Fragmentation functions for doubly heavy mesons

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In collaboration with Chao-Hsi Chang and Xing-Gang Wu

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Outline

1. Background

2. Fragmentation functions at NLO

3. Applications

4. Summary

1. Background

Top quark cannot form a hadron

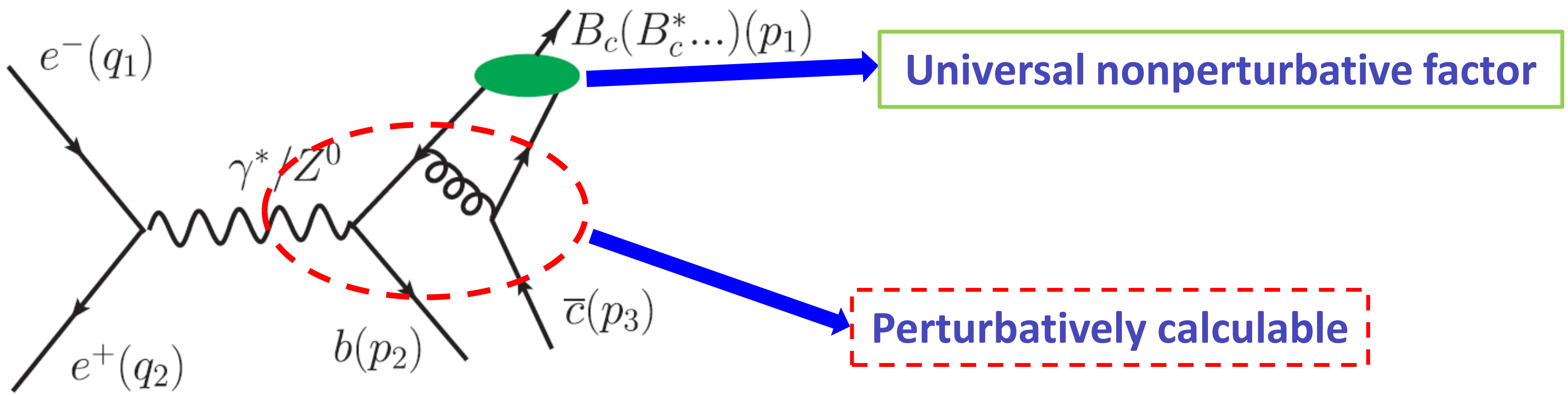
Doubly heavy mesons：
$c\bar{c}$, $b\bar{b}$ (J/ψ, Υ, etc.), $c\bar{b}$(B_c, B_c^*, etc.)

- **Production： perturbative, non-perturbative QCD**

  To test pQCD, NRQCD

- **Decay： weak interaction**

- NRQCD factorization



$$d\sigma(e^+ + e^- \to Bc + b + \bar{c})$$
$$= \sum_n d\hat{\sigma}(e^+ + e^- \to c\bar{b}[n] + b + \bar{c}) \langle O^{Bc}(n) \rangle$$

NRQCD factorization

Short-distance coefficients

Long-distance matrix elements

- NRQCD factorization

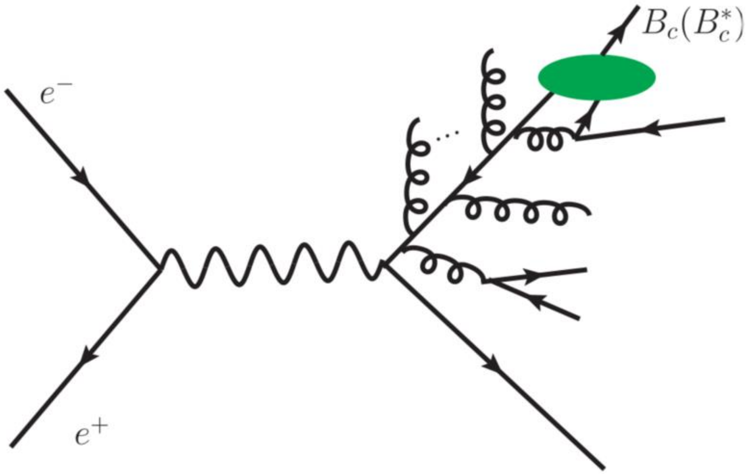
$$d\sigma(e^+ + e^- \to Bc + b + \bar{c})$$
$$= \sum_n d\sigma(e^+ + e^- \to (c\bar{b})[n] + b + \bar{c})\langle O^{Bc}(n)\rangle$$

Energy scales: $\sqrt{s}, m_Q$

Log-terms appear in short-distance coefficients:

