Anisotropic Flow from Multi-parton Scattering

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第六届重味物理与量子色动力学研讨会

2024年4月22日，青岛

Outline

- Motivation
- Calculation and results
  - Dipole-dipole scattering
  - pion-pion scattering
- Summary

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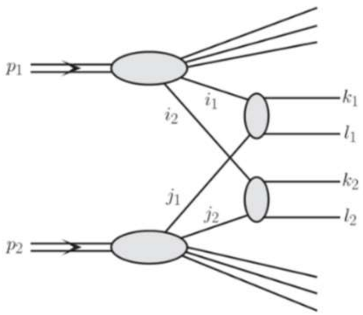
Double-Parton Scattering

- Tremendous success of single-parton scattering picture
- Double-parton scattering has attracted lots of attention



MPI@LHC, starts at 2008

14th International workshop on Multiple Partonic Interactions at the LHC (MPI@LHC 2023)
20-24 November 2023. Manchester, United Kingdom (C23-11-20.1)
Phenomenology-HEP | Theory-HEP | Experiment-HEP

- Double-parton PDF   **A double copy of single-parton pdf?**
  - ➢ Proton is a **quantum** wave packet.
  - ➢ Take two partons and integrate out the rest

    Quantum correlation in transverse position, spin, flavor and color…
  - ➢ Nonperturbative, can be calculated on lattice

    Zhang, Jian-Hui, arXiv:2304.12481

e.g. spin correlation of electrons

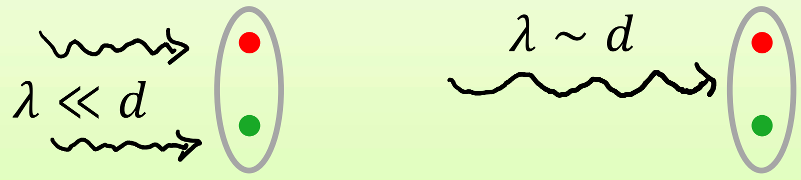
$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\Big)$$

Observables

- Observables:
  - ➢ Current calculations focus on double-hard scattering
  - ➢ Soft radiation is a simpler probe

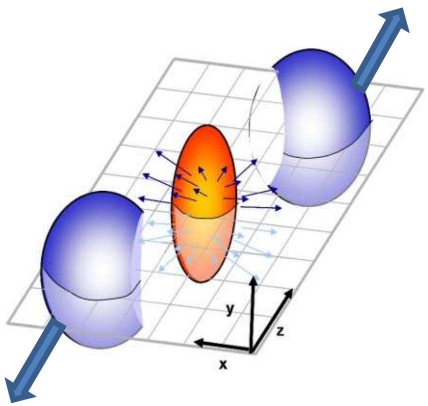
Li, Qian, Wu, **HZ**, JHEP 2023

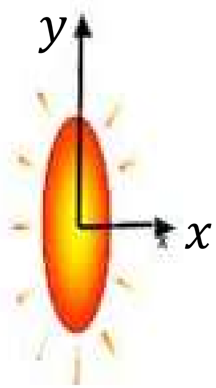
e.g. probe coupled electrons with light beams

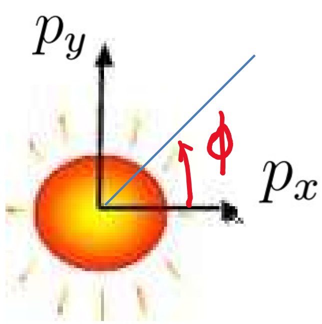


Observe interference pattern

- Anisotropic flow in heavy ion collisions

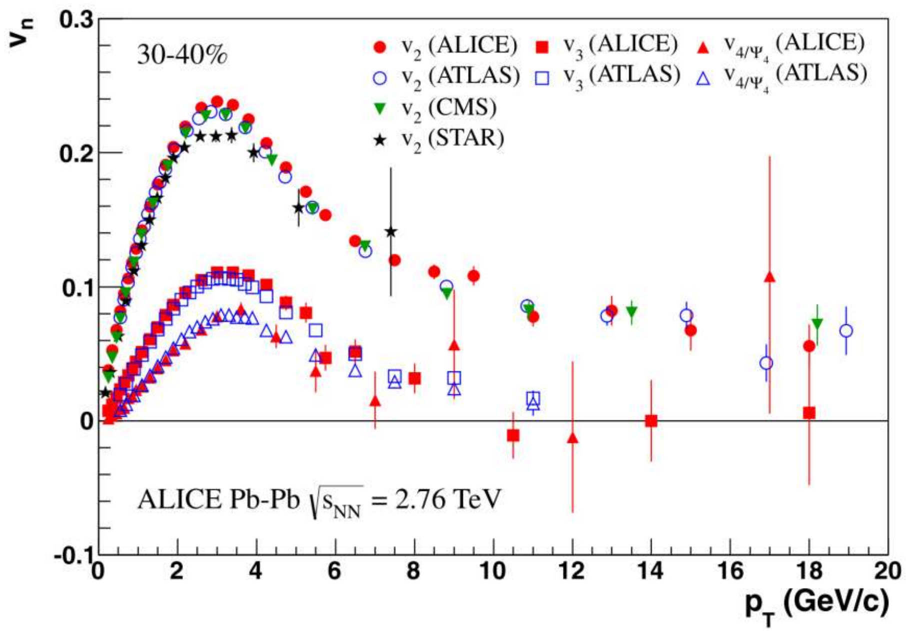






$$\frac{d^2N}{d^2p_T} \propto 1 + 2v_2\cos(2\phi) + \sum_{n=3}^{\infty} 2v_n\cos(n\phi)$$

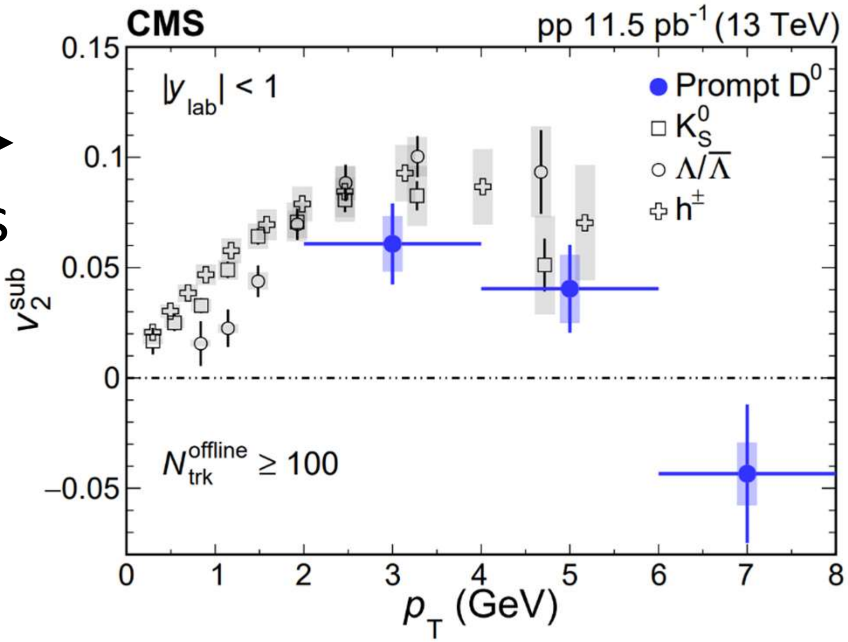
↑ elliptic flow



ALICE collaboration PLB (2013)

Surprise!

