Probing Dark Matter with Gravitational-Wave Interferometers in Space

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中国科学院大学

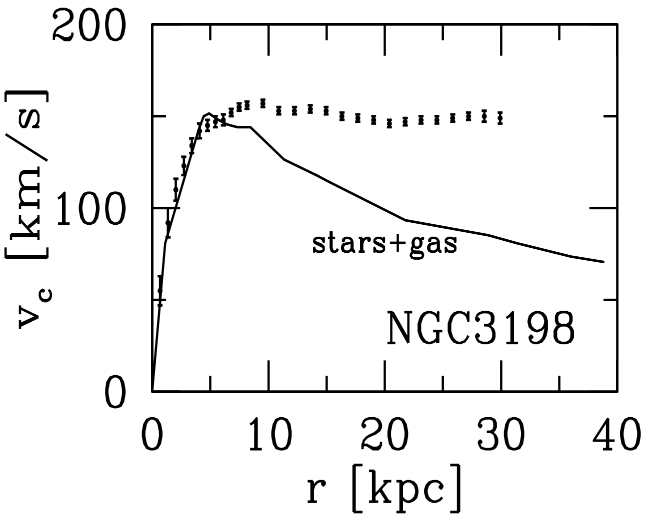
第十七届粒子物理、核物理和宇宙学交叉学科前沿问题研讨会

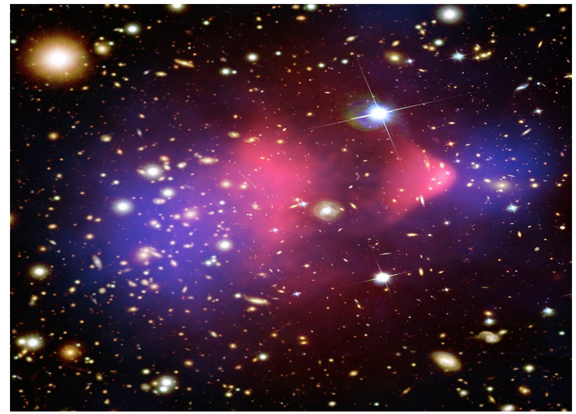
2024.7.12-16，贵阳

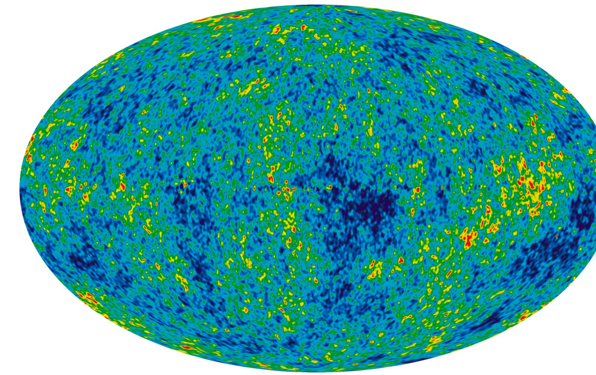
w/ Li, and Wu, arXiv:2112.14041; Yu, Yao, Wu, arXiv:2307.09197;
Zhang, arXiv:2403.13882; Yao, arXiv:2404.10494；Yu, Cao, Wu, arXiv:2404.04333

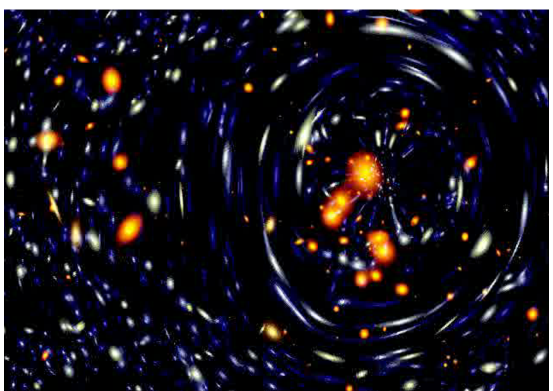
Contents

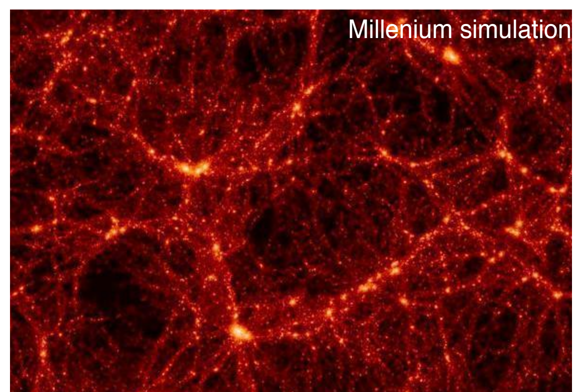
Evidence for Dark Matter







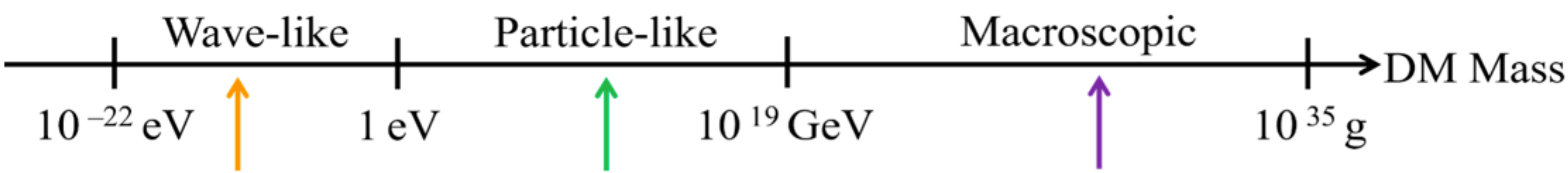




- Rotation curve
- Gravitational lensing
- Bullet cluster
- Large-scale structure
- Anisotropy of CMB

Dark Matter Candidates

- Primordial black holes
- Super heavy particles
- Asymmetric DM
- Hidden sector DM
- ……
- Weakly-interacting (WIMP)
- Strongly-interacting (SIMP)
- Sterile neutrino
- Axion (ALP), Ultralight DM



Axion, ALP, Ultralight DM, etc

WIMP, SIMP, ADM Sterile neutrino, etc

Primordial black holes etc

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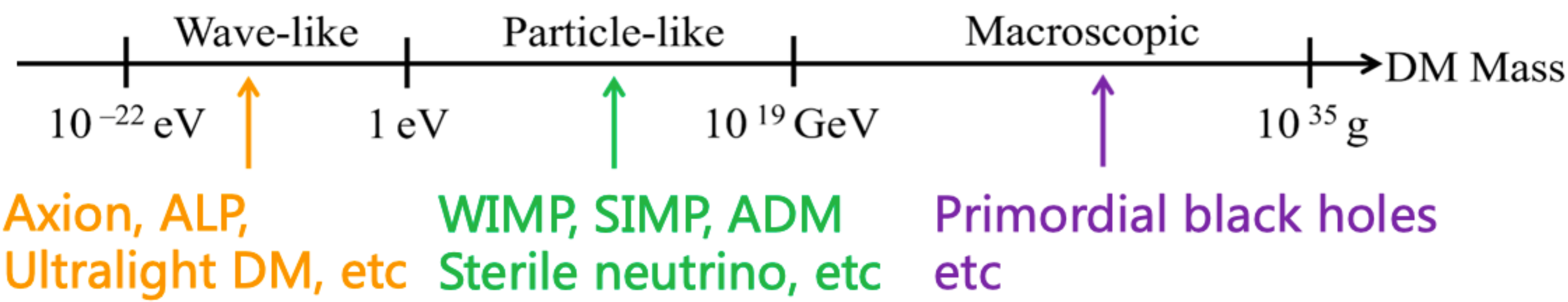
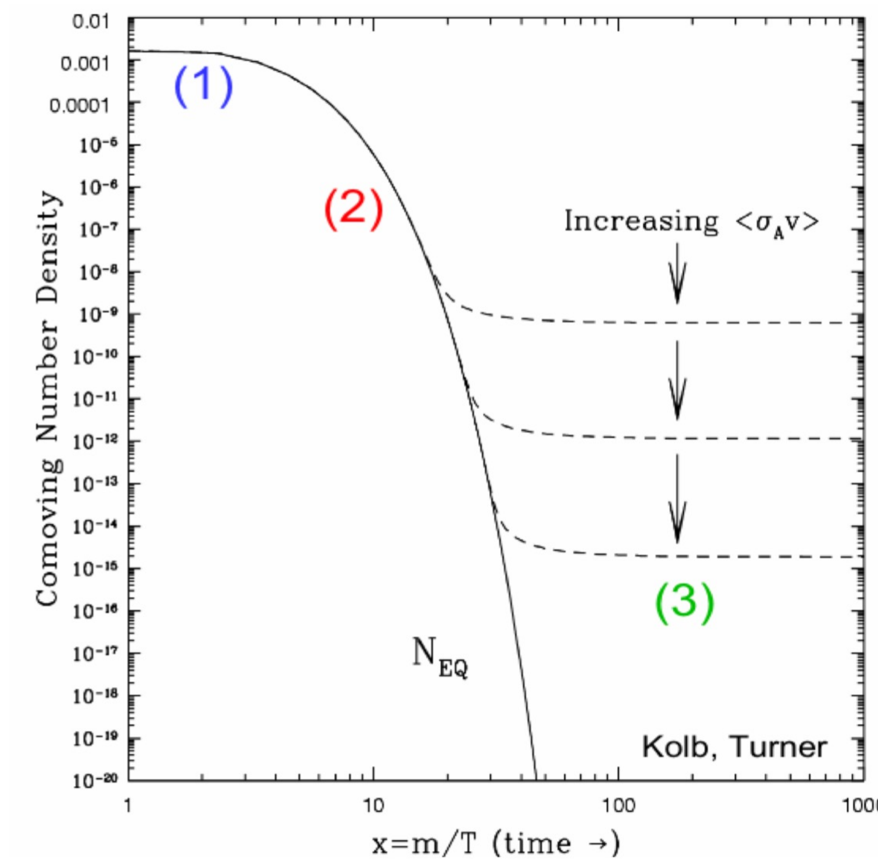


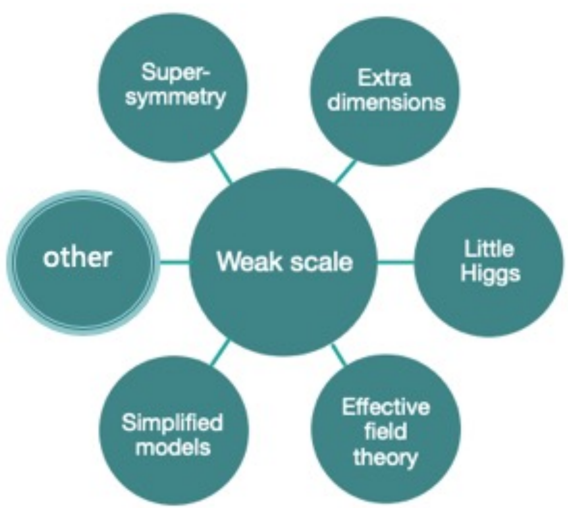
WIMP

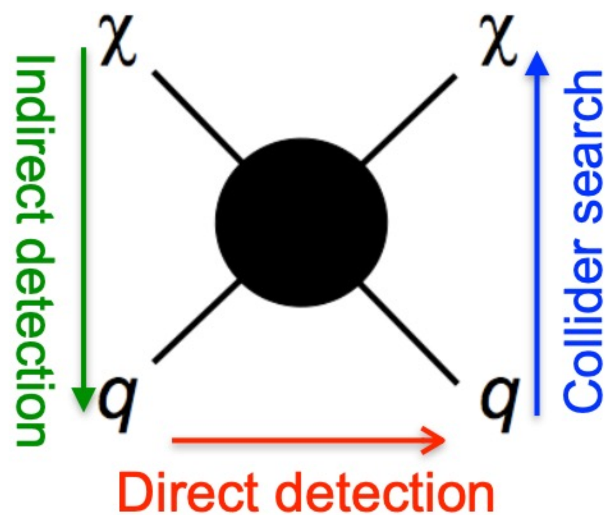
- Mass ~ 100 GeV
- Coupling constant ~ 0.5
- Relic abundance ~ 0.3
- Thermal history
  1. Equilibrium XX <> ff
  2. Equilibrium XX > ff
  3. Freeze out
- Cold dark matter
- The & Exp attractive

