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正德厚生
笃学敏行

底粲介子衰变和极化的理论研究

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2024年4月19-22日　　重味物理与QCD研讨会@青岛

Background

- **Bc meson physics**

  study of Bc meson family is poor

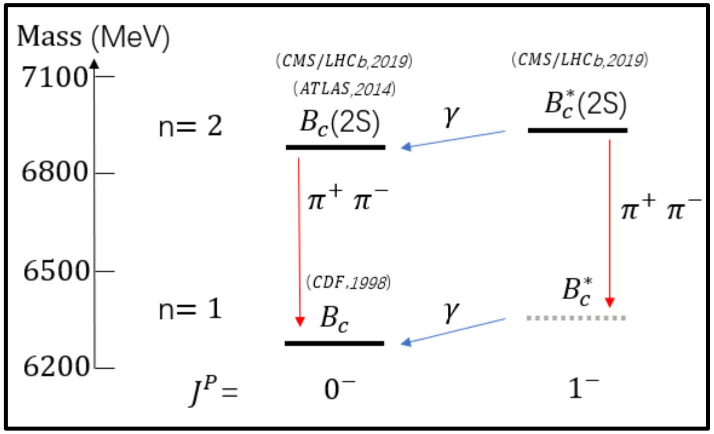
- **Polarization analysis in cascade decay process**

  provide new views and more information

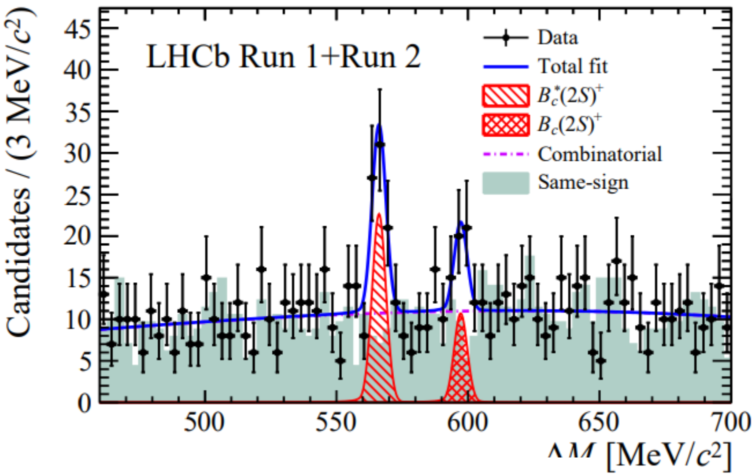
- **Quantum spin entanglement in Bc decays**

  1935 EPR Paradox

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle_A|-\rangle_B - |-\rangle_A|+\rangle_B) ,$$