$$\alpha_s^m \sum_{n=0}^{\infty} \alpha_s^n \ln^n(s/m_Q^2)$$

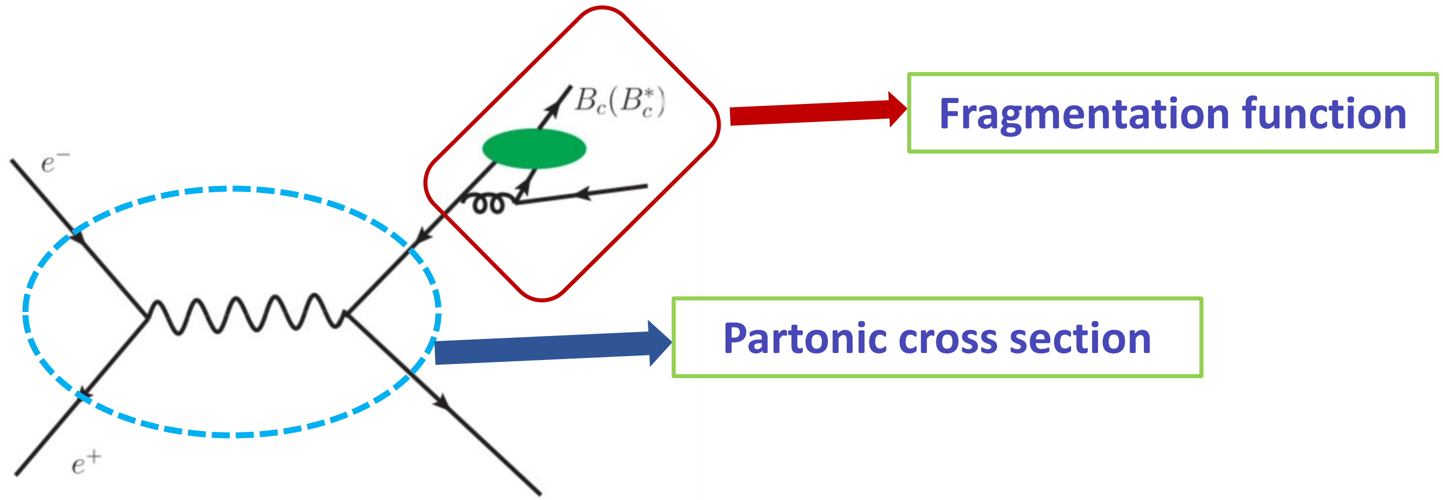
Collinear emission



Spoil or weak the convergence of the series

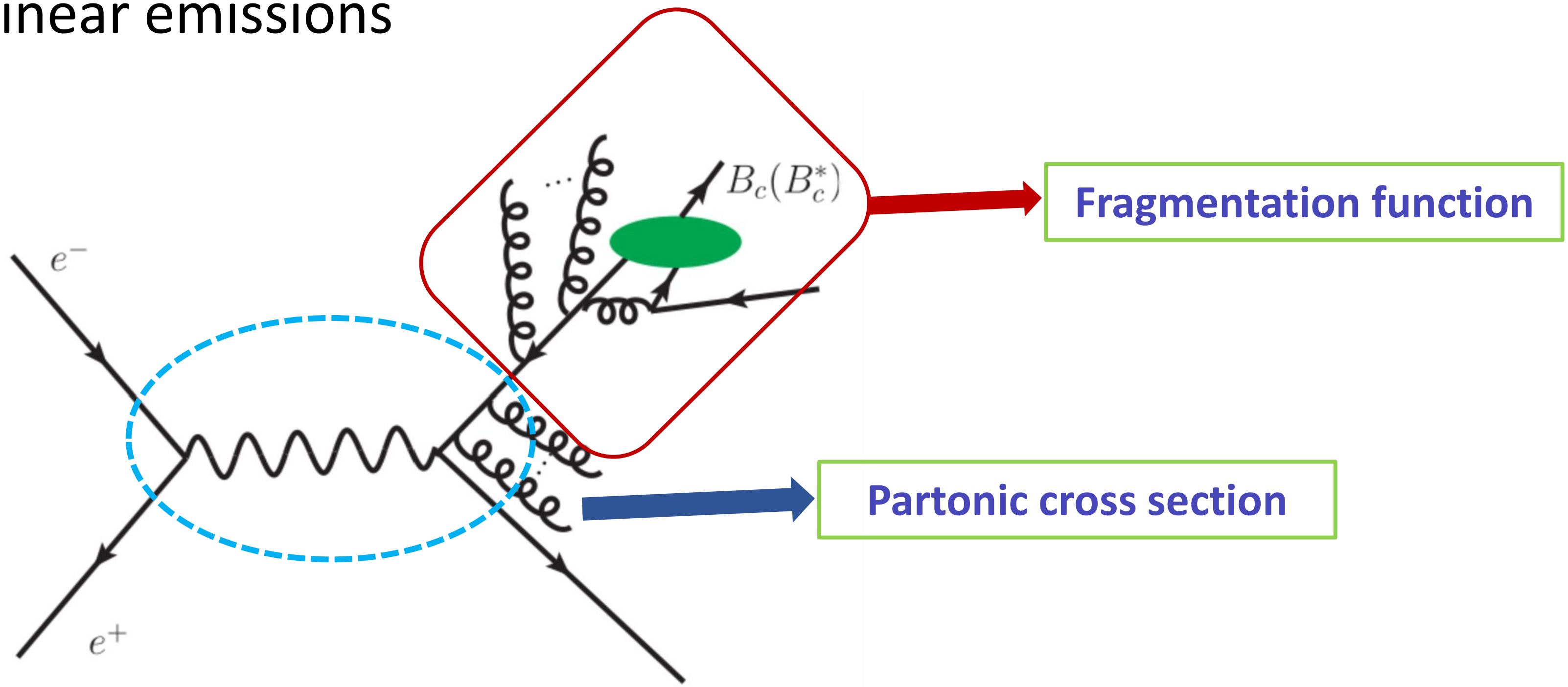
$\ln(p_t^2/m_Q^2)$ appearing in the production at a hadron collider

- Fragmentation-function approach



$$d\sigma(e^+ + e^- \to Bc(p) + b + \bar{c})$$
$$= \sum_i d\hat{\sigma}(e^+ + e^- \to i + X)(p/z, \mu_F) \otimes D_{i\to Bc}(z, \mu_F) + O(m_Q^2/s)$$

Collinear emissions



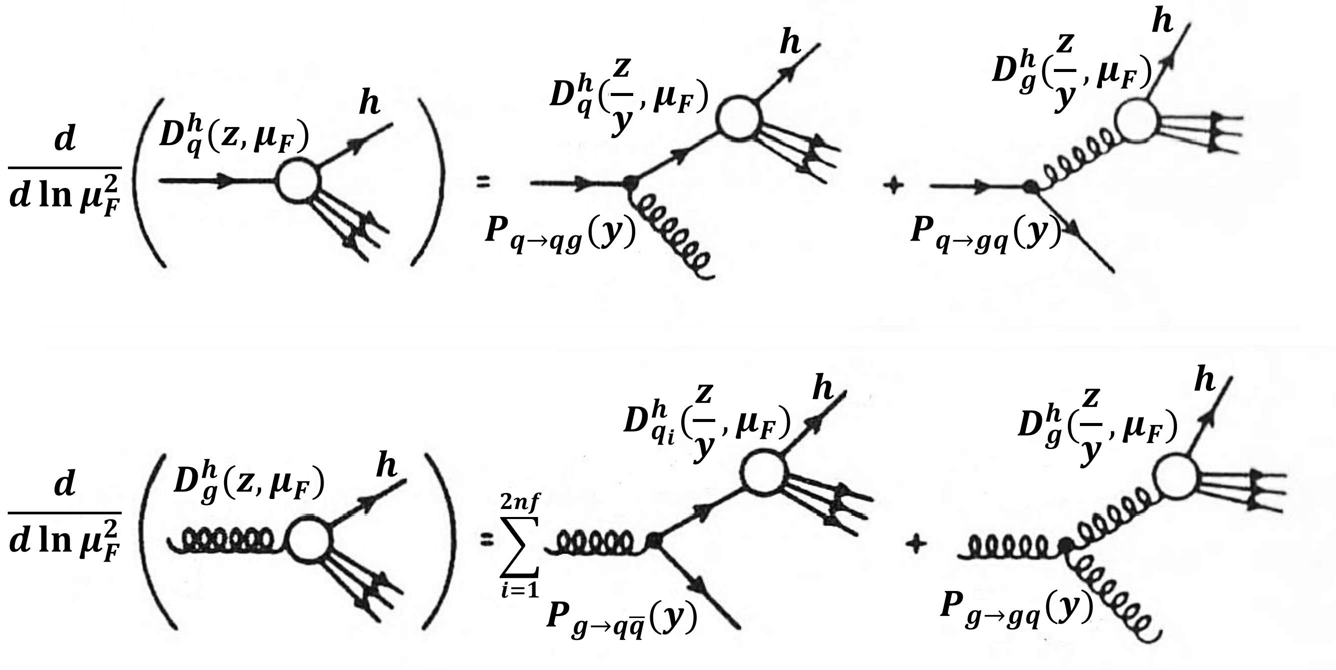
Large Logrithms of s/m_Q^2?

Involving $\ln(s/\mu_F^2)$ $\mu_F = O(\sqrt{s})$

$$d\sigma(e^+ + e^- \to Bc(p) + b + \bar{c})$$
$$= \sum_i d\hat{\sigma}(e^+ + e^- \to i + X)(p/z, \mu_F) \otimes D_{i\to Bc}(z, \mu_F) + O(m_Q^2/s)$$

Involving $\ln(\mu_F^2/m_Q^2)$

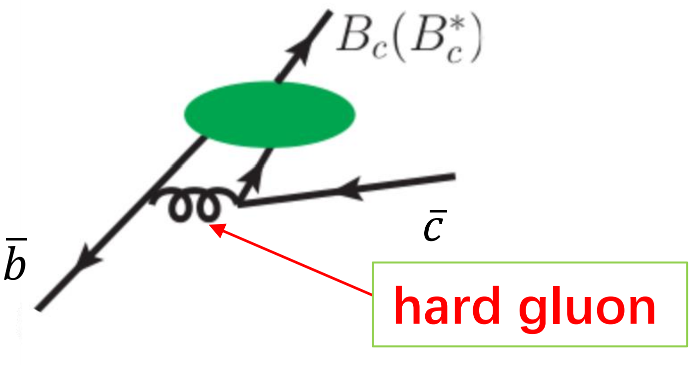
DGLAP evolution



Collinear log-terms can be resummed through the DGLAP evolution.

Boundary condition: $D_i^h(z,\mu_{F0})$ with $\mu_{F0} = O(m_Q)$

NRQCD factorization for FFs:



The fragmentation process contains perturbatively calculable information

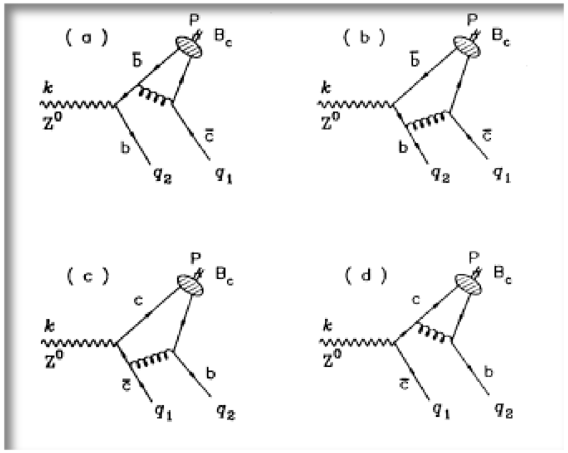
$$D_{i\to Bc}(z,\mu_{F0})=\sum_n d_{i\to c\bar{b}[n]}(z,\mu_{F0})\langle O^{\mathrm{Bc}}(n)\rangle$$

Involving $\ln(\mu_{F0}^2/m_Q^2)$ $\mu_{F0}=O(m_Q)$

The FFs at a higher scale can be obtained by solving the DGLAP equations

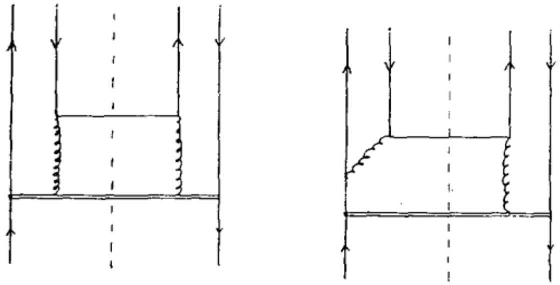
➢ LO fragmentation functions for the Bc production

- Extracting from the LO calculation of process $Z^0 \rightarrow Bc + b + \bar{c}$



C.-H. Chang, Y.-Q. Chen, PRD 46, 3845, (1992);
E. Braaten et al, PRD 48, R5049, (1993).