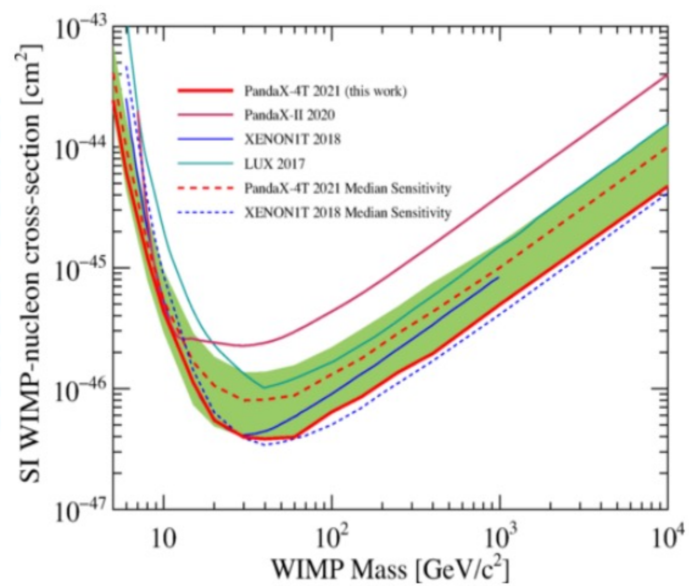


WIMP

- Theories: supersymmetry, extra dimensions, ……
- Direct, indirect and collider searches
- Impressive progress



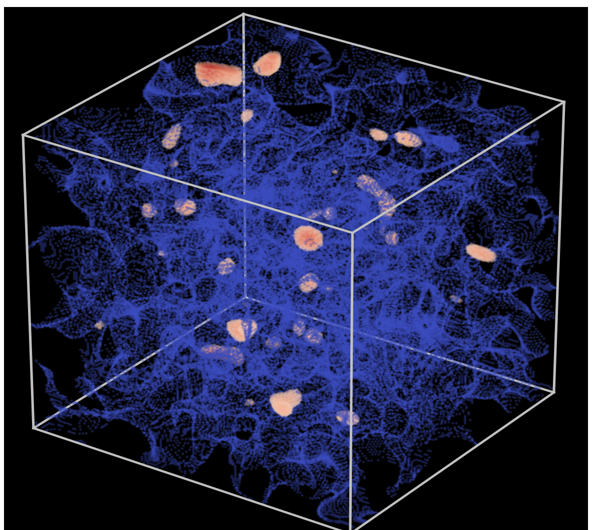


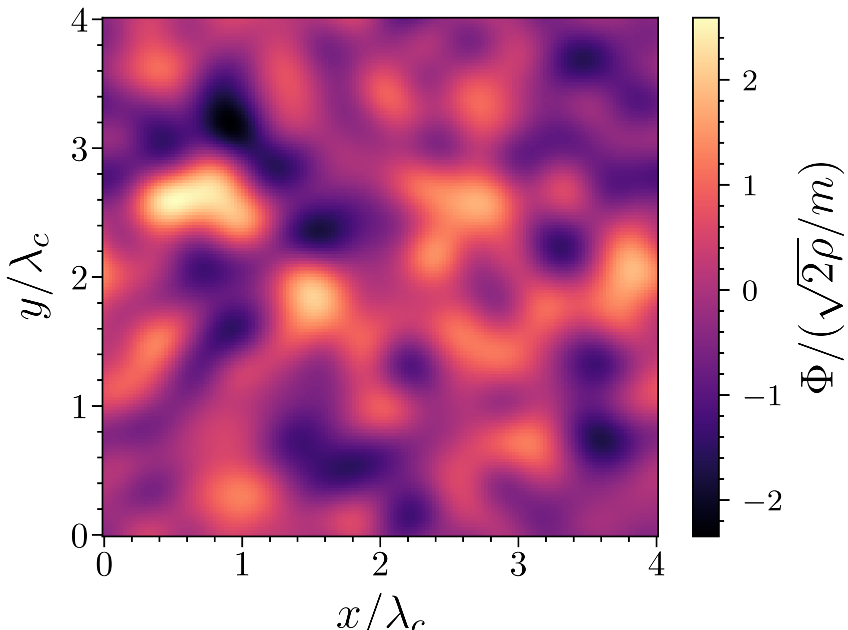


Ultralight Dark Matter

- Mass < 1 eV, QCD axion, axion-like-particles, bosonic
- Number density is very large, behaves as classical wave

$$\Phi(x) = \sum_{\mathbf{v}} \frac{\sqrt{2\rho/N}}{m} e^{i(\omega t - \mathbf{k}\cdot\mathbf{x} + \theta_{\mathbf{v}})},$$





Ultralight Dark Matter

- Scalar ϕ

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m_\phi^2\phi^2 - C\frac{\phi}{M_P}\mathcal{O}_{\rm SM}, \quad \phi\left(t,\vec{x}\right) = \phi_{\vec{k}}e^{i(\omega t - \vec{k}\cdot\vec{x}+\theta_0)},$$

- Interaction depending on the underlying theory, e.g.

$$C\frac{\phi}{M_P}m_\psi\bar{\psi}\psi \Rightarrow m_\psi \to \left(1 + C\frac{\phi}{M_P}\right)m_\psi, \quad S = -\int m(\phi)\sqrt{-\eta_{\mu\nu}dx^\mu dx^\nu}.$$

$$\delta x^i(t,\vec{x}) = \mathcal{M}_s\hat{k}^i e^{im_\phi(t-v\hat{k}\cdot\vec{x})}, \mathcal{M}_s \propto \phi_{\vec{k}}|\vec{k}|/m_\phi^2$$

- Vector A_μ

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_A^2A^\nu A_\nu - \epsilon_D e J_D^\nu A_\nu, \quad \vec{A}(t,\vec{x}) = |\vec{A}|\hat{e}_A e^{i(\omega t - \vec{k}\cdot\vec{x})},$$

$$\delta x^i(t,\vec{x}) = \mathcal{M}_v\hat{e}_A^i e^{im_A(t-v\hat{k}\cdot\vec{x})}, \mathcal{M}_v \propto \epsilon_D e q_{D,j}|\vec{A}|/m_A M_j$$

- DM property

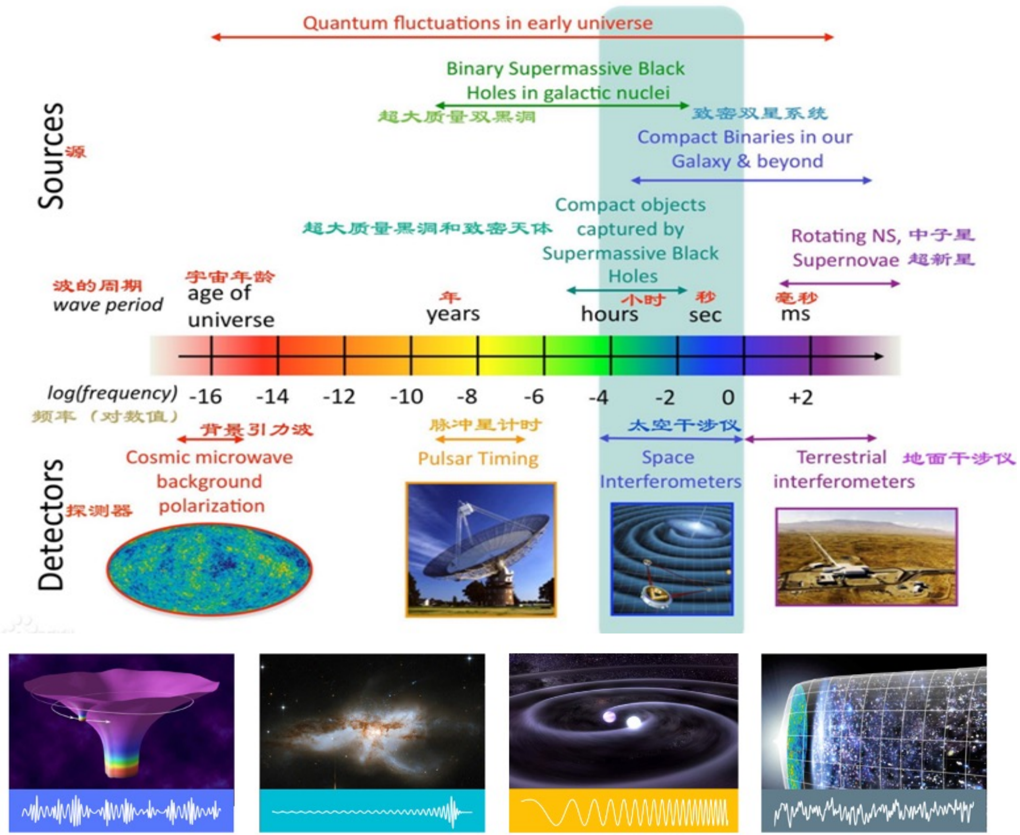
$$\phi_{\vec{k}} = \frac{\sqrt{2\rho_{\rm DM}}}{m_\phi}, \qquad |\vec{A}| = \frac{\sqrt{2\rho_{\rm DM}}}{m_A}, \quad v \sim 10^{-3}, \quad \vec{k} \approx m_\phi\vec{v} \text{ and } \omega \approx m_\phi$$

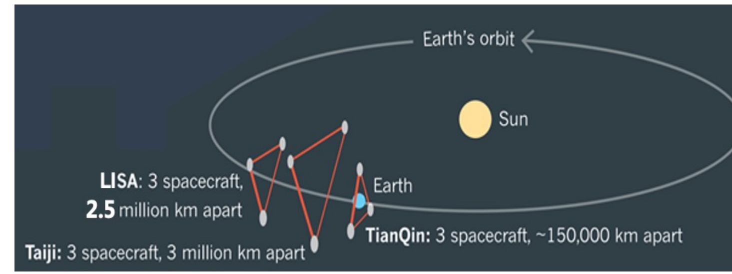
Physical Effects

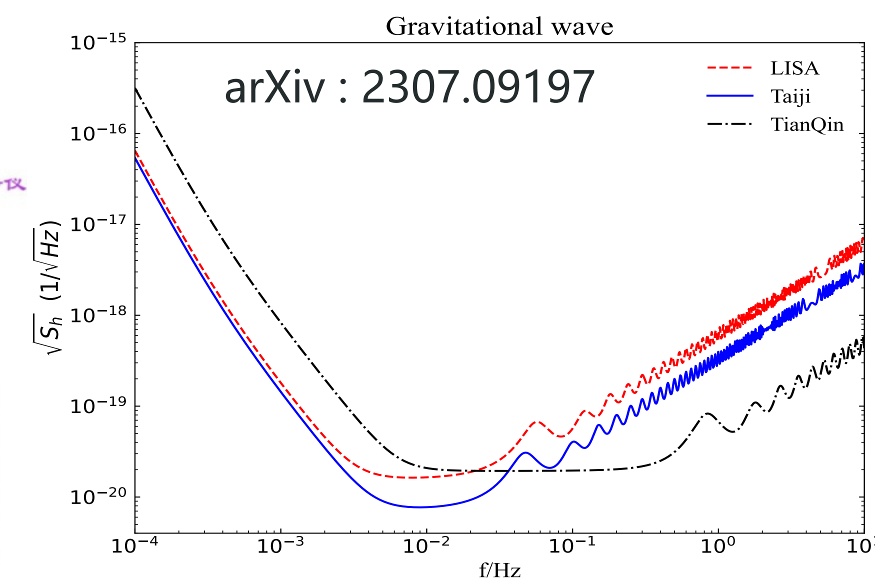
- Atomic physics
  - Arvanitaki, Huang & Tilburg (2014), Graham, Kaplan, Mardon, Rajendran & Terrano (2015), Safronova, Budker, DeMille, Kimball, Derevianko & Clark (2018),
  - Stadnik (2022), ……
- Astrophysical physics
  - Pierce, Riles & Zhao (2018), Morisaki & Suyama (2019), Guo, Riles, Yang & Zhao 2019 , Grote & Stadnik (2019),
  - An, Huang, Liu & Xue (2021), Chen, Shu, Xue, Yuan & Zhao (2019), Xia, Xu & Zhou (2020), Sun, Yang & Zhang (2021), Wu, Chen, & Huang (2023),
  - Liu, Lou & Ren (2021), Luu, Liu, Ren, Broadhurst, Yang, Wang & Xie (2023), ……
- Underground searches
  - Dark Matter Experiments, PandaX, XENONnT, …

