} f_{4}\left(\omega_{1}-\omega_{2}\right)+2 M^{2} f_{6} \omega_{2}}{M\left(m_{1} \omega_{2}+m_{2} \omega_{1}\right)} \\
f_{7}=\frac{M\left(\omega_{1}-\omega_{2}\right)}{m_{1} \omega_{2}+m_{2} \omega_{1}} f_{5}, & f_{8}=\frac{M\left(\omega_{1}+\omega_{2}\right)}{m_{1} \omega_{2}+m_{2} \omega_{1}} f_{6} \tag{3.6}
\end{array}
$$

T. Wang, H.-F. Fu, Y. Jiang, Q. Li, G.-L. Wang, Int. J. Mod. Phys. A 32, 1750035 (2017).

## III. Transition amplitude and relativistic wave function

- The partial wave of $3^{--}$state

The relativistic wave function in Eq.(3.5) for $3^{--}$state is not a pure $D$ wave. In terms of spherical harmonics $Y_{l m}$, we can rewrite

$$
\begin{equation*}
\epsilon_{\mu \nu \alpha} q_{\perp}^{\mu} q_{\perp}^{\nu} q_{\perp}^{\alpha}=2 i \sqrt{\frac{6 \pi}{35}}|\vec{q}|^{3}\left(Y_{s 2}-Y_{3-2}\right), \tag{3.7}
\end{equation*}
$$

so $f_{1}$ and $f_{2}$ terms are $F$ wave.
Similarly, for the $f_{3}$ and $f_{4}$ terms,

$$
\begin{align*}
\epsilon_{\mu \nu \alpha} q_{\perp}^{\mu} q_{\perp}^{\nu} q_{\perp}^{\alpha} q_{\perp}= & i|\vec{q}|^{4}\left[\frac{4}{7} \sqrt{\frac{3 \pi}{5}}\left(-Y_{21} \gamma^{+}+Y_{2-1} \gamma^{-}\right)+\frac{2}{7} \sqrt{\frac{6 \pi}{5}}\left(-Y_{22}+Y_{2-2}\right) \gamma^{\Delta}\right. \\
& +\frac{2}{7} \sqrt{\frac{2 \pi}{5}}\left(Y_{41} \gamma^{+}-Y_{4-1} \gamma^{-}\right)+\frac{4}{7} \sqrt{\frac{2 \pi}{5}}\left(-Y_{42}+Y_{4-2}\right) \gamma^{\Delta} \\
& \left.+2 \sqrt{\frac{2 \pi}{35}}\left(Y_{43} \gamma^{-}-Y_{4-3} \gamma^{+}\right)\right] \tag{3.8}
\end{align*}
$$

where, $\gamma^{+}=-\frac{\gamma^{1}+i \gamma^{2}}{\sqrt{2}}, \gamma^{-}=\frac{\gamma^{1}-i \gamma^{2}}{\sqrt{2}}$ and $\gamma^{\Delta}=\gamma^{3}$. So $f_{3}$ and $f_{4}$ terms include $D$ wave and $G$ wave, they are $D-G$ mixing.

## III. Transition amplitude and relativistic wave function

- The partial wave of $3^{--}$state

The $f_{5}$ and $f_{6}$ terms are $D$ wave, because,

$$
\begin{equation*}
\epsilon_{\mu \nu \alpha} q_{\perp}^{\mu} q_{\perp}^{\nu} \gamma_{\perp}^{\alpha}=2 i \sqrt{\frac{2 \pi}{15}}|\vec{q}|^{2}\left[\sqrt{2}\left(Y_{21} \gamma^{+}-Y_{2-1} \gamma^{-}\right)+\left(Y_{Y_{22}}-Y_{2-2}\right) \gamma^{\Delta}\right] . \tag{3.9}
\end{equation*}
$$

