Large CP violation in charmed baryon decays

arXiv : 2404.19166

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Why are there matters?

Where are antimatters?

Experimental status of charmed hadron decays

2019: First evidence of CP violation in charm sector

PRL **122**, 211803 (2019)

$$A^{dir}_{CP}(D^0 \to K^+K^-) - A^{dir}_{CP}(D^0 \to \pi^+\pi^-) = (-1.54 \pm 0.29) \times 10^{-3}$$

* First evidence of CP violation in charm hadron decays.

2022: The first measurement of CP violation in charmed baryon two-body decays

Sci. Bull. **68**, 583-592 (2023)

$$A_{CP}(\Lambda_c^+ \to \Lambda K^+) = 0.021 \pm 0.026$$

* The most precise CP violation measurement by far in charmed baryons.

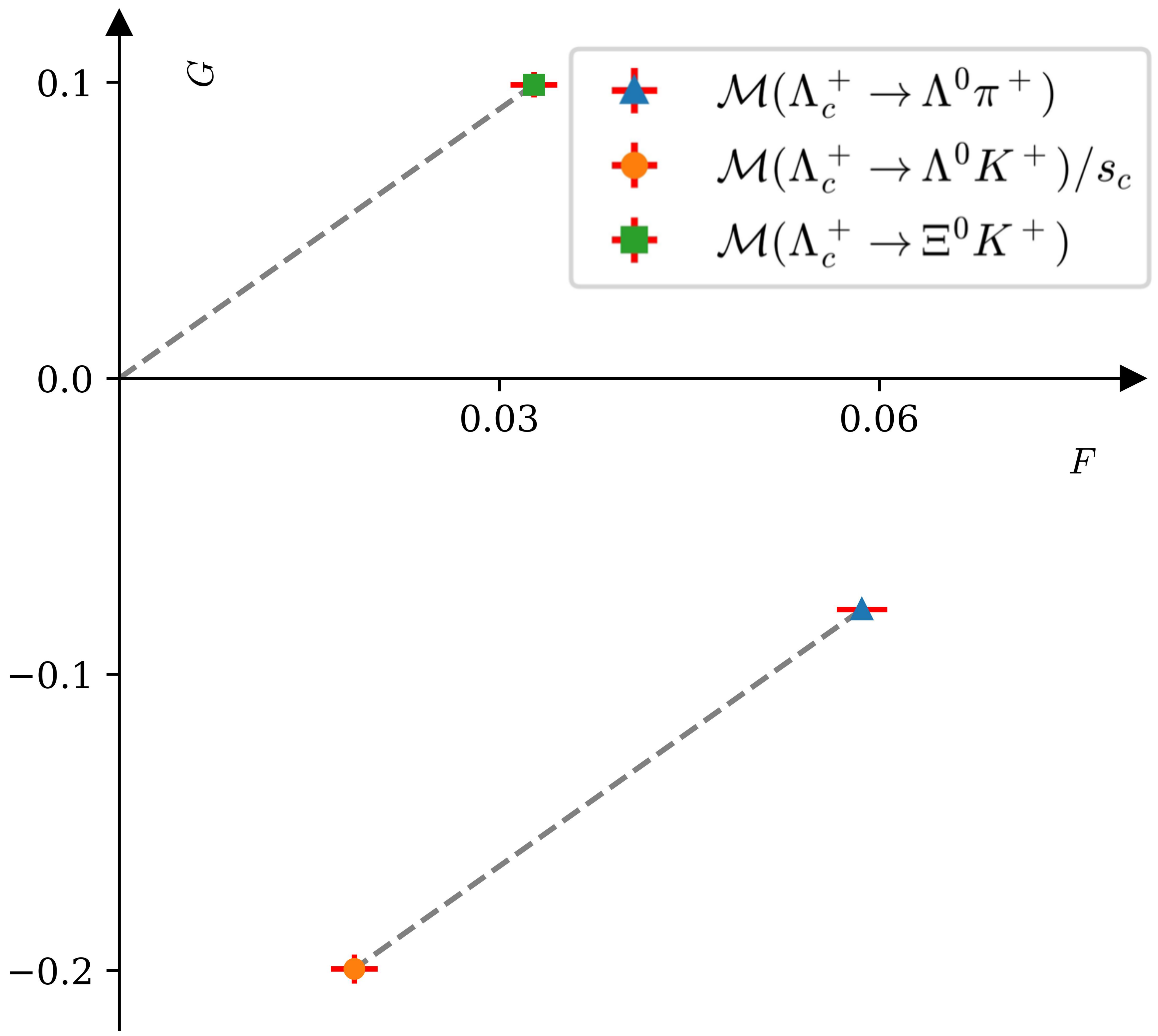
2023: Measurements of strong phases in charmed baryon decays

PRL **132**, 031801 (2024)

$$\alpha(\Lambda_c^+ \to \Xi^0 K^+) = 0.01 \pm 0.16\,, \quad \delta_P - \delta_S = -1.55 \pm 0.27(+\pi)$$

* CP even and Cabibbo-favored, but very important to studies of CP violation!

• Experimental status of charmed hadron decays



The SU(3) flavor relation:

$$\Gamma = \frac{p_f}{8\pi}\frac{\left(M_i + M_f\right)^2 - M_P^2}{M_i^2}\left(|F|^2 + \kappa^2|G|^2\right), \quad \alpha = \frac{2\kappa\,\mathrm{Re}\,(F^*G)}{|F|^2 + \kappa^2|G|^2}$$

$$F(\Lambda_c^+ \to \Xi^0 K^+) = \frac{2}{\sqrt{6}}F(\Lambda_c^+ \to \Lambda^0\pi^+) - \frac{1}{s_c}F(\Lambda_c^+ \to \Lambda^0 K^+)$$

If F and G are real, they are solvable from experimental Γ and α!

→ Leads to $|\alpha(\Lambda_c^+ \to \Xi^0 K^+)| \approx 1$

2023: Measurements of strong phases in charmed baryon decays

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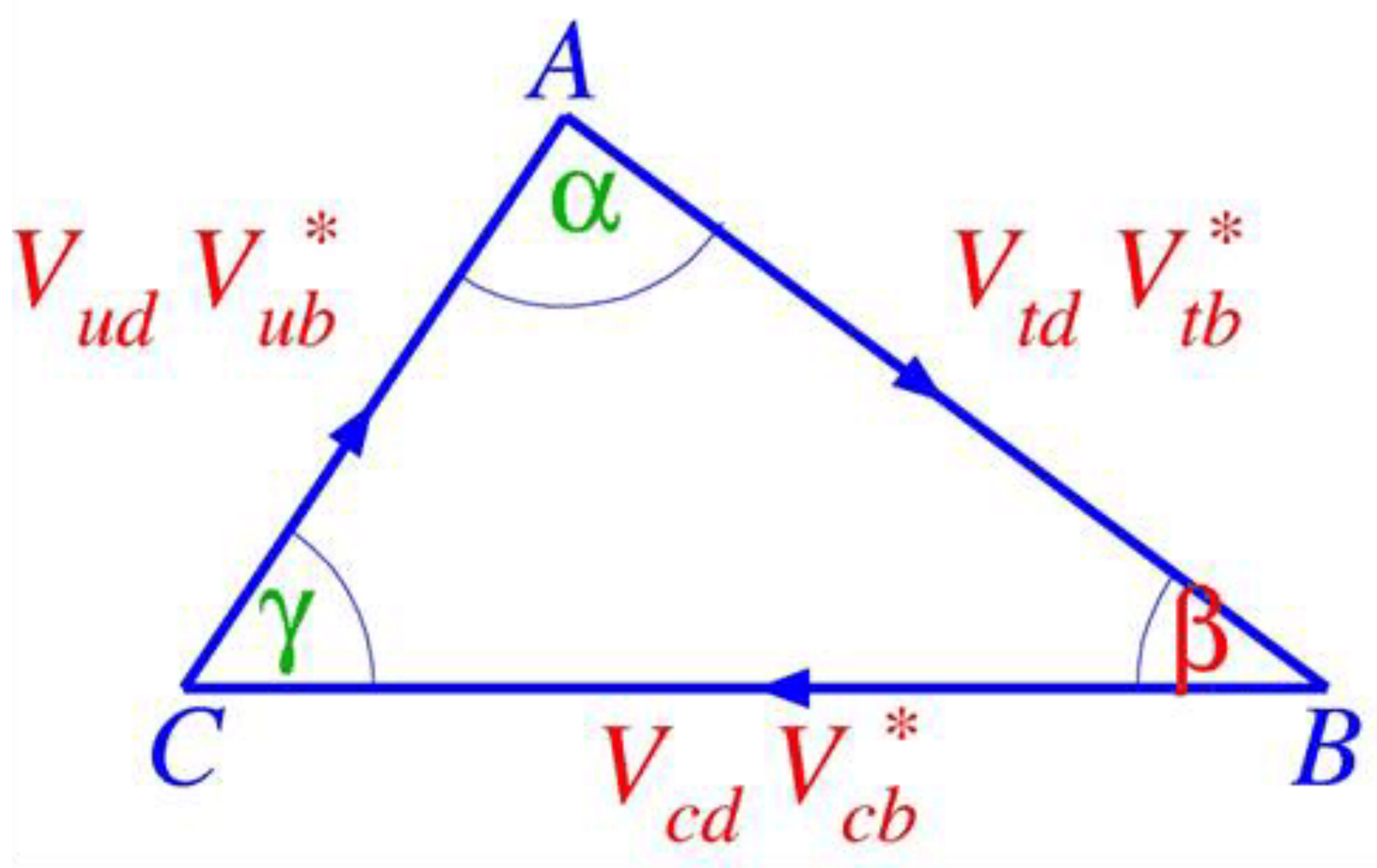
* CP even and Cabibbo-favored, but very important to studies of CP violation!

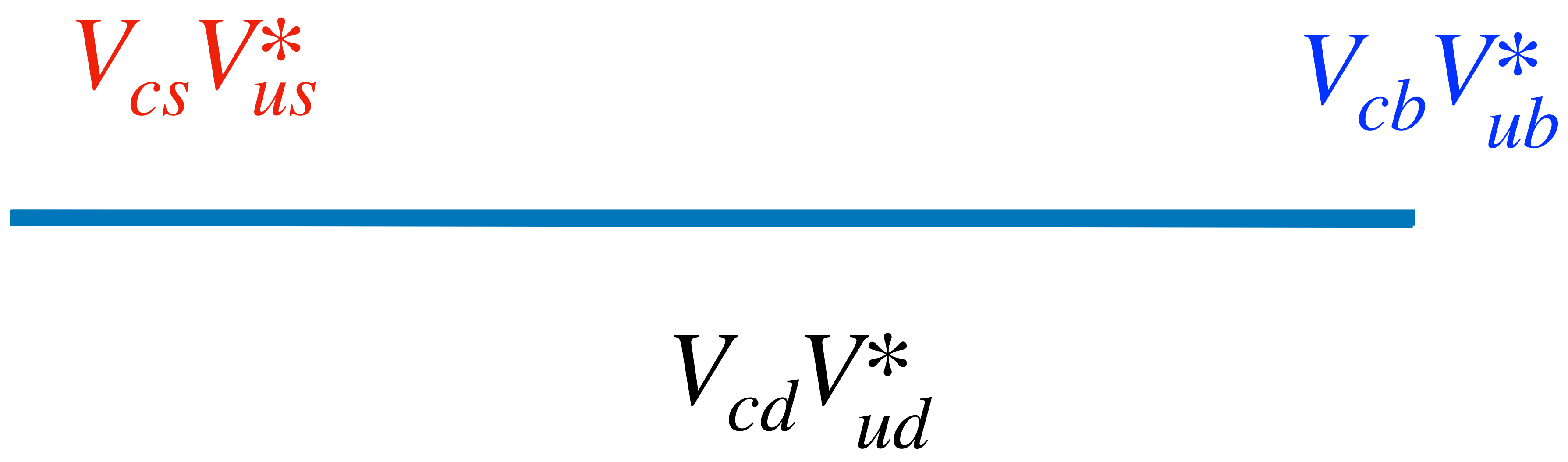
• SU(3) flavor perspective of charmed baryon decays

$$\text{S wave amplitude}: V_{cs}V^*_{us}\overbrace{F^{s-d}}^{\text{5 parameters}} + V_{cb}V^*_{ub}\overbrace{F^{b}}^{\text{4 parameters}}$$

Do not need to consider F^b in studying CP-even quantities. $\longleftrightarrow$ F^b cannot be determined with CP-even quantities.



CKM triangle for $b \to d$



CKM triangle for $c \to u$

SU(3) flavor perspective of charmed baryon decays

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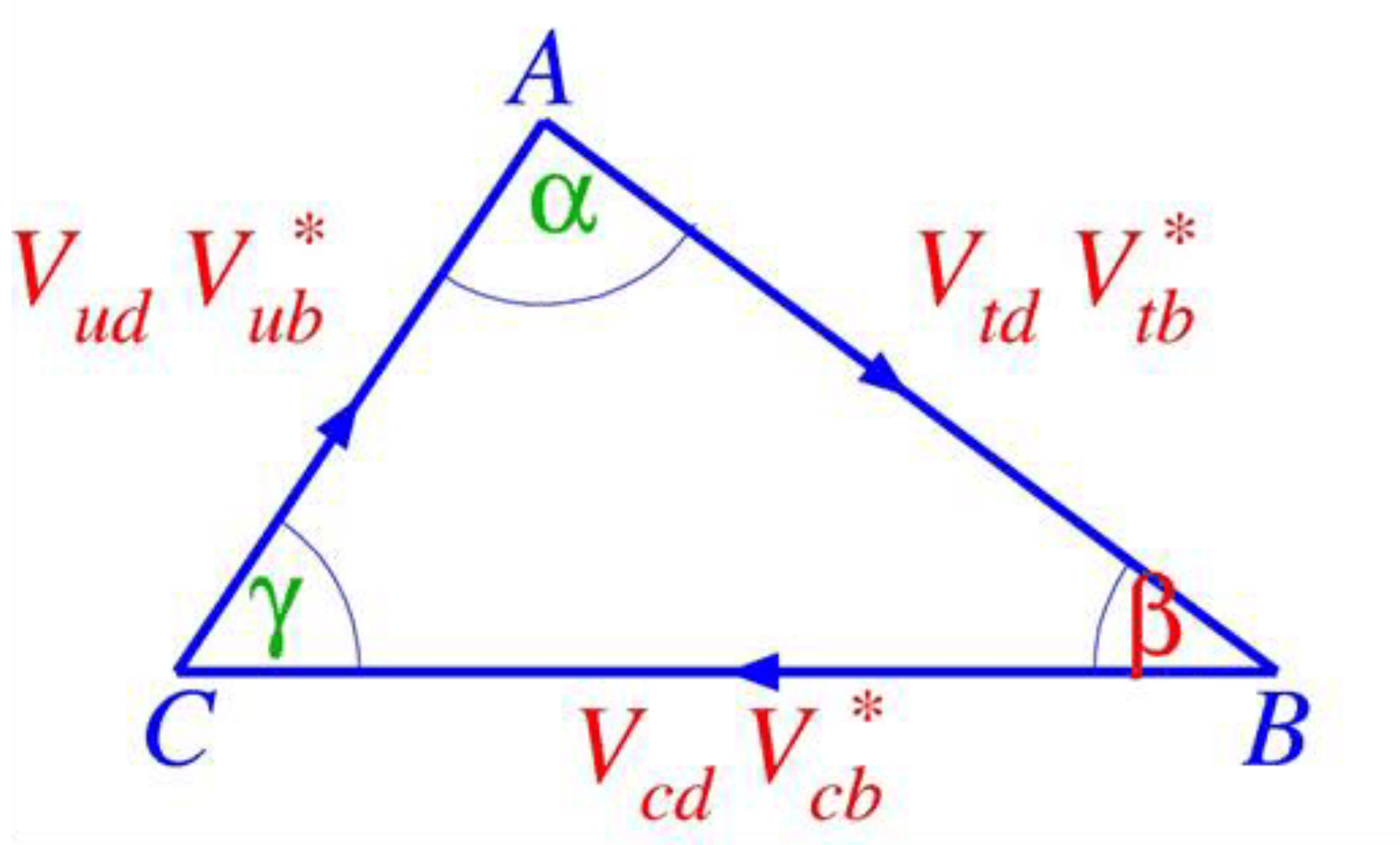

