The superkick effects in high-energy vortex state collisions

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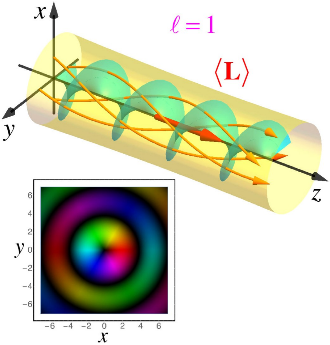
April 25, 2024 in Zhuhai

Phys. Rev. A 105, 013522 (2022) Igor P. Ivanov, Bei Liu and Pengming Zhang
Phys. Rev. A 107, 063110 (2023) Bei Liu and Igor P. Ivanov

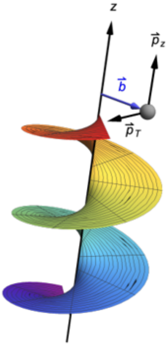
What is vortex states?

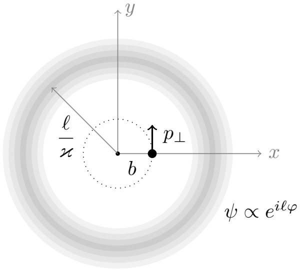
- **Non-plane-wave**.
- Possess a nonzero intrinsic orbital angular momentum (OAM) respect to propagation direction (z axis) **in free space**.
- Typically: $\psi \propto e^{i\ell\varphi_r}$, gives $\langle \hat{L}_z \rangle = \hbar\ell$, here $\ell = \pm 1, \pm 2 \cdots$.
- Vortex photons, electrons, neutrons, atoms...



Superkick effect

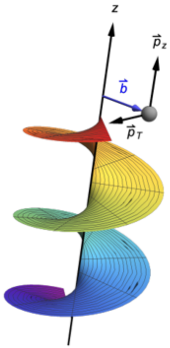
- Barnett and Berry [J. Opt, 2013] predicted a "**superkick**" effect. An atom placed in an optical vortex close to the axis may, upon absorbing a photon, acquire a much larger transverse momentum than vortex light field: $P_\perp \gg \varkappa$.
- Afanasev et al. [Phys. Rev. Res, 2021], [Ann. Phys, 2021] predicted the **shift of energy threshold** and **dramatic enhancement of cross section** during the production of heavy particles as $b \to 0$.





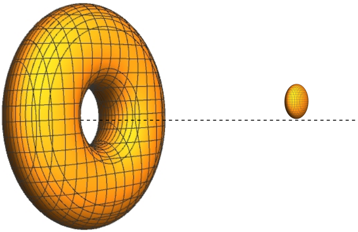
Parameterization

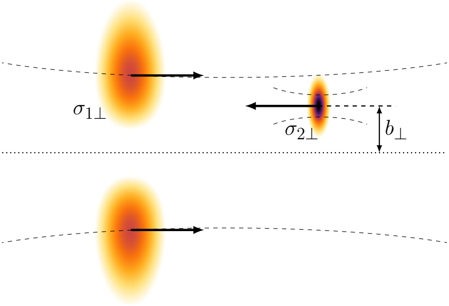
- Quantum-field-theoretical approach
- **vortex wavepacket** vs. **Gaussian wavepacket**.
- **Three key parameters**: the "size" of two wavepackets and impact parameter.
- Calculate the average total transverse momentum of final state to verify the "superkick effect".
- Calculate the cross section to confirm the relation between superkick and energy threshold.



Wavepacket collision

- Particle 1: Laguerre-Gaussian (LG) wavepacket with typical size $\sigma_{1\perp}$.
- Particle 2: Gaussian wavepacket with typical size $\sigma_{2\perp}$ and transverse offset $b_\perp$.
- Consider $m + m \to M + M$, calculate $\langle \boldsymbol{P}_\perp \rangle$, σ.



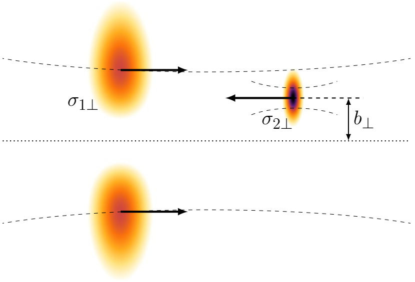


Wave function:

$$\varphi_1(\boldsymbol{k}_1) \propto \frac{(\sigma_{1\perp} k_{1\perp})^\ell}{\sqrt{\ell!}} \exp\left[-\frac{k_{1\perp}^2 \sigma_{1\perp}^2}{2} - \frac{(k_{1z}-p_{1z})^2 \sigma_{1z}^2}{2} + i\ell\varphi_k\right].$$

$$\varphi_2(\boldsymbol{k}_2) \propto \exp\left[-\frac{k_{2\perp}^2 \sigma_{2\perp}^2}{2} - \frac{(k_{2z}-p_{2z})^2 \sigma_{2z}^2}{2} - i\boldsymbol{b}_\perp \boldsymbol{k}_{2\perp}\right]$$