Space-based GW Interferometers

- LISA, Taiji, TianQin, DECIGO, BBO, LISAMax, ASTROD-GW, μAres

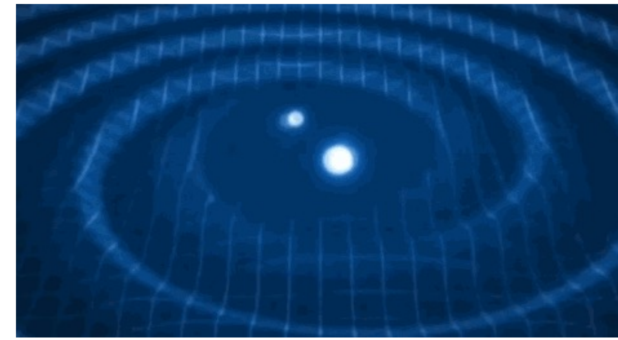




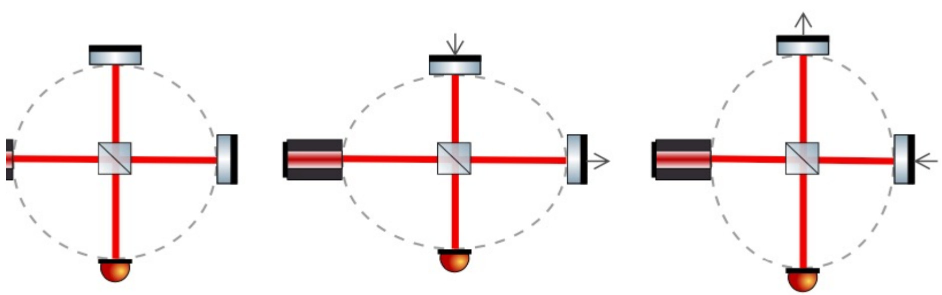


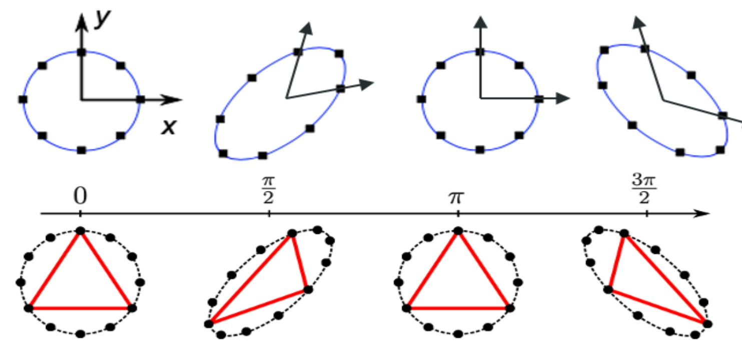
Signal Response

- Gravitational wave can change the structure of spacetime, and the physical distance between objects



- One can measure the phase by laser





- Response $\frac{\delta\nu(t)}{\nu_0} \equiv y_{BA} = -\frac{1}{2}\frac{n_i n_j}{1+\vec{k}\cdot\vec{n}}\left[h_{ij}\left(t-\frac{\vec{k}\cdot\vec{x}_B}{c}\right) - h_{ij}\left(t-\frac{\vec{k}\cdot\vec{x}_A+L}{c}\right)\right]$

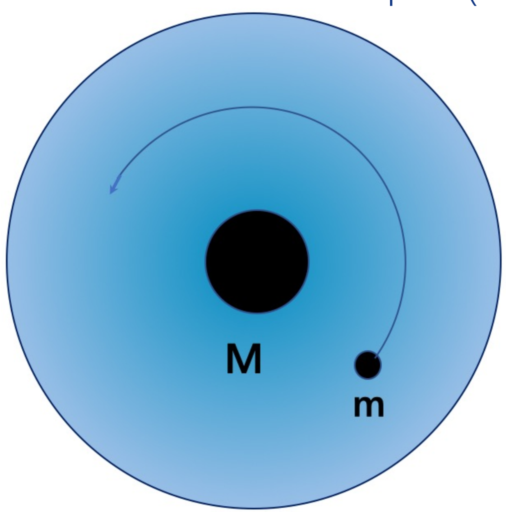
DM Spikes around Black Holes

- WIMP DM particles accretion around BH → DM spike
- NFW profile → spiky density profile **Gondolo & Silk (1999)**

$$\rho(r) \propto r^{-\gamma}, 0 \leq \gamma \leq 2 \Rightarrow \rho_{\rm spike}(r) = \rho_{\rm sp}\left(\frac{r_{\rm sp}}{r}\right)^{\alpha}, \quad \alpha = \frac{9-2\gamma}{4-\gamma}$$

- Dynamical friction → Gravitational wave

Extreme-mass-ratio Inspiral (EMRI)

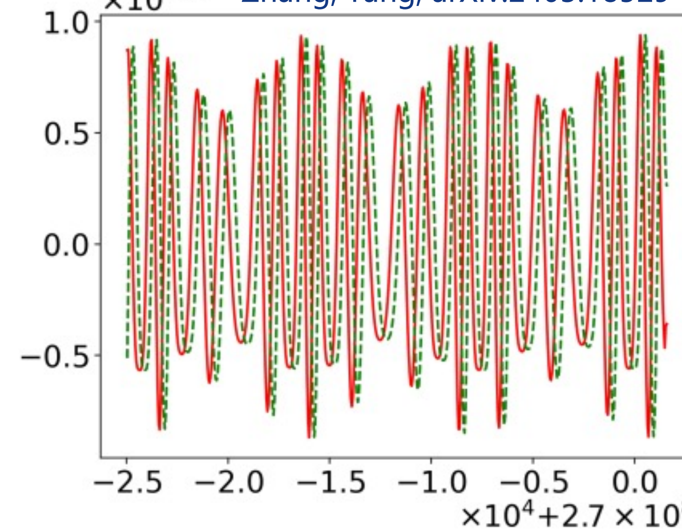


$$m\frac{d^2\mathbf{x}}{dt^2} = \mathbf{F}_{\rm G} + \mathbf{F}_{\rm DF}$$

$$h_{ij} \sim \frac{G}{d}\frac{d^2 Q_{ij}}{dt^2},$$

$$Q_{ij} \sim m\left(x_i x_j - \frac{1}{3}x^2\delta_{ij}\right)$$

Zhang, Tang, arXiv:2403.18529



Eda, Itoh, Kuroyanagi & Silk, Yue & Cao, Yue, Han & Chen , Bertone et al … …

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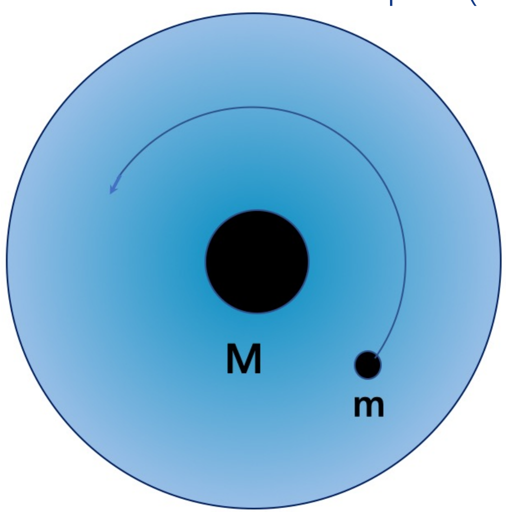

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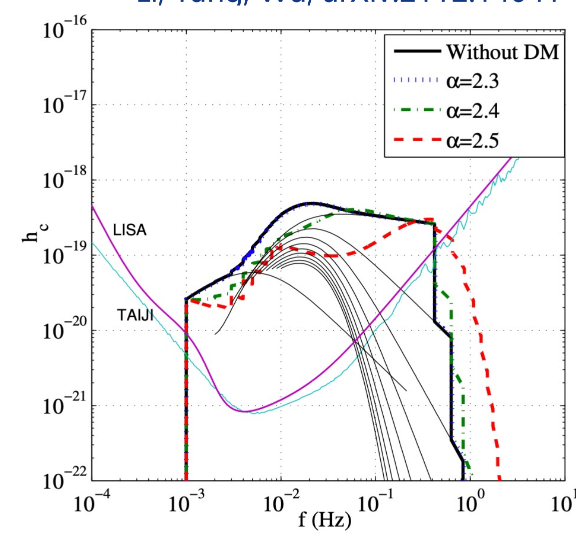
Li, Tang, Wu, arXiv:2112.14041



 Eda, Itoh, Kuroyanagi & Silk, Yue & Cao, Yue, Han & Chen , Bertone et al … …

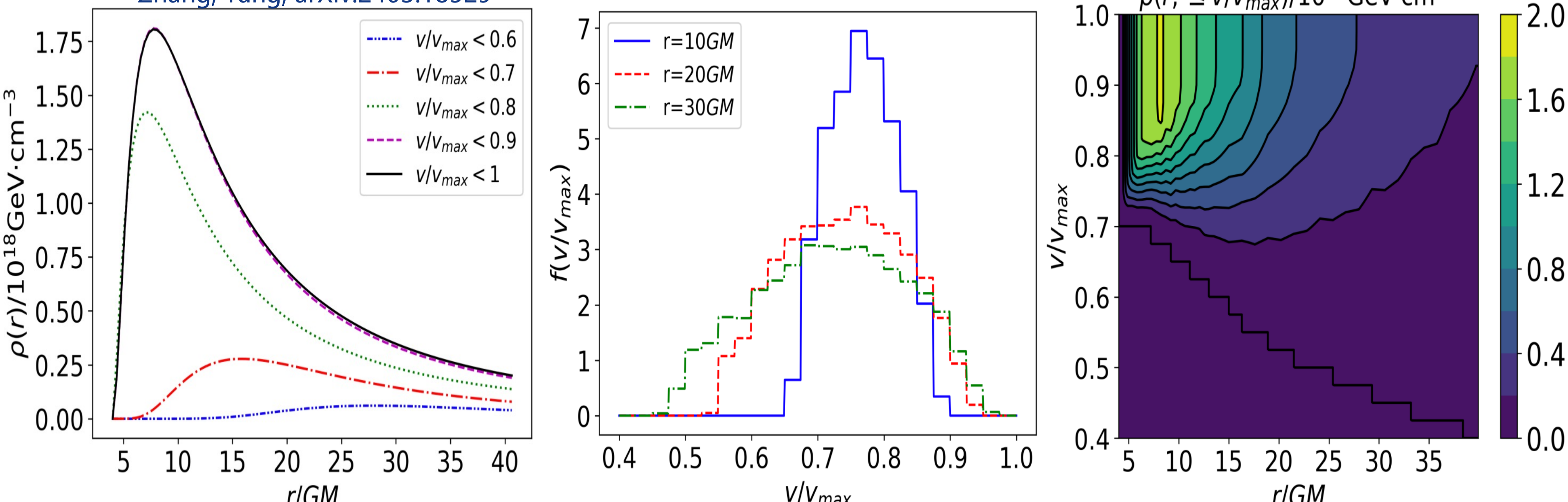
Density Distribution

- Relativistic treatment (Will et al)

$$\rho(r,\alpha=1)=\frac{\kappa}{(r/GM)^{\omega}}\left(1-\frac{4GM}{r}\right)^{\eta}.$$

Halo parameters (M_{halo}, r_s)	Index (γ)	Fitting parameters $(\kappa,\ \eta\ ,\ \omega)$
$(10^{12}M_{\odot}, 20\ \text{kpc})$	-	$(5.33\times10^{20}\ \text{GeV/cm}^3,\ 1.99,\ 2.07)$
$(4.5\times10^{8}M_{\odot}, 1.85\ \text{kpc})$	-	$(6.15\times10^{24}\ \text{GeV/cm}^3,\ 2.03,\ 2.11)$
$(7.3\times10^{8}M_{\odot}, 1.85\ \text{kpc})$	7/4	$(5.83\times10^{26}\ \text{GeV/cm}^3,\ 2.04,\ 2.16)$