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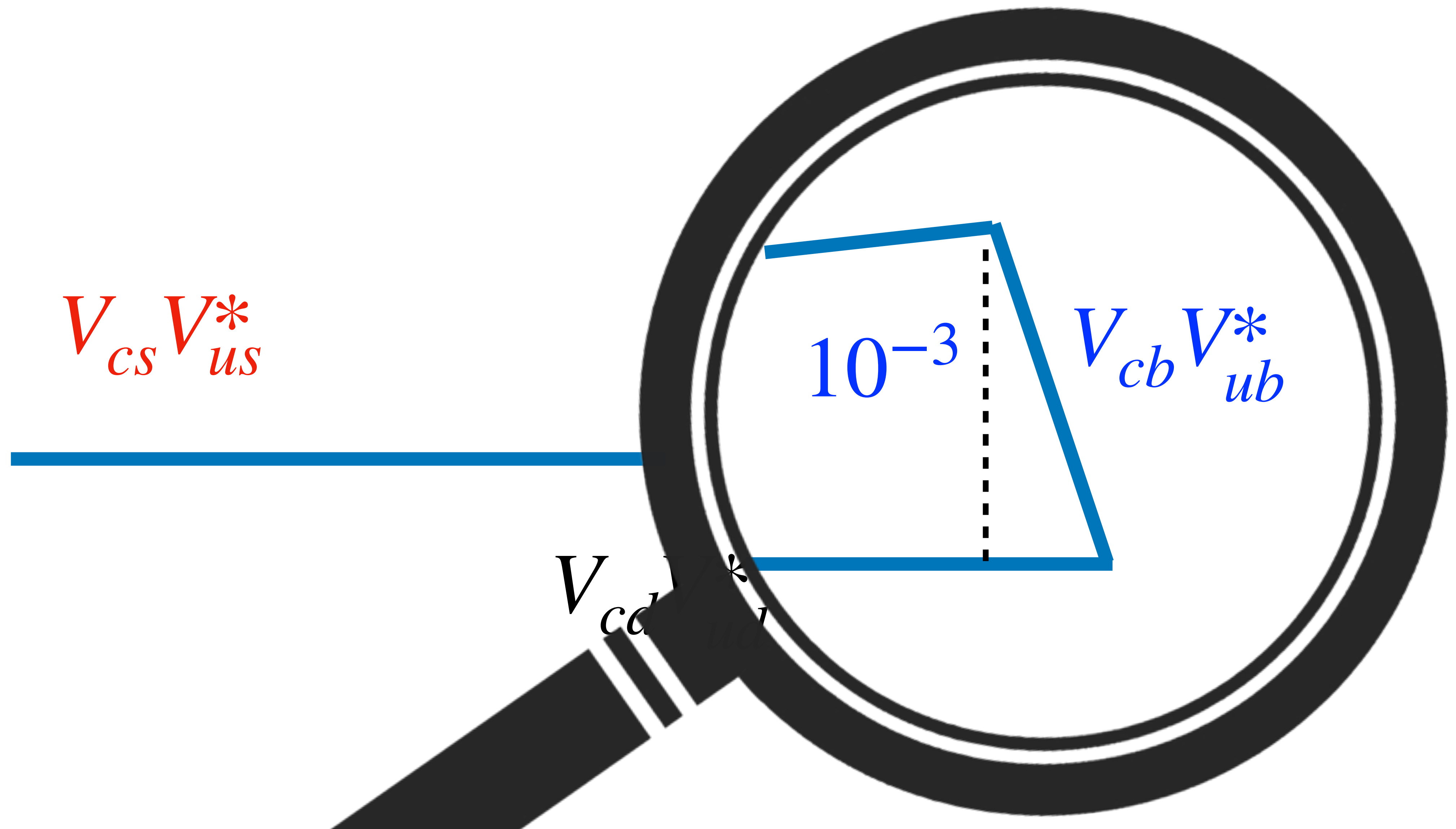


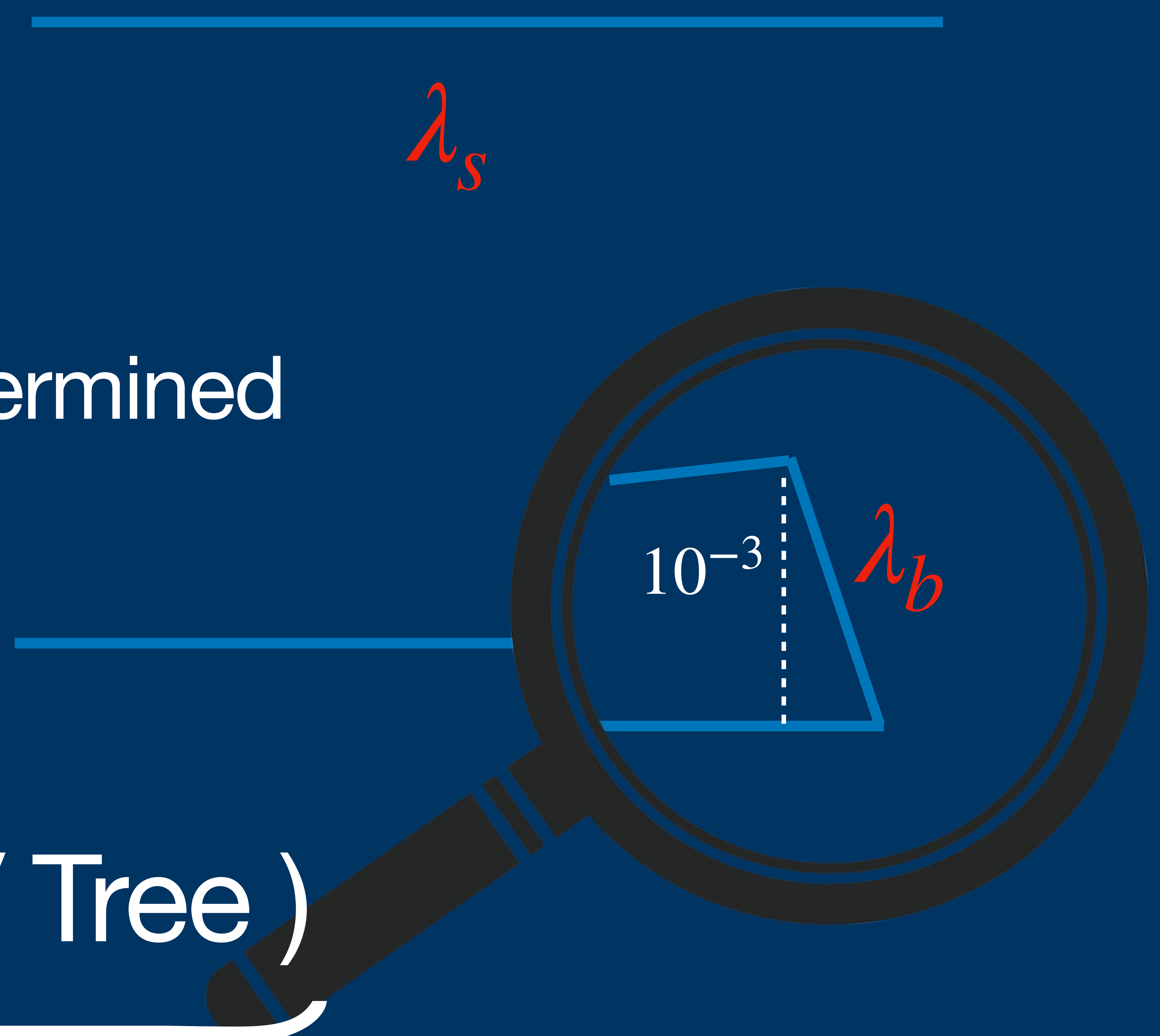
CKM triangle for $c \to u$

SU(3) flavor analysis

$\underbrace{\lambda_s \text{ Tree} + \lambda_b \text{ Penguin}}$

Insensitive to CP-even quantities & undetermined

$\lambda_q = V^*_{cq} V_{uq}$



Rescattering

$\lambda_s \text{ Tree} + \lambda_b \text{ Tree} \times \underbrace{(\text{ Penguin / Tree })}_{\text{Determined by the rescattering}}$

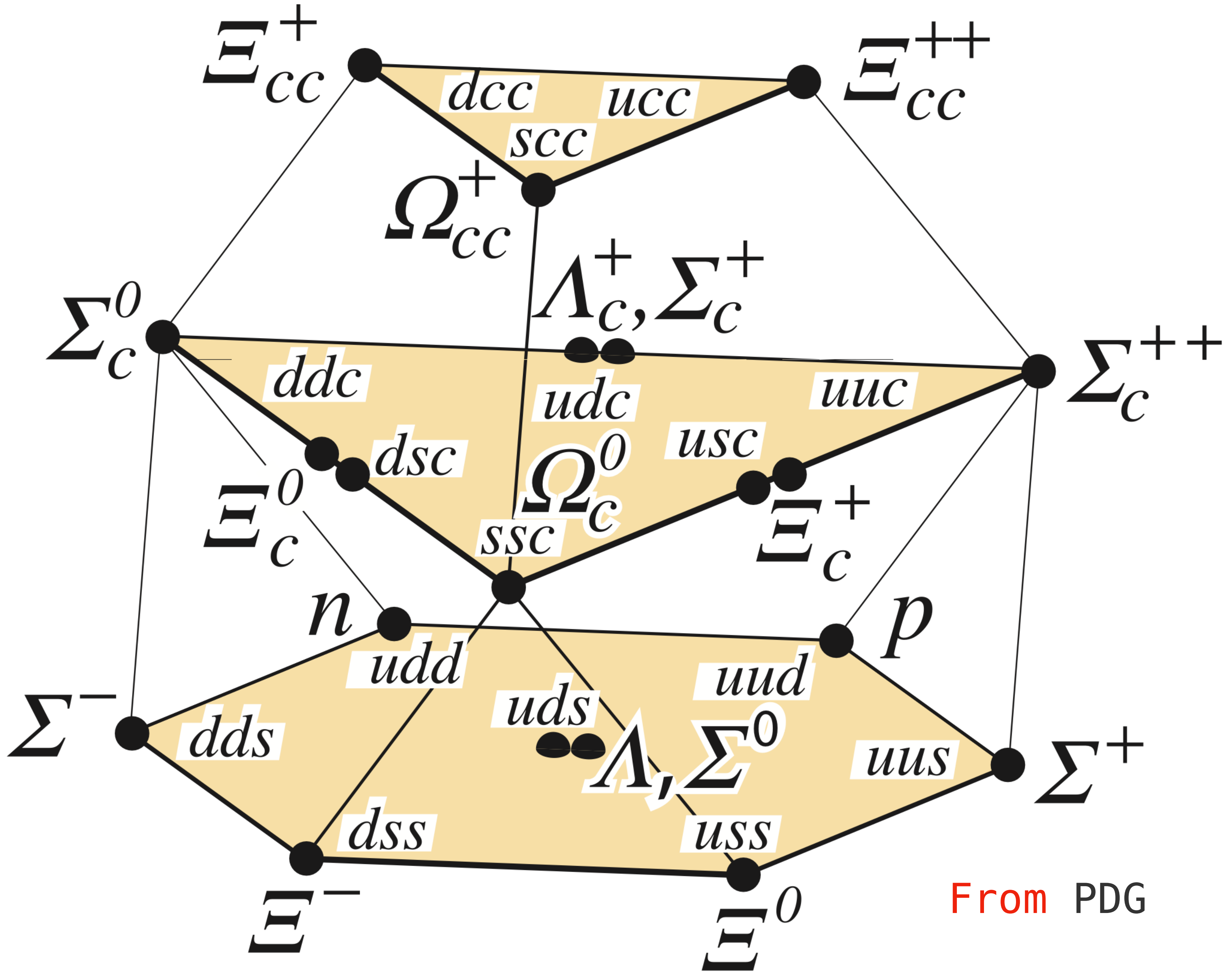
SU(3) flavor analysis — Tree

SU(3) flavor representations :

$$\mathbf{B}_c = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+)\,,$$

$$\mathbf{B} = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix},$$

$$\boldsymbol{P} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\pi^0 + c_\phi\eta + s_\phi\eta') & \pi^+ & K^+ \\ \pi^- & \frac{1}{\sqrt{2}}(-\pi^0 + c_\phi\eta + s_\phi\eta') & K^0 \\ K^- & \bar{K}^0 & -s_\phi\eta + c_\phi\eta' \end{pmatrix},$$



From PDG

- **SU(3) flavor analysis — Tree**

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\left[\sum_{q,q'=d,s} V_{cq}^{*}V_{uq'}\left(C_1^{qq'}Q_1^{qq'} + C_2^{qq'}Q_2^{qq'}\right) + \lambda_q \sum_{i=3\sim 6} C_i Q_i\right] + (H.c.)\,,$$

$Q_1^{qq'} = (\bar{u}q')(\bar{q}c)$ $\quad$ $Q_2^{qq'} = (\bar{q}q')(\bar{u}c)$

$$\underbrace{\mathbf{3}\otimes\mathbf{3}\otimes\bar{\mathbf{3}}}_{\mathscr{H}_{eff}} = \underbrace{(\mathbf{15}\oplus\mathbf{3}_+)}_{Q_1+Q_2}\oplus\underbrace{(\bar{\mathbf{6}}\oplus\mathbf{3}_-)}_{Q_1-Q_2}$$

*V – A dirac strucutre implied

Cabibbo-suppressed decays ($c \to u$)

$C_\pm = (C_1 \pm C_2)/2 \qquad \lambda_d + \lambda_s + \lambda_b = 0$

$$\begin{aligned}\mathcal{H}_{eff} &= \frac{G_F}{\sqrt{2}}\Bigg\{\frac{\lambda_s-\lambda_d}{2}\Big[C_+\big((\bar{u}s)(\bar{s}c)+(\bar{s}s)(\bar{u}c)-(\bar{d}d)(\bar{u}c)-(\bar{u}d)(\bar{d}c)\big)_{\mathbf{15}}\\ &+C_-\big((\bar{u}s)(\bar{s}c)-(\bar{s}s)(\bar{u}c)+(\bar{d}d)(\bar{u}c)-(\bar{u}d)(\bar{d}c)\big)_{\bar{\mathbf{6}}}\Big]\\ &-\frac{\lambda_b}{4}\Big[C_+\big((\bar{u}d)(\bar{d}c)+(\bar{d}d)(\bar{u}c)+(\bar{s}s)(\bar{u}c)+(\bar{u}s)(\bar{s}c)-2(\bar{u}u)(\bar{u}c)\big)_{\mathbf{15}}\\ &+C_+\sum_{q=u,d,s}\big((\bar{u}q)(\bar{q}c)+(\bar{q}q)(\bar{u}c)\big)_{\mathbf{3}_+}+2C_-\sum_{q=d,s}\big((\bar{u}q)(\bar{q}c)-(\bar{q}q)(\bar{u}c)\big)_{\mathbf{3}_-}\Big]\Bigg\}\end{aligned}$$

