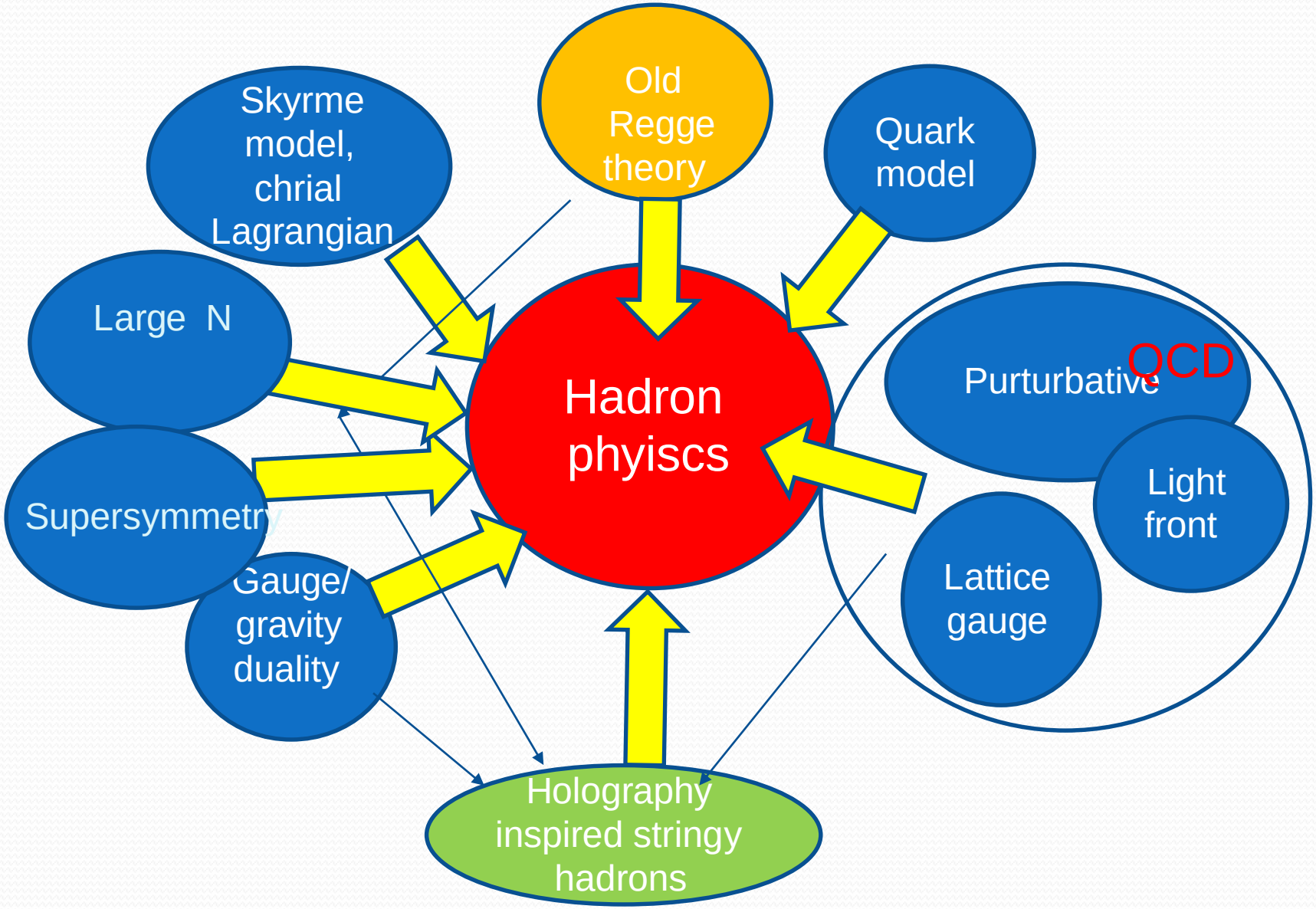
# Taming the Zoo of Tetraquarks and Pentaquarks using the HISH model

with Michael Green

Belle II, BES III January 2024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# Using HISH in the zoo of exotic hadrons

- Why do we think the HISH is an appropriate framework to describe tetraquarks and pentaquarks?
- Mainly because it can account for the spectra and the strong decay widths of mesons and baryons
- In other methods in particular lattice it is very difficult to determine the spectra of states with high angular momentum and high excitation number.
- Via holography the HISH is connected to the QCD.

# Summary of the HISH model

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- Let us briefly summarize the HISH ( Holography Inspired Stringy Hadron) model.
- HISH is a map from strings in 10d holographic background to strings in flat 4d

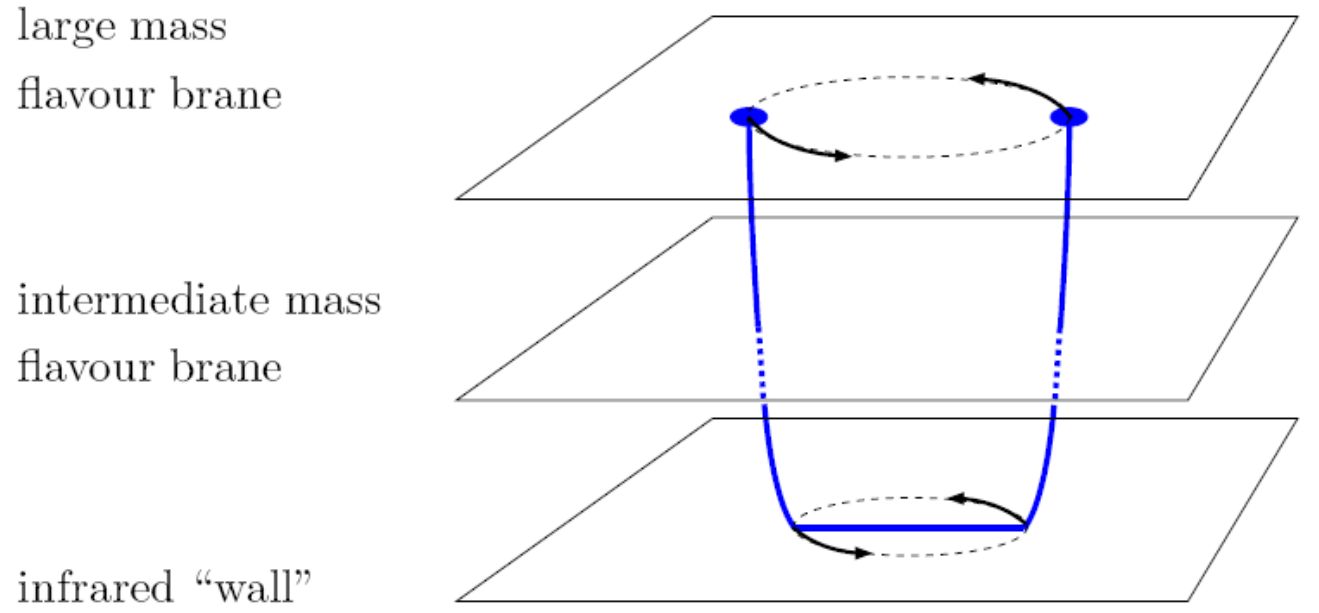
- The main results related to the spectra and strong decay processes are:
- A meson is described as an open string with massive quark and an anti-quark on its ends.
- A baryon is an open string with a quark on one side and a baryonic vertex (bv.) + a di-quark on the other side
- A glueball is a close string

# Hadrons in holography

- We can mimic the holographic background as a 5 dimensional space time with the ordinary 3+1 dimensions + a holographic fifth dimension.
- In this background there are manifolds reffered to as flavor D branes on which string can start and end.
- Each D brane associates with one flavor and is located at a different height in the holographic direction.
- A meson is a string that starts at one flavor brane stretches along ordinary space direction and ends on another flavor brane.

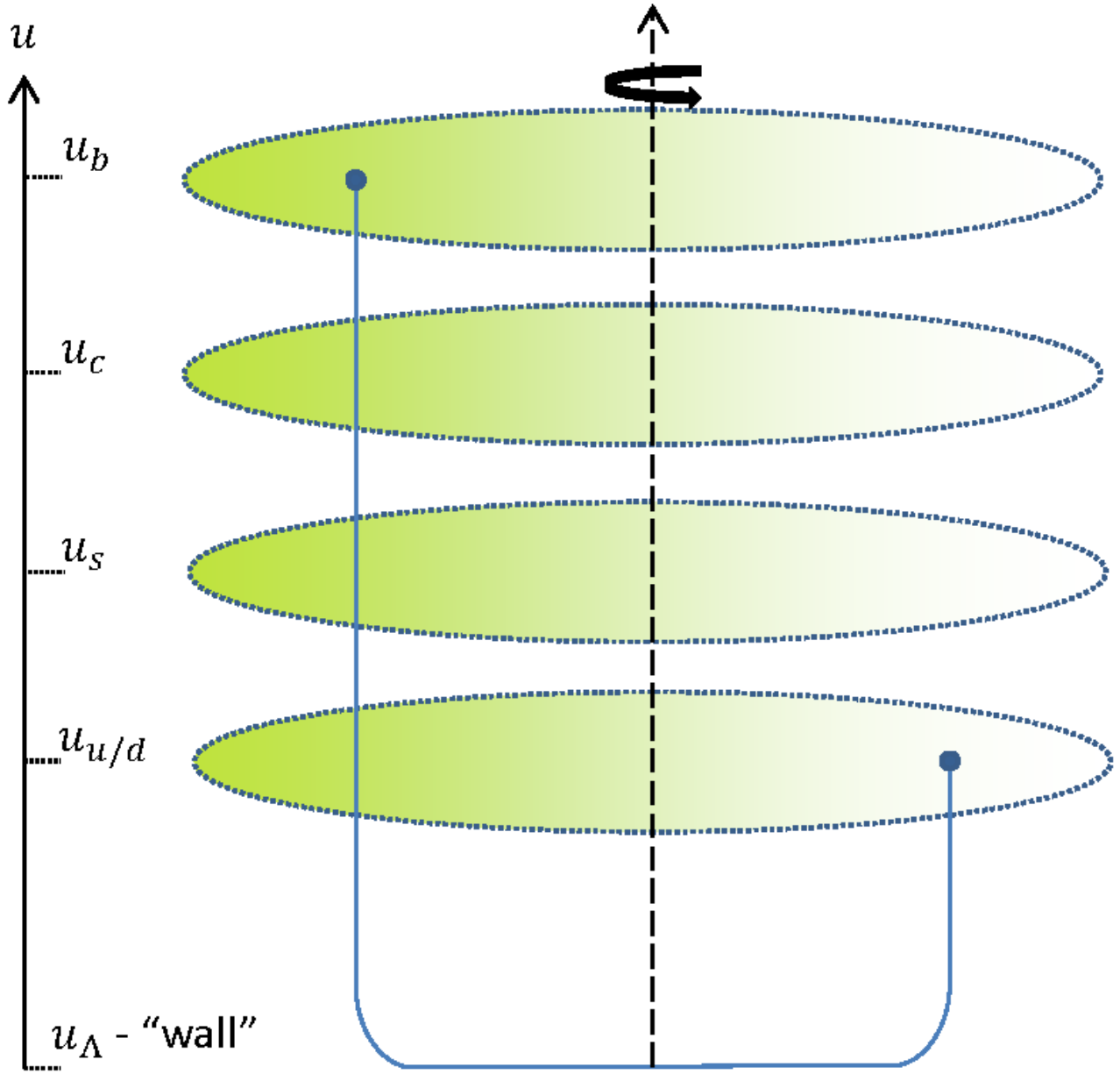
# (1) Messonsin holography

- The holographic meson with angular momentum is a rotating string connected to flavor branes




- The string is the classical solution of the Nambu - Goto action defined in a confining holographic background

# Example: The B meson



# (2) Stringy Baryons in holography

- How do we identify a baryon in holography ?

- Since a quark corresponds to an end of a string , the baryon has to be a structure with $N_c$ strings connected to it.

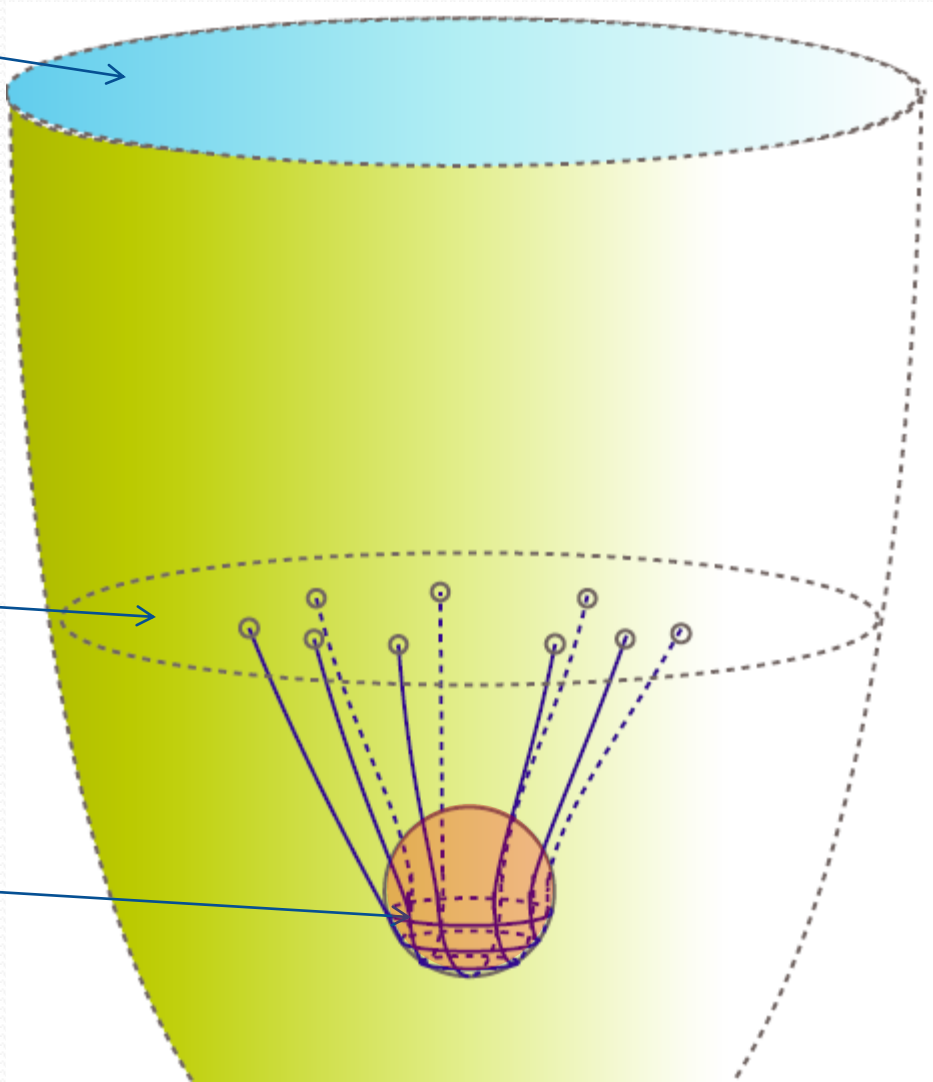
- The proposed baryonic vertex in holographic background is a wrapped Dp brane over a p cycle

Witten, Gross Ooguri

- Because of the RR flux in the background the wrapped brane has to be connected to Nc strings

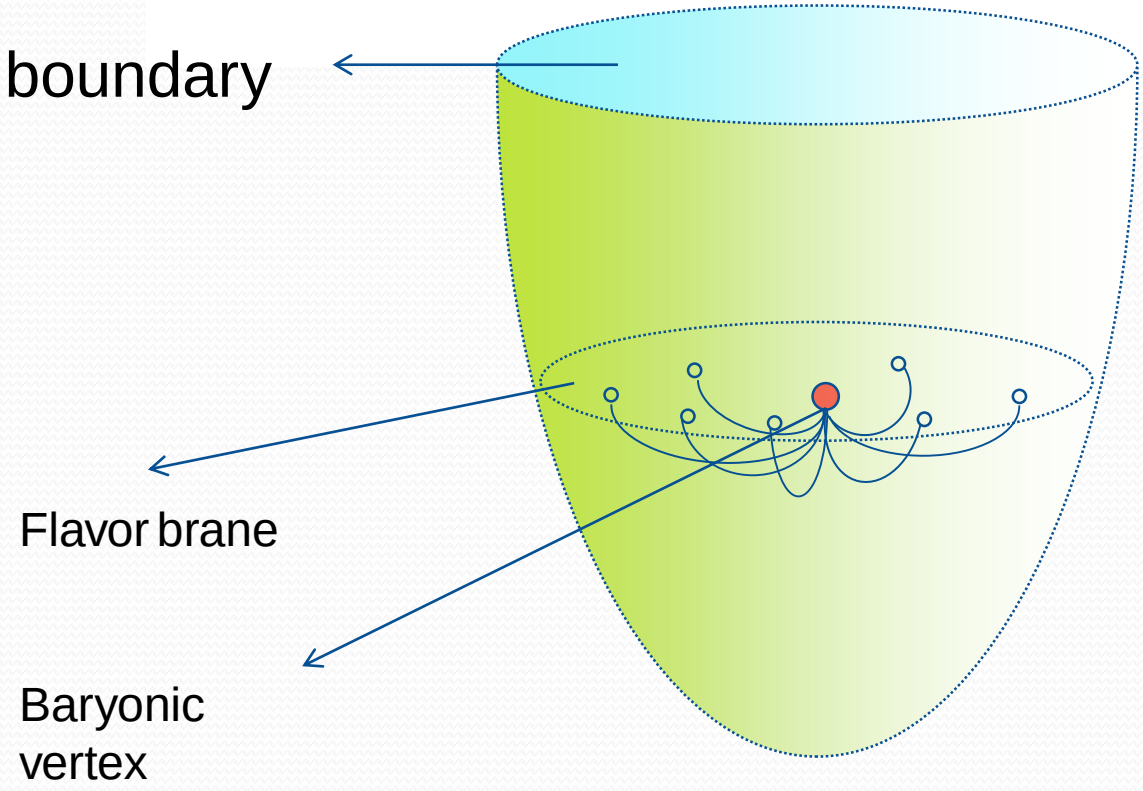
# Dynamical baryon

- Dynamical baryon ζNc strings connecting the baryonic vertex and flavor branes.



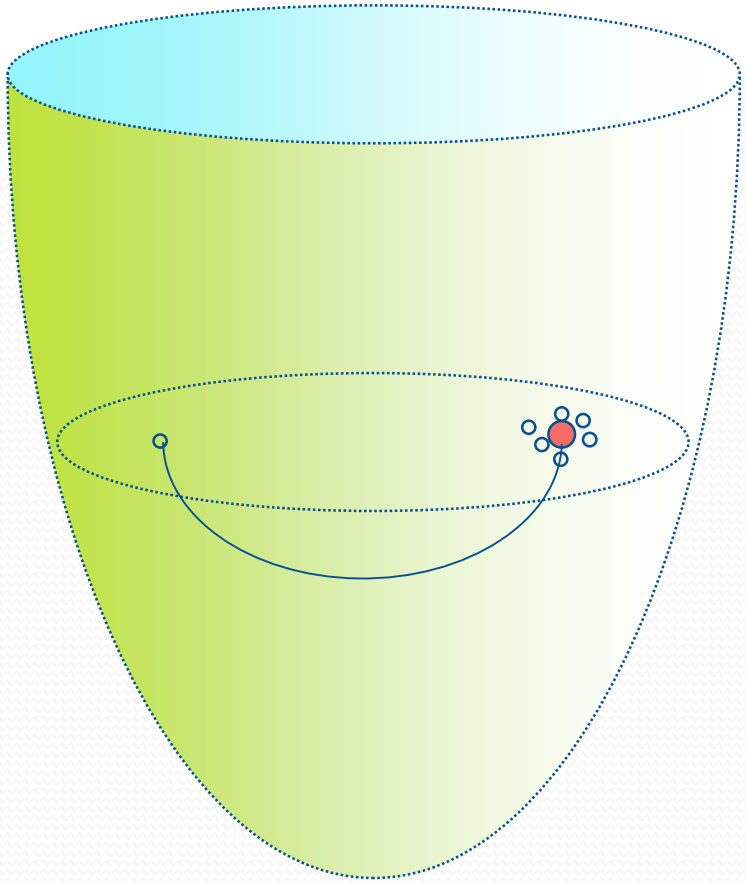
# A possible baryon : Symmetric layout

- A priori there are many possible layouts, in particular the maximal symmetric one. The preferred one has the lowest energy .



