Momentum space oscillation properties in vortex states collision

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Outline

Vortex states (scalar)

Coordinate space: $\psi(r_\perp,\, \varphi,\, z) = f(r_\perp,\, z)\, \mathrm{e}^{i\ell\varphi}$
Momentum space: $\phi(k_\perp,\, \varphi_k,\, k_z) = g(k_\perp,\, k_z)\, \mathrm{e}^{i\ell\varphi_k}$

The states:

- Solutions for equation of motion(*i.e.* Schrodinger, Klein-Gordon);
- Angular momentum eigenstates $\rightarrow L_z = \hbar\ell$.

Distribution $f(r_\perp,\, z)$ and $g(k_\perp,\, k_z)$:

- Rotational-symmetric along propagation direction;
- Multiple choices → Real (*i.e.* Bessel) or complex (*i.e.* Laguerre-Gaussian).

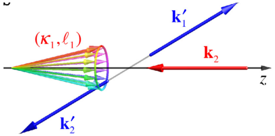
★ Topological charge ℓ:

- An integer quantum number → infinite intrinsic freedoms;
- Proportional to angular momentum.

Background: Collision/decay processes involving vortex particles

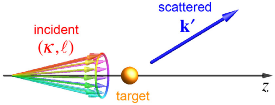
Final particles: Detected only by plane wave basis nowadays!
Initial particles: Several different schemes with different phenomenons.

a.



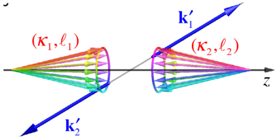
Collision of vortex particle and plane wave particle or decay of vortex particle: no interference.

b.



Collision of vortex particle with a target (size$\neq 0$) or wave packet(*i.e.* Gaussian): there is interference.

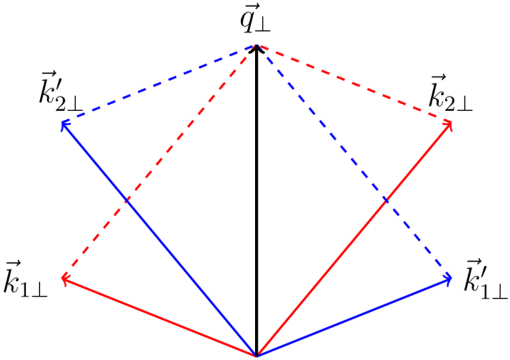
c.



Collision of two vortex particles: there is interference.

Pictures from [arXiv:1703.06879].

The interference (in momentum space) means:
For the same final plane wave particle, there is more than one combination of initial plane wave components which satisfy momentum conservation law.



Example: $\vec{k}_{1\perp}$ and $\vec{k}'_{1\perp}$ come from plane wave components in initial state "1", $\vec{k}_{2\perp}$ and $\vec{k}'_{2\perp}$ come from plane wave components in initial state "2".

Case a: Collision of vortex particle with plane wave particle or decay of vortex particle

Special kinetic phenomenons induced by special transverse momentum distribution(*i.e.* two-peak spectrum).

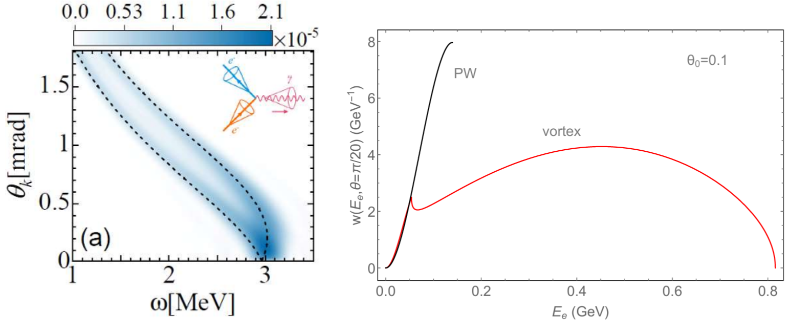


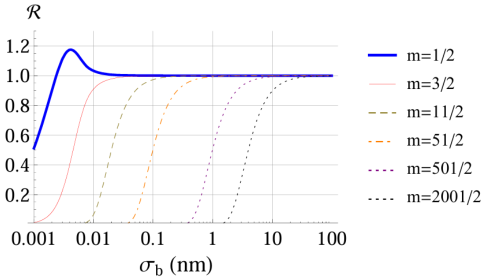
Figure 1: Two-peak spectrum of final particle: Radiation of charged vortex particle (Left: [arXiv:2302.05065]) & Decay of vortex particle (Right: [arXiv:2106.00345]).