- Calculating from the definition:



J.-P. Ma, PLB 332, 398, (1994).

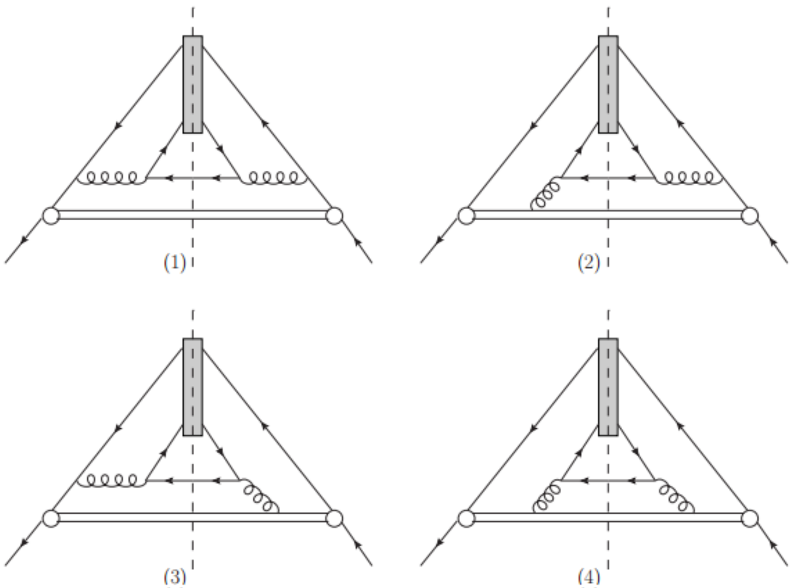
2. FFs at NLO

Based on the definition of FFs by **Collins and Soper.**

Nucl. Phys. B 194, 445, (1982).

Process independent approach

LO cut diagrams:



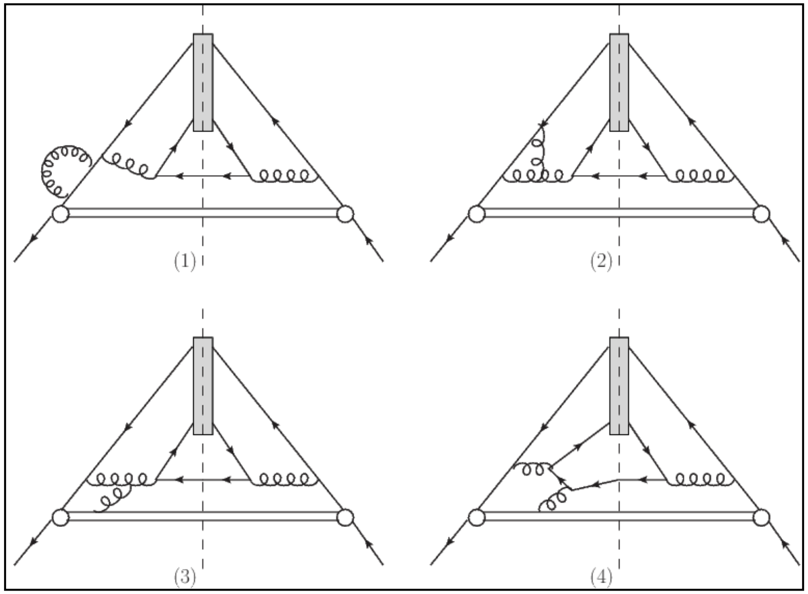
LO fragmentation functions:

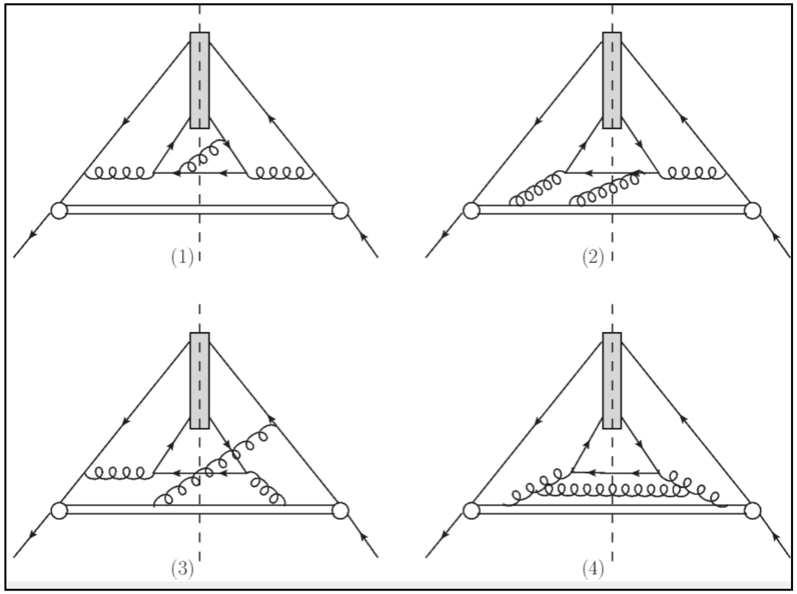
$$D^{\rm LO}_{\bar{b}\to B_c}(z) = \frac{2\alpha_s^2 z(1-z)^2|R_S(0)|^2}{81\pi r_c^2(1-r_b z)^6 M^3}[6-18(1-2r_c)z+(21-74r_c+68r_c^2)z^2 -2r_b(6-19r_c+18r_c^2)z^3+3r_b^2(1-2r_c+2r_c^2)z^4],$$

$$D^{\rm LO}_{\bar{b}\to B_c^*}(z) = \frac{2\alpha_s^2 z(1-z)^2|R_S(0)|^2}{27\pi r_c^2(1-r_b z)^6 M^3}[2-2(3-2r_c)z+3(3-2r_c+4r_c^2)z^2 -2r_b(4-r_c+2r_c^2)z^3+r_b^2(3-2r_c+2r_c^2)z^4].$$

NLO corrections

Sample NLO cut diagrams





54 virtual cut diagrams， 72 real cut diagrams.

- **Virtual corrections**

Loop-integral reduction

Many integrals containing an eikonal line, e.g,

$$\int \frac{d^D l}{[(l-p_1)^2-m_1^2+i\varepsilon][(l-p_2)^2-m_2^2+i\varepsilon][(l-p_3)^2-m_3^2+i\varepsilon](l\cdot n+i\varepsilon)}$$

- **Real corrections**

UV and IR divergences!

$$\mathrm{D}_{\bar{b}\to Bc}^{real}(z)=\int N_{CS}d\phi_{real}(A_{real}-A_S)+\int N_{CS}d\phi_{real}A_S$$

Calculated in 4 dimensions

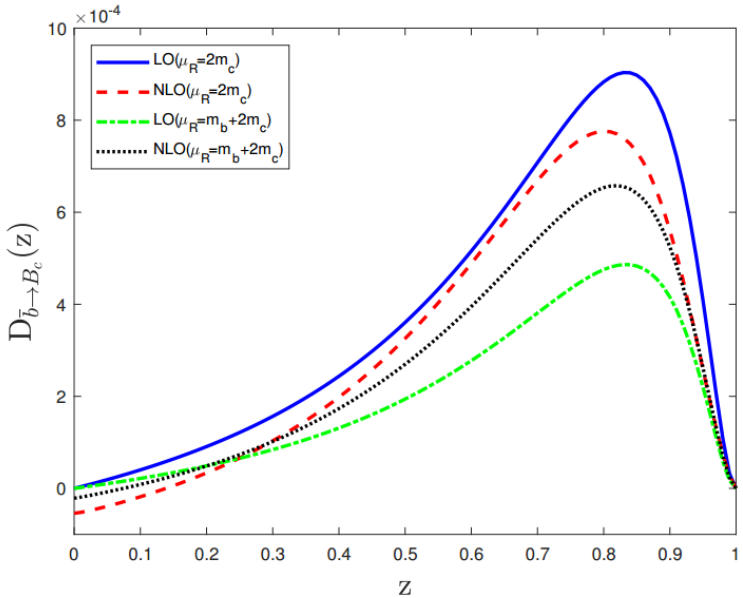
Calculated in d dimensions

Various types of subtraction terms need to be integrated!

