Constructing the holographic QCD model with machine learning

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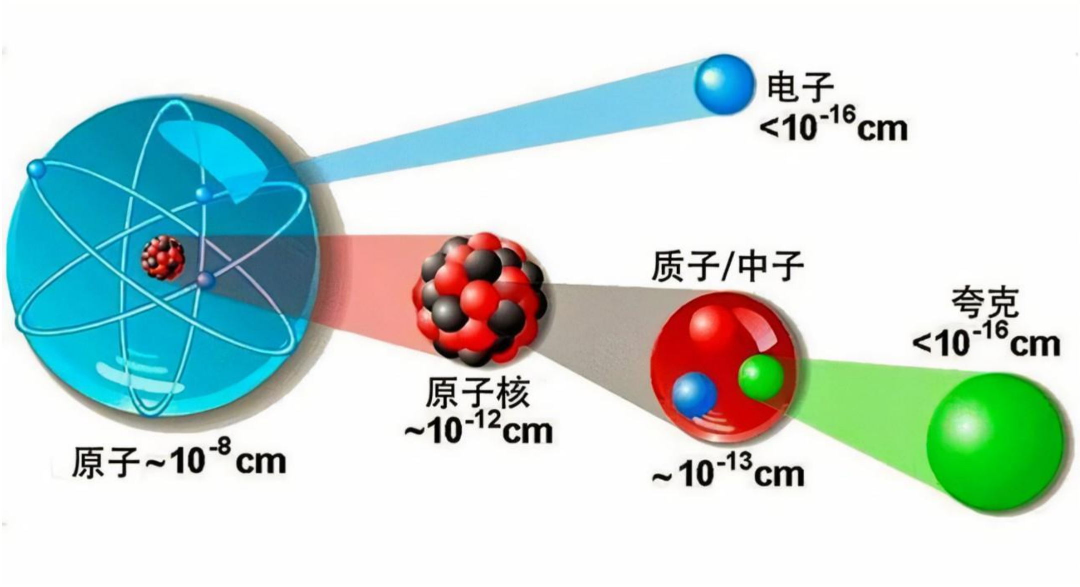
Collaborator: Mei Huang(黄梅) 、Defu Hou(侯德富)、Kai Zhou(周凯)、Long-Gang Pang(庞龙刚)…

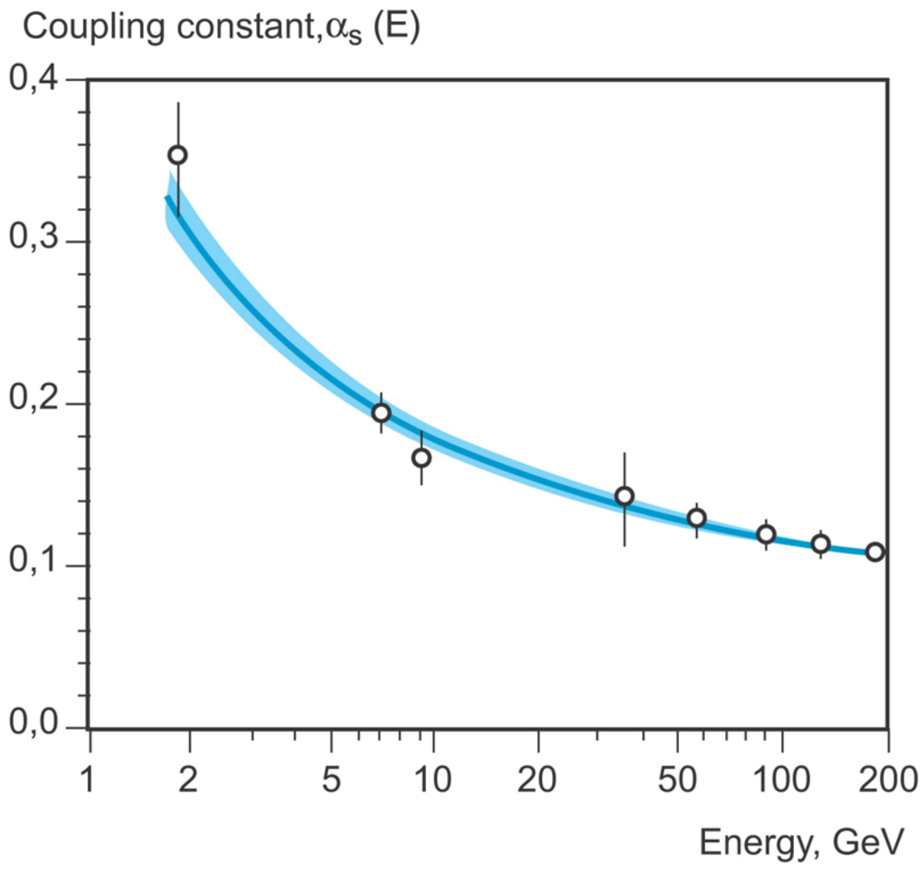
Content

- Introduction
- Holographic QCD and machine learning
- QCD phase transition
- Transport Properties of Quark-Gluon Plasma
- Heavy-quark potential
- Outlook

➢ Introduction

Quantum Chromodynamics





Lattice, PNJL, FRG….

Perturbation QCD

The origin of AdS/CFT

Proposed by Maldacena, 1997

The Large N limit of superconformal field theories and supergravity #1

Juan Martin Maldacena (Harvard U.) (Nov, 1997)

Published in: *Int.J.Theor.Phys.* 38 (1999) 1113-1133 (reprint), *Adv.Theor.Math.Phys.* 2 (1998) 231-252 • e-Print: hep-th/9711200 [hep-th]

pdf DOI cite claim reference search 19,793 citations

The strongest form of the AdS_5/CFT_4 correspondence

$\mathcal{N} = 4$ Super Yang–Mills (SYM) theory with gauge group SU(N) and is dynamically equivalent to type IIB superstring theory on $AdS_5 \times S^5$.

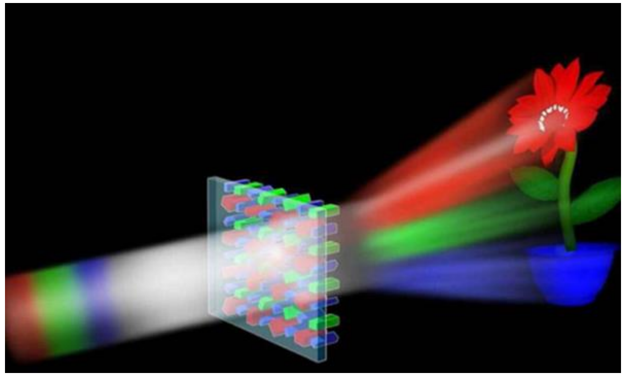
Further expression

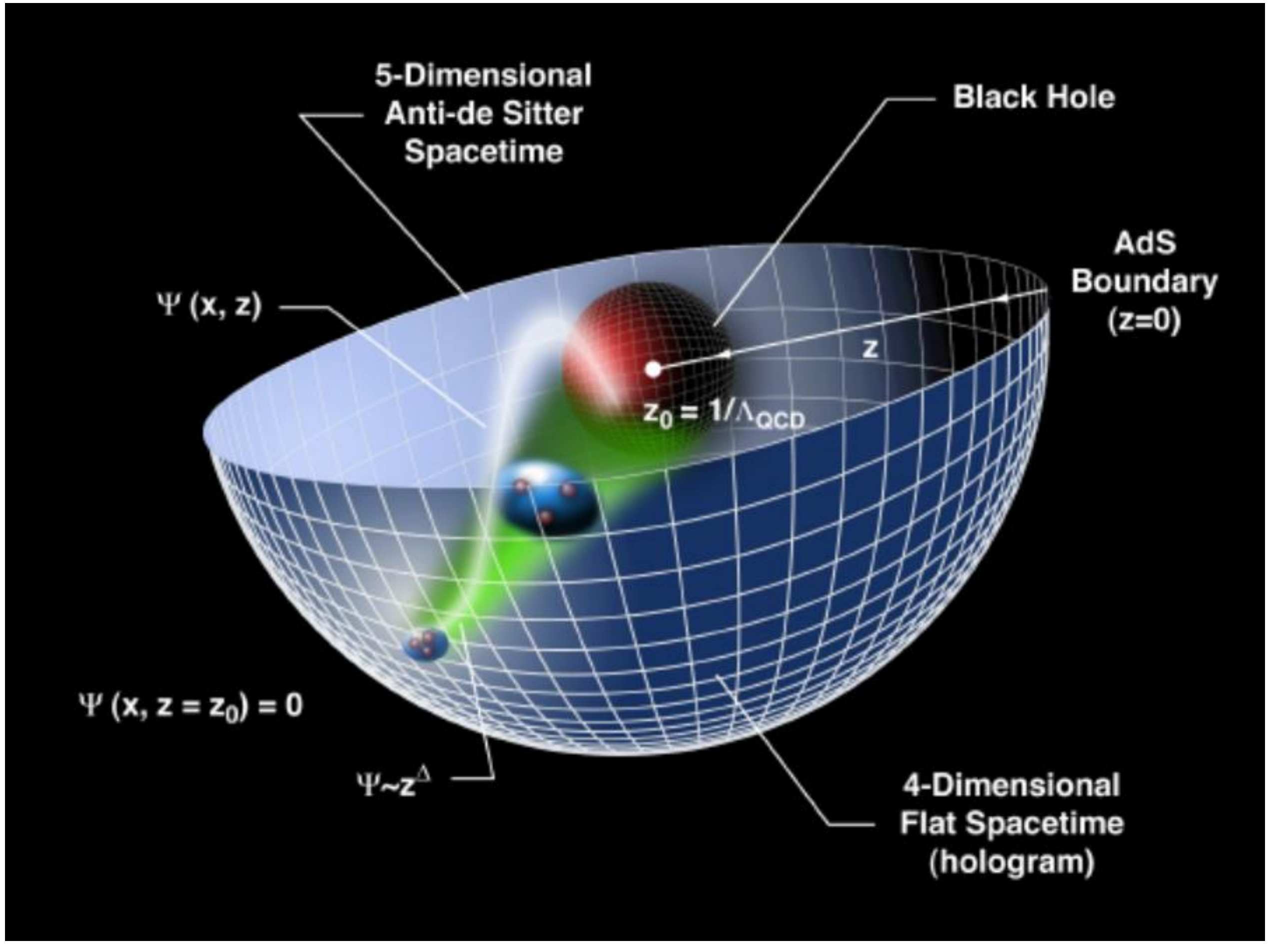
D+1 weakly-coupled classical gravitational theory in AdS spacetime = D dimension strongly-coupled gauge theory.

Holographic QCD (Gauge/gravity duality)

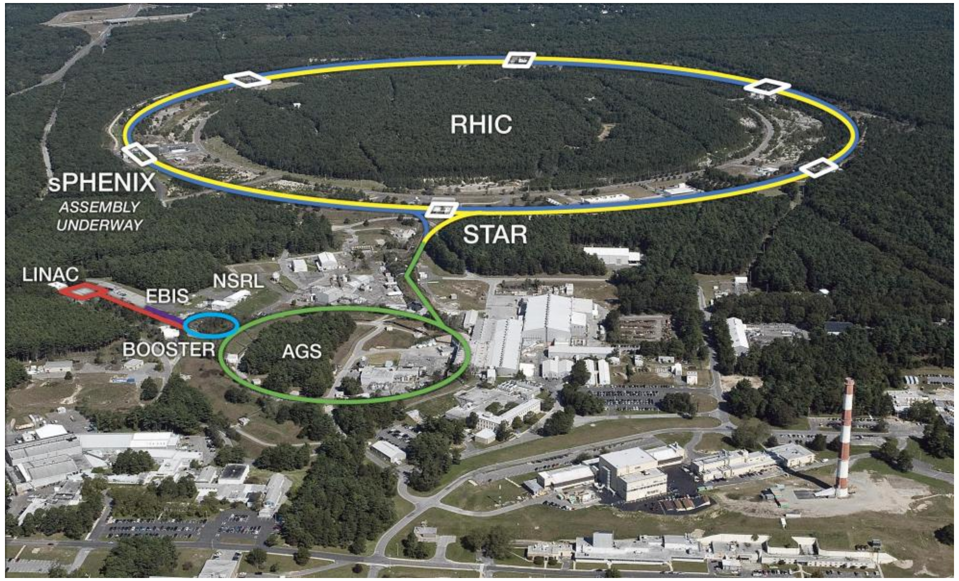
Black hole entropy: $S = \dfrac{\pi A k c^3}{2hG}$

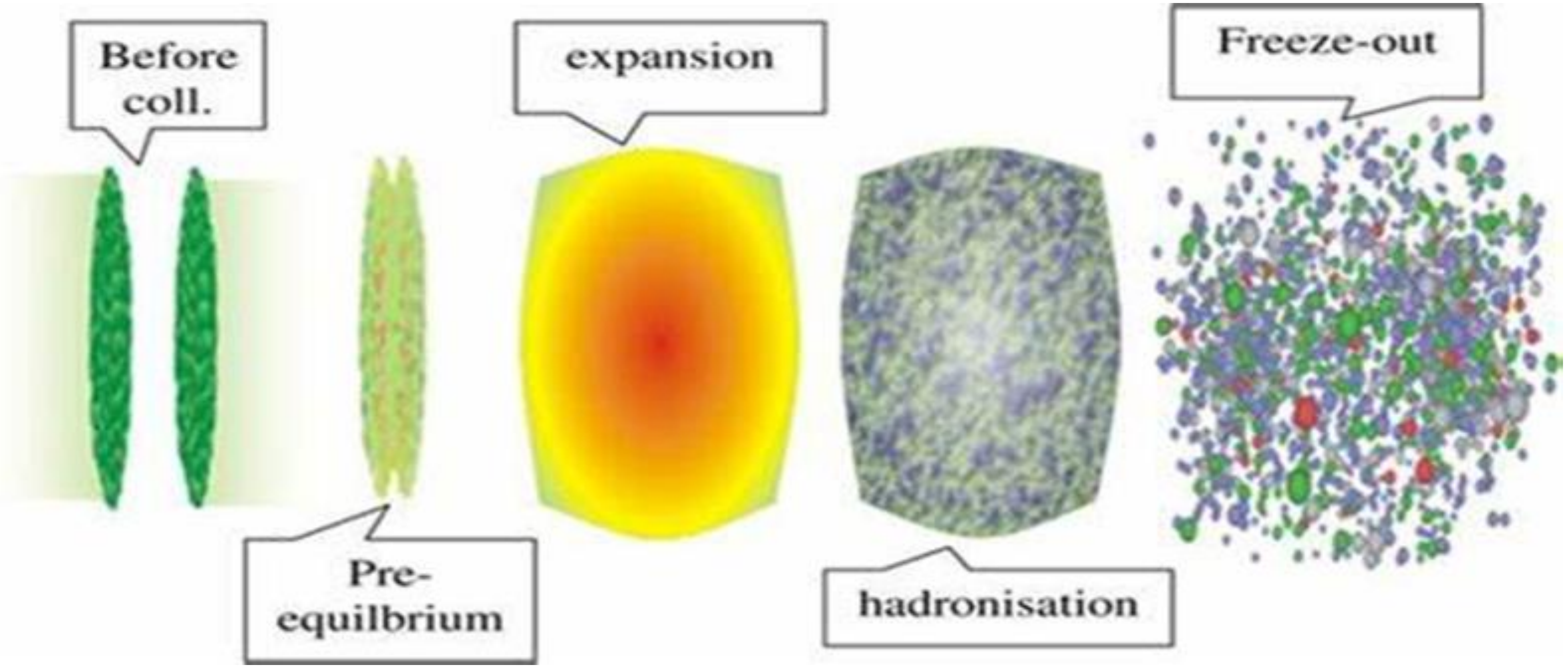
项目	每100毫升	营养素参考值
能量	190千焦	2%
蛋白质	0克	0%
脂肪	0克	0%
-饱和脂肪酸	0克	0%
碳水化合物	11.2克	4%
-糖	11.2克	
钠	12毫克	1%



