Λ_b 重子衰变及其形状因子

第六届重味物理与量子色动力学研讨会

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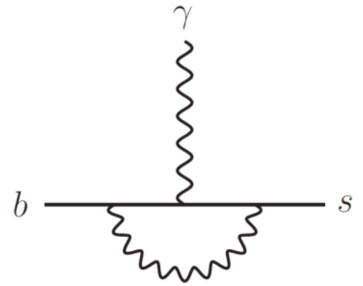

图1. 味道改变中性流

表1. $\Lambda_b\to\Lambda$ 领头幂次形状因子$q^2=0$

微扰	非微扰	
SCET【3】	SCET+LCSR【4】	LCSR【5】
$\xi=-0.012$ (LO)	$\xi=0.38$ (LO)	$f^0=0.12$ (NLO)

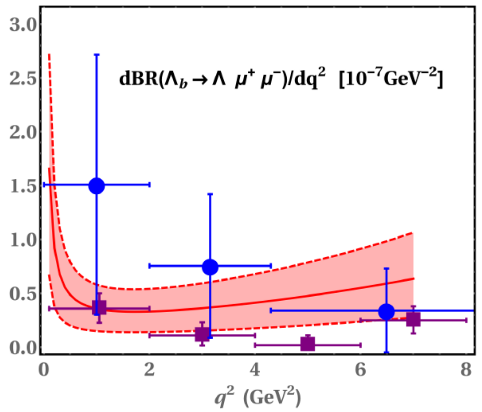

图2. NLO微分分支比q^2依赖【5】

【1】R. Aaij et al. [LHCb], Measmurement of matter-antimatter differences in beauty baryon decays, Nature Phys. 13, 391-396 (2017) [arXiv:1609.05216 [hep-ex]].

【2】R. Aaij et al. [LHCb], Deterination of the quark coupling strength |Vub| using baryonic decays, Nature Phys. 11, 743-747 (2015) [arXiv:1504.01568 [hep-ex]].

【3】W. Wang, Factorization of heavy-to-light baryonic transitions in SCET, Phys. Lett. B 708, 119 (2012) [arXiv:1112.0237 [hep-ph]].

【4】T. Feldmann and M. W. Y. Yip, Form factors for $\Lambda_b\to\Lambda$ transitions in the soft-collinear effective theory," Phys. Rev. D 85, 014035 (2012) [arXiv:1111.1844 [hep-ph]].

【5】Y. M. Wang and Y. L. Shen, Perturbative Corrections to $\Lambda_b\to\Lambda$ Form Factors from QCD Light-Cone Sum Rules," JHEP 02, 179 (2016) [arXiv:1511.09036 [hep-ph]].

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共线因子化：SCET　　存在量级差问题

【1】W. Wang, Factorization of heavy-to-light baryonic transitions in SCET," Phys. Lett. B 708, 119 (2012) [arXiv:1112.0237 [hep-ph]].

横向动量因子化：PQCD　　同样存在量级差问题，leading power < high power

【2】X. G. He, T. Li, X. Q. Li, and Y. M. Wang, PQCD calculation for $\Lambda_b \to \Lambda\gamma$ in the standard model, Phys. Rev. D 74, 034026 (2006) [arXiv:hep-ph/0606025 [hep-ph]] .
【3】C. D. Lu, Y. M. Wang, H. Zou, A. Ali, and G. Kramer, Anatomy of the pQCD approach to the baryonic decays $\Lambda_b \to p\pi, pK$, Phys. Rev. D 80, 034011 (2009). [arXiv:0906.1479 [hep-ph]].
【4】J. J. Han, Y. Li, H. n. Li, Y. L. Shen, Z. J. Xiao, and F. S. Yu, $\Lambda_b \to p$ transition form factors in perturbative QCD, Eur. Phys. J. C 82, 686 (2022) [arXiv:2202.04804 [hep-ph]].
【5】Z. Rui, C. Q. Zhang, J. M. Li, and M. K. Jia, Investigating the color-suppressed decays $\Lambda_b \to \Lambda\psi$ in the perturbative QCD approach, Phys. Rev. D 106, 053005 (2022) [arXiv:2206.04501 [hep-ph]].

光锥求和规则：LCSR　　非微扰的软形状因子

【6】Y. M. Wang, Y. L. Shen and C. D. Lu, $\Lambda_b \to p$; transition form factors from QCD light-cone sum rules, Phys. Rev. D 80, 074012 (2009) [arXiv:0907.4008 [hep-ph]].
【7】T. Feldmann and M. W. Y. Yip, Form factors for $\Lambda_b \to \Lambda$ transitions in the soft-collinear effective theory, Phys. Rev. D 85, 014035 (2012) [arXiv:1111.1844 [hep-ph]].
【8】Y. M. Wang and Y. L. Shen, Perturbative Corrections to $\Lambda_b \to \Lambda$ Form Factors from QCD Light-Cone Sum Rules, JHEP 02, 179 (2016) [arXiv:1511.09036 [hep-ph]].
【9】K. S. Huang, W. Liu, Y. L. Shen, and F. S. Yu, $\Lambda_b \to p, N*(1535)$ form factors from QCD Light-cone sum rules, [arXiv:2205.06095 [hep-ph]]..

光前夸克模型：LFQM

【10】Z. T. Wei, H. W. Ke and X. Q. Li, Evaluating decay Rates and Asymmetries of Λ_b into Light Baryons in LFQM, Phys. Rev. D 80, 094016 (2009) [arXiv:0909.0100 [hep-ph]].
【11】Z. X. Zhao, Weak decays of heavy baryons in the light-front approach, Chin. Phys. C 42, no.9, 093101 (2018) [arXiv:1803.02292 [hep-ph]].

二、领头幂次重到轻流形状因子的计算

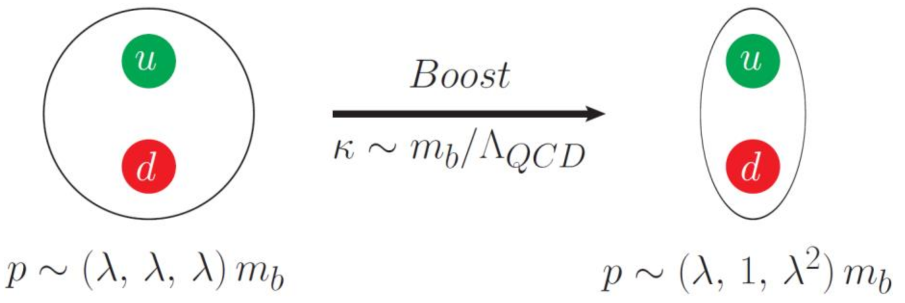
2.1 软共线有效理论—SCET

- 有效场论：

QCD：动力学性质 —— + 运动学性质 / + **Power counting** ——→ **有效场论：SCET，HQET，NRQCD**

- 能标分离，处理多标度问题。
- SCET 中场和算符按照幂次 $\lambda \sim \Lambda_{QCD}/m_b$ 展开。

π 介子价夸克动量变化：



SCET场：

$$\xi_{hc} \sim \lambda^{1/2}, \quad \xi_c \sim \lambda^{3/2}, \quad q_s \sim \lambda^{3/2}, \quad h_v \sim \lambda^{3/2}$$
$$A^\mu_{hc} \sim (\lambda, 1, \lambda^{1/2}), \quad A^\mu_c \sim (\lambda, 1, \lambda^2), \quad A^\mu_s \sim (\lambda, \lambda, \lambda) \tag{1}$$