- $v_2 \neq 0$ in high-multiplicity pp collisions
  - As large as 0.1
- Previous explanations:
  - Final state interaction (hydrodynamics or transport)
  - Initial state interaction (Color-Glass-Condensate)



CMS collaboration PLB (2021)

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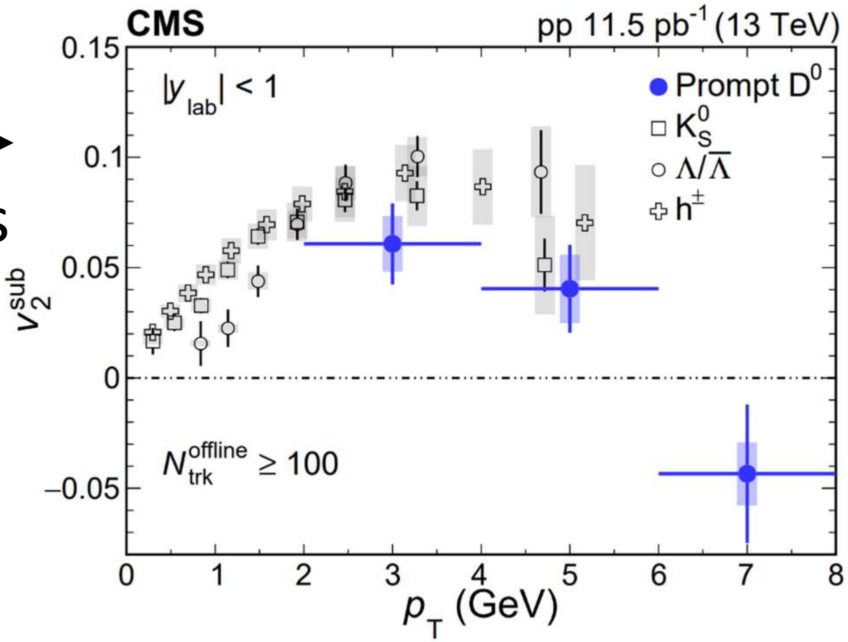
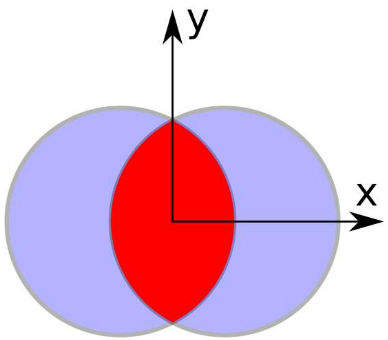


CMS collaboration PLB (2021)

- New idea: **overlap of wave functions**

Sensitive to geometry: needs multi-parton scattering



✓ **Large pT particles** can only see a single parton, causing small v_2

✓ **Small pT particles** cannot resolve the overlapped region, causing small v_2.

✓ **Transition** from single-parton process to multi-parton process

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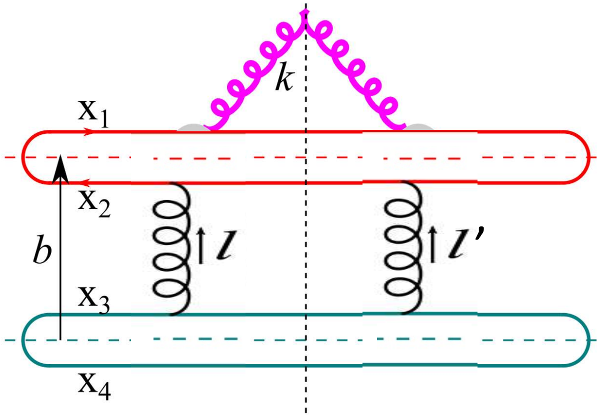
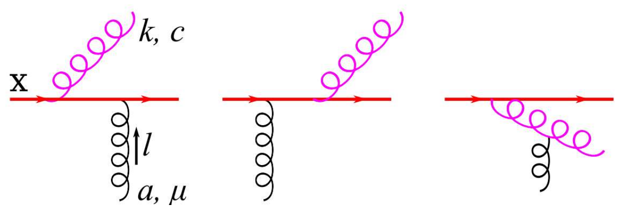
Theoretical Framework

- To study this effect, we consider $\pi\pi$ scattering
  - Approx. $|\pi\rangle = |q\bar{q}\rangle$, strongest interference
  - Focus on soft radiation with small rapidity, the eikonal appox. is qualified.





20 ways of connection in amplitude

Two partons involve due to **correlation**

$$\frac{d\hat{\sigma}}{d^2\mathbf{b}d\eta d^2\mathbf{k}} = 8\alpha_s^3 C_F |\mathbf{J}(\{\mathbf{x}_i\})|^2$$

$$\mathbf{J}(\{\mathbf{x}_i\}) \equiv \int \frac{d^2\mathbf{l}}{(2\pi)^2}\frac{1}{|\mathbf{l}|^2}\left(e^{-i\mathbf{l}\cdot\mathbf{x}_3} - e^{-i\mathbf{l}\cdot\mathbf{x}_4}\right)\left(\frac{\mathbf{k}}{|\mathbf{k}|^2} - \frac{\mathbf{k}-\mathbf{l}}{|\mathbf{k}-\mathbf{l}|^2}\right)\left[e^{i(\mathbf{l}-\mathbf{k})\cdot\mathbf{x}_1} - e^{i(\mathbf{l}-\mathbf{k})\cdot\mathbf{x}_2}\right]$$

The integral is finite with nontrivial divergence cancellation

- $l = 0$
- $l = k$
- $l \to \infty$

Small k limit of v_2

$$\frac{d\hat{\sigma}}{d^2\mathbf{b}d\eta d^2\mathbf{k}} = 8\alpha_s^3 C_F |\mathbf{J}(\{\mathbf{x}_i\})|^2 \propto A[1 + 2v_2\cos(2\phi) + \cdots]$$

$$\mathbf{J}(\{\mathbf{x}_i\}) \equiv \int \frac{d^2\mathbf{l}}{(2\pi)^2}\frac{1}{|\mathbf{l}|^2}\left(e^{-i\mathbf{l}\cdot\mathbf{x}_3} - e^{-i\mathbf{l}\cdot\mathbf{x}_4}\right)\left(\frac{\mathbf{k}}{|\mathbf{k}|^2} - \frac{\mathbf{k}-\mathbf{l}}{|\mathbf{k}-\mathbf{l}|^2}\right)\left[e^{i(\mathbf{l}-\mathbf{k})\cdot\mathbf{x}_1} - e^{i(\mathbf{l}-\mathbf{k})\cdot\mathbf{x}_2}\right]$$

$$\lim_{k_T\to 0}|\mathbf{J}|^2 = \frac{1}{|\mathbf{k}|^2}\left[B_0 + |\mathbf{k}|^2\sum_{i,j=1}^{4} D_{ij}\cos(\theta_i - \phi)\cos(\theta_j - \phi)\right] + \mathcal{O}(|\mathbf{k}|)$$

$\theta_i = \arg\vec{x}_i$, $\phi = \arg\vec{k}$, B_0 and D_{ij} do not depend on θ_i and ϕ

$$\lim_{k_T\to 0}\int_0^{2\pi} d\phi|\mathbf{J}|^2 = \frac{2\pi}{|\mathbf{k}|^2}B_0 + \mathcal{O}\left(|\mathbf{k}|^0\right)$$

$$\lim_{k_T\to 0}\int_0^{2\pi} d\phi|\mathbf{J}|^2\cos(2\phi) = \frac{\pi}{2}\sum_{i,j=1}^{4} D_{ij}\cos(\theta_i + \theta_j) + \mathcal{O}(|\mathbf{k}|).$$

$\Rightarrow$ $\lim_{k\to 0} v_2 \propto \boldsymbol{k}^2$

✓ **Small k particles** cannot resolve the overlapped region.