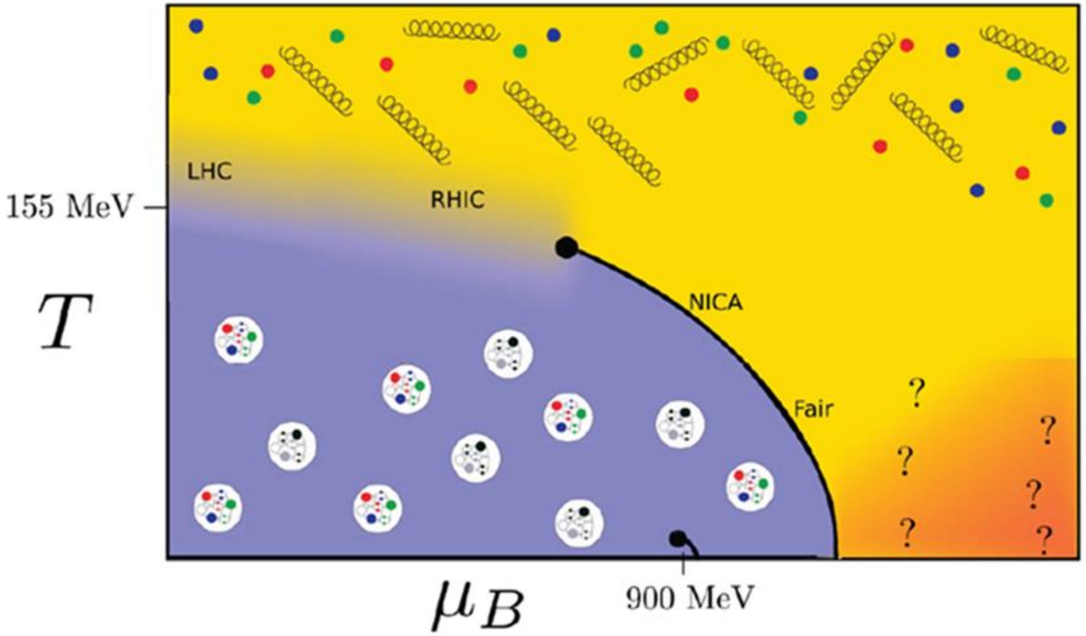


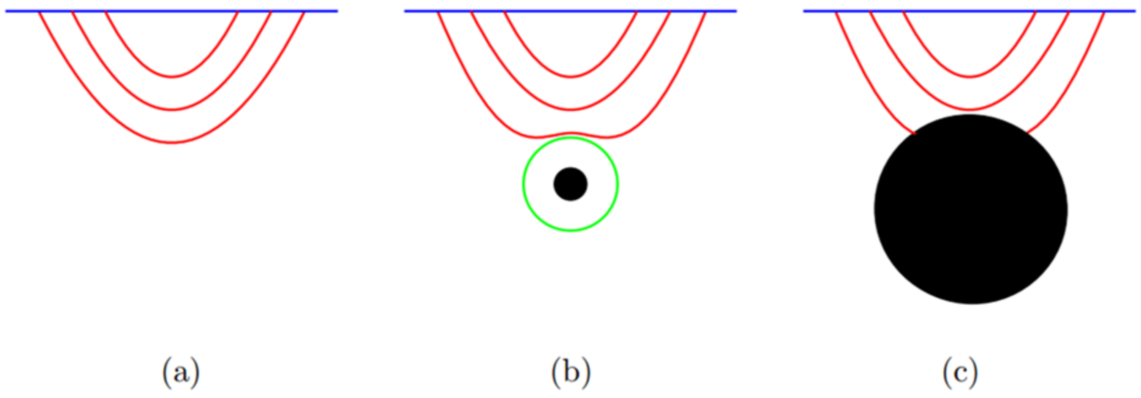
Relativistic heavy ion collisions





Holographic QCD and phase transition

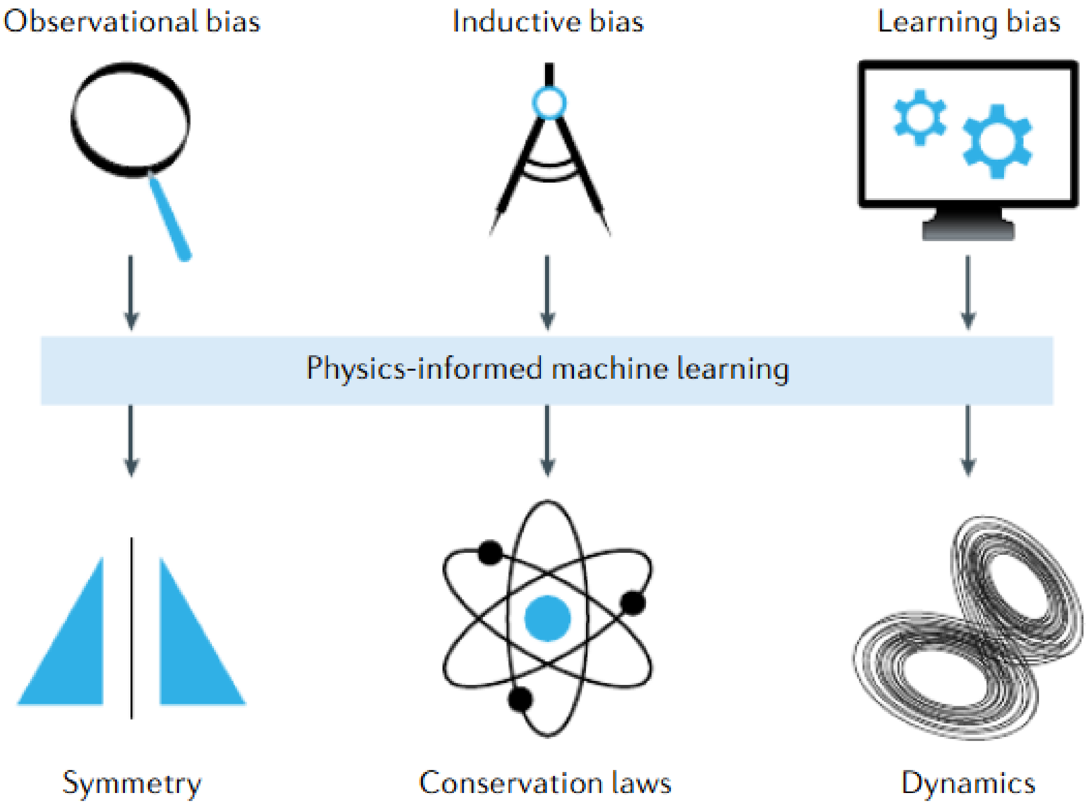




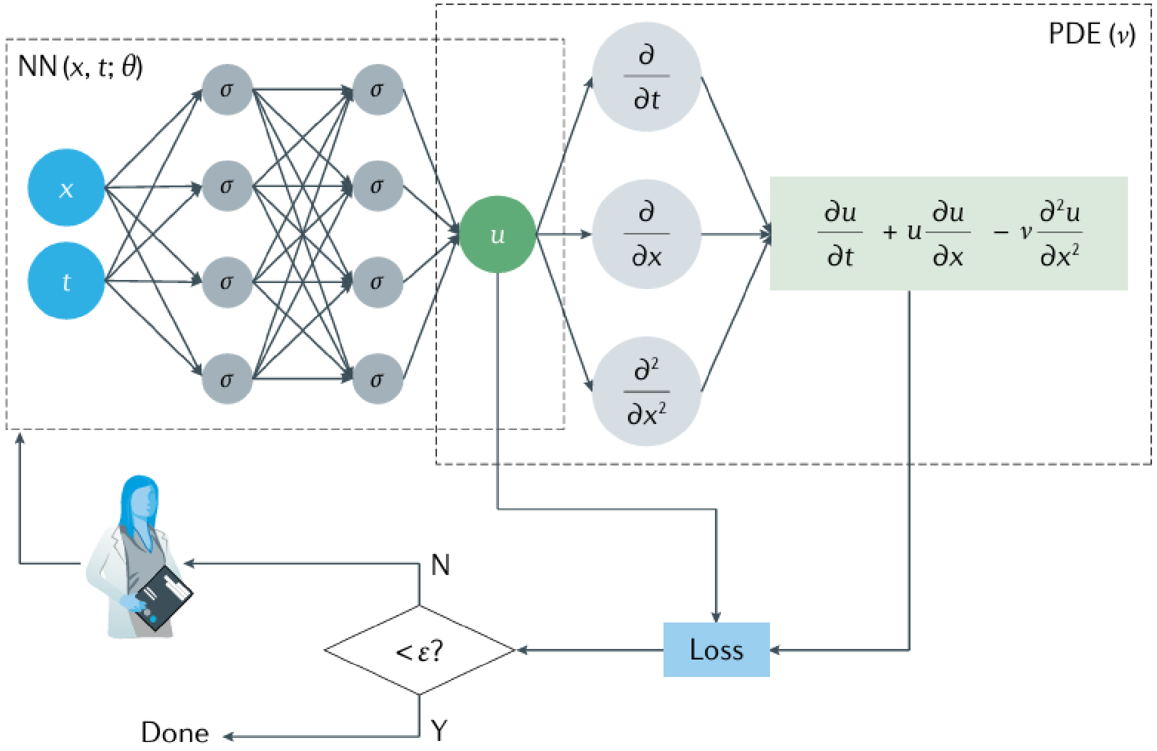
ArXiv: 1506.05930

Physics Informed Machine Learning (Nature review)

George Em Karniadakis, Ioannis G. Kevrekidis, Sifan Wang and Liu Yang



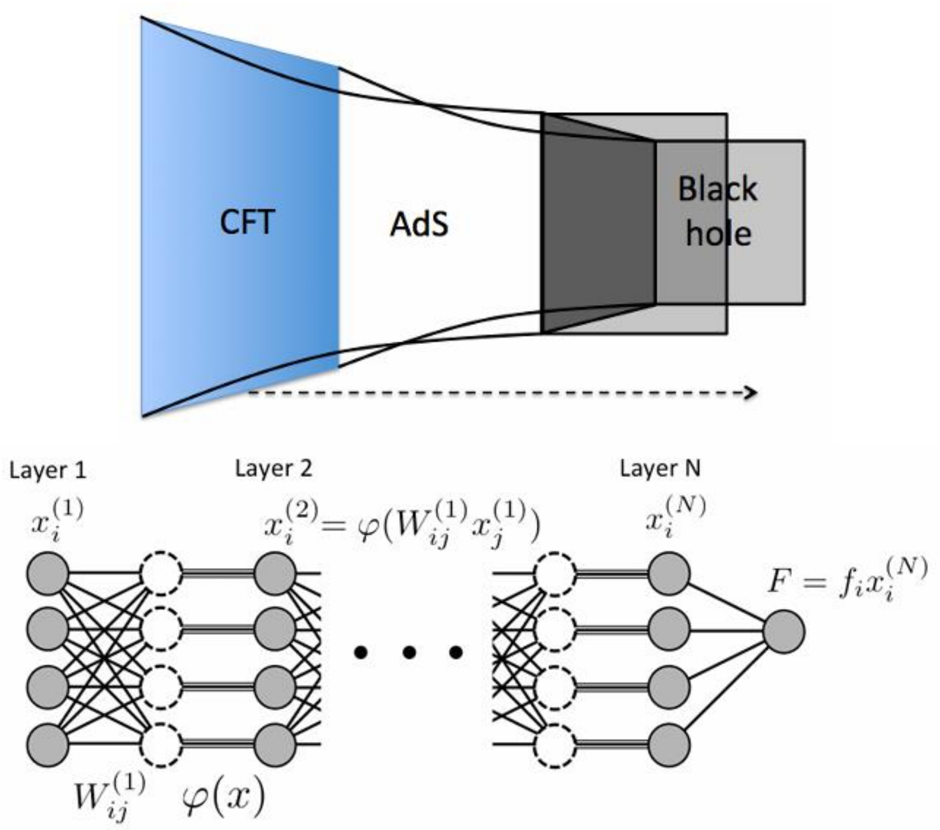
PINN



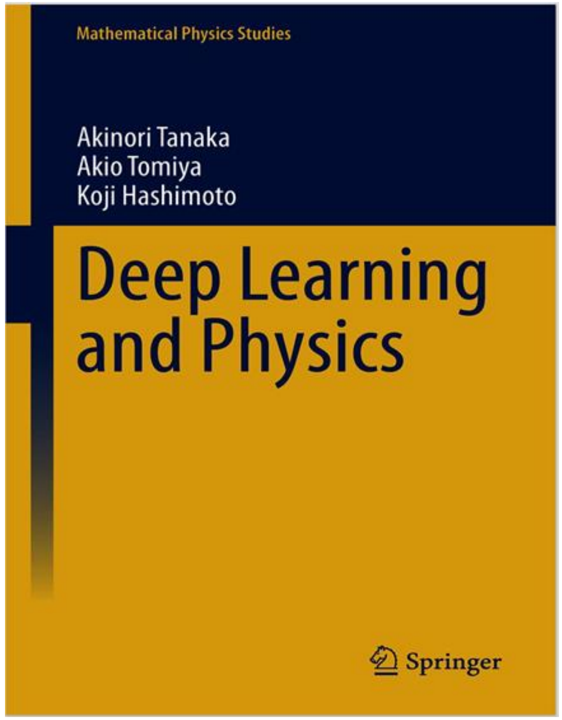
➢ Holographic QCD and machine learning

Deep learning and the AdS/CFT correspondence

Koji Hashimoto, Sotaro Sugishita, Akinori Tanaka, and Akio Tomiya ArXiv: 1802.08313



We provide a deep neural network representation of a scalar field equation in (d + 1) dimensional curved spacetime. The discretized holographic (AdS radial) direction is the deep layers, see Fig. 1. The weights of the neural network to be trained are identified with a metric component of the curved spacetime. The input response data is at the boundary of AdS, and the output binomial data is the black hole.



Holographic QCD and machine learning

1、K. Li, Y. Ling, P. Liu and M. H. Wu, Phys. Rev. D 107 (2023) no.6, 066021.
2、K. Hashimoto, K. Ohashi and T. Sumimoto, PTEP 2023, no.3, 033B01 (2023).
3、Y. K. Yan, S. F. Wu, X. H. Ge and Y. Tian, Phys. Rev. D 102, no.10, 101902.
4、B. Ahn, H.-S. Jeong, K.-Y. Kim, and K. Yun, JHEP 03 (2024) 141.
……

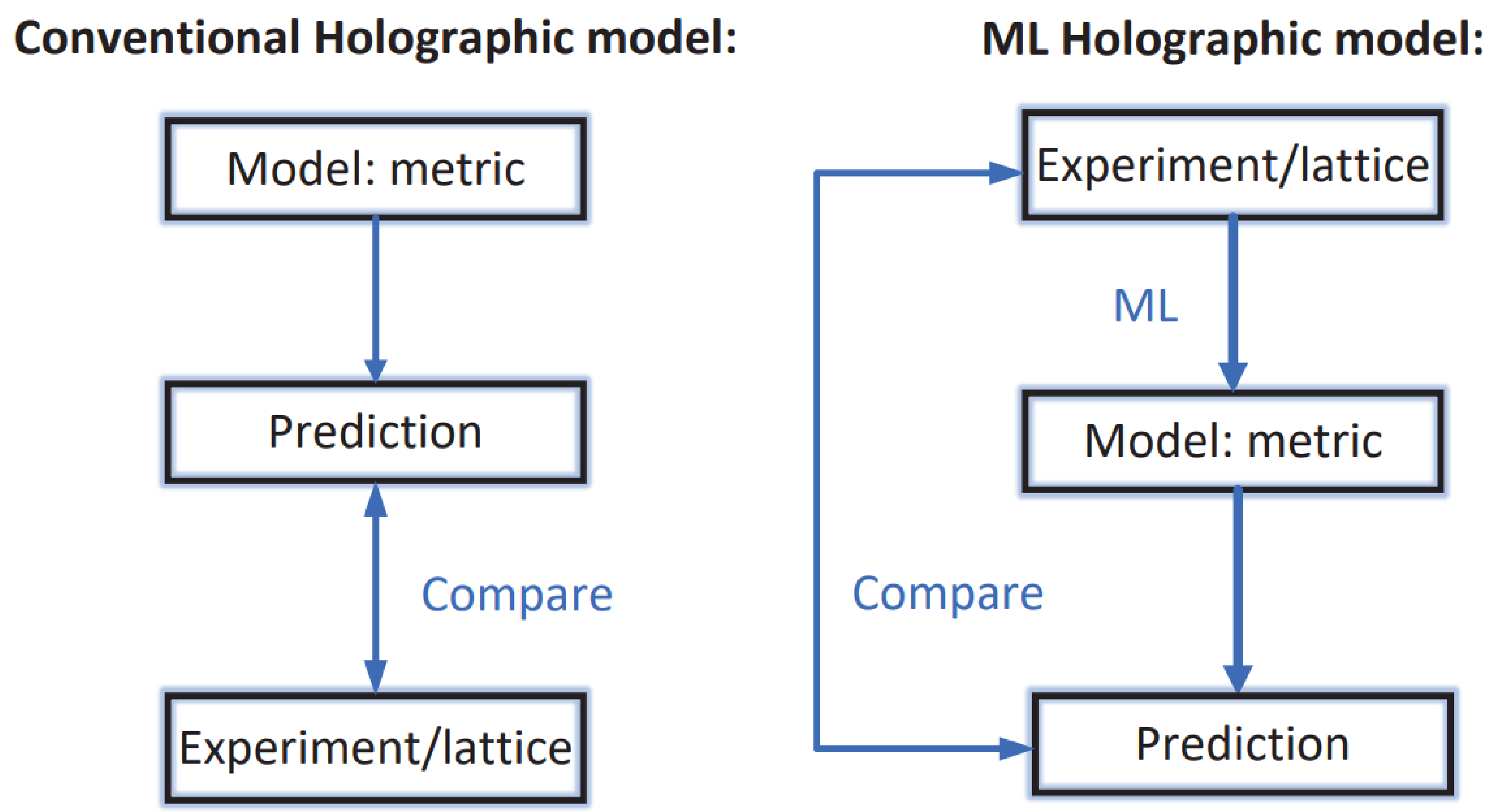
holographic conductivity,
Shear viscosity，
Chiral condensate，
Optical conductivity，
……

- **QCD phase transition**

ArXiv： 2401.06417 （Phys. Rev. D 109 (2024) 5, L051902）

2405.06179

Xun Chen, Mei Huang



Einstein-Maxwell-Dilaton model

Action: $$S_b = \frac{1}{16\pi G_5}\int d^5x \left[\sqrt{-g}R - \frac{f(\phi)}{4}F^2 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right].$$

Non-conformal

ϕ is dilaton

F is the tensor of gauge field

Metric ansatz: $$ds^2 = \frac{e^{2A(z)}}{z^2}\left[-g(z)dt^2 + \frac{dz^2}{g(z)} + d\vec{x}^2\right]$$

Potential reconstruction: Danning Li, Song He, Mei Huang, Qi-Shu Yan, JHEP 09 (2011) 041.
Rong-Gen Cai, Song He, Danning Li, JHEP 03 (2012) 033.