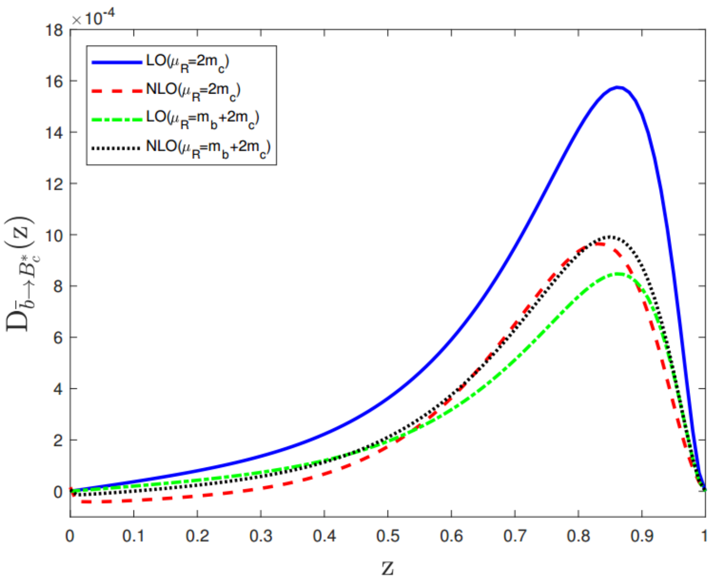
NLO results

PRD 100, 034004, (2019),
X.-C. Zheng, C.-H. Chang, T.-F Feng, X.-G. Wu.

NLO fragmentation functions for $\bar{b} \to B_c$ and $\bar{b} \to B_c^*$

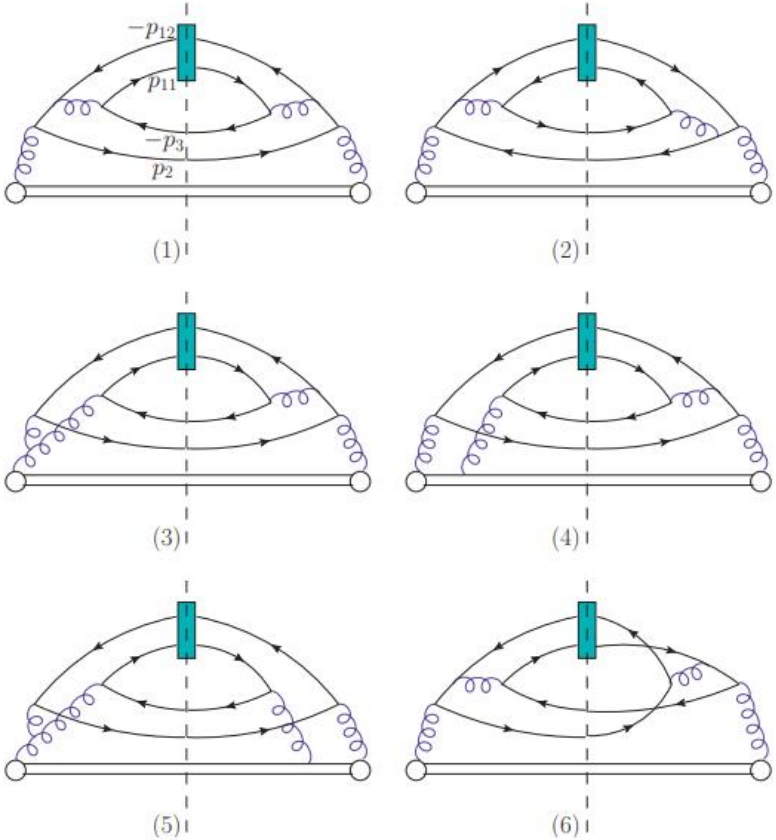


$$D_{\bar{b}\to Bc}(z,\mu_F = m_b + 2m_c)$$



$$D_{\bar{b}\to B_c^*}(z,\mu_F = m_b + 2m_c)$$

g->Bc(Bc*) FFs

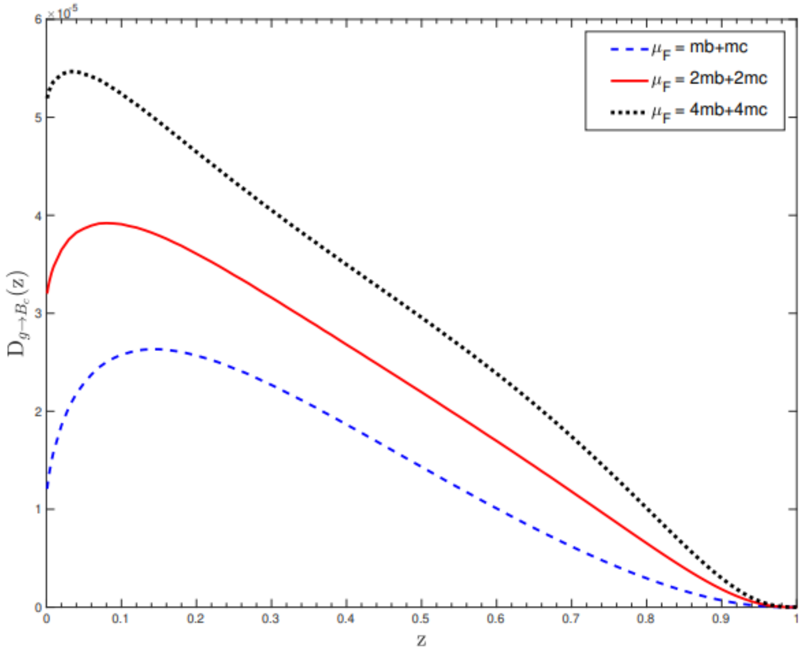


Six of the 49 cut diagrams

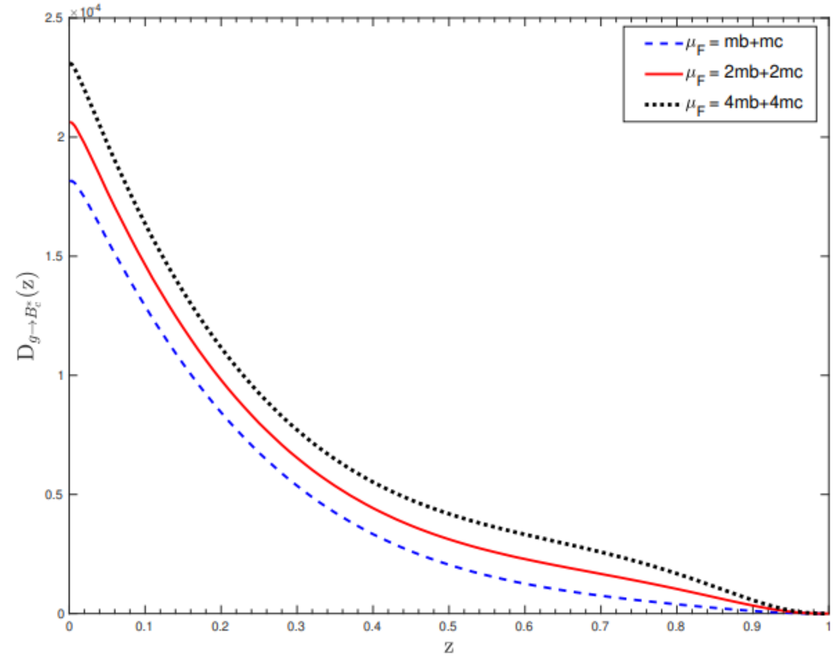
末态有三个粒子，且相空间积分存在紫外发散。

g->Bc(Bc*) FFs

JHEP 05, 036, (2022),
X.-C. Zheng, C.-H. Chang, X.-G. Wu.