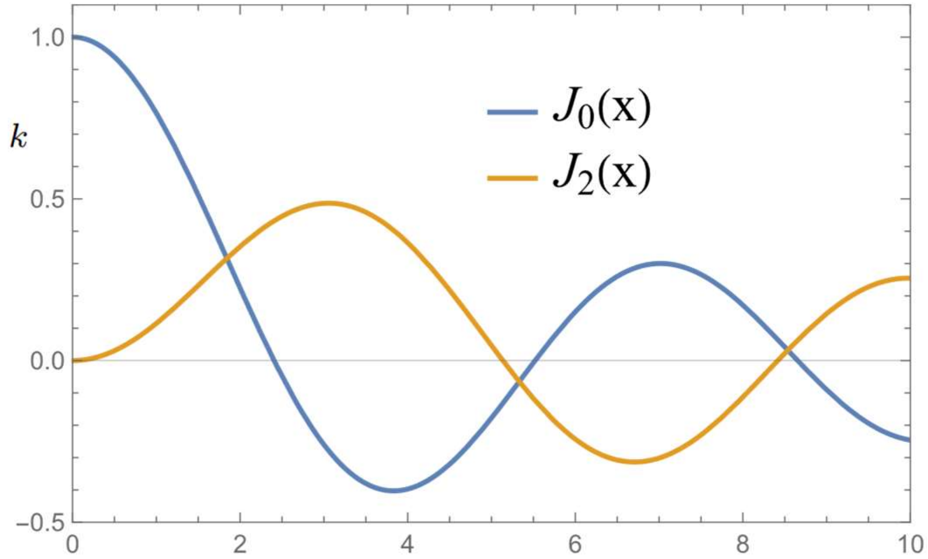
Large k limit of v_2

$$\lim_{k_T\to\infty}|\mathbf{J}|^2 = \frac{1}{|\mathbf{k}|^4}\left\{B_1 + \left[\sum_{\substack{m,n=1\\ m\neq n}}^{4} f_{mn} + \sum_{\substack{n=1,2\\ m=3,4}} g_{mn}\cos(2\phi) + \sum_{\substack{n=1,2\\ m=3,4}} h_{mn}\sin(2\phi)\right]\right.$$
$$\left.\times\ \cos\left[|\mathbf{k}||\mathbf{x}_m|\cos(\theta_m-\phi) - |\mathbf{k}||\mathbf{x}_n|\cos(\theta_n-\phi)\right]\right\} + \mathcal{O}\left(\frac{1}{|\mathbf{k}|^6}\right)$$

$\theta_m = \arg\vec{x}_m$, $\phi = \arg\vec{k}$, coefficients do not depend on θ_i and ϕ

$$\lim_{k_T\to\infty}\int_0^{2\pi} d\phi|\mathbf{J}|^2 = \frac{2\pi}{|\mathbf{k}|^4}B_1 + \frac{2\pi}{|\mathbf{k}|^4}\left[\sum_{m,n=1}^{4} f_{mn}J_0\left(|\mathbf{k}|\bar{x}_{mn}\right) - \sum_{\substack{n=1,2\\ m=3,4}} g_{mn}J_2\left(|\mathbf{k}|\bar{x}_{mn}\right)\right] + \mathcal{O}\left(\frac{1}{|\mathbf{k}|^6}\right)$$

$$_{mn}J_2\left(|\mathbf{k}|\bar{x}_{mn}\right) + \sum_{\substack{n=1,2\\ m=3,4}} g_{mn}J_0\left(|\mathbf{k}|\bar{x}_{mn}\right) + \mathcal{O}\left(\frac{1}{|\mathbf{k}|^6}\right)$$



Interference pattern

Large k limit of v_2

$$\lim_{k_T\to\infty}|\mathbf{J}|^2 = \frac{1}{|\mathbf{k}|^4}\Bigg\{B_1 + \Bigg[\sum_{\substack{m,n=1\\m\neq n}}^{4} f_{mn} + \sum_{\substack{n=1,2\\m=3,4}} g_{mn}\cos(2\phi) + \sum_{\substack{n=1,2\\m=3,4}} h_{mn}\sin(2\phi)\Bigg]$$

$$\times\ \cos\left[|\mathbf{k}||\mathbf{x}_m|\cos(\theta_m-\phi) - |\mathbf{k}||\mathbf{x}_n|\cos(\theta_n-\phi)\right]\Bigg\} + \mathcal{O}\left(\frac{1}{|\mathbf{k}|^6}\right)$$

$\theta_m = \arg\vec{x}_m$, $\phi = \arg\vec{k}$, coefficients do not depend on θ_i and ϕ

$$\lim_{k_T\to\infty}\int_0^{2\pi} d\phi|\mathbf{J}|^2 = \frac{2\pi}{|\mathbf{k}|^4}B_1 + \frac{2\pi}{|\mathbf{k}|^4}\Bigg[\sum_{\substack{m,n=1\\m\neq n}}^{4} f_{mn}J_0\left(|\mathbf{k}|\bar{x}_{mn}\right) - \sum_{\substack{n=1,2\\m=3,4}} g_{mn}J_2\left(|\mathbf{k}|\bar{x}_{mn}\right)\Bigg] + \mathcal{O}\left(\frac{1}{|\mathbf{k}|^6}\right)$$

$$\lim_{k_T\to\infty}\int_0^{2\pi} d\phi|\mathbf{J}|^2\cos(2\phi) = \frac{2\pi}{|\mathbf{k}|^4}\Bigg[-\sum_{\substack{m,n=1\\m\neq n}}^{4} f_{mn}J_2\left(|\mathbf{k}|\bar{x}_{mn}\right) + \sum_{\substack{n=1,2\\m=3,4}} g_{mn}J_0\left(|\mathbf{k}|\bar{x}_{mn}\right) + \mathcal{O}\left(\frac{1}{|\mathbf{k}|^6}\right)$$

$$\lim_{k_T\to\infty} v_2 = \frac{1}{B_1}\sqrt{\frac{2}{\pi|\mathbf{k}|}}\Bigg[\sum_{\substack{m,n=1\\m\neq n}}^{4}\frac{f_{mn}}{\sqrt{\bar{x}_{mn}}}\cos(|\mathbf{k}|\bar{x}_{mn}-\frac{\pi}{4}) + \sum_{\substack{n=1,2\\m=3,4}}\frac{g_{mn}}{\sqrt{\bar{x}_{mn}}}\cos(|\mathbf{k}|\bar{x}_{mn}-\frac{\pi}{4})\Bigg]$$