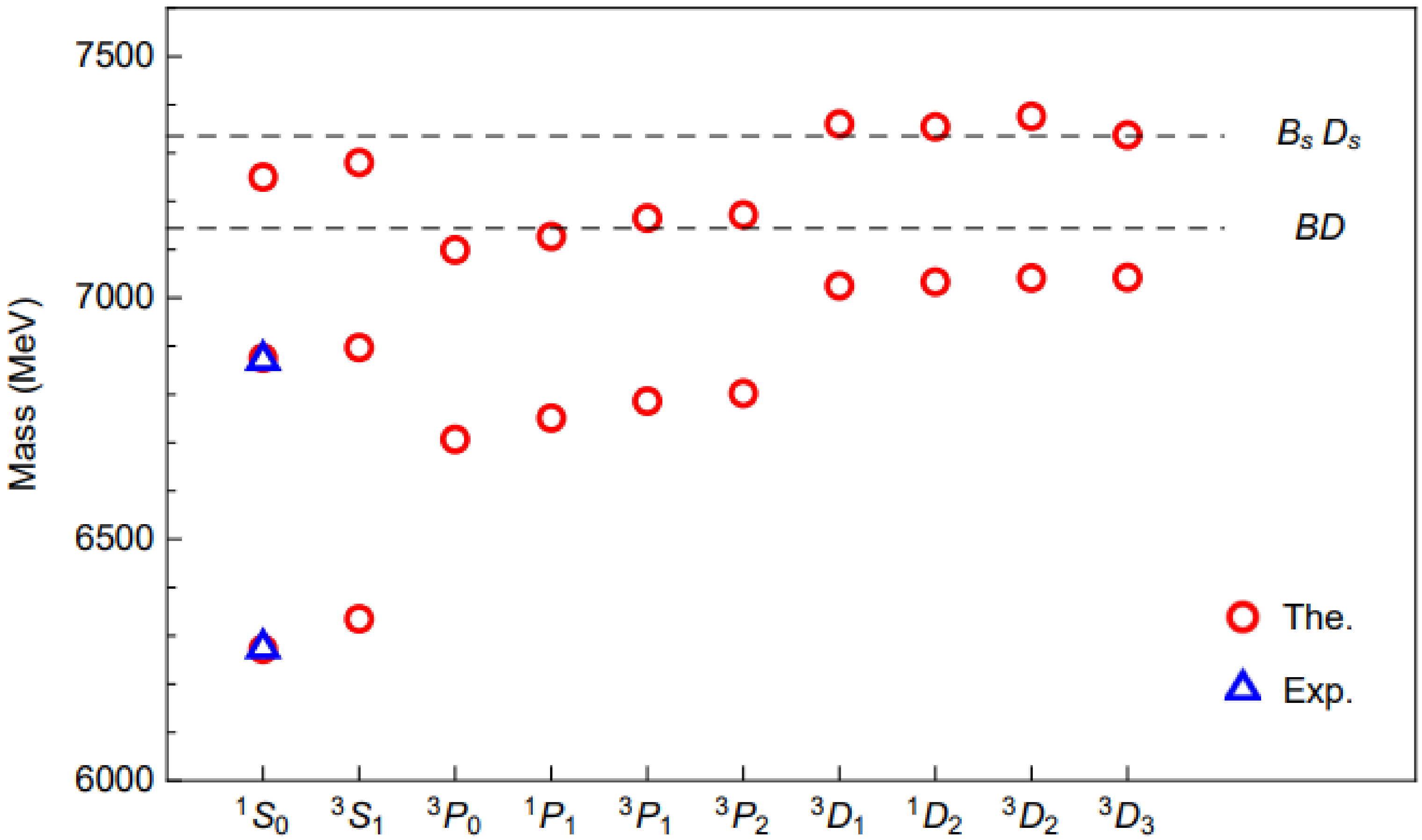


Bc meson family



$$\Delta M = M(B_c^+\pi^+\pi^-) - M(B_c^+)$$

Bc meson family spectrum

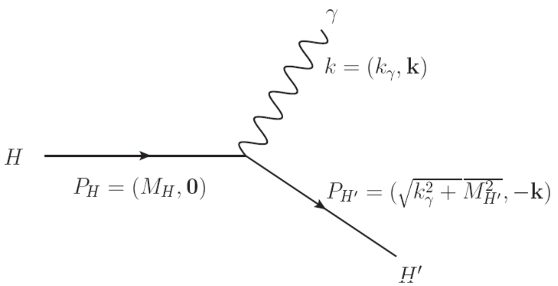


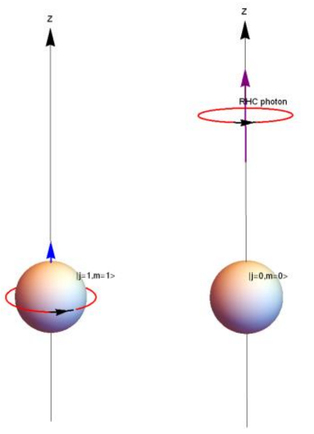
nonrelativistic potential model + Coupled channel analysis

Hao-Zhu, 2402.18898

Vector Bc* meson decay width

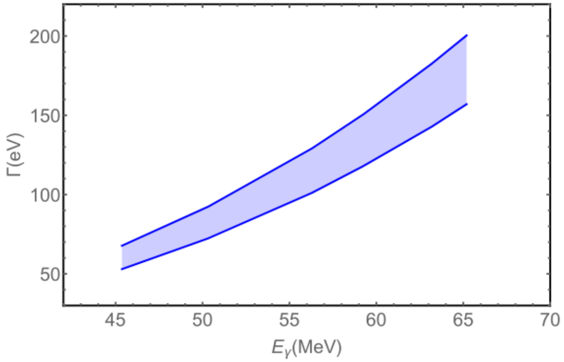
- **Bc* (1S) major (99.99%) electromagnetic decays to Bc(1S): M1 transition**





- **We generalize the pNRQCD to unequal mass case and obtain the effective Lagrangian**

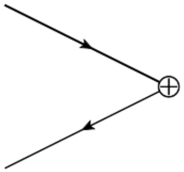
$$
\begin{aligned}
\mathcal{L}_{\gamma\text{pNRQCD}} = \int d^3r\, \mathrm{Tr}\Bigg[& e\frac{e_Q - e'_Q}{2} V_A^{\text{em}} \mathrm{S}^\dagger \mathbf{r}\cdot\mathbf{E}^{\text{em}}\mathrm{S} \\
& + e\left(\frac{e_Q m'_Q - e'_Q m_Q}{4 m_Q m'_Q}\right)\Bigg[V_S^{\frac{\sigma\cdot B}{m}} \left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\text{em}} \right\}\mathrm{S} \\
& + \frac{1}{8} V_S^{(r\cdot\nabla)^2 \frac{\sigma\cdot B}{m}} \left\{ \mathrm{S}^\dagger, \mathbf{r}^i\mathbf{r}^j \left(\boldsymbol{\nabla}^i\nabla^j \boldsymbol{\sigma}\cdot\mathbf{B}^{\text{em}}\right)\right\}\mathrm{S} \\
& + V_O^{\frac{\sigma\cdot B}{m}} \left\{\mathrm{O}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\text{em}}\right\}\mathrm{O}\Bigg] \\
& + e\left(\frac{e_Q m_{Q'}^2 - e'_Q m_Q^2}{32 m_Q^2 m_{Q'}^2}\right)\Bigg[4\frac{V_S^{\frac{\sigma\cdot B}{m^2}}}{r}\left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\text{em}}\right\}\mathrm{S} \\
& + 4\frac{V_S^{\frac{\sigma\cdot(r\times r\times B)}{m^2}}}{r}\left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot[\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\mathbf{B}^{\text{em}})]\right\}\mathrm{S} \\
& - V_S^{\frac{\sigma\cdot\nabla\times E}{m^2}}\left[\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot[-i\boldsymbol{\nabla}\times, \mathbf{E}^{\text{em}}]\right]\mathrm{S} \\
& - V_S^{\frac{\sigma\cdot\nabla_r\times r\cdot\nabla E}{m^2}}\left[\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\left[-i\boldsymbol{\nabla}_r\times, \mathbf{r}^i\left(\boldsymbol{\nabla}^i\mathbf{E}^{\text{em}}\right)\right]\right]\mathrm{S}\Bigg] \\
& + e\left(\frac{e_Q m_{Q'}^3 - e'_Q m_Q^3}{8 m_Q^3 m_{Q'}^3}\right)\Bigg[V_S^{\frac{\nabla_r^2\sigma\cdot B}{m^3}}\left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\text{em}}\right\}\nabla_r^2\,\mathrm{S} \\
& + V_S^{\frac{(\nabla r\cdot\sigma)(\nabla r\cdot B)}{m^3}}\left\{ \mathrm{S}^\dagger, \boldsymbol{\sigma}^i\mathbf{B}^{\text{em}\,j}\right\}\boldsymbol{\nabla}_r^i\nabla_r^j\,\mathrm{S}\Bigg]\Bigg],
\end{aligned}
$$



Vector Bc* meson decay constant

- **Bc* decay constants in QCD**

$$\left\langle 0 \left| \bar{b}\gamma^{\mu} c \right| B_c^*(P,\varepsilon) \right\rangle = f_{B_c^*}^{v} m_{B_c^*} \varepsilon^{\mu},$$