Einstein-Maxwell-Dilaton model

$$A(z) = d\ln\left(az^2+1\right) + d\ln\left(bz^4+1\right) \qquad f(z) = e^{cz^2 - A(z) + k}$$

Black hole entropy: $S_{\mathrm{BH}} = \frac{e^{3A(z_h)}}{4G_5 z_h^3}$ The second-order baryon susceptibility $\chi = \frac{1}{\mathrm{T}^2}\frac{\partial \rho}{\partial \mu}$

$$V(\phi) = -12\cosh[c_1\phi] + \left(6c_1^2 - \frac{3}{2}\right)\phi^2 + c_2\phi^6,$$

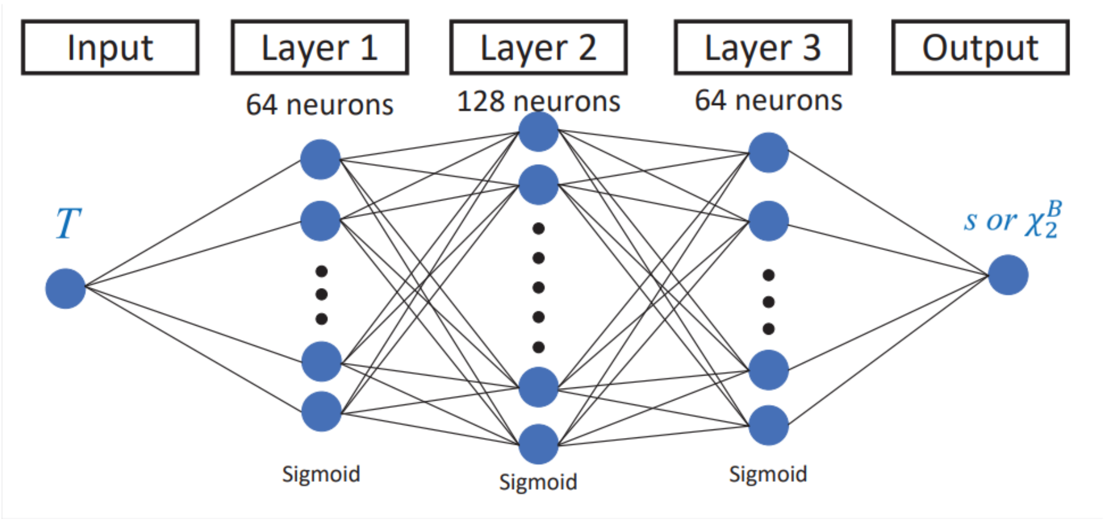
$$f(\phi) = \frac{1}{1+c_3}\mathrm{sech}[c_4\phi^3] + \frac{c_3}{1+c_3}e^{-c_5\phi},$$

Rong-Gen Cai, Song He, Li Li, Yuan-Xu Wang, Phys. Rev. D 106 (2022) 12, L121902

Y.-Q. Zhao, S. He, D. Hou, L. Li, and Z. Li (2023) arXiv:2310.13432 [hep-ph].

Emulator

First step：



2+1 flavor

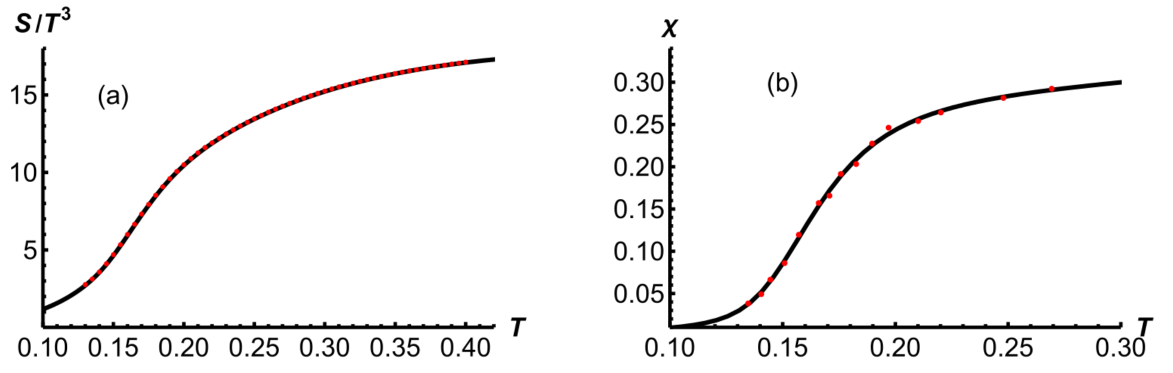


FIG. 1. (a) The entropy as a function of temperature. (b) The baryon susceptibility as a function of temperature. The dots are the results from the lattice and the black line is the prediction of the neural network. The unit of T is GeV.

We use ”TensorFlow” to build a neural network model for regression tasks.

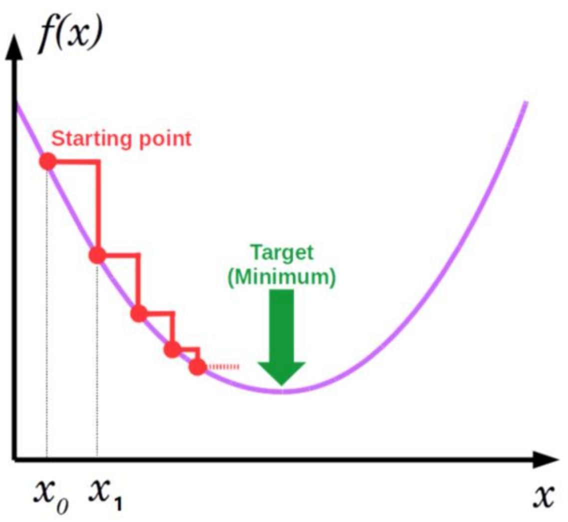
Activation function： Sigmoid

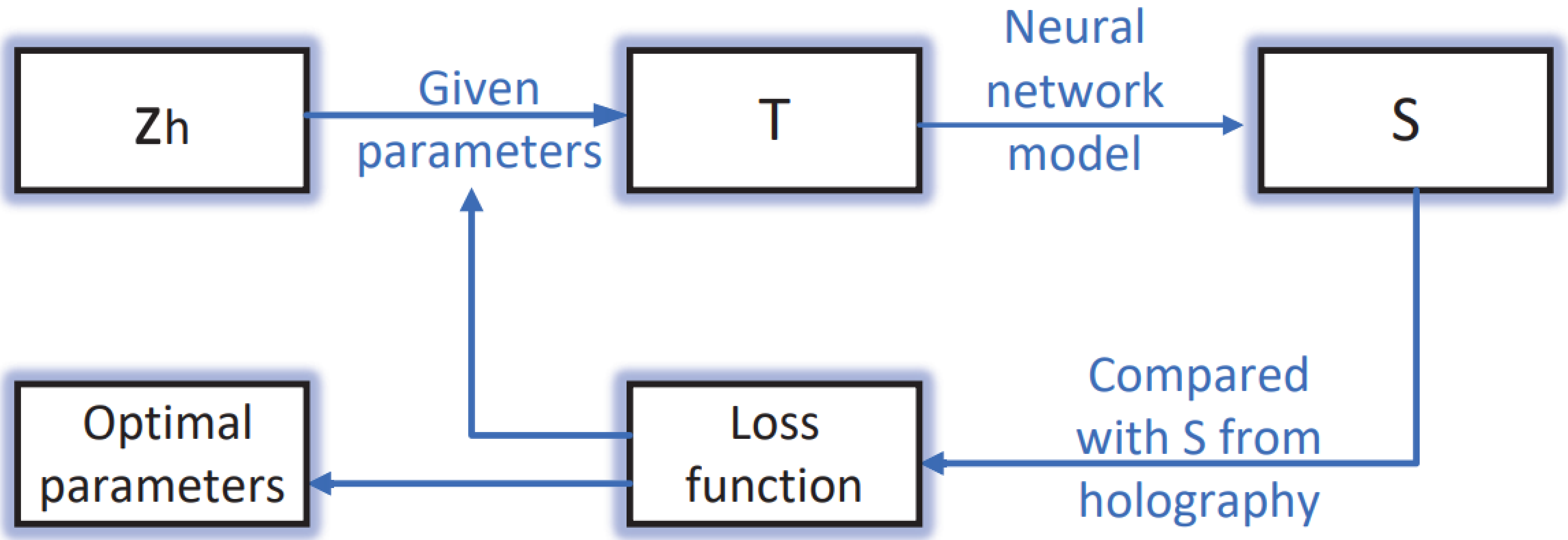
Optimizer: Adam

Learning progress

Second step：

Gradient descent algorithm



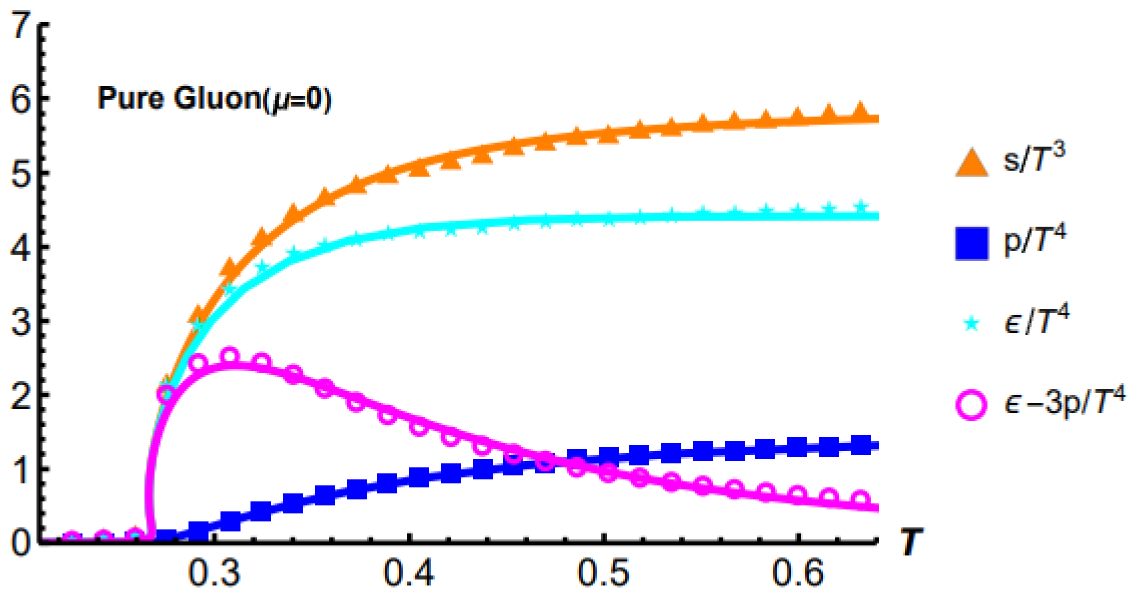


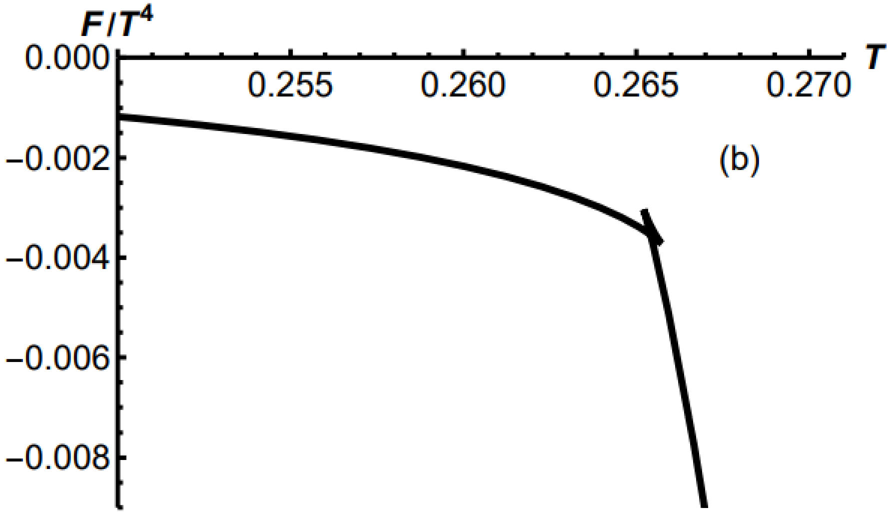
Fix parameters

	a	b	c	d	k	G_5	T_c (GeV)	CEP (GeV)
$N_f = 0$	0	0.072	0	-0.584	0	1.326	0.265	/
$N_f = 2$	0.067	0.023	-0.377	-0.382	0	0.885	0.189	(μ_B^c=0.46, T^c=0.147)
$N_f = 2+1$	0.204	0.013	-0.264	-0.173	-0.824	0.400	0.128	(μ_B^c=0.74, T^c=0.094)
$N_f = 2+1+1$	0.196	0.014	-0.362	-0.171	-0.735	0.391	0.131	(μ_B^c=0.87, T^c=0.108)

T_c is the critical temperature predicted by the model.

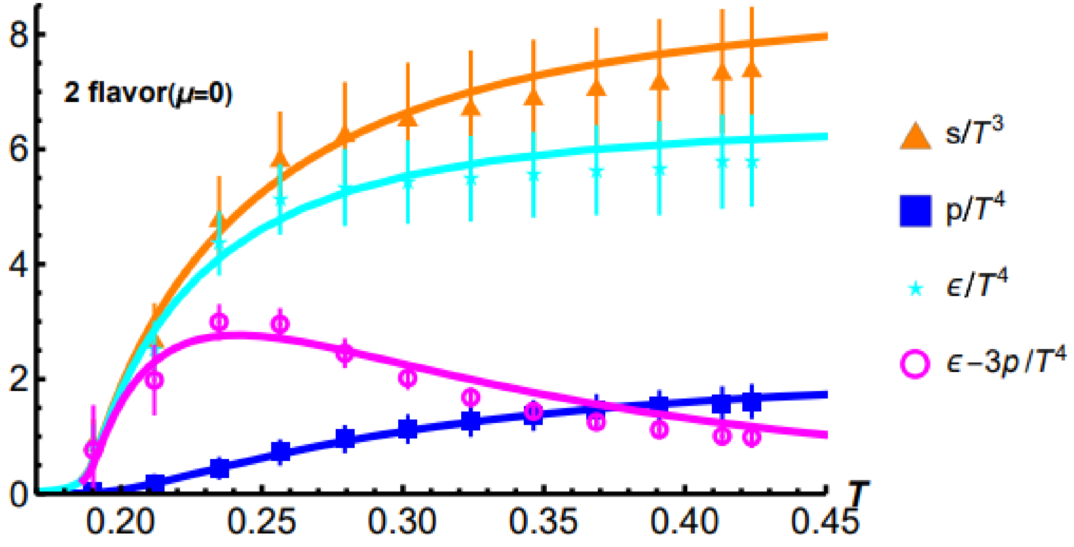
EoS and free energy of pure gluon

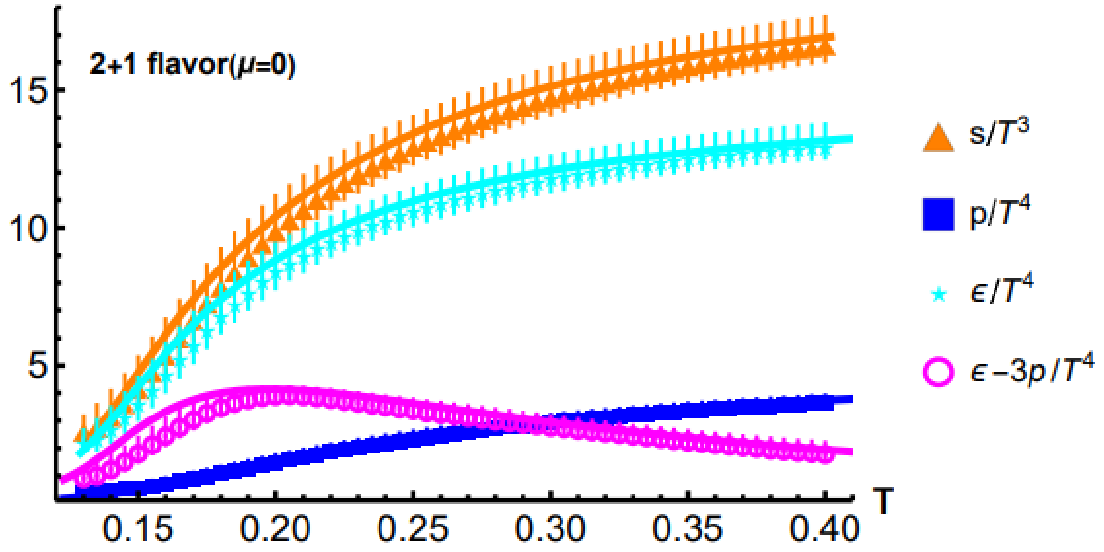


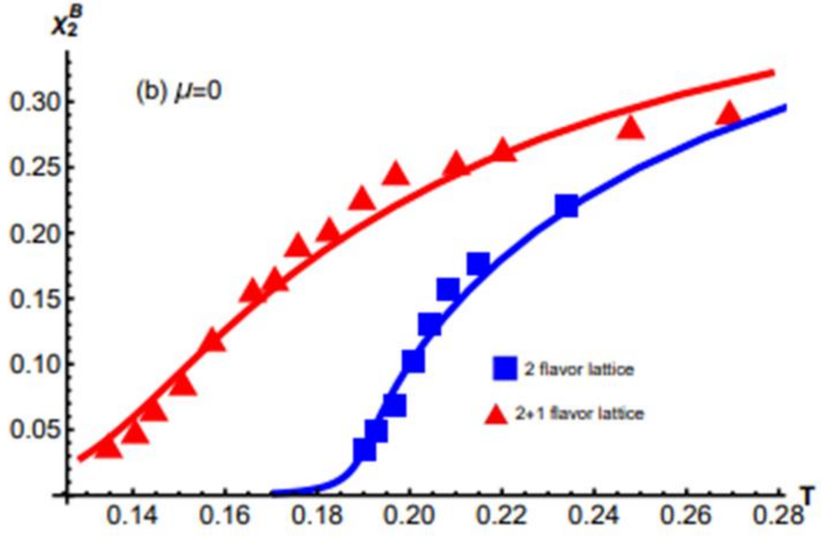


The phase transition is first order at vanishing chemical potential.