No effect relating to topological charge.

Case b: Collision of vortex particle with a target or Gaussian wave packet

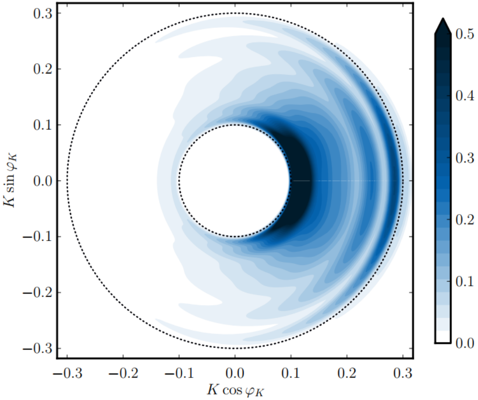
Effects which are dependent on topological charge

- Packet size effect [arXiv:2305.12419];
- Super-kick effect [arXiv:2109.01323];
- And so on.



Packet size effect.

Oscillatory distribution in momentum space, which is dependent on topological charge. [arXiv:1608.06551]



Oscillation of momentum distribution for final particle.

Motivation and Research plan

Motivation

Trying to find general observable quantity that:

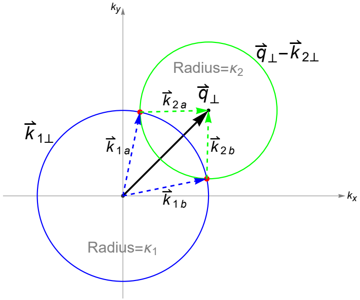
- show common features which are dependent on topological charge;
- are not sensitive to transverse and longitudinal distributions.

Research plan

- Choosing scheme of two vortex particle collision: Calculating $\mathcal{S}$ matrix for vortex particle collision (Scalar ϕ^4 model);
- Angular momentum eigenstates $\rightarrow$ Rotational-symmetric total momentum distribution.
- Comparation: Bessel vortex states vs General monochromatic vortex states vs Laguerre-Gaussian vortex states.

Bessel vortex state:

$$\psi^B_{E,\kappa,\ell}(\mathbf{x},\ t) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} a^B_{\kappa\ell}(\mathbf{k}) \exp(i\mathbf{k}\mathbf{x} - iEt),$$
$$a^B_{\kappa\ell}(\mathbf{k}) = N^B \delta(\vec{k}_\perp - \kappa)\delta(k_z - p_z)\exp(i\ell\varphi_k)$$



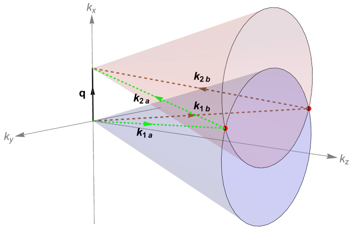


Figure 2: Interference space for Bessel vortex collision: two points

Results: 1D oscillatory distributions (more)

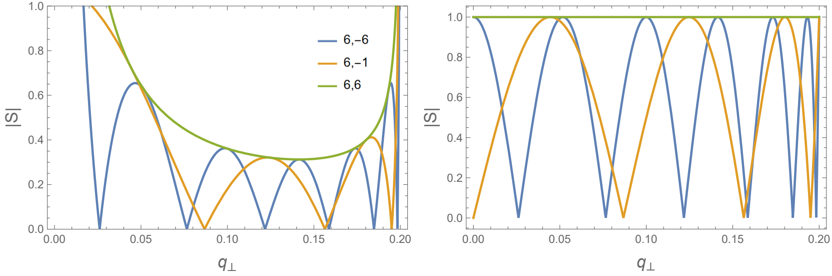


Figure 3: Oscillatory distribution for Bessel vortex collision with Δ(left) and without Δ(right). $\ell = 6$ and ℓ_2 is chosen to be -6, -1 or 6.

