Detecting the Vortex state of high-energy electrons through elastic electron scattering

Zhengjiang Li

In collaboration with: Shiyu Liu, Liangliang Ji, Bei Liu, Igor P. Ivanov

School of Physics and Astronomy, Sun Yat-Sen University

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Outline

Motivation

Detecting high-energy vortex particles

The failure of existing methods in high-energy senario:

1. Fork diffraction gratings (generation)—— very short de Broglie wavelength and high penetrating power

2.Interference of a vortex state with a reference plane wave——sufficiently large transverse coherence length, hardly be available for high energy vortex states

3.Vortex electron scattering on target atoms, observe characteristic ring-like structures—— loses the information about the phase factor, cannot distinguish it from a non-vortex ring shaped wave function.

4. Recover the phase vortex from outgoing particles wave front——require a completely new class of wavefront-sensitive detectors.

Verify particles are exactly in vortex state through **Superkick** effect. (electron, elastic scattering)

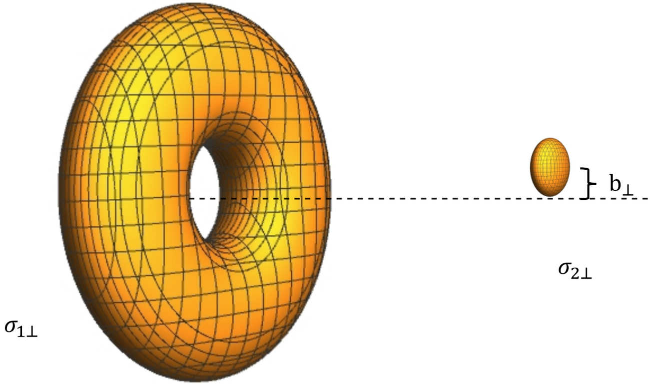
Benefits:

1. Superkick only happens in presence of phase vortex.
2. Can be detected easily in experiments
3. Can be achieved with the traditional detectors.

Møller Scattering of Wavepacket

Wavepacket Scattering

Collision of L-G Wavepacket and Gaussion Wavepacket.



Wave function and cross section:

$$\phi_1(\mathbf{k}_1) = (4\pi)^{\frac{3}{4}} \sigma_{1\perp} \sqrt{2E_1\sigma_{1z}} \frac{(\sigma_{1\perp} k_{1\perp})^\ell}{\sqrt{\ell!}} \exp\left(-\frac{k_{1\perp}^2 \sigma_{1\perp}^2 + (k_{1z} - p_{1z})^2 \sigma_{1z}^2}{2} + i\ell\phi_k \right)$$

$$\phi_2(\mathbf{k}_2) = (4\pi)^{\frac{3}{4}} \sigma_{2\perp} \sqrt{2E_2\sigma_{2z}} \times \exp\left(-\frac{k_{2\perp}^2 \sigma_{2\perp}^2 + (k_{2z} - p_{2z})^2 \sigma_{2z}^2}{2} - i\mathbf{b}_\perp \cdot \mathbf{k}_{2\perp} - ib_{2z}k_{2z} + i\tau E_2 \right).$$

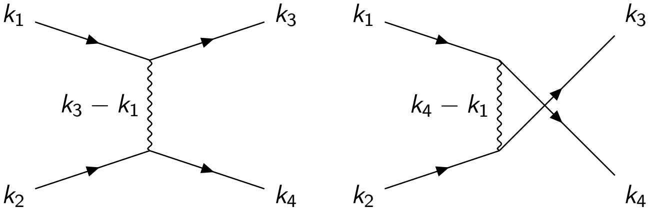
$$d\sigma = \frac{(2\pi)^8}{L} |\mathcal{I}|^2 \frac{d^3k_3}{(2\pi)^3 2E_3} \frac{d^3k_4}{(2\pi)^3 2E_4},$$

$$\mathcal{I} = \int \frac{d^3k_1}{(2\pi)^3 2E_1} \frac{d^3k_2}{(2\pi)^3 2E_2} \phi_1(\mathbf{k}_1)\phi_2(\mathbf{k}_2)\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{P})\delta(E_1 + E_2 - E_f)\mathcal{M}.$$

Bei Liu, Igor P. Ivanov, Phys.Rev.A 107 (2023) 6, 6.