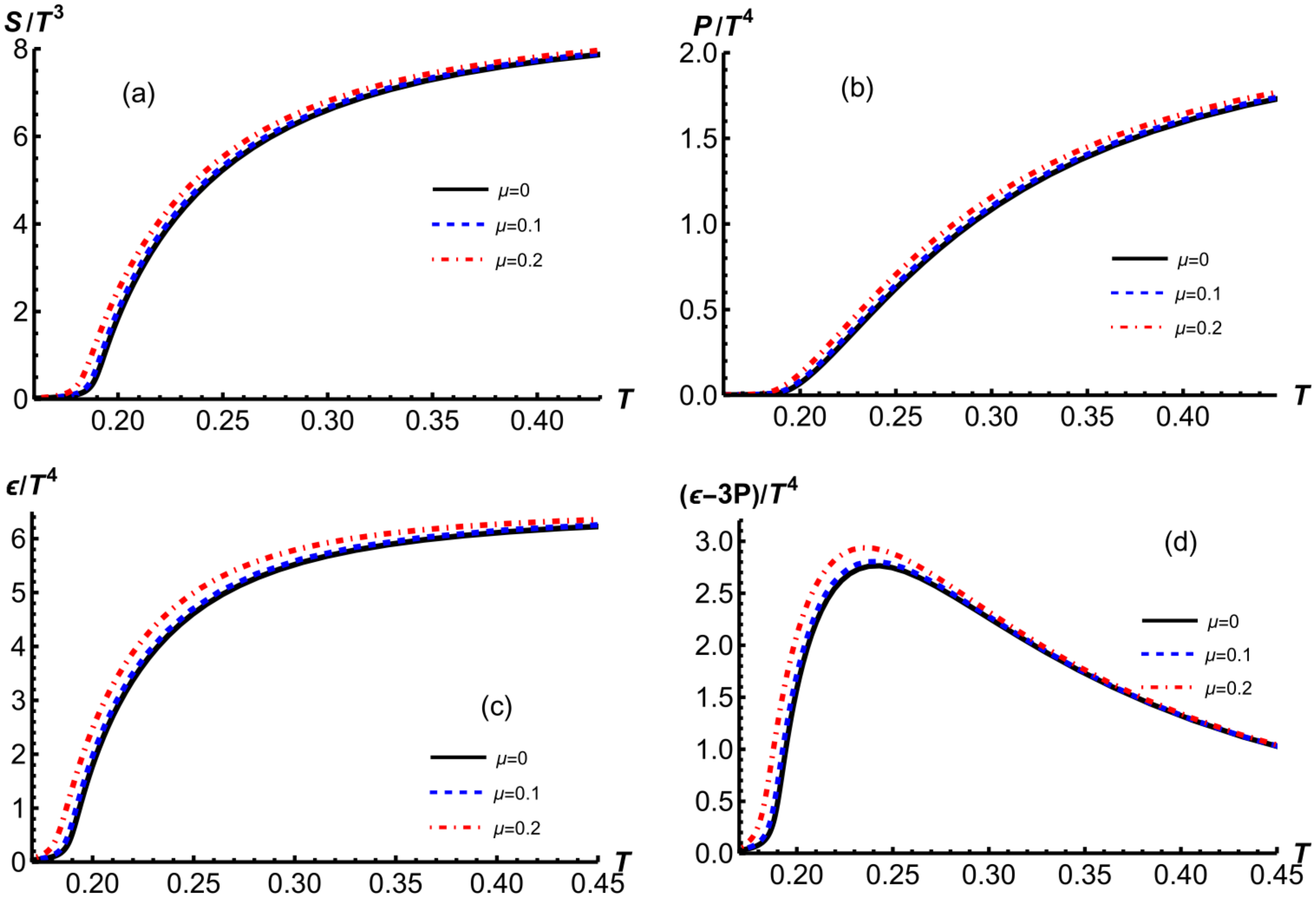
EoS and Baryon number susceptibility



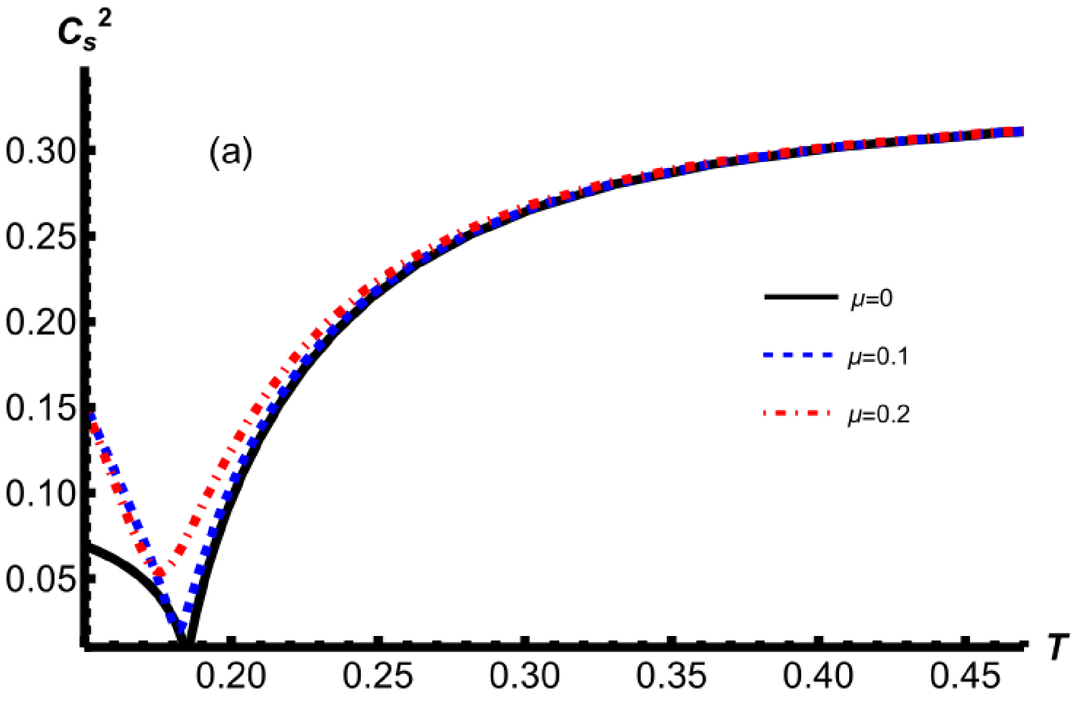


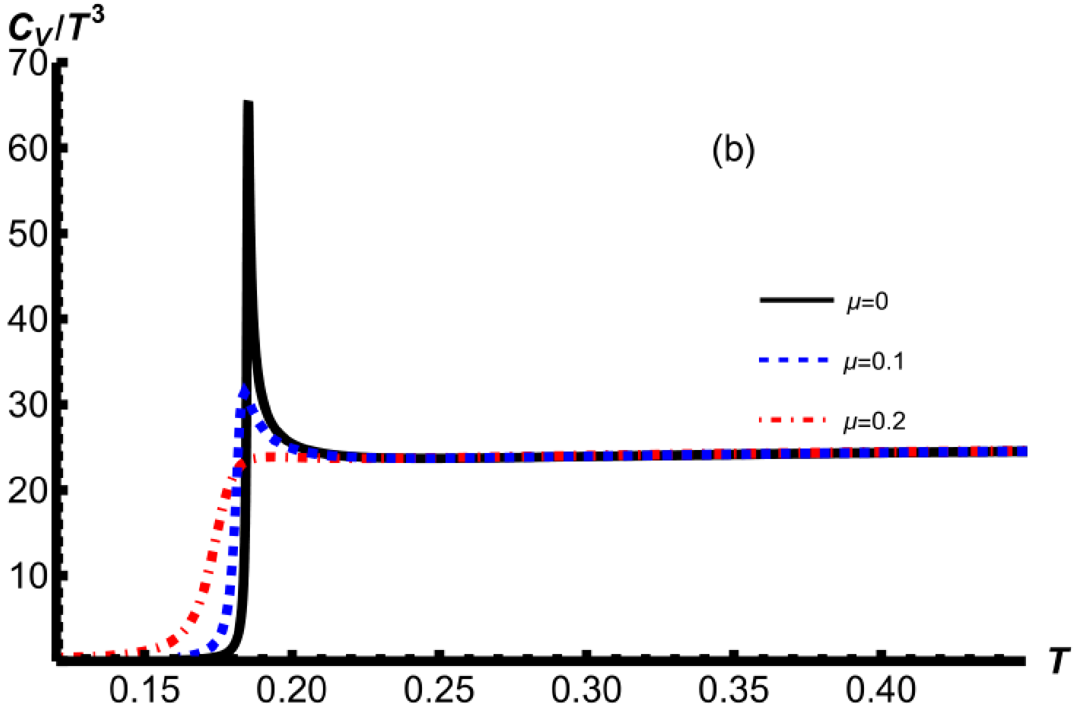


EoS at finite chemical potential for 2 flavor

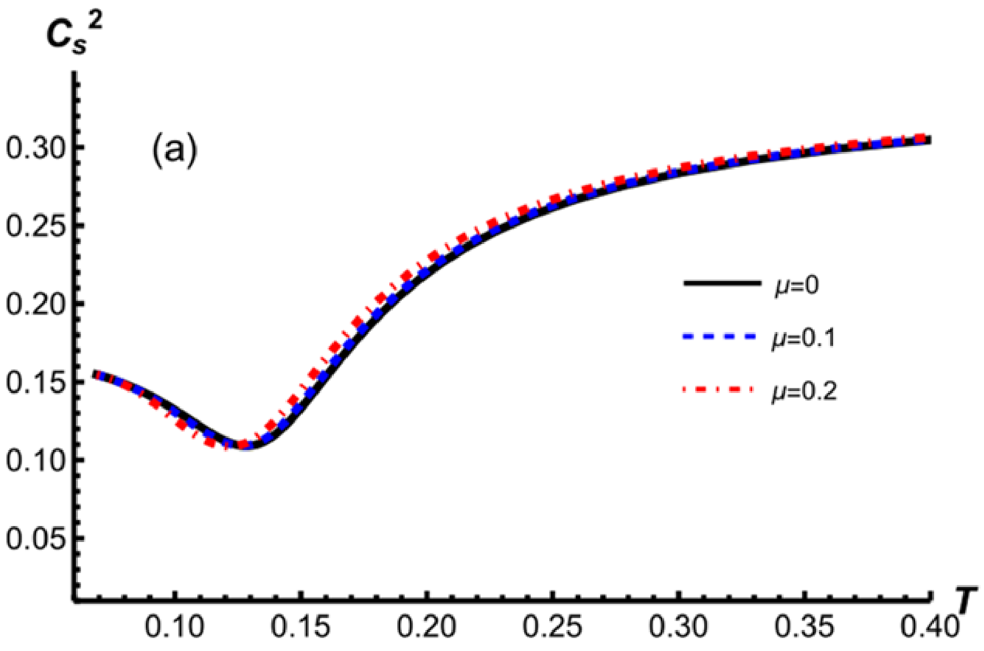


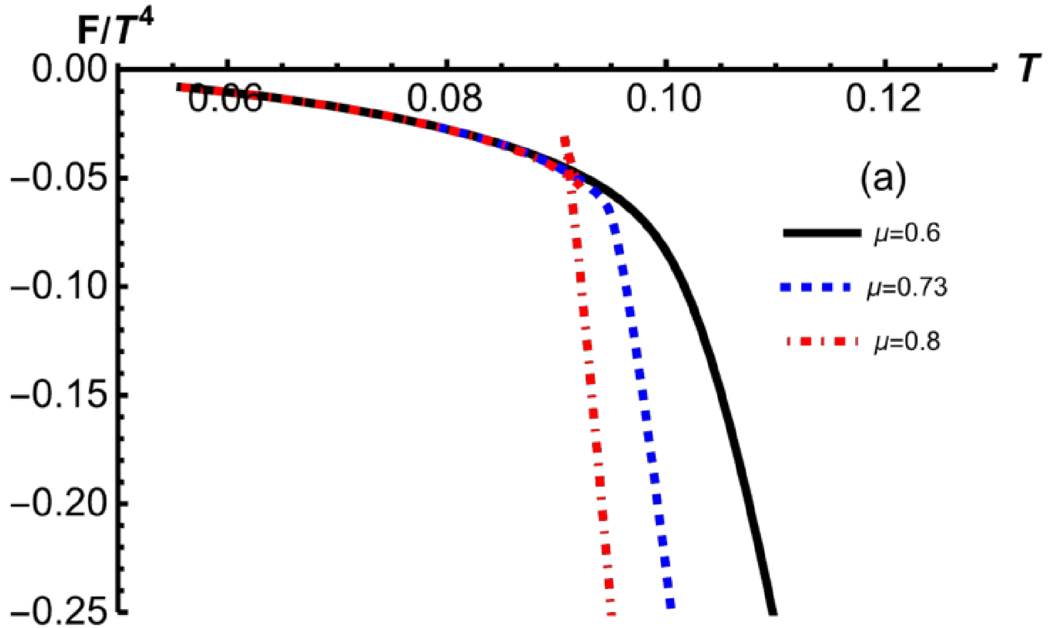
The square of the speed of sound and the specific heat at finite chemical potential for 2 flavor