Momentum value range: $|\kappa_1 - \kappa_2| < |\vec{q}_\perp| < |\kappa_1 + \kappa_2|$.

$$\mathcal{S}(B_1 + B_2 \to P_3 + P_4) \propto \frac{\cos(\ell_1\varphi_1 + \ell_2\varphi_2)}{\Delta}\,\delta(k_{1z} + k_{2z} - q_z)\delta(E_1 + E_2 - E_q),$$

Results: 1D oscillatory distributions (more)

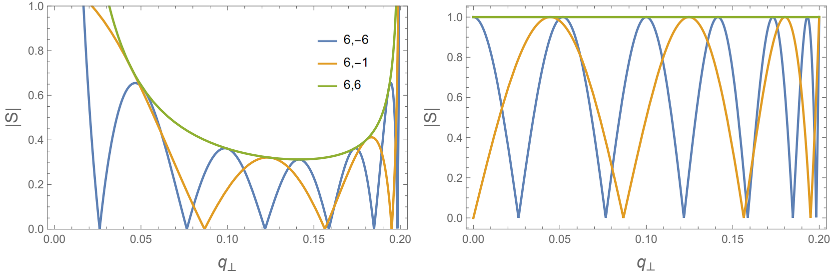


Figure 3: Oscillatory distribution for Bessel vortex collision with Δ(left) and without Δ(right). $\ell = 6$ and ℓ_2 is chosen to be -6, -1 or 6.

Momentum value range: $|\kappa_1 - \kappa_2| < |\vec{q}_\perp| < |\kappa_1 + \kappa_2|$.

$$\mathcal{S}(B_1 + B_2 \to P_3 + P_4) \propto \frac{\cos(\ell_1\varphi_1 + \ell_2\varphi_2)}{\Delta}\,\delta(k_{1z} + k_{2z} - q_z)\delta(E_1 + E_2 - E_q),$$

Oscillation rule

$$N^B = \text{Integer}[|\ell_1 - \ell_2|/2 + 1]$$

General monochromatic vortex collision

General monochromatic vortex state:

$$\psi^M_{E,\ell}(\mathbf{x},\ t) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, a^M_\ell(\mathbf{k}) \exp(i\mathbf{k}\mathbf{x} - iEt),$$

$$a^M_\ell(\mathbf{k}) = N^M f(\theta)\delta(\sqrt{\mathbf{k}^2 + m^2} - E) \exp(i\ell\varphi_k)$$

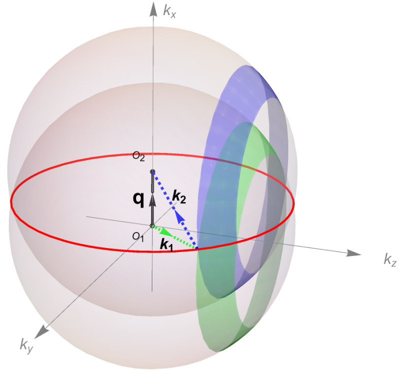


Figure 4: Interference space for general monochromatic vortex collision: a circle.

Results: 2D oscillatory distributions (more)

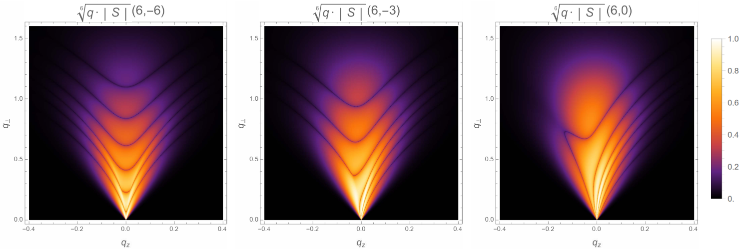


Figure 5: Oscillatory distribution for general monochromatic vortex collision with different (ℓ_1, ℓ_2).