Møller Scattering in plane wave

Initial and final electron: $k_i = (E_i, \mathbf{k}_i)$



$$M_t = \frac{e^2}{t} \bar{u}_3 \gamma^\mu u_1 \bar{u}_4 \gamma_\mu u_2, \;\; M_u = \frac{e^2}{u} \bar{u}_4 \gamma^\mu u_1 \bar{u}_3 \gamma_\mu u_2.$$

$$M = M_t - M_u.$$

Spin configeration

$$M_{s_1 s_2 s_3 s_4} = \frac{e^2}{t} \bar{u}_3^{s_3} \gamma^{\mu} u_1^{s_1} \bar{u}_4^{s_4} \gamma_{\mu} u_2^{s_2}, \qquad s_i = -\frac{1}{2},\ \frac{1}{2}.$$

non-zero elements(under high-energy limit):$s_1 = s_3,\ s_2 = s_4$
Spin bases:

$$\text{for } u_1^{s_1}: \quad \xi_1^{\frac{1}{2}} = \begin{pmatrix} \cos\frac{\theta_i}{2} \\ \sin\frac{\theta_i}{2} e^{i\phi_i} \end{pmatrix}, \quad \xi_1^{-\frac{1}{2}} = \begin{pmatrix} -\sin\frac{\theta_i}{2} e^{-i\phi_i} \\ \cos\frac{\theta_i}{2} \end{pmatrix}.$$

$$\text{for } u_2^{s_2}: \quad \xi_2^{\frac{1}{2}} = \begin{pmatrix} \cos\frac{\theta_i}{2} e^{-i\phi_i} \\ \sin\frac{\theta_i}{2} \end{pmatrix}, \quad \xi_2^{-\frac{1}{2}} = \begin{pmatrix} -\sin\frac{\theta_i}{2} \\ \cos\frac{\theta_i}{2} e^{i\phi_i} \end{pmatrix}.$$

Simplify Matrix Element

Small angle for incoming particles: $\theta_1 \sim 0$ and $\theta_2 \sim \pi$. Small angle scattering: $\theta_3 \leq 5^\circ$

$$M_{s_1,s_2,s_3,s_4} = \frac{8e^2}{t}\sqrt{E_1 E_2 E_3 E_4}.$$

$$t = (k_1 - k_3)^2 = (E_1 - E_3)^2 - (k_{1z} - k_{3z})^2 - (\mathbf{k}_{1\perp} - \mathbf{k}_{3\perp})^2$$
$$\approx -(\mathbf{k}_{1\perp} - \mathbf{k}_{3\perp})^2.$$

$$E_i \approx \epsilon_i = \sqrt{m^2 + p_{iz}^2}$$

Corss Section

Average over spins:

$$|\mathcal{I}|^2 = \frac{1}{4}\sum_{spins} |\mathcal{I}_{s_1 s_2 s_3 s_4}|^2,$$

Good quantum number:$J = \ell + s$. Fix J, average over spins:

$$|\mathcal{I}|^2 = \frac{1}{4}(|\mathcal{I}_{\ell,+,+,+,+}|^2 + |\mathcal{I}_{\ell,+,-,+,-}|^2 + |\mathcal{I}_{\ell+1,-,+,-,+}|^2 + |\mathcal{I}_{\ell+1,-,-,-,-}|^2).$$

$$\mathcal{I}_\ell = \frac{(4\pi)^{3/2}}{(2\pi)^7\sqrt{4\epsilon_1\epsilon_2}}\sqrt{\sigma_{1\perp}^2\sigma_{1z}\sigma_{2\perp}^2\sigma_{2z}}e^{-i\mathbf{b}_\perp\cdot\mathbf{P}_\perp}\mathcal{I}_{\perp\ell}\int dte^{i(\delta E - v_2\Delta P_z)t}\mathcal{I}_L(t).$$

$$\langle P_\perp\rangle = \frac{\int d^3k_3 d^3k_4 |\mathcal{I}|^2 \mathbf{P}_\perp}{\int d^3k_3 d^3k_4|\mathcal{I}|^2} = \frac{\int d^2P_\perp d^2k_{3\perp}\mathbf{P}_\perp|\mathcal{I}_\perp|^2}{\int d^2P_\perp d^2k_{3\perp}|\mathcal{I}_\perp|^2}.$$

Scattering Results and Superkick

Probability distribution

We choose $\sigma_{1\perp} = 10\text{nm}, \ \sigma_{2\perp} = 1\text{nm}, \ J = \frac{11}{2}$, considering the distribution of $|\mathcal{I}_\perp|^2$ under $(P_x, \ P_y)$ coordinates.

