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Studying time-like proton form factors using vortex $p\bar{p}$ annihilation

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Vortex states in nuclear and particle physics
Zhuhai, China,
based on arXiv:2403.08949 [hep-ph]

Motivation

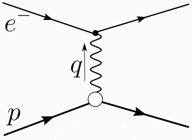
Until now: produced vortex states have small energy (photons, electrons, neutrons and atoms)
Why consider p and even $\bar{p}$?

- high p mass $\rightarrow$ access to many hadronic reactions
- a milestone on the road to use vortex states in DIS to probe of nucleon structure (spin crisis, OAM of quarks and gluons)

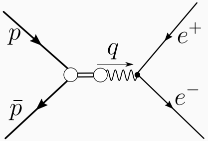
Proton electromagnetic form factors

Form factors describe non-point-like interaction: $\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{pl} |F(q)|^2$

Space-like ($q^2 < 0$)



Time-like ($q^2 > 0$)



- Real valued function
- Has intuitive interpretation: Fourier transform of electric and magnetization distributions

$$\rho(\vec{x}) = \frac{Ze}{(2\pi)^2} \int d^3q F(q) e^{-i\vec{q}\cdot\vec{x}}$$

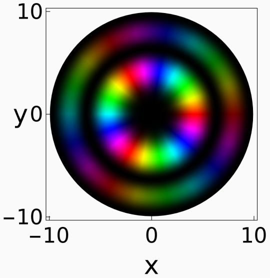
- No intuitive meaning
- Complex valued: sources of imaginary part — intermediate particles and excitations
- Relative phase is unknown. Can be measured in experiments with polarized beams (not done).

Simple scalar Bessel state

Coordinate space

In cylindrical coordinates (ρ, ϕ, z) for a scalar particle propagating along z direction:

$$\Psi(r,t) \propto \psi(\rho) e^{il\phi} e^{ik_z z} e^{-iEt}$$

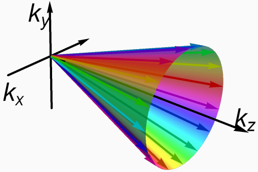


Momentum space

$$\Psi_{\varkappa m k_z \lambda}(r) = \frac{N_{\rm Bes}}{\sqrt{2E}} \int \frac{d^2\mathbf{k}}{(2\pi)^2}\, a_{\varkappa m}(\mathbf{k})\, u_{k\lambda}\, e^{-i\vec{k}\cdot\vec{r}},$$

where the Fourier amplitude is

$$a_{\varkappa m}(\mathbf{k}) = (-i)^m\, e^{im\phi_k} \sqrt{\frac{2\pi}{\varkappa}}\, \delta(|\mathbf{k}| - \varkappa)\,.$$



Vortex fermion

- Vortex Bessel fermion can be constructed using plane wave basis.
  PW basis: A plane wave fermion with $k^\mu = (E,\, \mathbf{k},\, k_z)$, where $\mathbf{k} = |\mathbf{k}|(\cos\phi_k,\, \sin\phi_k)$, $k_z = |\vec{k}|\cos\theta$, and helicity $\lambda = \pm 1/2$:

$$\Psi_{k\lambda}(r) = \frac{N_{\rm PW}}{\sqrt{2E}}\, u_{k\lambda}\, e^{-i\vec{k}\vec{r}}\,.$$

$$u_{k\lambda} = \begin{pmatrix} \sqrt{E+m}\, w^{(\lambda)} \\ 2\lambda\sqrt{E-m}\, w^{(\lambda)} \end{pmatrix},\, v_{k\lambda} = \begin{pmatrix} -\sqrt{E-m}\, w^{(-\lambda)} \\ 2\lambda\sqrt{E+m}\, w^{(-\lambda)} \end{pmatrix},$$

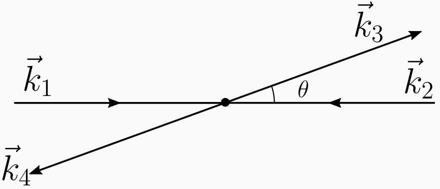
$$w^{(+1/2)} = \begin{pmatrix} c_i\, e^{-i\phi_i/2} \\ s_i\, e^{i\phi_i/2} \end{pmatrix},\, w^{(-1/2)} = \begin{pmatrix} -s_i\, e^{-i\phi_i/2} \\ c_i\, e^{i\phi_i/2} \end{pmatrix},$$

  where $c_i \equiv \cos(\theta_i/2)$, $s_i \equiv \sin(\theta_i/2)$.
- Construction of Bessel vortex state using PW basis:

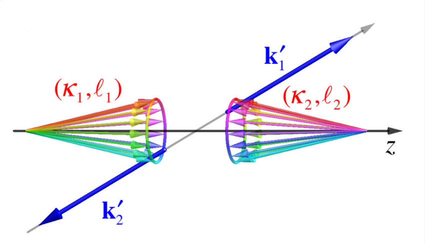
$$\Psi_{\varkappa m k_z \lambda}(r) = \frac{N_{\rm Bes}}{\sqrt{2E}} \int \frac{d^2\mathbf{k}}{(2\pi)^2}\, a_{\varkappa m}(\mathbf{k})\, u_{k\lambda}\, e^{-i\vec{k}\cdot\vec{r}}, \qquad a_{\varkappa m}(\mathbf{k}) = (-i)^m\, e^{im\phi_k} \sqrt{\frac{2\pi}{\varkappa}}\, \delta(|\mathbf{k}| - \varkappa)\,.$$

Kinematics of final 2 particle state

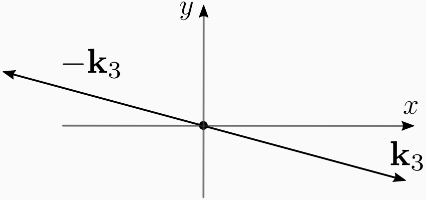
Plane wave scattering



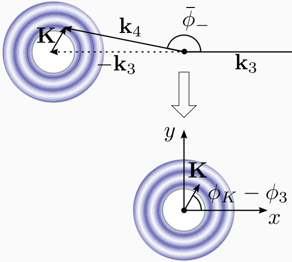
Double vortex scattering



Transverse momenta plane



Transverse momenta plane

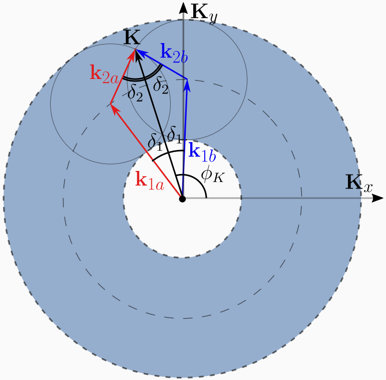


Scattering of two scalar Bessel states

1st particle: $k_1 = (E_1, \mathbf{k}_1, k_{1z})$ with OAM m_1; 2nd particle: $k_2 = (E_2, \mathbf{k}_2, k_{2z})$ with OAM m_2

Vortex scattering amplitude:

$$\mathcal{J} = \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 e^{i(m_1\phi_1 - m_2\phi_2)}\delta(|\mathbf{k}_1| - \varkappa_1)\delta(|\mathbf{k}_2| - \varkappa_2)\delta^{(2)}(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{K})\mathcal{M}(k_1, k_2, k_3, k_4)$$



The integral in $\mathcal{J}$ is nonzero when:
$|\kappa_1 - \kappa_2| \le |\mathbf{K}| < \kappa_1 + \kappa_2$

configuration a: $\phi_1 \to \phi_K + \delta_1 \quad \phi_2 \to \phi_K - \delta_2$

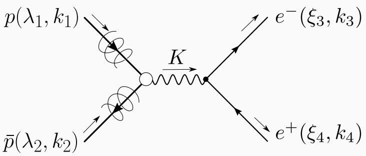
configuration b: $\phi_1 \to \phi_K - \delta_1 \quad \phi_2 \to \phi_K + \delta_2$

$$\mathcal{J} = \frac{e^{i(m_1 - m_2)\phi_K}}{\sin(\delta_1 + \delta_2)}\left(\mathcal{M}_a e^{i(m_1\delta_1 + m_2\delta_2)} + \mathcal{M}_b e^{-i(m_1\delta_1 + m_2\delta_2)}\right).$$

Cross section: $d\sigma \propto |\mathcal{J}|^2 d^2\mathbf{k}_3 d^2\mathbf{k}_4$

Proton electromagnetic form factors

The plane wave amplitude is product of hadron J^μ and lepton L_μ currents.