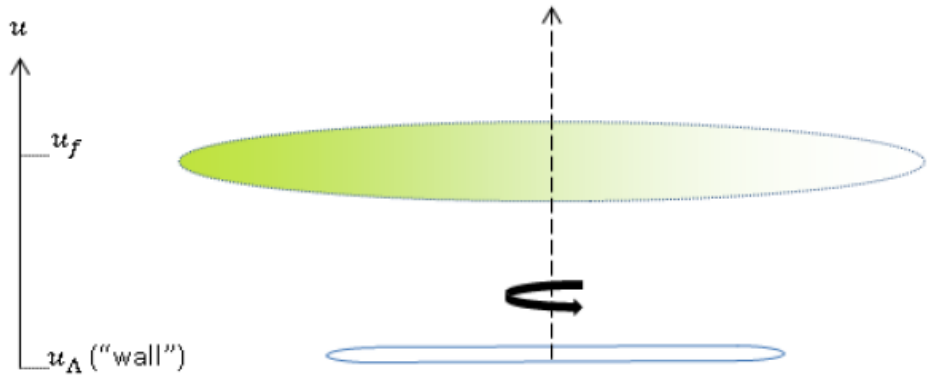
# Asymmetric layout

- An asymmetric possible layout is that of one quark connected with a string to the baryonic vertex to which the rest of the Nc-1quarks are attached.



# (3) Glueballs as closed strings

- Mesons are open strings connected to flavor branes.
- Baryons are Nc open strings connected to a baryonic vertex on one side and to a flavor brane on the other one.
- What are glue balls ?
- Since they do not incorporate quarks it is natural to assume that they are rotating closed strings
- Angular momentum associates with rotation of folded closed strings



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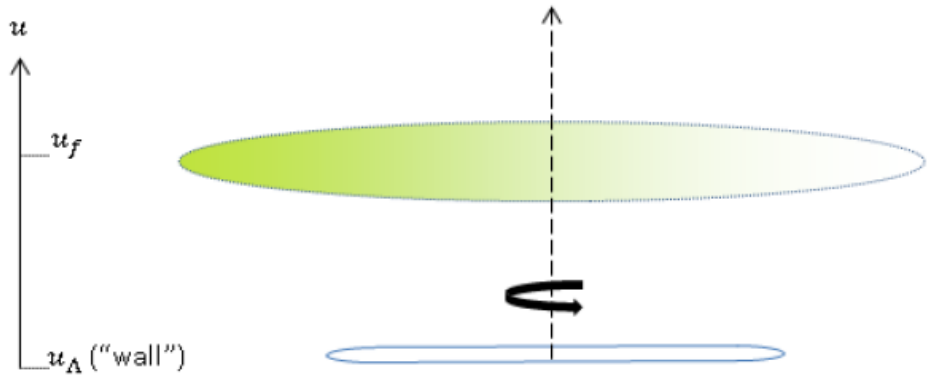
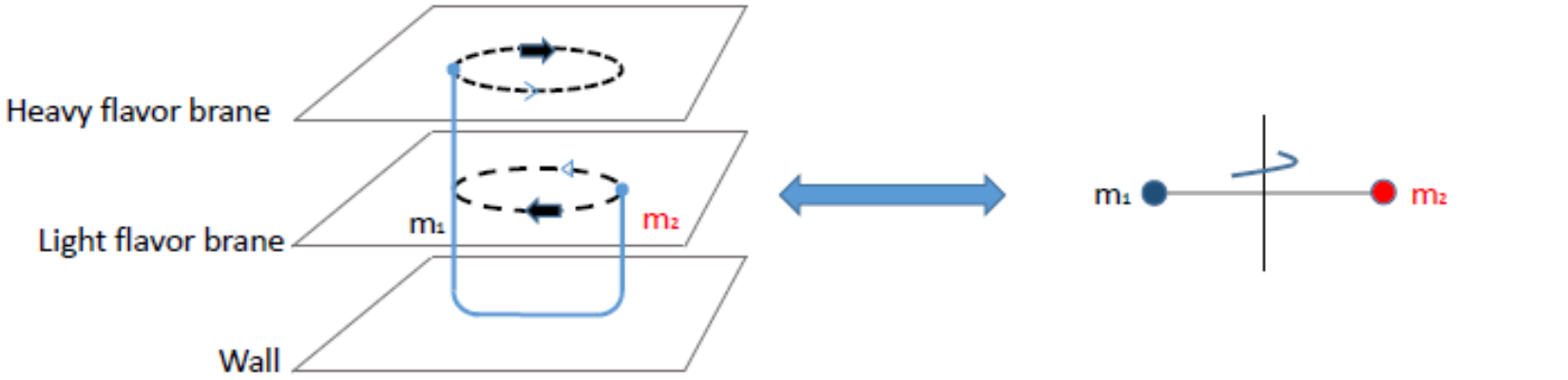


# The HISH map of stringy hadrons

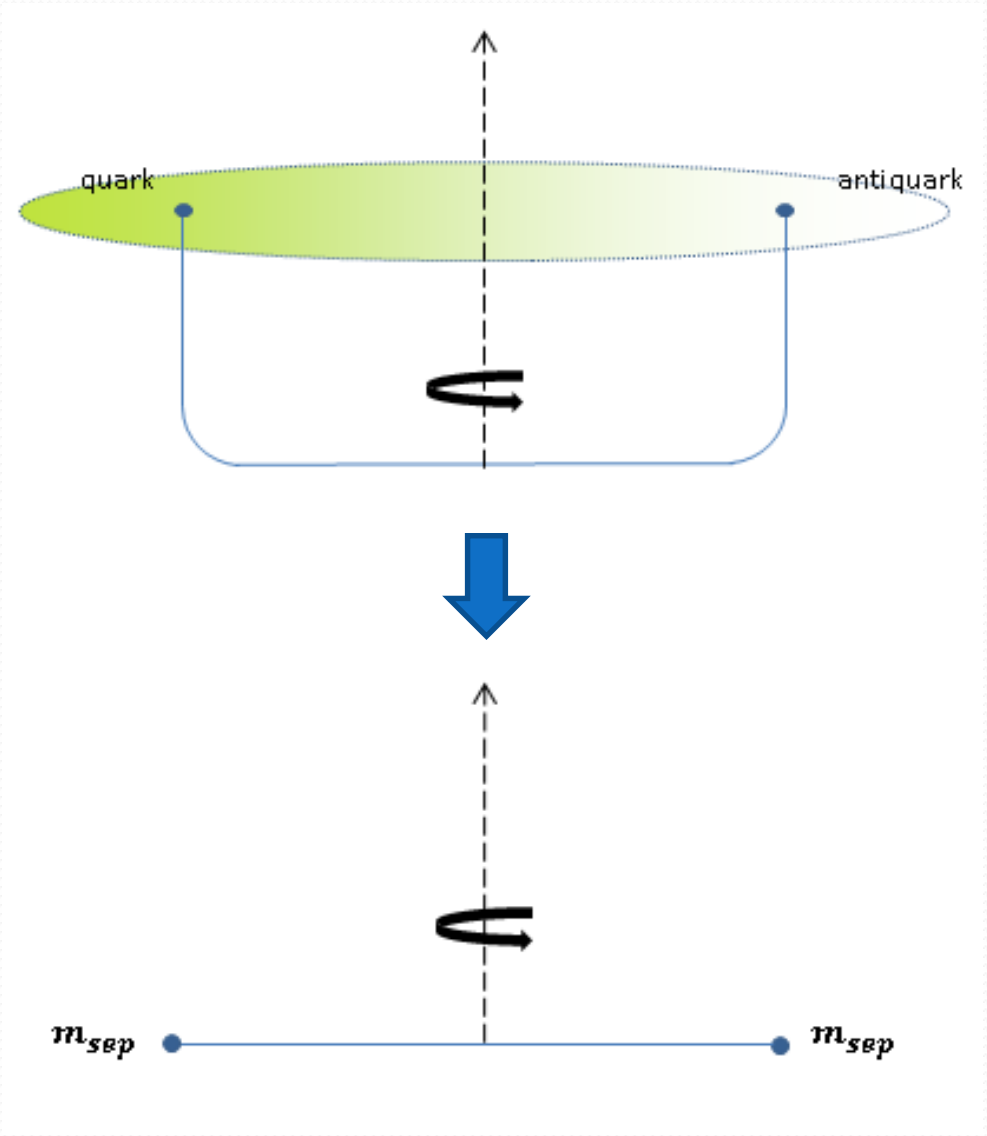
- The HISH map



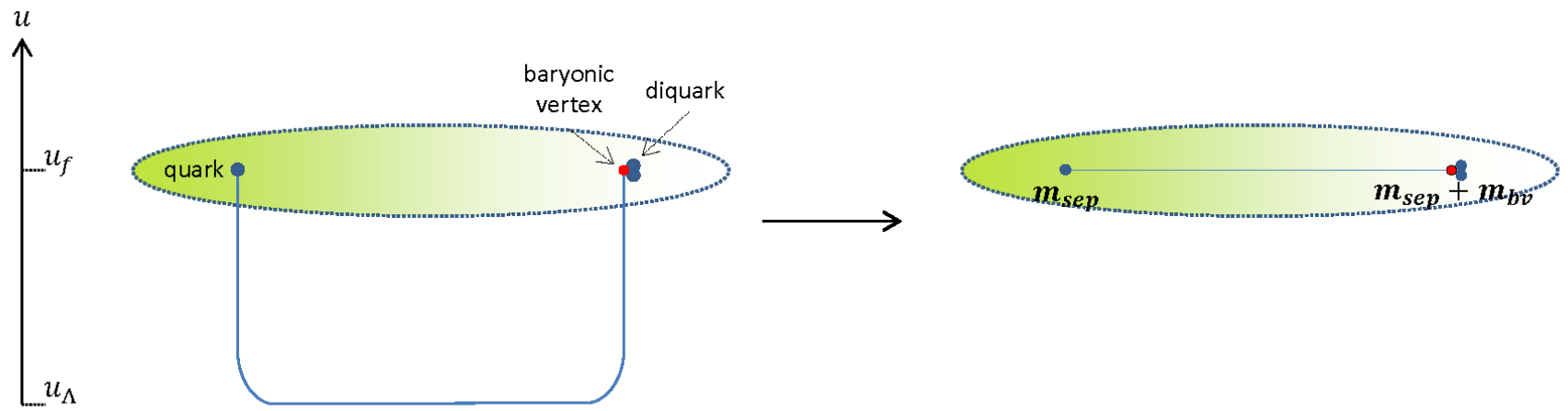

Rotating holographic string
Connected to different flavor branes

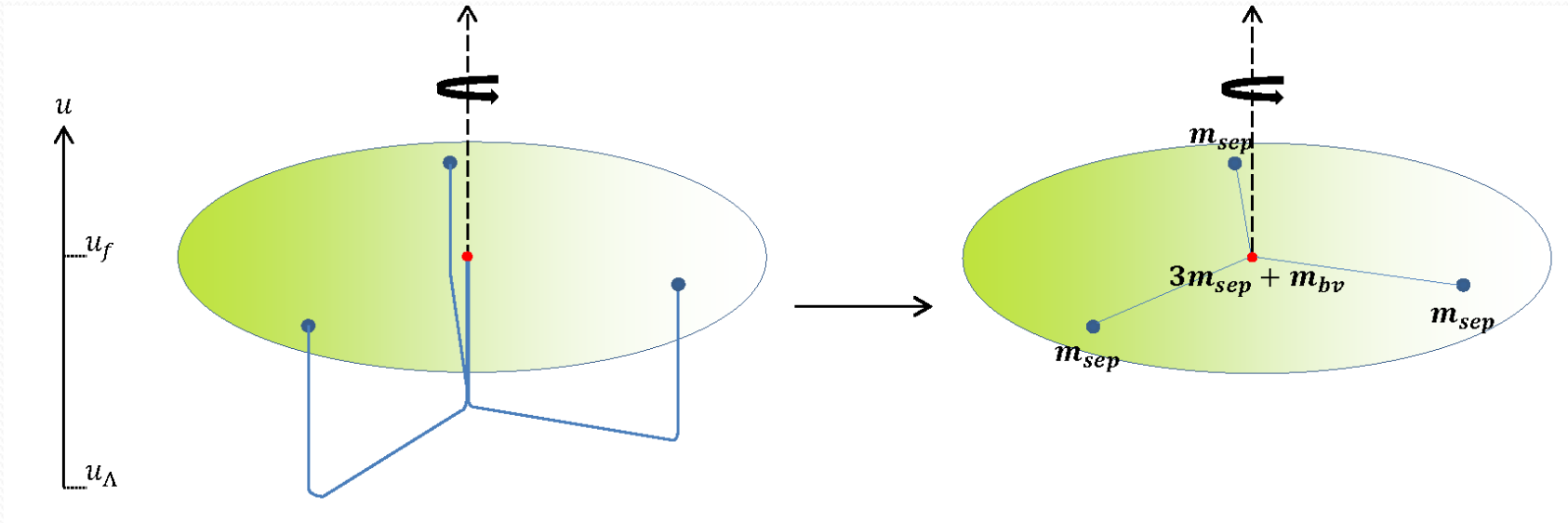
Rotating string in flat space-time
with massive endpoints

# (i) The HISH map of mesons



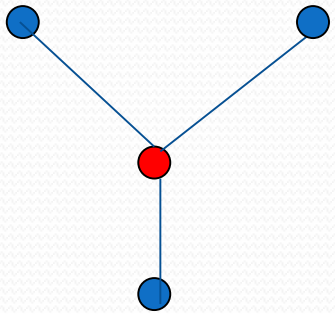
# (ii) The HISH map of Baryons



# From large Nc to three colors

- Naturally the analog at Nc=3 of the symmetric configuration with a central baryonic vertex is the old Y shape baryon

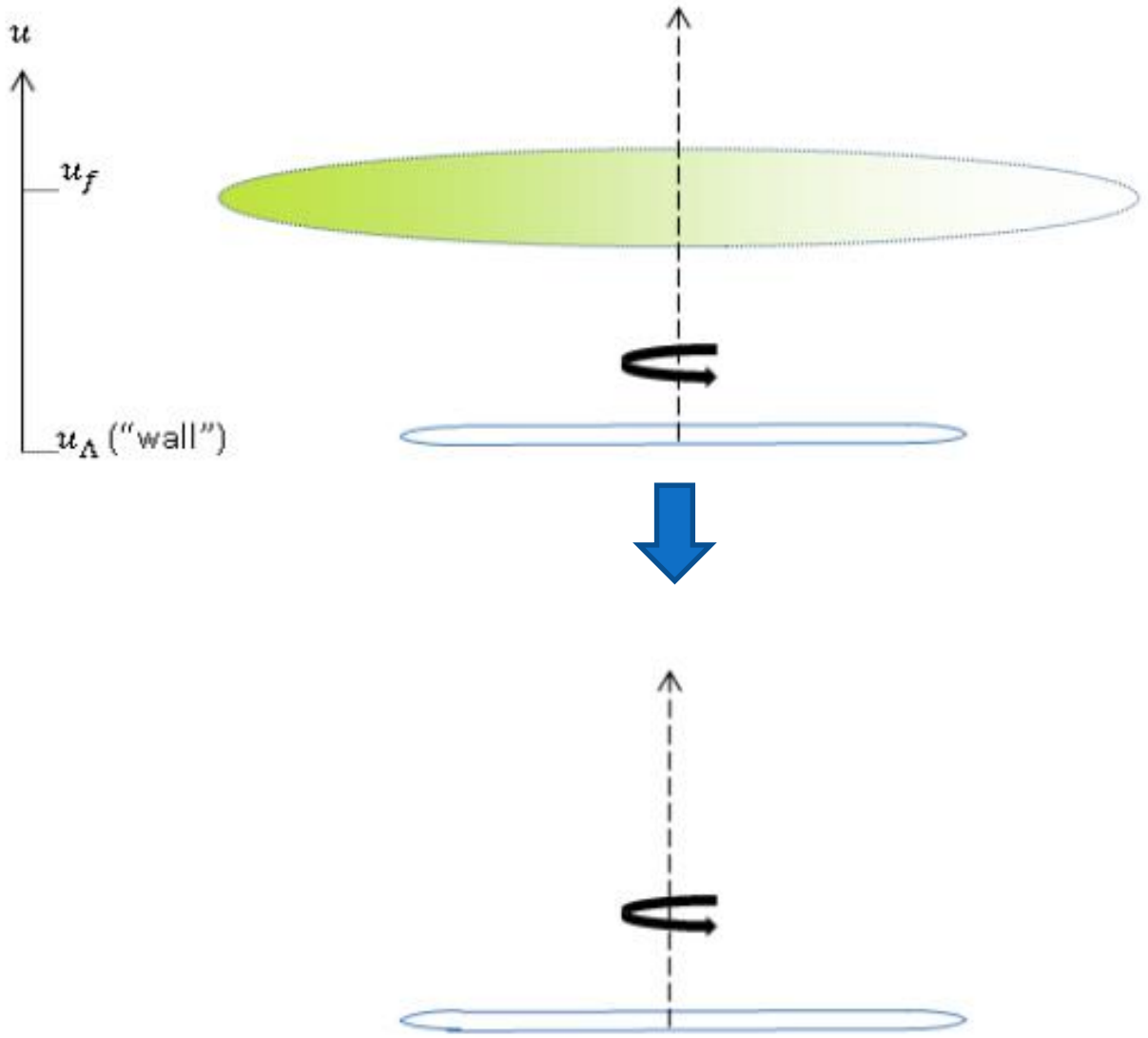


- The analog of the asymmetric setup with one quark on one end and Nc-1 on the other is a straight string with quark and a di-quark on its ends.

# Stability of an excited baryon

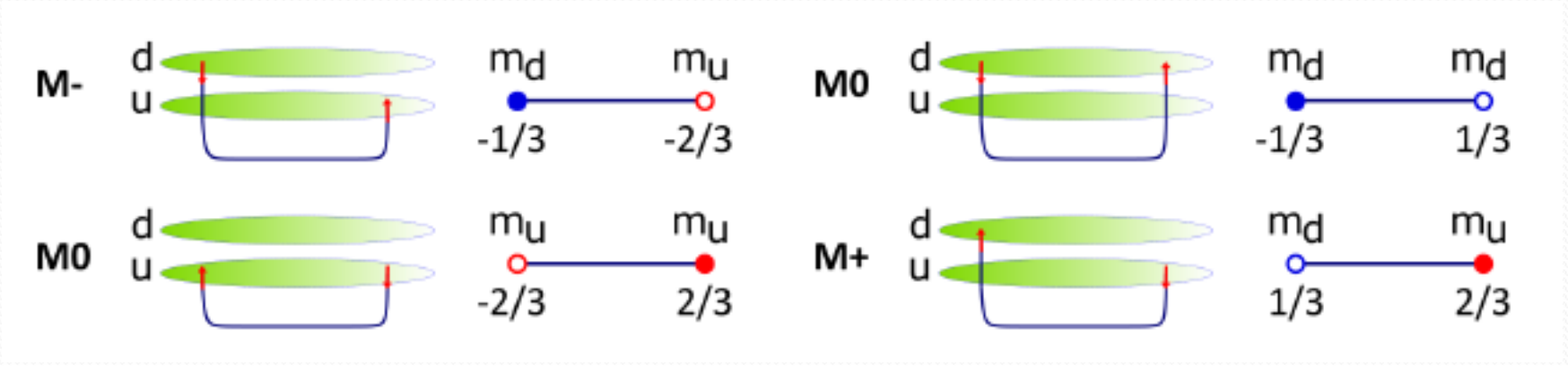
- Sharovand ŐHooft showed that the classicalY shape three string configuration is unstable . An arm that is slightly shortened will eventually shrink to zero size.
- We also examined Y shape strings with massive endpoints and with a baryonic vertex in the middle.
- The analysis included numerical simulations of the motions of mesons and Y shape baryons under the influence of symmetric and asymmetric disturbance.
- We indeed detected the instability
- We also performed a perturbative analysis where the instability does not show up.
- The Y shape is also ruled out because experimentally the slopes of baryons and mesons are the same ( up to 5%)