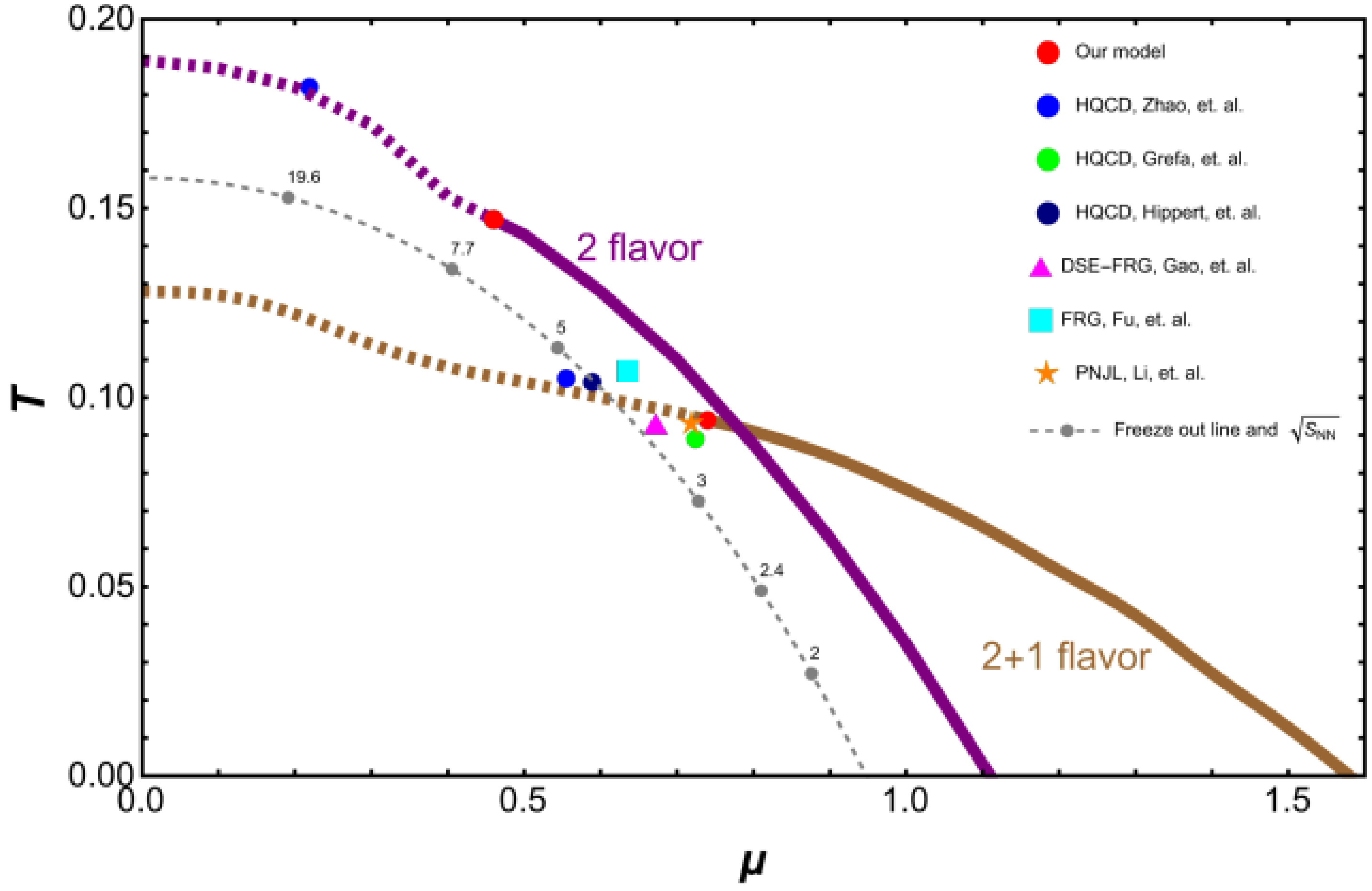


Determine the phase transition temperature





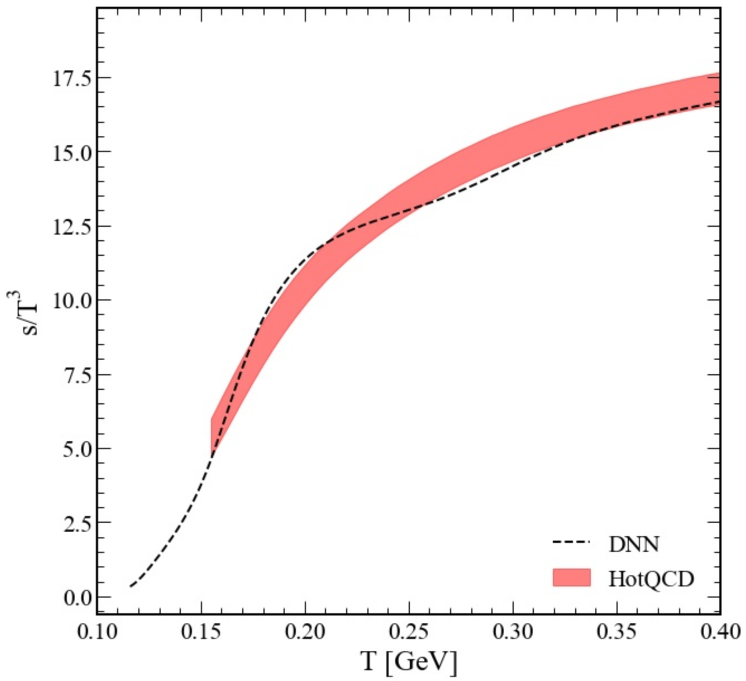
QCD Phase diagram

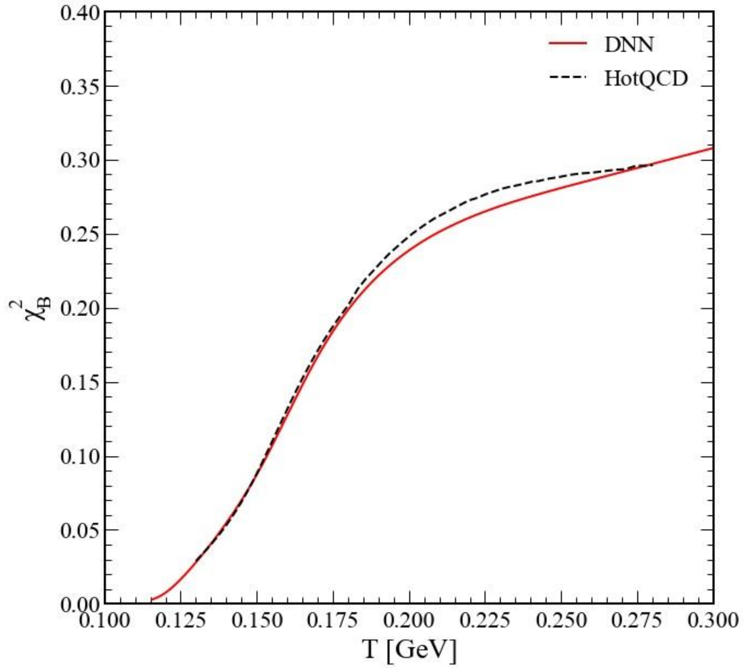


An idea

$$A(z) = d\ln(az^2+1) + d\ln(bz^4+1) \qquad f(z) = e^{cz^2 - A(z) + k}$$

Collaborates : Fu-peng Li, Long-Gang Pang, Defu Hou

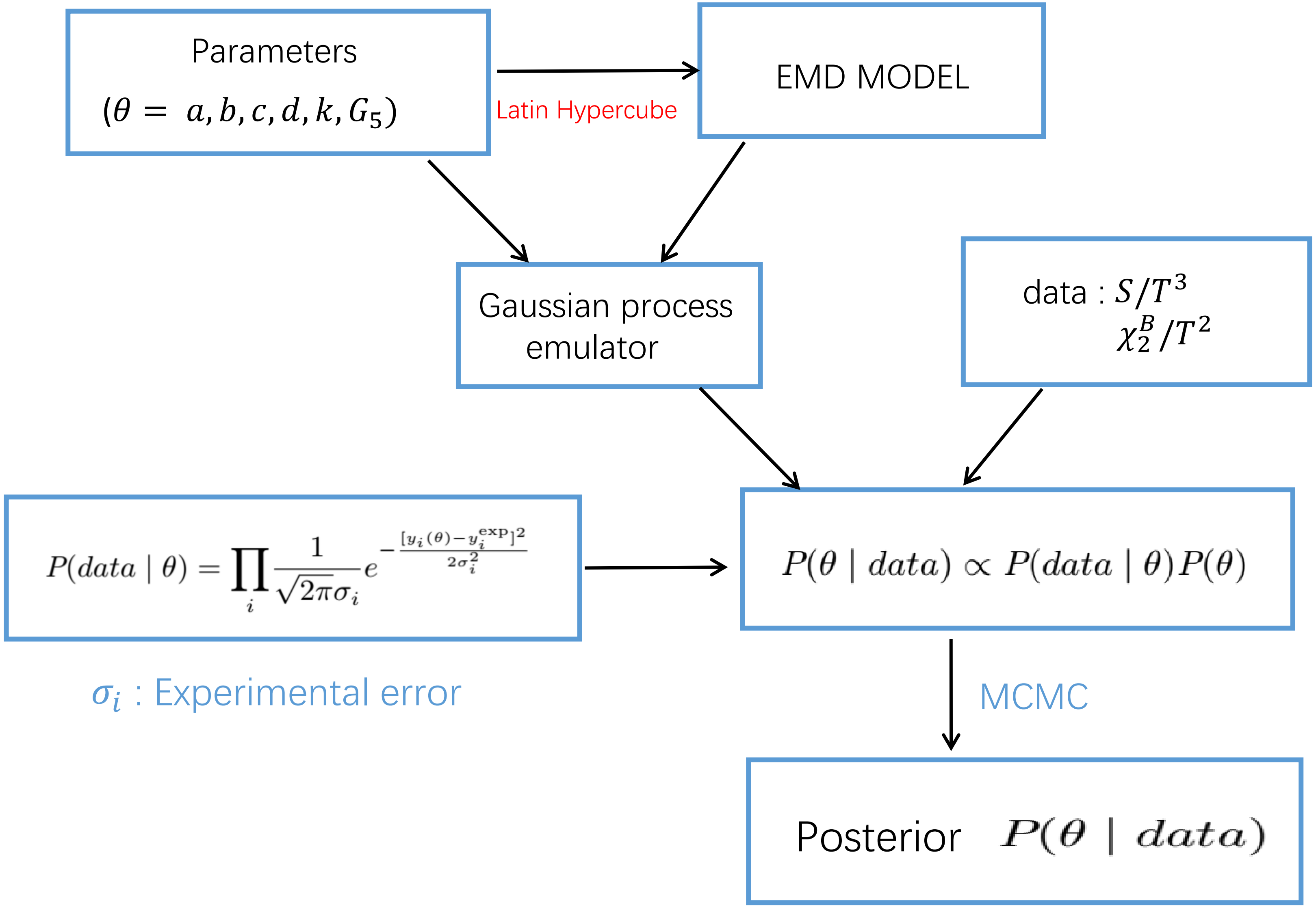




Bayesian location of the critical endpoint from a holographic perspective in 2+1 flavor QCD

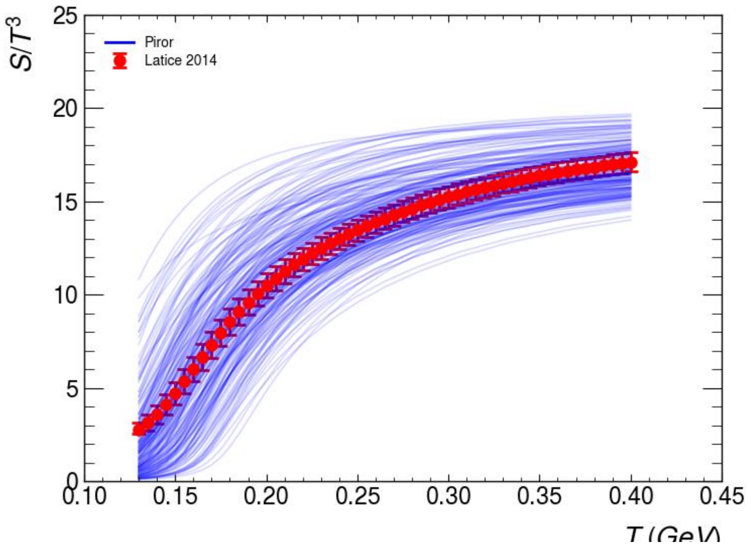
Xun Chen, Liqiang Zhu, Hanzhong Zhang, Kai Zhou, Mei Huang

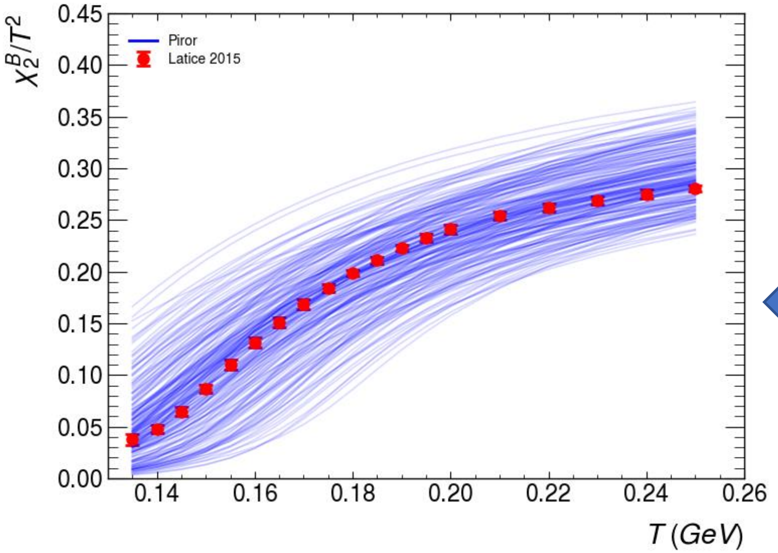
Bayesian analysis



Calibration of the EMD calculation

Piror





Piror paraments range:

a： 0.005 ~ 0.180

b： 0.005 ~ 0.027

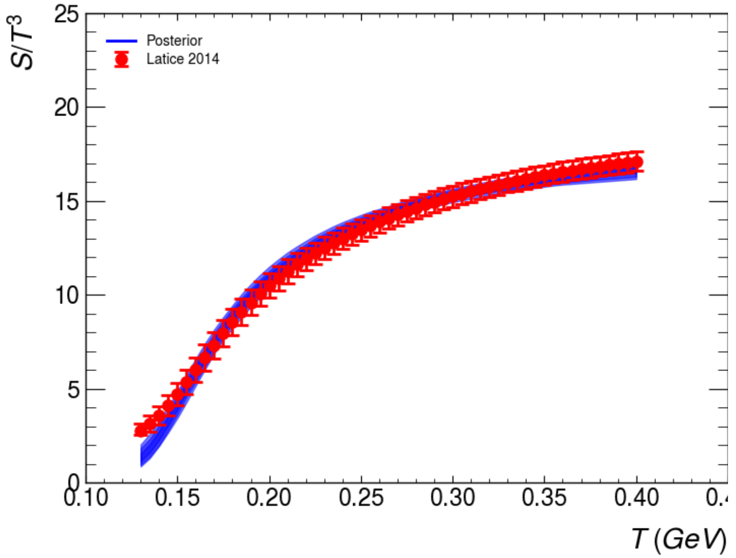
c： -0.260 ~ -0.170

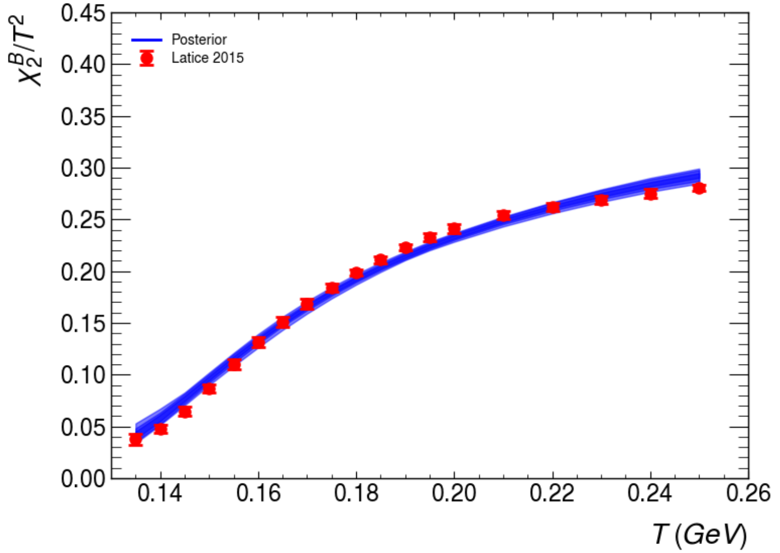
d： -0.270 ~ -0.160

k： -0.940 ~ 0.780

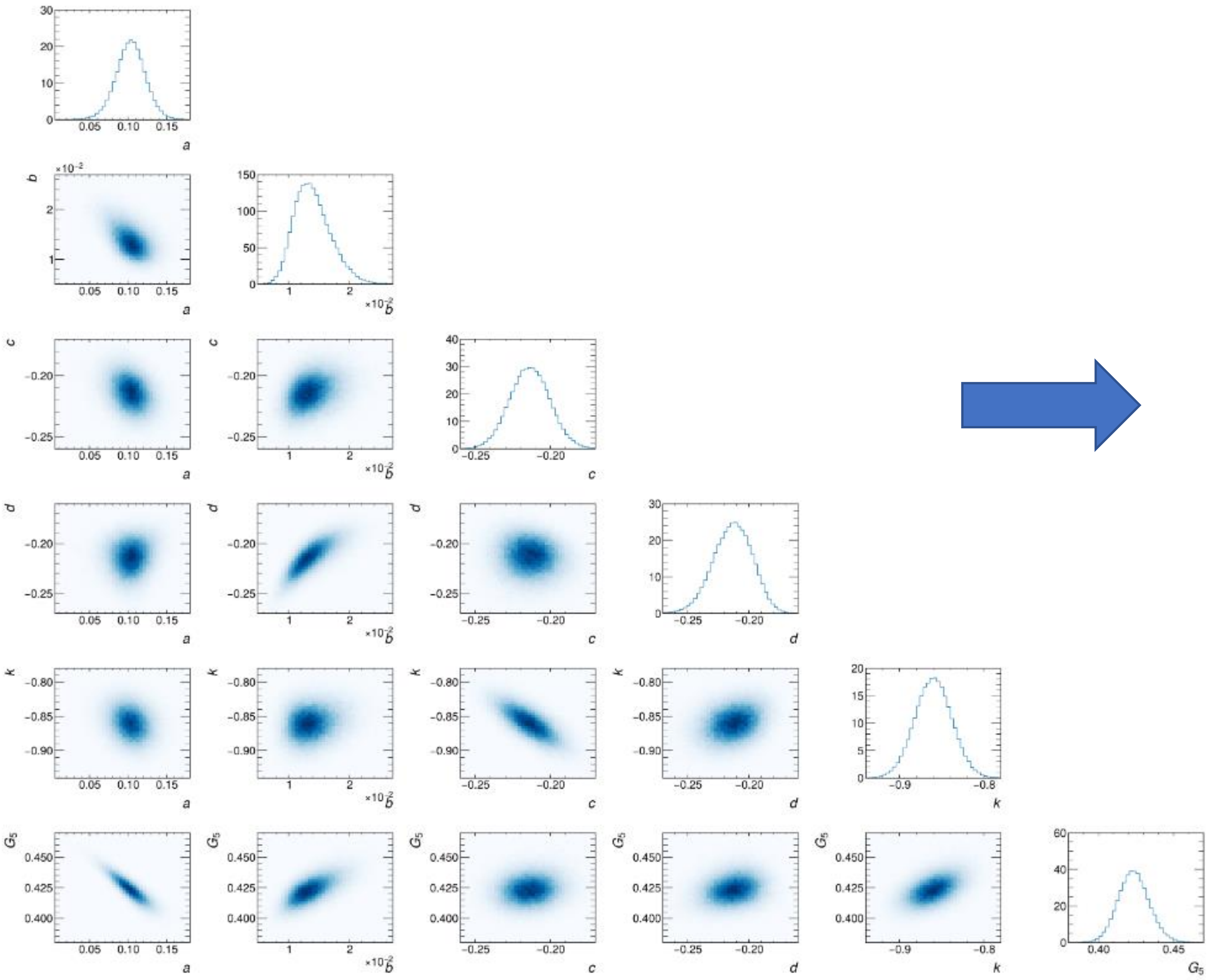
G_5： 0.380 ~ 0.470

Posteiror





Posterior distributions of the model parameters



Over 200 sets of θ for prior calculation.

