



北京航空航天大学
BEIHANG UNIVERSITY

Probing **radiative electroweak symmetry breaking** with colliders and gravitational waves

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Beihang University

2024.7.13 @第17届粒子物理、核物理和宇宙学交叉学科前沿问题研讨会

with Wei Liu, [arXiv: 2407.xxxxx](https://arxiv.org/abs/2407.xxxxx) (maybe next week)

Mystery of electroweak symmetry breaking

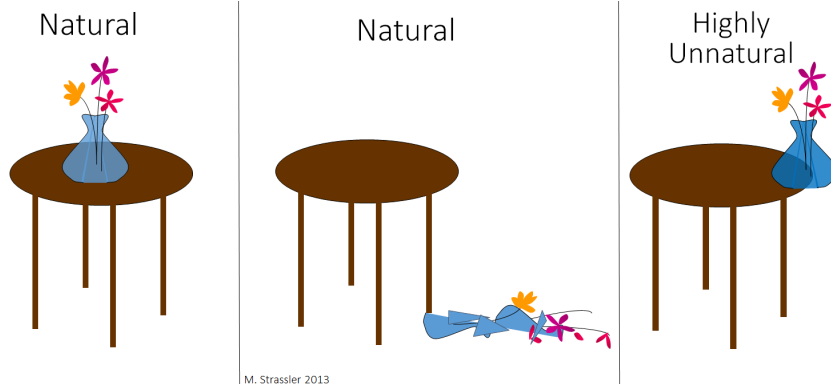
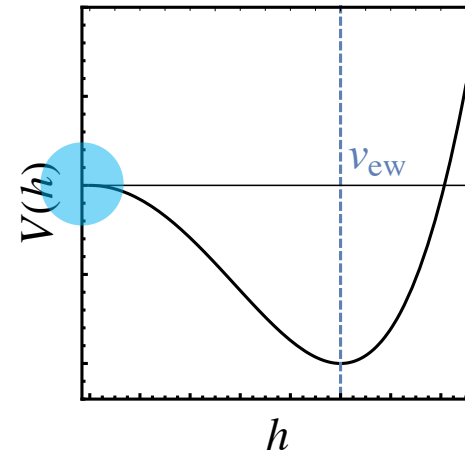
The Standard Model:

$$V(h) = \frac{\mu_h^2}{2} h^2 + \frac{\lambda_h}{4} h^4$$

Two issues:

1. What is **the origin** of $\mu_h^2 < 0$?
2. The **hierarchy problem**

$$\delta m_h^2 \sim \Lambda_{\text{NP}}^2 \gg (100 \text{ GeV})^2$$



... the flush toilet in a stately mansion.

S. Glashow



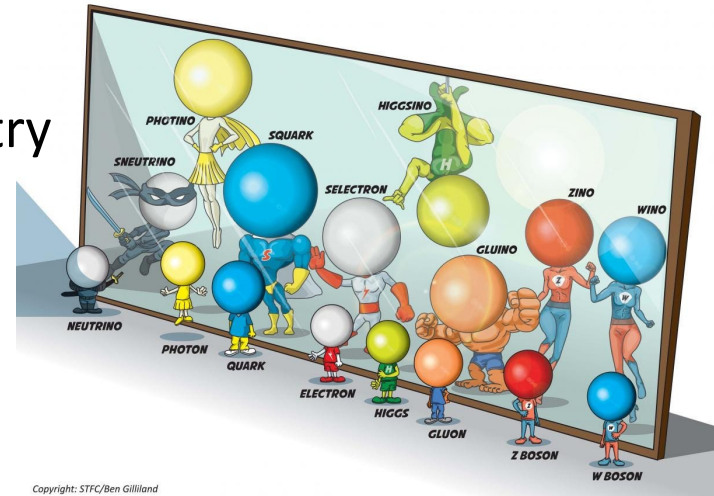
Efforts to solve the mystery

Supersymmetry

Cancellation guaranteed by symmetry

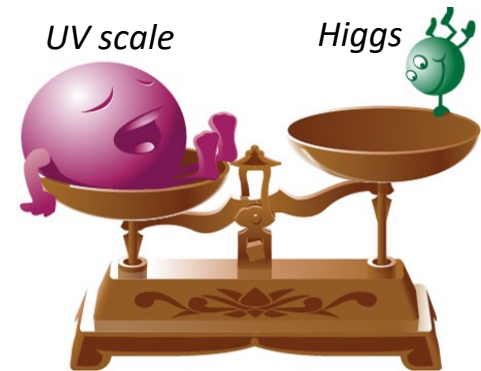
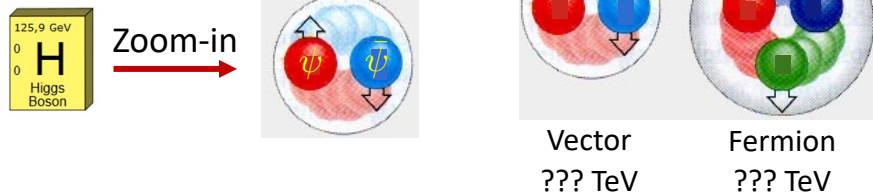
$$\delta\mu_B^2 - \delta\mu_F^2 = 0$$

Prediction: *sparticles*



Composite Higgs

The Higgs boson is a pNGB, like π mesons in the QCD
Fundamental strong dynamics



Prediction: *composite resonances*

Radiative symmetry breaking [This talk]

Radiative symmetry breaking

Originates from Coleman & Weinberg [\[Phys.Rev.D 7 \(1973\) 1888-1910\]](#)