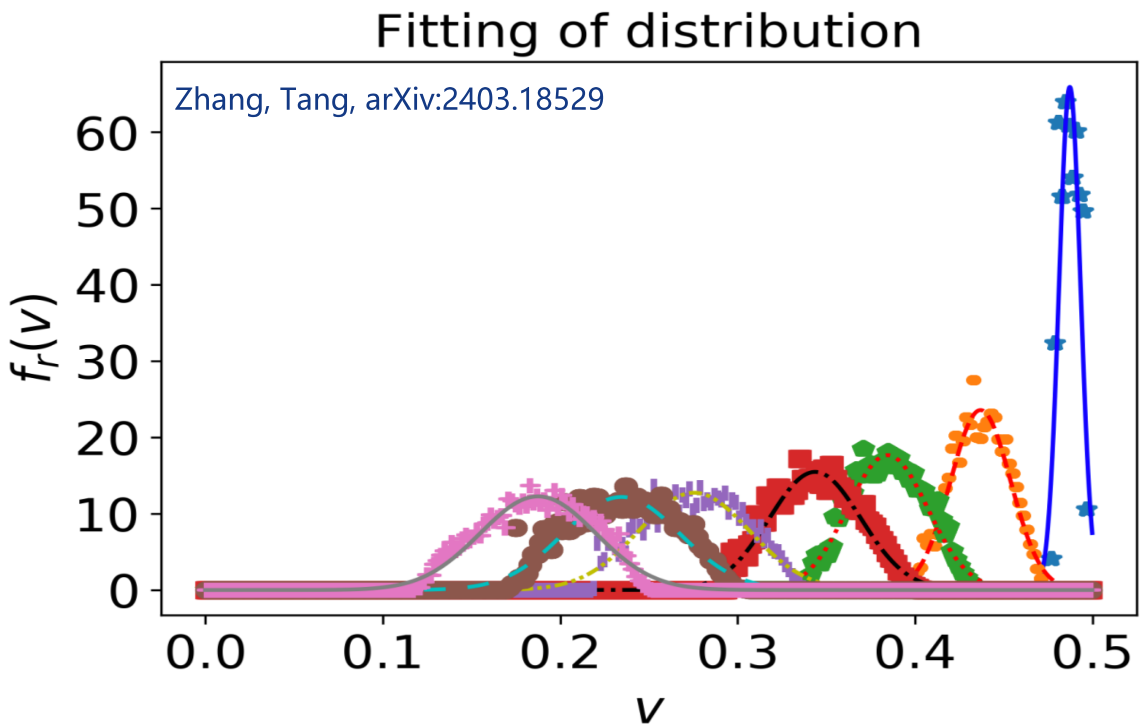
Zhang, Tang, arXiv:2403.18529

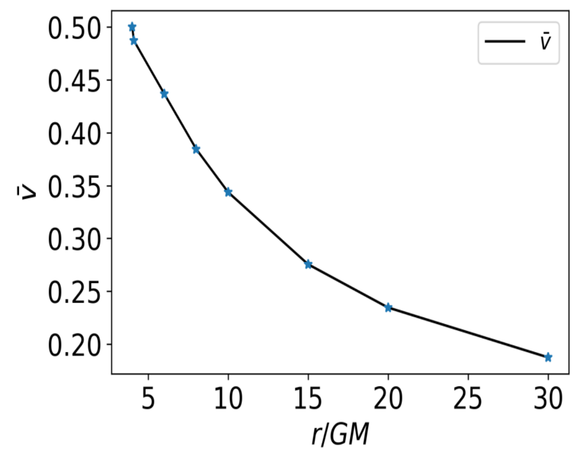


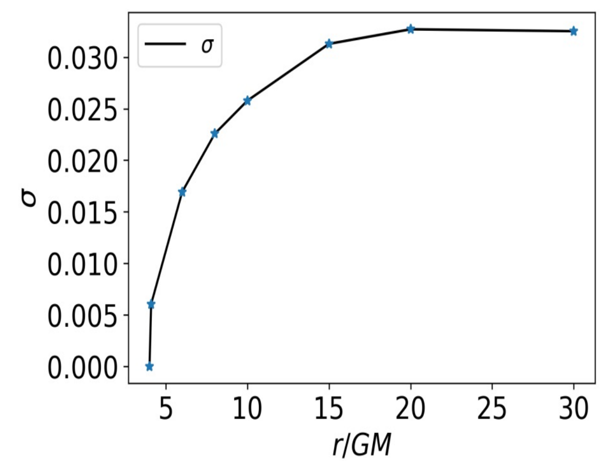
Velocity Distribution

- Fitted with Gaussian distribution

$$f_r(v) = \frac{1}{\sqrt{2\pi}\sigma(r)} \exp\left(-\frac{(v-\bar{v}(r))^2}{2\sigma^2(r)}\right),$$



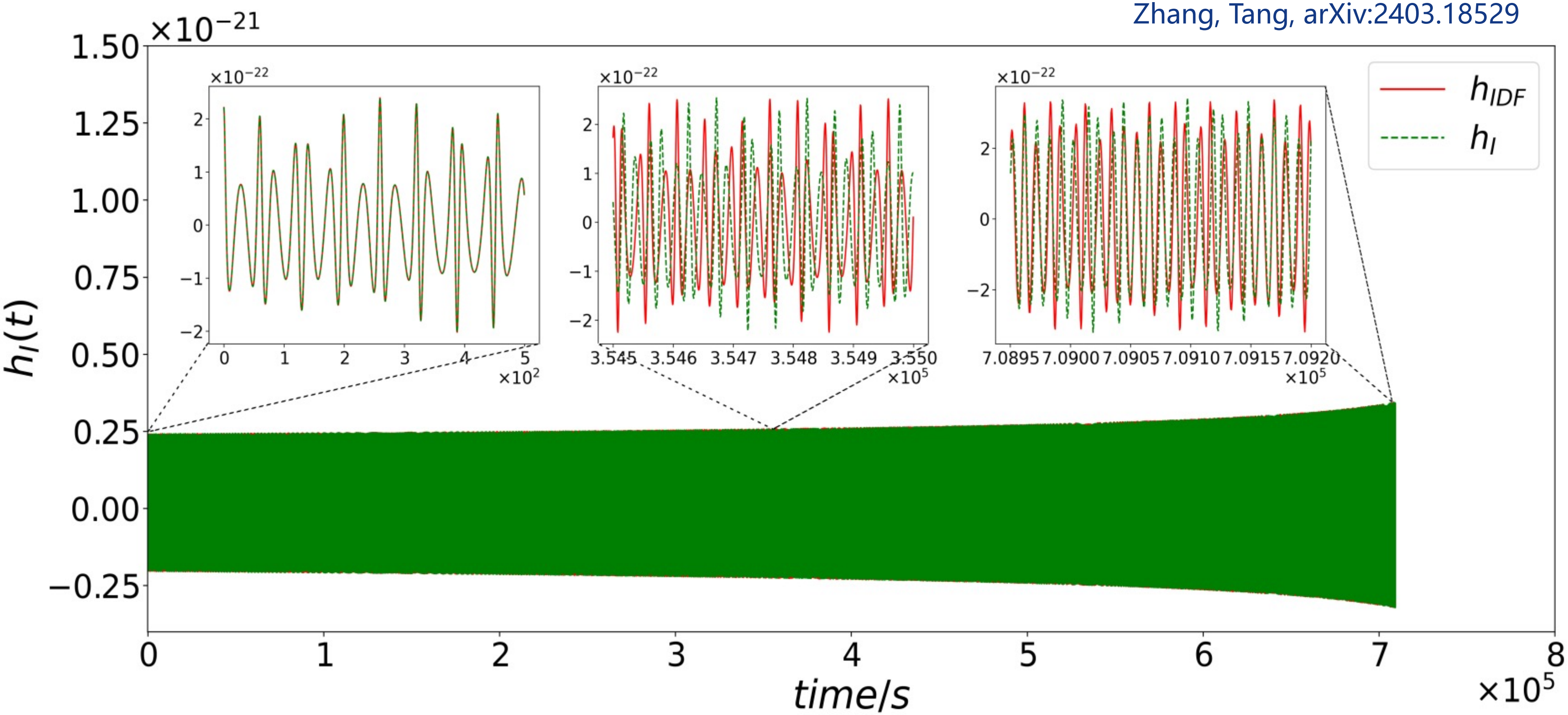




Effects on EMRI

➤ Phase shift of the waveform of GW

Zhang, Tang, arXiv:2403.18529



Ultralight/Wave DM - Signal Response

- DM couples to SM particles, inducing oscillations of test mass, effectively changing the length

- One-way Doppler shift

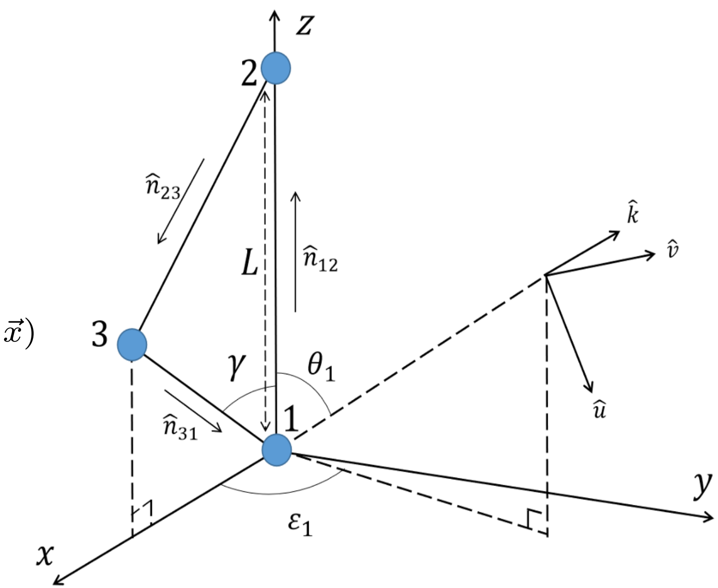
$$\delta t_{rs} = -\hat{n}_{rs} \cdot [\delta \vec{x}(t, \vec{x}_r) - \delta \vec{x}(t - L, \vec{x}_s)] \, ,$$

$$\frac{\delta \nu_{rs}}{\nu_0} = \frac{\nu_{rs} - \nu_0}{\nu_0} = -\frac{d\, \delta t_{rs}}{dt} .$$

- Fractional frequency change

$$y_{rs}(t) \equiv \frac{\delta \nu_{rs}}{\nu_0} = \mu_{rs} \left[h(t, \vec{x_r}) - h(t - L, \vec{x_s}) \right], \; h(t, \vec{x}) \propto e^{im(t - v\hat{k}\cdot\vec{x})}$$

$$\mu_{rs} = \begin{cases} \hat{k} \cdot \hat{n}_{rs} & \text{for scalar field,} \\ \hat{e}_A \cdot \hat{n}_{rs} & \text{for vector field,} \\ \dfrac{\hat{n}^i_{rs} \hat{n}^j_{rs} e_{ij}(\hat{k}, \psi)}{2(1 + \hat{n}_{rs} \cdot \hat{k})} & \text{for gravitational wave,} \end{cases}$$

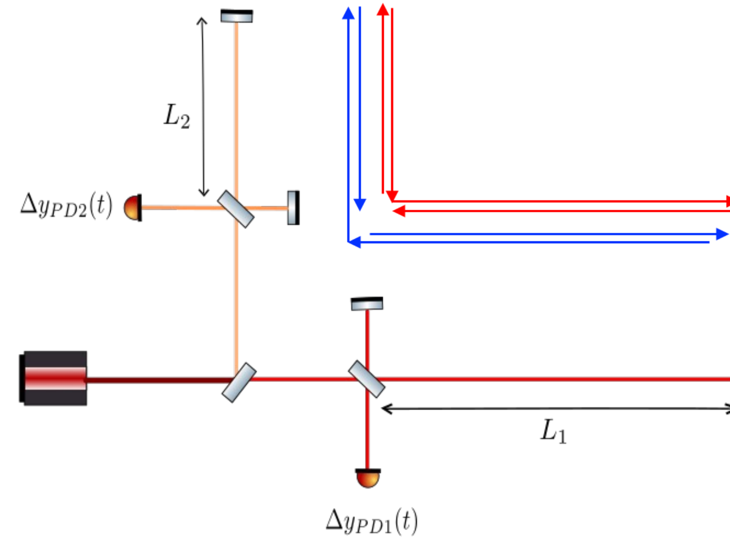


Yu, Yao, Tang, Wu, arXiv: 2307.09197

Time-Delay Interferometry

Tinto & Dhurandhar

- The arm lengths are not equal
- Laser frequency noises dominate



$$
\begin{aligned}
X(t) &\equiv [\Delta y_{PD1}(t) - \Delta y_{PD2}(t)] - [\Delta y_{PD1}(t - T_2) - \Delta y_{PD2}(t - T_1)] \\
&= [H_1(t) - H_2(t) + p(t - T_1) - p(t - T_2)] \\
&- [H_1(t - T_2) - H_2(t - T_1) + p(t - T_1) - p(t - T_2)] \\
&= H_1(t) - H_2(t) - H_1(t - T_2) + H_2(t - T_1),
\end{aligned}
$$