(a) LO

- **Bc* decay constants in NRQCD**

mathing coefficients — NRQCD LDMEs

$$f_{B_c^*}^{v} = \sqrt{\frac{2}{m_{B_c^*}}} \underline{C_v(m_b, m_c, \mu_f)} \underline{\left\langle 0 \left| \chi_b^{\dagger} \sigma \cdot \varepsilon \psi_c \right| B_c^*(\mathbf{P}) \right\rangle (\mu_f)} + O(v^2)$$

- **Matching Formulae**

Braaten-Fleming,PRD52,181(1995);
Lee-Sang-Kim,JHEP01,113(2011)

$$Z_J Z_{2,b}^{\frac{1}{2}} Z_{2,c}^{\frac{1}{2}} \Gamma_J = C_J \tilde{Z}_J^{-1} \tilde{Z}_{2,b}^{\frac{1}{2}} \tilde{Z}_{2,c}^{\frac{1}{2}} \tilde{\Gamma}_J$$

$\tilde{Z}_J$:NRQCD $\overline{\text{MS}}$ current renormalization constants

Matching coefficients up to three loops

- **for vector current**

$$\mathcal{C} = 1 - 2.29\left(\frac{\alpha_s^{(n_l)}}{\pi}\right) - 35.44\left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^2 - 1686.27\left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^3 + \mathcal{O}(\alpha_s^4),$$

for $n_l = 3, n_c = 1, n_b = 0,$

Sang-Zhang-Zhou, arXiv:2210.02979

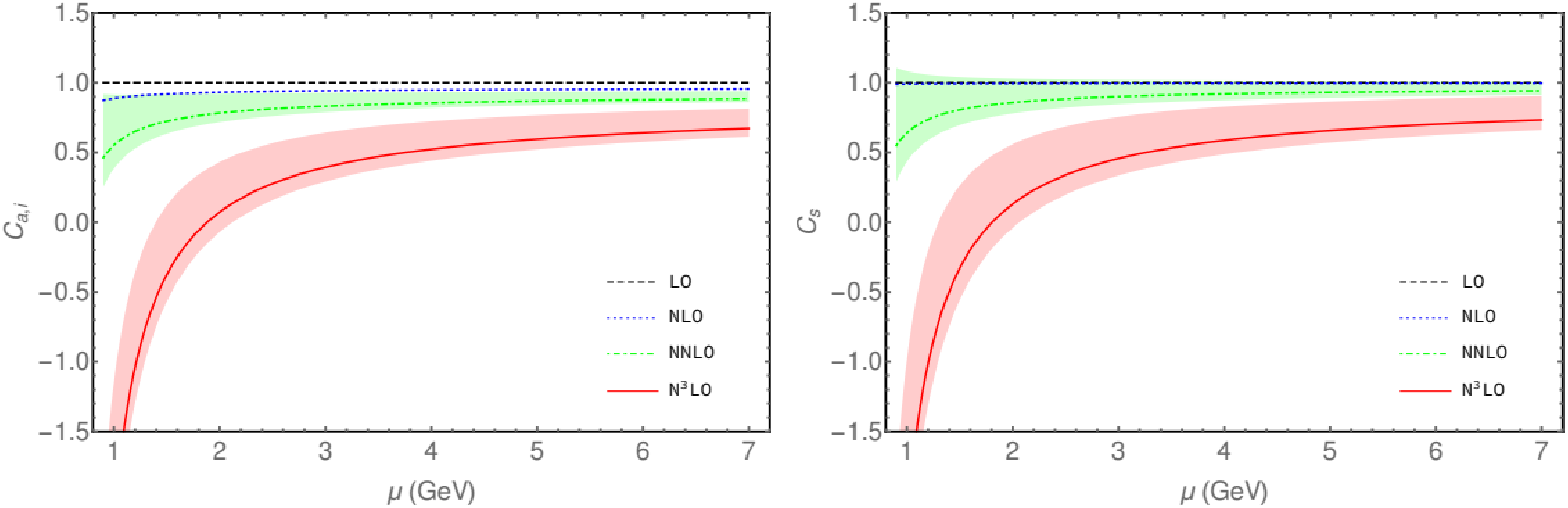
- **for pseudoscalar current**

$$\mathcal{C}(x_{\text{phys}}) = 1 - 1.62623\left(\frac{\alpha_s^{(n_l)}(m_r)}{\pi}\right) - 6.51043\left(\frac{\alpha_s^{(n_l)}(m_r)}{\pi}\right)^2 - 1520.59\left(\frac{\alpha_s^{(n_l)}(m_r)}{\pi}\right)^3 + \mathcal{O}(\alpha_s^4)$$

Feng-Jia-Mo-Pan-Sang-Zhang, arXiv:2208.04302

for axial-vector and scalar currents

- **Matching coefficients for axial-vector and scalar up to three loops**



Nonconvergence behaviors also in other two currents

Multi-loop integral calculation performed by AMFlow (Ma et al)

Sub-leading Contribution

- **Relativistic corrections**

$$\langle 0|\overline{Q_1}\gamma^5 Q_2\,|Q_2\overline{Q_1}\rangle_{\rm QCD}$$

$$= \sqrt{2M_H}\left[C_0^P\left\langle 0\left|\chi_1^\dagger\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle_{\rm NRQCD}+C_2^P\left\langle 0\left|(\mathbf{D}\chi_1)^\dagger\cdot\mathbf{D}\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle_{\rm NRQCD}+\cdots\right]$$

Employing EOM: $\left\langle 0\left|(\mathbf{D}\chi_1)^\dagger\mathbf{D}\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle = -2m_r E\left\langle 0\left|\chi_1^\dagger\psi_2\right|Q_2\overline{Q_1}(\mathbf{p})\right\rangle.$

$$f_{B_c^*} = 2\sqrt{\frac{N_c}{m_{B_c^*}}}\left[\mathcal{C}_v+\frac{d_v E_{B_c^*}}{12}\left(\frac{8}{M}-\frac{3}{m_r}\right)\right]|\Psi_{B_c^*}(0)|,$$

$$f_{B_c} = 2\sqrt{\frac{N_c}{m_{B_c}}}\left[\mathcal{C}_p-\frac{d_p E_{B_c}}{4m_r}\right]|\Psi_{B_c}(0)|,$$