Posterior distributions of the model parameters, together with their correlations.

Maximum a posteriori (MAP) values for parameters:

a：0.103

b：0.013

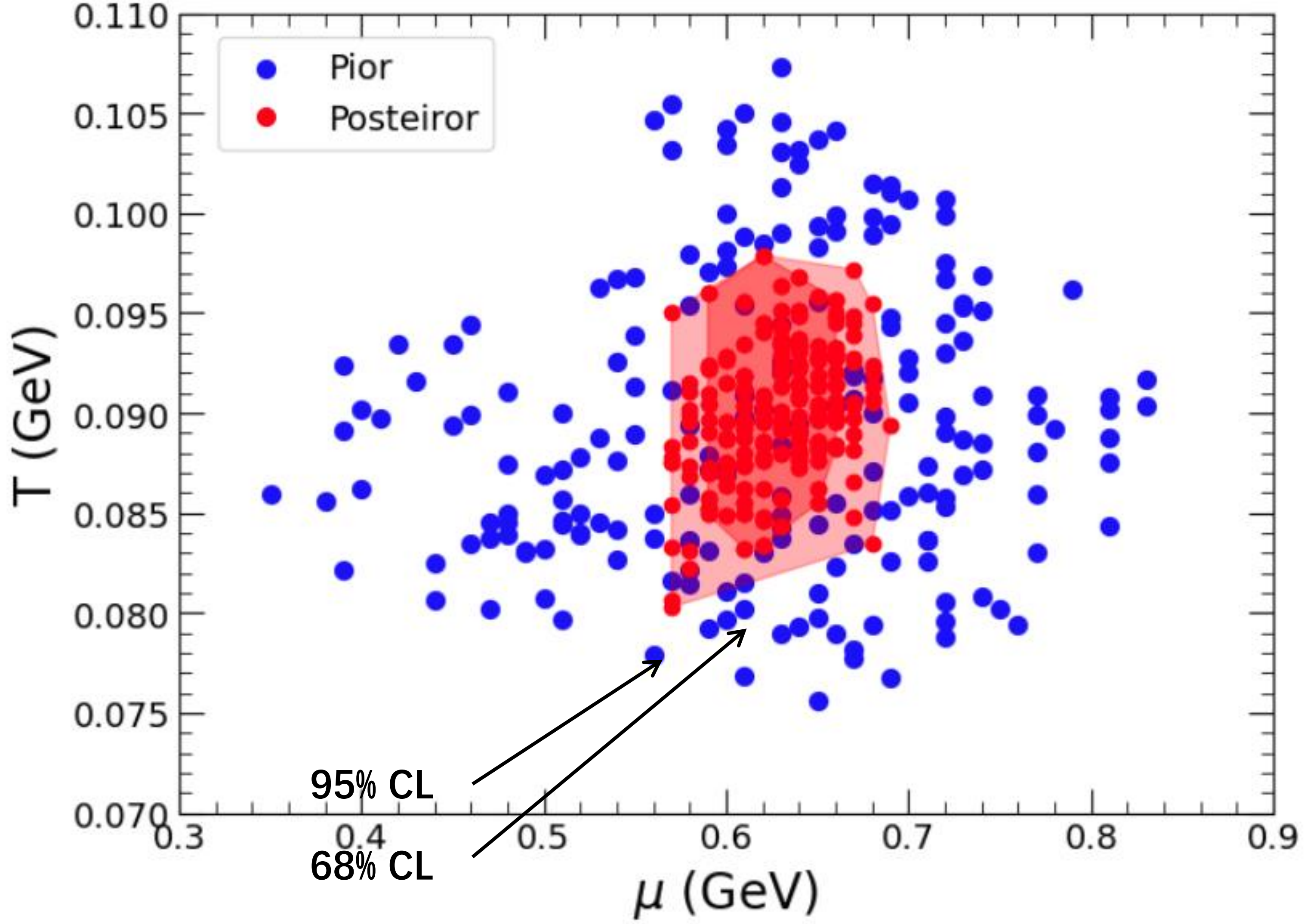
c：-0.213

d：-0.212

k：-0.860

G_5：**0.423**

Bayesian location of the QCD critical endpoint



Predictions for the QCD critial endpointfor the piror and posteiror:

Piror: blue dots

Posteiror: red dots represent respectively 68% and 95% confidence levels

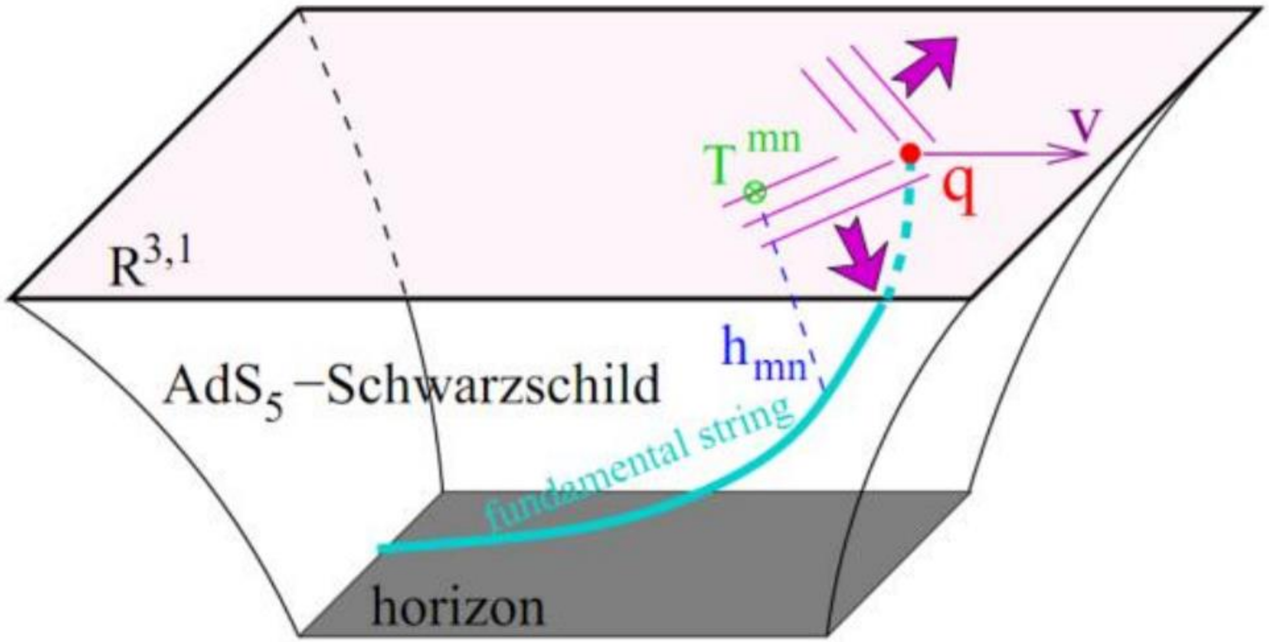
➢ Transport Properties of Quark-Gluon Plasma

Exploring Transport Properties of Quark-Gluon Plasma with a Machine-Learning assisted Holographic Approach

Bing Chen, Xun Chen, Xiaohua Li, Zhou-Run Zhu, Kai Zhou
ArXiv: 2404.18217

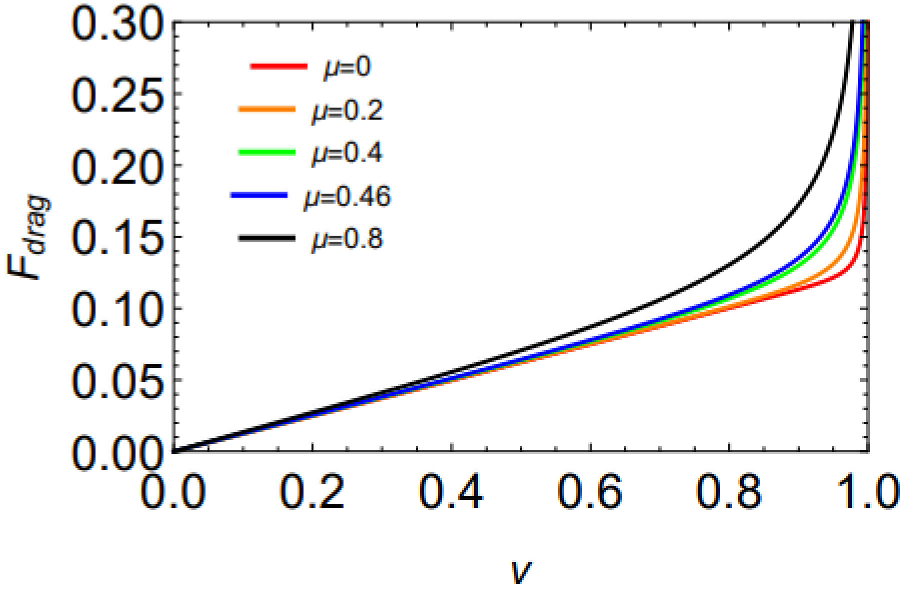
Drag force in the QGP

Quark with constant velocity

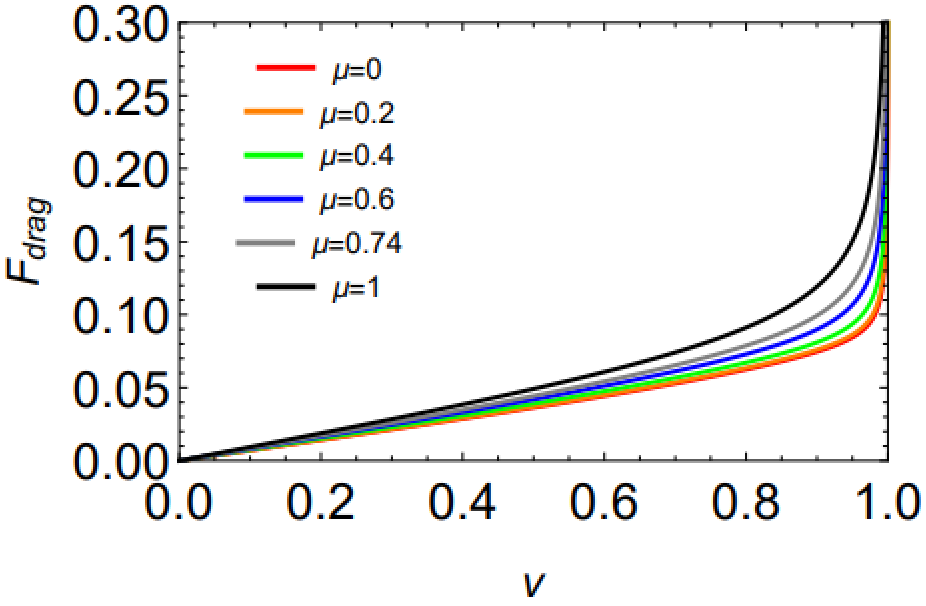


Drag force in the QGP

2 flavor



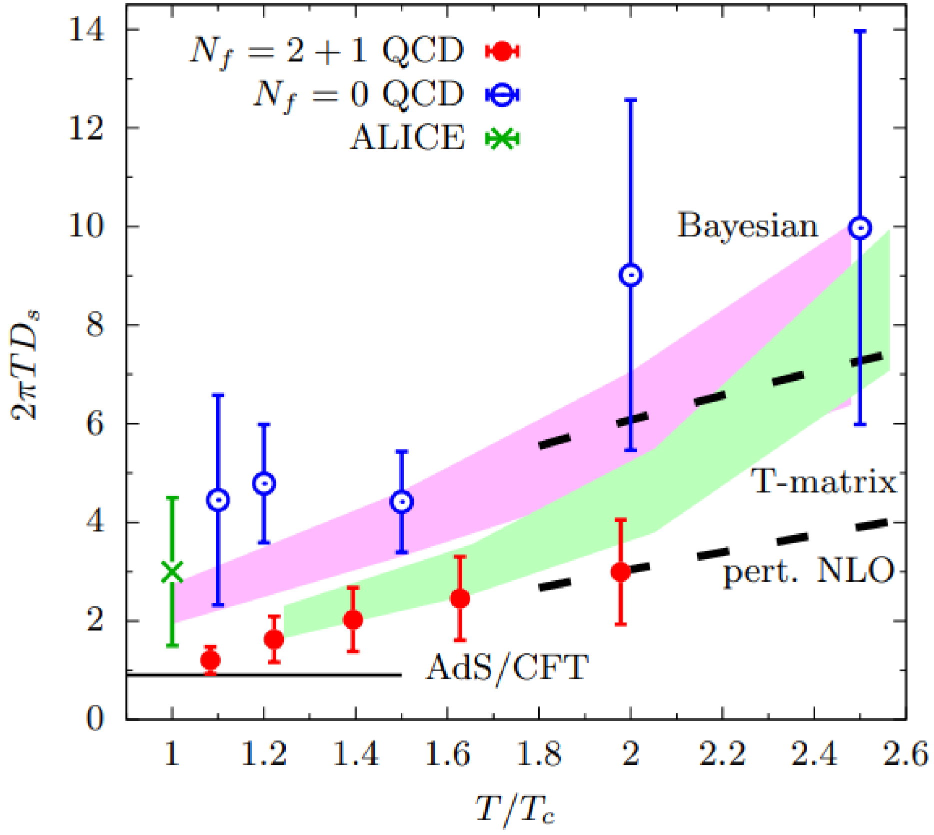
2+1 flavor

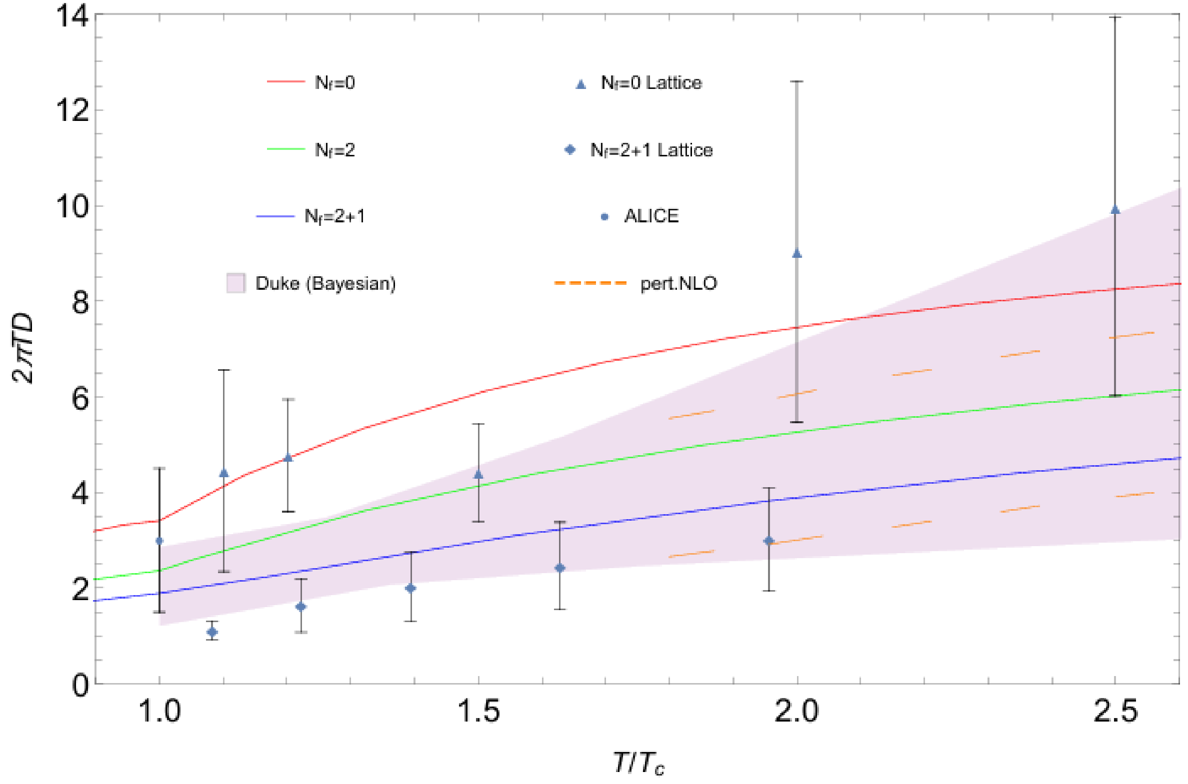


The diffusion coefficient D

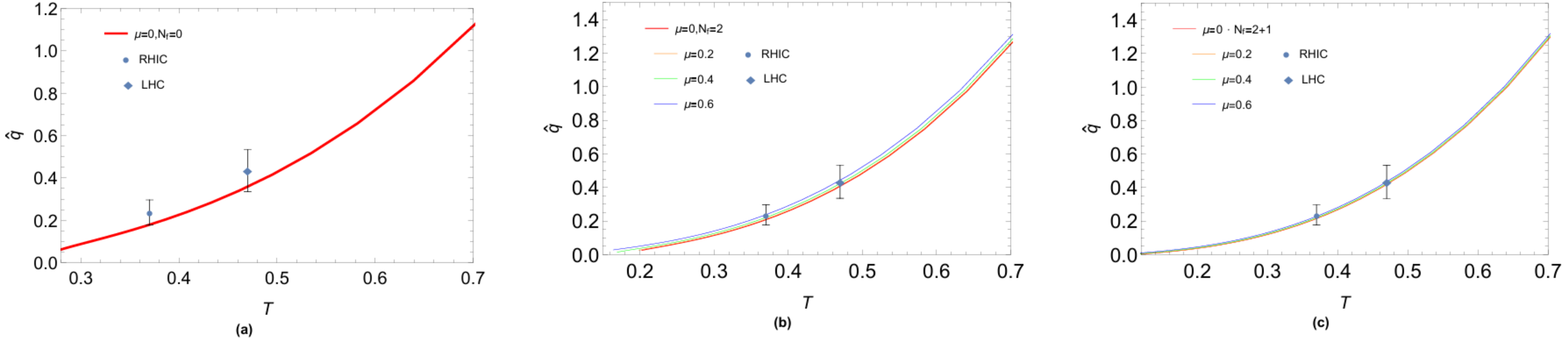
HotQCD, Phys. Rev. Lett. 130 (2023) 23, 231902

ArXiv: 2404.18217





Jet quenching parameter



Dots with error bars represent experimental values from RHIC and LHC.