- ## SU(3) flavor analysis — Tree

S wave amplitude : $\frac{\lambda_s-\lambda_d}{2}\, F^{s-d} + \lambda_b\, F^b$

<u>**Generalized Wigner-Eckart theorem**</u>

$\tilde{f}$:Free parameters

$$F^{s-d} = \tilde{f}^a (P^\dagger)^l_l \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}_c)^{ik} (\mathbf{B}^\dagger)^j_k + \tilde{f}^b \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}_c)^{ik} (\mathbf{B}^\dagger)^l_k (P^\dagger)^j_l + \tilde{f}^c \underbrace{\mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}_c)^{ik} (P^\dagger)^l_k (\mathbf{B}^\dagger)^j_l}_{SU(3)_F \text{ tensors}}$$

$$+ \tilde{f}^d \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}^\dagger)^i_k (P^\dagger)^j_l (\mathbf{B}_c)^{kl} + \tilde{f}^e (\mathbf{B}^\dagger)^j_i \mathcal{H}(\mathbf{15}^{\mathbf{C}})^{\{ik\}}_l (P^\dagger)^l_k (\mathbf{B}_c)_j \,,$$

$$F^b = \tilde{f}^e (\mathbf{B}^\dagger)^j_i \mathcal{H}(\mathbf{15}^b)^{\{ik\}}_l (P^\dagger)^l_k (\mathbf{B}_c)_j + \tilde{f}^a_{\mathbf{3}} (\mathbf{B}_c)_j \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_i (P^\dagger)^k_k + \tilde{f}^b_{\mathbf{3}} (\mathbf{B}_c)_k \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_i (P^\dagger)^k_j$$

$$+ \tilde{f}^c_{\mathbf{3}} (\mathbf{B}_c)_i \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_k (P^\dagger)^k_j + \tilde{f}^d_{\mathbf{3}} (\mathbf{B}_c)_j \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_k (P^\dagger)^k_i \,,$$

$$\mathcal{H}(\overline{\mathbf{6}}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & V_{cs}^* V_{ud} & -\lambda_s - \frac{\lambda_b}{2} \\ 0 & -\lambda_s - \frac{\lambda_b}{2} & V_{cd}^* V_{us} \end{pmatrix} \qquad \mathcal{H}(\mathbf{15})^{ij}_k = \left(\begin{pmatrix} \frac{\lambda_b}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij}, \begin{pmatrix} 0 & -\lambda_s - \frac{3\lambda_b}{4} & V_{cs}^* V_{ud} \\ -\lambda_s - \frac{3\lambda_b}{4} & 0 & 0 \\ V_{cs}^* V_{ud} & 0 & 0 \end{pmatrix}_{ij}, \begin{pmatrix} 0 & V_{cd}^* V_{us} & \lambda_s + \frac{\lambda_b}{4} \\ V_{cd}^* V_{us} & 0 & 0 \\ \lambda_s + \frac{\lambda_b}{4} & 0 & 0 \end{pmatrix}_{ij} \right)_k$$

Equivalent to quark diagrams analysis; see arXiv:1811.03480, 2404.01350

arXiv:2310.05491 [hep-ph]

- **SU(3) flavor analysis — Tree**

S wave amplitude : $\frac{\lambda_s - \lambda_d}{2} F^{s-d} + \lambda_b F^b$

Generalized Wigner-Eckart theorem $\tilde{f}$:Free parameters

Channels	F^{s-d}	F^b
$\Lambda_c^+ \to \Sigma^+ K_S$	$\frac{\sqrt{2}\left(\tilde{f}^b - \tilde{f}^d\right)}{2}$	$\frac{\sqrt{2}\tilde{f}_{\mathbf{3}}^b}{2}$
$\Lambda_c^+ \to \Sigma^0 K^+$	$\frac{\sqrt{2}\left(\tilde{f}^b - \tilde{f}^d\right)}{2}$	$\frac{\sqrt{2}\tilde{f}_{\mathbf{3}}^b}{2}$
$\Lambda_c^+ \to p\pi^0$	$\frac{\sqrt{2}\left(\tilde{f}^c + \tilde{f}^d + \tilde{f}^e\right)}{2}$	$\frac{\sqrt{2}\cdot\left(4\tilde{f}_{\mathbf{3}}^d + 3\tilde{f}^e\right)}{8}$
$\Lambda_c^+ \to p\eta$	$\frac{\sqrt{6}c_\phi\left(-2\tilde{f}^b + \tilde{f}^c - \tilde{f}^d - 3\tilde{f}^e\right)}{6} + \frac{\sqrt{3}s_\phi\left(-3\tilde{f}^a - \tilde{f}^b - \tilde{f}^c + \tilde{f}^d\right)}{3}$	$\sqrt{6}c_\phi\left(-\frac{\tilde{f}_{\mathbf{3}}^b}{3} + \frac{\tilde{f}_{\mathbf{3}}^d}{6} + \frac{\tilde{f}^e}{8}\right) + \frac{\sqrt{3}s_\phi\left(-3\tilde{f}_{\mathbf{3}}^a - \tilde{f}_{\mathbf{3}}^b - \tilde{f}_{\mathbf{3}}^d\right)}{3}$
$\Lambda_c^+ \to n\pi^+$	$\tilde{f}^c + \tilde{f}^d - \tilde{f}^e$	$\tilde{f}_{\mathbf{3}}^d - \frac{\tilde{f}^e}{4}$
$\Lambda_c^+ \to \Lambda K^+$	$\frac{\sqrt{6}\left(\tilde{f}^b - 2\tilde{f}^c + \tilde{f}^d - 2\tilde{f}^e\right)}{6}$	$\frac{\sqrt{6}\cdot\left(2\tilde{f}_{\mathbf{3}}^b - 4\tilde{f}_{\mathbf{3}}^d + \tilde{f}^e\right)}{12}$

$$\mathcal{H}(\overline{\mathbf{6}}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & V_{cs}^* V_{ud} & -\lambda_s - \frac{\lambda_b}{2} \\ 0 & -\lambda_s - \frac{\lambda_b}{2} & V_{cd}^* V_{us} \end{pmatrix} \qquad \mathcal{H}(\mathbf{15})_k^{ij} = \left(\begin{pmatrix} \frac{\lambda_b}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij}, \begin{pmatrix} 0 & -\lambda_s - \frac{3\lambda_b}{4} & V_{cs}^* V_{ud} \\ -\lambda_s - \frac{3\lambda_b}{4} & 0 & 0 \\ V_{cs}^* V_{ud} & 0 & 0 \end{pmatrix}_{ij}, \begin{pmatrix} 0 & V_{cd}^* V_{us} & \lambda_s + \frac{\lambda_b}{4} \\ V_{cd}^* V_{us} & 0 & 0 \\ \lambda_s + \frac{\lambda_b}{4} & 0 & 0 \end{pmatrix}_{ij} \right)_k$$

Equivalent to quark diagrams analysis; see arXiv:1811.03480, 2404.01350

arXiv:2310.05491 [hep-ph]

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S wave amplitude : $\frac{\lambda_s - \lambda_d}{2} F^{s-d} + \lambda_b F^b$

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$$+ \tilde{f}^d \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}^\dagger)^i_k (P^\dagger)^j_l (\mathbf{B}_c)^{kl} + \tilde{f}^e (\mathbf{B}^\dagger)^j_i \mathcal{H}(\mathbf{15}^{\mathbf{C}})^{\{ik\}}_l (P^\dagger)^l_k (\mathbf{B}_c)_j ,$$

$$\cancel{F^b = \tilde{f}^e (\mathbf{B}^\dagger)^j_i \mathcal{H}(\mathbf{15}^b)^{\{ik\}}_l (P^\dagger)^l_k (\mathbf{B}_c)_j + \tilde{f}^a_{\mathbf{3}} (\mathbf{B}_c)_j \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_i (P^\dagger)^k_k + \tilde{f}^b_{\mathbf{3}} (\mathbf{B}_c)_k \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_i (P^\dagger)^k_j}$$
$$\cancel{+ \tilde{f}^c_{\mathbf{3}} (\mathbf{B}_c)_i \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_k (P^\dagger)^k_j + \tilde{f}^d_{\mathbf{3}} (\mathbf{B}_c)_j \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_k (P^\dagger)^k_i},$$

To date, there are in total $\underbrace{\textbf{30}}_{\text{CP-even}}$ data points but $\overset{\mathbf{5}}{\cancel{9}} \times 2$(S & P waves) $\times\, 2$(complex) $- 1 = 3\overset{\mathbf{19}}{\cancel{5}}$

$\tilde{f}^{a,b,c,d,e}, \cancel{\tilde{f}^{a,b,c,d}_{\mathbf{3}}}$

arXiv:2310.05491 [hep-ph]

- **SU(3) flavor analysis — Tree**

- Eliminate 4 redundancies in $\mathscr{H}(\mathbf{15})$

$$\mathscr{B}(\Lambda_c^+ \to \Sigma^0 K^+) = \mathscr{B}(\Lambda_c^+ \to \Sigma^+ K_S^0)$$

PLB **794**, 19(2019)

$$(4.7 \pm 1.0) \times 10^{-4} \approx (4.8 \pm 1.4) \times 10^{-4}$$

BESIII PRD **106**, 052003 (2022)

- Works without considering color-symmetry:

PRD **93**, 056008 (2016), PRD **97**, 073006 (2018), NPB **956**, 115048 (2020)

Lü, Wang, Yu — Geng, Hsiao, Liu, Tsai — Jia, Wang, Yu

JHEP **09**, 035 (2022), JHEP **03**, 143 (2022)

Hsiao, Wang, Zhao — Huang, Xing, He

Not able to determine both partial waves and complex amplitudes.

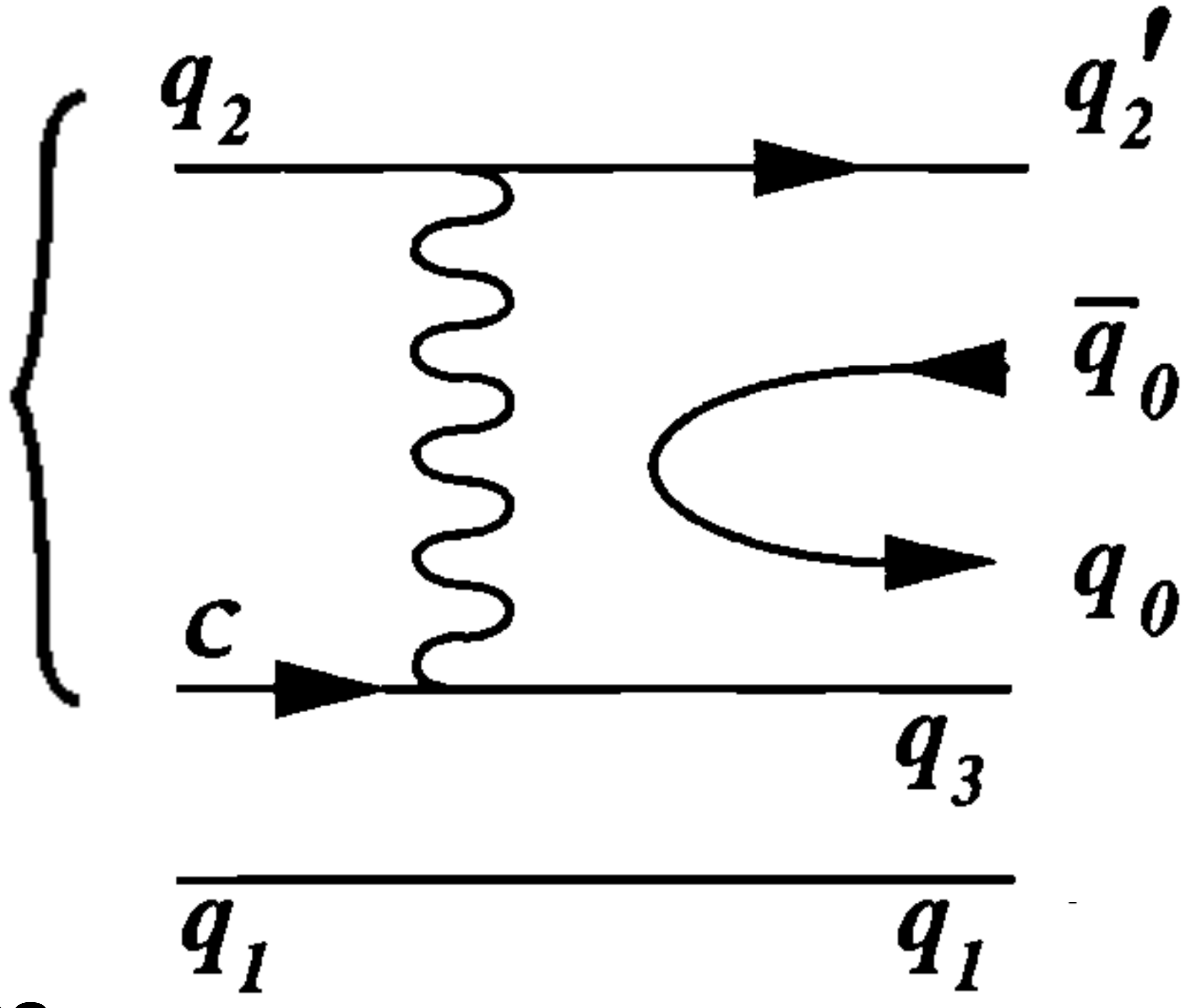
- Körner-Pati-Woo theorem:

$$\langle q_a q_b q_c | O_+^{qq'} | \mathbf{B}_i \rangle = 0$$

15

Color symmetric — Color singlet

$$O_+^{qq'} = \frac{1}{2}\left[(\bar{u}q')_{V-A}(\bar{q}c)_{V-A} + (\bar{q}q')_{V-A}(\bar{u}c)_{V-A}\right],$$



$= 0$

PRD **54**, 2132 (1996)

Chau, Cheng, Tseng

arXiv:2310.05491 [hep-ph]

• SU(3) flavor analysis — Tree

• Sizable strong phases

$$\frac{\tau_{\Lambda_c^+}}{\tau_{\Xi_c^0}}\mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+) = \mathcal{B}(\Lambda_c^+ \to \Sigma^0\pi^+) + 3\mathcal{B}(\Lambda_c^+ \to \Lambda\pi^+) - \frac{1}{s_c^2}\mathcal{B}(\Lambda_c^+ \to n\pi^+)$$

$$\mathscr{B}(\Xi_c^0 \to \Xi^- e^+\nu_e) = (2.38 \pm 0.44)\,\%$$

$$\times$$

LQCD, CPC **46**, 011002 (2022);
also Wang's talk in this morning.

$$\frac{\mathscr{B}(\Xi_c^0 \to \Xi^-\pi^+)}{\mathscr{B}(\Xi_c^0 \to \Xi^- e^+\nu_e)} = 1.37 \pm 0.08$$