$$\mathcal{M} = \frac{e^2}{s} J^\mu L_\mu,$$

Nucleon current J^μ is

$$\bar{v}_{\lambda_2}(k_2)\Big[\gamma^\mu F_1(s) + \frac{F_2(s)}{2M}\sigma^{\mu\nu}K_\nu\Big]u_{\lambda_1}(k_1) = \bar{v}_{\lambda_2}(k_2)\left[\gamma^\mu G_M(s) + \frac{P^\mu}{2M}\frac{G_M(s) - G_E(s)}{1-\tau}\right]u_{\lambda_1}(k_1),$$

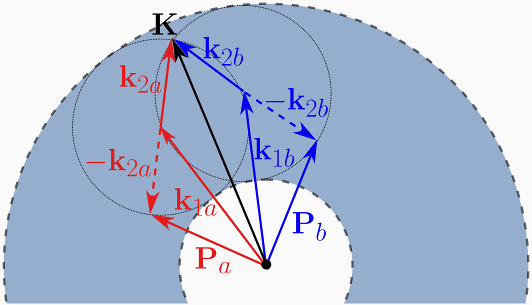
$$G_E = F_1 + \tau F_2, \qquad G_M = F_1 + F_2.$$

where $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2$, $K = k_1 + k_2$, $s = K^2$, $P = k_2 - k_1$, and $\tau = K^2/4M^2$.

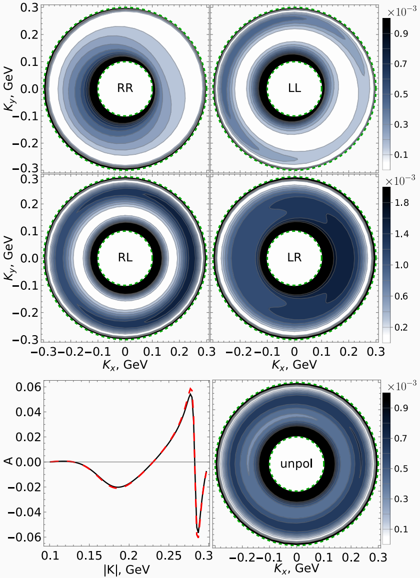
Proton electromagnetic form factors

$$J^\mu = \bar{v}_{\lambda_2}(k_2)\left[\gamma^\mu G_M(s) + \frac{P^\mu}{2M}\frac{G_M(s) - G_E(s)}{1-\tau}\right]u_{\lambda_1}(k_1),$$

$$P = k_2 - k_1$$



Results



Parameters:

$$m_1 = 7/2; \qquad m_2 = 3/2;$$

$$E_1 = 1.2 \text{ GeV}, \qquad E_2 = \sqrt{\kappa_2^2 + M^2 + p_{1z}^2}$$

$$\kappa_1 = 0.2 \text{ GeV}, \qquad \kappa_2 = 0.1 \text{ GeV};$$

$$K_z = 0, \qquad |\mathbf{k}_3| = 0.8 \text{ GeV}$$

$$A = \frac{\int d\phi_K |\mathcal{J}|^2 \sin(\phi_K - \phi_3)}{\int d\phi_K |\mathcal{J}|^2}.$$

Few observations:

- number of fringes is defined by angular momenta of initial states (m_1, m_2)
- Different Mandelstam s in one setup $s = (E_1 + E_2)^2 - \mathbf{K}^2$

Helicity structure

$$\mathcal{M} = \mathcal{M}_{\parallel} + \mathcal{M}_{\perp}$$

$$|\mathcal{J}|^2 = |\mathcal{J}_{\parallel} + \mathcal{J}_{\perp}|^2 = |\mathcal{J}_{\parallel}|^2 + |\mathcal{J}_{\perp}|^2 + 2\mathrm{Re}[\mathcal{J}_{\parallel}\mathcal{J}_{\perp}^{\dagger}].$$

$$\mathfrak{C}^{m_1}_{m_2} = \cos(m_1\delta_1 + m_2\delta_2), \quad V_{\pm} = \sqrt{E_1^+ E_2^-} \pm \sqrt{E_1^- E_2^+}, \quad W_{\pm} = \sqrt{E_1^+ E_2^+} \pm \sqrt{E_1^- E_2^-}.$$