Wave function scale dependence

- Wave function at origin

For Power-law potential $$V(r) = Ar^a + C$$

Exact solution $$|\psi_\mu^n(0)|^2 = f(n,a)(A\mu)^{3/(2+a)}$$

Scale relation $$|\Psi_{B_c^*}(0)| = |\Psi_{J/\psi}(0)|^{1-y}\ |\Psi_\Upsilon(0)|^y,$$

$$y = y_c = \ln((1 + m_c/m_b)/2)/\ln(m_c/m_b)$$

Collins-Imbo-King-Martell, PLB 393 (1997) 155–160

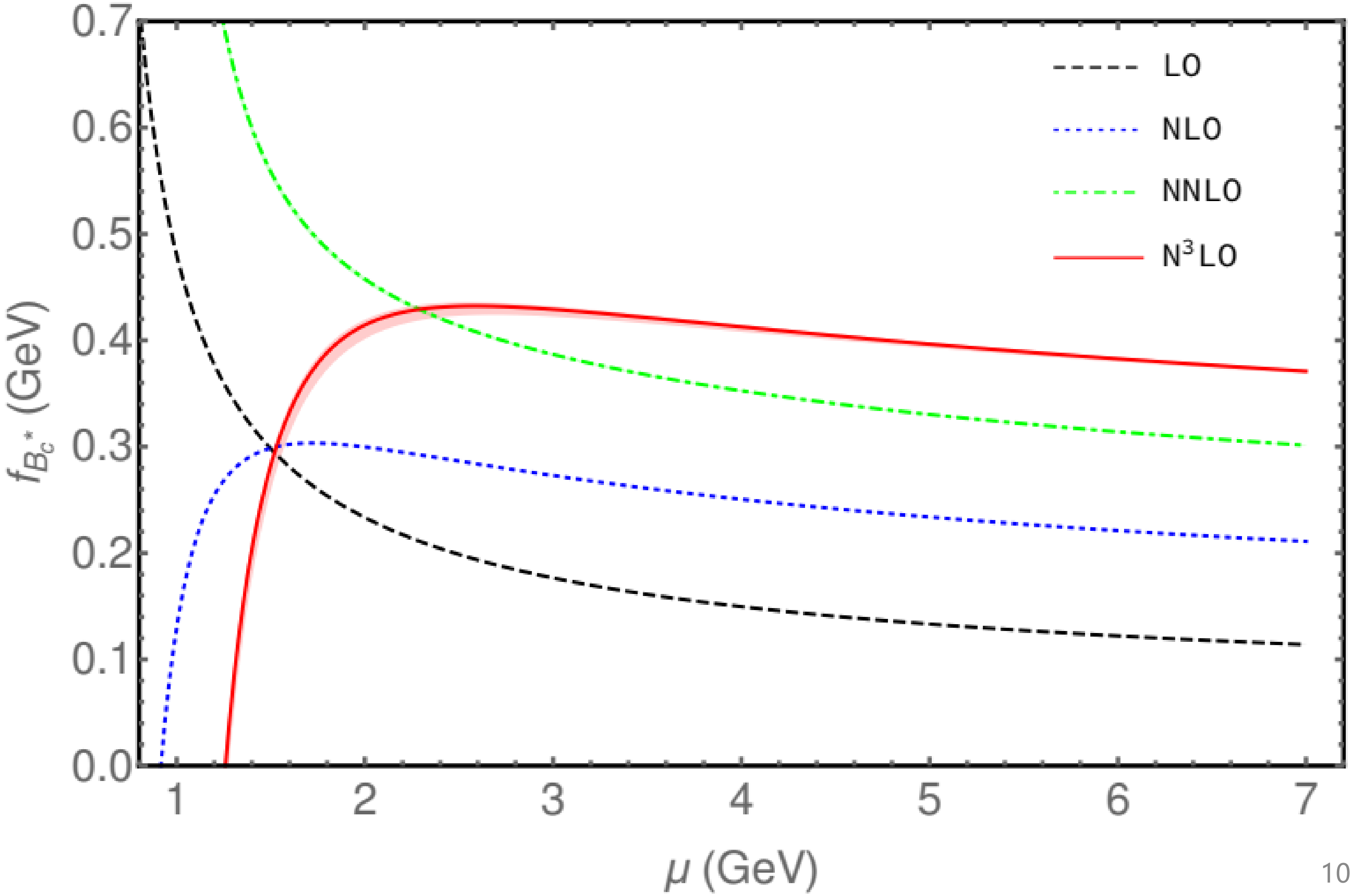
$$|\psi_1(0)|^2 = |\psi_1^{(0)}(0)|^2 \left(1 + \sum_{k=1}^{n} f_k a_s^k\right).$$

$$\left|\psi_1^{(0)}(0)\right|^2 = \frac{(m_b C_F \alpha_s)^3}{8\pi},$$

$$E_1^{(0)} = -\frac{1}{4} m_b (C_F \alpha_s)^2,$$

Beneke et al., PRL. 112, 151801 (2014)