The $f_{7}$ and $f_{8}$ terms are $F$ wave, since

$$
\begin{gather*}
\epsilon_{\mu \nu \alpha} q_{\perp}^{\mu} q_{\perp}^{\nu} \gamma_{\perp}^{\alpha} \phi_{\perp}=2 i|\vec{q}|^{3}\left[\frac{2}{5} \sqrt{\frac{\pi}{7}} Y_{30}\left(\gamma^{+2}-\gamma^{-2}\right)+\frac{1}{5} \sqrt{\frac{6 \pi}{7}}\left(-Y_{31} \gamma^{+}+Y_{3-1} \gamma^{-}\right) \gamma^{\Delta}\right. \\
\left.\quad+\sqrt{\frac{2 \pi}{105}}\left(Y_{32}-Y_{3-2}\right)\left(2 \gamma^{+} \gamma^{-}-\gamma^{\Delta 2}\right)+\sqrt{\frac{2 \pi}{35}}\left(Y_{33} \gamma^{-}-Y_{3-3} \gamma^{+}\right) \gamma^{\Delta}\right] \tag{3.10}
\end{gather*}
$$

Then the complete $D$ wave in Eq.(3.5) is

$$
\epsilon_{\mu \nu \alpha} q_{\perp}^{\mu} q_{\perp}^{\nu} M \gamma^{\alpha}\left(f_{5}+\frac{\not P}{M} f_{6}\right)-\frac{3}{7} \epsilon_{\mu \nu \alpha} q_{\perp}^{\mu} q_{\perp}^{\nu} M \gamma^{\alpha} \frac{|\vec{q}|^{2}}{M^{2}}\left(f_{3}-\frac{\not p}{M} f_{4}\right)
$$

The pure $G$ wave in Eq.(3.5) is

$$
\epsilon_{\mu \nu \alpha} q_{\perp}^{\mu} q_{\perp}^{\nu} q_{\perp}^{\alpha}\left(\frac{q_{\perp}}{M} f_{3}+\frac{\not p q_{\perp}}{M^{2}} f_{4}\right)+\frac{3}{7} \epsilon_{\mu \nu \alpha} q_{\perp}^{\mu} q_{\perp}^{\nu} M \gamma^{\alpha} \frac{|\vec{q}|^{2}}{M^{2}}\left(f_{3}-\frac{p}{M} f_{4}\right) .
$$

## III. Transition amplitude and relativistic wave function

- For $J^{P C}=0^{-}$state, the relativistic wave function

$$
\begin{equation*}
\varphi_{0^{-}}\left(q_{f_{\perp}}\right)=M_{f}\left(\frac{P_{f}}{M_{f}} g_{1}+g_{2}+\frac{q_{f_{\perp}}}{M_{f}} g_{3}+\frac{P_{f} \phi_{f_{\perp}}}{M_{f}^{2}} g_{4}\right) \gamma^{5}, \tag{3.11}
\end{equation*}
$$

The relationships between $g_{i}$ ss follows

$$
\begin{equation*}
g_{3}=\frac{g_{2} M_{f}\left(\omega_{2 f}-\omega_{1,}\right)}{\left(m_{1,} \omega_{2 f}+m_{2 f} \omega_{1,}\right)}, \quad g_{4}=-\frac{g_{1} M_{f}\left(\omega_{2 f}+\omega_{1,}\right)}{\left(m_{1,} \omega_{2 f}+m_{2 f} \omega_{1, f}\right)} . \tag{3.12}
\end{equation*}
$$

Similarly, the relativistic $0^{-}$state wave function is not a pure $S$ wave (the terms including $g_{1}$ and $g_{2}$ ), but also contains a small component of $P$ wave (terms including $g_{3}$ and $g_{4}$ ).