1. $|\mathcal{I}_\perp|^2$ distribution, an example.

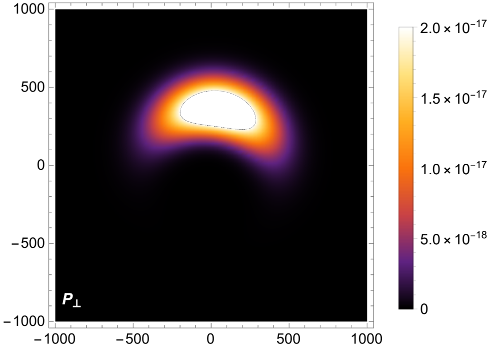


Figure: $|\mathcal{I}_\perp|^2$ with $k_{3\perp} = 3$ keV, $\phi_{k_3} = 0, \ b_x = 1$ nm, $\ b_y = 0$

2. $|\mathcal{I}_\perp|^2$ distribution with different b.

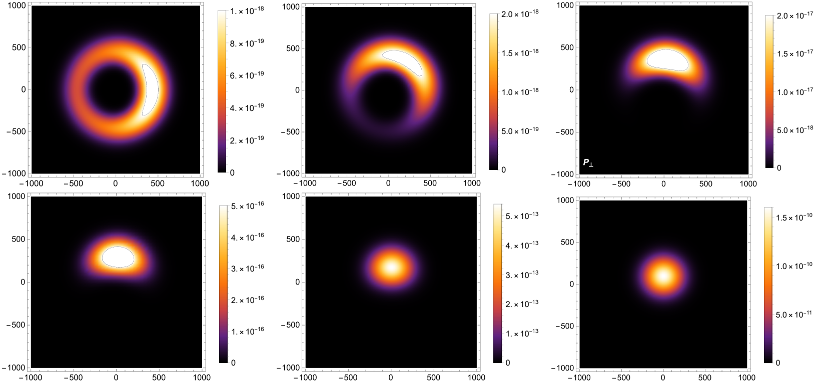


Figure: $|\mathcal{I}_\perp|^2$ with different b. $k_{3\perp} = 3$ keV, $\phi_{k_3} = 0$, $\phi_b = 0$, $b_y = 0$, $b_x = 0,\ 0.3,\ 1,\ 2,\ 5,\ 10$nm

3. $|\mathcal{I}_\perp|^2$ distribution with different $k_{3\perp}$.

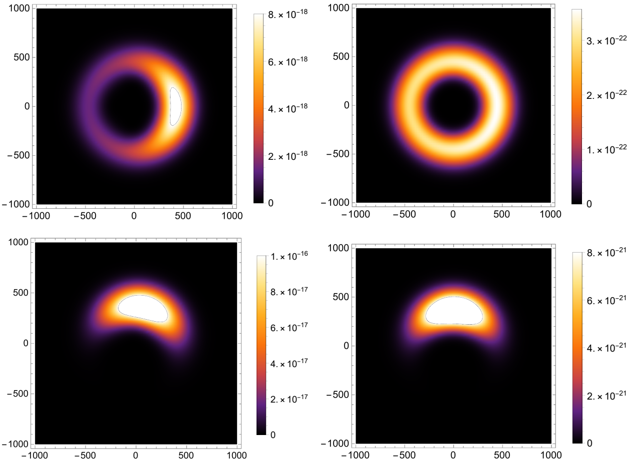


Figure: $|\mathcal{I}_\perp|^2$ with different $k_{3\perp}$. $b_x = 0$, 1nm, $\phi_{k_3} = 0$, $b_y = 0$, $k_{3\perp}$ =2 keV, 10 keV

Total Average Transverse Momentum

we can calculate $|\langle P_\perp \rangle|$ with different b.