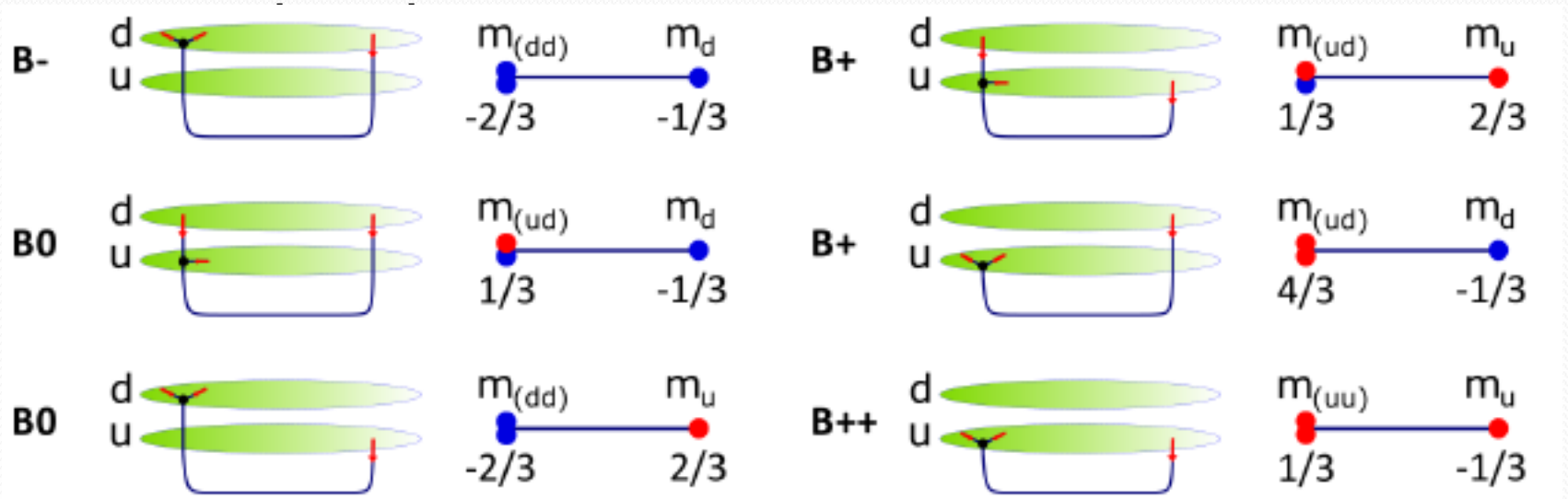
# (iii) The HISH map of glueballs



# Light stringy hadrons in holography and HISH

- Light stringy mesons and baryons in holography and HISH



# Summary of the HISH model

- The spectra of the HISH stringy hadrons include:

(i) Ground state -

- The tension is balanced by a `repulsive Casimir force ō

$$T = 2\pi \frac{|a|}{L^2} \rightarrow \qquad TL = \sqrt{\frac{|a|}{2\pi T}} = \sqrt{\frac{|a|}{\alpha'}}$$

Tension Length Intercept slope

- The total mass of the state is

$$M = m_1 + m_2 + TL$$

Hadron mass endpoint mass

$$M = m_1 + m_2 + \sqrt{\frac{|a|}{2\pi T}} = m_1 + m_2 + \sqrt{\frac{|a|}{\alpha'}}$$

# Summary of the HISH model

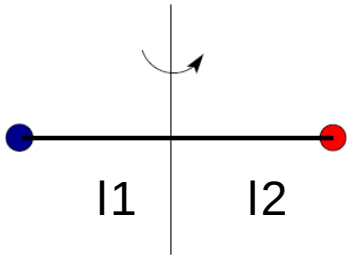
(iii) Excited states with angular momentum -

- Classicallyfor a rotating string with massive endpoints

$$M = E = \sum_{i=1,2} \left( \gamma_i m_i + T \ell_i \frac{\arcsin \beta_i}{\beta_i} \right)$$

length of an arm

velocity of endpoint particle




$$J = \sum_{i=1,2} \left[ \gamma_i m_i \beta_i \ell_i + \frac{1}{2} T \ell_i^2 \left( \arcsin \beta_i - \beta_i \sqrt{1 - \beta_i^2} \right) \right]$$

Quantum Mechanically

$$J_{cl} \rightarrow J_{QM} = J_{cl} + a$$

# Small and large $m_{sep}$ approximations

- For small $m_{sep}$, and $\beta_i \to 1$ the modified Regge trajectory is

$$J = \alpha' E^2 \times \left(1 - \sum_{i=1}^{2} \left( \frac{4\sqrt{\pi}}{3} \left(\frac{m_i}{E}\right)^{3/2} + \frac{2\sqrt{\pi^3}}{10\sqrt{2}} \left(\frac{m_i}{E}\right)^{5/2} + \cdots \right)\right)$$

- For large $m_{sep}$ and $\beta_i \to 0$,

$$J = \frac{4\pi}{3\sqrt{3}}\alpha' \sqrt{\frac{m_1 m_2}{m1 + m_2}} (E - m_1 - m_2)^{3/2} + \frac{7\sqrt{2}\pi}{27\sqrt{3}}\alpha' \frac{m_1^2 - m_1 m_2 + m_2^2}{m_1 m_2 \sqrt{(m_1 + m_2)^3}} (E - m_1 - m_2)^{5/2} + \cdots$$

# Summary of the HISH model

- The intercept is related to the Casimir energy of the quantized string. For an ordinary string with no massive endpoints

$$E_{Casimir} \equiv \frac{1}{2}\sum_{n=1}^{n=\infty} w_n = \frac{\pi(D-2)}{2L}\sum_{n=1}^{n=\infty} n = -\frac{(D-2)}{24}\frac{\pi}{L} = a\frac{\pi}{L}$$

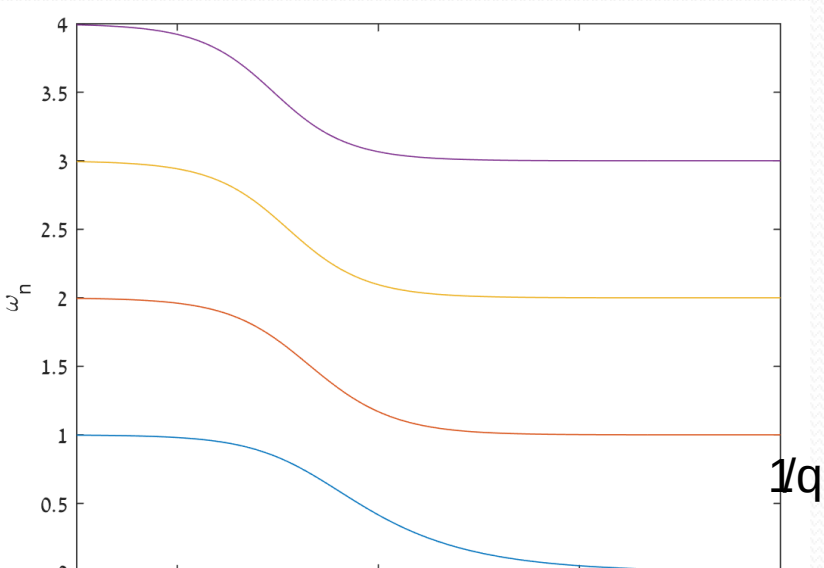
- For a string with quarks on its ends the eigen frequencies are given by

q=m/TL

$$\tan(\omega_n) = \frac{2q\omega_n}{q^2\omega_n^2 - 1}$$

We computed the intercept $a_m$

Using a contour integral.



# Summary of the HISH model

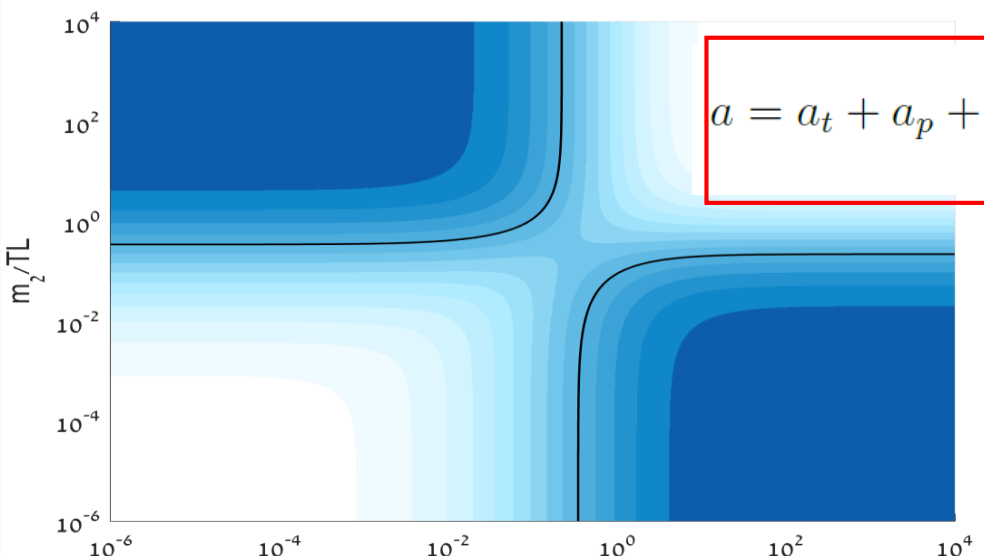
(iii) Radial excited string states

- The excited states of ordinary string with no angular momentum have a spectrum of

$$N = \alpha' M^2 + a$$

- For string with massive endpoints

$$w_N = \alpha'(M - m_1 - m_2)^2 + a_m$$

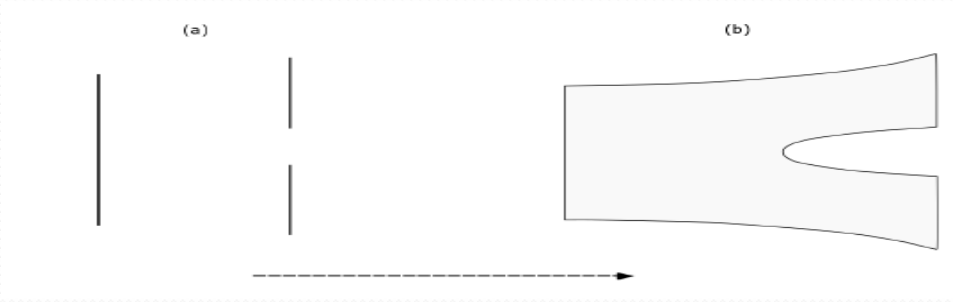



$$a = a_t + a_p + a_{PS} \sim 1 - \frac{11}{6\pi}\left[\frac{2m}{TL}\right]^{1/2} + \frac{143}{240\pi} + \left[\frac{2m}{TL}\right]^{3/}$$

# The strong decay width

- A string calculation for the break of the string yields linear dependence on the length L

$$\Gamma = \frac{\pi}{2} A T L(M, m_1, m_2, T) \,.$$




- For short strings with important role of the massive endpoints we add a phase space factor

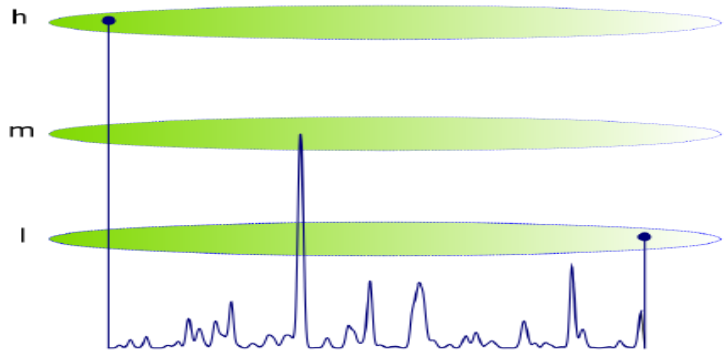
$$\Gamma = \frac{\pi}{2} A \times \Phi(M) \times T L(M, m_1, m_2, T) \,.$$

- The phase space factor

$$\Phi(M, M_1, M_2) \equiv 2\frac{|p_f|}{M} = \sqrt{\left(1 - (\frac{M_1 + M_2}{M})^2\right)\left(1 - (\frac{M_1 - M_2}{M})^2\right)}$$

# The branching ratios

- What is the probability that the string breaksand a pair of quark anti-quark is created.
- In holography this translates to the probability that a hadronic string hit a flavor brane




- The result is

$$\Gamma \sim \exp\left(-1.0\,\frac{z_B^2}{\alpha'_{\rm eff}}\right) = \exp\left(-2\pi\,\frac{m_{sep}^2}{T_{\rm eff}}\right)$$

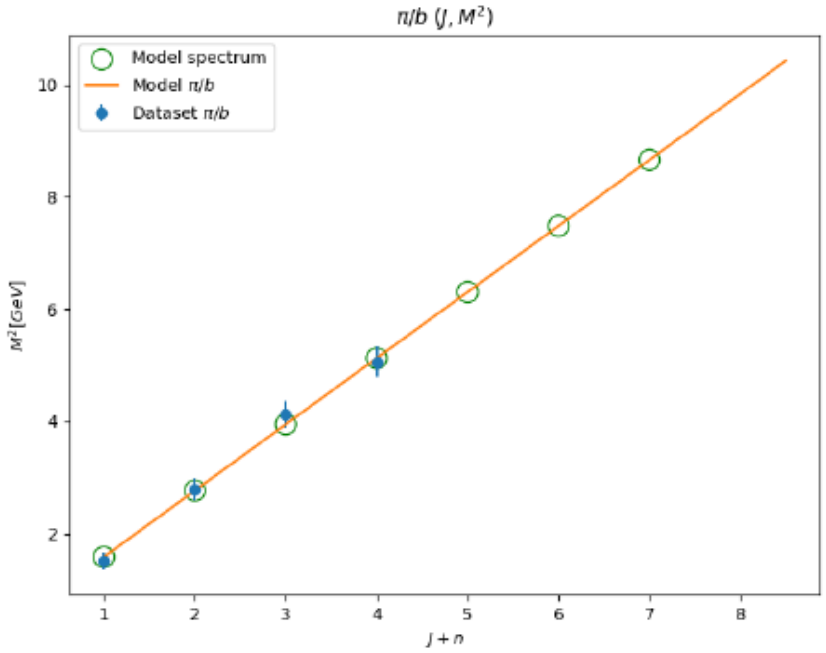
- This is the similar to the result of Casher Neuberger and Nussinov

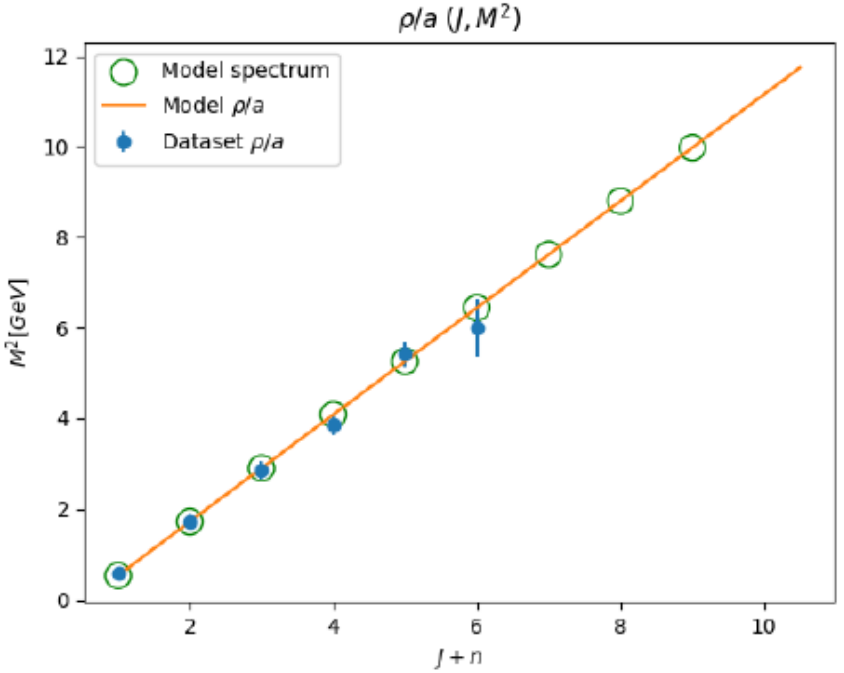
# Outline:

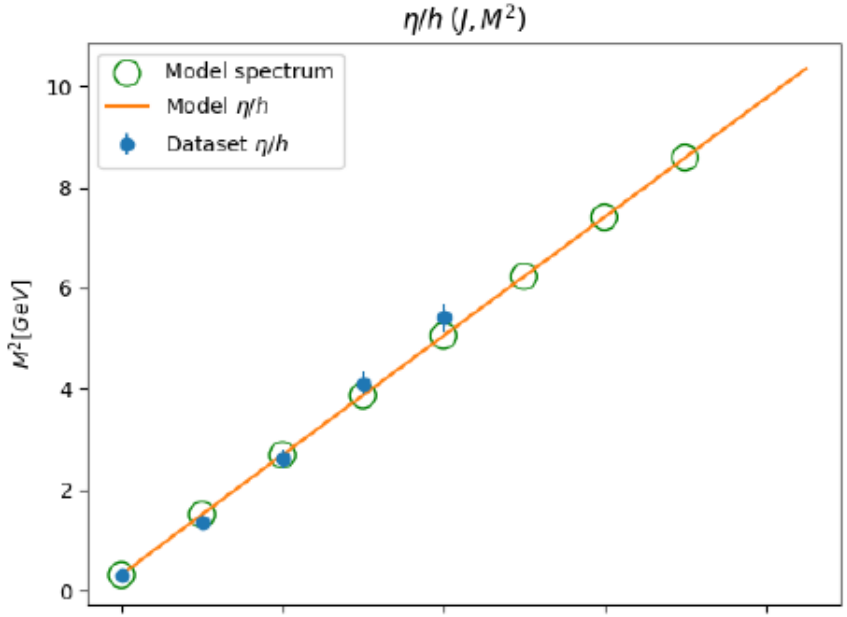
- I. A brief review of the HISH model
- II. Fitting with PDG meson and baryon data
- The structure of HISH tetraquarks
- Decay mechanisms of tetraquarks
- Structure of pentaquarks
- Decays of pentaquark
- Fitting procedure
- Terraquark candidates
- Test case
- Predictions of tetraquarks
- Pentaquark candidates
- Test case and predictions of pentaquarks