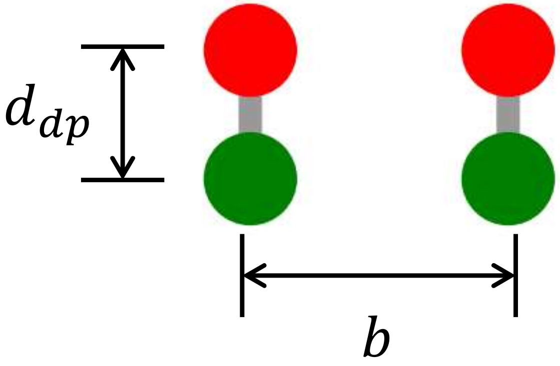
$$+\,\mathcal{O}(\frac{1}{|\mathbf{k}|})$$

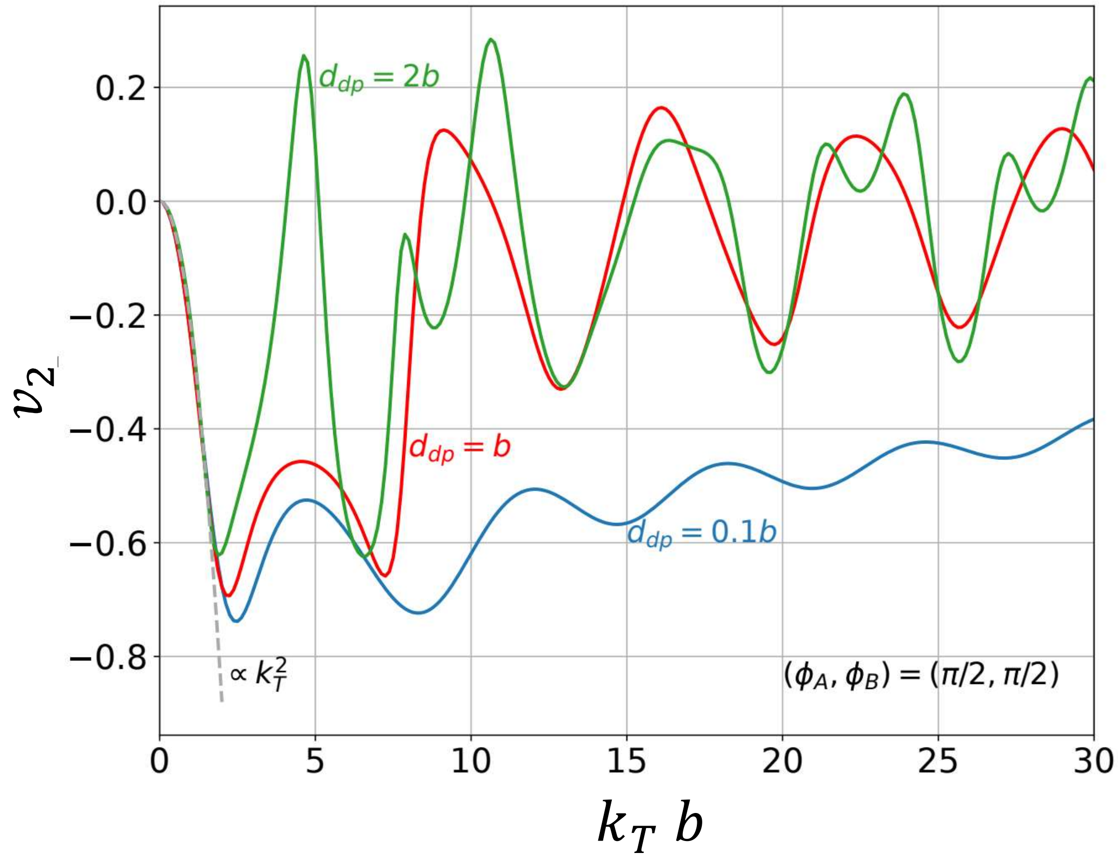
$\Rightarrow$ $\lim_{k\to\infty} v_2 \propto 1/\sqrt{k}$

Dipole-dipole Scattering I

- **Strong interference** exists!

Example:



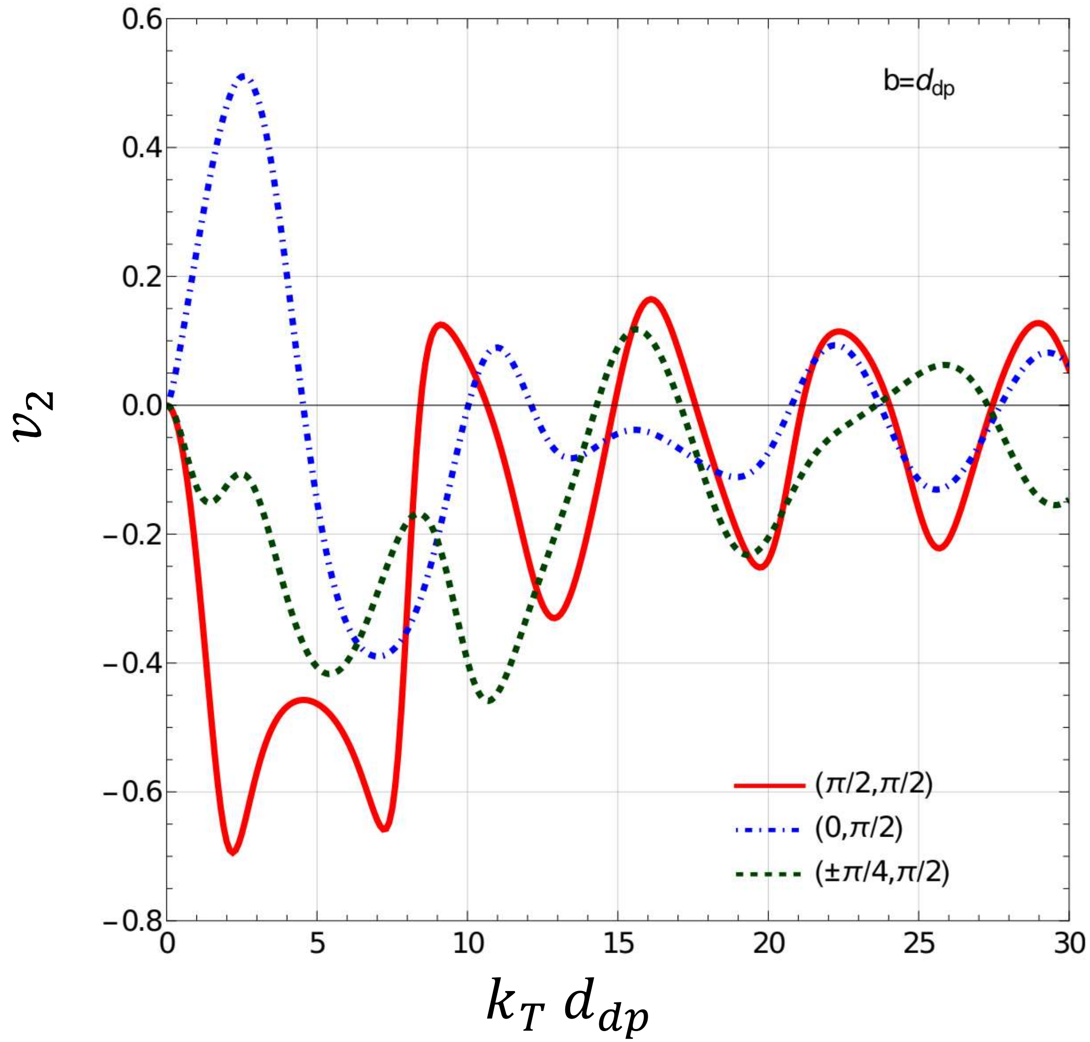


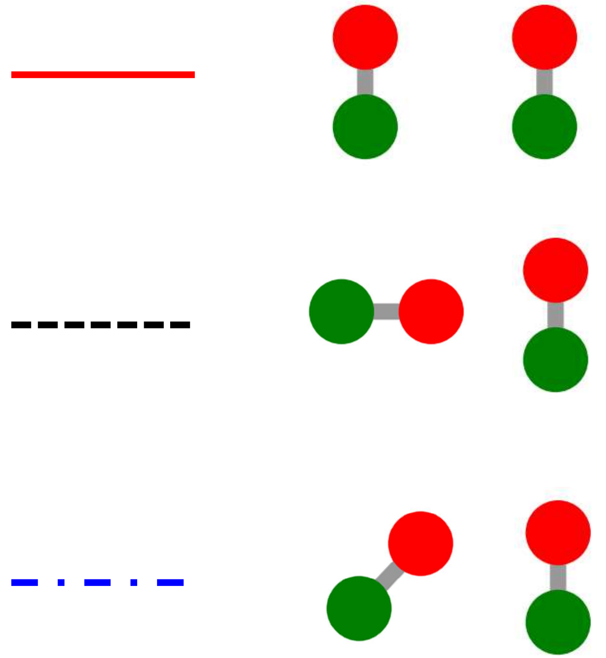
- With $d_{dp} = 0.1b, b, 2b$
- For all curves, $\boldsymbol{v_2 \propto -(k_T b)^2}$ at small $k_T b$
- Strong oscillation exists at large $k_T b$
- With d_{dp} fixed, **$\boldsymbol{v_2}$ changes sign for small $\boldsymbol{b}$**
- With d_{dp} fixed, **$\boldsymbol{v_2}$ is always negative with large $\boldsymbol{b}$**.

Dipole-dipole Scattering II

- **Strong destructive interference** for dipoles with different angles.

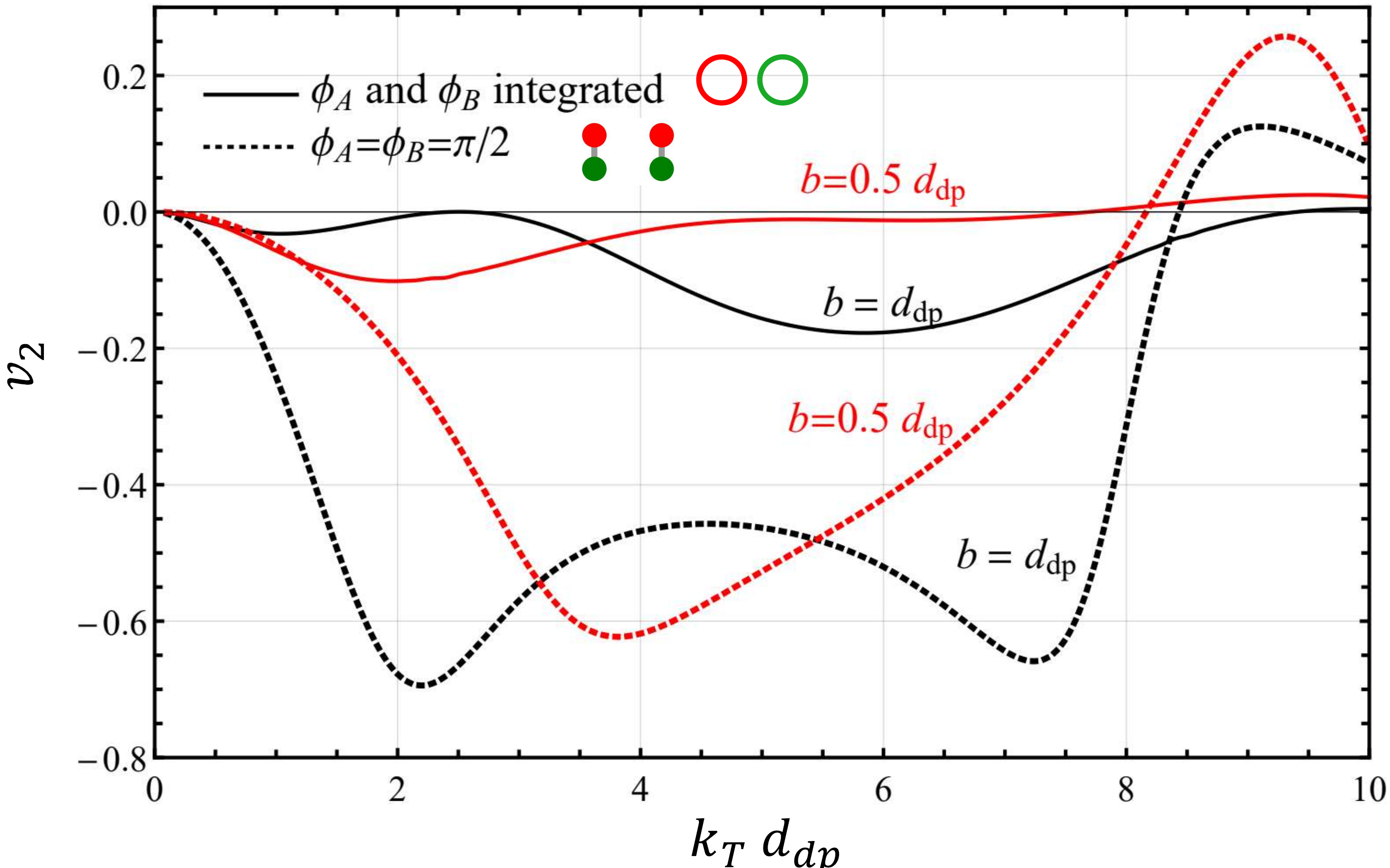
Fix $b = d_{dp}$





Average over Angles

- v_2 decreases by a factor of 4 with dipole angle integrated
- v_2 with angle integrated is mainly negative
- **Interference pattern is smeared:** one peak for small b
  two peaks for large b