- Particle 1: $\sigma_{1\perp}, \ell$
- Particle 2: $\sigma_{2\perp}, b_\perp$
- Average energy: $\bar{E}_i = \sqrt{m_i^2 + p_{iz}^2}$



Cross section: $d\sigma = \frac{dW}{L}$

- Probability:

$$\boldsymbol{dW} = (2\pi)^8 |\mathcal{I}|^2 \frac{d^3 k_1'}{(2\pi)^3 2E_1'} \frac{d^3 k_2'}{(2\pi)^3 2E_2'}$$

$$\mathcal{I} = \int \frac{d^3 k_1}{(2\pi)^3 2E_1} \frac{d^3 k_2}{(2\pi)^3 2E_2} \varphi_1(\boldsymbol{k}_1)\, \varphi_2(\boldsymbol{k}_2) \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{P})\, \delta(E_1 + E_2 - E_f)\, \mathcal{M}$$

- Luminosity:

$$\begin{aligned}\boldsymbol{L} =& |v_1 - v_2| \int d^3 r\, dt\, |\psi_1(\boldsymbol{r}, t)|^2\, |\psi_2(\boldsymbol{r}, t)|^2 \\ =& \frac{1}{\pi} \frac{1}{\sigma_{1\perp}^2 + \sigma_{2\perp}^2} \left(\frac{\sigma_{2\perp}^2}{\sigma_{1\perp}^2 + \sigma_{2\perp}^2} \right)^{\ell} \exp\left(- \frac{b_\perp^2}{\sigma_{1\perp}^2 + \sigma_{2\perp}^2} \right) \\ & \times L_\ell \left(- \frac{b_\perp^2}{\sigma_{2\perp}^2} \frac{\sigma_{1\perp}^2}{\sigma_{1\perp}^2 + \sigma_{2\perp}^2} \right)\end{aligned}$$

- $\boldsymbol{P} = \boldsymbol{k}_1' + \boldsymbol{k}_2', \quad E_f = E_1' + E_2'$

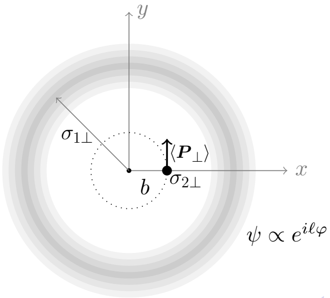
Analytical result

Average transverse momentum:

$$d\sigma = \frac{dW}{L} \quad \langle \boldsymbol{P}_\perp \rangle = \frac{\int \boldsymbol{P}_\perp \, d\sigma}{\int d\sigma} = \frac{\int \boldsymbol{P}_\perp \, dW}{\int dW}$$

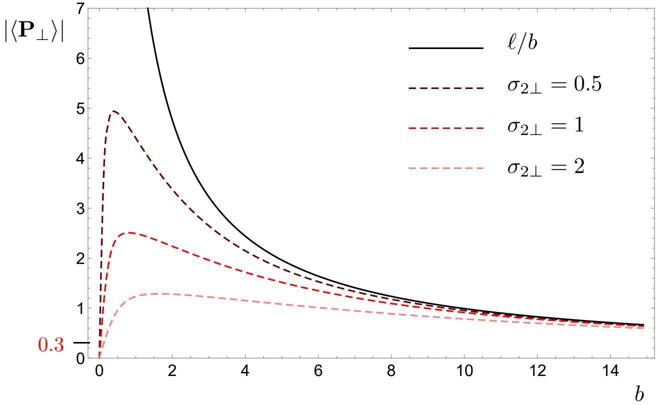
when $\boldsymbol{b}_\perp = b\,\boldsymbol{e}_x$,

$$\langle \boldsymbol{P}_\perp \rangle = \frac{\boldsymbol{e}_y}{2} \frac{\sigma_{1\perp}^2 + \sigma_{2\perp}^2}{\sigma_{1\perp}^2} \, \partial_b \, \ln \left[L_\ell \left(-\frac{b^2}{\sigma_{2\perp}^2} \frac{\sigma_{1\perp}^2}{\sigma_{1\perp}^2 + \sigma_{2\perp}^2} \right) \right].$$



$$\langle \boldsymbol{P}_\perp \rangle = \frac{\boldsymbol{e}_y}{2} \frac{\sigma_{1\perp}^2 + \sigma_{2\perp}^2}{\sigma_{1\perp}^2} \partial_b \ln \left[L_\ell \left(-\frac{b^2}{\sigma_{2\perp}^2} \frac{\sigma_{1\perp}^2}{\sigma_{1\perp}^2 + \sigma_{2\perp}^2} \right) \right].$$