SCET拉氏量：

$$\mathcal{L}^{(0)}_{\xi\xi} = \bar{\xi}_n \left(in \cdot D + i\not{D}_{n\perp} \frac{1}{i\bar{n} \cdot D_n} i\not{D}_{n\perp} \right) \frac{\not{\bar{n}}}{2} \xi_n \tag{2}$$

动量变化 —引起→ 场的变化 —引起→ 拉氏量变化

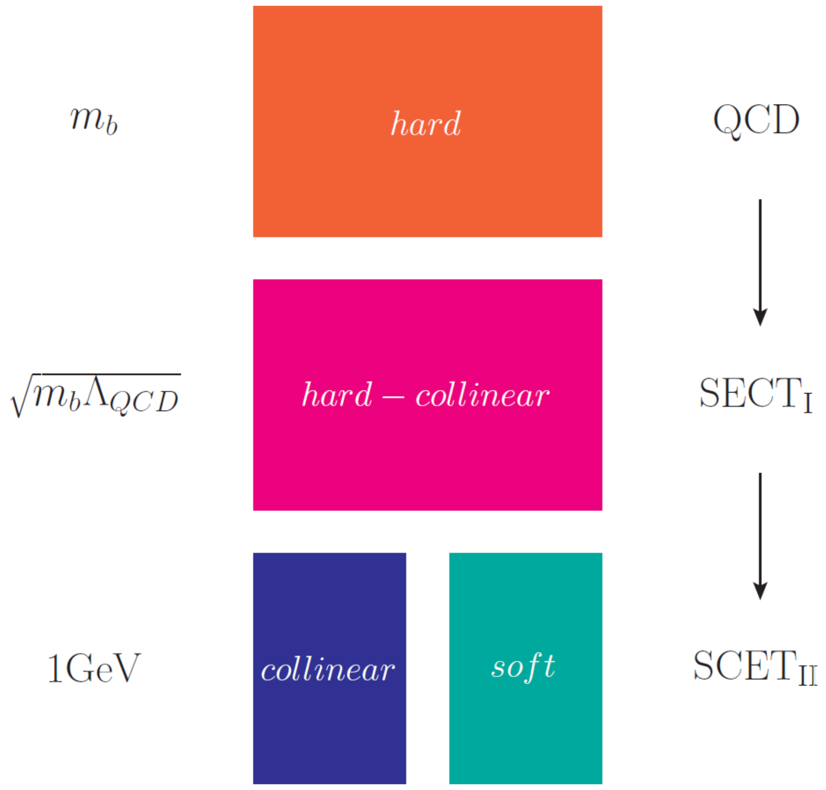


图3. 重子因子化能标分布

2.2 SCET在介子中的因子化证明

$B\to D\pi$ 因子化形式[1]：

$$\langle D_{v'}\pi_n|Q_{\mathbf{0}}^1|B_v\rangle = N F^{B\to D}(0)\int_0^1 dx\, T(x,\mu)\,\phi_\pi(x,\mu)\,. \qquad (3)$$

$B\to D$ 形状因子：

$$F^{B\to D}(0) = \frac{1}{2}\sqrt{\frac{m_B}{m_D}}\left(1+\frac{m_B}{m_D}\right)\xi(v\cdot v') \qquad (4)$$

其中，$\xi(v\cdot v')$ 为 Isgur-Wise函数。

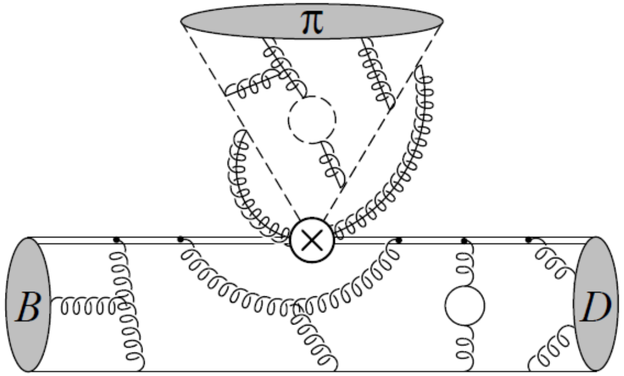


图4. $B\to D\pi$ 因子化图像[1]

介子重到轻流形状因子： $B\to\pi$ [2,3]

$$f_i(q^2) = C_i(q^2)\,\xi_\pi(q^2) + \int_0^\infty d\omega \int_0^1 du\, T_i(q^2;\omega,u)\,\phi_{B+}(\omega)\,\phi_\pi(u) \qquad (5)$$

其中，上式左边软形状因子 $\xi_\pi(q^2)$，吸收了非微扰效应；右边代表微扰的因子化效应

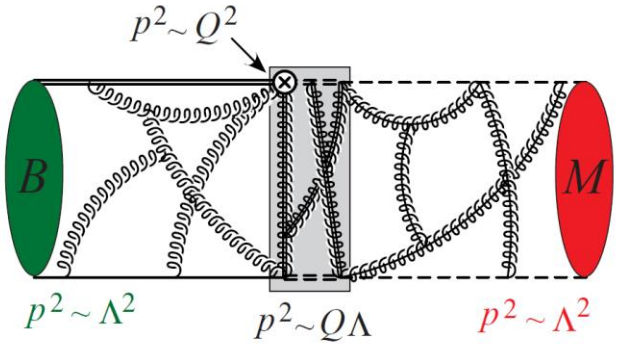


图5. $B\to\pi$ 因子化图像[2]

【1】C.W.Bauer, D.Pirjol and I.W.Stewart, A Proof of factorization for $B\to D\pi$, Phys. Rev. Lett. 87 (2001), 201806 [arXiv:hep-ph/0107002 [hep-ph]].
【2】C.W.Bauer, D. Pirjol and I.W.Stewart, Factorization and endpoint singularities in heavy to light decays, Phys. Rev. D 67 (2003), 071502 [arXiv:hep-ph/0211069 [hep-ph]].
【3】M.Beneke and D.Yang, Heavy-to-light B meson form-factors at large recoil energy: Spectator-scattering corrections," Nucl. Phys. B 736 (2006), 34-81 [arXiv:hep-ph/0508250 [hep-ph]].