Convergent vector Bc* decay constant



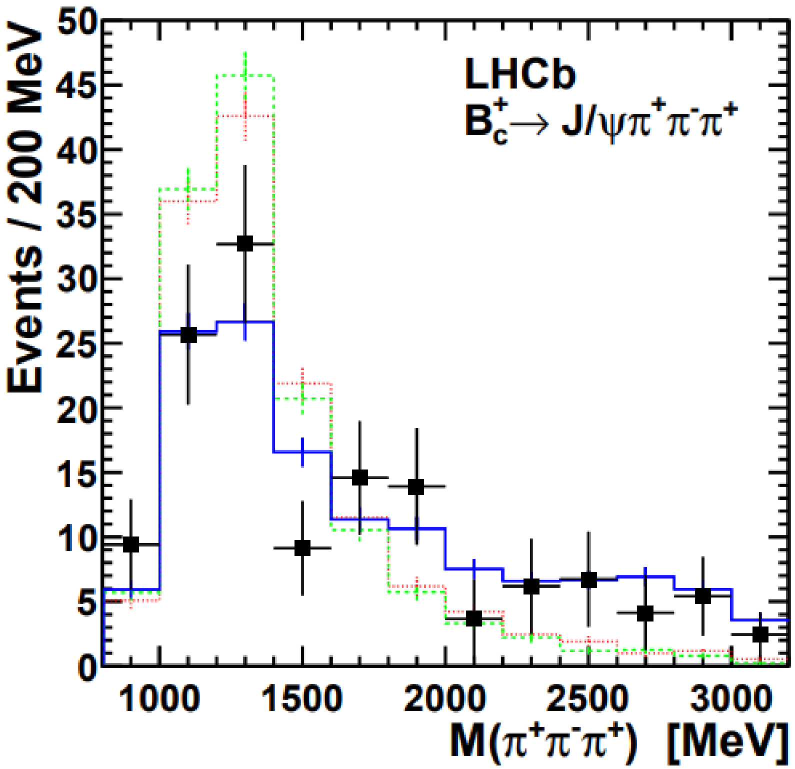
Leptonic decay branching ratios

Branching ratios	N^{3}LO
$\mathcal{B}(B_c^{*+} \to e^+ \nu_e)$	$(3.85^{+0.29-0.07-1.35}_{-0.46+0.03+0.37}) \times 10^{-6}$
$\mathcal{B}(B_c^{*+} \to \mu^+ \nu_\mu)$	$(3.85^{+0.29-0.07-1.35}_{-0.46+0.03+0.37}) \times 10^{-6}$
$\mathcal{B}(B_c^{*+} \to \tau^+ \nu_\tau)$	$(3.40^{+0.25-0.06-1.19}_{-0.41+0.03+0.33}) \times 10^{-6}$
$\mathcal{B}(B_c^{+} \to e^+ \nu_e)$	$(1.91^{+0.15-0.19-0.70}_{-0.23+0.12+0.22}) \times 10^{-9}$
$\mathcal{B}(B_c^{+} \to \mu^+ \nu_\mu)$	$(8.18^{+0.63-0.83-2.99}_{-1.00+0.52+0.94}) \times 10^{-5}$
$\mathcal{B}(B_c^{+} \to \tau^+ \nu_\tau)$	$(1.96^{+0.15-0.20-0.72}_{-0.24+0.12+0.23}) \times 10^{-2}$

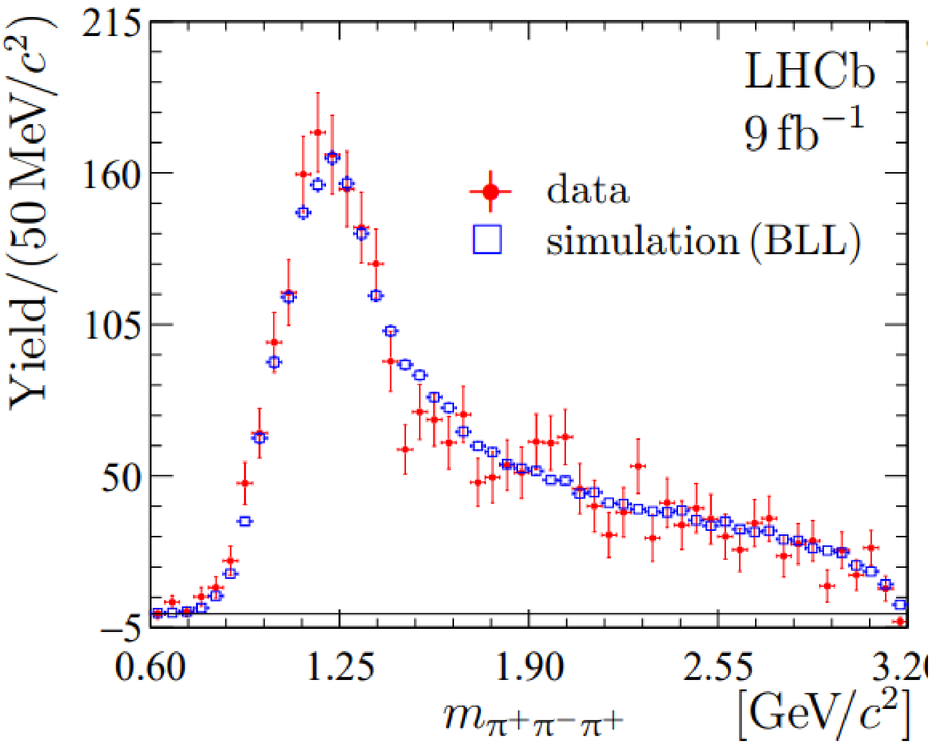
$$\Gamma(B_c^*(\lambda = \pm 1) \to \ell \nu_\ell) = \frac{|V_{cb}|^2}{12\pi} G_F^2 f_{B_c^*}^2 \left(1 - \frac{m_\ell^2}{m_{B_c^*}^2}\right)^2 \times m_{B_c^*}^3,$$

$$\Gamma(B_c^*(\lambda = 0) \to \ell \nu_\ell) = \frac{m_\ell^2 \Gamma(B_c^{*+}(\lambda = \pm 1) \to \ell \nu_\ell)}{2 m_{B_c^*}^2},$$