Momentum value range: $||\mathbf{k}_1| - |\mathbf{k}_2|| < q_z < ||\mathbf{k}_1| + |\mathbf{k}_2||$.

$$\mathcal{S}(M_1 + M_2 \to P_3 + P_4) \propto \int_0^\pi d\hat{\varphi}_1 \frac{k_1 k_2}{q} f_1(\theta_1) f_2(\theta_2) \cos(\ell_1\varphi_1 + \ell_2\varphi_2) \mathcal{M}\, \delta(E_1 + E_2 - E_q)$$

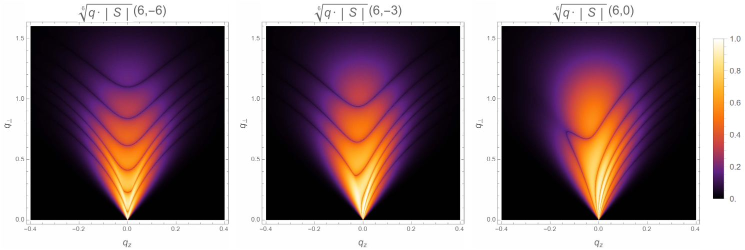


Figure 5: Oscillatory distribution for general monochromatic vortex collision with different (ℓ_1, ℓ_2).

Momentum value range: $||\mathbf{k}_1| - |\mathbf{k}_2|| < q_z < ||\mathbf{k}_1| + |\mathbf{k}_2||$.

$$\mathcal{S}(M_1 + M_2 \to P_3 + P_4) \propto \int_0^\pi d\hat{\varphi}_1 \frac{k_1 k_2}{q} f_1(\theta_1) f_2(\theta_2) \cos(\ell_1\varphi_1 + \ell_2\varphi_2)\mathcal{M}\,\delta(E_1 + E_2 - E_q)$$

Oscillation rule

$$N_\perp^M = (|\ell_1| + |\ell_2| - |\ell_1 + \ell_2|)/2 + 1, \quad N_z^M = |\ell_1 + \ell_2| + 1$$

Limitation at $q_\perp = 0$

$$\mathcal{S}(M_1 + M_2 \to P_3 + P_4) \propto \int_0^\pi d\varphi_1 \, \cos[(\ell_1 + \ell_2)\varphi_1)] \, \mathcal{M},$$

$$\lim_{q_\perp \to 0} |\mathcal{S}| = \begin{cases} \text{Nonzero} & \text{if } \ell_1 + \ell_2 \equiv 0 \,\&\, ||\mathbf{k}_1| - |\mathbf{k}_2|| < q < ||\mathbf{k}_1| + |\mathbf{k}_2||; \\ 0 & \text{if } \ell_1 + \ell_2 \neq 0 \,. \end{cases}$$

Limitation at $q = 0$

$$\lim_{q \to 0} |\mathcal{S}| = \begin{cases} \infty & \text{if } \ell_1 + \ell_2 \equiv 0 \,; \\ 0 & \text{if } \ell_1 + \ell_2 \neq 0 \,. \end{cases}$$

Laguerre-Gaussian vortex collision

Laguerre-Gaussian(LG) vortex state:

$$\psi^L_{n,\ell}(\mathbf{x},\ t) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} a^L_{n\ell}(\mathbf{k}) \exp(i\mathbf{kx} - iEt),$$

$$a^L_{n\ell}(\mathbf{k}) = N^L k_\perp^{|\ell|} L_n^{|\ell|}(\frac{k_\perp^2}{\sigma_\perp^2}) \exp(-\frac{k_\perp^2}{2\sigma_\perp^2} - \frac{(k_z - k_{z0})^2}{2\sigma_z^2} + i\ell\varphi_k)$$

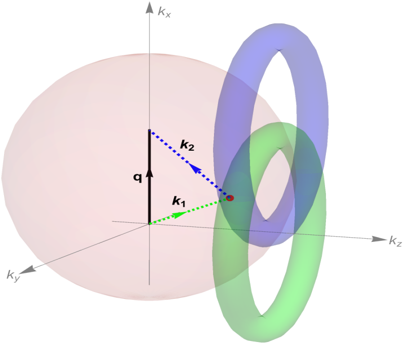


Figure 6: Interference space for LG vortex collision: a surface.