- Michelson interferometry

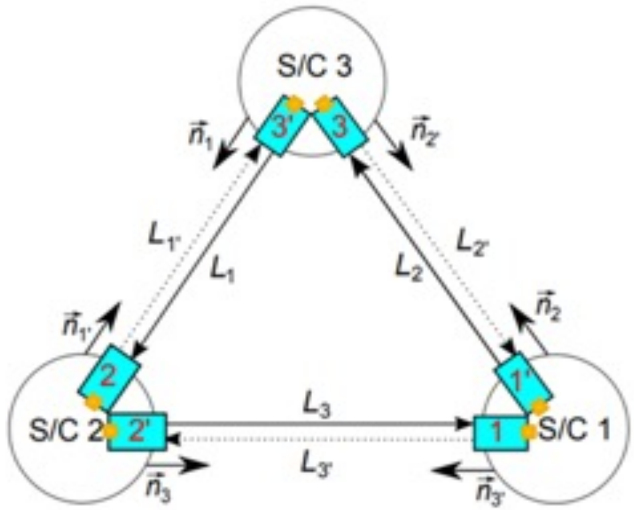
$$
\begin{aligned}
X(t) \equiv &[\Delta y_{PD2}(t - T_1) + \Delta y_{PD1}(t)] \\
&-[\Delta y_{PD1}(t - T_2) + \Delta y_{PD2}(t)]
\end{aligned}
$$

$$
\Delta y_{PD1}(t) = H_1(t) + p(t - T_1) - p(t),
$$
$$
\Delta y_{PD2}(t) = H_2(t) + p(t - T_2) - p(t),
$$

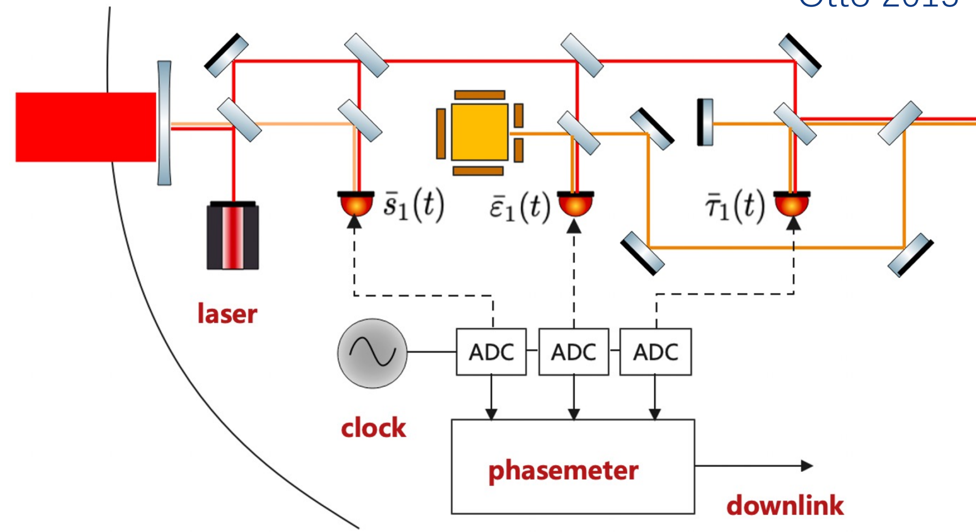
$$
T_1 = 2L_1 \qquad T_2 = 2L_2
$$

- TDI-virtual equal-arm interference

Time-Delay Interferometry



Otto 2015



- Input for TDI

$$\eta_{i'} \equiv s_{i'} + \frac{\varepsilon_{i'} - \tau_{i'}}{2} + D_{i+1'}\frac{\varepsilon_{i-1} - \tau_{i-1}}{2} + \frac{\tau_i - \tau_{i'}}{2}$$

$$\eta_i \equiv s_i + \frac{\varepsilon_i - \tau_i}{2} + D_{i-1}\frac{\varepsilon_{i+1'} - \tau_{i+1'}}{2} - D_{i-1}\frac{\tau_{i+1} - \tau_{i+1'}}{2}$$

$$\eta_{1'} \sim D_{2'}p_3 - p_1,\ \eta_1 \sim D_3 p_2 - p_1,$$
$$\eta_{2'} \sim D_{3'}p_1 - p_2,\ \eta_2 \sim D_1 p_3 - p_2,$$
$$\eta_{3'} \sim D_{1'}p_2 - p_3,\ \eta_3 \sim D_2 p_1 - p_3.$$

- TDI cancels laser frequency noise

Time-Delay Interferometry

- There are multiple combinations
- Michelson channels

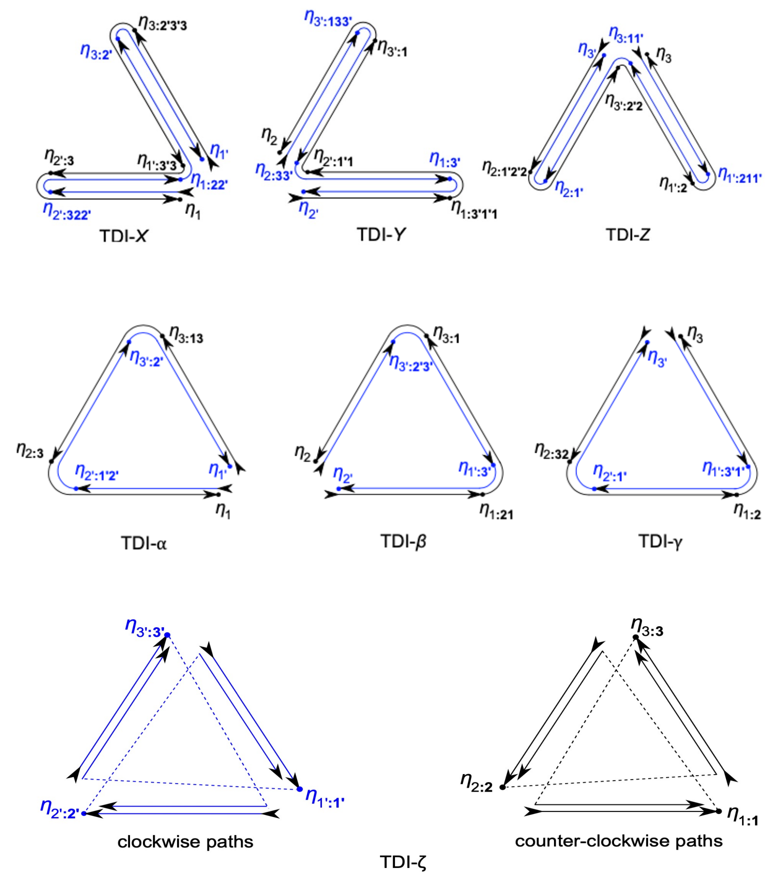
$$
\begin{aligned}
X(t) &= (\eta_{2':322'} + \eta_{1:22'} + \eta_{3:2'} + \eta_{1'}) - (\eta_{3:2'3'3} + \eta_{1':3'3} + \eta_{2':3} + \eta_1),\\
Y(t) &= (\eta_{3':133'} + \eta_{2:33'} + \eta_{1:3'} + \eta_{2'}) - (\eta_{1:3'1'1} + \eta_{2':1'1} + \eta_{3':1} + \eta_2),\\
Z(t) &= (\eta_{1':211'} + \eta_{3:11'} + \eta_{2:1'} + \eta_{3'}) - (\eta_{2:1'2'2} + \eta_{3':2'2} + \eta_{1':2} + \eta_3).
\end{aligned}
$$

- Sagnac channels

$$
\begin{aligned}
\alpha(t) &= (\eta_{2':1'2'} + \eta_{3':2'} + \eta_{1'}) - (\eta_{3:13} + \eta_{2:3} + \eta_1)\,,\\
\beta(t) &= (\eta_{3':2'3'} + \eta_{1':3'} + \eta_{2'}) - (\eta_{1:21} + \eta_{3:1} + \eta_2)\,,\\
\gamma(t) &= (\eta_{1':3'1'} + \eta_{2':1'} + \eta_{3'}) - (\eta_{2:32} + \eta_{1:2} + \eta_3)\,.
\end{aligned}
$$

- ζ channel

$$
\zeta(t) = (\eta_{1':1'} + \eta_{2':2'} + \eta_{3':3'}) - (\eta_{1:1} + \eta_{2:2} + \eta_{3:3})\,.
$$



Credit: Otto 2015

Transfer Functions

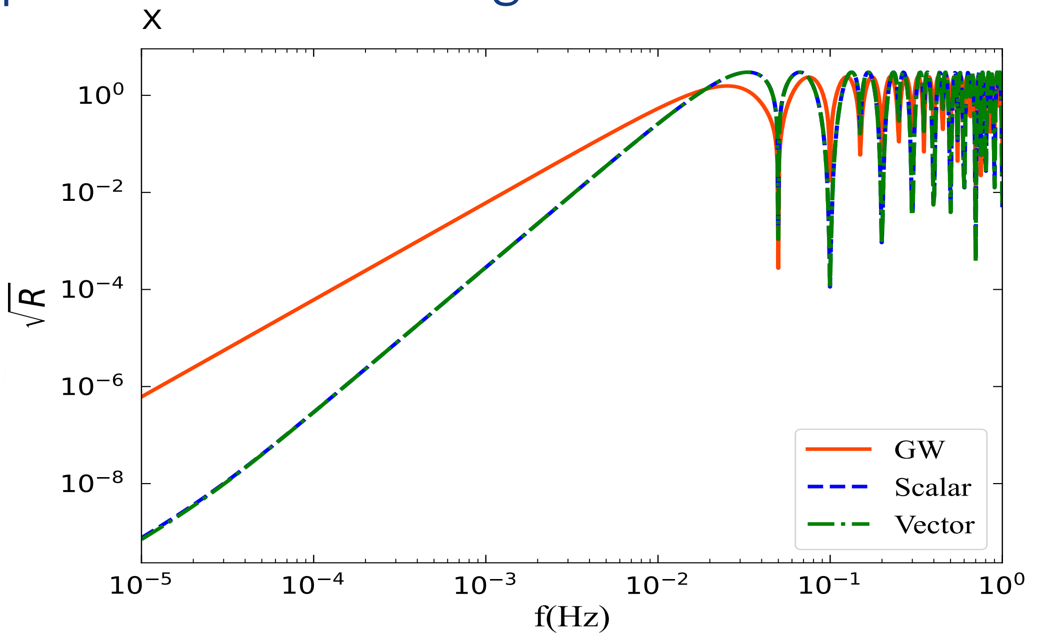
- Fourier transform $$h(t) = \frac{\sqrt{T}}{2\pi}\int_0^\infty \tilde{h}(\omega)e^{i\omega t}d\omega$$
- One-way single link $$y_{rs}(t) = \mu_{rs}\,\frac{\sqrt{T}}{2\pi}\int_0^\infty d\omega\;\tilde{h}(\omega)e^{i\omega t}\left[e^{-i\vec{k}\cdot\vec{x}_r} - e^{-i(\tau+\vec{k}\cdot\vec{x}_s)}\right],$$
$$\tilde{y}_{rs}(\omega) = \mu_{rs}\,\tilde{h}(\omega)\left[e^{-i(\vec{k}\cdot\vec{x}_r)} - e^{-i(\tau+\vec{k}\cdot\vec{x}_s)}\right].$$
- Transfer function, sky and polarization averaged

$$R(\omega) = \overline{\left|\frac{\tilde{y}_{rs}(\omega)}{\tilde{h}(\omega)}\right|^2},$$

$$I_s \equiv \frac{1}{4\pi}\int_{-1}^{1} d\cos\theta_1 \int_0^{2\pi} d\epsilon_1 \cdots,$$