g->Bc FFs



g->Bc* FFs

FFs at NLO

PHYSICAL REVIEW D **106,** 054030 (2022)

Gluon fragmentation into $B_c^{(*)}$ in NRQCD factorization

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[3]School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

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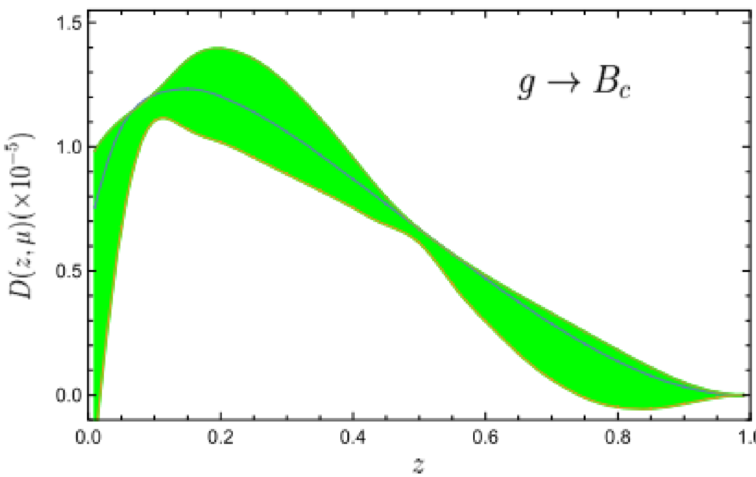
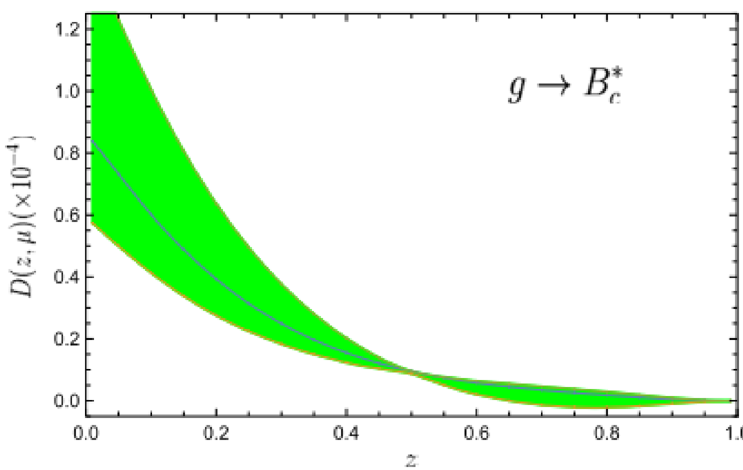
The universal fragmentation functions of gluon into the flavored quarkonia B_c and (polarized) B_c^* are computed within NRQCD factorization framework at the lowest order in velocity expansion and strong coupling constant. It is mandatory to invoke the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi renormalization program to render the NRQCD short-distance coefficients UV finite in a pointwise manner. The calculation is facilitated with the sector decomposition method, with the final results presented with high numerical accuracy. This knowledge is useful to enrich our understanding toward the large-p_T behavior of $B_c^{(*)}$ production at LHC experiment.

DOI: 10.1103/PhysRevD.106.054030

$c_1^{Bc}(z)$	z=0.1	Z=0.2	Z=0.5	Z=0.8	Z=0.9
Zheng et al	0.2323	0.2311	0.1282	0.02612	0.006144
Feng et al	0.2324	0.2311	0.1282	0.02612	0.006143

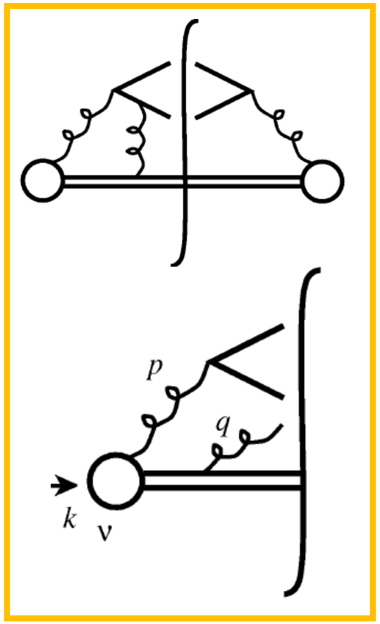
$c_1^{Bc^*}(z)$	z=0.1	Z=0.2	Z=0.5	Z=0.8	Z=0.9
Zheng et al	1.155	0.7554	0.1822	0.03412	0.009589
Feng et al	1.155	0.7550	0.1822	0.03411	0.009586

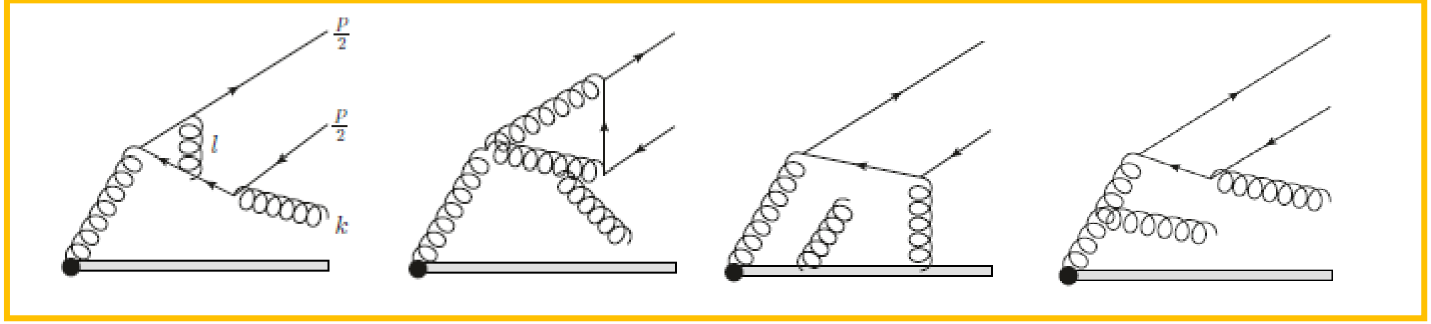




➤ Quarkonium FFs at NLO

- E. Braaten et al, NPB 586, 427, (2000) , $g \to Q\bar{Q}(\,^3S_1^{[8]})$
- E. Braaten et al, JHEP 04, 121, (2015) , $g \to Q\bar{Q}(\,^1S_0^{[1]})$
- E. Braaten et al, JHEP 01, 227, (2019) , $g \to Q\bar{Q}(\,^1S_0^{[8]})$
- Y. Jia et al, arXiv:1810.04138(2018) , $g \to Q\bar{Q}(\,^1S_0^{[1,8]})$
- Y. –Q. Ma et al, JHEP 04, 116, (2019) , $g \to Q\bar{Q}(\,^1S_0^{[1,8]})$
- Y. –Q. Ma et al, JHEP 08, 111, (2021) , $g \to Q\bar{Q}(\,^3P_J^{[1,8]})$