Results: 2D oscillatory distributions (more)

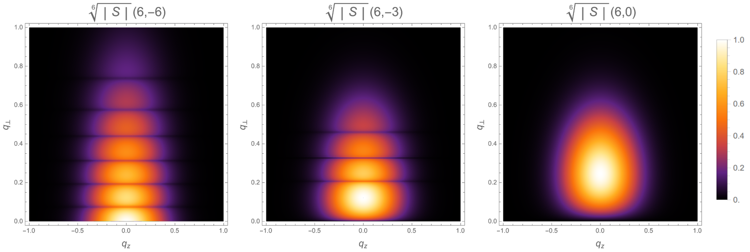


Figure 7: Oscillatory distribution for LG vortex collision

$$\mathcal{S}(L_1 + L_2 \to P_3 + P_4)$$
$$\propto \int_0^\pi d\hat{\varphi}_1 \int_0^\pi d\hat{\theta}_1 \, \frac{\sin\hat{\theta}_1 k_1^2 E_1 E_2 \, g_1(k_{1\perp}, k_{1z}) g_2(k_{2\perp}, k_{2z})}{|E_q k_1 - E_1 \, q \cos\hat{\theta}_1|} \cos(\ell_1\varphi_1 + \ell_2\varphi_2) \, \mathcal{M}$$

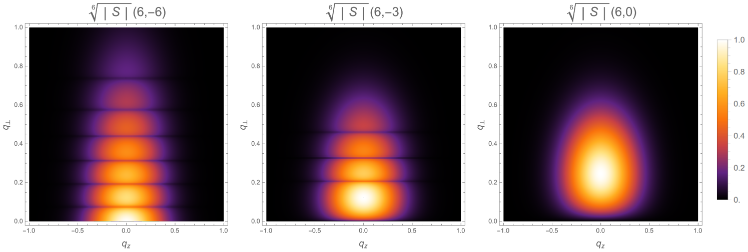


Figure 7: Oscillatory distribution for LG vortex collision

$$\mathcal{S}(L_1 + L_2 \to P_3 + P_4)$$
$$\propto \int_0^{\pi} d\hat{\varphi}_1 \int_0^{\pi} d\hat{\theta}_1 \frac{\sin\hat{\theta}_1 k_1^2 E_1 E_2\, g_1(k_{1\perp}, k_{1z}) g_2(k_{2\perp}, k_{2z})}{|E_q k_1 - E_1 q \cos\hat{\theta}_1|} \cos(\ell_1\varphi_1 + \ell_2\varphi_2)\,\mathcal{M}$$

oscillation rule

$$N_\perp^L = (|\ell_1| + |\ell_2| - |\ell_1 + \ell_2|)/2 + 1, \quad N_z^L = 1$$

LG vortex collision: 1D oscillatory distributions

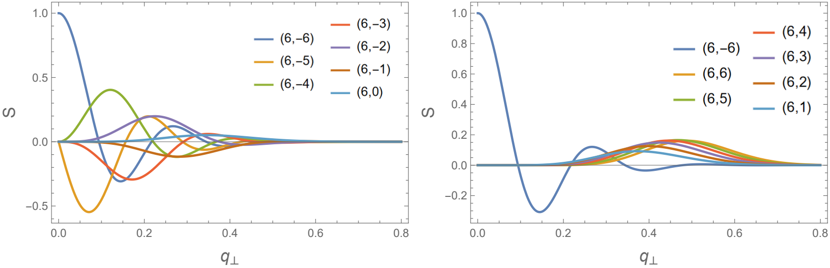


Figure 8: Distribution of total transverse momentum at fixed total longitudinal momentum ($q_z = 0$).