Bc and Bc* decays along with 3 pions



LHCb, arXiv:1204.0079



LHCb, arXiv:2111.03001

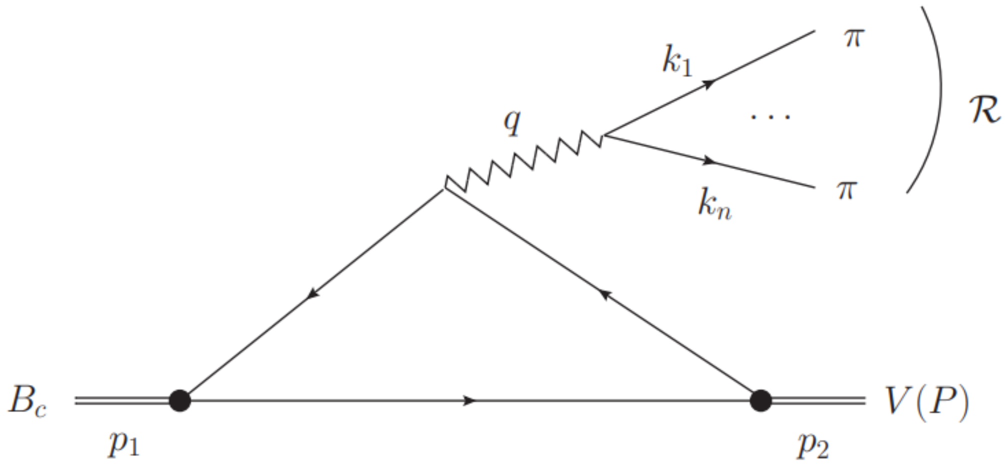
Around 10^5 Bc to Jpsi+X events

Invariant mass distribution in Bc* decays to J/psi +n h

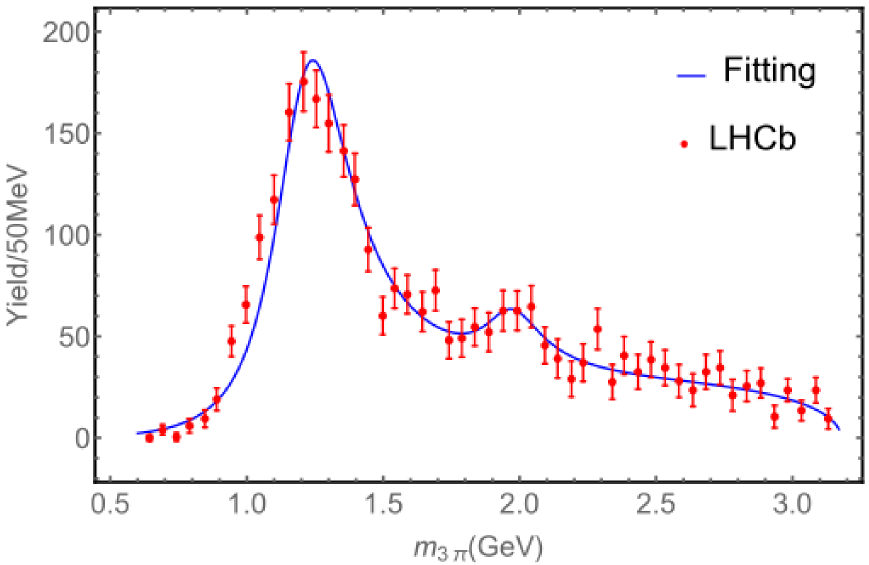
➢ Helicity decomposition of weak decay width

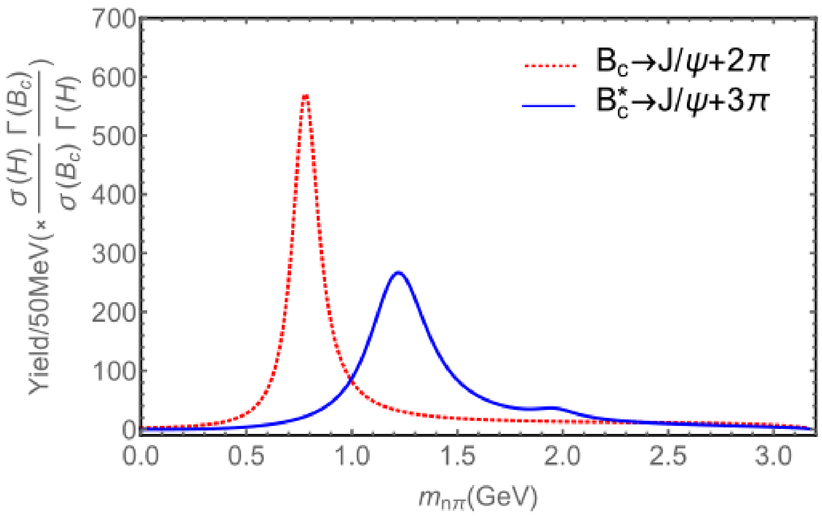
$$\frac{d\Gamma(B_c^{(*)} \to J/\psi + nh)}{dq^2} = \sum_{\lambda_i} \frac{|V_{cb}|^2 G_F^2 a_1^2 |\mathbf{p}'|}{32\pi M^2} \Gamma_{J_1\lambda_1 J_2\lambda_2\lambda_{nh}},$$

$$\Gamma_{11110} = 2\left[V_1^2\left((M-M')^2 - q^2\right)\left((M'+M)^2 - q^2\right) + \left(A_1\left(M^2 - M'^2\right) + A_2 q^2\right)^2\right] \rho_T^{nh}(q^2),$$