## III. Transition amplitude and relativistic wave function

- For $J^{P C}=2^{++}$state, the relativistic wave function

$$
\begin{align*}
\varphi_{2++}\left(q_{f_{\perp}}\right)= & \epsilon_{\mu \nu} q_{f \perp}^{\mu}\left[q_{f \perp}^{\nu}\left(h_{1}+\frac{P_{f}}{M_{f}} h_{2}+\frac{q_{f_{\perp}}}{M_{f}} h_{3}+\frac{P_{f} q_{f_{\perp}}}{M_{f}^{2}} h_{4}\right)\right. \\
& \left.+M_{f} \gamma^{\nu}\left(h_{5}+\frac{P_{f}}{M_{f}} h_{6}+\frac{q_{f_{\perp}}}{M_{f}} h_{\gamma}\right)+\frac{i}{M_{f}} h_{8} \epsilon^{\mu \alpha \beta \gamma} P_{f_{\alpha}} q_{f \perp \beta} \gamma_{\gamma} \gamma_{5}\right] . \tag{3.13}
\end{align*}
$$

The relationships between $h_{i} s$ as follows

$$
\begin{gather*}
h_{1}=\frac{\left(q_{f}^{2} h_{3}+M_{f}^{2} h_{5}\right)\left(\omega_{1_{f}}+\omega_{2_{f}}\right)-M_{f}^{2} h_{5}\left(\omega_{1_{f}}-\omega_{2_{f}}\right)}{M_{f}\left(m_{1_{f}} \omega_{2_{f}}+m_{2_{f}} \omega_{1_{f}}\right)}, h_{7}=\frac{M_{f}\left(\omega_{1_{f}}-\omega_{2_{f}}\right)}{m_{1_{f}} \omega_{2_{f}}+m_{2_{f}} \omega_{1_{f}}} h_{5}, \\
h_{2}=\frac{\left(q_{f \perp}^{2} h_{4}-M_{f}^{2} h_{6}\right)\left(\omega_{1_{f}}-\omega_{2_{f}}\right)}{M_{f}\left(m_{1_{f}} \omega_{2_{f}}+m_{2_{f}} \omega_{1_{f}}\right)}, \quad h_{8}=\frac{M_{f}\left(\omega_{1_{f}}+\omega_{2_{f}}\right)}{m_{1_{f}} \omega_{2_{f}}+m_{2_{f}} \omega_{1_{f}}} h_{6} . \tag{3.14}
\end{gather*}
$$

In Eq. (3.13), the terms including $h_{5}$ and $h_{6}$ are $P$ waves, those including $h_{3}$ and $h_{4}$ are $P-F$ mixing waves, others are $D$ waves.

## III.Transition amplitude and relativistic wave function

- The form factors expression of amplitude Integrate internal momentum $q_{\perp}$ over the initial and final state wave functions, and finishing the trace, then we obtain the decay amplitude described using form factor.
(1) For the strong decay $X(3842) \rightarrow D \bar{D}$

$$
\begin{equation*}
\mathcal{M}(X(3842) \rightarrow D \bar{D})=\epsilon_{\mu \nu \alpha} P_{f}^{\mu} P_{f}^{\nu} P_{f}^{\alpha} t_{1} \tag{3.15}
\end{equation*}
$$

where $t_{1}$ is the form factor.
(2) For the EM decay $X(3842) \rightarrow \chi_{c 2}\left({ }^{3} P_{2}\right) \gamma$,

$$
\begin{align*}
& \mathcal{M}^{\xi}\left(X(3842) \rightarrow \chi_{c 2}(1 P) \gamma\right)=P^{\xi} \epsilon_{P_{f} \rho_{f} P_{f}} \epsilon_{f, p P} s_{1}++P^{\xi} \epsilon_{\rho P_{f} P_{f}} \epsilon_{f, p}^{\rho} s_{2}+\epsilon_{P_{f} p_{f}}^{\xi} \epsilon_{f, P P} s_{3} \\
& +\epsilon_{P_{f} \rho_{f} \rho_{f}} \epsilon_{f, P}^{\xi} s_{4}+P^{\xi} \epsilon_{\rho \sigma \rho_{f}} \epsilon_{f}^{\rho \sigma} s_{5}+\epsilon_{\rho p_{f}}^{\xi} \epsilon_{f, p}^{\rho} s_{6} \\
& +\epsilon_{\rho_{f} \rho_{f} \rho_{f}} \epsilon_{f}^{\rho \rho} s_{7}+\epsilon_{\rho \sigma}^{\xi} \epsilon_{f}^{\rho \sigma} s_{8}, \tag{3.16}
\end{align*}
$$