2.3 重到轻流形状因子的定义

$b\to s\gamma$ 重子跃迁形状因子定义【1,2】：

$$\begin{aligned}\langle\mathcal{B}(p',s')|\bar{s}\,\gamma_\mu\,b|\Lambda_b(p,s)\rangle =& \bar{u}_\mathcal{B}(p',s')\Big[f^0(q^2)\frac{m_{\Lambda_b}-m_\mathcal{B}}{q^2}q_\mu \\ &+f^+(q^2)\frac{m_{\Lambda_b}+m_\mathcal{B}}{s_+}\left((p+p')_\mu-\frac{m^2_{\Lambda_b}-m^2_\mathcal{B}}{q^2}q_\mu\right) \\ &+f^T(q^2)\left(\gamma_\mu-\frac{2m_\mathcal{B}}{s_+}p_\mu-\frac{2m_{\Lambda_b}}{s_+}p'_\mu\right)\Big]u_{\Lambda_b}(p,s), \\ \langle\mathcal{B}(p',s')|\bar{s}\,\gamma_\mu\gamma_5\,b|\Lambda_b(p,s)\rangle =& -\bar{u}_\mathcal{B}(p',s')\gamma_5\Big[g^0(q^2)\frac{m_{\Lambda_b}+m_\mathcal{B}}{q^2}q_\mu \\ &+g^+(q^2)\frac{m_{\Lambda_b}-m_\mathcal{B}}{s_-}\left((p+p')_\mu-\frac{m^2_{\Lambda_b}-m^2_\mathcal{B}}{q^2}q_\mu\right) \\ &+g^T(q^2)\left(\gamma_\mu+\frac{2m_\mathcal{B}}{s_-}p_\mu-\frac{2m_{\Lambda_b}}{s_-}p'_\mu\right)\Big]u_{\Lambda_b}(p,s), \\ \langle\Lambda(p',s')|\bar{s}\,i\sigma_{\mu\nu}q^\nu\,b|\Lambda_b(p,s)\rangle =& -\bar{u}_\mathcal{B}(p',s')\Big[h^+(q^2)\frac{q^2}{s_+}\left((p+p')_\mu-\frac{m^2_{\Lambda_b}-m^2_\mathcal{B}}{q^2}q_\mu\right) \\ &+(m_{\Lambda_b}+m_\mathcal{B})\,h^T(q^2)\left(\gamma_\mu-\frac{2m_\mathcal{B}}{s_+}p_\mu-\frac{2m_{\Lambda_b}}{s_+}p'_\mu\right)\Big]u_{\Lambda_b}(p,s), \\ \langle\Lambda(p',s')|\bar{s}\,i\sigma_{\mu\nu}q^\nu\gamma_5\,b|\Lambda_b(p,s)\rangle =& -\bar{u}_\mathcal{B}(p',s')\gamma_5\Big[\tilde{h}^+(q^2)\frac{q^2}{s_-}\left((p+p')_\mu-\frac{m^2_{\Lambda_b}-m^2_\mathcal{B}}{q^2}q_\mu\right) \\ &+(m_{\Lambda_b}-m_\mathcal{B})\,\tilde{h}^T(q^2)\left(\gamma_\mu+\frac{2m_\mathcal{B}}{s_-}p_\mu-\frac{2m_{\Lambda_b}}{s_-}p'_\mu\right)\Big]u_{\Lambda_b}(p,s),\end{aligned} \tag{6}$$

SCET极限下，形状因子可以**约化成一个**【1】：

$$\langle\mathcal{B}(p',s')|\bar{\xi}W_c\Gamma h_v|\Lambda_b(p,s)\rangle=\xi(\bar{n}\cdot p')\bar{u}_\mathcal{B}(p',s')\Gamma u_{\Lambda_b}(p,s) \tag{7}$$

QCD→ SCET 匹配【3】：

$$\begin{aligned}J^{(9)}=&\bar{\psi}^{(6)}\Gamma h_v \\ =&-\bar{\xi}_c\left(g\not{A}^{(3)}_{\perp hc}\frac{1}{-in_+\overleftarrow{\partial}}g\not{A}^{(3)}_{\perp hc}+gn_-A^{(6)}_{hc}\right)\frac{1}{-in_-\overleftarrow{\partial}}\Gamma h_v\end{aligned} \tag{8}$$

积分掉 **hard-collinear** 胶子：

$$A^{(3)}_{\perp hc}=gT^A\frac{1}{in_+\partial in_-\partial}\{\bar{q}_s\gamma_\perp T^A\xi_c+h.c.\} \tag{9}$$

QCD形状因子：

$$\xi_\Lambda=f_{\Lambda_b}\Phi_{\Lambda_b}(x_i)\otimes J(x_i,y_i)\otimes f_\Lambda\Phi_\Lambda(y_i) \tag{10}$$

LP形状因子没有**端点发散**【3】。

【1】T. Feldmann and M. W. Y. Yip, Form factors for $\Lambda_b\to\Lambda$ transitions in the soft-collinear effective theory, Phys. Rev. D 85, 014035 (2012) [arXiv:1111.1844 [hep-ph]].
【2】Y. M. Wang and Y. L. Shen, Perturbative Corrections to $\Lambda_b\to\Lambda$ Form Factors from QCD Light-Cone Sum Rules, JHEP 02, 179 (2016) [arXiv:1511.09036 [hep-ph]].
【3】W. Wang, Factorization of heavy-to-light baryonic transitions in SCET, Phys. Lett. B 708, 119 (2012) [arXiv:1112.0237 [hep-ph]].
【4】 **L.Y.Li**, C.D.Lu, J. Wang and Y.B.Wei, $\Lambda_b\to Pl$ factorization in QCD, [arXiv:2401.11978 [hep-ph]].
【5】 Y.Zheng, J.N.Ding, D.H.Li, **L.Y.Li**, C.D.Lu and F.S.Yu, Invisible and Semi-invisible Decays of Bottom Baryons, [arXiv:2404.04337 [hep-ph]].