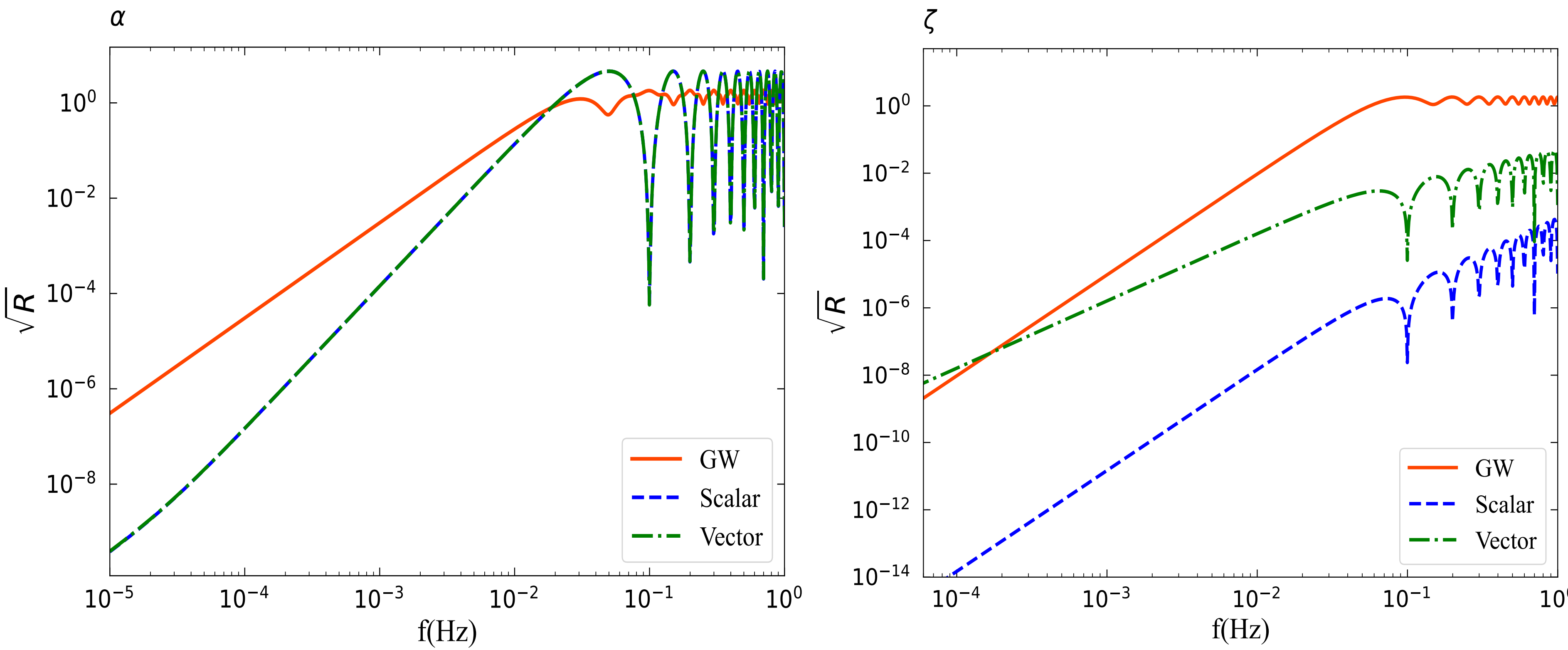
$$I_v \equiv \frac{1}{16\pi^2}\int_{-1}^{1} d\cos\theta_1 \int_0^{2\pi} d\epsilon_1 \int_{-1}^{1} d\cos\theta_2 \int_0^{2\pi} d\epsilon_2 \cdots$$

$$I_{GW} \equiv \frac{1}{8\pi^2}\int_{-1}^{1} d\cos\theta_1 \int_0^{2\pi} d\epsilon_1 \int_0^{2\pi} d\psi \cdots.$$



Transfer Functions

- Different channels have different transfer functions
- DM is also different from gravitational wave, velocity effect, ⋯



Sensitivity

➢ Defined by $S_O(f) = \frac{N_O(f)}{R_O(f)}, \quad N_X = 16\sin^2(\tau)\left\{[3+\cos(2\tau)]\,S_{acc} + S_{oms}\right\}, \quad \tau = 2\pi f L$

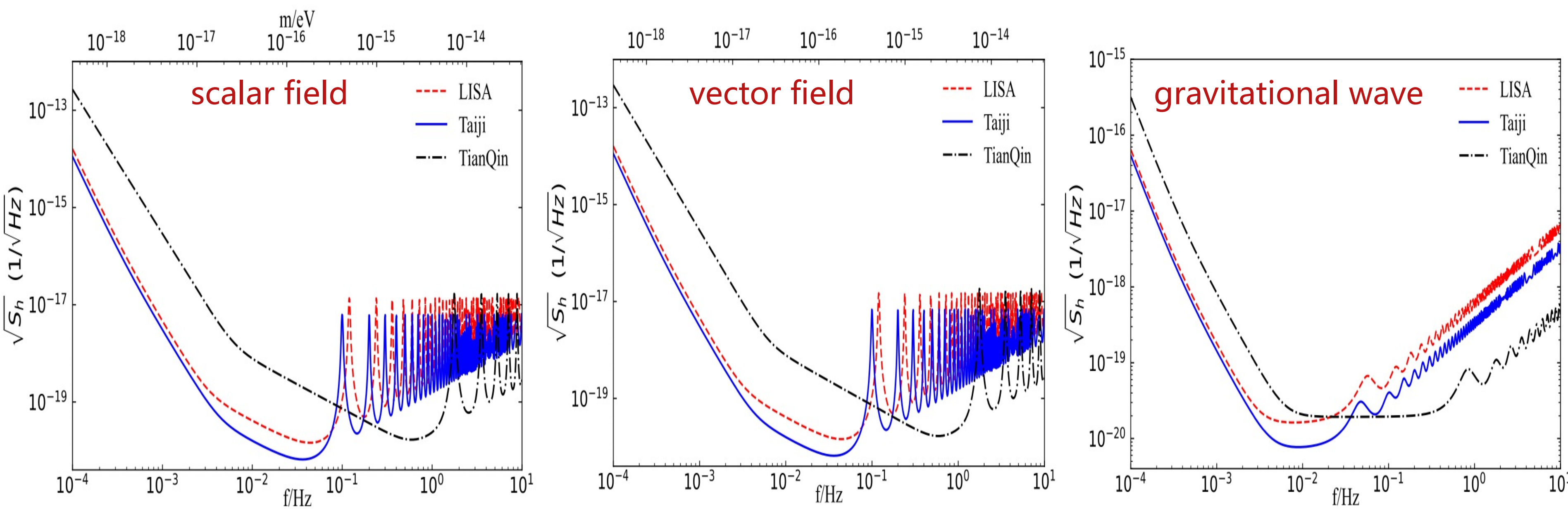
$$S_{oms}(f) = \left(s_{oms}\,\frac{2\pi f}{c}\right)^2 \left[1 + \left(\frac{2\times 10^{-3}\ \mathrm{Hz}}{f}\right)^4\right] \frac{1}{\mathrm{Hz}},$$

$$S_{acc}(f) = \left(\frac{s_{acc}}{2\pi f c}\right)^2 \left[1 + \left(\frac{0.4\times 10^{-3}\ \mathrm{Hz}}{f}\right)^2\right] \left[1 + \left(\frac{f}{8\times 10^{-3}\ \mathrm{Hz}}\right)^4\right] \frac{1}{\mathrm{Hz}},$$

$$\text{LISA}: \quad s_{oms} = 15\times 10^{-12}\ \mathrm{m},\ s_{acc} = 3\times 10^{-15}\ \mathrm{m/s^2},$$

$$\text{Taiji}: \quad s_{oms} = 8\times 10^{-12}\ \mathrm{m},\ s_{acc} = 3\times 10^{-15}\ \mathrm{m/s^2},$$

$$\text{TianQin}: \quad s_{oms} = 1\times 10^{-12}\ \mathrm{m},\ s_{acc} = 1\times 10^{-15}\ \mathrm{m/s^2}.$$

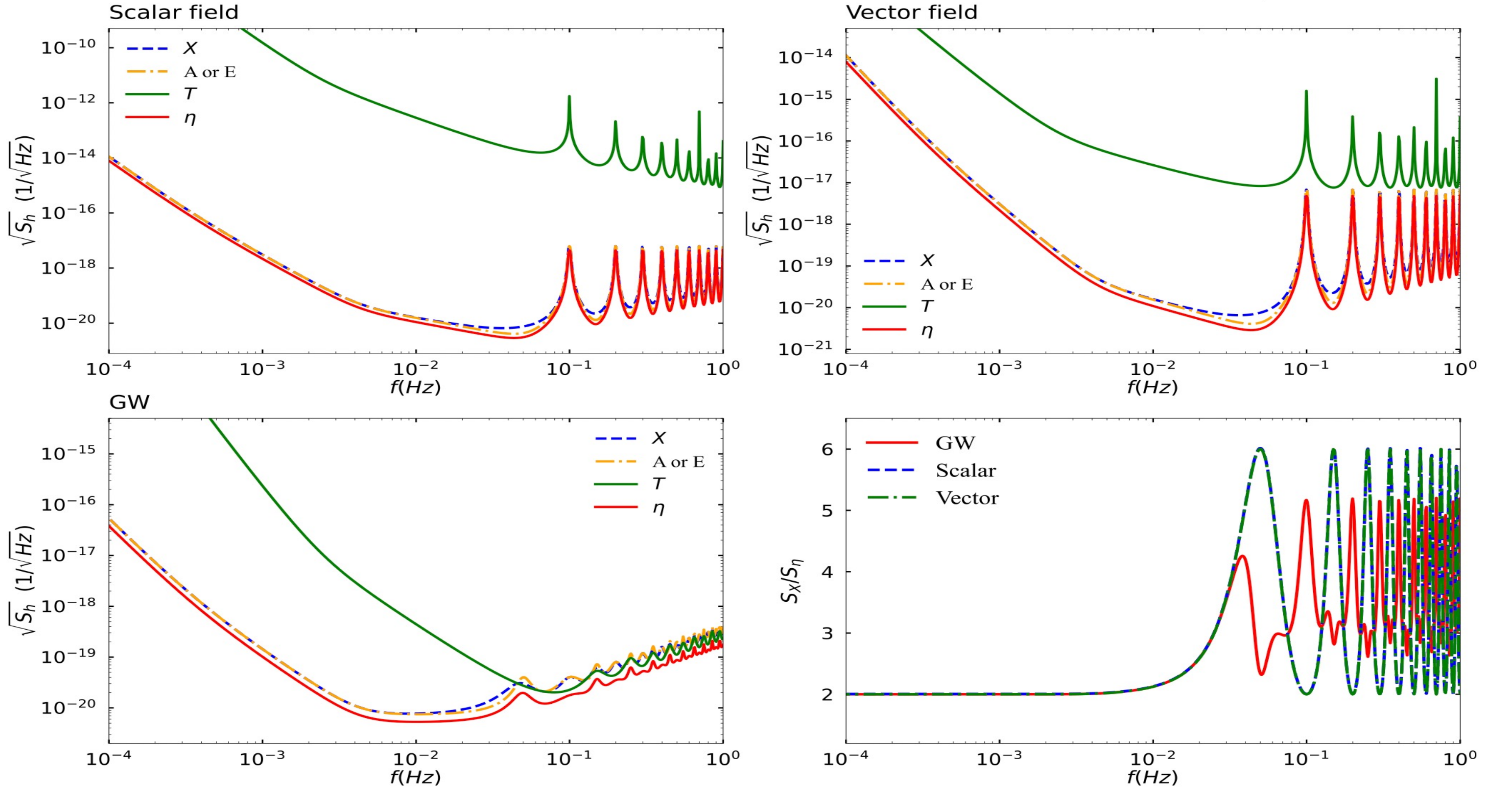


Yu, Yao, Tang, Wu, arXiv: 2307.09197

Sensitivity

Prince, Tinto, Larson & Armstrong

➢ Optimal channels $A = \frac{1}{\sqrt{2}}[Z - X], E = \frac{1}{\sqrt{6}}[X - 2Y + Z], T = \frac{1}{\sqrt{3}}[X + Y + Z].$ $\frac{1}{S_\eta} = \frac{1}{S_A} + \frac{1}{S_E} + \frac{1}{S_T}$