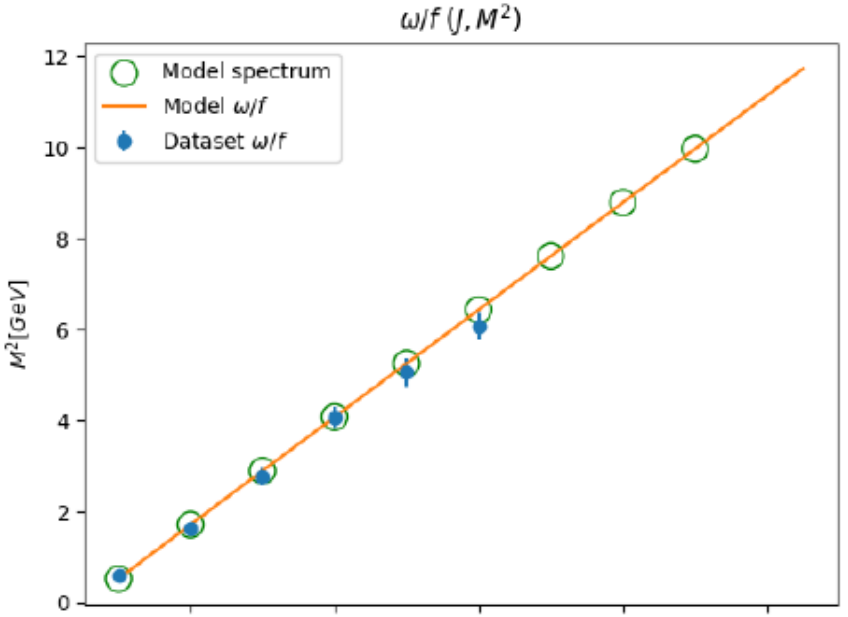
# Fits of HISH for mesons and baryons

# Fits of p/b, r /a, h/h and w/f

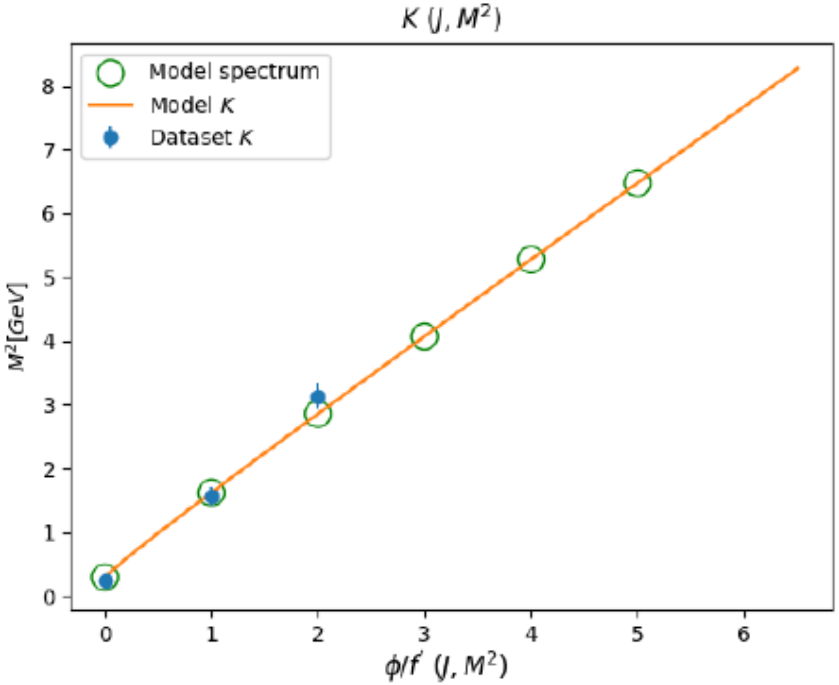


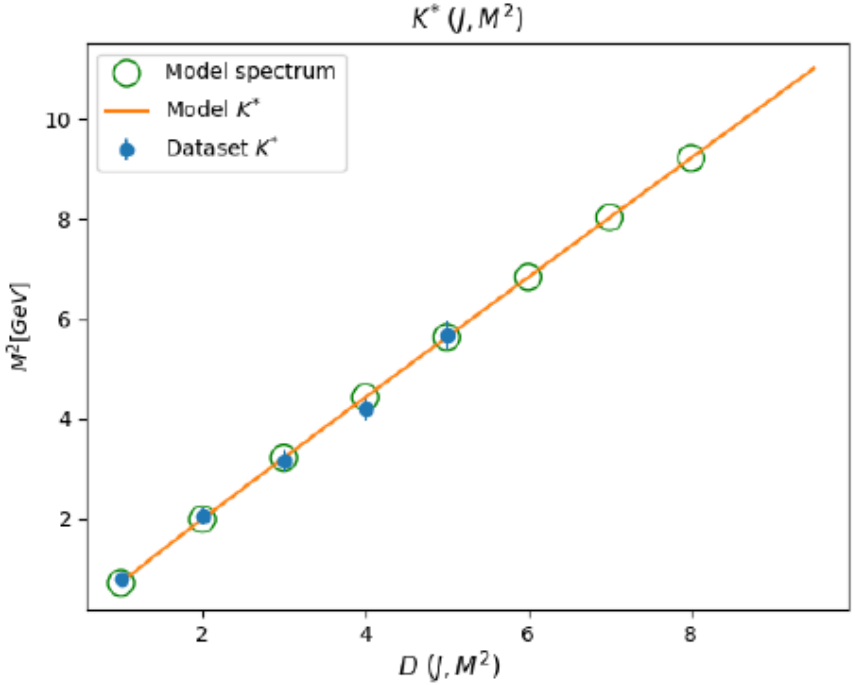



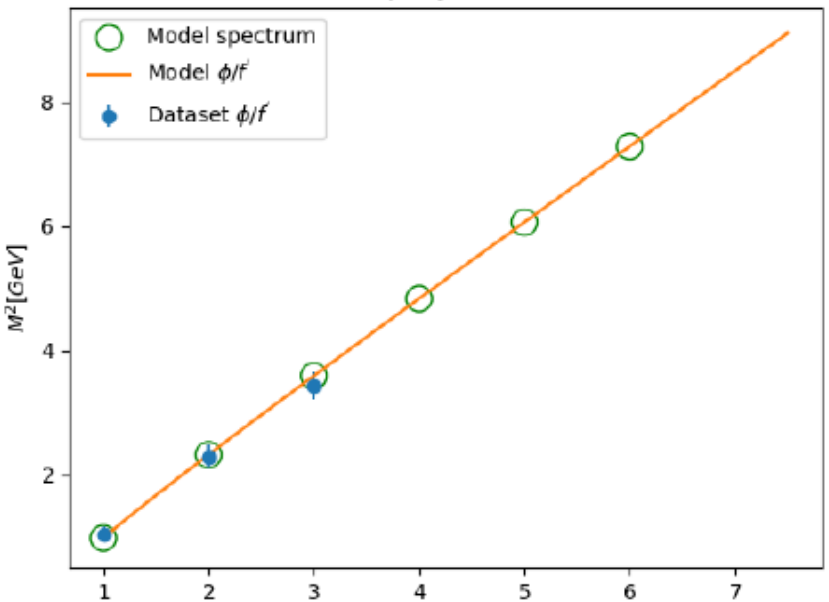



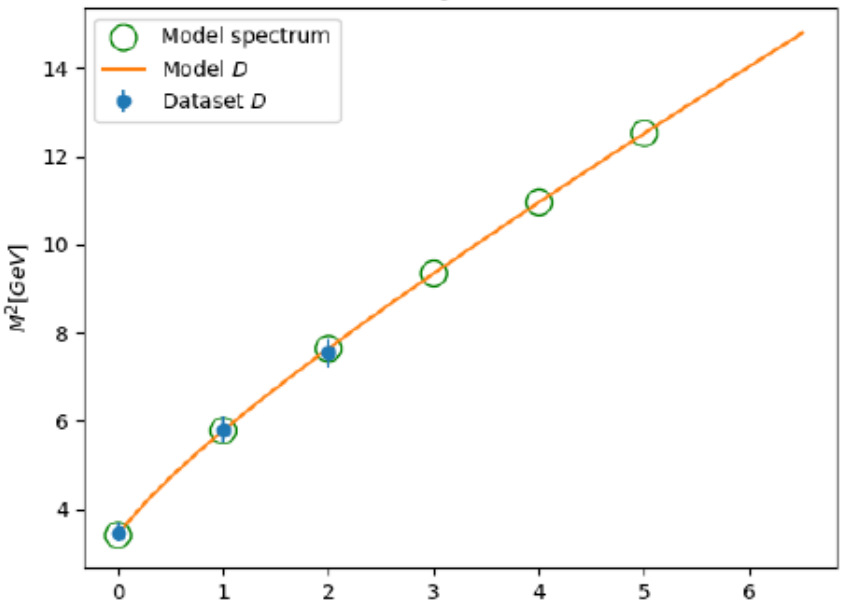



# Fits of K,K*, f /f and D

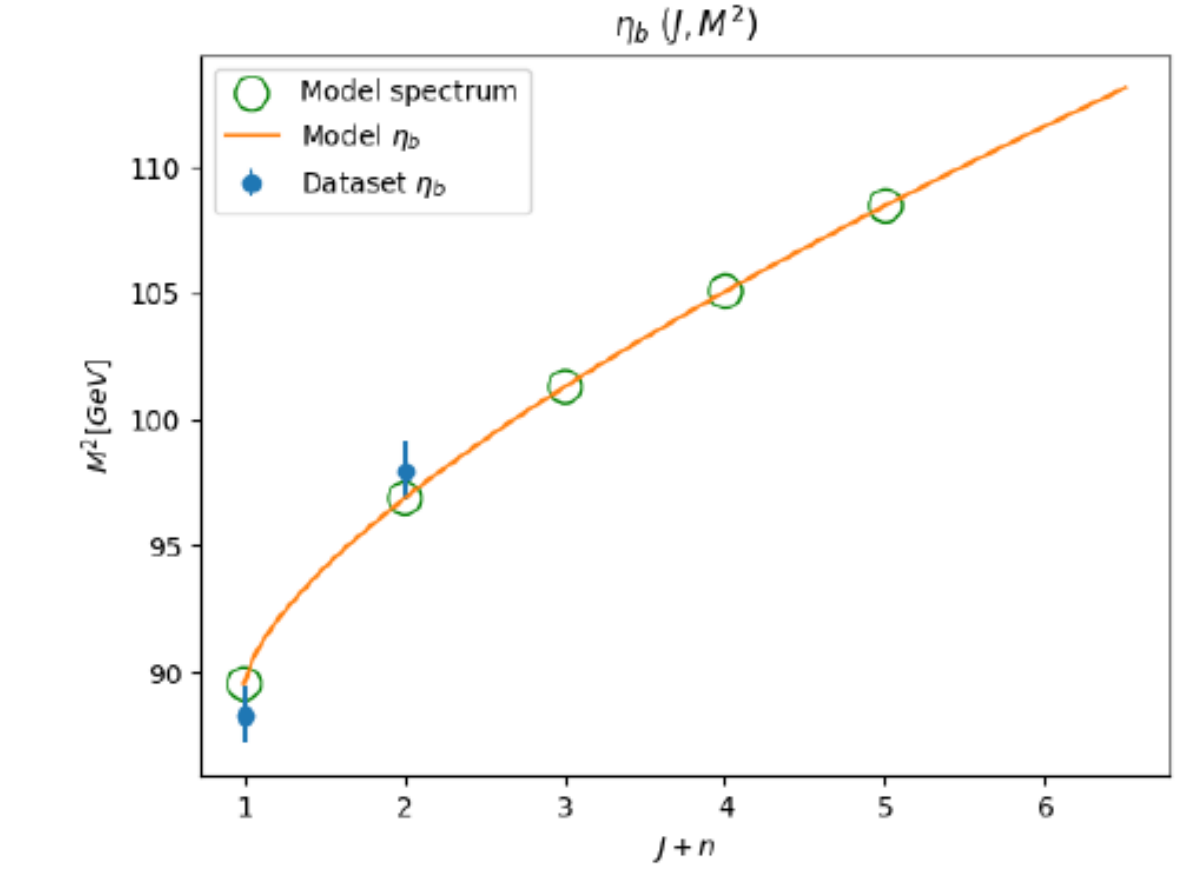


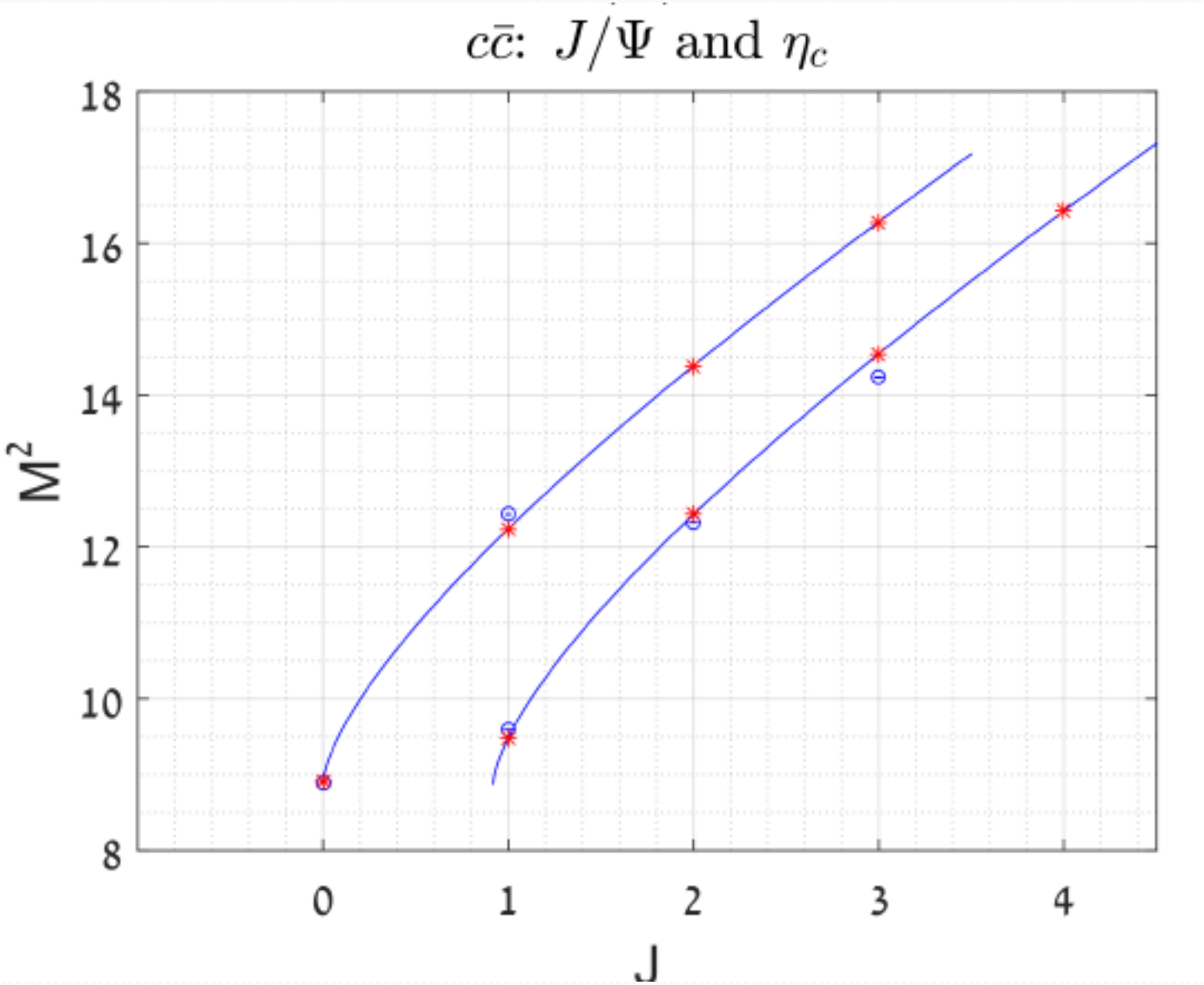



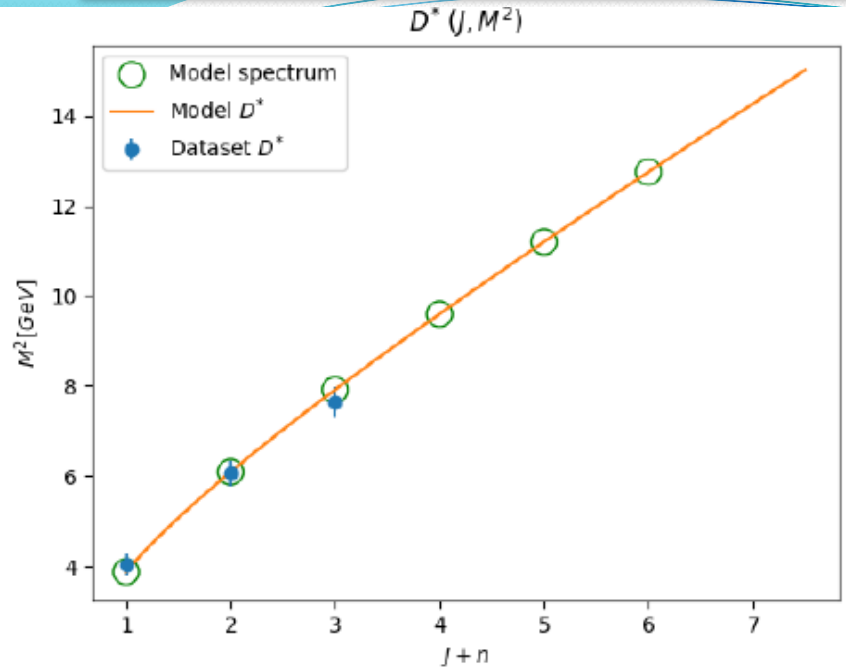
# Fit of hb

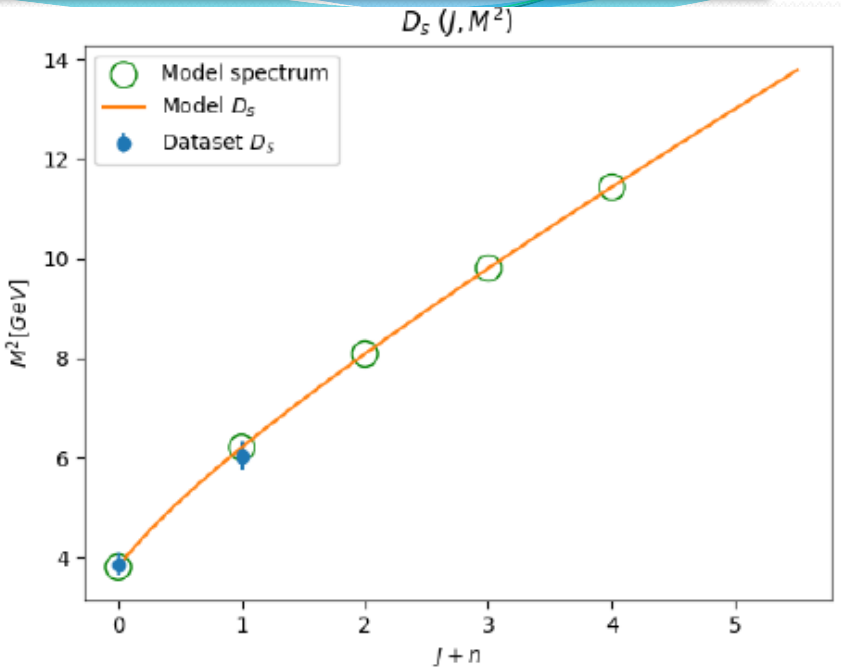

$\eta_b$ $(J, M^2)$
Model spectrum
Model $\eta_b$
Dataset $\eta_b$
110
105
100
95
90
$M^2$[GeV]
1
2
3
4
5
6
$J+n$