Results of invariant mass distribution

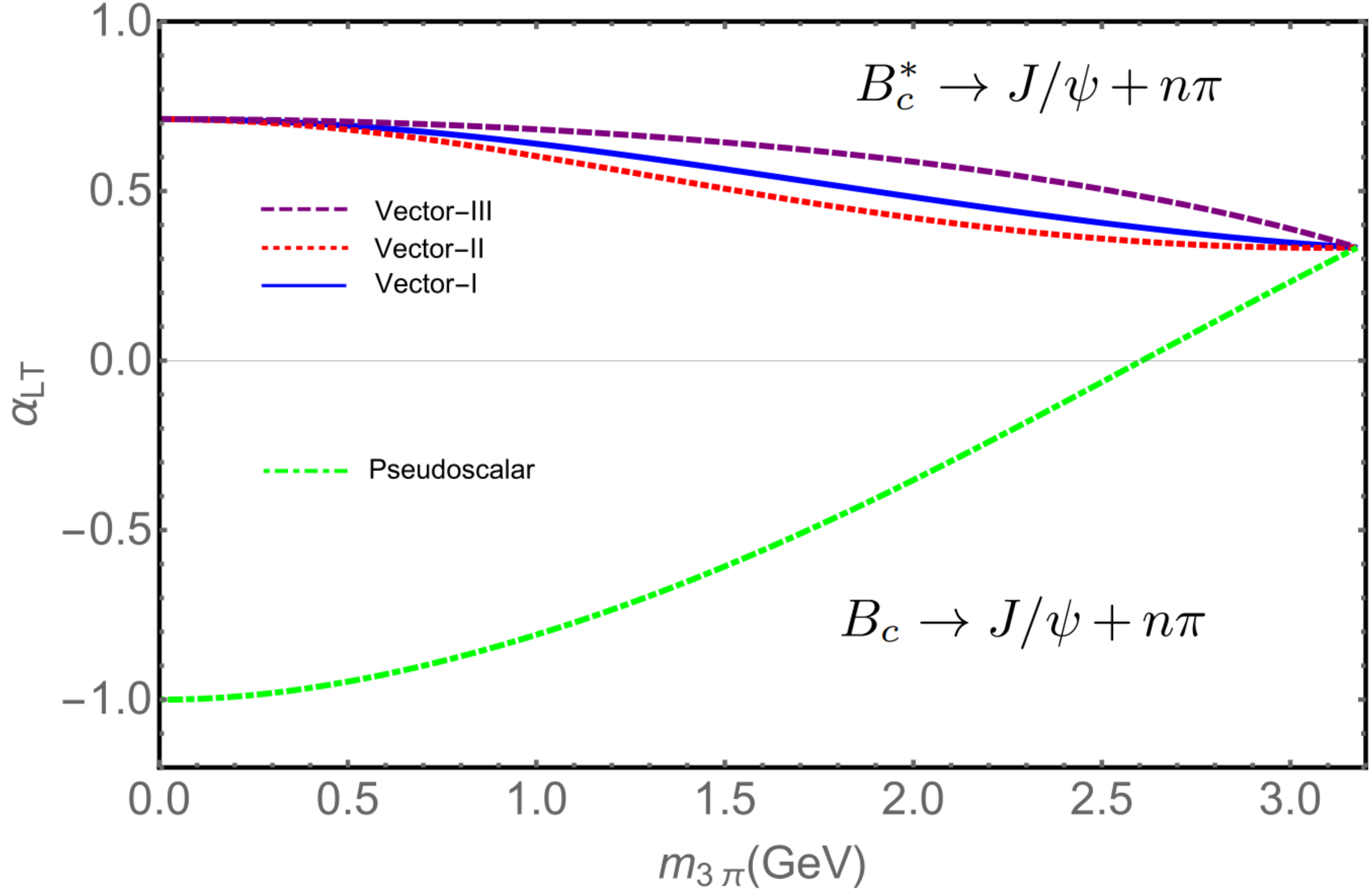




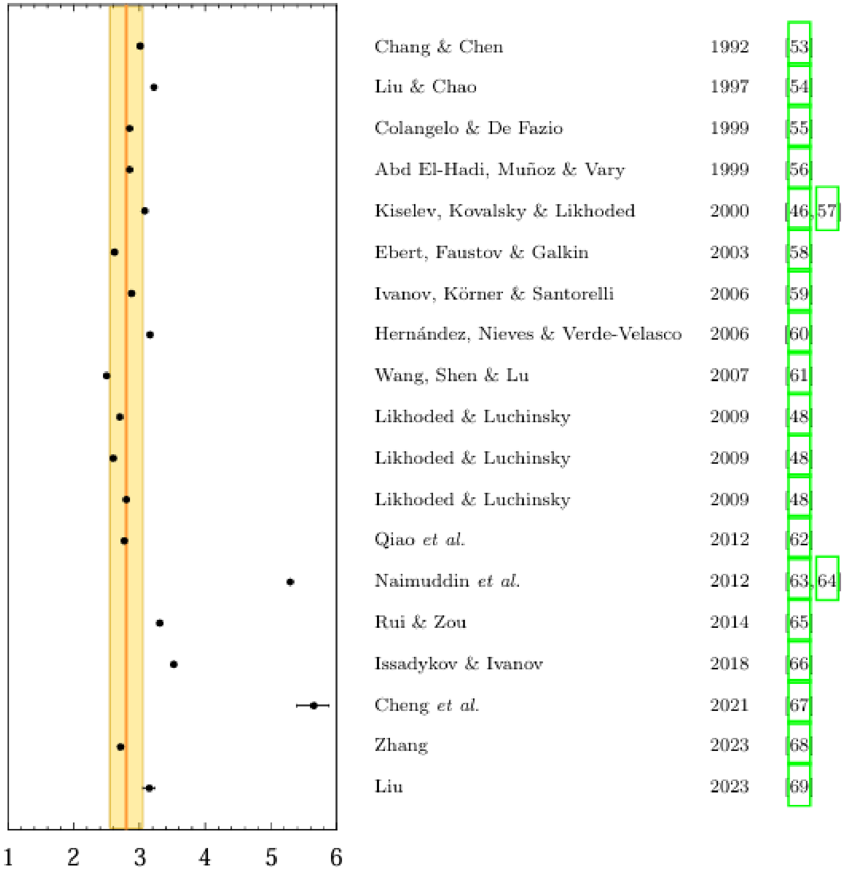
LHCb, arXiv:2111.03001

Polarization Asymmetry(A general law in V(P) to V transitions)

$$\alpha_{LT} = \sum_{\lambda_1,\lambda_{nh}} \frac{\Gamma_{J_1\lambda_1 11\lambda_{nh}} - \Gamma_{J_1\lambda_1 10\lambda_{nh}}}{\Gamma_{J_1\lambda_1 11\lambda_{nh}} + \Gamma_{J_1\lambda_1 10\lambda_{nh}}},$$