Sensitivity on scalar DM

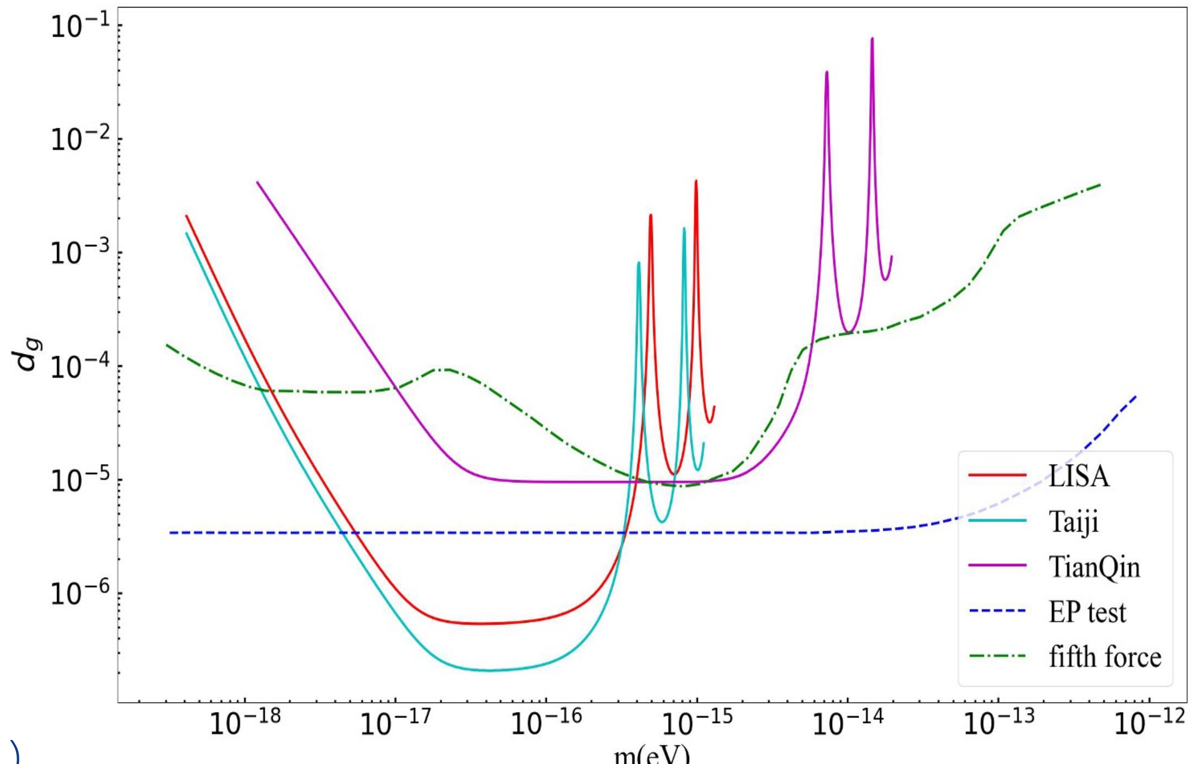
- Strong sector $\delta\mathcal{L} = \frac{\phi}{M_P}\left[-\frac{d_g\beta_3}{2g_3}F^A_{\mu\nu}F^{A\mu\nu} - \sum_{i=u,d}(d_{m_i}+\gamma_{m_i}d_g)\,m_i\bar{\psi}_i\psi_i\right]$ Damour & Donoghue

$$d_{\hat{m}} \equiv \frac{d_{m_d}m_d + d_{m_u}m_u}{m_d+m_u},$$

$$d_g^* \approx d_g + 0.093(d_{\hat{m}} - d_g).$$

assuming $d_m = 0$ and $d_g^* \approx 0.9 d_g$.

- Equivalence principle is violated.
- MICROSCOPE
- Pulsars Shao, Wex, Kramer, arXiv:1805.08408(PRL)



Yu, Yao, Tang, Wu, arXiv: 2307.09197

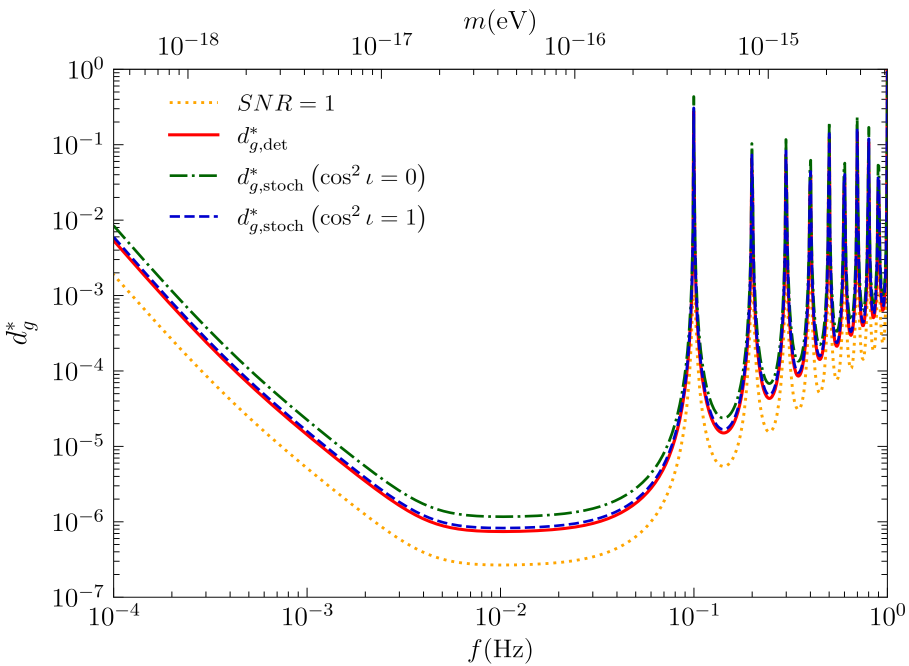
Statistical Effects

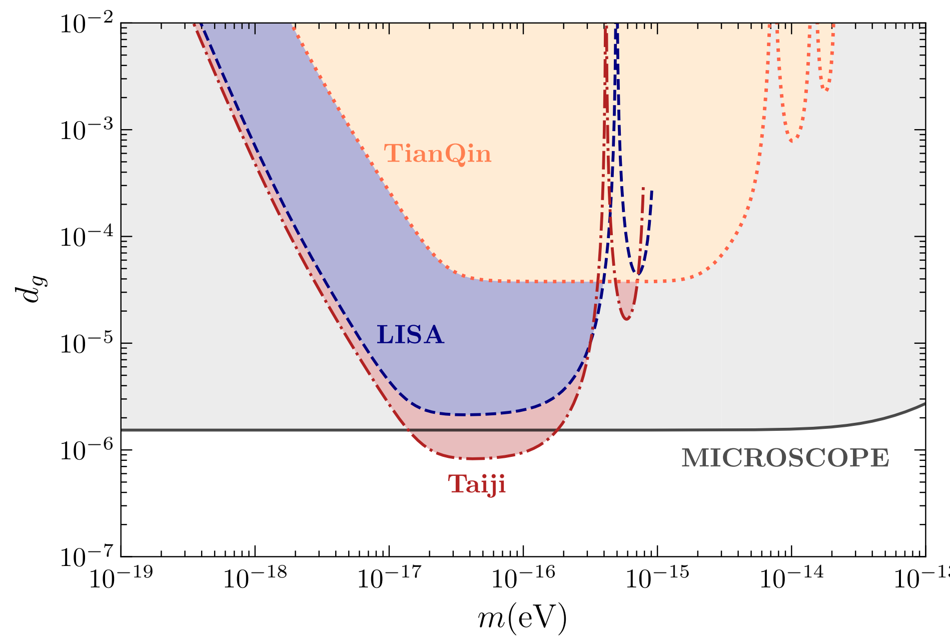
- Velocity distributions, likelihood analysis

$$S_O(\lambda_{\min}) = \left(\frac{\ln\alpha}{\ln\gamma} - 1\right) N_O, \quad \lambda_{\min}^2 = \frac{N_O}{\Gamma_O S_\Phi}\left(\frac{\ln\alpha}{\ln\gamma} - 1\right).$$

$$\gamma = \int_{P_*(\alpha)}^{\infty} dP\, \mathcal{L}_{\rm stoch}\left(\tilde{d}\middle| S_O(\lambda_{\min}), N_O\right),$$ detection probability

$$\alpha = \int_{P_*}^{\infty} dP\, \mathcal{L}_{\rm stoch}\left(P\middle|\lambda = 0, N_O\right).$$ false alarm rate

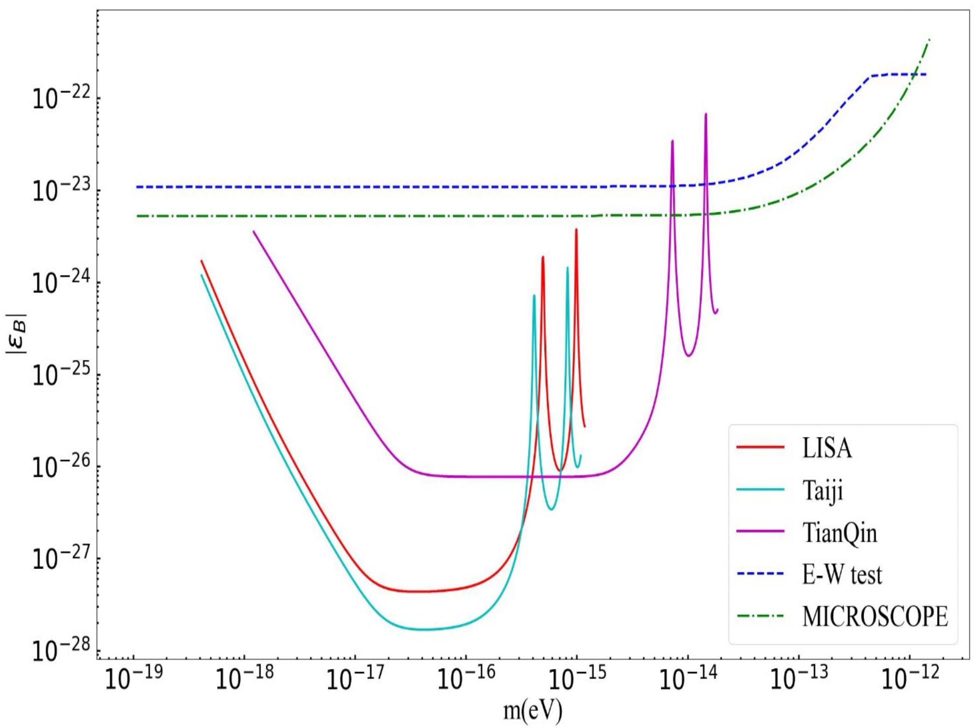


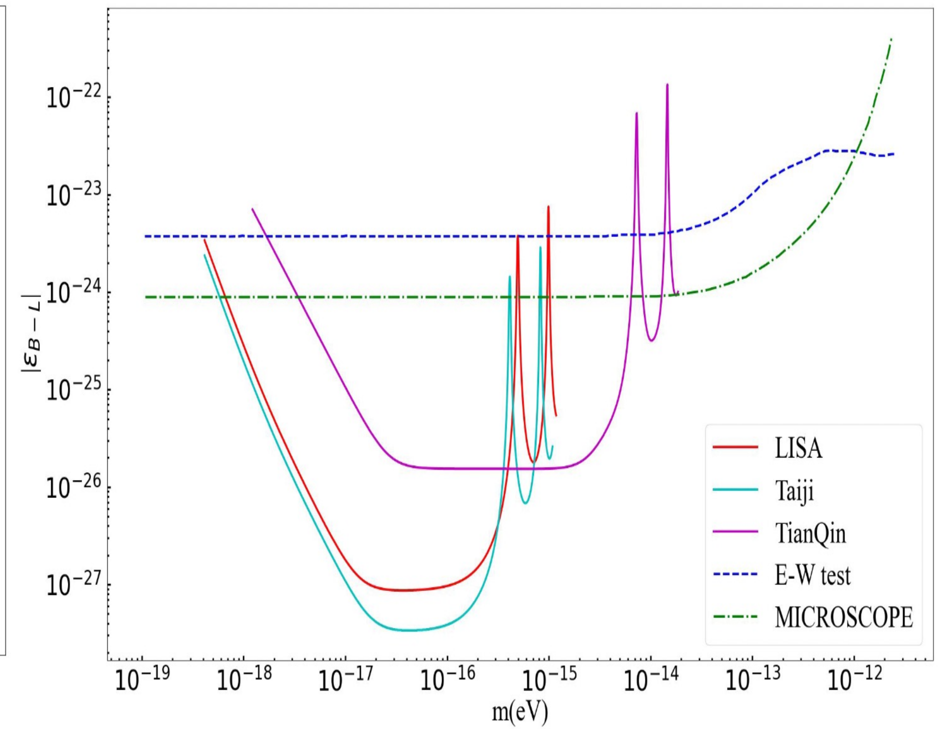


Yao, Tang, arXiv: 2404.01494

Sensitivity on vector DM

- For example, vector fields couple to baryon number *B*, or *B-L*, effectively neutron number. Sensitivity on ratio $\epsilon_D = e_D/e$





Yu, Yao, Tang, Wu, arXiv: 2307.09197(PRD)

Detecting Ultralight Dark Matter Gravitationally

- Metric perturbation in solar system

$$ds^2 = -(1+2\Psi)dt^2 + (1-2\Phi)\delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j$$