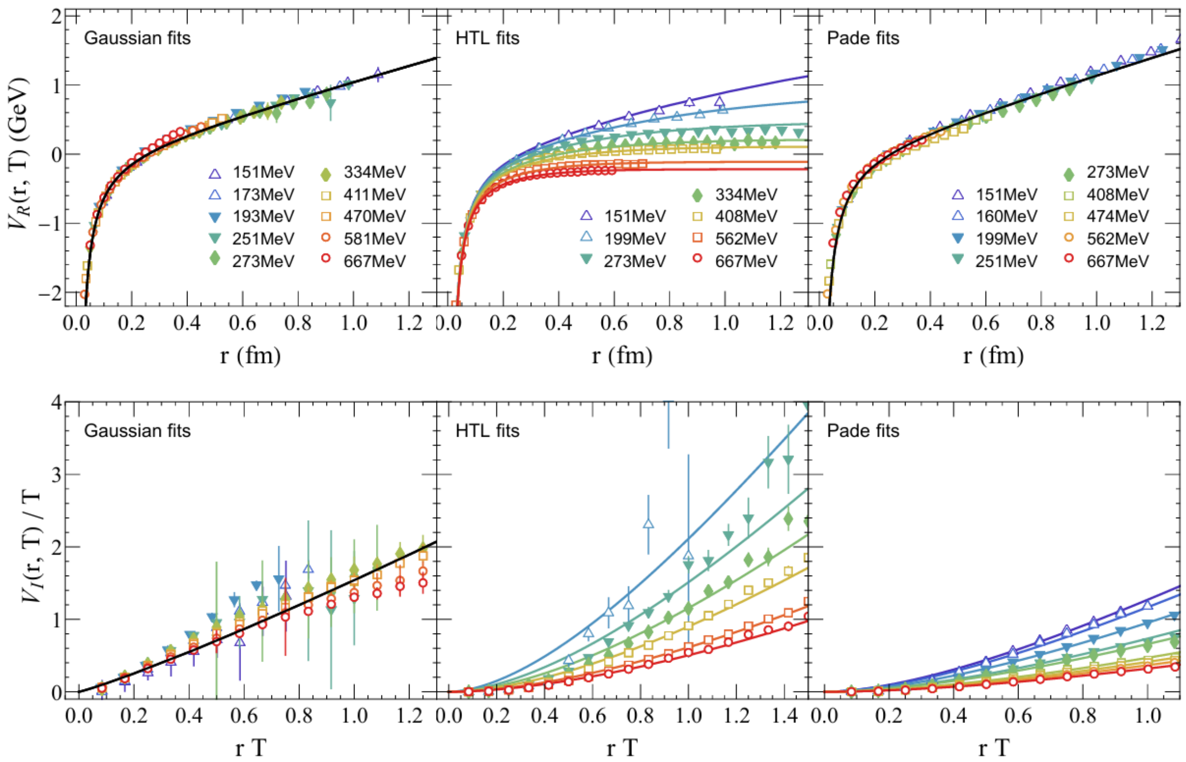
K. M. Burke et al. [JET Collaboration], Phys. Rev. C 90, no. 1, 014909 (2014)

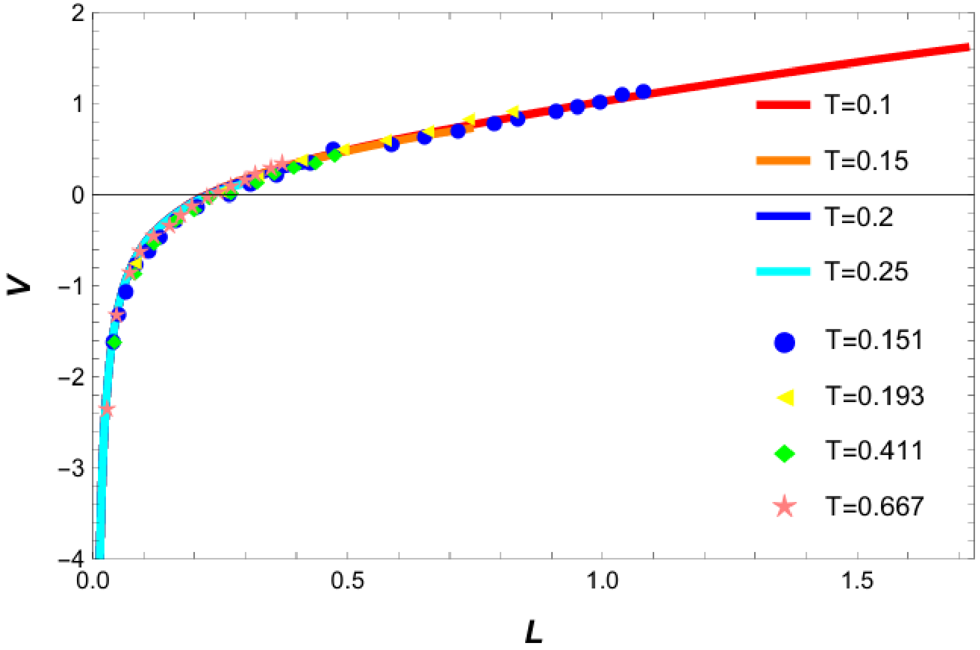
➢ Heavy-quark potential

The Potential Energy of Heavy Quarkonium in Flavor-Dependent Systems from a Holographic Model

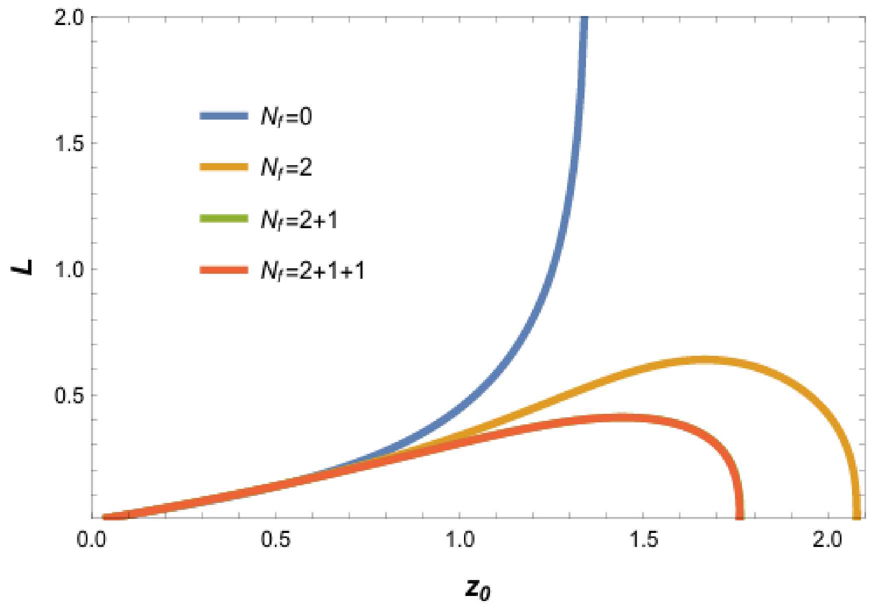
ArXiv: 2406.04650

ArXiv: 2402.11316 Lattice QCD

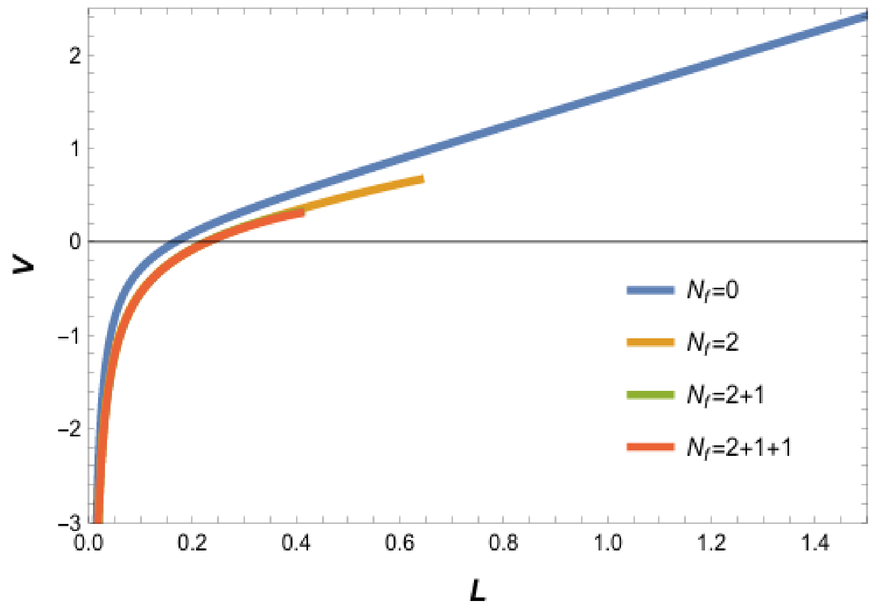




Flavor-Dependent Systems



(a)



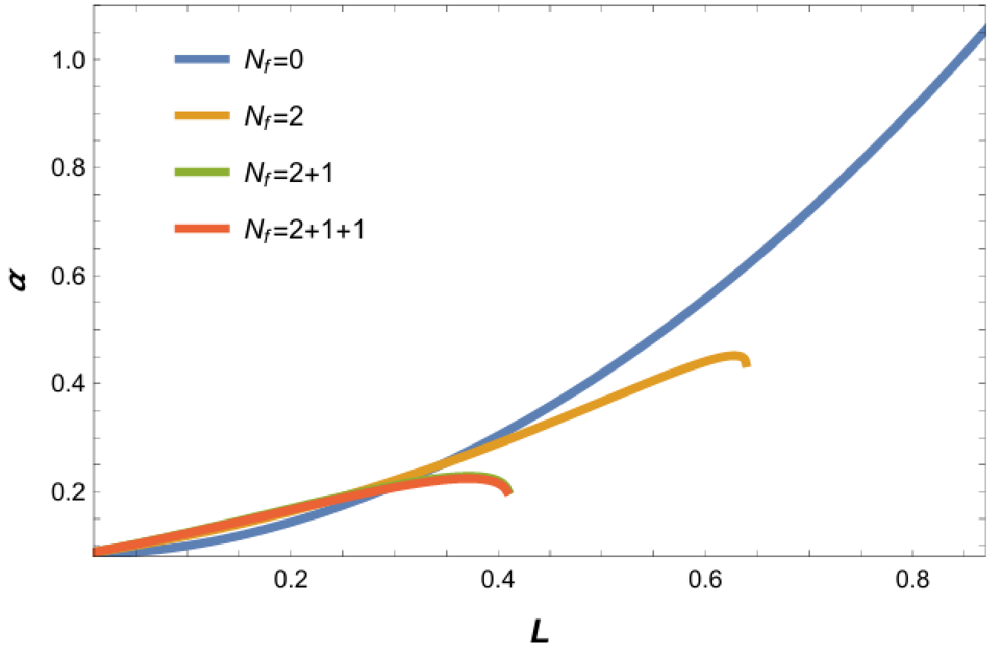
(b)

The force between two quarks

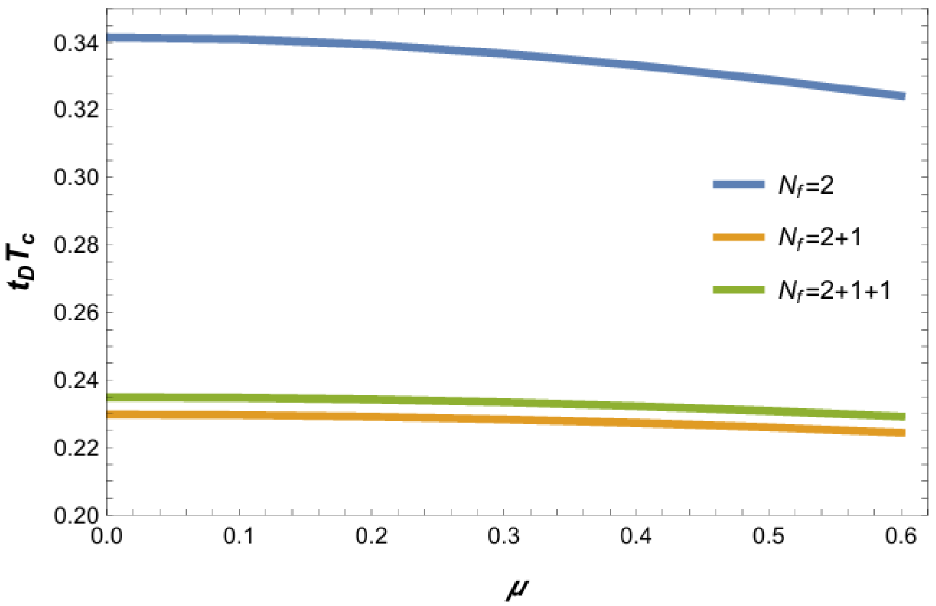
O. Kaczmarek and F. Zantow, Phys. Rev. D 71, 114510 (2005), arXiv:hep-lat/0503017.

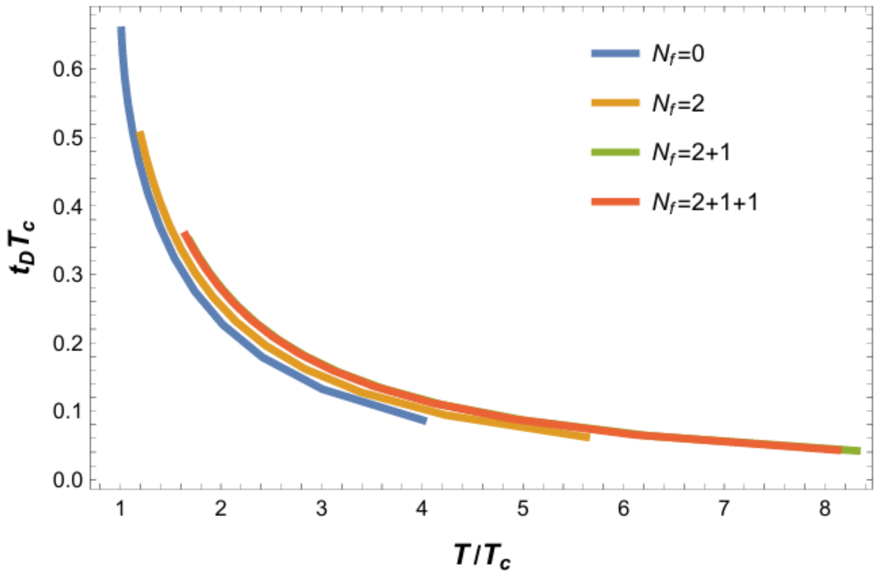
Effective running coupling

$$\alpha_{Q\bar{Q}} = \frac{3L^2}{4}\frac{\mathrm{d}V_{Q\bar{Q}}}{\mathrm{d}L}.$$