Light-front Wave Function I

- Consider pion-pion scattering
- The dominant Fock state: $|q\bar{q}\rangle$

ξ : longitudinal momentum fraction of q
$\vec{k}$: transverse momentum of q

$$|\psi_h(P,j,m_j)\rangle = \sum_{s_q,s_{\bar{q}}} \int_0^1 \frac{\mathrm{d}\xi}{2\xi(1-\xi)} \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^3} \psi^{(m_j)}_{s_q,s_{\bar{q}}/h}(\mathbf{k},\xi)$$

Wave function

$$\times \frac{1}{\sqrt{N_c}} \sum_{i=1}^{N_c} b^\dagger_{s_q i}(\xi P^+, \mathbf{k}+\xi\mathbf{P}) d^\dagger_{s_{\bar{q}} i}((1-\xi)P^+, -\mathbf{k}+(1-\xi)\mathbf{P})|0\rangle.$$

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- Fourier transform to coordinate space in the transverse plane

$$\tilde{\psi}_{s\bar{s}/h}(\mathbf{r},\xi) = \sqrt{\xi(1-\xi)} \sum_{n.m.l} \psi_h(n,m,l,s,\bar{s}) \tilde{\phi}_{nm}(\sqrt{\xi(1-\xi)}\mathbf{r}) \chi_l(\xi)$$

with

$$\phi_{nm}(\mathbf{q}) = \frac{1}{\kappa}\sqrt{\frac{4\pi n!}{(n+|m|)!}} \left(\frac{|\mathbf{q}|}{\kappa}\right)^{|m|} e^{-\frac{|\mathbf{q}|^2}{2\kappa^2}} L_n^{|m|}\left(\frac{|\mathbf{q}|^2}{\kappa^2}\right) e^{im\theta_q},$$

2d harmonic oscillator basis

$$\chi_l(\xi;\alpha,\beta) = \xi^{\frac{\beta}{2}}(1-\xi)^{\frac{\alpha}{2}} P_l^{(\alpha,\beta)}(2\xi-1)\sqrt{4\pi(2l+\alpha+\beta+1)} \sqrt{\frac{\Gamma(l+1)\Gamma(l+\alpha+\beta+1)}{\Gamma(l+\alpha+1)\Gamma(l+\beta+1)}}$$

$$\alpha = \beta = 4m_q^2/\kappa^2$$

- Only two parameters: m_q and κ

Light-front Wave Function II

- By fitting the pion and rho masses,

  $m_q = 480$ MeV, $\kappa = 610$ MeV

Qian, Jia, Li, Vary,
PRC 102, 055207(2020)

No free parameter in the calculation in this work.

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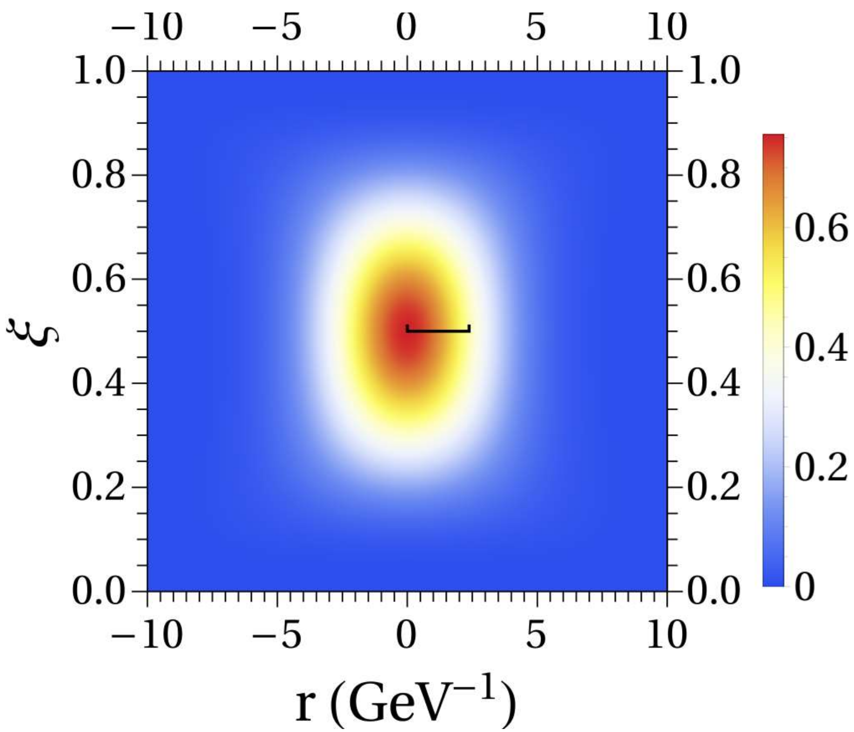
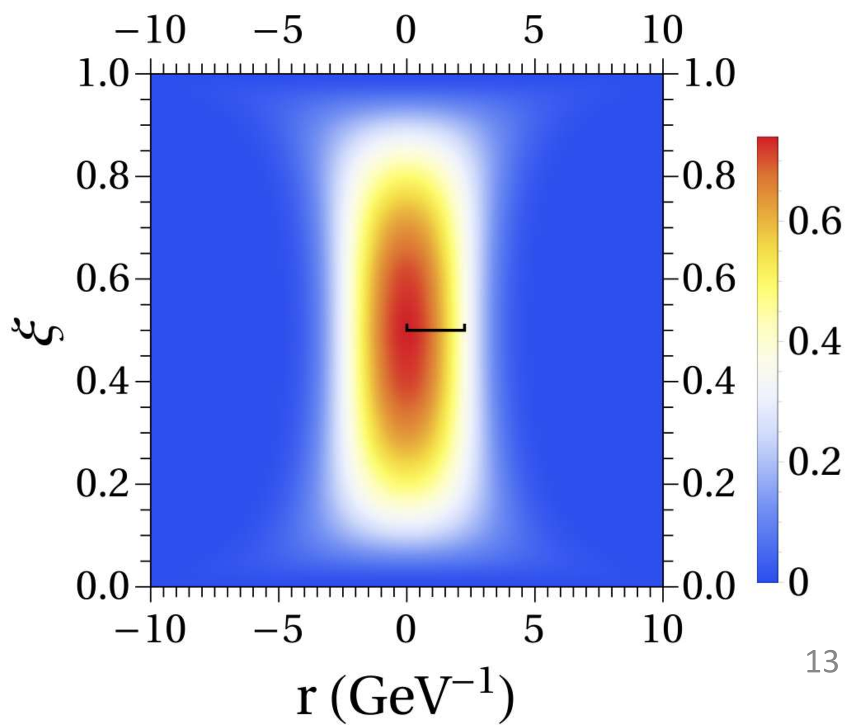
- Probability density for q in a pion