2.4 领头幂次的重到轻流形状因子

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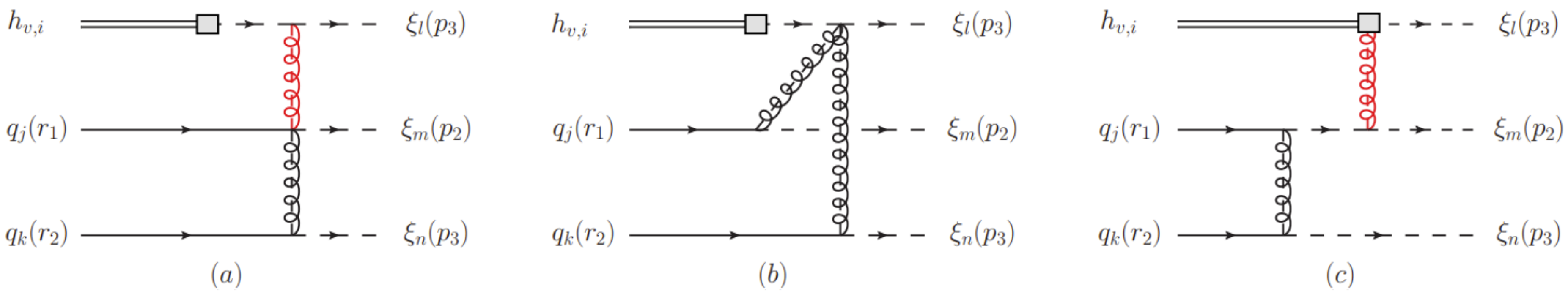


图6. $\Lambda_b \to \Lambda/p$ 领头幂次的费曼图

- 领头幂次关联函数:

$$T^F_{\mathcal{B},0} = \int d^4x \int d^4y \, \mathrm{T} \left\{ J^{(0)}(0), i\mathcal{L}^{(1)}_{\xi q}(x), i\mathcal{L}^{(1)}_{\xi q}(y) \right\} \tag{11}$$

$$J^{(0)}(0) = (\bar{\xi}_n W_n)\Gamma h_v(0) \qquad i\mathcal{L}^{(1)}_{\xi q}(x) = \bar{q}_s W_c^\dagger i\not{D}_{c\perp}\xi(x) - \bar{\xi}\overleftarrow{\not{D}}_{\perp c} W_c q_s(x),$$

- 领头扭度的 Λ_b重子LCDA:

$$\langle 0| [q_i(t_1 n)]_A \, [0, t_1 n] \, [q_j(t_2 n)]_B \, [0, t_2 n] \, [h_{v,k}(0)]_C \, |\Lambda_b(v)\rangle$$
$$= \frac{\epsilon_{ijk}}{4N_c!} f^{(2)}_{\Lambda_b}(\mu) \, [u_{\Lambda_b}(v)]_C \left[\frac{\not{\bar{n}}}{2} \gamma_5 \, C^T \right]_{BA} \int_0^\infty d\omega \, \omega \int_0^1 du \, e^{-i\omega(t_1 u + i t_2 \bar{u})} \, \psi_2(x, \omega) \tag{12}$$

- 领头扭度的 Λ重子/质子LCDA:

$$\langle \mathcal{B}(p)| [\bar{\xi}_l(t_1\bar{n})]_A \, [t_1\bar{n}, 0] \, [\bar{\xi}_m(t_2\bar{n})]_B \, [t_2\bar{n}, 0] \, [\bar{\xi}_n(0)]_C \, |0\rangle$$
$$= \frac{\epsilon_{lmn}}{4N_c!} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \, \delta(1 - x_1 - x_2 - x_3) \, e^{-i\bar{n}\cdot p(t_1 x_1 + i t_2 x_2)} \tag{13}$$
$$\times \left[V_1(x_i)(C\not{p})_{BA}(\bar{N}^+_u \gamma_5)_C + A_1(x_i)(C\gamma_5\not{p})_{BA}(\bar{N}^+_u)_C - T_1(x_i)(Ci\sigma_{\perp p})_{BA}(\bar{N}^+_u \gamma_5 \gamma_\perp)_C \right]$$

2.5 抽取 Jet 函数

$\Lambda_b \to \Lambda$ 形状因子的因子化形式:

$$\xi_\Lambda(\bar n\cdot p)=f^{(2)}_{\Lambda_b}\bar n\cdot p\int du\int d\omega\omega\int[\mathcal{D}x_i]\times\mathcal{J}_\Lambda(x_i,u,\omega,\mu)A_1(x_i)\psi_2(u,\omega) \tag{14}$$

Jet 函数:

$$\mathcal{J}_\Lambda(x_i,u,\omega,\mu)=\frac{g^2T_c}{4(\bar n\cdot p)^3}\left[\frac{4}{u\omega^3x_1(x_1+x_2)^2}-\frac{1}{u\omega^3x_1x_2(x_2+x_3)}\right]+(x_1\leftrightarrow x_2,u\leftrightarrow\bar u) \tag{15}$$

$\Lambda_b \to p$ 形状因子的因子化形式:

$$\xi_p(\bar n\cdot p)=f^{(2)}_{\Lambda_b}\bar n\cdot p\int du\int d\omega\omega\int[\mathcal{D}x_i]\big[\mathcal{J}_V(u,\omega,x_i,\mu)V_1(x_i)+\mathcal{J}_A(u,\omega,x_i,\mu)A_1(x_i)+\mathcal{J}_T(u,\omega,x_i,\mu)T_1(x_i)\big]\psi_2(u,\omega) \tag{16}$$

Jet 函数:

$$\begin{aligned}\mathcal{J}_V(u,\omega,x_i,\mu)&=\frac{g^4T_c}{4(\bar n\cdot p)^3}\left[\frac{2}{\bar u\omega^3x_3(x_2+x_3)^2}+\frac{2}{u\omega^3x_2(x_2+x_3)^2}+\frac{1}{u\bar u\omega^3x_2x_3(x_1+x_3)}\right]\\ \mathcal{J}_T(u,\omega,x_i,\mu)&=\frac{2g^4T_c}{4(\bar n\cdot p)^3}\left[\frac{2}{\bar u\omega^3x_3(x_2+x_3)^2}+\frac{2}{u\omega^3x_2(x_2+x_3)^2}+\frac{1}{u\bar u\omega^3x_2x_3(x_2+x_3)}\right]\\ \mathcal{J}_A(u,\omega,x_i,\mu)&=-\mathcal{J}_V(u,\omega,x_i,\mu)\end{aligned} \tag{17}$$

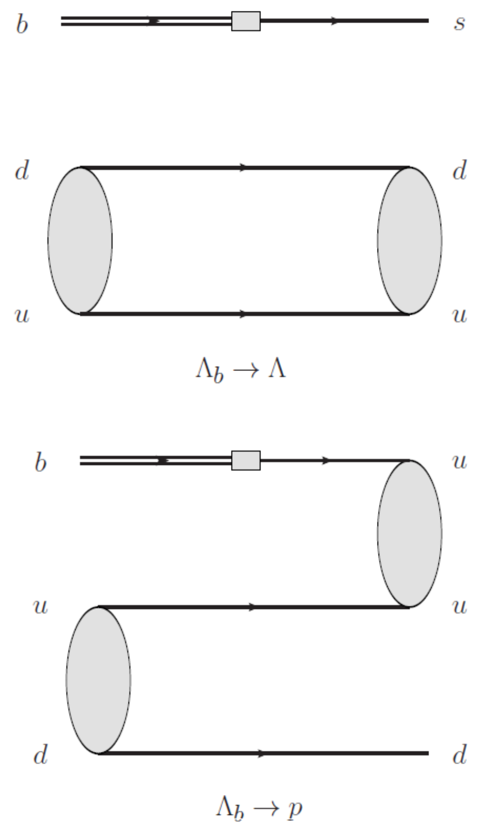


图7. 不同末态的di-quark 结构

Λ重子/质子波函数:

$$\begin{aligned}V_1(x_i,\mu)&=120\,x_1x_2x_3(x_1-x_2)\,\phi_3^-(\mu)\,,\\ A_1(x_i,\mu)&=-120\,x_1x_2x_3[\phi_3^0(\mu)+\phi_3^+(\mu)(1-x_3)]\,,\\ T_1(x_i,\mu)&=120\,x_1x_2x_3[t_1^0(\mu)+t_1^-(\mu)(x_1-x_2)+t^+(\mu)(1-3x_3)]\,,\end{aligned} \tag{18}$$

$$\begin{aligned}V_1(x_i,\mu)&=120\,f_p(\mu)\,x_1x_2x_3\left[1+\frac{7}{2}(1-3V_1^d)(1-3x_3)\right],\\ A_1(x_i,\mu)&=120\,f_p(\mu)\,x_1x_2x_3\left[\frac{21}{2}A_1^u(x_2-x_1)\right],\\ T_1(x_i,\mu)&=120\,f_p(\mu)\,x_1x_2x_3\left[1-\frac{7}{4}(1-3V_1^d-3A_1^u)(1-3x_3)\right],\end{aligned} \tag{19}$$

2.6 领头幂次形状因子的数值结果

Λ_b重子波函数：

Exponential 模型：
$$\psi_2(\omega,u)=\frac{\omega^2u(1-u)}{\omega_0^4}e^{-\omega/\omega_0} \qquad (20)$$

Gegenbauer 模型：
$$\psi_2(\omega,u)=u\bar{u}\omega^2\left[\frac{1}{\epsilon_0^4}+a_2C_2^{3/2}(2u-1)\frac{1}{\epsilon_1^4}e^{-\omega/\epsilon_1}\right] \qquad (21)$$

表2. LP形状因子q^2=0

SCET	Gegenbauer model		Exponential model	
$\xi_\Lambda(\bar{n}\cdot p)$	$8.5^{+17.2}_{-6.6}\times10^{-3}$	This work	$3.1^{+1.7}_{-1.1}\times10^{-3}$	This work
	$-1.2^{+0.9}_{-2.3}\times10^{-2}$	[1]	-	
$\xi_p(\bar{n}\cdot p)$	$3.7^{+6.7}_{-2.7}\times10^{-2}$	This work	$1.7^{+0.7}_{-0.5}\times10^{-2}$	This work
LCSR	Gegenbauer model		Exponential model	
$\xi_\Lambda(\bar{n}\cdot p)$	-		0.38	[2]
	-		0.18	[3]
$\xi_p(\bar{n}\cdot p)$	-		0.277 ± 0.125	[4]

量级差变大

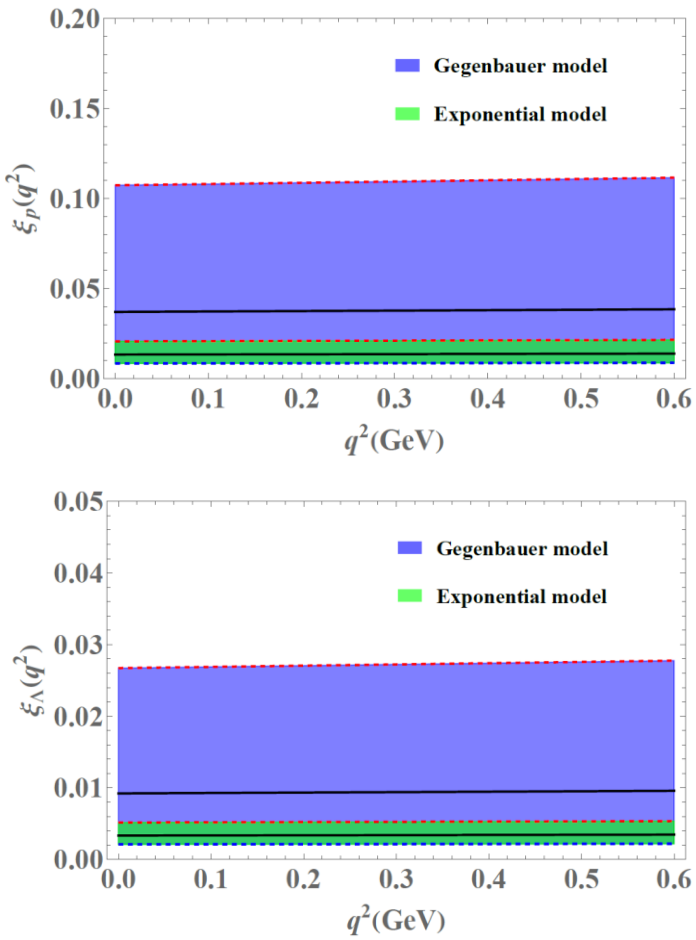


图8. 形状因子 q^2 依赖

Gegenbauer 模型的**误差较大**，Exponential 模型的**精度较高**。形状因子**依赖非微扰模型**。

【1】W. Wang, Factorization of heavy-to-light baryonic transitions in SCET, Phys. Lett. B 708, 119 (2012) [arXiv:1112.0237 [hep-ph]].
【2】T. Feldmann and M. W. Y. Yip, Form factors for $\Lambda_b\to\Lambda$ transitions in the soft-collinear effective theory, Phys. Rev. D 85, 014035 (2012) [arXiv:1111.1844 [hep-ph]].
【3】Y. M. Wang and Y. L. Shen, Perturbative Corrections to $\Lambda_b\to\Lambda$ Form Factors from QCD Light-Cone Sum Rules, JHEP 02, 179 (2016) [arXiv:1511.09036 [hep-ph]].
【4】K. S. Huang, W. Liu, Y. L. Shen, and F. S. Yu, $\Lambda_b\to p, N*$ (1535) form factors from QCD Light-cone sum rules, [arXiv:2205.06095 [hep-ph]]..