- Einstein equations

$$\partial_i\partial^i\Phi = 4\pi G\, T_{00},$$

$$3\ddot{\Phi} + \partial_i\partial^i(\Psi - \Phi) = 4\pi G\, T^k_k,$$

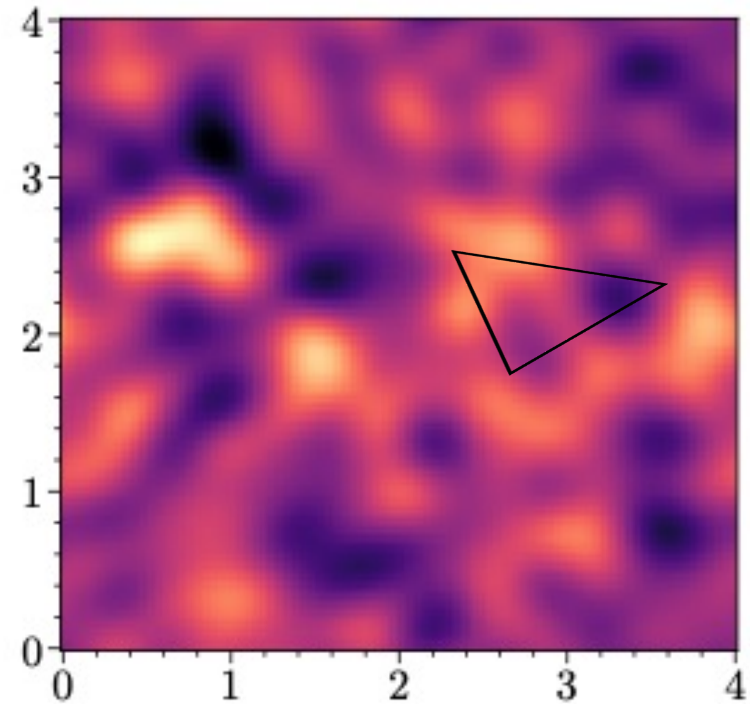
$$\ddot{h}_{ij} = 16\pi G\left(T_{ij} - \frac{1}{3}\delta_{ij}T^k_k\right),$$

$$\Psi^j \simeq \Phi^j \simeq \pi G\frac{\rho}{m^2} = \frac{7\times10^{-26}\rho}{0.4\,\mathrm{GeV/cm^3}}\left(\frac{10^{-18}\mathrm{eV}}{m}\right)^2,$$

$$h^v_{ij} \propto h_0 \simeq \frac{8}{3}\pi G\frac{\rho}{m^2} = \frac{2\times10^{-25}\rho}{0.4\,\mathrm{GeV/cm^3}}\left(\frac{10^{-18}\mathrm{eV}}{m}\right)^2,$$

$$h^s_{ij} \simeq h_0 v^2/2$$

- Tensor perturbation for scalar and tensor ULDM is suppressed.



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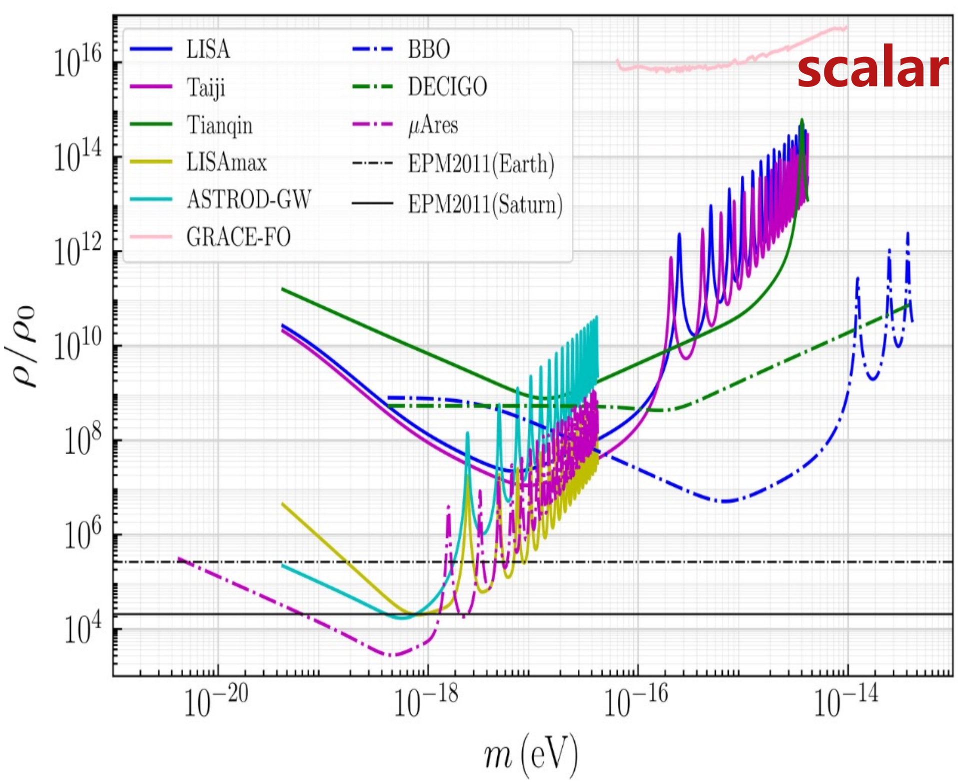
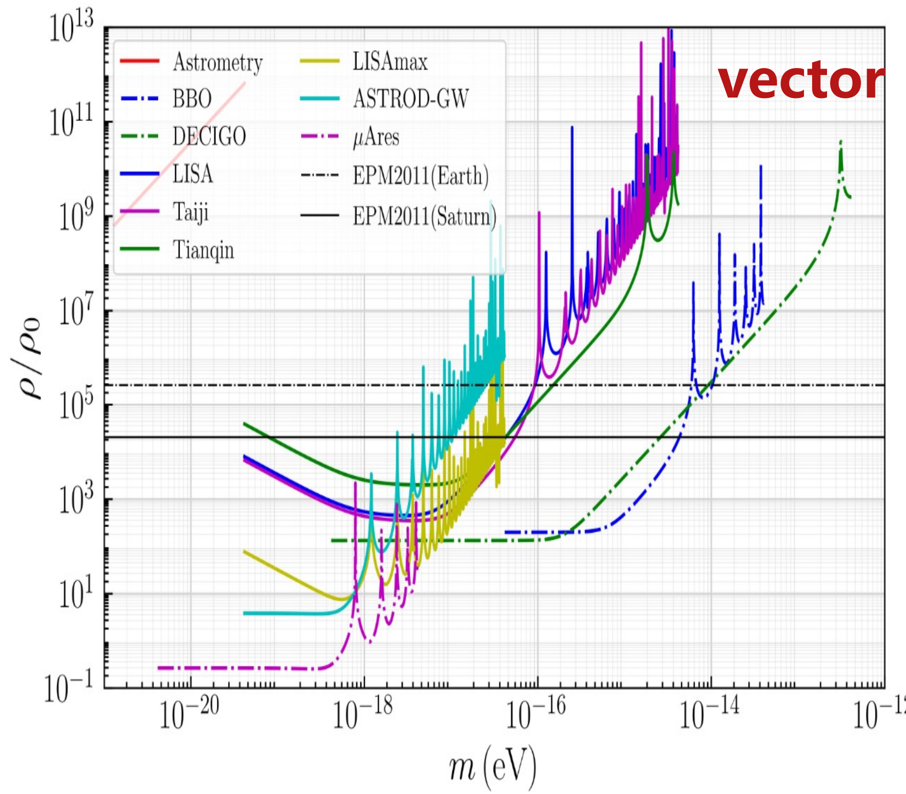




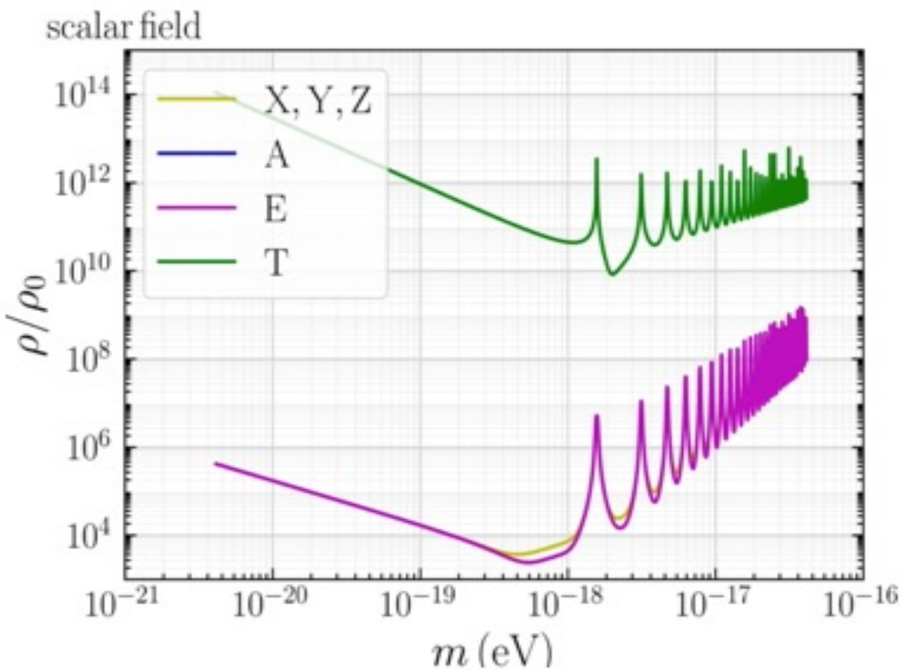
Yu, Cao, Tang, Wu, arXiv:2404.04333 (PRD)

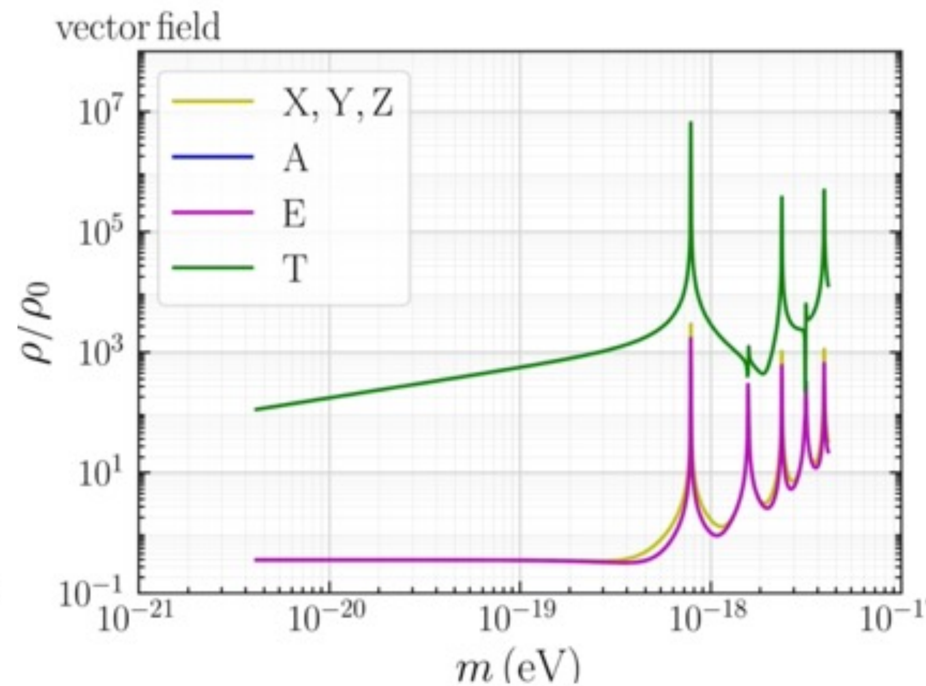
Detecting Ultralight Dark Matter Gravitationally

- Metric perturbation in solar system

$$ds^2 = -(1+2\Psi)dt^2 + (1-2\Phi)\delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j$$

- Different channels have different sensitivities





Yu, Cao, Tang, Wu, arXiv:2404.04333 (PRD)

Summary

- We discuss how gravitational-wave detectors in space may help to understand the nature of dark matter.
- Weakly-interacting massive particles
  - can form spikes around black holes and affect the orbiting compact objects, imprinting on the waveform of GW.
  - Relativistic effect, velocity distribution, ···
- Ultralight bosonic, wave like
  - can couple to normal matter and induce the oscillation of test masses, leading to signals in detectors and very good sensitivities.
  - Metric perturbation by vector ULDM may be detectable in future.

Thanks!