$n = l = m = 0$ state

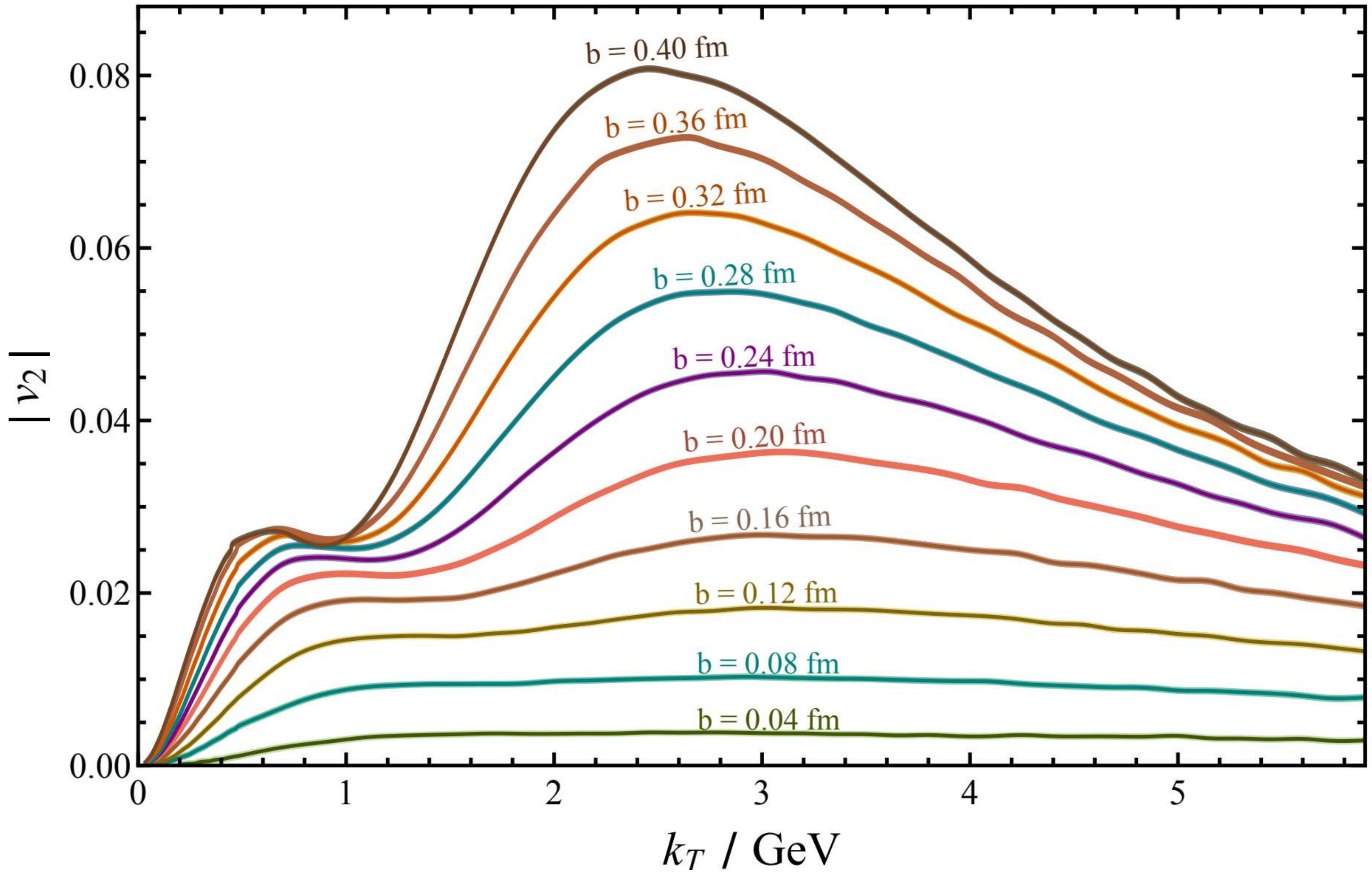


First 400 (nlm) states



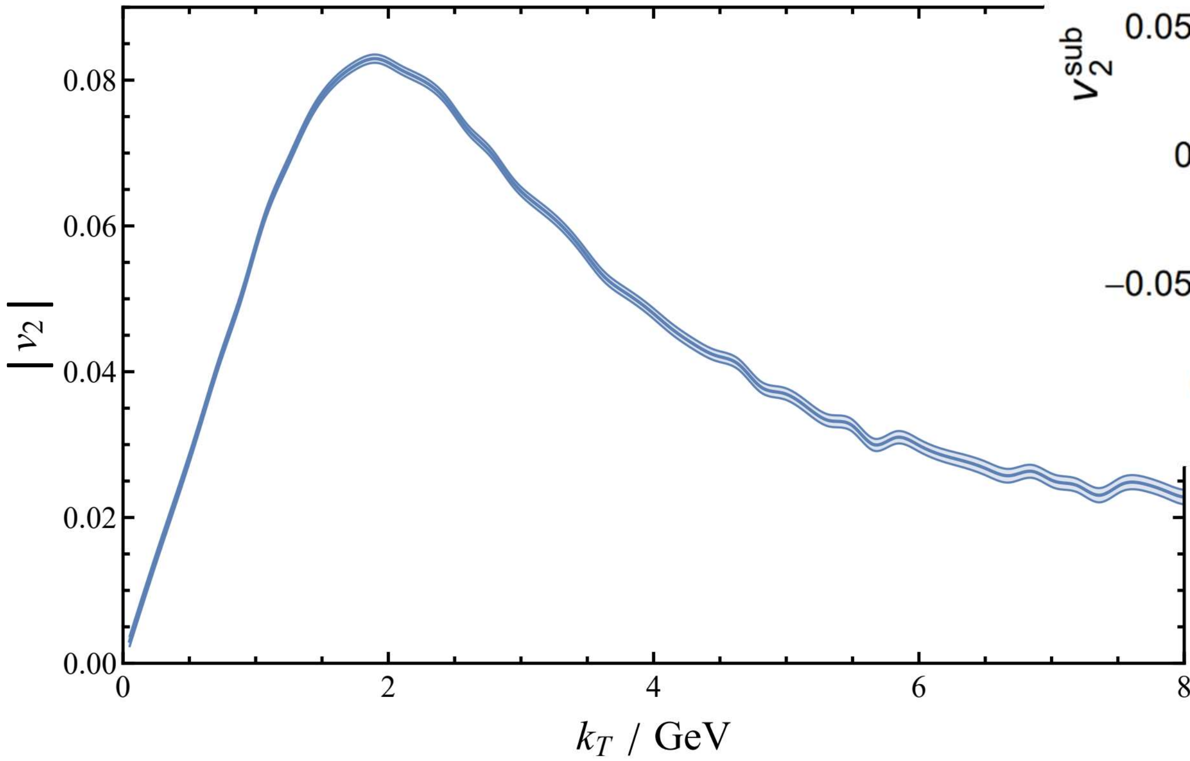
Pion-pion Scattering with b Fixed

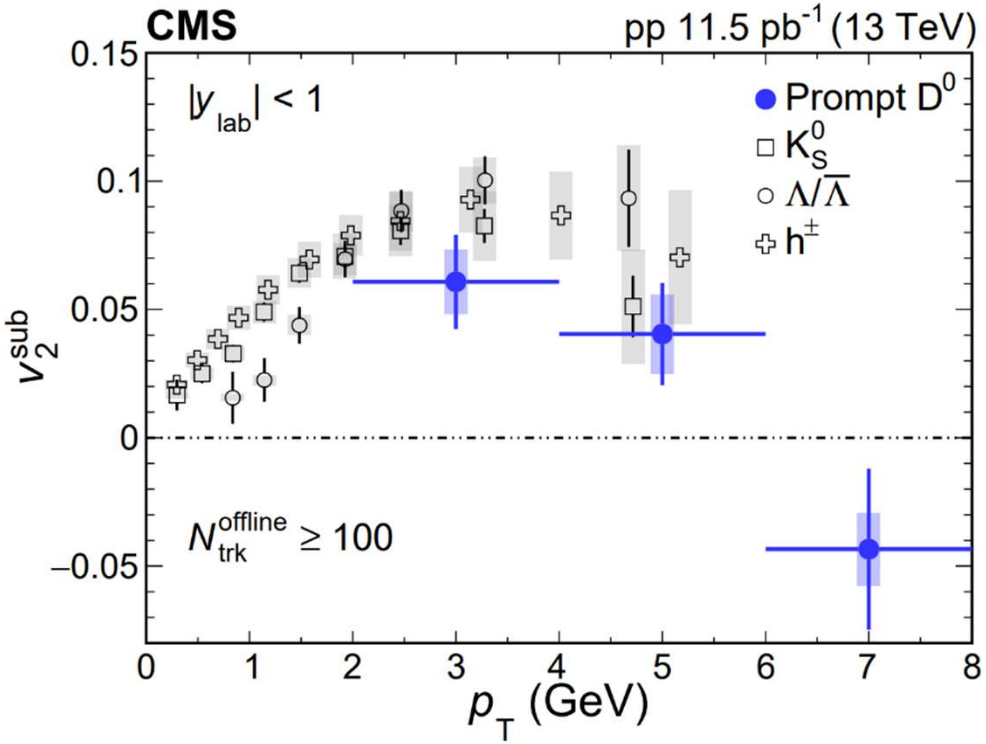
- **Double-Peak structure** with $b \geq 0.1$ fm, residue of interference
- Interference effect is invisible with $b < 0.1$ fm



Pion-pion Scattering with b-integrated

- $d\sigma/db^2$ is dominant at small b
- **Interference effect is invisible** with b integrated





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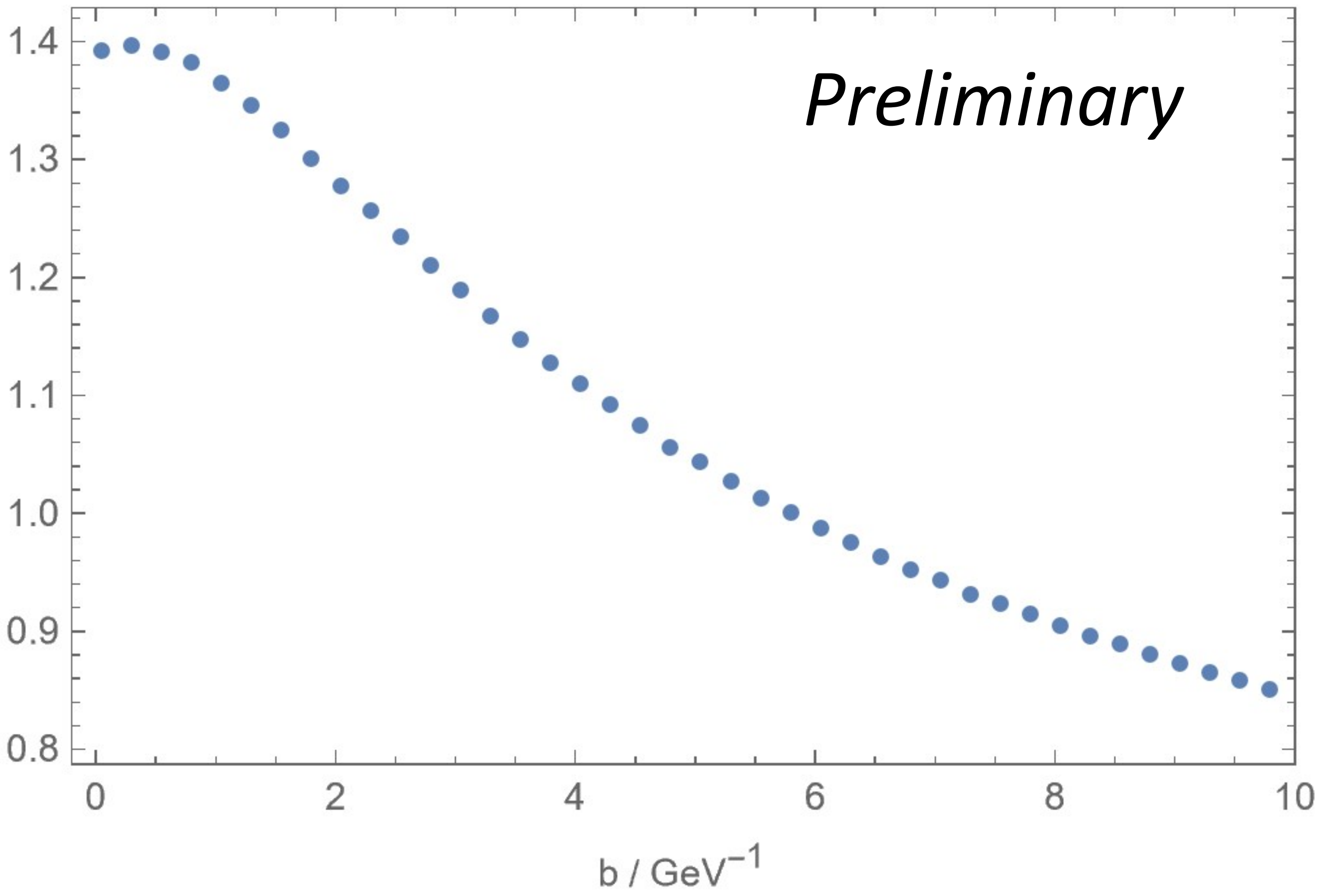
Select Events with Large b

- Events with large b tend to have smaller $\langle k_T \rangle$

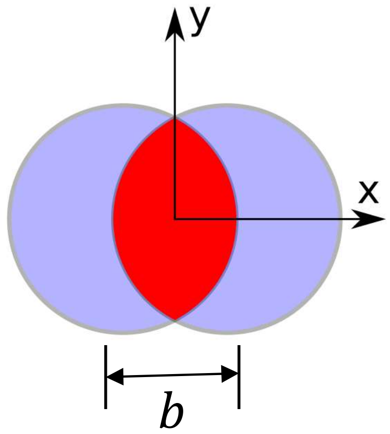
$$\frac{d\langle k_T \rangle / d^2 b}{d\sigma / d^2 b}$$

$k_T \geq 0.2$ GeV



- Events with small $\langle k_T \rangle$ may show double-peak structure in v_2

Summary



- We calculate the anisotropic flow v_2 in $\pi\pi$ collisions with eikonal approximation.
- The **transition** from single-parton scattering to multi-parton scattering is clearly visible in v_2.
- With no free parameter, the obtained v_2 **agrees qualitatively** with data in pp collisions.
- The interference pattern is smeared after integrating over b. The **double-peak structure** may be revived by selecting events with small $\langle k_T \rangle$.
- Anisotropic flow can be applied in other processes to probe multi-parton correlation.

Thank you!