三、次领头幂次关联函数与端点发散

3.1 $\Lambda_b \to p$ 次领头幂次计算前提

次领头幂次SCET重到轻流和拉氏量：

$$
\begin{aligned}
J^{(0)} &= \bar\xi_n W \Gamma h_v \\
J^{(1a)} &= -\bar\xi_n \overleftarrow{\not D}_{\perp c}\frac{\not{\bar n}}{2}\frac{1}{i\bar n\cdot \overleftarrow{D}_c}\Gamma h_v \\
J^{(1b)} &= -\frac{1}{n\cdot v m_b}\bar\xi_n \Gamma\frac{\not n}{2}\frac{1}{i\bar n\cdot D_c} ig\not B_{\perp c} W h_v
\end{aligned}
\qquad
\begin{aligned}
\mathcal{L}^{(1)}_{\xi q} &= ig\bar\xi_n \frac{1}{i\bar n\cdot D_n}\not B_\perp W_n q_{us} + \text{h.c.}\,, \\
\mathcal{L}^{(2a)}_{\xi q} &= ig\bar\xi_n \frac{1}{i\bar n\cdot D_n}\not M W_n q_{us} + \text{h.c.}\,, \\
\mathcal{L}^{(2b)}_{\xi q} &= ig\bar\xi_n i\not D_{n\perp}\frac{1}{i\bar n\cdot D_n}\not M W_n q_{us} + \text{h.c.}\,,
\end{aligned}
\qquad
\begin{aligned}
\mathcal{L}^{(0)}_{\xi\xi} &= \bar\xi_n\left(in\cdot D + i\not D_{n\perp}\frac{1}{i\bar n\cdot D_n} i\not D_{n\perp}\right)\frac{\not{\bar n}}{2}\xi_n \\
\mathcal{L}^{(1)}_{\xi\xi} &= \bar\xi_n i\not D_{\perp us}\frac{1}{i\bar n\cdot D_c} i\not D_{\perp c}\frac{\not{\bar n}}{2}\xi_n + h.c.
\end{aligned}
\tag{22}
$$

重夸克极限：$\boldsymbol{p}_\perp, \boldsymbol{r}_\perp \to 0$，需要引入高twist的LCDA。

包含横向动量 $p_\perp$ 依赖的质子 LCDA

$$
\begin{aligned}
&\langle p|[\bar\xi_i(z_1)]_\alpha[\bar\xi_j(z_2)]_\beta[\bar\xi_k(z_3)]_\gamma|0\rangle \\
=&\frac{\epsilon_{ijk}}{4N_c!}\Big[V_1^*(C\not p)_{\beta\alpha}(\bar N_u^+\gamma_5)_\gamma + V_{2,1}^*\frac{M}{2}(C\not p)_{\beta\alpha}(\bar N_u^+\gamma_5\not\partial_{p_{1\perp}})_\gamma \\
&+V_{2,2}^*\frac{M}{2}(C\not p)_{\beta\alpha}(\bar N_u^+\gamma_5\not\partial_{p_{1\perp}})_\gamma + A_1^*(C\gamma_5\not p)_{\beta\alpha}(\bar N_u^+)_\gamma \\
&+A_{2,1}^*(C\gamma_5\not p)_{\beta\alpha}(\bar N_u^+\not\partial_{p_{1\perp}})_\gamma + A_{2,2}^*(C\gamma_5\not p)_{\beta\alpha}(\bar N_u^+\not\partial_{p_{2\perp}})_\gamma \\
&+T_1^*(Ci\sigma_{\perp p})_{\beta\alpha}(\bar N_u^+\gamma_5\gamma^\perp)_\gamma - T_{2,1}^*\frac{M}{2}\partial^\mu_{p_{1\perp}}(Ci\sigma_{\mu p})_{\beta\alpha}(\bar N_u^+\gamma_5)_\gamma \\
&-T_{2,2}^*\frac{M}{2}\partial^\mu_{p_{2\perp}}(Ci\sigma_{\mu p})_{\beta\alpha}(\bar N_u^+\gamma_5)_\gamma + T_{4,1}^*\frac{M}{2}\partial_{p_{1\perp}\mu}(Ci\sigma_{\perp p})_{\beta\alpha}(\bar N_u^+\gamma_5\sigma^{\perp\mu})_\gamma \\
&+T_{4,2}^*\frac{M}{2}\partial_{p_{2\perp}\mu}(Ci\sigma_{\perp p})_{\beta\alpha}(\bar N_u^+\gamma_5\sigma^{\perp\mu})_\gamma\Big]
\end{aligned}
\tag{23}
$$

包含横向动量 $r_\perp$ 依赖的 Λ_b-LCDA

$$
\begin{aligned}
&\langle 0|q^a_\alpha(r_1)q^b_\beta(r_2)h^c_{v,\gamma}(0)|\Lambda_b\rangle \\
=&\frac{\epsilon^{abc}}{4N_c!}\left\{f^{(1)}_{\Lambda_b}\left[\tilde M^{(1)}(v,z_1,z_2)\gamma_5 C^T\right]_{\beta\alpha} + f^{(2)}_{\Lambda_b}\left[\tilde M^{(1)}(v,z_1,z_2)\gamma_5 C^T\right]_{\beta\alpha}u_{\Lambda_b}(v,s)\right\}
\end{aligned}
\tag{24}
$$

动量空间的projector

$$
\begin{aligned}
M^{(1)}(\omega_1,\omega_2) =& \frac{\not{\bar n}\not n}{4}\phi_3^{+-}(\omega_1,\omega_2) + \frac{\not n\not{\bar n}}{4}\phi_3^{-+}(\omega_1,\omega_2) \\
&-\frac{1}{2}\gamma_{\perp\mu}\int_0^{\omega_1} d\eta_1\phi_3^{(i)}(\eta_1,\omega_2)\not n\frac{\partial}{\partial k_{1\perp\mu}} - \frac{1}{2}\gamma_{\perp\mu}\int_0^{\omega_1} d\eta_2\phi_3^{(ii)}(\omega_1,\eta_2)\not n\frac{\partial}{\partial k_{2\perp\mu}} \\
&-\frac{1}{2}\gamma_{\perp\mu}\int_0^{\omega_1} d\eta_1\phi_Y(\eta_1,\omega_2)\not n\frac{\partial}{\partial k_{1\perp\mu}} - \frac{1}{2}\gamma_{\perp\mu}\int_0^{\omega_1} d\eta_1\phi_Y(\omega_1,\eta_2)\not n\frac{\partial}{\partial k_{2\perp\mu}}
\end{aligned}
\tag{25}
$$

$M^{(2)}(\omega_1,\omega_2) = \ldots\ldots$

3.2 重到轻流关联函数

介子重到轻流形状因子[1]：

纵向动量依赖

Leading power

$$T_0^F = \mathrm{T}\left[J^{(0)}, i\mathcal{L}_{\xi q}^{(1)}\right], \quad = 0$$