Belle, PRL **127** 121803 (2021)

$$||$$

$$\mathscr{B}(\Xi_c^0 \to \Xi^-\pi^+) = (3.26 \pm 0.63)\,\%$$

$$\beta = \frac{2\,\mathrm{Im}\,(S^*P)}{|S|^2 + |P|^2}$$

Channels	$\mathcal{B}_{\rm exp}(\%)$	$\alpha_{\rm exp}$	$\mathcal{B}(\%)$	α	β
$\Lambda_c^+ \to pK_S$	1.59(8)	*0.18(45)	1.55(7)	−0.40(49)	0.32(29)
$\Lambda_c^+ \to \Lambda^0\pi^+$	1.30(6)	−0.755(6)	1.29(5)	−0.75(1)	−0.13(19)
$\Lambda_c^+ \to \Sigma^0\pi^+$	1.27(6)	−0.466(18)	1.27(5)	−0.47(2)	0.88(2)
$\Lambda_c^+ \to \Sigma^+\pi^0$	1.25(10)	−0.48(3)	1.27(5)	−0.47(2)	0.88(2)
$\Lambda_c^+ \to \Xi^0K^+$	**0.55(7)	0.01(16)	0.40(3)	−0.15(14)	−0.29(22)
$\Lambda_c^+ \to \Lambda^0K^+$	0.064(3)	−0.585(52)	0.063(3)	−0.56(5)	0.82(5)
$\Lambda_c^+ \to \Sigma^0K^+$	0.0382(25)	−0.54(20)	0.0365(21)	−0.52(10)	0.48(24)
$\Lambda_c^+ \to n\pi^+$	0.066(13)		0.067(8)	−0.78(12)	−0.63(15)
$\Lambda_c^+ \to \Sigma^+K_S$	0.048(14)		0.036(2)	−0.52(10)	0.48(24)
$\Lambda_c^+ \to p\pi^0$	<0.008		0.02(1)		−0.82(32)
$\Lambda_c^+ \to \Sigma^+\eta$	0.32(4)	−0.99(6)	0.32(4)	−0.93(4)	−0.32(16)
$\Lambda_c^+ \to p\eta$	0.142(12)		0.145(26)	−0.42(61)	0.64(40)
$\Lambda_c^+ \to \Sigma^+\eta'$	0.437(84)	−0.46(7)	0.420(70)	−0.44(25)	0.86(6)
$\Lambda_c^+ \to p\eta'$	0.0484(91)		0.0520(114)	−0.59(9)	0.76(14)
$\Xi_c^+ \to \Xi^0\pi^+$	1.6(8)		0.90(16)	−0.94(6)	0.32(21)
$\Xi_c^0 \to \Xi^-\pi^+$	****1.43(32)	$^*-0.64(5)$	2.72(9)	−0.71(3)	0.36(20)
Channels	$\mathcal{R}_X^{\rm exp}$	$\alpha_{\rm exp}$	$\mathcal{R}_X$	α	β
$\Xi_c^0 \to \Lambda^0K_S$	0.225(13)		0.233(9)	−0.47(29)	0.66(20)
$\Xi_c^0 \to \Xi^-K^+$	**0.0275(57)		0.0410(4)	−0.75(4)	0.38(20)
$\Xi_c^0 \to \Sigma^0K_S$	0.038(7)		0.038(7)	−0.07(117)	−0.83(28)
$\Xi_c^0 \to \Sigma^+K^-$	0.123(12)		0.132(11)	−0.21(18)	−0.39(29)

arXiv:2310.05491 [hep-ph]

- **SU(3) flavor analysis — Tree**

PDG $> 4\sigma$

$(1.43 \pm 0.32)\,\%$

Belle $< 2\sigma$

$(1.80 \pm 0.52)\,\%$

SU(3)

$(2.72 \pm 0.09)\,\%$

$\mathscr{B}(\Xi_c^0 \to \Xi^- e^+ \nu_e) = (2.38 \pm 0.44)\,\%$

$\times$

LQCD, CPC **46**, 011002 (2022);
also Wang's talk in this morning.

$$\frac{\mathscr{B}(\Xi_c^0 \to \Xi^- \pi^+)}{\mathscr{B}(\Xi_c^0 \to \Xi^- e^+ \nu_e)} = 1.37 \pm 0.08$$

Belle, PRL **127** 121803 (2021)

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$\Lambda_c^+ \to p\pi^0$	< 0.008		0.02(1)		−0.82(32)
$\Lambda_c^+ \to \Sigma^+\eta$	0.32(4)	−0.99(6)	0.32(4)	−0.93(4)	−0.32(16)
$\Lambda_c^+ \to p\eta$	0.142(12)		0.145(26)	−0.42(61)	0.64(40)
$\Lambda_c^+ \to \Sigma^+\eta'$	0.437(84)	−0.46(7)	0.420(70)	−0.44(25)	0.86(6)
$\Lambda_c^+ \to p\eta'$	0.0484(91)		0.0520(114)	−0.59(9)	0.76(14)
$\Xi_c^+ \to \Xi^0\pi^+$	1.6(8)		0.90(16)	−0.94(6)	0.32(21)
$\Xi_c^0 \to \Xi^-\pi^+$	****1.43(32)	* − 0.64(5)	2.72(9)	−0.71(3)	0.36(20)
Channels	$\mathcal{R}_X^{\rm exp}$	$\alpha_{\rm exp}$	$\mathcal{R}_X$	α	β
$\Xi_c^0 \to \Lambda^0 K_S$	0.225(13)		0.233(9)	−0.47(29)	0.66(20)
$\Xi_c^0 \to \Xi^- K^+$	**0.0275(57)		0.0410(4)	−0.75(4)	0.38(20)
$\Xi_c^0 \to \Sigma^0 K_S$	0.038(7)		0.038(7)	−0.07(117)	−0.83(28)
$\Xi_c^0 \to \Sigma^+ K^-$	0.123(12)		0.132(11)	−0.21(18)	−0.39(29)

arXiv:2310.05491 [hep-ph]

- **SU(3) flavor analysis — Tree**

S wave amplitude : $\frac{\lambda_s - \lambda_d}{2} F^{s-d} + \lambda_b F^b$

Generalized Wigner-Eckart theorem

$\tilde{f}$:Free parameters

$$F^{s-d} = \tilde{f}^a (P^\dagger)^l_l \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}_c)^{ik} (\mathbf{B}^\dagger)^j_k + \tilde{f}^b \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}_c)^{ik} (\mathbf{B}^\dagger)^l_k (P^\dagger)^j_l + \tilde{f}^c \underbrace{\mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}_c)^{ik} (P^\dagger)^l_k (\mathbf{B}^\dagger)^j_l}_{SU(3)_F \text{ tensors}}$$
$$+ \tilde{f}^d \mathcal{H}(\overline{\mathbf{6}}^{\mathbf{C}})_{ij} (\mathbf{B}^\dagger)^i_k (P^\dagger)^j_l (\mathbf{B}_c)^{kl} + \tilde{f}^e (\mathbf{B}^\dagger)^j_i \mathcal{H}(\mathbf{15}^{\mathbf{C}})^{\{ik\}}_l (P^\dagger)^l_k (\mathbf{B}_c)_j ,$$

$$F^b = \tilde{f}^e (\mathbf{B}^\dagger)^j_i \mathcal{H}(\mathbf{15}^b)^{\{ik\}}_l (P^\dagger)^l_k (\mathbf{B}_c)_j \cancel{+ \tilde{f}^a_{\mathbf{3}} (\mathbf{B}_c)_j \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_i (P^\dagger)^k_k + \tilde{f}^b_{\mathbf{3}} (\mathbf{B}_c)_k \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_i (P^\dagger)^k_j}$$
$$\cancel{+ \tilde{f}^c_{\mathbf{3}} (\mathbf{B}_c)_i \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_k (P^\dagger)^k_j + \tilde{f}^d_{\mathbf{3}} (\mathbf{B}_c)_j \mathcal{H}(\mathbf{3}^b)^i (\mathbf{B}^\dagger)^j_k (P^\dagger)^k_i},$$

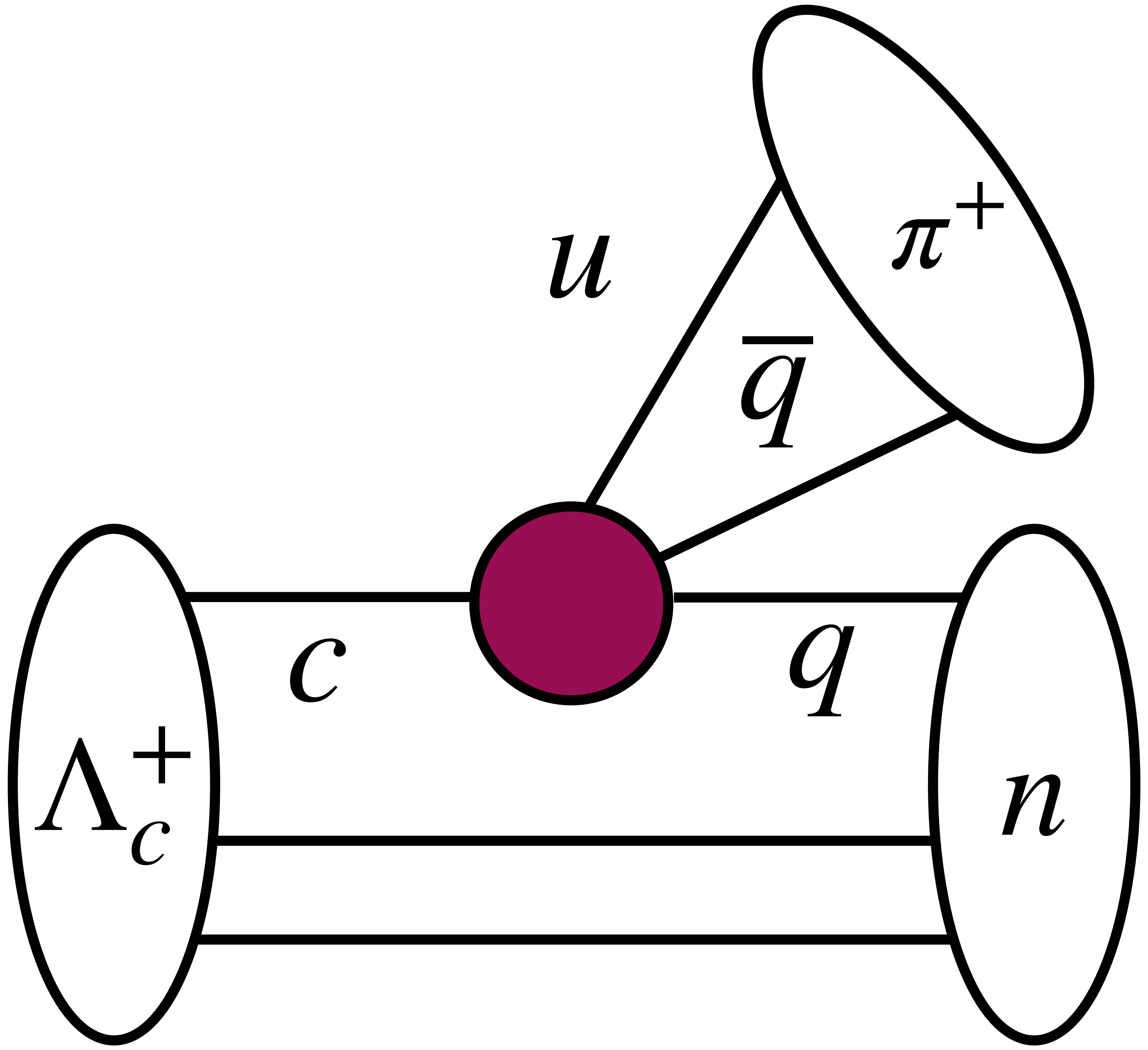
Naive assumption: $\tilde{f}^{a,b,c,d}_{\mathbf{3}} \to 0$

To date, there are in total $\underbrace{\textbf{30}}_{\text{CP-even}}$ data points and $\mathbf{5} \times 2$(S & P waves) $\times\, 2$(complex) $- 1 = \mathbf{19}$

arXiv:2310.05491 [hep-ph]

• SU(3) flavor analysis — Tree

CP asymmetries of $\Lambda_c^+ \to n\pi^+$ do not vanish, as parts of the tree interaction contain penguin topology.



$$\mathscr{H}_{eff}^{\text{Tree}} = \frac{G_F}{\sqrt{2}}\lambda_b\Bigg(C_+ \sum_{q=u,d,s} ((\bar{u}q)(\bar{q}c) + (\bar{q}q)(\bar{u}c)) + 2C_- \sum_{q=d,s} ((\bar{u}q)(\bar{q}c) - (\bar{q}q)(\bar{u}c)) \Bigg)_{\mathbf{3}} \cdots$$

Too small compared to *D* meson's:

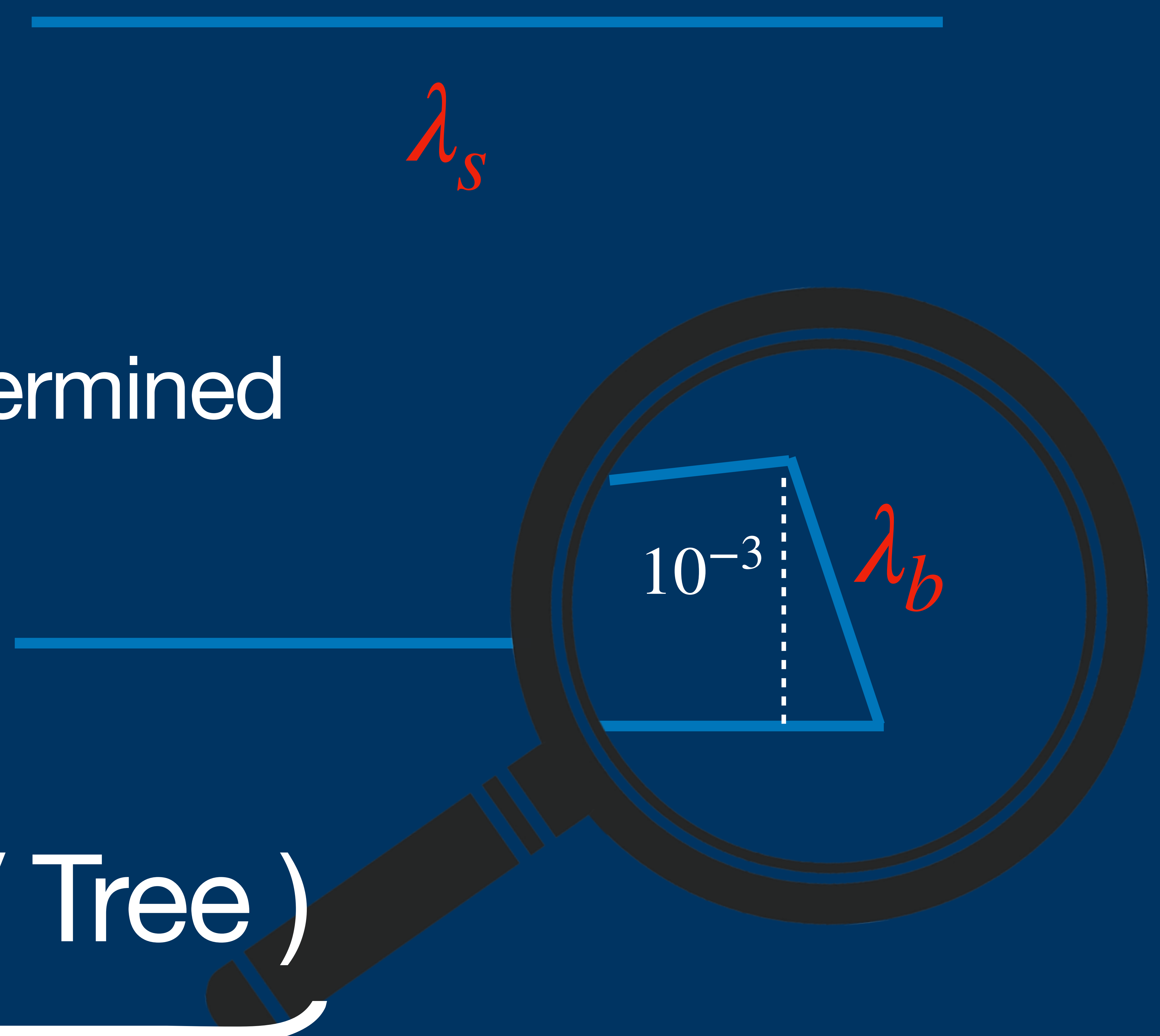
$$A_{CP}^{dir}(D^0 \to K^+K^-) - A_{CP}^{dir}(D^0 \to \pi^+\pi^-) = (-1.54 \pm 0.29) \times 10^{-3}$$