For the EM decay, not all the form factors are independent, due to the Ward identity $\left(P_{\xi}-P_{f, \xi}\right) \mathcal{M}^{\xi}=0$, we have the following relations

$$
s_{3}=\left(M^{2}-M E_{f}\right) s_{1}+s_{4}, s_{6}=\left(M^{2}-M E_{f}\right) s_{2}+s_{7}, s_{8}=\left(M^{2}-M E_{f}\right) s_{5} .
$$

## IV. Results and discussions

1. Strong decay widths

- Strong decay widths of $X(3842)$ as the $\psi_{3}\left(1^{3} D_{3}\right)$ state

The strong decay width of $X(3842)$ decays to $D^{0} \bar{D}^{0}$ and $D^{+} D^{-}$are calculated as

$$
\begin{equation*}
\Gamma\left[X(3842) \rightarrow D^{0} \bar{D}^{0}\right]=1.27 \mathrm{MeV}, \quad \Gamma\left[X(3842) \rightarrow D^{+} D^{-}\right]=1.08 \mathrm{MeV} \tag{4.1}
\end{equation*}
$$

TABLE I: The strong decay widths $(\mathrm{MeV})$ of the $X(3842) \rightarrow D D$, and the ratio of $\frac{\mathcal{B}\left[X(3842) \rightarrow D^{+} D^{-}\right]}{\mathcal{B}\left[X(3842) \rightarrow D^{0} D^{0}\right]}$.

|  | $[15]$ | $[16]$ | $[17]$ | $[23]$ | $[56]$ | Ours | $\mathrm{EX}[13]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $M_{\left(1^{3} D_{3}\right)}(\mathrm{MeV})$ | 3868 | 3849 | 3872 | 3806 | $3762-3912$ | 3872 | 3902 |
| $\Gamma\left(X(3842) \rightarrow D^{+} D^{-}\right)$ |  |  |  | $2-3$ | 0.39 | 0.72 | 1.08 |
| $\Gamma\left(X(3842) \rightarrow D^{0} \bar{D}^{0}\right)$ |  |  |  |  | $2.5-3.5$ | 0.47 | 0.84 |
| $\Gamma(X(3842) \rightarrow D D)$ | 1.27 |  |  |  |  |  |  |
| $\frac{\Gamma\left(X(3842) \rightarrow D^{+} D^{-}\right)}{\Gamma\left(X(3842) \rightarrow D^{0} \bar{D}^{0}\right)}$ | 0.82 | 2.27 | 4.04 | 0.5 | $4.5-6.5$ | 0.86 | 1.56 |

## IV. Results and discussions

## 2. EM decay width

- EM decay width of $X(3842)$ as $1^{3} D_{3}$ state
$\psi_{3}\left(1^{3} D_{3}\right) \rightarrow \chi_{c 2}(1 P) \gamma$ is dominant in the EM decays of $\psi_{3}\left(1^{3} D_{3}\right)$, and the result is

$$
\begin{equation*}
\Gamma\left[X(3842) \rightarrow \chi_{c 2}(1 P) \gamma\right]=288 \mathrm{keV} \tag{4.2}
\end{equation*}
$$

TABLE II: The radiative partial decay widths (keV) of the $X_{\left(1^{3} D_{3}\right)}(3842) \rightarrow \chi_{c 2} \gamma$.