Next leading power

$$T_1^F = \mathrm{T}\left[J^{(1a)}, i\mathcal{L}_{\xi q}^{(1)}\right],$$
$$T_2^F = \mathrm{T}\left[J^{(1b)}, i\mathcal{L}_{\xi q}^{(1)}\right],$$
（微扰）

$$T_3^F = \mathrm{T}\left[J^{(0)}, i\mathcal{L}_{\xi q}^{(2b)}\right],$$
$$T_4^{NF} = \mathrm{T}\left[J^{(0)}, i\mathcal{L}_{\xi q}^{(2a)}\right],$$
$$T_5^{NF} = \mathrm{T}\left[J^{(0)}, i\mathcal{L}_{\xi\xi}^{(1)}, i\mathcal{L}_{\xi q}^{(1)}\right],$$
$$T_6^{NF} = \mathrm{T}\left[J^{(0)}, i\mathcal{L}_{ng}^{(1)}, i\mathcal{L}_{\xi q}^{(1)}\right].$$
（非微扰）

LP 形状因子

推广 → 完备

重子重到轻流形状因子：

纵向动量依赖

Leading power

$$T_{B,0}^F = \mathrm{T}\left[J^{(0)}, i\mathcal{L}_{\xi q}^{(1)}, i\mathcal{L}_{\xi q}^{(1)}\right], \quad \neq 0$$
（微扰）LP 形状因子

纵向 + 横向动量依赖

Next leading power

$$T_{B,1}^F = \mathrm{T}\left[J^{(1a)}, i\mathcal{L}_{\xi q}^{(1)}, i\mathcal{L}_{\xi q}^{(1)}\right],$$
$$T_{B,2}^F = \mathrm{T}\left[J^{(1b)}, i\mathcal{L}_{\xi q}^{(1)}, i\mathcal{L}_{\xi q}^{(1)}\right],$$
$$T_{B,3}^F = \mathrm{T}\left[J^{(0)}, i\mathcal{L}_{\xi q}^{(1)}, i\mathcal{L}_{\xi q}^{(2b)}\right],$$
$$T_{B,4}^{NF} = \mathrm{T}\left[J^{(0)}, i\mathcal{L}_{\xi q}^{(1)}, i\mathcal{L}_{\xi q}^{(2a)}\right],$$
$$T_{B,5}^{NF} = \mathrm{T}\left[J^{(0)}, i\mathcal{L}_{\xi\xi}^{(1)}, i\mathcal{L}_{\xi q}^{(1)}, i\mathcal{L}_{\xi q}^{(1)}\right],$$
$$T_{B,6}^{NF} = \mathrm{T}\left[J^{(0)}, i\mathcal{L}_{ng}^{(1)}, i\mathcal{L}_{\xi q}^{(1)}, i\mathcal{L}_{\xi q}^{(1)}\right].$$
NLP非微扰？

SCET 重到轻流形状因子的因子化：

$$\boldsymbol{B \to V}: \quad f(q^2) = \underbrace{C \cdot \xi_M}_{\text{非微扰}} + \underbrace{\phi_B \otimes T \otimes \phi_M}_{\text{微扰}}$$

微扰：非微扰： = 1：10 [2]

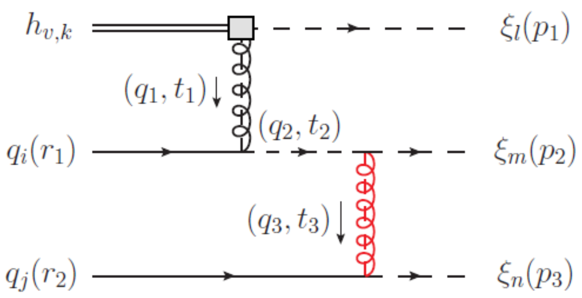
(26)

$$\boldsymbol{\Lambda_b \to \Lambda/p}: \quad f(q^2) = \underbrace{\phi_{\Lambda_b} \otimes T \otimes \phi_\Lambda}_{\text{微扰}}$$

重子非微扰贡献？NLP？

【1】C.W.Bauer, D. Pirjol and I.W.Stewart, Factorization and endpoint singularities in heavy to light decays, Phys. Rev. D 67 (2003), 071502 [arXiv:hep-ph/0211069 [hep-ph]].
【2】H.Deng, J.Gao, **L.Y.Li**, C.D.Lu, Y.L.Shen and C.X.Yu, Study on pure annihilation type $B \to V\gamma$ decays PhysRevD.103.076004 [arXiv:2101.01344 [hep-ph]].

3.3 次领头幂次端点发散



$\bar{n}\cdot q_1=\bar{n}\cdot p_2+\bar{n}\cdot p_3$ $\quad n\cdot t_1=-n\cdot r_1-n\cdot r_2$ $\quad q_{1\perp}=p_{2\perp}+p_{3\perp}-r_{1\perp}-r_{2\perp}$

$\bar{n}\cdot q_2=\bar{n}\cdot p_2+\bar{n}\cdot p_3$ $\quad n\cdot t_2=-n\cdot r_2$ $\quad q_{2\perp}=p_{2\perp}+p_{3\perp}-r_{2\perp}$

$\bar{n}\cdot q_3=\bar{n}\cdot p_3$ $\quad n\cdot t_3=-n\cdot r_2$ $\quad q_{3\perp}=p_{3\perp}-r_{2\perp}$

图9. $J^{(0)}$ 流构建的NLP关联函数 $T^F_{B,1}$ 的费曼图之一

振幅：

$$i\mathcal{M}_{T_1}=f^{(2)}_{\Lambda_b}\bar{n}\cdot p\int_0^\infty d\omega\omega\int_0^1 du\int_0^1[\mathcal{D}x_i]\times\mathcal{J}_{NLP}(u,\omega,x_i,\mu)$$
$$\times[V_1(x_i,\mu)+A_1(x_i,\mu)]B(u,\omega)\times\bar{N}^+_u\frac{\not{\bar{n}}}{2}\Gamma u_{\Lambda_b} \tag{26}$$

Jet函数为：

$$\mathcal{J}_{NLP}(u,\omega,x_i,\mu)=\frac{ig^4T_c}{8m_b(\bar{n}\cdot p)^3}\frac{1}{u\bar{u}^2\omega^3x_3^2x_2} \tag{27}$$

质子波函数与Jet函数在**端点处**的行为：

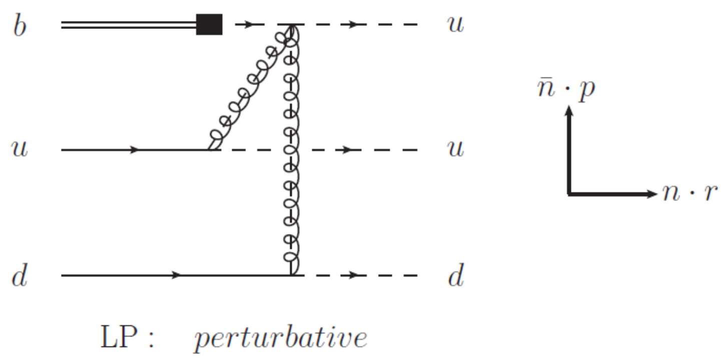
$$V_1(x_i)=A_1(x_i)\overset{x_1,x_2\to0}{\sim}x_1x_2 \tag{28}$$

$$\mathcal{J}_{NLP}(x_i,u,\omega,\mu)\overset{x_1,x_2\to0}{\sim}\frac{1}{x_1^2(x_1+x_2)} \tag{29}$$