Channels	$\mathcal{B}(10^{-3})$	$A_{CP}^{\alpha}(10^{-3})$	$A_{CP}^{\beta}(10^{-3})$	$A_{CP}^{\gamma}(10^{-3})$	$A_{CP}(10^{-3})$
$\Lambda_c^+ \to p\pi^0$	0.16(2)	−0.61(39)	−0.43(48)	0.53(1.45)	0.42(1.15)
$\Lambda_c^+ \to p\eta$	1.45(25)	0.05(17)	0.04(14)	−0.07(22)	−0.24(26)
$\Lambda_c^+ \to p\eta'$	0.52(11)	−0.02(7)	0.01(4)	0.00(4)	0.08(2)
$\Lambda_c^+ \to n\pi^+$	0.67(8)	0.12(20)	0.13(26)	−0.28(40)	−0.15(42)
$\Lambda_c^+ \to \Lambda^0 K^+$	0.63(2)	−0.03(10)	0.03(5)	0.04(24)	0.19(18)

SU(3) flavor analysis

$$\lambda_s\ \text{Tree} + \underbrace{\lambda_b\ \text{Penguin}}_{\text{Insensitive to CP-even quantities \& undetermined}}$$

$$\lambda_q = V_{cq}^{*} V_{uq}$$

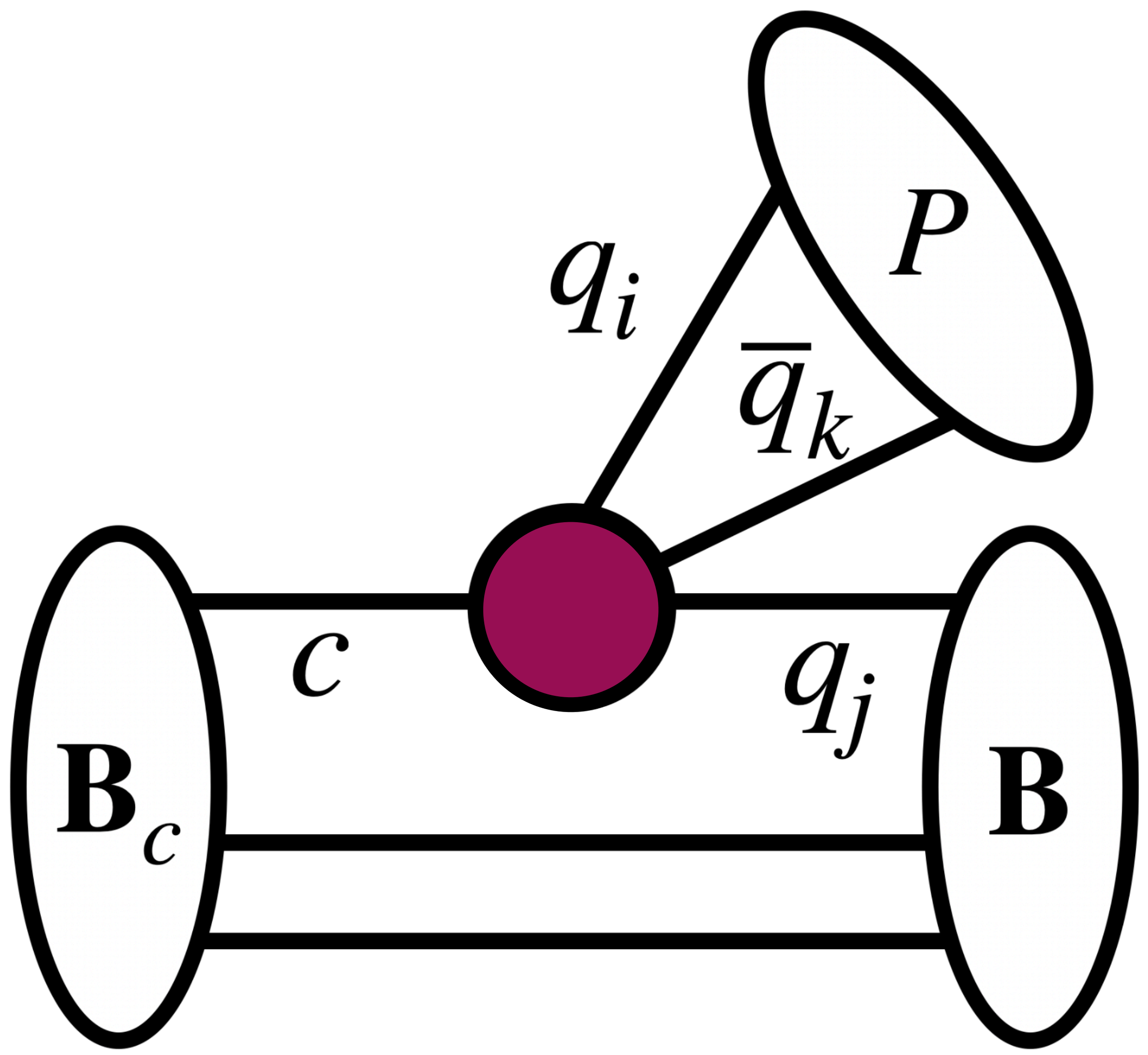


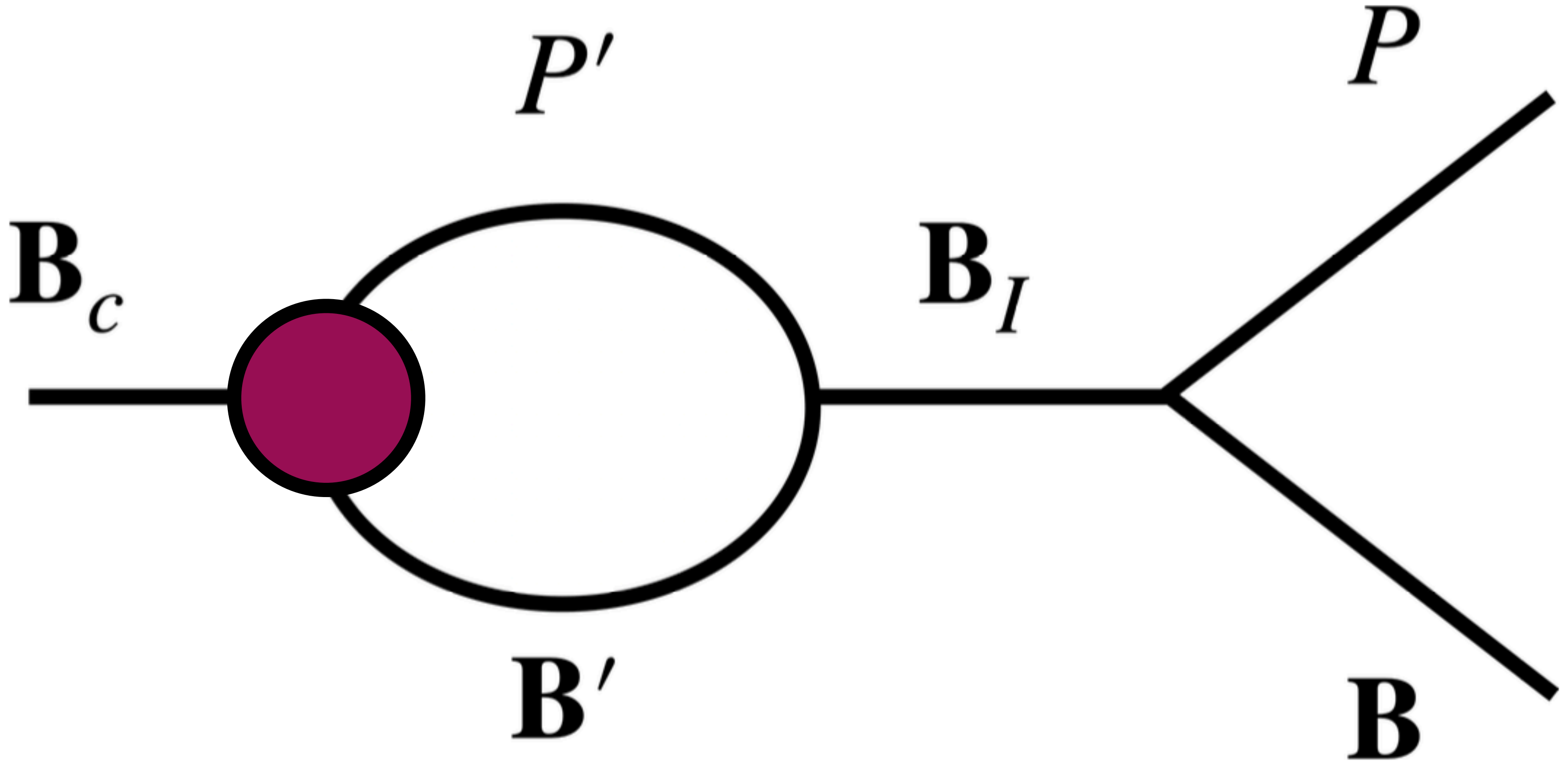
Rescattering

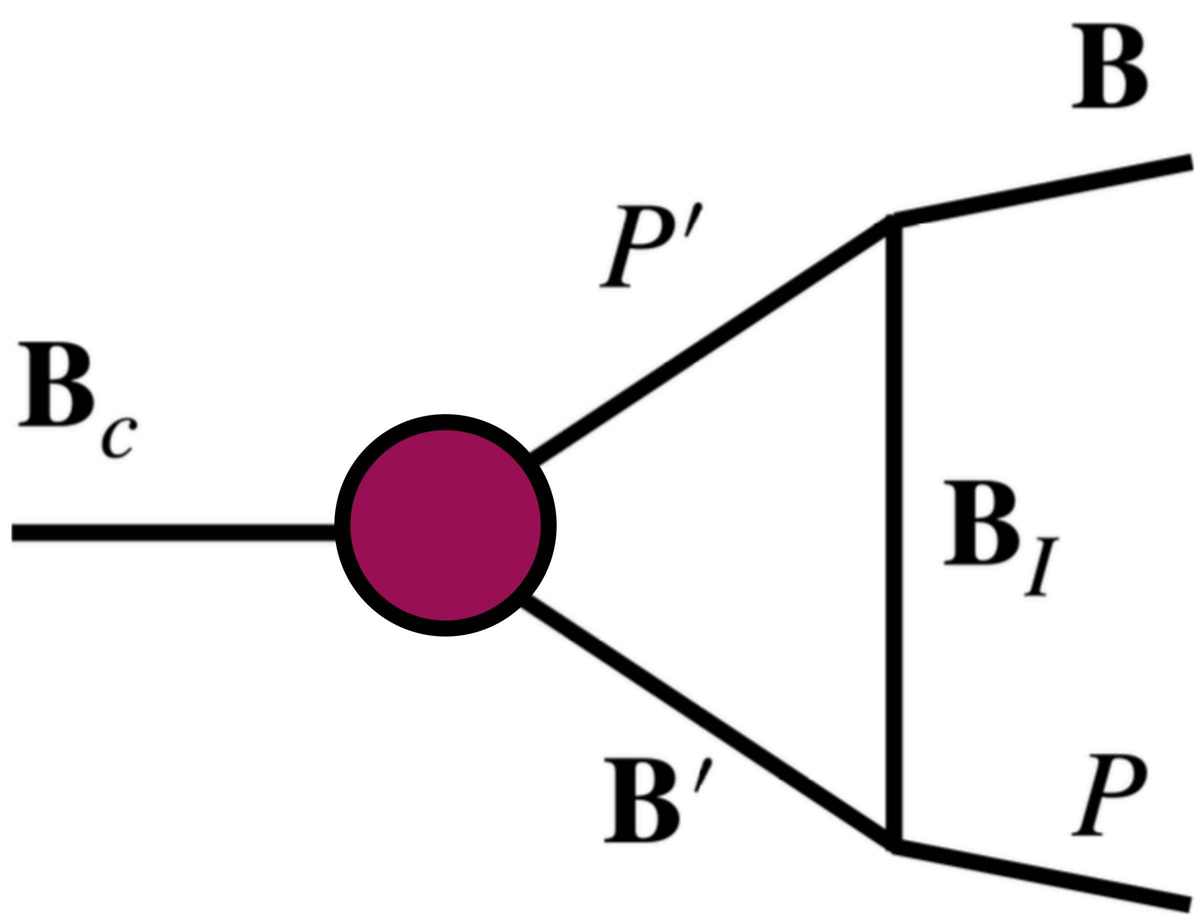
$$\lambda_s\ \text{Tree} + \lambda_b\ \text{Tree} \times \underbrace{(\ \text{Penguin / Tree}\ )}_{\text{Determined by the rescattering}}$$

Rescattering

$$\mathscr{L}_{\mathbf{B}_c\mathbf{B}P} = \mathscr{L}^{\mathrm{Tree}}_{\mathbf{B}_c\mathbf{B}P} + \mathscr{L}^{\mathrm{FSR-s}}_{\mathbf{B}_c\mathbf{B}P} + \mathscr{L}^{\mathrm{FSR-t}}_{\mathbf{B}_c\mathbf{B}P}$$