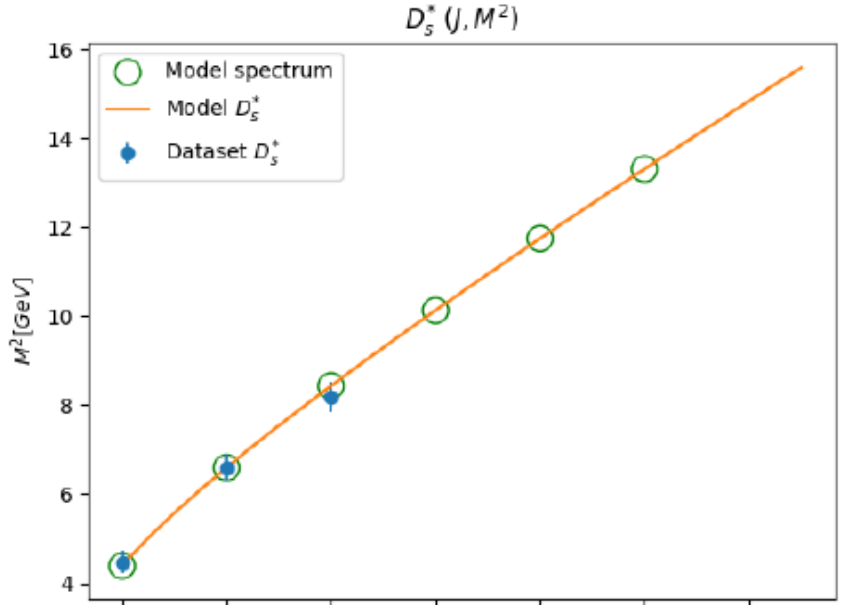
# Fitted trajectories of mesons

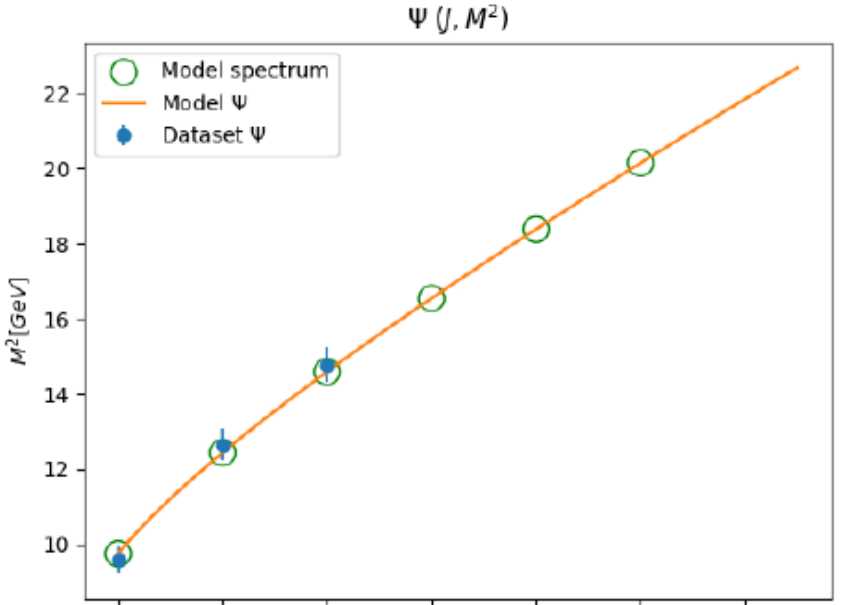


# Fits of D*, Ds,Ds* and y

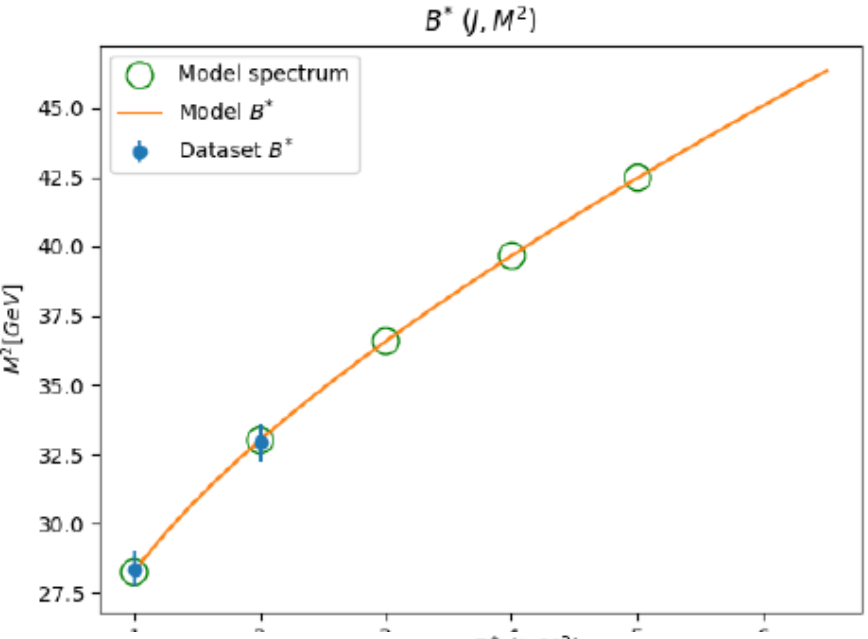


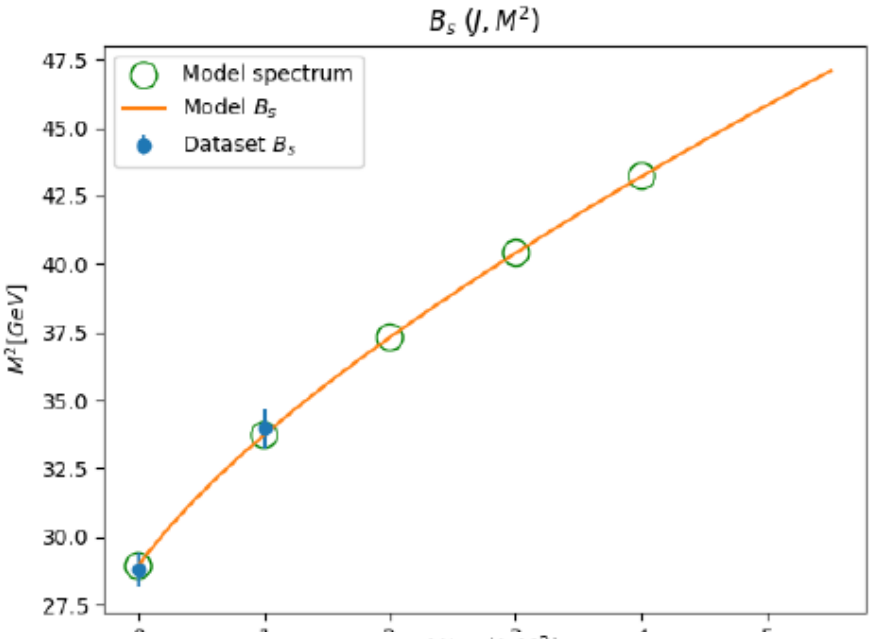



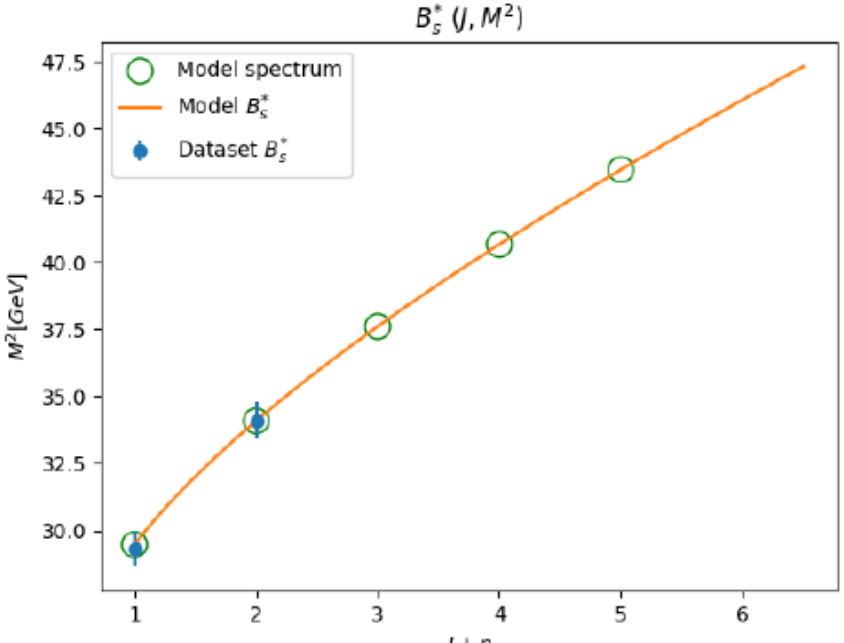



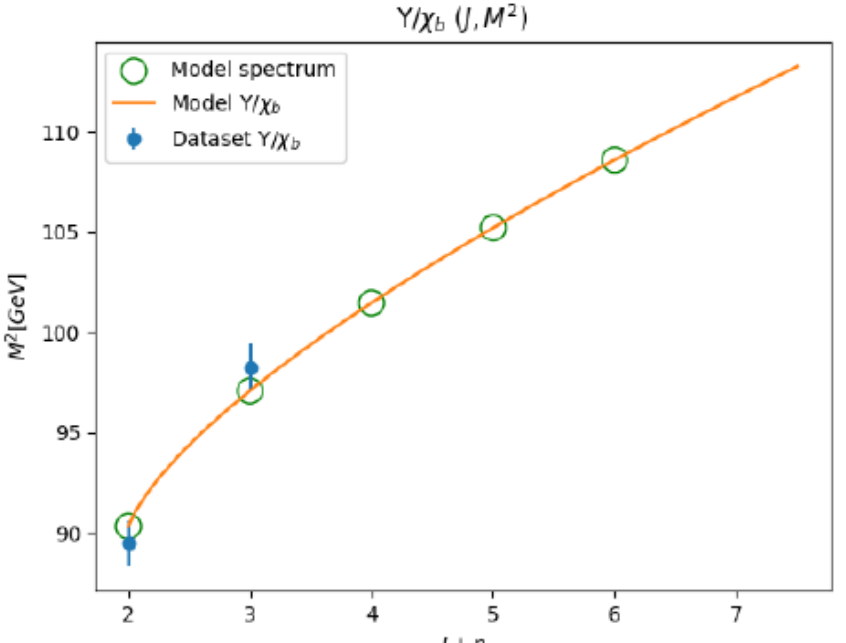



# Fits of B*, Bs, Bs*, U



# Toward a universal model

- The fit results for several trajectories simultaneously.
  The $(J, M^2)$ trajectories of $\rho, \omega, K^*, \phi$ $D$, and $\Psi$ mesons
- We take the string endpoint masses in MeV

$$m_{u/d} = 60, m_s = 220, m_c = 1500$$

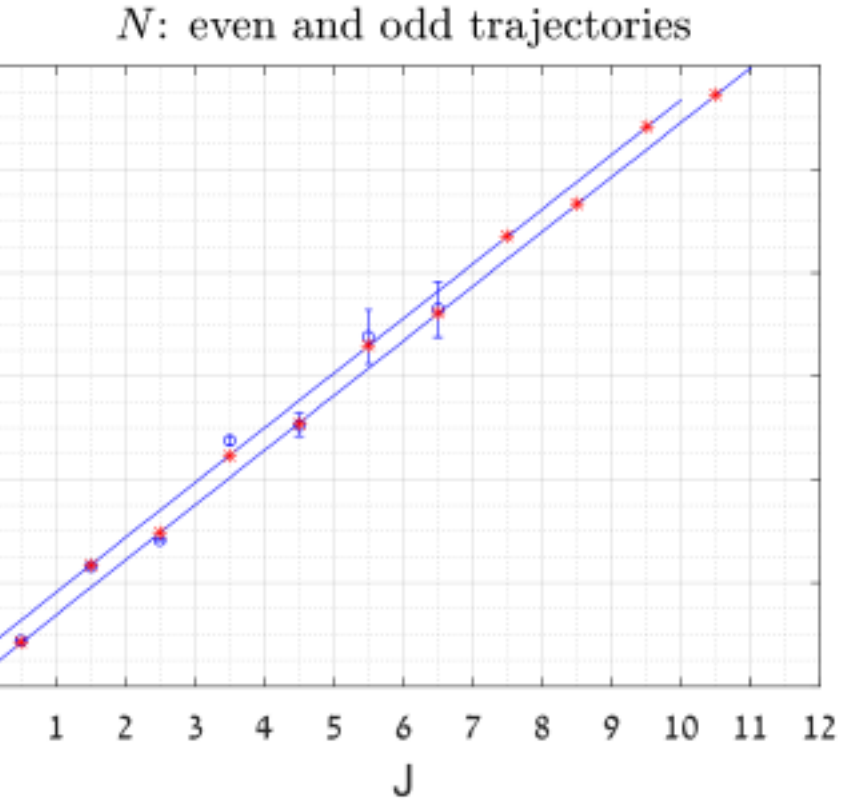
- Only the intercept was allowed tochange. We got

$$\alpha' = 0.899$$

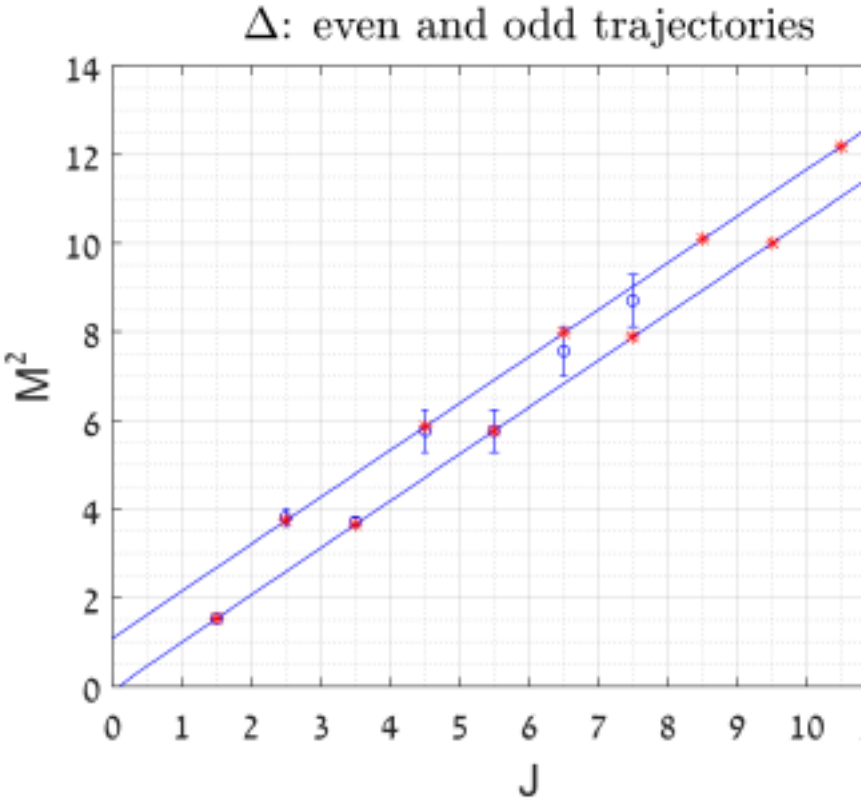
$$a_\rho = 0.51, a_\omega = 0.52, a_{K^*} = 0.49$$

$$a_\phi = 0.44, a_D = 0.80, a_\Psi = 0.94$$

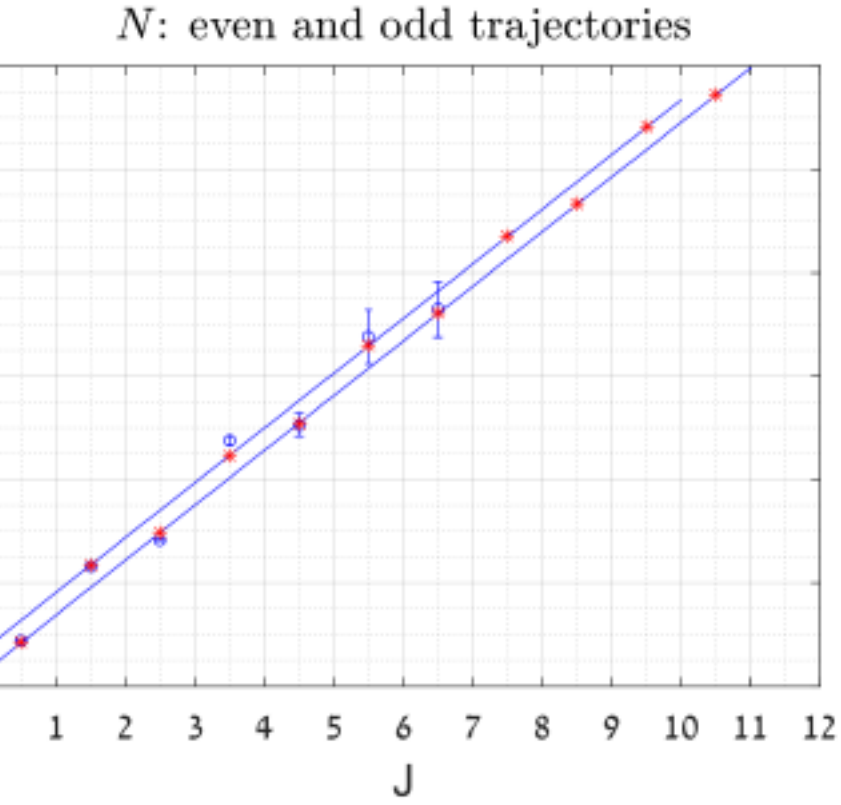
# Trajectories of N and Δ



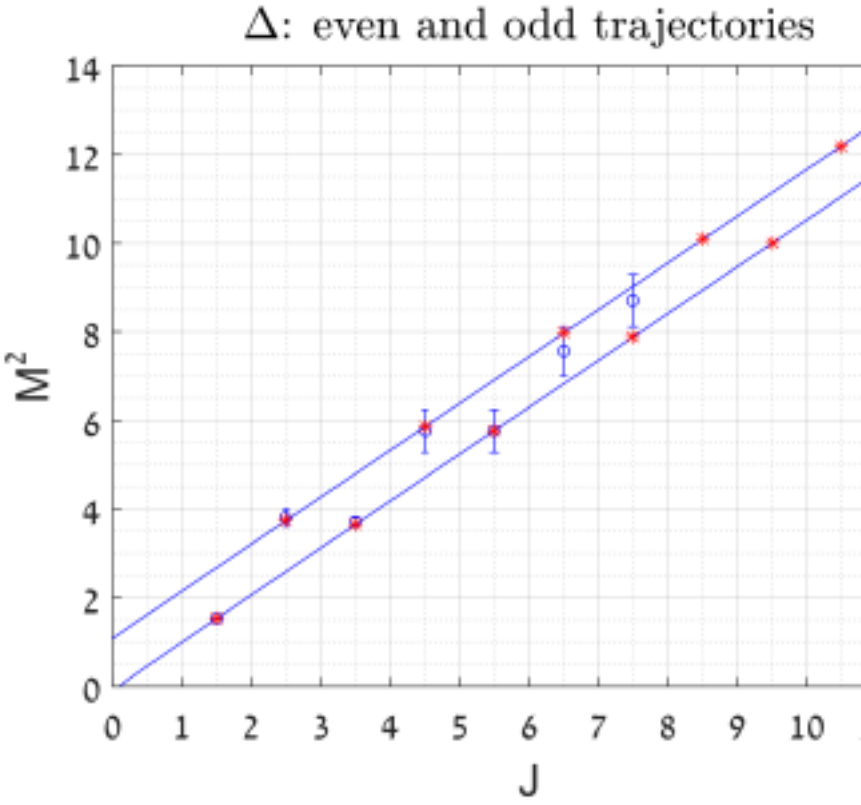

(b)



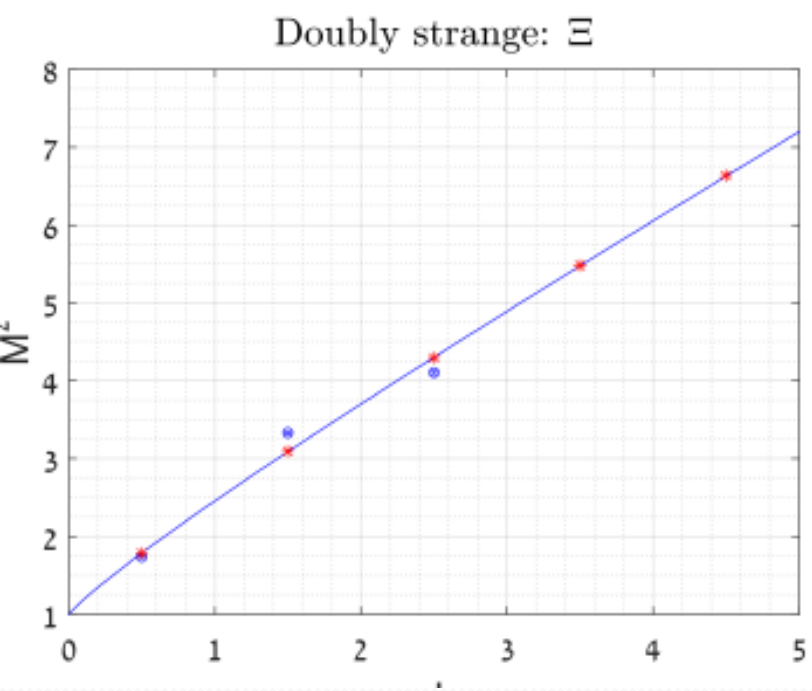
# Trajectories of N and Δ

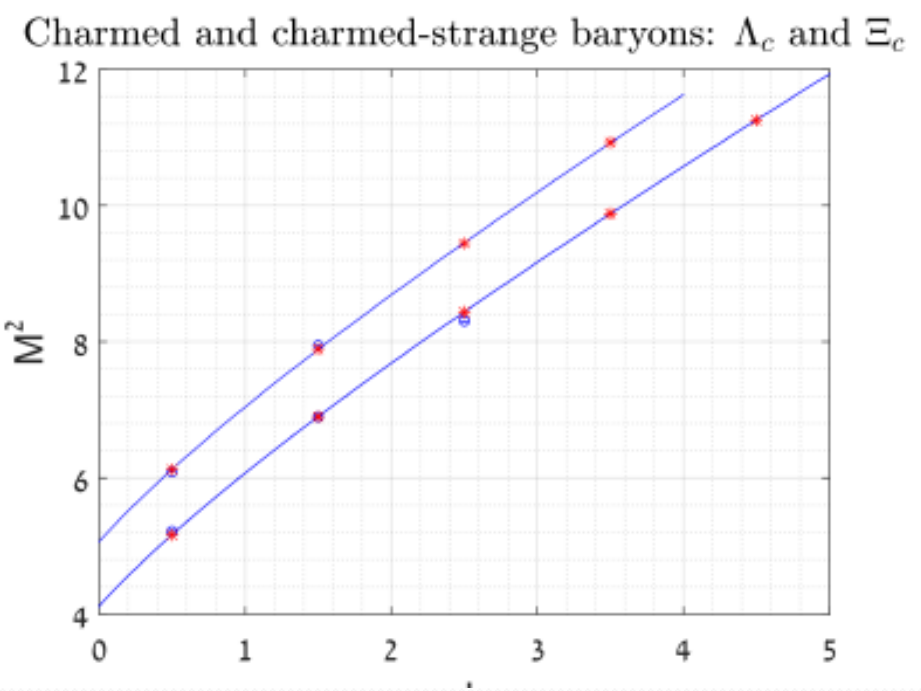



(b)