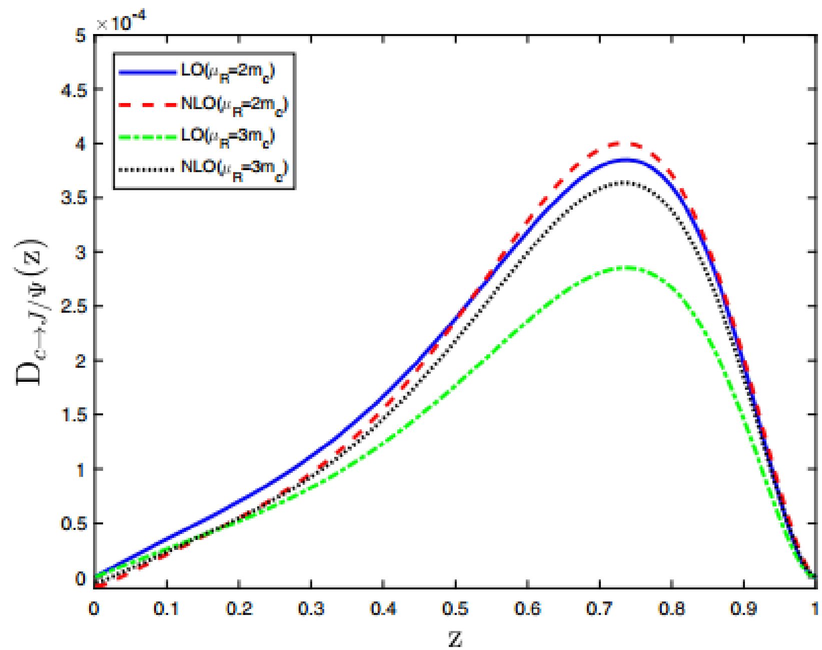


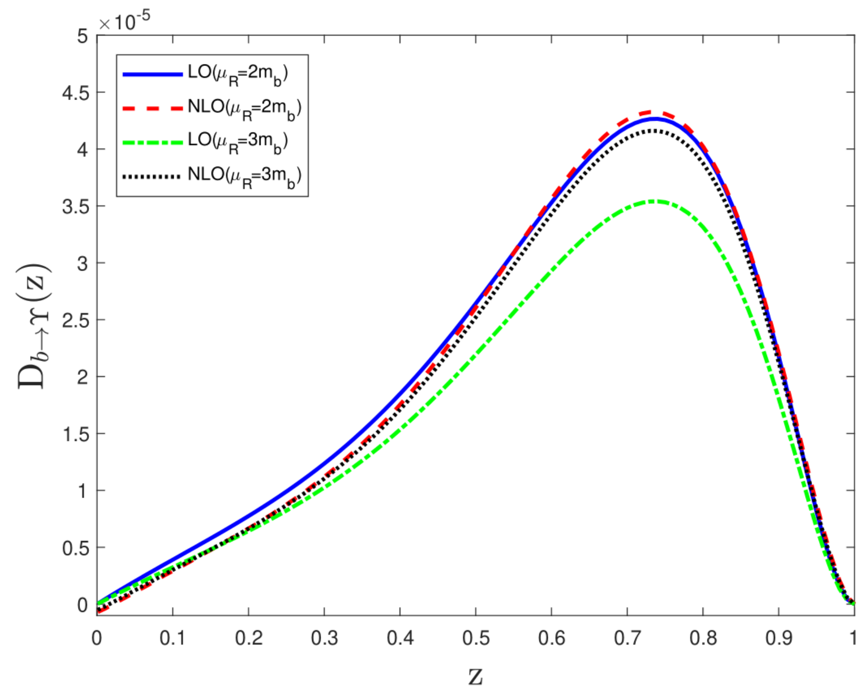
Several typical diagrams

Quarkonium FFs

PRD 100, 014005, (2019),
X.-C. Zheng, C.-H. Chang, X.-G. Wu.

NLO fragmentation functions for $c \to J/\psi$ and $b \to \Upsilon$





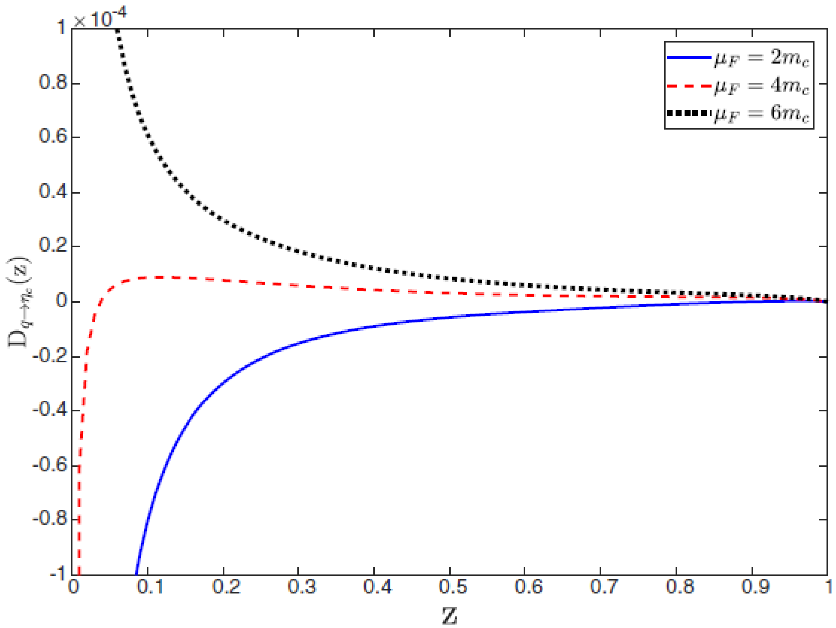
$$D_{c\to J/\psi}(z,\mu_F=3m_c)$$

$$D_{b\to\Upsilon}(z,\mu_F=3m_b)$$

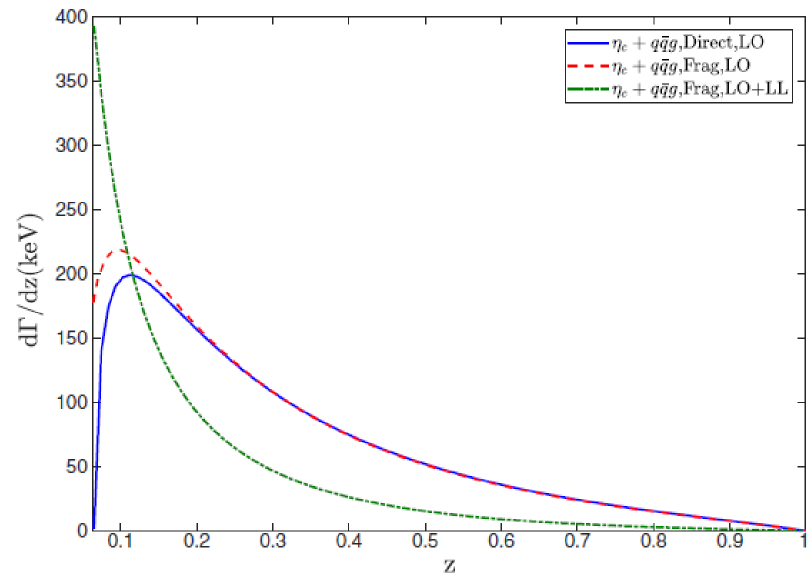
Quarkonium FFs

PRD 103, 074004, (2021),
X.-C. Zheng, Z.-Y. Zhang, X.-G. Wu.

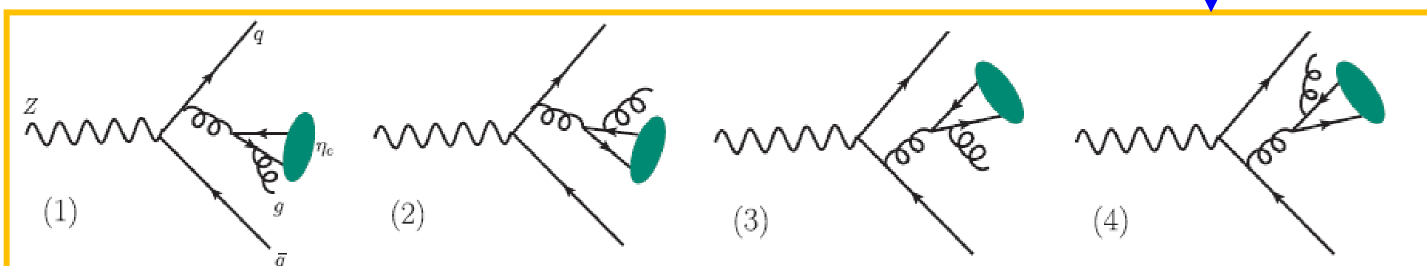
Fragmentation functions for $q \to \eta_Q (q \neq Q)$



$D_{q\to\eta_c}(z,\mu_F)$



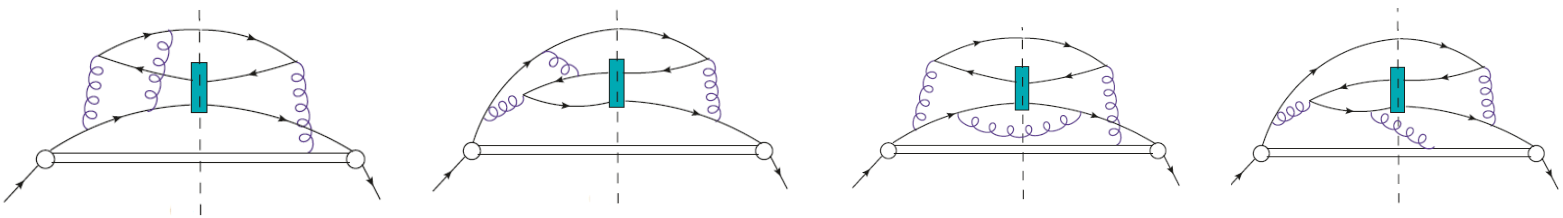
$d\Gamma/dz$ for $Z \to \eta_c + q\bar{q}g$