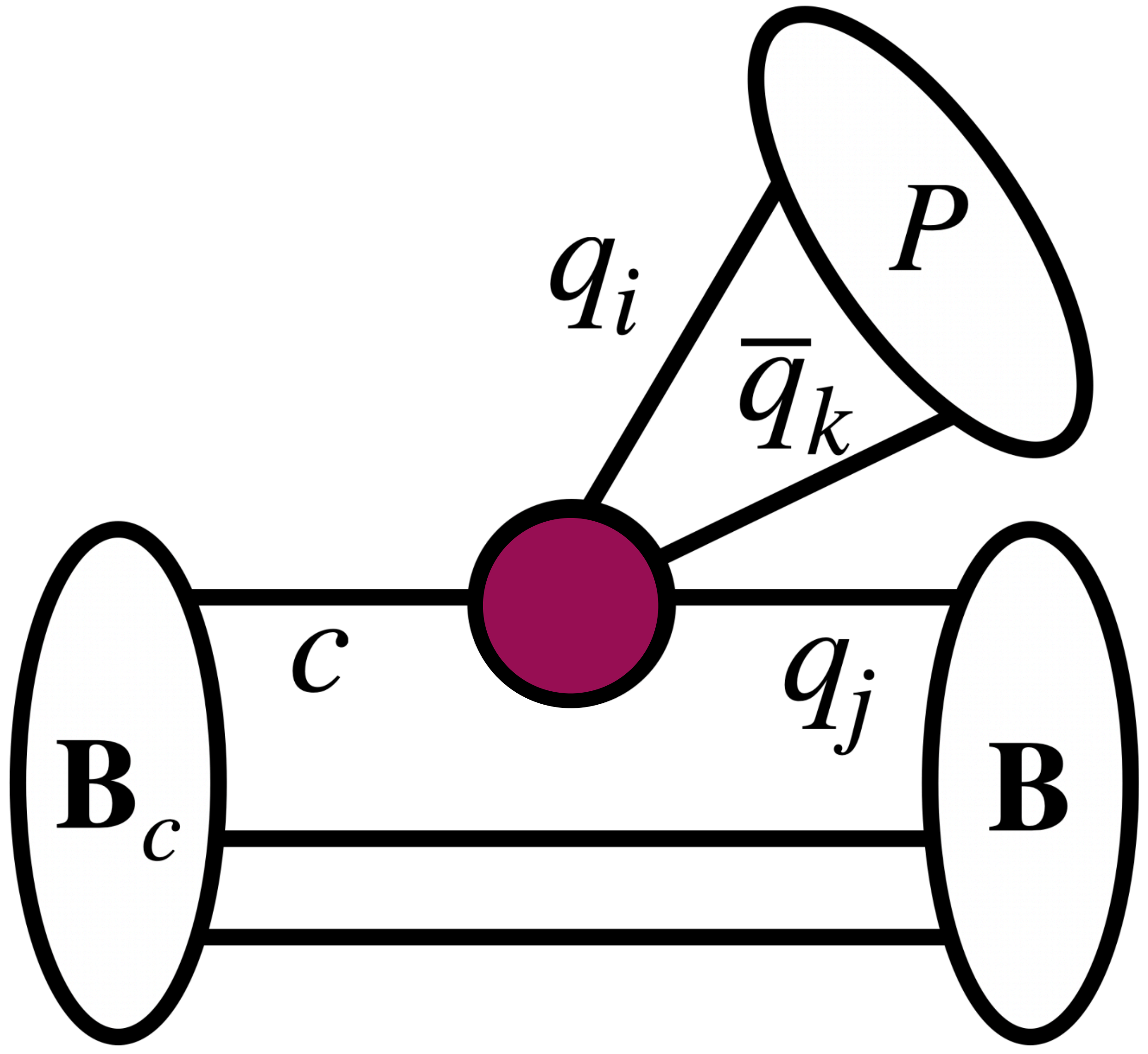
<u>**Assumptions:**</u>

1. Short distance transitions are dominated by the W-emission, including both color-enhanced and color-suppressed.
2. $\mathbf{B}_I \in$ lowest-lying baryons of both parities.
3. The re-scattering is closed, *i.e.* $\mathbf{B}'P'$ belong to the same $SU(3)_F$ group of $\mathbf{B}P$.

See Yu's talks for $\mathbf{B}_c \to \mathbf{B}V$.

• Rescattering — $\mathscr{L}^{\text{Tree}}_{\mathbf{B}_c\mathbf{B}P}$

$$
\begin{aligned}
\langle \mathbf{B}P|\, \mathscr{L}^{\text{Tree}}_{\mathbf{B}_c\mathbf{B}P}\, |\mathbf{B}_c\rangle \;=\; & \frac{G_F}{\sqrt{2}}\left[C_+(\mathscr{H}_+)^{ij}_k + C_-(\mathscr{H}_-)^{ij}_k\right]\langle P|\bar{q}_i\gamma_\mu\gamma_5 q^k|0\rangle\langle \mathbf{B}|\bar{q}_j\gamma^\mu(1-\gamma_5)c|\mathbf{B}_c\rangle \\
+ & \frac{G_F}{\sqrt{2}}\left[C_+(\mathscr{H}_+)^{ij}_k - C_-(\mathscr{H}_-)^{ij}_k\right]\langle P|(\bar{q}_i)_\sigma\gamma_\mu\gamma_5(q^k)_\rho|0\rangle\langle \mathbf{B}|(\bar{q}_j)_\rho\gamma^\mu(1-\gamma_5)c_\sigma|\mathbf{B}_c\rangle\,.
\end{aligned}
$$



From above, we deduce:

$$\mathscr{L}^{\text{Tree}}_{\mathbf{B}_c\mathbf{B}P} = \left(P^\dagger\right)^k_i(\overline{\mathbf{B}})^l_j\left(\tilde{F}^+_V\left(\mathscr{H}_+\right)^{ij}_k + \tilde{F}^-_V\left(\mathscr{H}_-\right)^{ij}_k\right)\left(\mathbf{B}_c\right)_l$$

where

$$(\mathscr{H}_+)^{ij}_k = \frac{\lambda_s-\lambda_d}{2}\mathscr{H}(\mathbf{15}^{s-d})^{ij}_k + \lambda_b\left(\mathscr{H}(\mathbf{15}^b)^{ij}_k + \mathscr{H}(\mathbf{3}_+)^i\delta^j_k + \mathscr{H}(\mathbf{3}_+)^j\delta^i_k\right)$$

$$(\mathscr{H}_-)^{ij}_k = \frac{\lambda_s-\lambda_d}{2}\mathscr{H}(\overline{\mathbf{6}})_{kl}\epsilon^{lij} + 2\lambda_b\left(\mathscr{H}(\mathbf{3}_-)^i\delta^j_k - \mathscr{H}(\mathbf{3}_-)^j\delta^i_k\right)$$

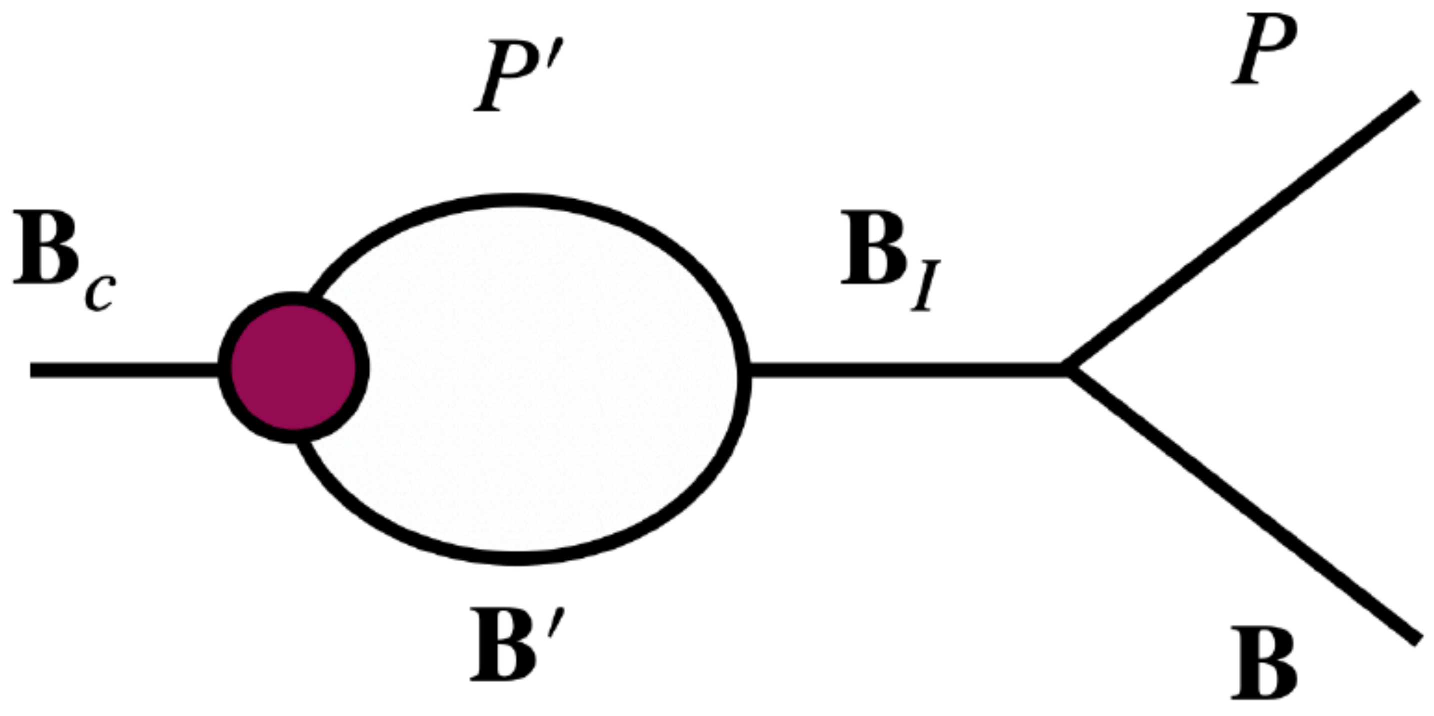
It is very important that $\mathbf{15}$, $\overline{\mathbf{6}}$ and $\mathbf{3}$ share two parameters $\tilde{F}^\pm_V$!

• Rescattering — $\mathscr{L}^{\mathrm{FSR-s}}_{\mathbf{B}_c\mathbf{B}P}$

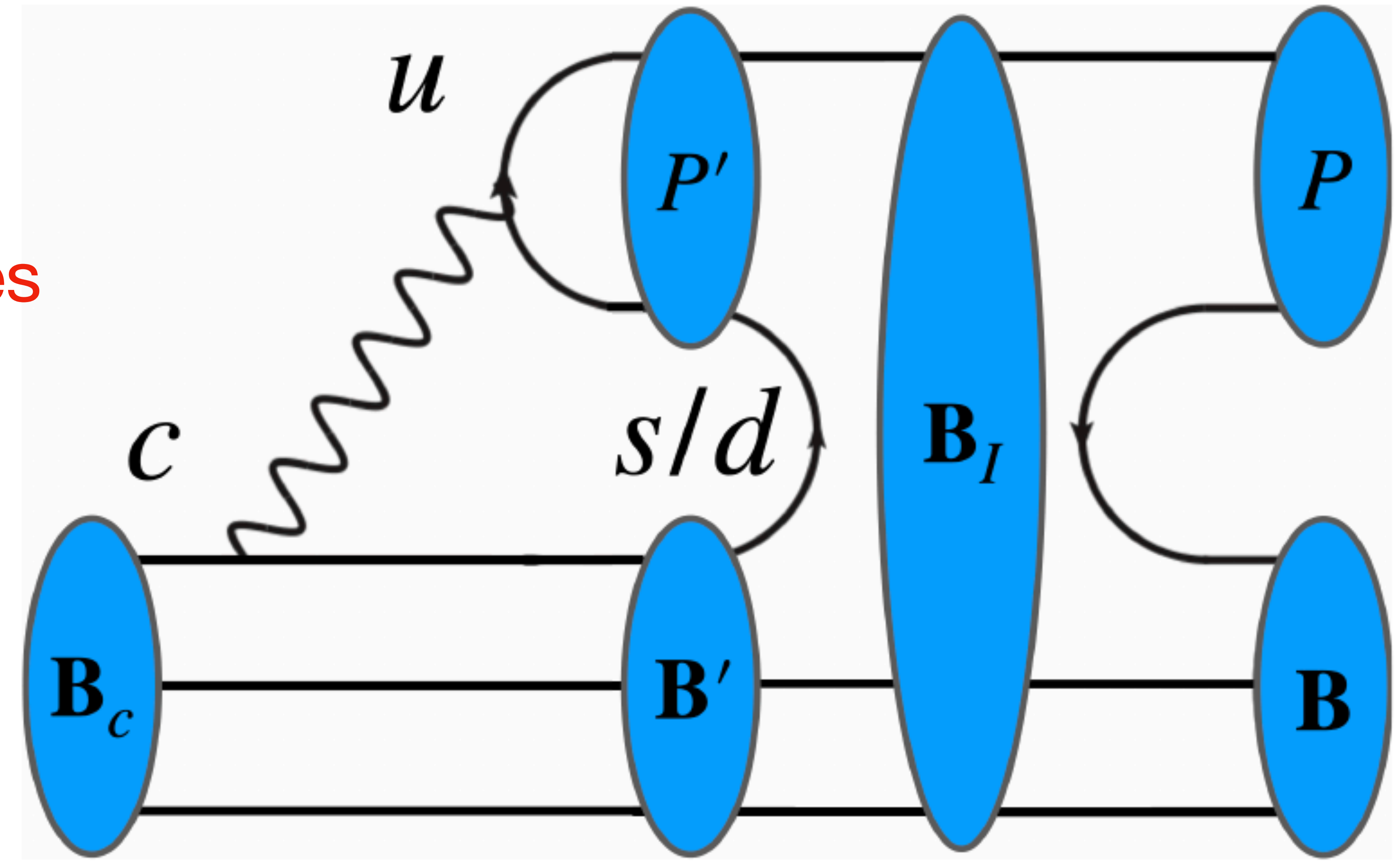
$$\left\langle \mathscr{L}^{\mathrm{FSR-s}}_{\mathbf{B}_c\mathbf{B}P} \right\rangle = -\left\langle \left(\mathscr{L}^{\mathrm{strong}}_{\mathbf{B}_I\mathbf{B}P}\right)\left(\mathscr{L}^{\mathrm{strong}}_{\mathbf{B}_I\mathbf{B}'P'}\right)\left(\mathscr{L}^{\mathrm{Tree}}_{\mathbf{B}_c\mathbf{B}'P'}\right) \right\rangle_{s-\mathrm{contract}}$$

$$\propto \sum_{\mathbf{B}',P',\mathbf{B}_I} \left\langle \left((P^\dagger)^{i_1}_{j_1}(\overline{\mathbf{B}})^{j_1}_{k_1}(\mathbf{B}_I)^{k_1}_{i_1}\right)\left((P')^{i_2}_{j_2}(\overline{\mathbf{B}}_I)^{j_2}_{k_2}(\mathbf{B}')^{k_2}_{i_2}\right)\left((P'^\dagger)^{k}_{i}(\overline{\mathbf{B}}')^{l}_{j}(\mathscr{H}_-)^{ij}_{k}(\mathbf{B}_c)_l\right) \right\rangle \cdots$$

We substitute $\sum_{\mathbf{B}_I} (\overline{\mathbf{B}}_I)^{k_1}_{i_1}(\mathbf{B}_I)^{j_2}_{k_2}$ with $\sum_{\lambda_a} (\lambda_a)^{k_1}_{i_1}(\lambda_a)^{j_2}_{k_2} = \delta^{j_2}_{i_1}\delta^{k_1}_{k_2} - \frac{1}{3}\delta^{k_1}_{i_1}\delta^{j_2}_{k_2}$



At quark level generates penguin topology



- **Rescattering — Net results**

Amplitudes $\sim \frac{\lambda_s - \lambda_d}{2}\tilde{f} + \lambda_b \tilde{f}_3$

$$\tilde{f}^b = \tilde{F}_V^- + \tilde{S}^- - \sum_{\lambda=\pm}(2r_\lambda^2 - r_\lambda)\tilde{T}_\lambda^- ,$$

$$\tilde{f}^c = r_-\tilde{S}^- - \sum_{\lambda=\pm}(r_\lambda^2 - 2r_\lambda + 3)\tilde{T}_\lambda^- ,$$

$$\tilde{f}^d = \tilde{F}_V^- - \sum_{\lambda=\pm}(2r_\lambda^2 - 2r_\lambda - 4)\tilde{T}_\lambda^- , \quad \tilde{f}^e = \tilde{F}_V^+ ,$$

$$\tilde{f}_3^b = \frac{7-2r_-}{8r_- + 2}\tilde{S}^- - \sum_{\lambda=\pm}(r_\lambda^2 - 5r_\lambda/2 + 1)\tilde{T}_\lambda^- ,$$

$$\tilde{f}_3^c = \frac{(r_- + 1)(2r_- - 7)}{24r_- + 6}\tilde{S}^- + \sum_{\lambda=\pm}\frac{1}{6}(r_\lambda^2 + 11r_\lambda + 1)\tilde{T}_\lambda^- ,$$

$$\tilde{f}_3^d = \frac{r_-(7-2r_-)}{8r_- + 2}\tilde{S}^- - \sum_{\lambda=\pm}\frac{1}{2}(r_\lambda + 1)^2\tilde{T}_\lambda^- - \frac{1}{4}\left(\tilde{F}_V^+ + 2\tilde{F}_V^-\right) ,$$

1. $\tilde{f}$ and $\tilde{f}_3$ share the same unknown $\tilde{F}_V^\pm, \tilde{S}^-, \tilde{T}^-$.