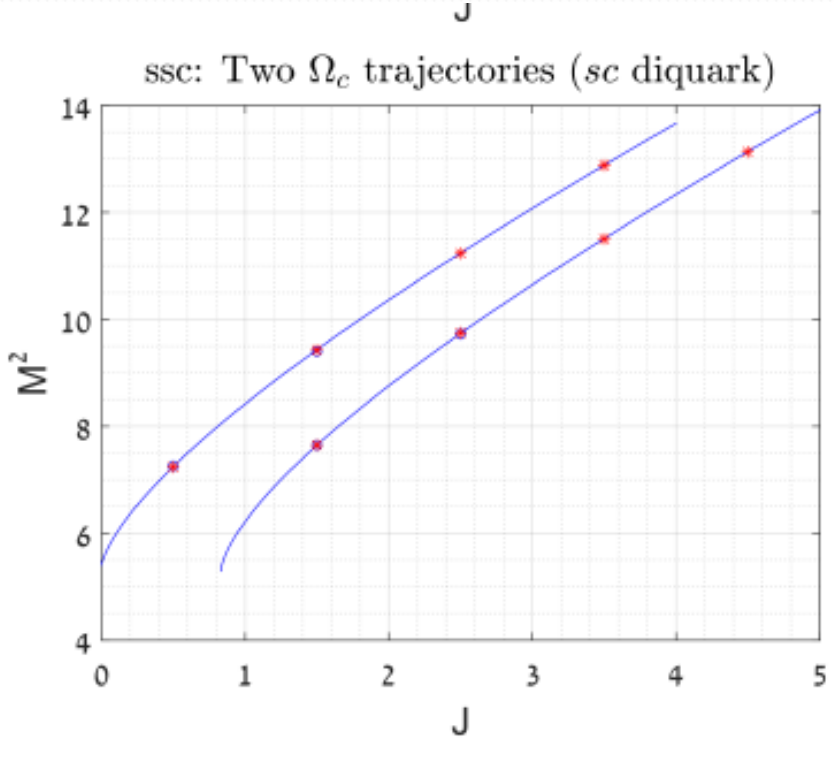


# Trajectories of $\Xi$ $\Lambda_c$ and $\Xi_c$

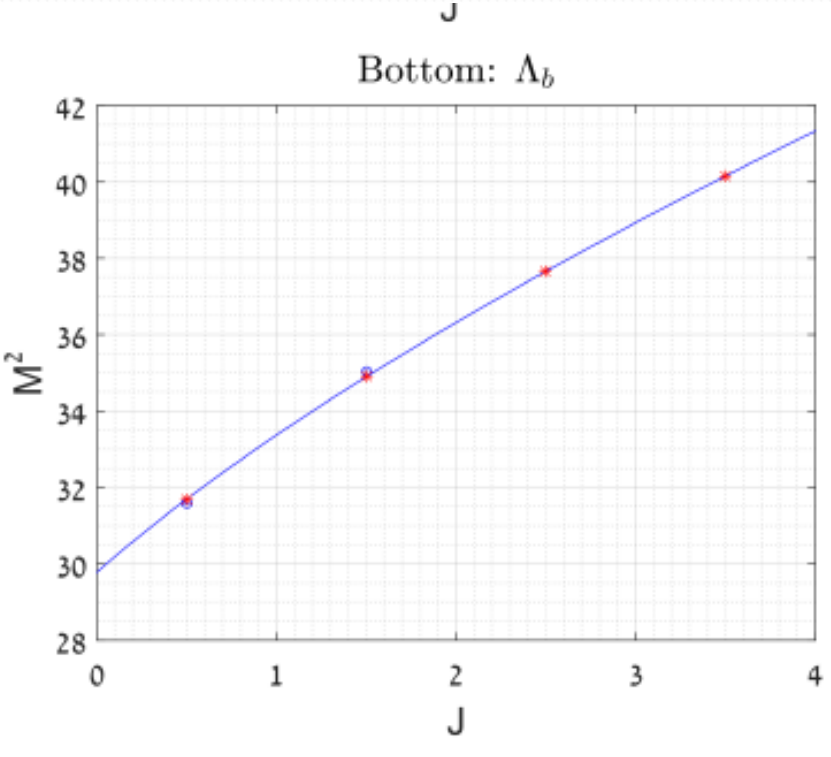


# Trajectories of $\Omega_c$ and $\Lambda_b$




(i)

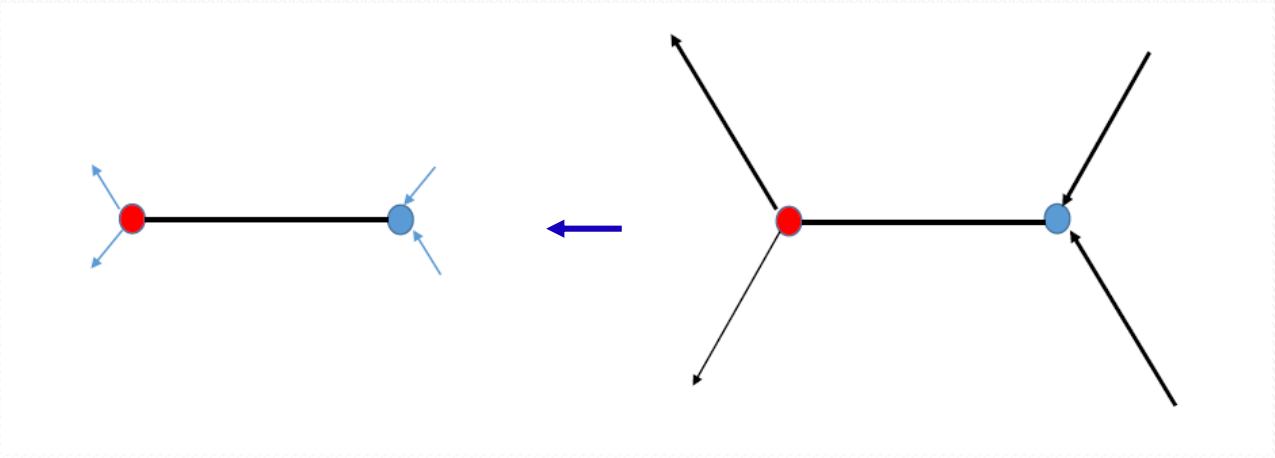



(h)

# Tetraquarks HISH

# The HISH structure of a tetraquark

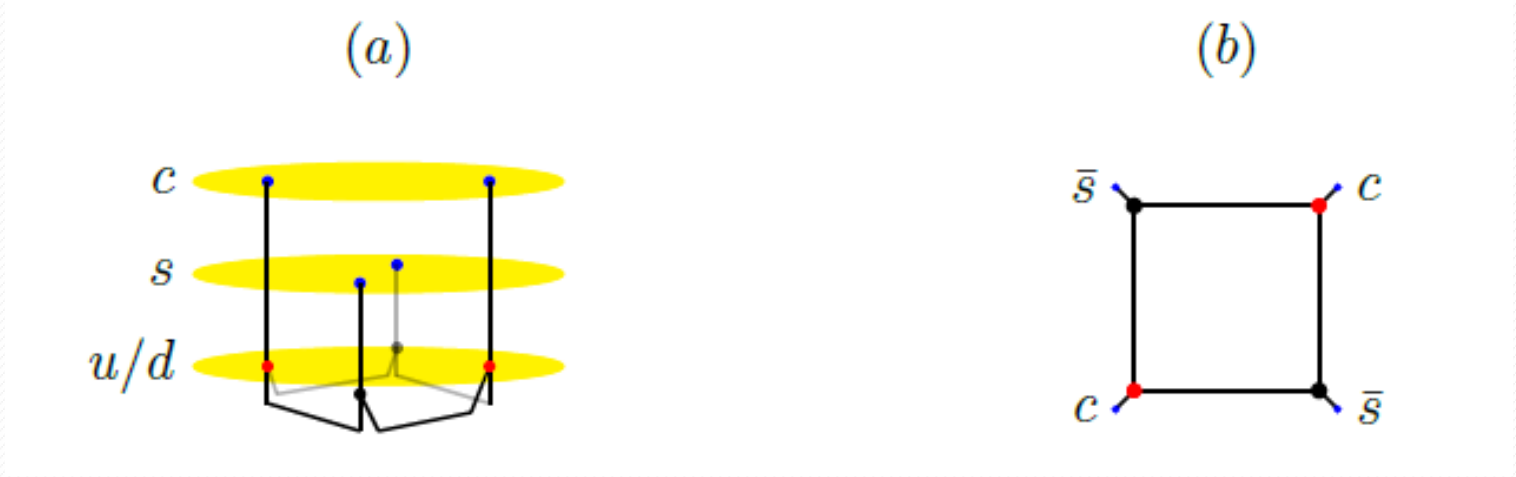
- A HISH tetraquark has four string endpoints, two with outgoing orientation and two incoming ones.
- It can have a layout of5 strings but from similar arguments as in the case of the baryon, it is more probable that it is build from a single string with a diquark on one end and an anti -diquark in the other.



# The HISH structure of a tetraquark

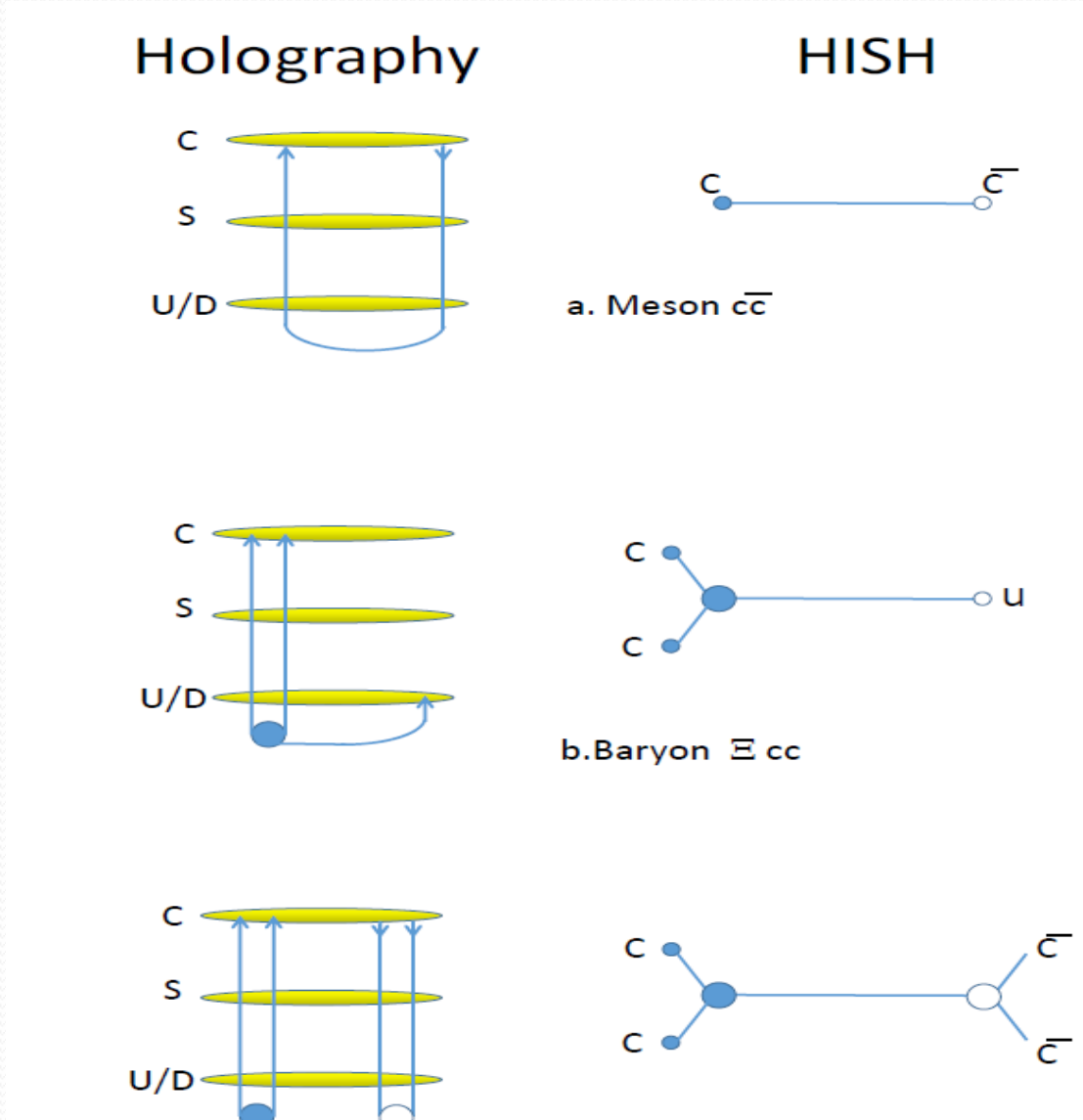
- An example of a holographic (a) and HISH (b) tetraquar

- In fact it can have also a more complicated structure



# The HISH structure of a tetraquark

- The structures of meson baryon and a tetraquark



# The HISH structure of a tetraquark

- We consider here mainly charmed tetraquarks which include one c or anti-c and up to $c\bar{c}c\bar{c}$

- Altogether there are 125 charmed tetraquarks, 40 with single c, 40 with anti -c, 10 with two charmed quarks and 10 with two anti -charmed quarks, 16 with $c\bar{c}$. 4 with $cc\bar{c}$ and 4 with $\bar{c}\bar{c}c$ and one $c\bar{c}c\bar{c}$

- There are 5 symmetric states with $q_1\bar{q}_1q_2, \bar{q}_2$

  44 semi-symmetric with $q_1\bar{q}_1q_2\bar{q}_3$

  60 asymmetric genuine with $q_1\bar{q}_2q_3\bar{q}_4$

# The HISH structure of a tetraquark

- There are separate trajectories fo pseudo scalar and vector meson , for spin ½ and spin 3/2 baryon . For instance the trajectories of (K,K*) (D,D*) (B,B*) and (N, D) are different.

- In a similar manner there should be different trajectories of spin 0 T, spin 1 T* and spin 2 T** tetraquarks.
- The mass difference is

$$M_{T^*} - M_T = \sqrt{\frac{|a_{T^*}|}{\alpha'}} - \sqrt{\frac{|a_T|}{\alpha'}}$$

# The HISH structure of a tetraquark

- Since the tetraquark is a single string it must have a HMRT .

- The ground sate that has vanishing orbital angular momentum has a mass

$$M_T = M_{diquark_L} + M_{BV_L} + M_{diquark_R} + M_{BV_R} + \sqrt{\frac{|a_T|}{\alpha'}}$$

slope intercept

# Decay mechanisms of HISH tetraquarks

- Is it possible that a tetraquark is stable against strong decays?
- Assuming that the only decay channels are the breakup and annihilation and since the threshold to decay into two mesons is lower the stability condition is

$$M_T - (M_{m_a} + M_{m_b}) < 0$$

- Where the ma and mb are the mesons resulting from the annihilation

$$(q_L^1 \bar{q}_R^1) and ((q_L^2 \bar{q}_R^2) \text{ or } (q_L^1 \bar{q}_R^2) and ((q_L^2 \bar{q}_R^1)$$

# Decaymechanisms of HISHtetraquarks

- Inserting the masses of the mesons we get

$$M_T - (M_{m_a} + M_{m_b}) = M_{BV_L} + M_{BV_R}\sqrt{\frac{|a_T|}{\alpha'_T}} - \sqrt{\frac{|a_{m_a}|}{\alpha'_{m_a}}} - \sqrt{\frac{|a_{m_b}|}{\alpha'_{m_b}}}$$

- mass of the BV  mass of the anti-BV

$(\alpha'_T, \alpha'_{m_a}, \alpha'_{m_b})$ slope of tetraquark and mesons

$(a_T, a_{m_a}, a_{m_b})$ intercept of tetraquark and mesons

- Note the masses of the quarks cancel out .

# Decay mechanisms of HISH tetraquarks

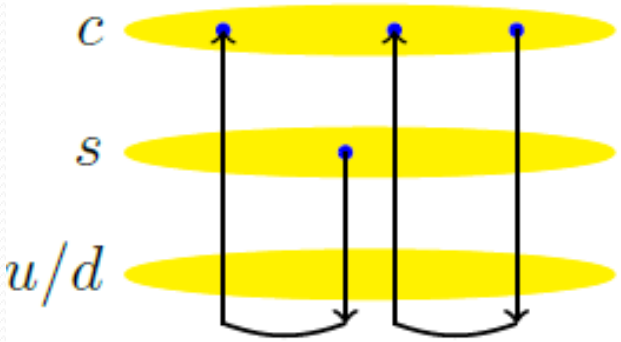
- Using a baryon that includes the same diquark as the tetraquark and a third quark the stability condition reads

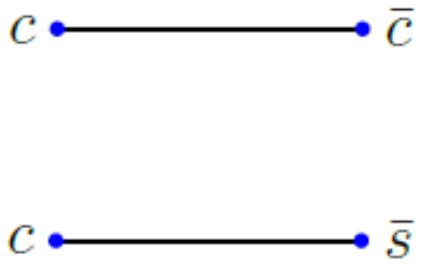
$$M_T-(M_{m_a}+M_{m_b})=+\sqrt{\frac{|a_T|}{\alpha'_T}}-(\sqrt{\frac{|a_{B_L}|}{\alpha'_{B_L}}}+\sqrt{\frac{|a_{B_R}|}{\alpha'_{B_R}}})+(M_{B_L}+M_{B_R})-(m_{\tilde{q}_L}+m_{\tilde{q}_L})-(M_{m_a}+M_{m_b})$$

- Once we have relaible values for the intercepts we will be able to check whether there are stable tetraquarks.

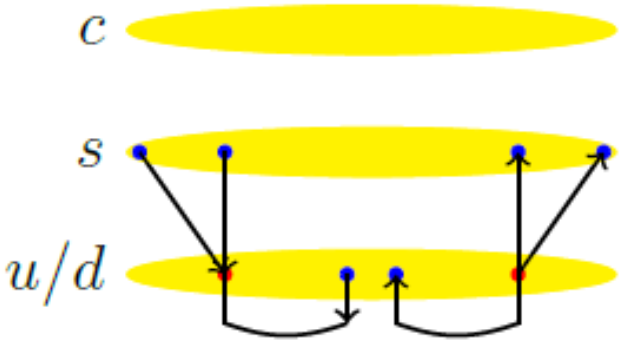
# Decay mechanisms of HISH tetraquarks

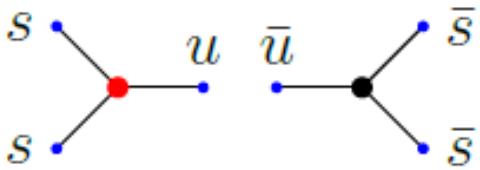
- Decay of $(cc\bar{c}\bar{s})$ via annihilation in holography and HISH



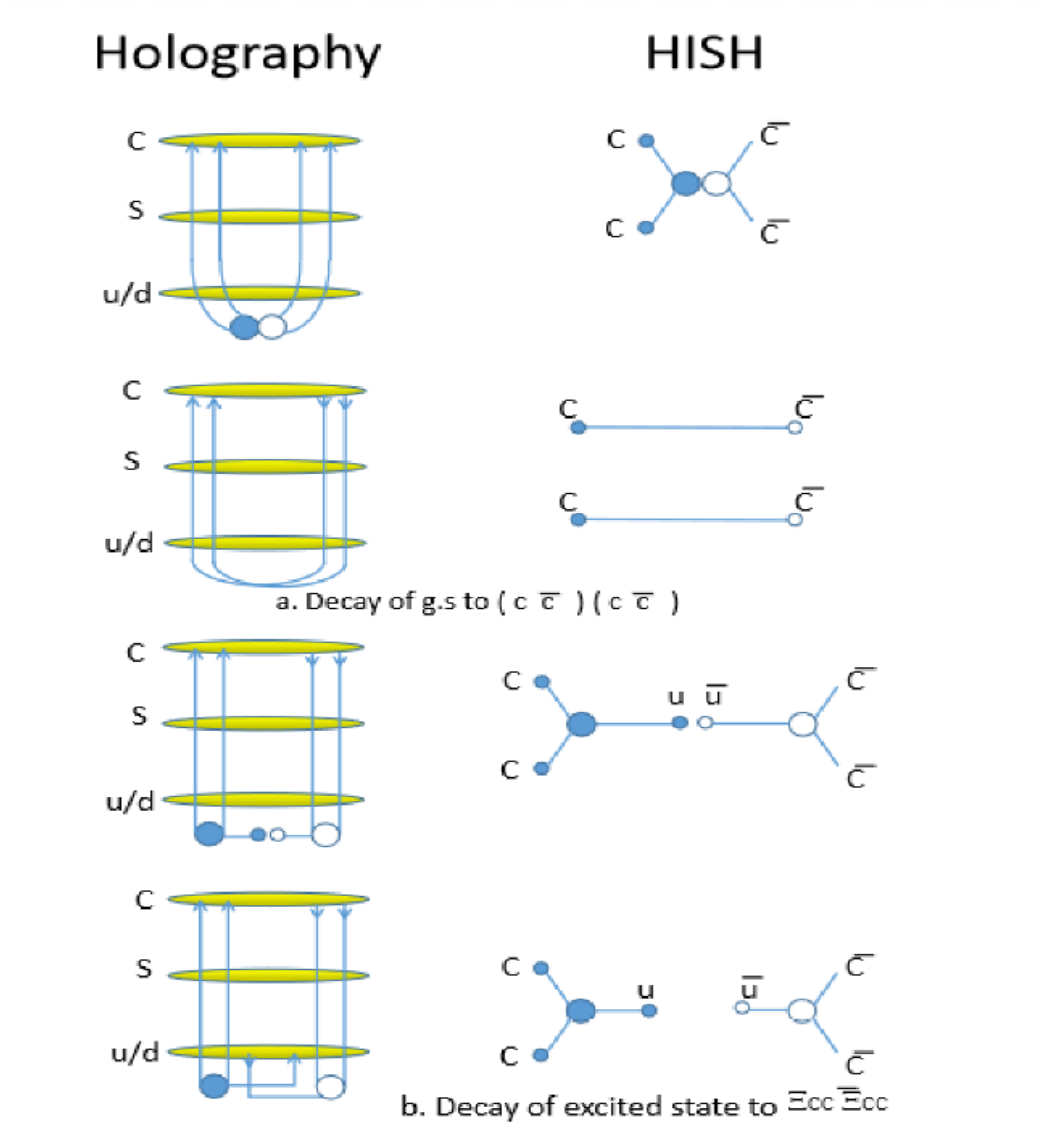




- Decay of $s\bar{s}s\bar{s}$ via string breakup



# Possible decays of the tetraquarks



# Pentaquarks: HISH

# Pentaquarks

- Tetraquarks are the simplest exotic hadrons related to mesons since they carry Baryon #0.
- In a similar way there are exotics related to baryons with B=1. These are referred to as pentaquarks since they include 4 quarks and one anti-quark.
- Excluding the top quark there are altogether 350 possible pentaquarks . 5 possibilities of the antiquark. 5 different combination for 4 different quarks so altogether 25.
- $\bar{q}_1 q_2 q_2 q_3 q_4$ 150 states $\bar{q}_1 q_2 q_2 q_2 q_3$ 100 states
- $\bar{q}_1 q_2 q_2 q_3 q_3$ 50 states $\bar{q}_1 q_2 q_2 q_2 q_2$ 25 states

# Pentaquarks

- Charmed pentaquarks there are 210 possible states

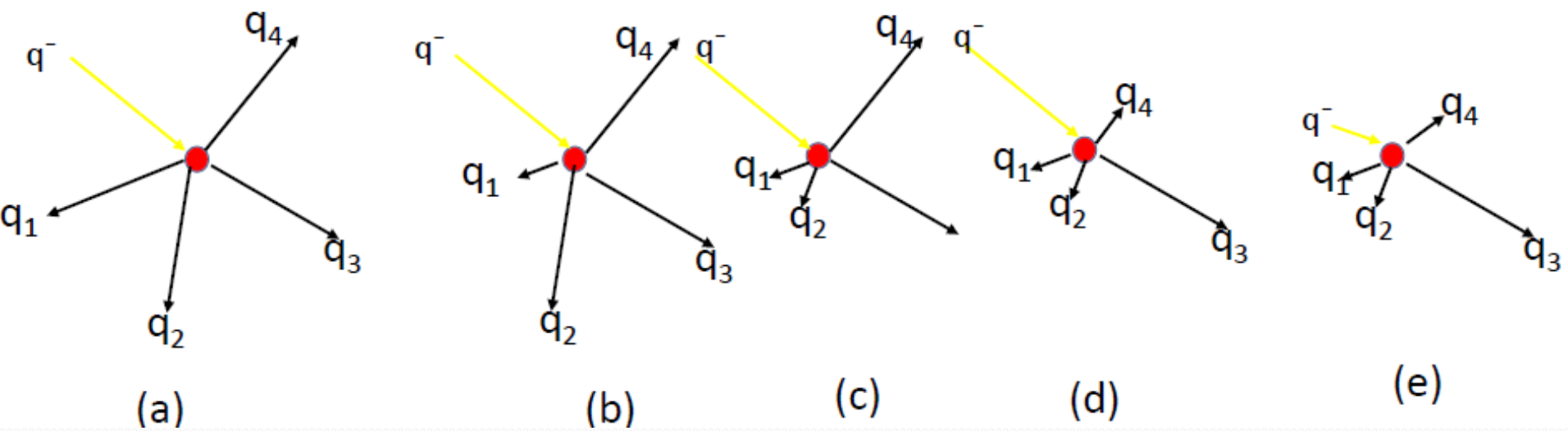
| 70 | 80 | 40 | 16 | 4 |
|---|---|---|---|---|
| one $\bar{c}$, | one c | two c | three c | four c |

- There 115 genuine pentaquarks that do not contain a quark and anti-quark of the same flavor.