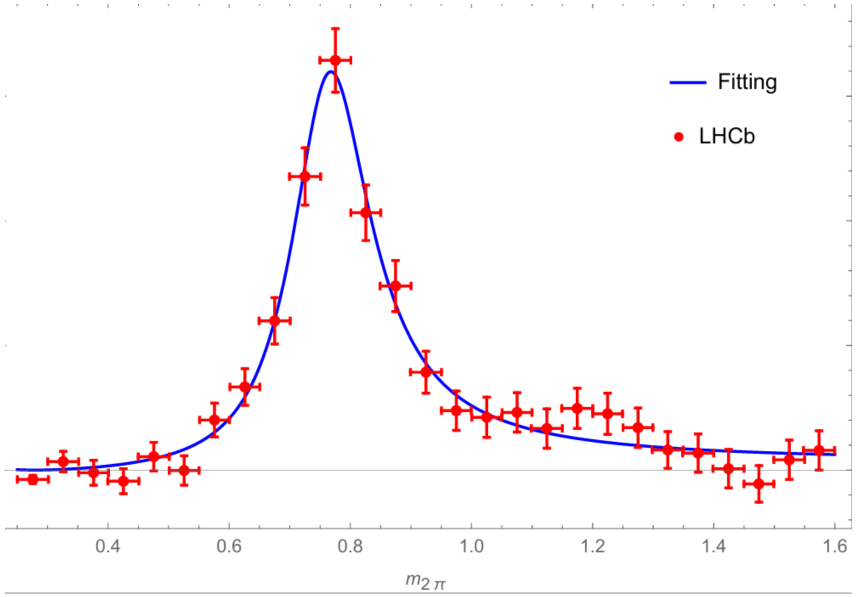


Bc decays along with 2 pions



$$\frac{\mathcal{B}_{B_c^+\to J/\psi\pi^+\pi^0}}{\mathcal{B}_{B_c^+\to J/\psi\pi^+}}$$

$$\mathcal{R} = \frac{\mathcal{B}_{B_c^+\to J/\psi\pi^+\pi^0}}{\mathcal{B}_{B_c^+\to J/\psi\pi^+}} = 2.80 \pm 0.15 \pm 0.11 \pm 0.16\,,$$



[62]Qiao-Sun-Yang-Zhu, 1209.5859

LHCb, arXiv: 2402.05523

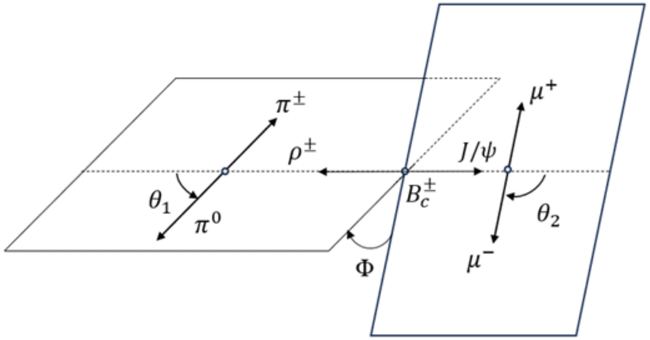
2404.06221, 2310.03425

Helicity angular distributions

➢ weak decay amplitude

$$h_\lambda \equiv \langle \rho(p_1,\lambda) J/\psi(p_2,\lambda) | \mathcal{H}_{eff} | B_c(p) \rangle$$
$$= \epsilon_{1\mu}(\lambda)^* \epsilon_{2\nu}(\lambda)^* \left(a g^{\mu\nu} + \frac{b p^\mu p^\nu}{m_1 m_2} + \frac{i c \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_\beta}{m_1 m_2} \right),$$

➢ Helicity angular distribution



$$\frac{d^3\Gamma\left(B_c \to J/\psi(\mu^+\mu^-) + \rho(\pi\pi)\right)}{d\cos\theta_1 d\cos\theta_2 d\phi} =$$

$$\frac{9p_m}{128\pi^2 M^2}\Big\{ \cos^2\theta_1 \sin^2\theta_2 H_{00} + \frac{1}{4}\sin^2\theta_1\left(1+\cos^2\theta_2\right)\left(H_{11}+H_{-1-1}\right)$$
$$-\frac{1}{2}\sin^2\theta_1\sin^2\theta_2\left[\cos 2\phi\,\mathrm{Re}\left(H_{1-1}\right) - \sin 2\phi\,\mathrm{Im}\left(H_{1-1}\right)\right]$$
$$-\frac{1}{4}\sin 2\theta_1 \sin 2\theta_2\left[\cos\phi\,\mathrm{Re}\left(H_{10}+H_{-10}\right) - \sin\phi\,\mathrm{Im}\left(H_{10}-H_{-10}\right)\right]\Big\},$$

$$f_L = \frac{|h_0|^2}{|h_{+1}|^2 + |h_{-1}|^2 + |h_0|^2}. \qquad \alpha_{LT} = 1 - 2f_L. \qquad f_L(J/\psi) = f_L(\rho) \simeq 0.877,$$

Quantum spin entanglement and Von Neumann entropy

➢ Quantum spin entanglement state

$$|\Psi\rangle = \frac{1}{\sqrt{|H|^2}} [h_{+1}|J/\psi(+1)\rho(+1)\rangle + h_0|J/\psi(0)\rho(0)\rangle + h_{-1}|J/\psi(-1)\rho(-1)\rangle],$$

➢ Von Neumann entropy

$$\varrho = \frac{1}{|H|^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{+1}h_{+1}^* & 0 & h_{+1}h_0^* & 0 & h_{+1}h_{-1}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0h_{+1}^* & 0 & h_0h_0^* & 0 & h_0h_{-1}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{-1}h_{+1}^* & 0 & h_{-}h_0^* & 0 & h_{-1}h_{-1}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\varepsilon = -Tr[\varrho_A \ln \varrho_A] = -Tr[\varrho_B \ln \varrho_B].$$

$$\varrho_{J/\psi} = \varrho_\rho = \frac{1}{|H|^2}\begin{pmatrix} h_{+1}h_{+1}^* & 0 & 0 \\ 0 & h_0h_0^* & 0 \\ 0 & 0 & h_{-1}h_{-1}^* \end{pmatrix}.$$

$$\varepsilon = 0.405.$$

Summary

- ✓ Bc family spectrum and Bc* decay width is studied in QCD effective approaches
- ✓ Convergent Bc* decay constant up to three-loop accuracy is obtained
- ✓ Distinguishing vector Bc* meson at LHC is possible by helicity decomposition
- ✓ Angular distribution and quantum spin entanglement is studied in Bc decays along with two pions.

Thank you a lot!