Much more complicated compared to $P^{LD} = E$ in *D* mesons !

PRD **100**, 093002 (2019) also H. Y. Cheng talk

2. $\tilde{f}$ can be determined by CP-even data.

- ## Rescattering — Net results

$$\text{Amplitudes} \sim \frac{\lambda_s - \lambda_d}{2}\tilde{f} + \lambda_b \tilde{f}_3$$

$$\tilde{f}^b = \tilde{F}_V^- + \tilde{S}^- - \sum_{\lambda=\pm}(2r_\lambda^2 - r_\lambda)\tilde{T}_\lambda^- \,,$$

$$\tilde{f}^c = r_-\tilde{S}^- - \sum_{\lambda=\pm}(r_\lambda^2 - 2r_\lambda + 3)\tilde{T}_\lambda^- \,,$$

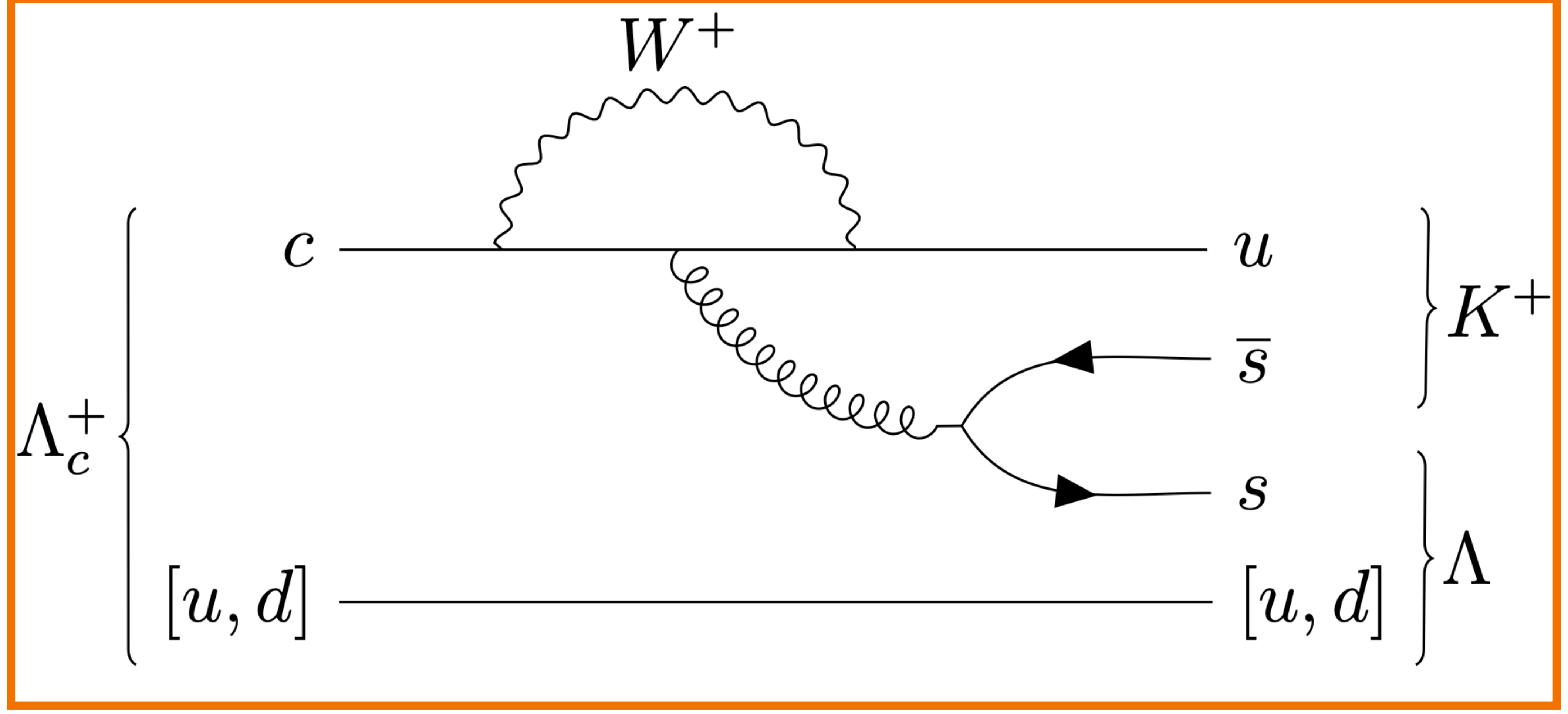
$$\tilde{f}^d = \tilde{F}_V^- - \sum_{\lambda=\pm}(2r_\lambda^2 - 2r_\lambda - 4)\tilde{T}_\lambda^- \,, \quad \tilde{f}^e = \tilde{F}_V^+ \,,$$

$$\tilde{f}_3^b = \frac{7 - 2r_-}{8r_- + 2}\tilde{S}^- - \sum_{\lambda=\pm}(r_\lambda^2 - 5r_\lambda/2 + 1)\tilde{T}_\lambda^- \,,$$

$$\tilde{f}_3^c = \frac{(r_- + 1)(2r_- - 7)}{24r_- + 6}\tilde{S}^- + \sum_{\lambda=\pm}\frac{1}{6}(r_\lambda^2 + 11r_\lambda + 1)\tilde{T}_\lambda^- \,,$$

$$\tilde{f}_3^d = \frac{r_-(7 - 2r_-)}{8r_- + 2}\tilde{S}^- - \sum_{\lambda=\pm}\frac{1}{2}(r_\lambda + 1)^2\tilde{T}_\lambda^- - \frac{1}{4}\left(\tilde{F}_V^+ + 2\tilde{F}_V^-\right)$$

Corrections to A_{CP} are around 10%



$$\left(1 + \frac{(3C_4 + C_3)\,m_c - \frac{2m_K^2}{m_s + m_u}(3C_6 + C_5)}{(C_+ + C_-)m_c}\right)$$

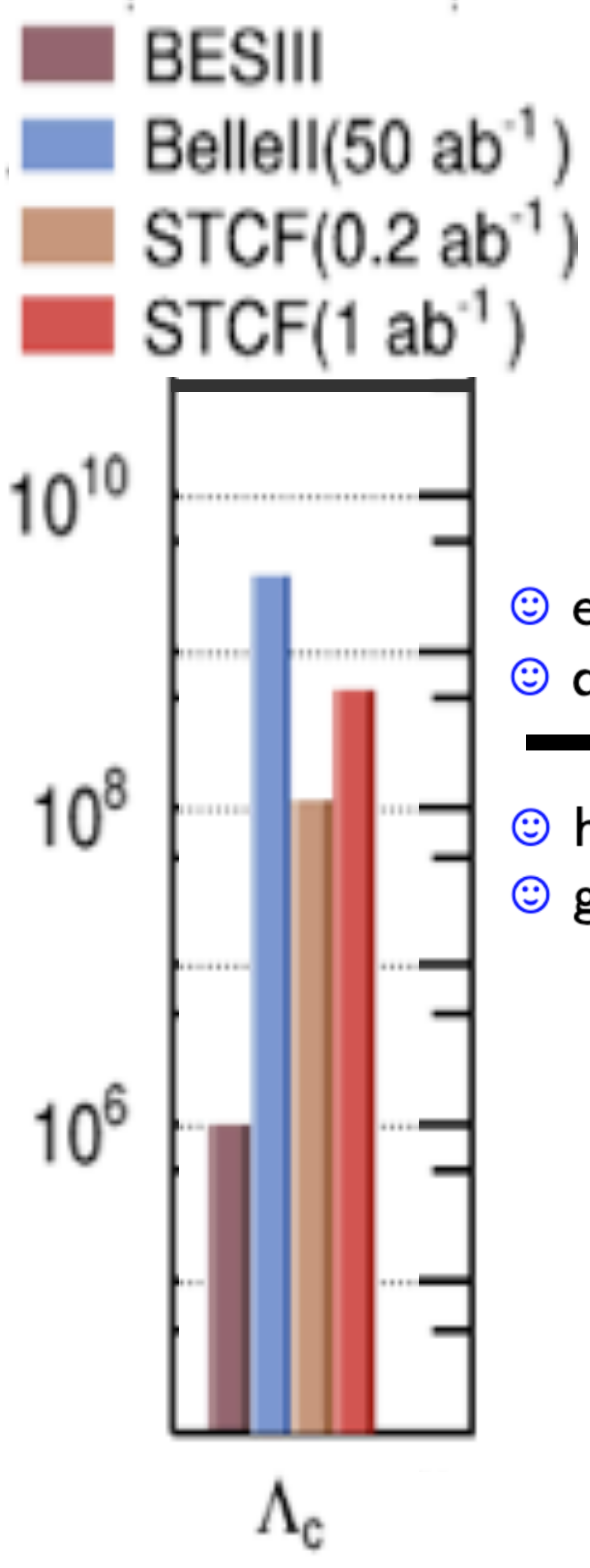
Rescattering — Net results

$$A_{CP}(\Xi_c^0 \to pK^-) - A_{CP}(\Xi_c^0 \to \Sigma^+\pi^-) = (1.87 \pm 0.57) \cdot 10^{-3}$$

Channels	$\mathcal{B}$	A_{CP}^{α}	A_{CP}	Channels	$\mathcal{B}$	A_{CP}^{α}	A_{CP}
$\Lambda_c^+ \to p\pi^0$	0.16(2)	−0.61(39) −3.39(1.05)	0.42(1.15) 1.08(1.55)	$\Xi_c^0 \to \Sigma^+\pi^-$	0.21(2)	0 2.27(22)	0 −0.86(25)
$\Lambda_c^+ \to n\pi^+$	0.67(8)	0.12(20) −1.27(1.07)	−0.15(42) 1.41(68)	$\Xi_c^0 \to \Sigma^0\pi^0$	0.34(3)	−0.04(12) 0.35(9)	−0.12(43) −0.11(11)
$\Lambda_c^+ \to \Lambda^0 K^+$	0.63(2)	−0.03(10) −0.46(14)	0.19(18) −0.13(25)	$\Xi_c^0 \to \Sigma^-\pi^+$	1.83(6)	0.02(7) −0.03(4)	0.13(21) 0.05(7)
$\Xi_c^+ \to \Sigma^+\pi^0$	2.12(14)	0.06(13) 0.21(10)	−0.32(36) 0.02(18)	$\Xi_c^0 \to \Xi^0 K_{S/L}$	0.43(2)	0 0.28(2)	0 −0.12(3)
$\Xi_c^+ \to \Sigma^0\pi^+$	3.04(10)	−0.01(6) 0.15(7)	0.17(19) 0.08(13)	$\Xi_c^0 \to \Xi^- K^+$	1.12(3)	0.02(5) 0.03(4)	0.10(14) −0.06(6)
$\Xi_c^+ \to \Xi^0 K^+$	1.04(14)	0.02(14) 1.22(78)	−0.19(28) −1.39(46)	$\Xi_c^0 \to pK^-$	0.20(2)	0 −2.67(35)	0 1.01(32)
$\Xi_c^+ \to \Lambda^0\pi^+$	0.32(9)	0.0(19) 1.40(34)	−0.15(49) −0.02(43)	$\Xi_c^0 \to nK_{S/L}$	0.71(6)	0 −0.27(2)	0 0.11(3)
$\Xi_c^0 \to \Lambda^0\pi^0$	0.09(1)	0.07(20) 0.89(22)	0.12(38) −0.39(23)				

• Wish list on future experiments

*Rough estimate from statistics only



A_{CP} at $\mathcal{O}(10^{-3})$

☺ extremely clean environment
☺ quantum coherence

☺ high-efficiency detection of neutrals
☺ good trigger efficiency

Belle : A_{CP} at $\mathcal{O}(10^{-2})$

↓ Upgrade

Belle II : A_{CP} at $\mathcal{O}(10^{-3})$

A_{CP} at $\mathcal{O}(10^{-3})$

↓ Upgrade

A_{CP} at $\mathcal{O}(10^{-4})$

☺ very large production cross-section
☺ large boost, excellent time resolution

☺ extremely clean environment
☺ quantum coherence

Measurements on β and γ
extract important information
of strong phases !