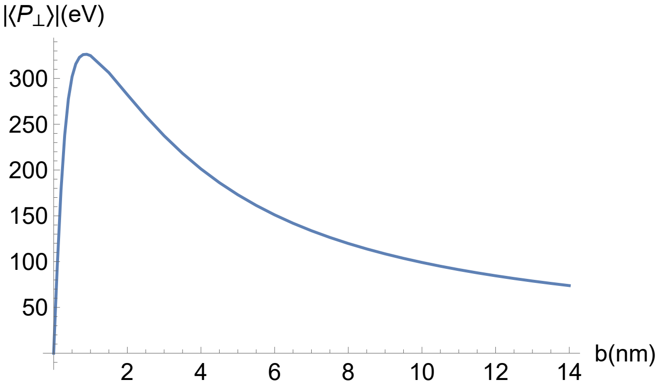


Figure: Total Average Transverse Momentum

Experimental Capability

1. Under above parameters, our analytical result is accurate enough for $k_{3\perp} \geq \frac{100}{\sigma_{1\perp}}$.

2. For vortex wavepacket with E=10MeV, above limit ($\frac{100}{\sigma_{1\perp}}$) corresponds to 0.01°, which means the only requirements for the detectors is to be placed above 0.01°.

3. Fully numerical result is achieved by our collaboration group from Shanghai Institute of Optics and Fine Mechanics (SIOM), also confirm our analytical results.

4. For statistical significance, we find that for 10^{14} electrons, there are hundreds of events.

Comparing with Non-vortex particles

Considering ring-like wavepacket:

$$\psi(\mathbf{r}) = \sqrt{\frac{\sigma_{1\perp}^{n-1}}{\pi^{3/2}\sqrt{\sigma_{1z}}\Gamma\left(n+1\right)}}\left(\frac{r_\perp}{\sigma_{1\perp}}\right)^n \exp\left(-\frac{r_\perp^2}{2\sigma_{1\perp}} - \frac{z^2}{2\sigma_{1z}^2}\right)$$

$$\phi(\mathbf{k}_1) = \frac{4\sqrt{E_1}\pi\sqrt{\Gamma\left(\frac{n}{2}+1\right)}\sigma_{1\perp}\sqrt{\sigma_{1z}}}{\sqrt{\Gamma\left(\frac{n+1}{2}\right)}} L_{-1-n/2}\left(-\frac{k_{1\perp}^2\sigma_{1\perp}^2}{2}\right)\exp\left(-\frac{(k_{1z}-p_{1z})^2\sigma_z^2}{2}\right)$$

Non-vortex $|\mathcal{I}_\perp|^2$ distribution with different b. Choose n=1.

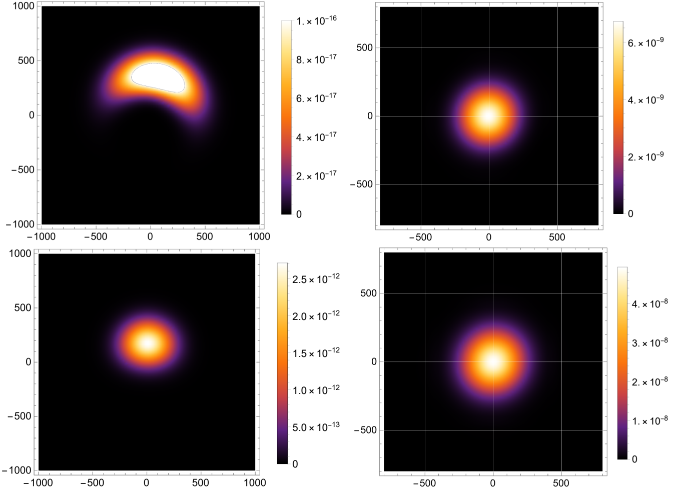


Figure: vortex vs non-vortex. $|\mathcal{I}_\perp|^2$ with different b_x. Keep $k_{3\perp} = 2keV$, $\phi_{k_3} = 0$, $b_y = 0$. First line: $b_x = 1$, 1nm Second line: $b = 5$nm.

Conclusions

1. Studied the elastic scattering of high-energy vortex electrons, and theoretically predicted the existence of Superkick effect under this scenario.

2. Superkick effect can be utilized to detect whether high-energy electrons are in a vortex state, by performing elastic scattering of a compact Gaussian probe electron with a vortex electron state. Detect scattered electrons, and measure $|\langle P_\perp \rangle|$.

3. It can be detected with the existing technologies.