- There are 50 that are semi-genuine containing a heavy pair

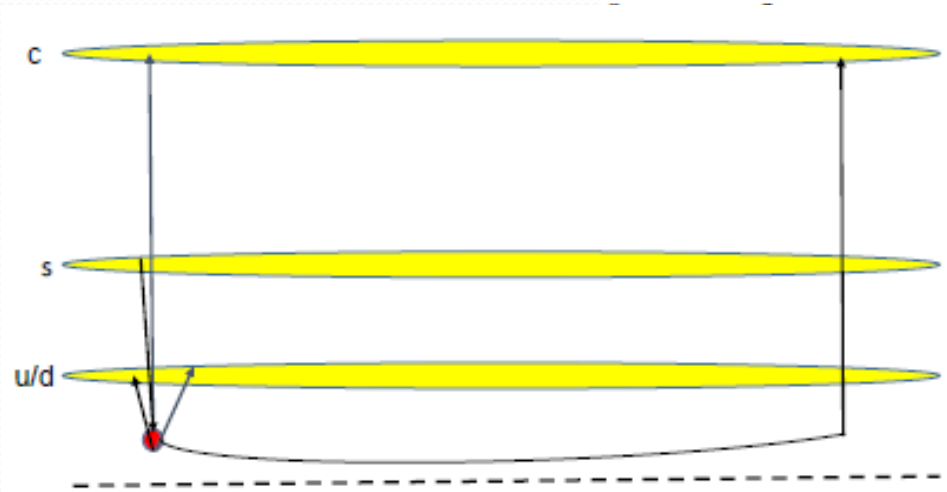
# A single BV pentaquark

- There are two elementary configurations for pentaquarks.
- Since by construction the BV has to have a net number of Nc=3 strings coming out of it one can add pairs of string anti-string to the BV.
- Thus there is a structure based on a single BV . There are several options for that

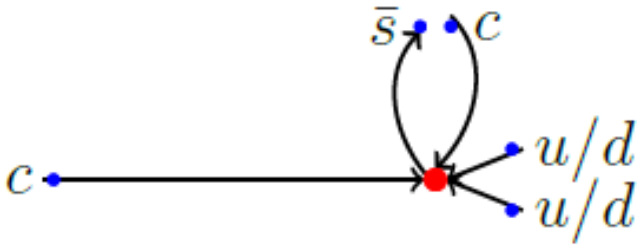


# A single BV pentaquark

- Based on the analogy with the string structure of the baryon we assume that the only stable configuration is (e)
- For a pentaquark with content of $(c, c, \bar{s}u/d, u/d)$
- In holography




- In HISH



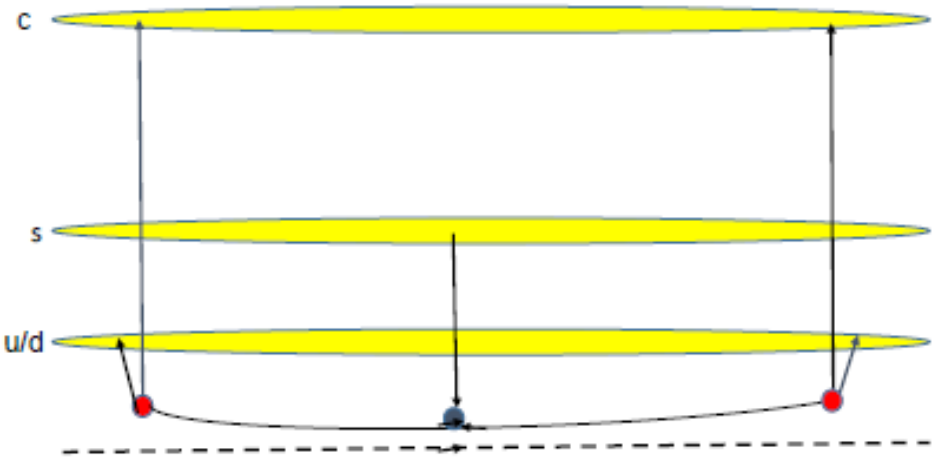
# Pentaquarkwith a BV anti-BV and BV

- The other option is adding a pair of BV anti -BV to the BV of a baryon.

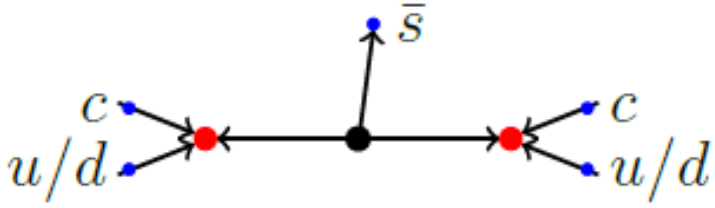
- The pentaquark is built from two finite size strings with a BV and a diquark on one side of them and an anti -BV on the other side with an anti -diquark attached to it.
- For a content of $(q_1, q_2, q_3, q_4, \bar{q}$ ) there are 6 possibilities of forming two diquarks.
- It is not necessary that the two strings are collinear.
- Since the contribution of the BVs are small pentaquarkswith this setup are not heavier than the those with a single BV.

# Pentaquarkwith a BV anti-BV and BV

- In holography this type of pentaquark looks



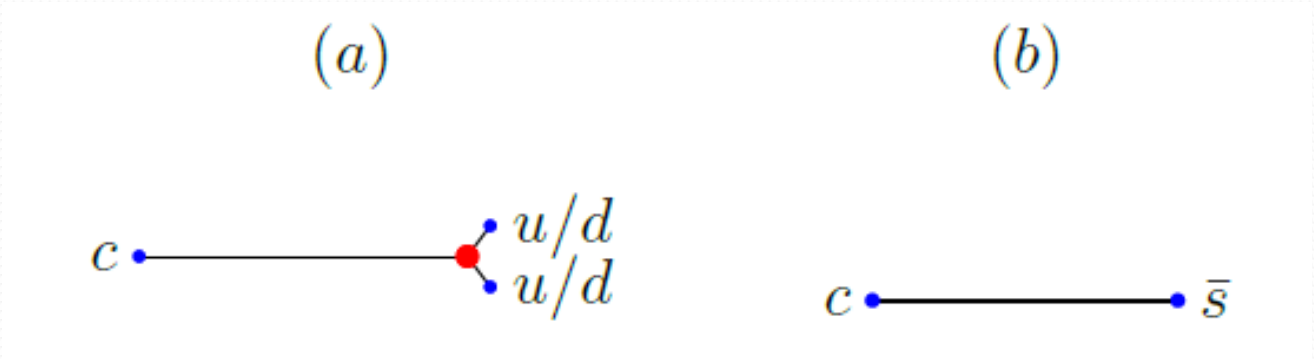

- In HISH



# Decays ofpentaquarks

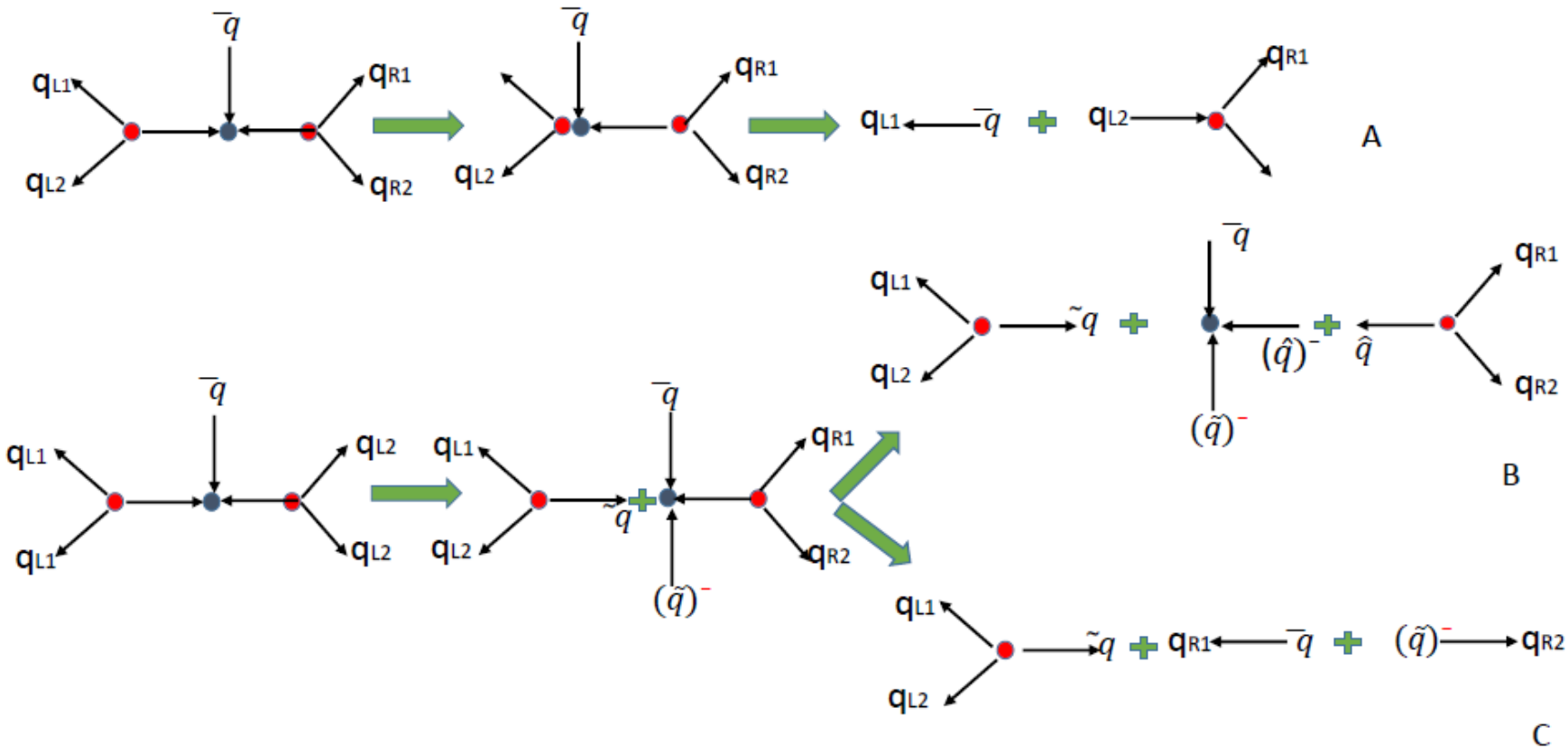
- The decay of a pentaqurk built from a single BV can be via a breaking of the long string producing a penaquark and a meson.

- A decay via a detachment of the endpoints of an outgoing and incoming strings on the BV



# Decays ofPentaquarks

- Decays of a (BV anti-BV BV) pentaquark



# Fitting procedure

- The mass of the exotic hadron

$$E = E(m_1, m_2, \beta_1, \beta_2(m_1, m_2, \beta_1), \alpha', a)$$

$$E = \sum_{i=1,2} m_i \left( \frac{\beta_i \arcsin \beta_i + \sqrt{1-\beta_i}}{1-\beta_i^2} \right)$$

- The angular momentum + excitation #

$$J + n = (J + n)(\alpha', \beta_1, \beta_2(m_1, m_2, \beta_1), m_1, m_2, a)$$

$$J = \sum_{i=1,2} \pi \alpha' m_i^2 \frac{\beta_i^2}{(1-\beta_i^2)^2} \left( \arcsin \beta_i + \beta_i \sqrt{1-\beta_i} \right)$$

- The direct dependence

$$E = E(J + n, m_1, m_2, \alpha', a) \rightarrow \boxed{M^2 = E^2(J + n, m_1, m_2, \alpha', a)}$$

# Fitting procedure

- The fit degree of conformity

$$\chi_r^2 = \frac{1}{\nu} \sum_{i=1}^{n} \frac{(M_{i,exp}^2 - M_{i,thry}^2)^2}{\sigma_i^2}$$

Estimated measured mass

predicted mass

total number of DOF

total error of $M_{exp}^2$

# Fitting procedure

- We make the following assumptions:

$$M_{BV} + M_{diquark} = m_{q1} + m_{q2}$$

- For diquark composed of q1and q2. This means that the BV is in the vicinity of the wall and its mass is negligible .

- The slope of a tetraquark (pentaquark ) is the sameas that of the meson(baryon) with similar endpoints.

# Fitting procedure

- We use 4 different fitting methods :
- Free fit- all the parameters $(\alpha', a, m_1 \text{ and } m_2)$
- m1 and m2 fixed a and a free to vary.
- m1 and m2 free to vary
- A the intercept free to vary

# Tetraquark candidates and predictions

# Tetraquark candidates

| Candidate | Quarks Content | Selected Decay Channels | $J^{PC}$ | Mass $[MeV]$ | Width $[MeV]$ | Ref. |
|---|---|---|---|---|---|---|
| $X(1855)$ | Molecule | $\bar{p}n$ | $?^{??}$ | $1856.6 \pm 5$ | $20 \pm 5$ | [56] |
| $X(1835)$ | Molecule | $p\bar{p}$ | $0^{-+}$ | $1826.5^{+13.0}_{-3.4}$ | $242^{+14}_{-15}$ | [57], [58], [59] |
| $f_4(2300)$ | $u\bar{d}\bar{u}d$ or $d\bar{d}\bar{d}d$ | $N\bar{N}$ | $4^{++}$ | $2320 \pm 60$ | $250 \pm 80$ | [60], [61], [62], [63] |
| $f_2(1640)$ | $u\bar{s}\bar{u}s$ or $d\bar{s}\bar{d}s$ | $K\bar{K}$ | $2^{++}$ | $1639 \pm 6$ | $99^{+60}_{-40}$ | [64], [65], [66] |
| $f_2(1750)$ | $u\bar{s}\bar{u}s$ or $d\bar{s}\bar{d}s$ | $K\bar{K}$ | $2^{++}$ | $1755 \pm 10$ | $67 \pm 12$ | [67] |
| $f_2(2300)$ | $u\bar{s}\bar{u}s$ or $d\bar{s}\bar{d}s$ | $\Lambda\bar{\Lambda}$ | $2^{++}$ | $2297 \pm 28$ | $149 \pm 40$ | [68] |

# Tetraquark candidates

| | | | | | | |
|---|---|---|---|---|---|---|
| $\omega(2290)$ | $u\bar{s}\bar{u}s$ or $d\bar{s}\bar{d}s$ | $\Lambda\bar{\Lambda}$ | $1^{--}$ | $2290 \pm 20$ | $275 \pm 35$ | [69], [70], [71] |
| $f_3(2300)$ | $u\bar{s}\bar{u}s$ or $d\bar{s}\bar{d}s$ | $\Lambda\bar{\Lambda}$ | $3^{++}$ | $2334 \pm 25$ | $200 \pm 20$ | [69], [70] |
| $X(3250)$ | $\bar{u}su\bar{s}$ or $\bar{d}sd\bar{s}$ | $\Lambda\bar{p}K^+$ | $?^{??}$ | $3250 \pm 8 \pm 20$ | $45 \pm 18$ | [72] |
| $X(3250)$ | $\bar{u}su\bar{s}$ or $\bar{d}sd\bar{s}$ | $\Lambda\bar{p}K^+\pi^\pm$ $K^0_S p\bar{p}K^\pm$ | $?^{??}$ | $3245 \pm 8 \pm 20$ | $25 \pm 11$ | [72] |
| $K(3100)^0$ | $u\bar{s}\bar{u}d$ | $\Sigma(1385)^+\bar{p}$ | $?^{??}$ | $\sim 3100$ | $\sim 10 - 70$ | [73] |
| $X(2632)$ | $c\bar{u}u\bar{s}$ | $D^0K^+$ $D_s^+\eta$ | $?^{??}$ | $2635.2 \pm 3.3$ | $< 17$ | [74] |
| $T_{cs0}(2900)^0$ | $\bar{c}du\bar{s}$ | $D^-K^+$ | $0^+$ | $2866 \pm 7$ | $57 \pm 13$ | [75] |
| $T_{cs1}(2900)^0$ | $\bar{c}d\bar{u}s$ | $D^-K^+$ | $1^-$ | $2904 \pm 5$ | $110 \pm 12$ | [75] |
| $T^a_{c\bar{s}0}(2900)^0$ | $c\bar{s}\bar{u}d$ | $D_s^+\pi^-$ | $0^?$ | $2892 \pm 14 \pm 15$ | $119 \pm 26 \pm 13$ | [76] |
| $T^a_{c\bar{s}0}(2900)^{++}$ | $c\bar{s}u\bar{d}$ | $D_s^+\pi^+$ | $0^?$ | $2921 \pm 17 \pm 20$ | $137 \pm 32 \pm 17$ | [76] |
| $X(3350)$ | $c\bar{d}d\bar{u}$ | $\Lambda_c^+\bar{p}$ | $0or1^{??}$ | $3350^{+10}_{-20} \pm 20$ | $70^{+40}_{-30} \pm 40$ | [77], [78] |
| $X(3800)^6$ | $c\bar{d}d\bar{u}$ | $\Lambda_c^+\bar{p}$ | $?^{??}$ | $3840 \pm 10$ | $30 \pm 30$ | [77] |