RR/LL

$$\sum_{\xi} |\mathcal{J}_{\parallel}|^2 \propto 32 E_3 E_4 \sin^2\theta_3 (\mathfrak{C}^{m_1-\lambda}_{m_2-\lambda})^2 |G_M|^2 \times |W_- - \frac{1 - G_E/G_M}{2M(1-\tau)} V_+ P_z|^2.$$

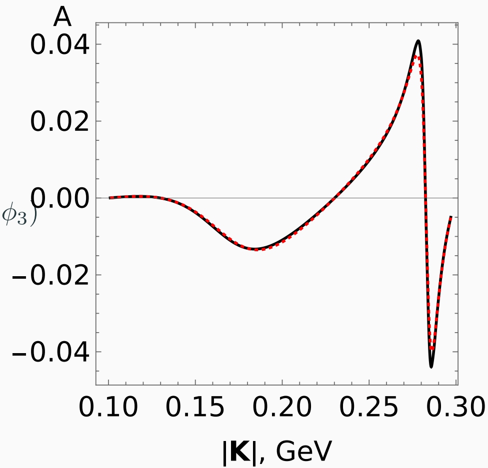
RL/LR

$$\sum_{\xi} |\mathcal{J}_{\gamma,\perp}|^2 = 32 |G_M|^2 W_+^2 E_3 E_4 (C^{m_1-\lambda}_{m_2+\lambda})^2 (1 + \cos^2\theta_3).$$

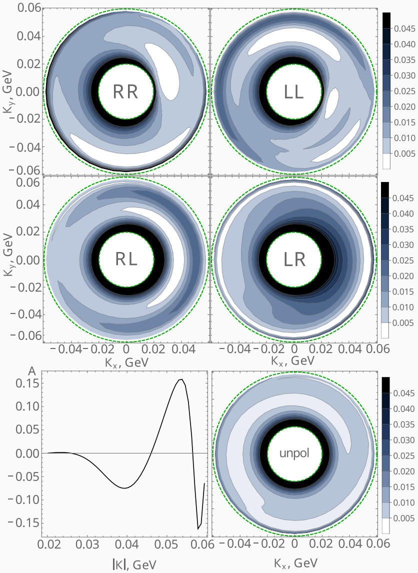
Numerator, RR/LL keeping only terms $\propto \sin(\phi_K - \phi_3)$

$$\sum_\xi \mathrm{Re}\{\mathcal{J}_\parallel \mathcal{J}_\perp^*\} \propto 16 E_3 E_4 \sin(2\theta_3) |G_M|^2 \sin(\phi_K - \phi_3)$$
$$\times 2\lambda |\mathbf{K}| \left(\mathfrak{C}^{m_1-\lambda}_{m_2-\lambda}\right)^2 \mathrm{Im}\{G_E/G_M\} \frac{(|\vec{k}_1| + |\vec{k}_2|)}{1-\tau}.$$

in approximation $\mathbf{K} \ll \kappa_{3,4}$



Non-relativistic regime



Parameters:

$$m_1 = 7/2, \qquad m_2 = 3/2,$$
$$E_1 = 939 \text{ MeV}, \qquad E_2 = \sqrt{\varkappa_2^2 + M^2 + k_{1z}^2},$$
$$\varkappa_1 = 40 \text{ MeV}, \qquad \varkappa_2 = 20 \text{ MeV},$$
$$K_z = 0,$$

The asymmetry is not suppressed by the hadron mass in non-relativistic regime, but defined by how non-paraxial are initial vortex states. Therefore, this method is feasible in low energy experiments.

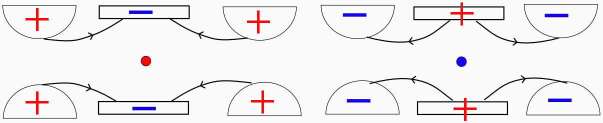
Conclusions

- It was shown that it is possible to extract the relative phase shift of proton electromagnetic form factors in $p\bar{p}$ annihilation. Analytical calculation in paraxial limit are in agreement with numerical calculation
- The asymmetry of differential cross section is proportional to relative form factor phase
- allows measurement at different Mandelstam s in one setup, useful if phase changes rapidly
- The asymmetry is not suppressed in non-relativistic limit, therefore can be measured in experiments with low energy vortex hadrons.

referring to yesterday discussion

is it feasible?

Penning trap



although extremely low energies can be used, but $s \approx 4M_p^2$, $G_{E,M}(4M_p^2) \to 1$