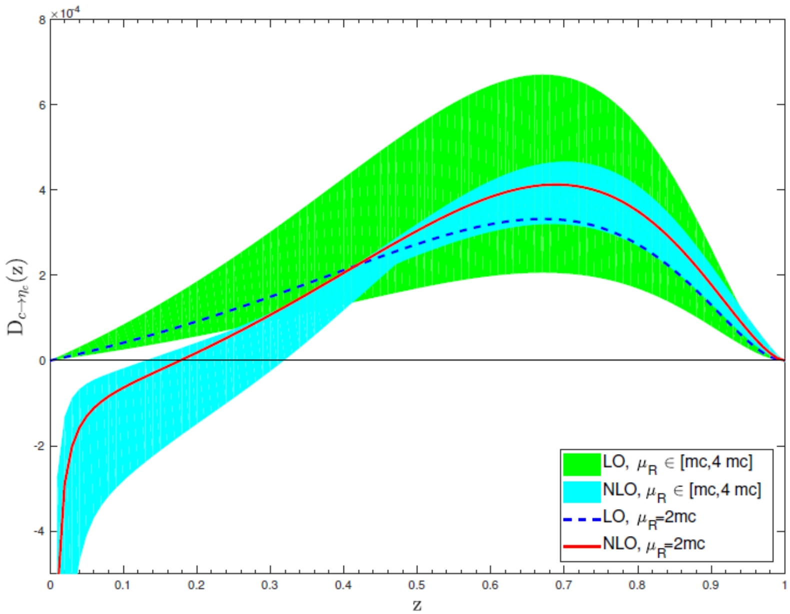


Quarkonium FFs

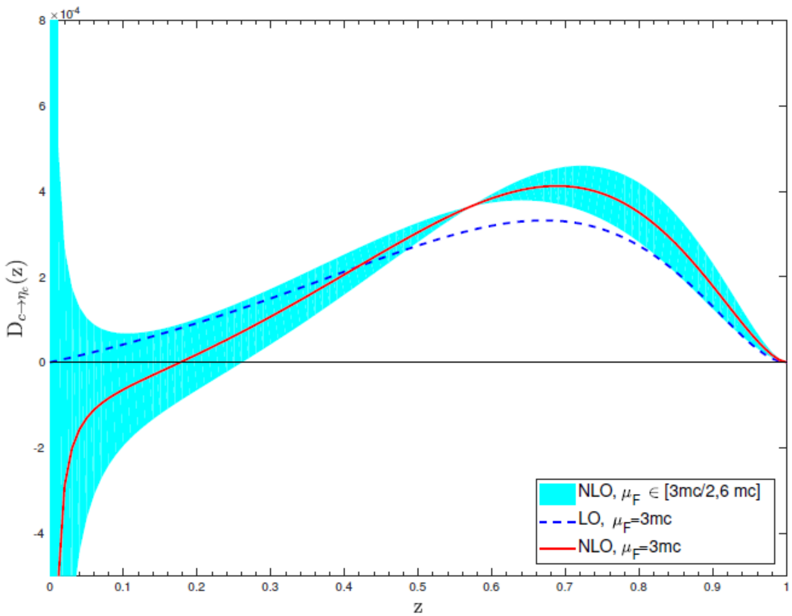
JHEP 07, 014, (2021),
X.-C. Zheng, X.-G. Wu, X.-D. Huang.

NLO fragmentation functions for $Q \to \eta_Q$





$$D_{c\to\eta_c}(z,\mu_F)$$



$$D_{c\to\eta_c}(z,\mu_F)$$

Quarkonium FFs

Eur. Phys. J. C (2021) 81:597
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THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Next-to-leading-order QCD corrections to heavy quark fragmentation into ${}^1S_0^{(1,8)}$ quarkonia

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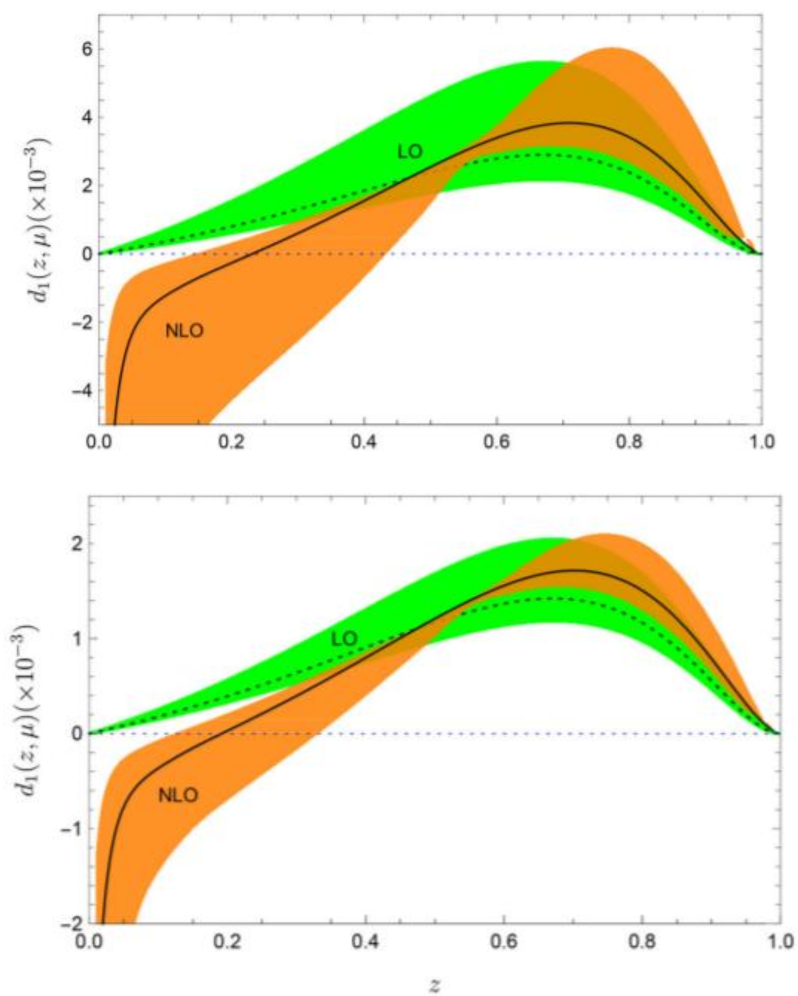
Abstract Within NRQCD factorization framework, in this work we compute, at the lowest order in velocity expansion, the next-to-leading-order (NLO) perturbative corrections to the short-distance coefficients associated with heavy quark fragmentation into the ${}^1S_0^{(1,8)}$ components of a heavy quarkonium. Starting from the Collins and Soper's operator definition of the quark fragmentation function, we apply the sector decomposition method to facilitate the numerical manipulation. It is found that the NLO QCD corrections have a significant impact.

hadronization mechanism. Similar to parton distribution functions, the scale dependence of FFs is governed by the celebrated Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equation:

$$\frac{d}{d\ln\mu^2} D_{i\to H}(z,\mu) = \sum_i \int_z^1 \frac{dy}{y} P_{ji}(y,\alpha_s(\mu)) D_{j\to H}\left(\frac{z}{y},\mu\right), \tag{2}$$