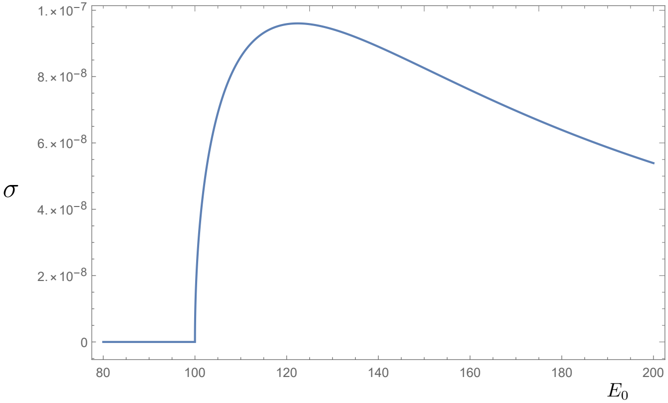
For $\ell = 10, \sigma_{1\perp} = 10$, with typical transverse momentum $\sqrt{\ell}/\sigma_{1\perp} \approx 0.3$ and $1/\sigma_{2\perp}$.
(Note: $[M] = [P] = [E], \quad [\sigma_{i\perp}] = [b] = [E]^{-1}$)



Significant superkick phenomenon condition: $\sigma_{2\perp} \ll \sigma_{1\perp}, b \approx \sigma_{2\perp}$

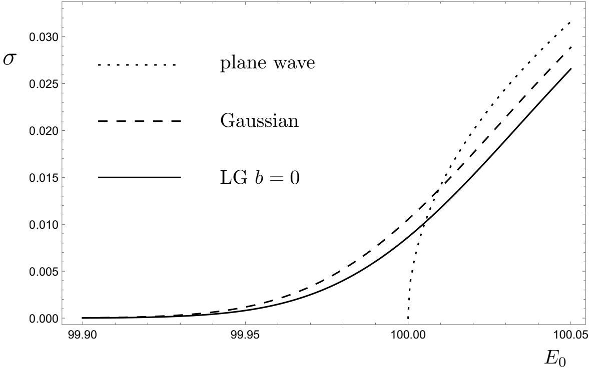
Plane wave case

For $m = 10, M = 100, E_1 = E_2 = E_0$, the cross section of plane wave scattering.



Near the threshold

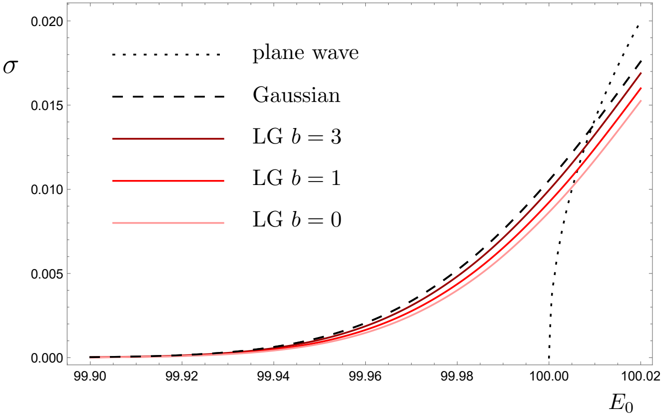
For $m = 10, M = 100, \overline{E}_1 = \overline{E}_2 = E_0$,
$\sigma_{1\perp} = 10, \sigma_{2\perp} = 1$ and $\ell = 10$ for LG wavepacket.



No certain threshold and, $\sigma_{LG} < \sigma_G$.

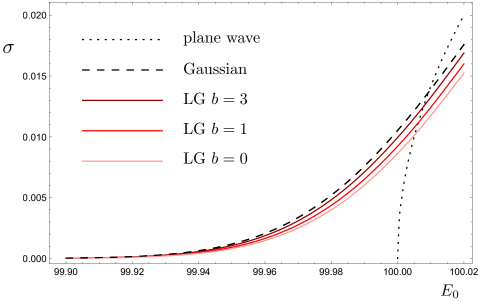
Near the threshold

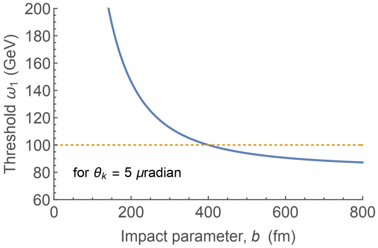
For $m = 10, M = 100, \overline{E}_1 = \overline{E}_2 = E_0$, and $\sigma_{1\perp} = 10, \sigma_{2\perp} = 1, \ell = 10$.

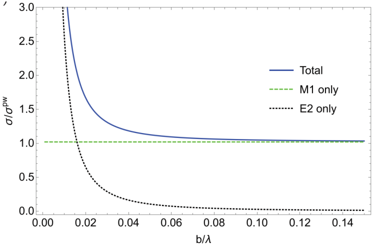


As $b \uparrow$, $\sigma_{LG} \to \sigma_G$.

Difference







As $b \to 0$, $\sigma_{LG} \downarrow$.

Opposite to previous papers.

Conclusion

- Significant superkick effect condition: $\sigma_{2\perp} \ll \sigma_{1\perp}, b \approx \sigma_{2\perp}$
- No energy threshold shift!
- No increase in scattering cross section!

Thank you!