Diagram from 周小蓉

Special thanks to 李龙科

• Some puzzles in 3-body decays…

There is not enough data to perform a comprehensive study. However…

$$A\left(\Xi_c^+ \to p\pi^+K^-\right) = A\left(\Lambda_c^+ \to \Sigma^+\pi^-K^+\right)$$

$$\Gamma \propto \int \left(A^2 + \kappa^2\left(m_{23}^2\right)B^2\right) dm_{12}^2 dm_{23}^2 \propto \Delta E^4$$

$$\Gamma\left(\Xi_c^+ \to p\pi^+K^-\right) \approx 10\,\Gamma\left(\Lambda_c^+ \to \Sigma^+\pi^-K^+\right)$$

*Provided that A, B are structureless

$(16 \pm 6) \times 10^{-15}$ GeV

PRD **102**, 071101 (2020) **LHCb**

$(6.2 \pm 3.3) \times 10^{-15}$ GeV

PRD **100**, 031101 (2019) **Belle**

$(6.6 \pm 1.3) \times 10^{-15}$ GeV

JHEP **09**, 125 (2023) **BESIII**

Ours $(42.2 \pm 3.9) \times 10^{-15}$ GeV $(5.3 \pm 1.0) \times 10^{-15}$ GeV

A simple solution would be replacing A^2, B^2 with δ functions, i.e. resonance.

arXiv: 2403.06469 accepted by PRD; credit to 柳盛麟

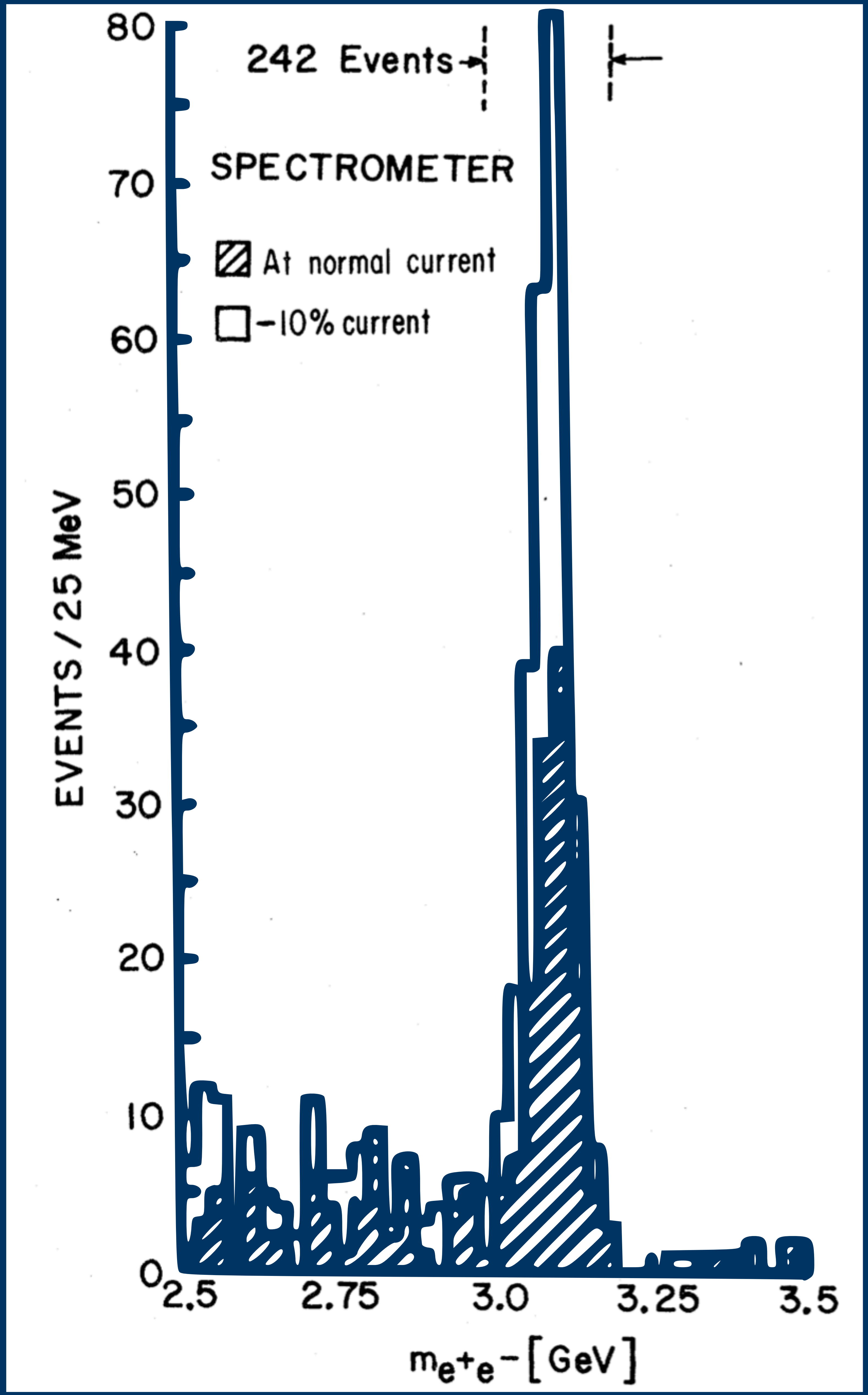
What have been done

$$\underbrace{\text{Tree}}_{\text{SU(3) flavor symmetry}}[\,\lambda_{d,s} + \lambda_b \underbrace{(\text{Penguin / Tree})}_{\text{Rescattering}}\,]$$

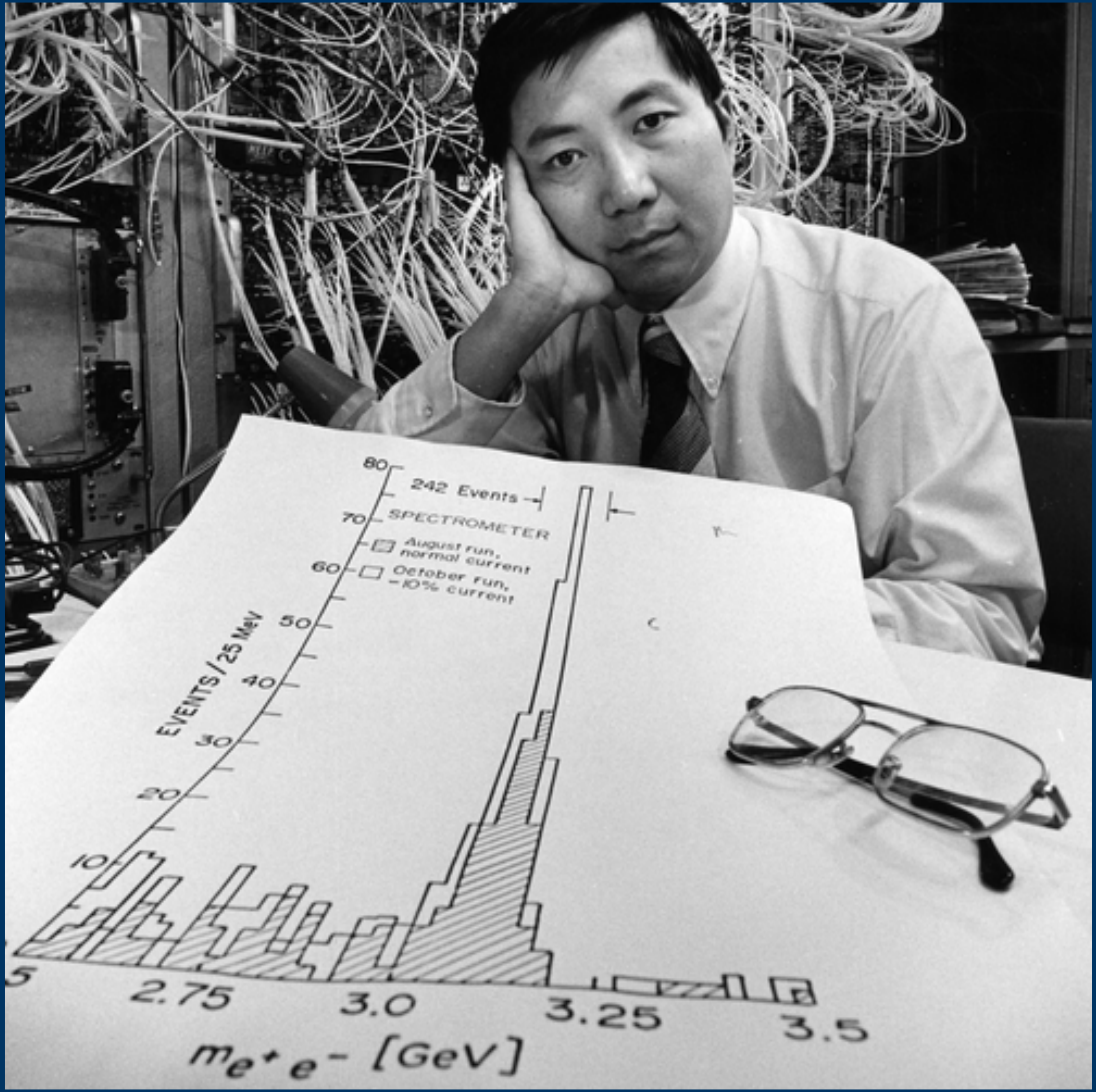
What we need

Measurements of β and γ in near future

Measurements of A_{CP} in STCF, Belle II, LHCb



Backup slides



Backup slide

PHYSICAL REVIEW D **81,** 074021 (2010)

Two-body hadronic charmed meson decays

Hai-Yang Cheng[1,2] and Cheng-Wei Chiang[1,3]

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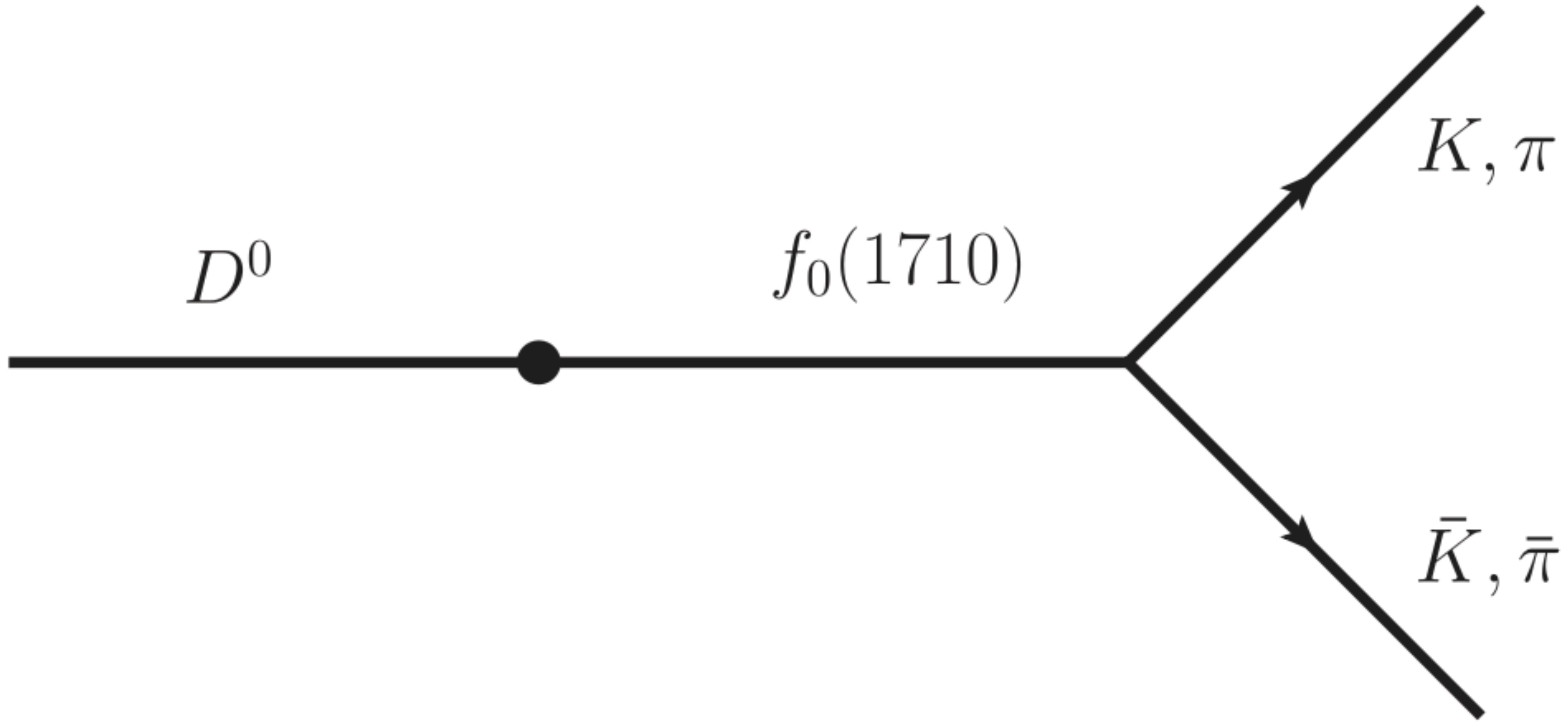

Enhancement of charm CP violation due to nearby resonances

Stefan Schacht[a,*], Amarjit Soni[b]

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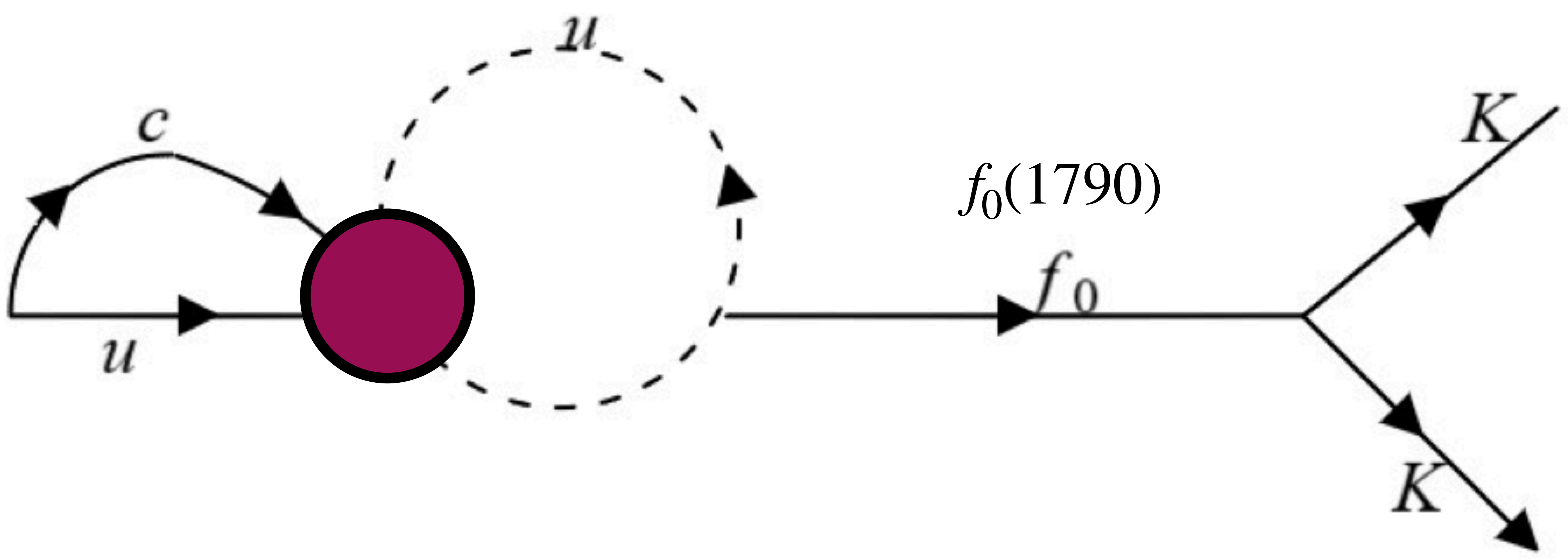

1. f_0 might be a glueball which mainly decays to KK.
2. Its mass is too close to D meson, enhancing SU(3) breaking effects from mass spectrum.
3. There do not have known intermediate baryons play the same rule (?)

- **Backup slide**

$$g^{-}_{n\Sigma^- K^-} : g^{-}_{pN\pi^-} : g^{-}_{p\Lambda K^-} : g^{-}_{\Sigma^-\Lambda\pi^+} : g^{-}_{\Sigma^-\Sigma^0\pi^+} : g^{-}_{\Lambda\Sigma^0\pi^0}$$
$$= 1 : g_s^- : \frac{1}{\sqrt{6}}\left(1 - 2g_s^-\right) : \frac{1}{\sqrt{6}}\left(1 + g_s^-\right) : \frac{1}{\sqrt{2}}(g_s^- - 1) : \frac{1}{\sqrt{6}}(1 + g_s^-).$$

$$\Gamma^{N\pi}_{N(1535)} : \Gamma^{\Lambda\pi}_{\Sigma(1620)} : \Gamma^{\Sigma\pi}_{\Sigma(1620)} : \Gamma^{N\overline{K}}_{\Sigma(1620)} : \Gamma^{N\overline{K}}_{\Lambda(1670)} : \Gamma^{\Sigma\pi}_{\Lambda(1670)}$$
$$= 44.1 \pm 14.8 : 3.51 \pm 1.53 : 6.63 \pm 2.70 : 13.7 \pm 10.5 : 8.0 \pm 1.9 : 12.8 \pm 5.1\,,$$