|  | [17] |  | [22] |  |  | [56] |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Model | $N R$ | GI | $N R$ | $V S$ |  |  | $S C^{3}$ |  |  | $C^{3}$ |  |
| $M_{\left(1^{3} D_{3}\right)}(\mathrm{MeV})$ | 3806 | 3849 |  | 3815 |  | 3815 | 3868 | 3972 | 3815 | 3868 | 3972 |
| $\Gamma\left(X(3842) \rightarrow \chi_{c 2} \gamma\right)$ | 272 | 296 | 252 | 63170 | 156 | 199 | 329 | 341 | 179 | 286 | 299 |


|  | $[57]$ |  | $[58]$ |  | $[59]$ |  | $[60]$ | Ours |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Model | $N R$ | $S N_{0}$ | $S N_{1}$ | $N R$ | $M N R$ |  | $C C P_{v}$ | $N R$ | $R E$ | $B S$ |
| $M_{\left(1^{3} D_{3}\right)}(\mathrm{MeV})$ | 3799 |  | 3815 | 3813 | 3520 | 3653 | 3831 | 3805 | 3842.7 |  |
| $\Gamma\left(X(3842) \rightarrow \chi_{c 2} \gamma\right)$ | 272 | 284 | 223 | 340 | 302 | 138 | 246 | 432 | 271 | 298 |

## IV. Results and discussions

## 3. Contributions of different partial waves of EM decay

- For $\psi_{3}(1 D) \rightarrow \chi_{c_{2}} \gamma$, for the initial state, compared to $F$ and $G$ waves, the $D$ wave have the dominant contribution. And the main contribution of the final state comes from the $P$ wave which provides the nonrelativistic result, and the relativistic correction ( $D$ and $F$ wave in $2^{++}$state) contribute relatively small.

TABLE III: The EM decay width (keV) of different partial waves for $X_{\left(1^{3} D_{3}\right)}(3842) \rightarrow \chi_{c 2}(1 P) \gamma$.

| $2^{++}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $3^{--}$ | complete | $P$ wave | $D$ wave | $F$ wave |
| complete | 288 | 215 | 18.2 | 0.197 |
| $D$ wave | 234 | 232 | 13.9 | 0.186 |
| $F$ wave | 13.7 | 13.4 | 0.180 | 0.0187 |
| $G$ wave | 0.540 | 0.245 | 0.00373 | 0.000358 |

## IV. Results and discussions

## 4. Discussions

- We have estimated total decay width of $X(3842)$ can be estimated as.

$$
\Gamma[X(3842)] \approx \Gamma(D D)+\Gamma\left(\chi_{c 2} \gamma\right)+\Gamma(J / \psi \pi \pi)+\Gamma(g g g)+\Gamma(g g \gamma)=2.87 \mathrm{MeV}
$$

This result is in good agreement with the experimental data $2.79_{-0.86}^{+0.86} \mathrm{MeV}$.

- For the EM decay of $\psi_{3}(1 D) \rightarrow \chi_{c 2} \gamma$, using the non-relativistic result 232 keV , and the relativistic one 288 keV , we obtain the relativistic effect is $19 \%$.


## V. Summary

- We studied the decays of the new particle $X(3842)$ as $\psi_{3}\left(1^{3} D_{3}\right)$ by using the relativistic Bethe-Salpeter equation method and ${ }^{3} P_{0}$ model. And estimated total decay width of $X(3842)$ is 2.87 MeV .
- Our results show that
$\Gamma\left[X(3842) \rightarrow D^{0} \bar{D}^{0}\right]=1.27 \mathrm{MeV}, \quad \Gamma\left[X(3842) \rightarrow D^{+} D^{-}\right]=1.08 \mathrm{MeV}$, this is the dominant strong decay channel. The decay ratio $\frac{\mathcal{B}\left[X(3842) \rightarrow D^{+} D^{-}\right]}{\mathcal{B}\left[X(3842) \rightarrow D^{0} D^{0}\right]}=0.84$, this indicating that the difference between the decay widths of $X(3842) \rightarrow D^{0} \bar{D}^{0}$ and $X(3842) \rightarrow D^{+} D^{-}$is almost purely from the phase space difference.
- Decay width of the dominant EM decay channel
$\Gamma\left[X(3842) \rightarrow \chi_{c 2}(1 P) \gamma\right]=288 \mathrm{keV}$. In addition, we calculated the contributions of different partial waves and obtained the relativistic effect $19 \%$.


## Thanks for your attention!