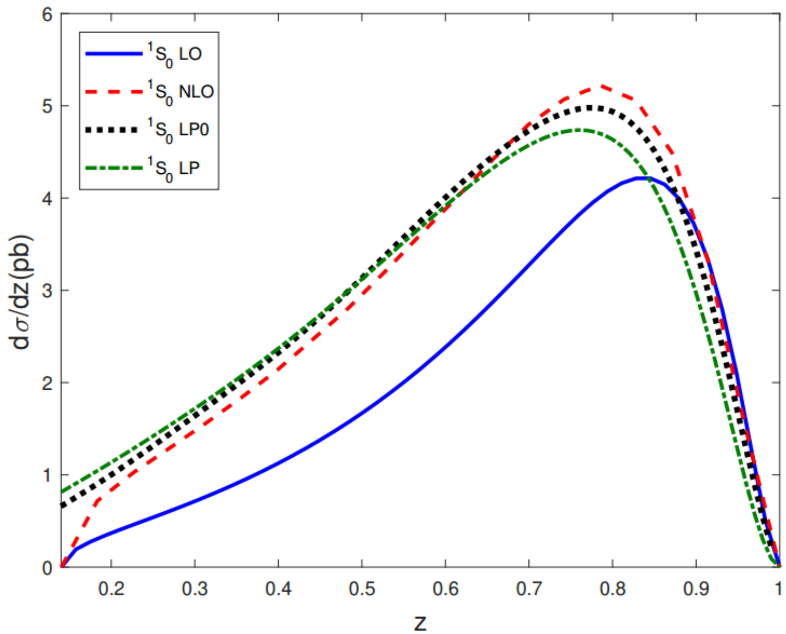
Note added After this work was completed and while we were preparing the manuscript, very recently a preprint [52] has appeared, which also computes the NLO perturbative corrections to the heavy quark fragmentation into a ${}^1S_0^{(1)}$ quarkonium. Their numerical results appear to be compatible with ours in this color-singlet channel.



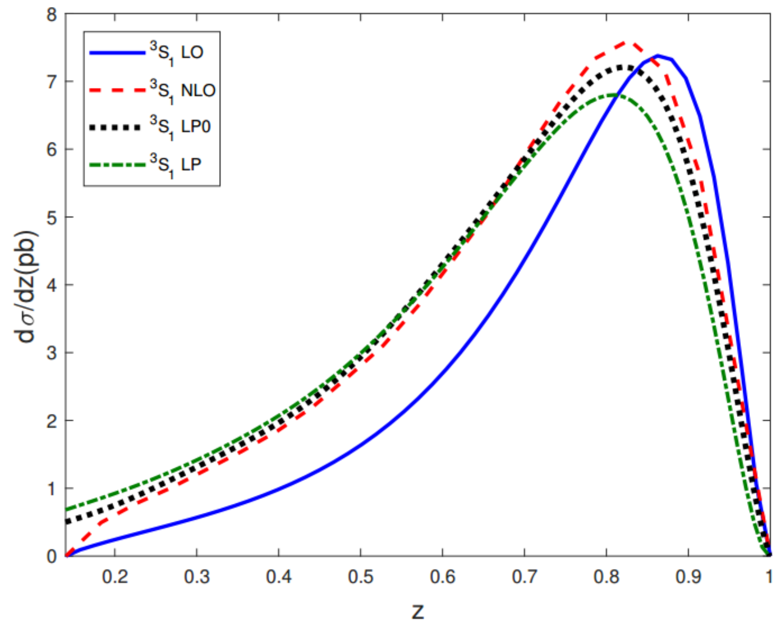
3. Applications

PRD 100, 034004, (2019),
X.-C. Zheng, C.-H. Chang, X.-G. Wu.

- Bc production at the Z pole



$$d\sigma / dz(Bc)$$



$$d\sigma / dz(Bc^*)$$

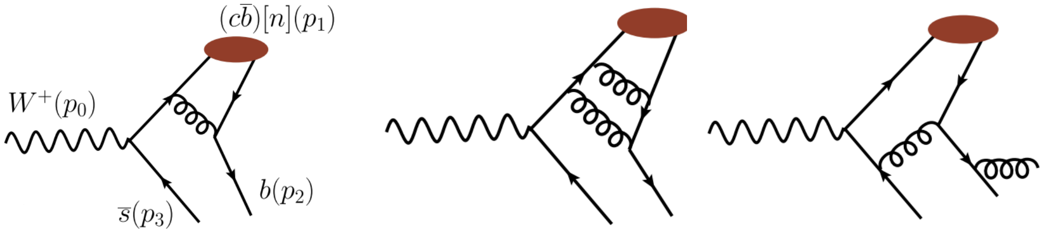
LO,NLO: direct NRQCD approach

LP0： fragmentation approach, no DGLAP evolution.

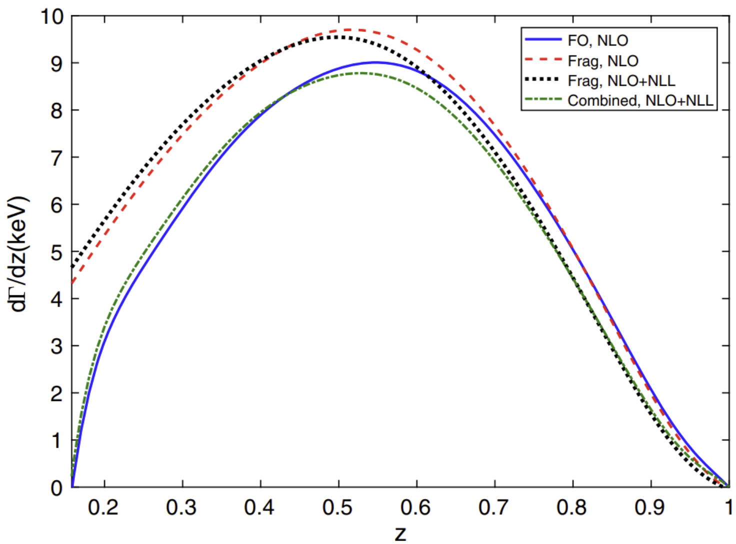
LP： fragmentation approach, evolved with DGLAP equation.

- Bc production via W⁺-boson decay

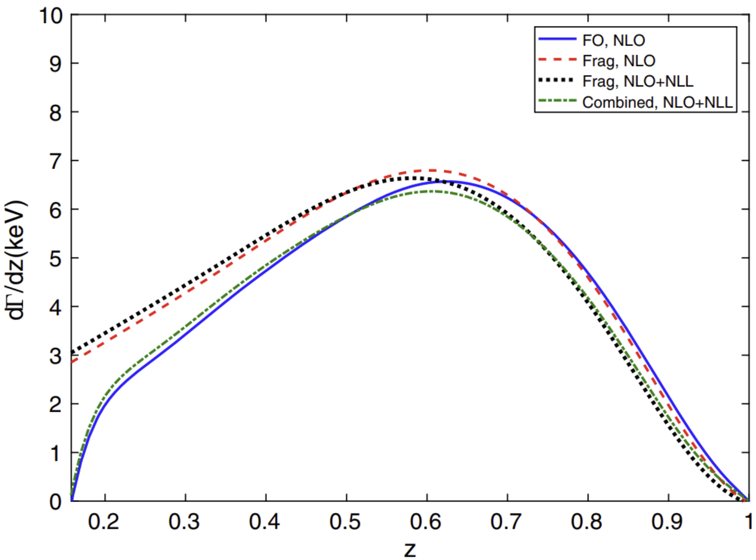
PRD101, 034029, (2020),
X.-C. Zheng, C.-H. Chang, X.-G. Wu, et al.



Typical Feynman diagrams under the Fixed-order approach



$d\Gamma / dz(Bc)$



$d\Gamma / dz(Bc^*)$

- Bc production via Higgs-boson decay

PRD 107, 074005, (2023), X.-C. Zheng, X.-G. Wu et al.

Two sources of large logarithms:

- Renormalization of the Yukawa couplings;
- Collinear emission of gluons and quarks.

Running quark masses

DGLAP evolution

Contributions	Direct NRQCD	FF approach
$\bar{b}$-fragmentation	1.20	1.22
c-fragmentation	4.13×10^{-3}	4.26×10^{-3}
Interference	1.25×10^{-2}	
Total	1.22	1.22

Decay width under the fixed-order calculation

Contributions	B_c
$\bar{b}$-fragmentation	0.673
c-fragmentation	1.47×10^{-3}
g-fragmentation	-1.80×10^{-3}
Triangle top-loop	4.59×10^{-2}
Total	0.719

Decay width after resuming the large log terms

Summary

- **Fragmentation-function approach can be used to resum the large logarithms arising from the collinear emissions;**
- **The NLO fragmentation functions for a quark or gluon into a doubly heavy meson (Bc, J/ψ, Υ, η_c, η_b) have been obtained;**
- **These fragmentation functions can be studied at the high energy colliders, such as CEPC, FCC-ee, etc.**

Thank you！