LG vortex collision: 1D oscillatory distributions

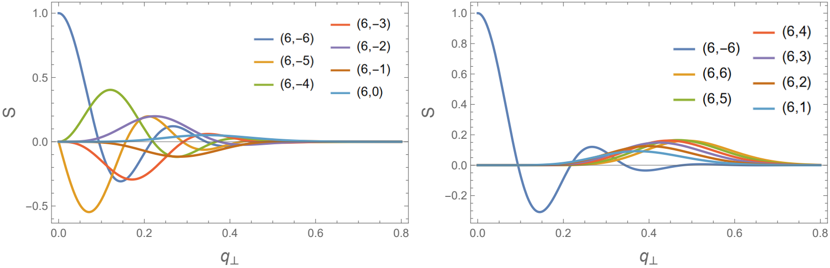


Figure 8: Distribution of total transverse momentum at fixed total longitudinal momentum $(q_z = 0)$.

Uniqueness of zero total angular momentum case

- $\ell_1 + \ell_2 = 0 \rightarrow$ Maximum of $|\mathcal{S}|$ appears on collision axis.
- $\ell_1 + \ell_2 \neq 0 \rightarrow \mathcal{S} = 0$ on $q_\perp = 0$.

Conclusions and Prospects

Conclusions

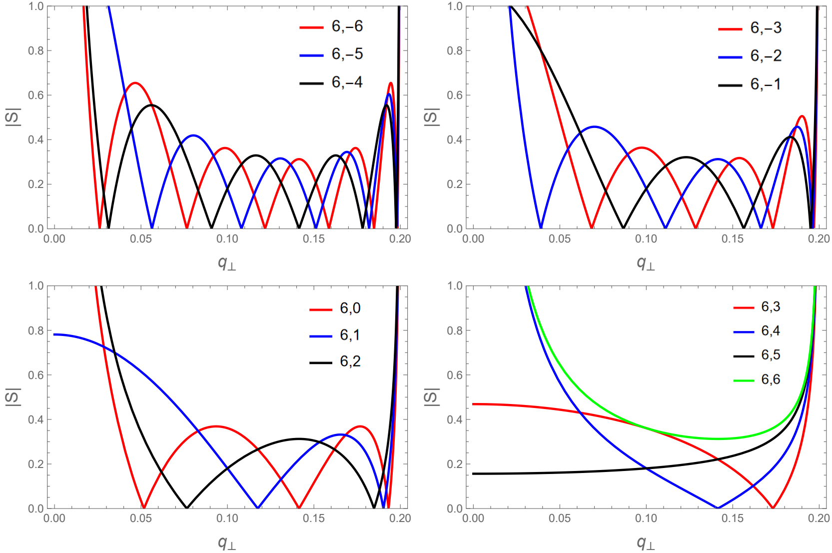
- Final total momentum distributions of vortex particle collision (in the scalar model) have topological structures that are determined by initial topological charges.
- The longitudinal topological number is different for different vortex states, while the transverse one show general dependence on topological charges ($N_\perp = (|\ell_1| + |\ell_2| - |\ell_1 + \ell_2|)/2 + 1$).
- Distributions of two cases with $\ell_1 + \ell_2 = 0$ or $\ell_1 + \ell_2 \neq 0$ have significant difference when total transverse momentum tends to zero. This may also be a general feature that is dependent on topological charges.

Prospects

Both features mentioned above can be used for detection of vortex states' topological charge.