Real-time dynamics of quark dissociation



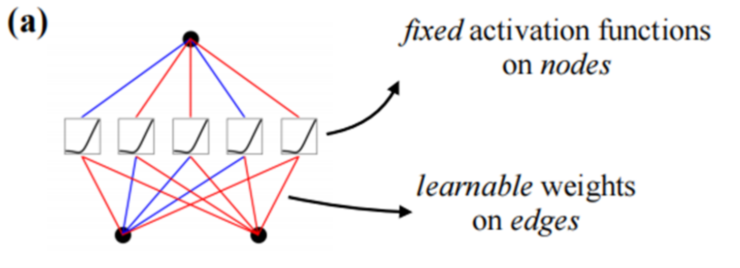
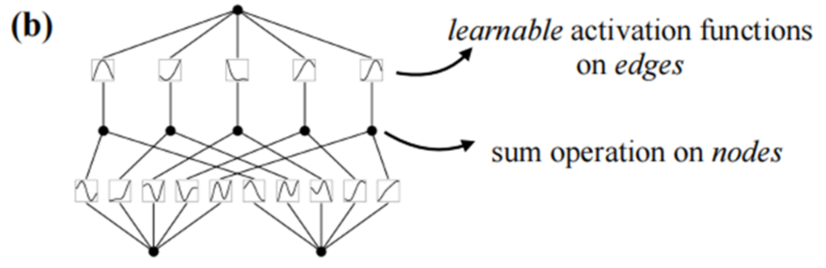
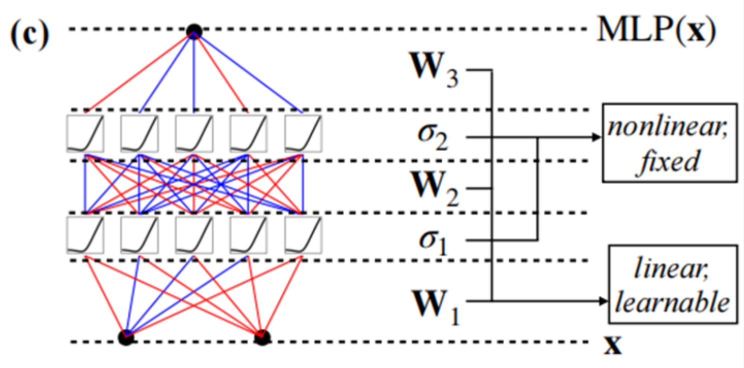
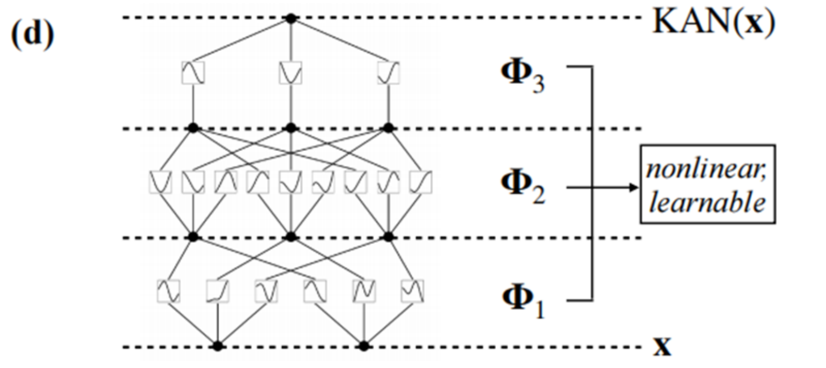


➢ Outlook

KAN: Kolmogorov-Arnold Networks

ArXiv: 2404.19756

刘子鸣

Model	Multi-Layer Perceptron (MLP)	Kolmogorov-Arnold Network (KAN)
Theorem	Universal Approximation Theorem	Kolmogorov-Arnold Representation Theorem
Formula (Shallow)	$f(\mathbf{x}) \approx \sum_{i=1}^{N(\epsilon)} a_i \sigma(\mathbf{w}_i \cdot \mathbf{x} + b_i)$	$f(\mathbf{x}) = \sum_{q=1}^{2n+1} \Phi_q \left(\sum_{p=1}^{n} \phi_{q,p}(x_p) \right)$
Model (Shallow)		
Formula (Deep)	$\mathrm{MLP}(\mathbf{x}) = (\mathbf{W}_3 \circ \sigma_2 \circ \mathbf{W}_2 \circ \sigma_1 \circ \mathbf{W}_1)(\mathbf{x})$	$\mathrm{KAN}(\mathbf{x}) = (\mathbf{\Phi}_3 \circ \mathbf{\Phi}_2 \circ \mathbf{\Phi}_1)(\mathbf{x})$
Model (Deep)		

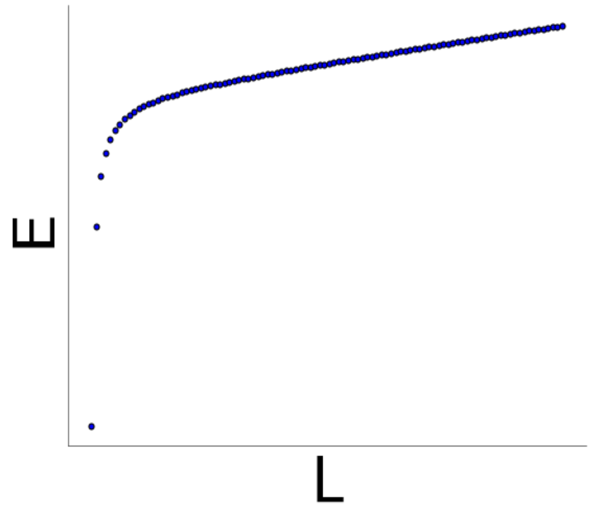
if f is a multivariate continuous functionon a bounded domain, then f can be written as a finite composition of continuous functions of a single variable and the binary operation of addition.

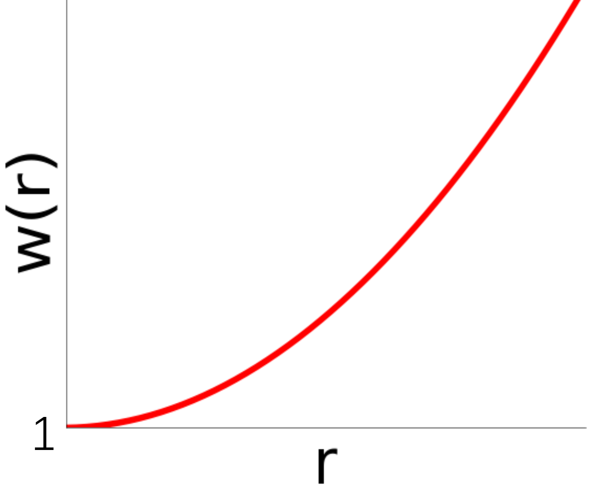
如果f是有界域上的多元连续函数，则f可以写成有限个连续函数的复合单变量和加法的二元运算。

$$loss = \frac{1}{m}\sum_{i=1}^{m}|h(r_i) - E_i|$$

Obeservations

Functions





$$L(r) = 2\int_0^{r_0}\sqrt{\frac{w^2(r_0)/r_0^4}{w^2(r)/r^4 - w^2(r_0)/r_0^4}}dr$$

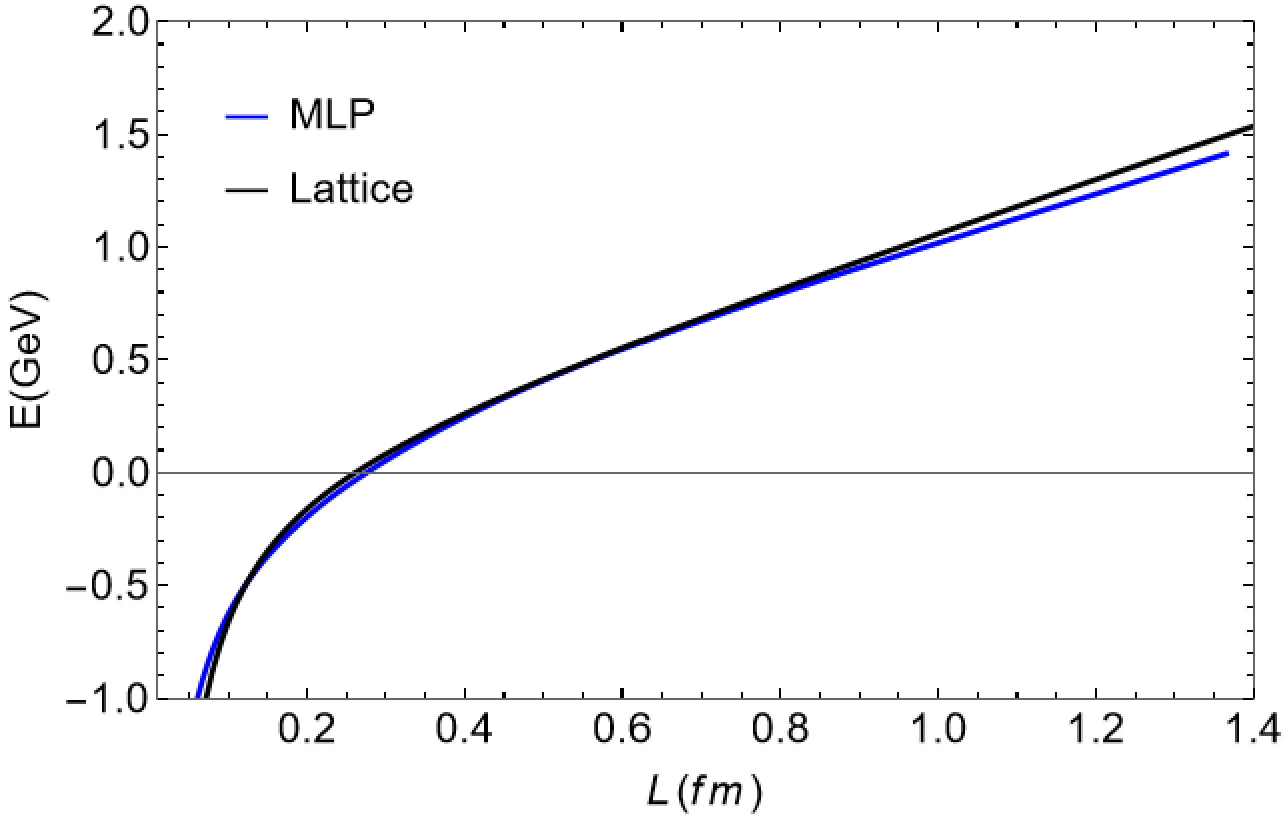
$$E(r) = 2g\int_0^{r_0}\frac{w(r)}{r^2}\sqrt{1+\frac{w^2(r_0)/r_0^4}{w^2(r)/r^4 - w^2(r_0)/r_0^4}} - \frac{1}{r^2}dr - 2\frac{g}{r_0}$$

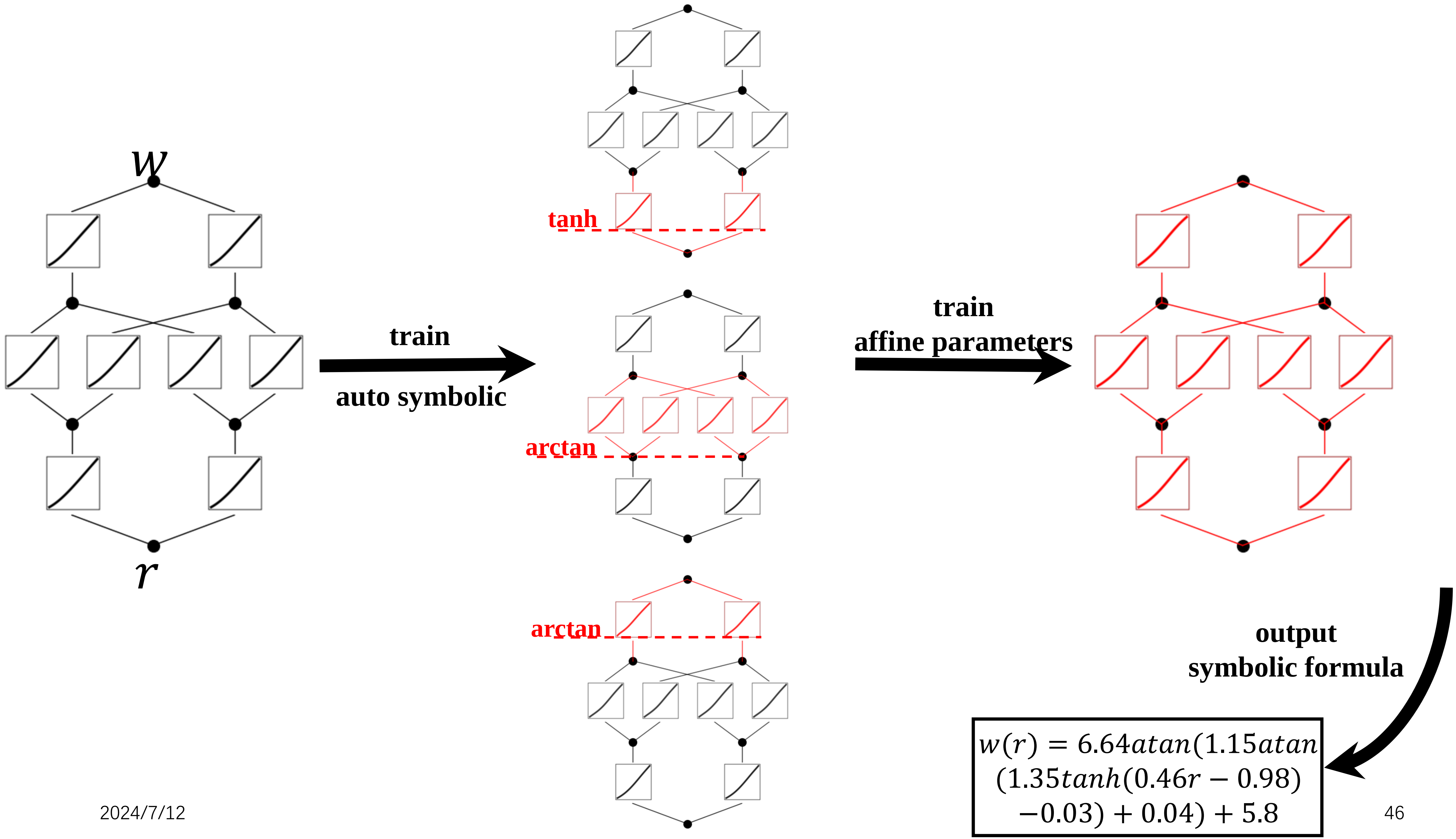
← BP

→ Forward

MLP: zero temperature

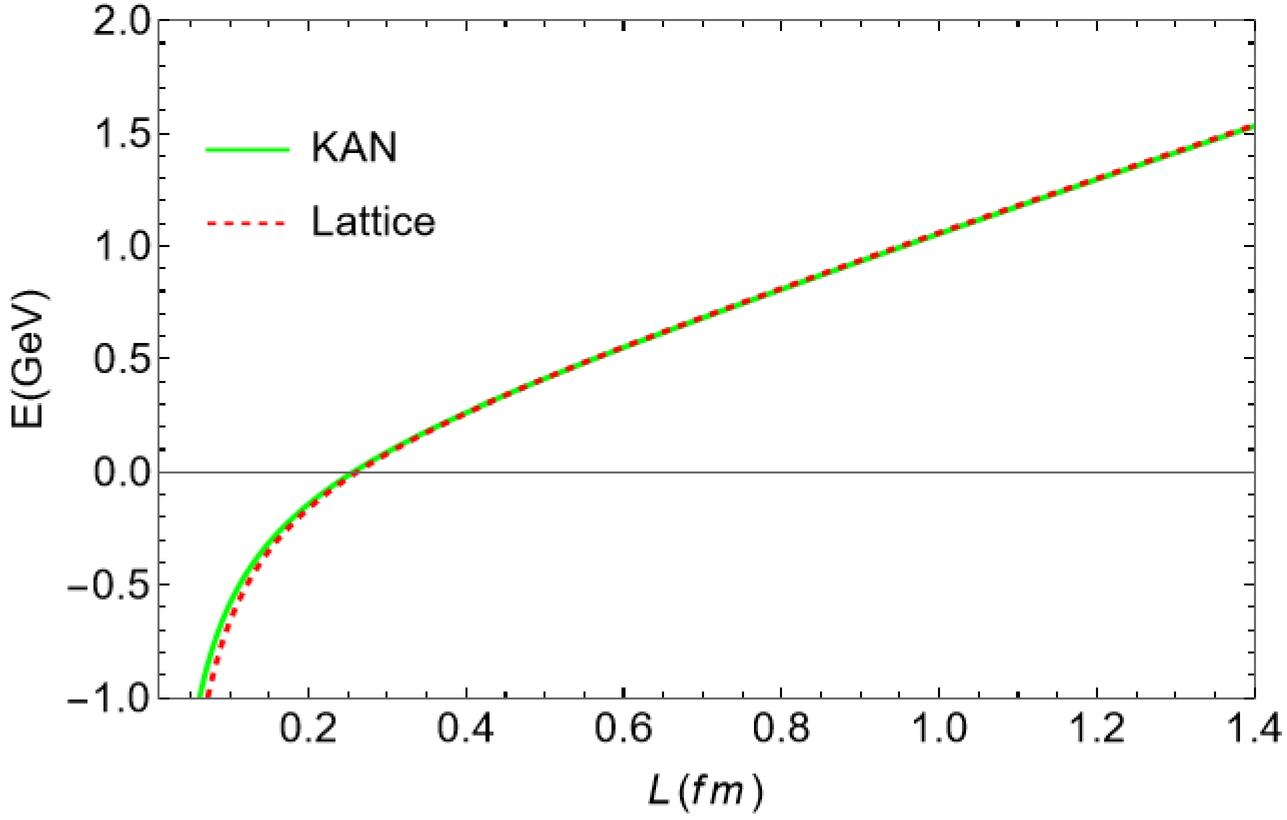
$$ds^2 = \frac{w(r)}{r^2}(dt^2 + d\vec{x}^2 + dr^2)$$





$$w(r) = 6.64atan(1.15atan(1.35tanh(0.46r - 0.98) - 0.03) + 0.04) + 5.8$$

$$w(r) = 6.64\arctan(1.15\arctan(1.35\tanh(0.46r - 0.98) - 0.03) + 0.04) + 5.8$$



Thank you for your time！

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