$J^{(0)}$，$J^{(1a)}$和$J^{(1b)}$流对应的重子NLP的关联函数 $T^F_{B,1}\sim T^{NF}_{B,5}$ 均存在**端点发散**

LP：纵向动量依赖：

NLP：纵向 + 横向动量依赖：



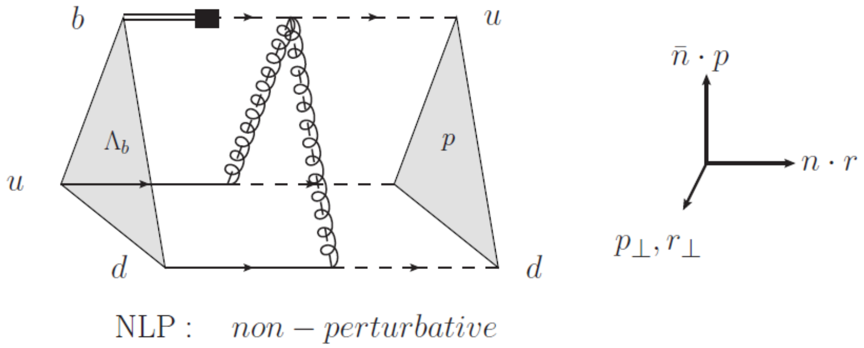
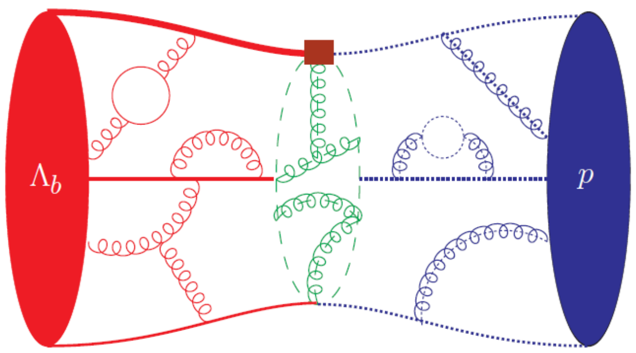


图10. 相同费曼图不同维度动量的贡献



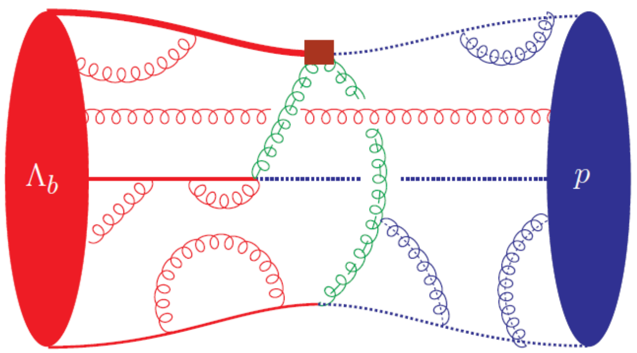


图10. 重子 $J^{(0)}$ 流因子化因子化图像

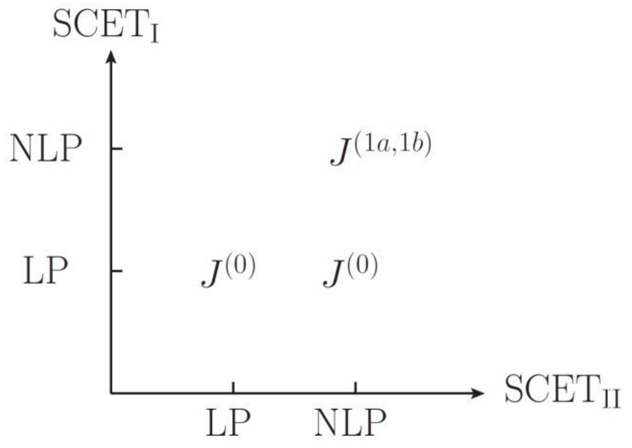


图11. 重到轻流的power counting

在**共线因子化**框架下，通过 $\mathrm{SCET_I}$ 的流定义**软形状因子**，吸收**端点发散**【1】效应：

$$\langle p|J^{(0)}|\Lambda_b\rangle = [\zeta^{(0)}(\bar{n}\cdot p) + \xi(\bar{n}\cdot p)]\,\bar{N}_u^+\Gamma u_{\Lambda_b}$$
$$\langle p|J^{(1a)}|\Lambda_b\rangle = \zeta^{(1a)}(\bar{n}\cdot p)\,\bar{N}_u^+\frac{\not{n}}{2}\Gamma u_{\Lambda_b}$$
$$\langle p|J^{(1b)}|\Lambda_b\rangle = \zeta^{(1b)}(\bar{n}\cdot p)\,\bar{N}_u^+\Gamma\frac{\not{n}}{2}u_{\Lambda_b} \quad (30)$$

LP： $J^{(0)}$ **微扰**

NLP： $J^{(0)}$，$J^{(1a)}$，$J^{(1b)}$ **非微扰**

$\zeta : \zeta^{(1a)} : \zeta^{(1b)} = 1 : \lambda : \lambda$

与 $\mathrm{SCET_I}$ 的 power 一致

$$\xi(\bar{n}\cdot p) = f^{(2)}_{\Lambda_b}\bar{n}\cdot p\int du\int d\omega\omega\int[\mathcal{D}x_i]\left[\mathcal{J}_V(u,\omega,x_i,\mu)V_1(x_i) + \mathcal{J}_A(u,\omega,x_i,\mu)A_1(x_i) + \mathcal{J}_T(u,\omega,x_i,\mu)T_1(x_i)\right]\psi_2(u,\omega) \quad (31)$$

将**软形状因子**的定义从**介子**推广到**重子**中。对比LCSR【2】的结果，**非微扰的效应**占主导贡献。

【1】Z.L.Liu, B.Mecaj, M.Neubert and X.Wang, Factorization at subleading power, Sudakov resummation, and endpoint divergences in soft-collinear effective theory, Phys. Rev. D 104 (2021) no.1, 014004 [arXiv:2009.04456 [hep-ph]].
【2】K. S. Huang, W. Liu, Y. L. Shen, and F. S. Yu, $\Lambda_b \to$ p, N∗ (1535) form factors from QCD Light-cone sum rules, [arXiv:2205.06095 [hep-ph]].

总结

1. 我们在SCET框架下，计算了领头幂次的重到轻流形状因子，抽取 Jet 函数。

2. 我们对 $\Lambda_b \to p$ 次领头幂次的关联函数进行计算，发现均存在端点发散。

3. 我们将软形状因子，从介子推广到重子中。

谢谢!