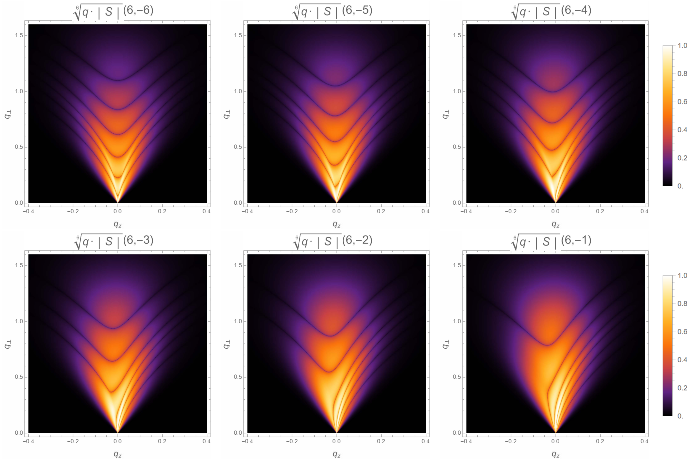
Thanks!

More results of Bessel vortex collision



Return

More results of monochromatic vortex collision



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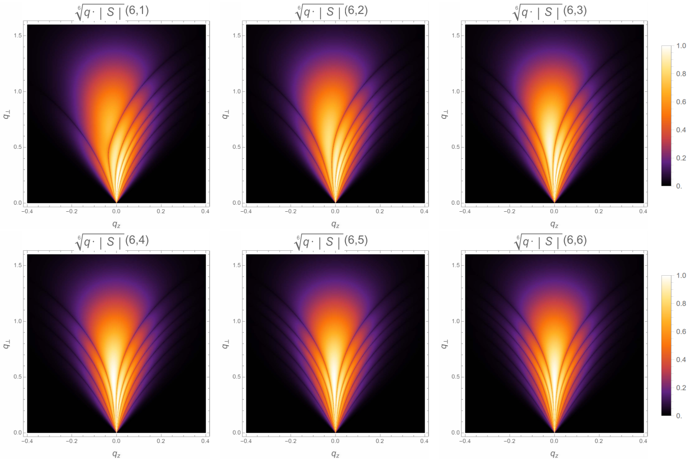
More results of monochromatic vortex collision



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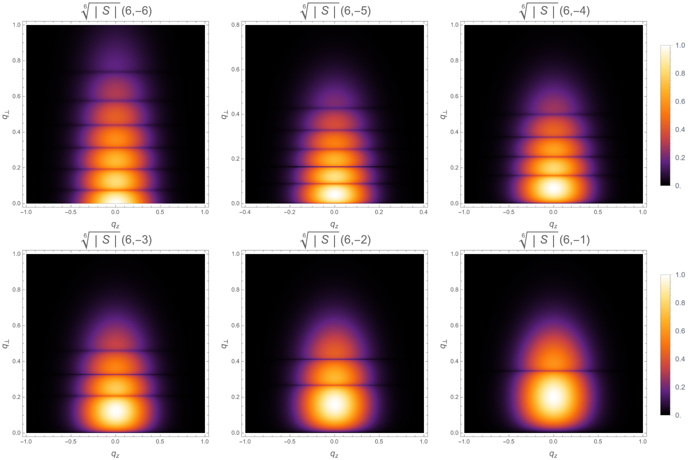
More results of LG vortex collision



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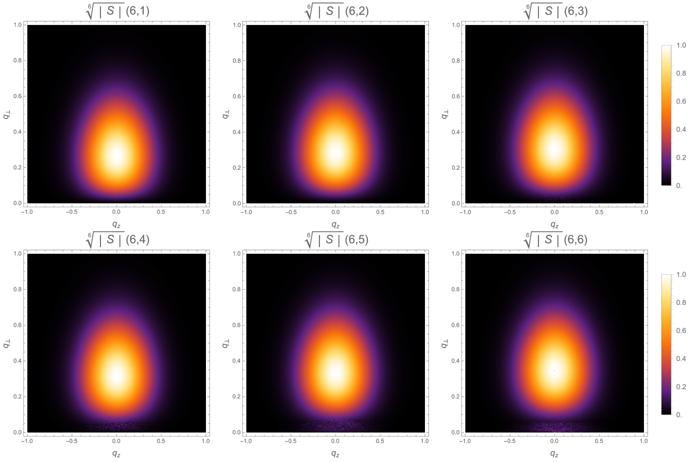
More results of LG vortex collision



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