# Tetraquark candidates

| | | | | | | |
|---|---|---|---|---|---|---|
| $T_{cc}(3875)$ | $cc\bar{u}\bar{d}$ | $D^{*+}D^{0}$<br>$D^{*0}D^{+}$ | $?^{?}$ | $3874 \pm 0.11$ | $0.41 \pm 0.17$ | [79] |
| $Z_c(3900)$ | $c\bar{c}u\bar{d}$<br>$c\bar{c}d\bar{u}$ | $J/\psi\pi^{\pm}$<br>$D\bar{D}^{*\pm}$ | $1^{+-}$ | $3887.1 \pm 2.6$ | $28.4 \pm 2.6$ | Many, including [80] |
| $Z_c(4200)^{\pm}$ | $c\bar{c}u\bar{d}$<br>or<br>$c\bar{c}d\bar{u}$ | $J/\psi\pi^{+}$ | $1^{+-}$[7] | $4196^{+35}_{-32}$ | $370^{+100}_{-150}$ | [81] |
| $Z_c(4430)^{\pm}$ | $c\bar{c}u\bar{d}$<br>or<br>$c\bar{c}d\bar{u}$ | $\pi^{+}\psi(2S)$<br>$\pi^{+}J/\psi$ | $1^{+-}$[7] | $4478^{+15}_{-18}$ | $181 \pm 31$ | [82], [83], [84], [85] |
| $X(4020)^{\pm}$ | $c\bar{c}ud$<br>or<br>$c\bar{c}d\bar{u}$ | $D^{*}\bar{D}^{*}$<br>$h_c(1P)\pi^{\pm}$ | $?^{?-}$ | $4024.1 \pm 1.9$ | $13 \pm 5$ | [86], [87] [88], [88] |
| $X(4051)^{\pm}$ | $c\bar{c}u\bar{d}$<br>or<br>$c\bar{c}d\bar{u}$ | $\pi^{+}\chi_{c1}(1P)$ | $?^{?+}$ | $4051^{+24}_{-40}$ | $82^{+50}_{-28}$ | [89] |
| $X(4055)^{\pm}$ | $c\bar{c}u\bar{d}$<br>or<br>$c\bar{c}d\bar{u}$ | $\pi^{+}\psi(2S)$ | $?^{?-}$ | $4054 \pm 3.2$ | $45 \pm 13$ | [90], [91] [92] |
| $X(4100)^{\pm}$ | $c\bar{c}u\bar{d}$<br>or<br>$c\bar{c}d\bar{u}$ | $\pi^{-}\eta(1S)$ | $?^{??}$ | $4096 \pm 28$ | $152^{+80}_{-70}$ | [93] |

# Tetraquark candidates

| | | | | | | |
|---|---|---|---|---|---|---|
| $R_{c0}(4240)^{\pm}$ | $c\bar{c}u\bar{d}$ or $c\bar{c}d\bar{u}$ | $\pi^-\psi(2S)$ | $0^{--}$ | $4239^{+50}_{-21}$ | $220^{+120}_{-90}$ | [94] |
| $X(4250)^{\pm}$ | $c\bar{c}ud$ or $c\bar{c}d\bar{u}$ | $\pi^+\chi(1P)$ | $?^{?-7}$ | $4248^{+190}_{-50}$ | $177^{+320}_{-70}$ | [89] |
| $T^{\theta}_{\psi s1}(4000)^+$ also $Z_{cs}(4000)^+$ | $c\bar{c}u\bar{s}$ | $J/\psi K^+$ $D_s^+\bar{D}^{*0}$ $D_s^{*+}\bar{D}^0$ | $1^+$ | $3980-4010$ | $5-150$ | [94] |
| $T_{\psi s1}(4220)^+$ also $Z_{cs}(4220)^+$ | $c\bar{c}u\bar{s}$ | $J/\psi K^+$ | $1^+$ | $4216^{+50}_{-40}$ | $233^{+110}_{-90}$ | [94] |
| $T^{\theta}_{\psi s1}(4000)^0$ | $c\bar{c}d\bar{s}$ | $J/\psi K^0_S$ | $1^+$ | $3991^{+14}_{-20}$ | $105^{+12+9}_{-10-17}$ | [95] |
| $X(3960)$ | $c\bar{c}s\bar{s}$ | $D_s^+\ D_s^-$ | $0^{++}$ | $3956\pm5\pm10$ | $43\pm13\pm8$ | [96] |
| $X(4140)$ | $c\bar{c}s\bar{s}$ | $J/\psi\phi$ | $1^{++}$ | $4146.5\pm3.0$ | $19^{+7}_{-5}$ | [97] |
| $X(4274)$ | $c\bar{c}s\bar{s}$ | $J/\psi\phi$ | $1^{++}$ | $4286^{+8}_{-9}$ | $51\pm7$ | [98] |
| $\chi_{c0}(4500)$ | $c\bar{c}s\bar{s}$ | $J/\psi\phi$ | $0^{++}$ | $4474\pm4$ | $77^{+12}_{-10}$ | [98] |
| $\chi_{c1}(4685)$ | $c\bar{c}s\bar{s}$ | $J/\psi\phi$ | $1^{++}$ | $4684^{+15}_{-17}$ | $126\pm40$ | [94] |
| $\chi_{c0}(4700)$ | $c\bar{c}s\bar{s}$ | $J/\psi\phi$ | $0^{++}$ | $4694^{+16}_{-5}$ | $87^{+18}_{-10}$ | [98] |
| $X(4350)^8$ | $c\bar{c}s\bar{s}$ | $J/\psi\phi$ | $?^{?+}$ | $4351\pm5$ | $13^{+18}_{-10}$ | [99] |
| $X(4630)$ | $c\bar{c}s\bar{s}$ | $J/\psi\phi$ | $1^{-+9}$ | $4626^{+24}_{-110}$ | $174^{+140}_{-80}$ | [94] |
| $\psi(4360)$ | $c\bar{c}s\bar{s}$ | $J/\psi\eta$ | $1^{--}$ | $4374\pm7$ | $118\pm12$ | [100], [101] |
| $\psi(4660)$ | $c\bar{s}\bar{c}s$ | $\Lambda_c^+\bar{\Lambda}_c^-$ $D_s^+D_{s1}^-(2536)$ $D_s^+D_{s2}^{*-}(2573)$ | $1^{--}$ | $4630\pm6$ | $72^{+14}_{-12}$ | [102] |
| $T_{\psi\psi}(6600)$ | $c\bar{c}c\bar{c}$ | $J/\psi J/\psi$ | $?^{??}$ | $6630\pm90$ | $350\pm11^{+110}_{-40}$ | [103] |
| $T_{\psi\psi}(6600)$ | $c\bar{c}c\bar{c}$ | $J/\psi J/\psi$ | $?^{??}$ | $6552\pm10\pm12$ | $124^{+32}_{-26}\pm33$ | [104] |
| $T_{\psi\psi}(6900)$ | $c\bar{c}c\bar{c}$ | $J/\psi J/\psi$ | $?^{??}$ | $6886\pm16$ | $168\pm80$ | [105] |
| $T_{\psi\psi}(6900)$ | $c\bar{c}c\bar{c}$ | $J/\psi J/\psi$ | $?^{??}$ | $6927\pm9\pm4$ | $122^{+24}_{}\pm18$ | [104] |

# A test case : Ty y

- The states Ty y discovered that we analyze are

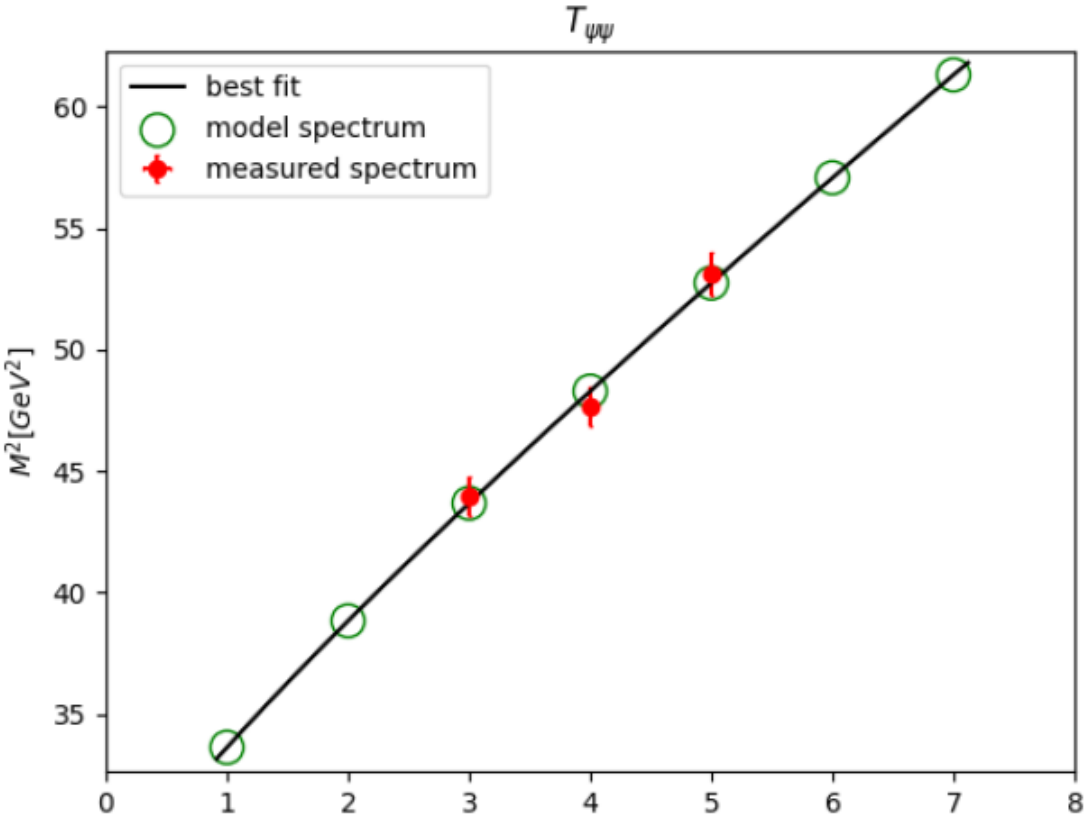
| Candidate | Quarks Content | Selected Decay Channels | $J^{PC}$ | Mass $[MeV]$ | Width $[MeV]$ | Ref. |
|---|---|---|---|---|---|---|
| $T_{\psi\psi}(6600)$ | $c\bar{c}c\bar{c}$ | $J/\psi J/\psi$ | $?^{??}$ | $6630 \pm 90$ | $350 \pm 11^{+110}_{-40}$ | |
| $T_{\psi\psi}(6600)$ | $c\bar{c}c\bar{c}$ | $J/\psi J/\psi$ | $?^{??}$ | $6552 \pm 10 \pm 12$ | $124^{+32}_{-26} \pm 33$ | |
| $T_{\psi\psi}(6900)$ | $c\bar{c}c\bar{c}$ | $J/\psi J/\psi$ | $?^{??}$ | $6886 \pm 16$ | $168 \pm 80$ | |
| $T_{\psi\psi}(6900)$ | $c\bar{c}c\bar{c}$ | $J/\psi J/\psi$ | $?^{??}$ | $6927 \pm 9 \pm 4$ | $122^{+24}_{-21} \pm 18$ | |
| $T_{\psi\psi}(7300)$ | $c\bar{c}c\bar{c}$ | $J/\psi J/\psi$ | $?^{??}$ | $7287^{+20}_{-18} \pm 5$ | $95^{+59}_{-40} \pm 19$ | |

- The decay width of Ty y(7300) gets contribution from the annihilation decay $\Gamma_{annihilation} = 47MeV$ and the breakup decay giving a total of 78 Mev in comparison to the measured one 95 Mev

# A test case : Ty y the HMRT spectrum

- The HMRT spectrum

| # | Candi-dates | $J^{PC}$ | Quarks | Thry Width $[MeV]$ | Mesons Pair | Meson Threshold $[MeV]$ | Baryon Anti-baryon | Baryonic Threshold $[MeV]$ | $J$ Spec | $n$ Spec | $a_{T_j}$ $a_{T_n}$ | Thry $J+n$ | Gen-uine |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 37 | $T_{\psi\psi}(6600)$<br>$T_{\psi\psi}(6600)$<br>$T_{\psi\psi}(6900)$<br>$T_{\psi\psi}(6900)$<br>$T_{\psi\psi}(7300)$ | $?^{??}$<br>$?^{??}$<br>$?^{??}$<br>$?^{??}$<br>$?^{??}$ | $c\bar{c}c\bar{c}$ | $6-21$<br>$29-58$ | $\eta_c(1S)\eta_c(1S)$ | 5968 | $\Xi_{cc}^{+}\bar{\Xi}_{cc}^{-}$<br>$\Xi_{cc}^{++}\bar{\Xi}_{cc}^{--}$<br>$\Omega_{cc}^{+}\bar{\Omega}_{cc}^{-}$ | 7038<br>7244 | $6768-6887$<br>$6926-7040$<br>$7078-7188$<br>$7225-7331$<br>$7366-7469$ | $6622-6745$<br>$6857-6973$<br>$7078-7188$<br>$7288-7393$<br>$7489-7589$ | $-5.9--5.1$<br>$-4.2--3.7$ | 6<br>4 | * |



# : A test case: Ty y

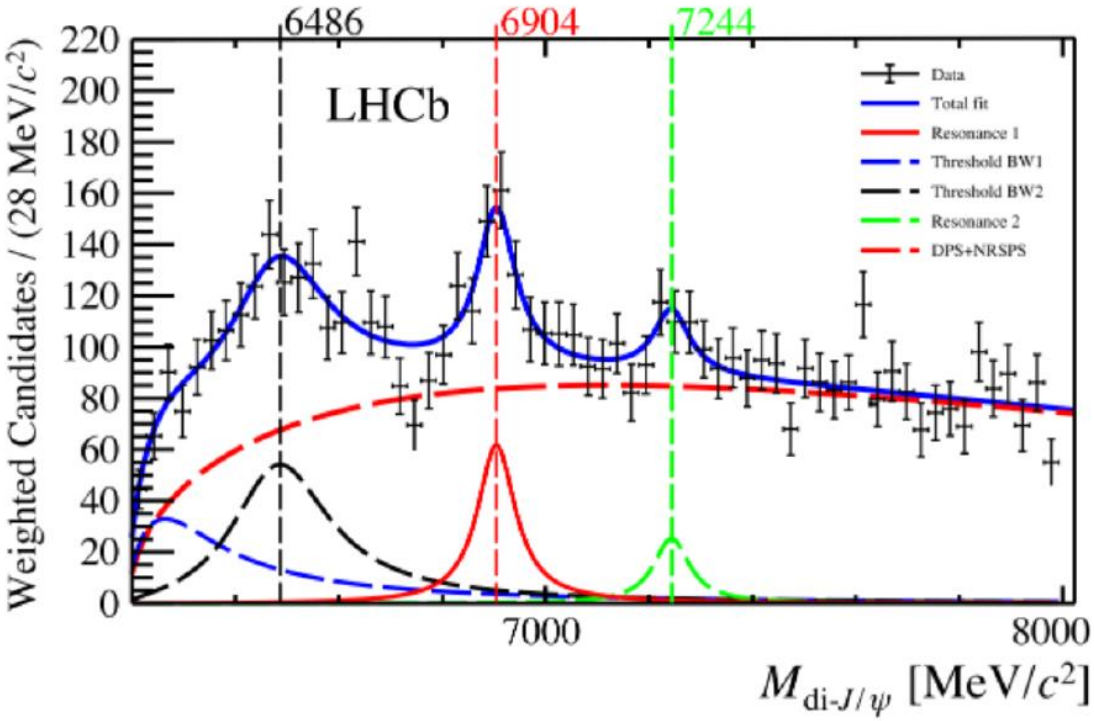



Figure 1: Location of peaks in the LHCb data. Adapted from figure 7 in [1]. The 7.2 GeV state appears to be almost exactly on the $\Xi_{cc}\bar{\Xi}_{cc}$ threshold, which is at 7242 MeV.

# Holography modified Regge trajectories (HMRT)

- We summarized all the predictions in the following table.

| # | Candi-dates | $J^{PC}$ | Quarks | Thry Width $[MeV]$ | Mesons Pair | Meson Threshold $[MeV]$ | Baryon Anti-baryon | Baryonic Threshold $[MeV]$ | $J$ Spec | $n$ Spec | $a_{T_j}$ $a_{T_n}$ | Thry $J+n$ | Gen-uine |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

Resonnances already detected

Thry width both for $(J, M^2)$ and $(n, M^2)$

Threshold for creation the mason pair

Threshold for baryon anti-baryon

- The J spec and n spec were computed as follows:
- Determining the baryonic threshold and let it vary 40-150 Mev ( based on available data)
- Calculating the # of states between the mesonic and baryonic thresholds.

# HMRT tetraquark predictions

- We derived the range of the spectrum for two states below and two states above the $j+n=3$ for the two spectra.
- When there is no baryonic threshold, since the baryons were not discovered yet, we started from the mesonic threshold and from there up for 12 states.
- "Genuine" refers to if the tetraquark is genuine or not according :
- If it includes (u,d,s) and their anti-particles the products could be reached from a tear up of a meson hence not genuine.
- Genuine * contain a pair only of c or b.
- Genuine** does not contain a pair.

# Holography modified Regge trajectories (HMRT)

| # | Candi-dates | $J^{PC}$ | Quarks | Thry Width $[MeV]$ | Mesons Pair | Meson Threshold $[MeV]$ | Baryon Anti-baryon | Baryonic Threshold $[MeV]$ | $J$ Spec | $n$ Spec | $a_{T_J}$ $a_{T_n}$ | Thry $J+n$ | Gen-uine |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | $c\bar{u}u\bar{u}$ | $79-118$<br>$30-61$ | $D^0\pi_u^0$ | 2000 | $\Lambda_c^+\bar{p}$<br>$\Xi_c^+\bar{\Sigma}^-$<br>$\Sigma_c^{++}\bar{\Delta}^{--}$ | 3224<br>3657<br>3686 | $2634-2779$<br>$2972-3095$<br>$3264-3374$<br>$3526-3626$<br>$3766-3858$ | $2270-2448$<br>$2829-2961$<br>$3264-3374$<br>$3635-3731$<br>$3964-4051$ | $-1.6--1.2$<br>$-1.2--0.92$ | 3<br>2 | |
| 2 | $X(3500)$<br>$X(3800)$ | $(0 \text{ or } 1)^{??}$<br>$?^{??}$ | $c\bar{d}d\bar{u}$ | $79-118$<br>$30-61$ | $D^0\pi_d^0$<br>$D^+\pi^-$ | 2000<br>2010 | $\Lambda_c^+\bar{p}$<br>$\Sigma_c^0\bar{n}$<br>$\Xi_c^0\bar{\Lambda}^0$ | 3224<br>3394<br>3586 | $2634-2779$<br>$2972-3095$<br>$3264-3374$<br>$3526-3626$<br>$3766-3858$ | $2270-2448$<br>$2829-2961$<br>$3264-3374$<br>$3635-3731$<br>$3964-4051$ | $-1.6--1.2$<br>$-1.2--0.92$ | 3<br>2 | |
| 3 | | | $c\bar{d}u\bar{u}$ | $78-117$<br>$31-61$ | $D^+\pi_u^0$<br>$D^0\pi^+$ | 2005<br>2005 | $\Lambda_c^+\bar{n}$<br>$\Sigma_c^{++}\bar{p}$<br>$\Xi_c^+\bar{\Lambda}^0$ | 3226<br>3392<br>3584 | $2637-2782$<br>$2974-3098$<br>$3266-3376$<br>$3528-3628$<br>$3767-3860$ | $2273-2451$<br>$2831-2963$<br>$3266-3376$<br>$3637-3733$<br>$3966-4053$ | $-1.6--1.2$<br>$-1.2--0.93$ | 3<br>2 | |
| 4 | | | $c\bar{u}d\bar{u}$ | $41-72$<br>$32-63$ | $D^0\pi^-$ | 2005 | $\Sigma_c^0\bar{p}$<br>$\Lambda_c^+\bar{\Delta}^{--}$<br>$\Xi_c^0\bar{\Sigma}^-$ | 3392<br>3518<br>3659 | $2854-2992$<br>$3160-3282$<br>$3432-3542$<br>$3679-3780$<br>$3908-4002$ | $2537-2698$<br>$3029-3157$<br>$3432-3542$<br>$3783-3881$<br>$4098-4187$ | $-2.3--1.8$<br>$-1.7--1.4$ | 3<br>2 | ** |
| 5 | | | $c\bar{d}d\bar{d}$ | $78-117$<br>$31-61$ | $D^+\pi_d^0$ | 2005 | $\Lambda_c^+\bar{n}$<br>$\Xi_c^0\bar{\Sigma}^+$<br>$\Sigma_c^0\bar{\Delta}^+$ | 3226<br>3667<br>3686 | $2637-2782$<br>$2974-3098$<br>$3266-3376$<br>$3528-3628$<br>$3767-3860$ | $2273-2451$<br>$2831-2963$<br>$3266-3376$<br>$3637-3733$<br>$3966-4053$ | $-1.6--1.2$<br>$-1.2--0.93$ | 3<br>2 | |
| 6 | | | $c\bar{d}u\bar{d}$ | $40-71$<br>$32-63$ | $D^+\pi^+$ | 2010 | $\Sigma_c^{++}\bar{n}$<br>$\Lambda_c^+\bar{\Delta}^+$ | 3394<br>3518 | $2856-2994$<br>$3162-3284$<br>$3434-3544$ | $2540-2701$<br>$3031-3160$<br>$3434-3544$ | $-2.3--1.9$<br>$-1.7--1.4$ | 3<br>2 | ** |