Example: **massless scalar** QED

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_{\mu}S^{\dagger}D^{\mu}S - \lambda|S|^4$$

$$S = (\phi + i\eta)/\sqrt{2} \text{ and } D_{\mu} = \partial_{\mu} + ieA_{\mu}$$

Radiative symmetry breaking

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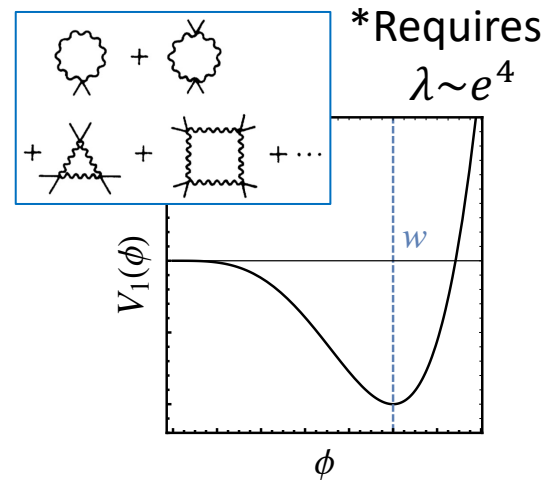
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One-loop level

$$V_1(\phi) = \frac{3e^4}{32\pi^2} \phi^4 \left(\log \frac{\phi}{w} - \frac{1}{4} \right)$$

Spontaneous symmetry breaking!

$$m_{\phi} = \sqrt{6}e^2w/4\pi \text{ and } m_A = ew$$



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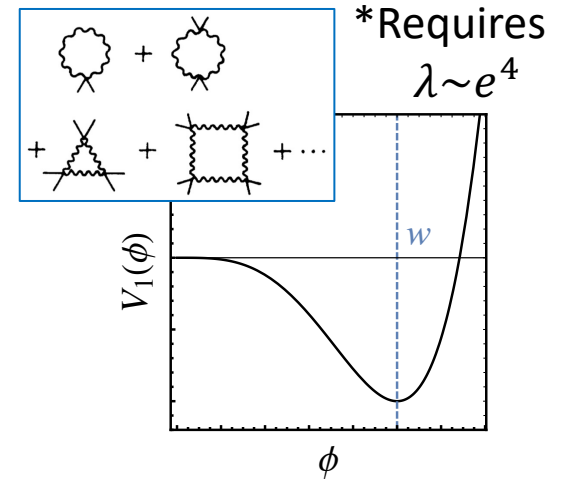
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Other perspectives

1. Scale anomaly, or **conformal anomaly**, or trace anomaly
2. Dimensional transmutation – from λ to w

Application to the EWSB

Get rid of the **quadratic divergence**

[Bardeen, FERMILAB-CONF-95-391-T; Meissner *et al*, PLB 660 (2008) 260-266]

Can it be **directly** applied to the SM **Higgs**? **No!**

1. EW W & Z induces: $\frac{m_h^2}{m_W^2} \sim \frac{g_W^2}{16\pi^2} \Rightarrow m_h \lesssim 10 \text{ GeV}$

2. Including t : $V_1(h) \sim B_h h^4 \log h$, with $B_h \sim (9g_W^2 - 48y_t^2) < 0$

Application to the EWSB

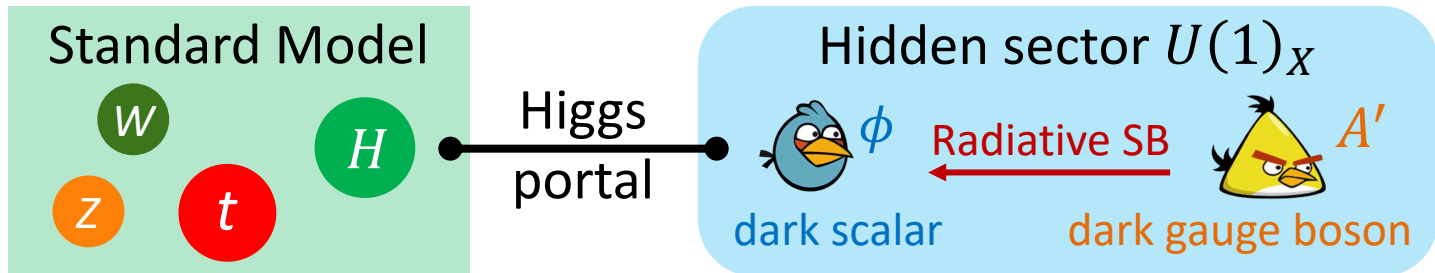
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A next-to-minimal approach [Hempfling, Phys.Lett.B 379 (1996) 153-158]



The framework of a realistic model

$$\text{Tree-level } V_0(H, S) = \lambda_h |H|^4 + \lambda_s |S|^4 - \lambda' |S|^2 |H|^2$$

Only dimensionless parameters

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Loop-level, unitary gauge

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Radiative symmetry breaking
at $w_0 \gg 100$ GeV via A'

$\langle \phi \rangle \approx w_0$

$$V_h(h) \approx -\frac{\lambda' w_0^2}{4} h^2 + \frac{\lambda_h}{4} h^4$$

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$$\langle \phi \rangle \approx w_0$$

$$V_h(h) \approx -\frac{\lambda' w_0^2}{4} h^2 + \frac{\lambda_h}{4} h^4$$

Taking $\lambda' \approx \frac{m_h^2}{w_0^2} \approx 1.6 \times 10^{-4} \times \left(\frac{10^4 \text{ GeV}}{w_0} \right)^2$ and $\lambda_h \approx \frac{m_h^2}{2v_{\text{ew}}^2} \approx 0.13$ --

Correct Higgs potential & EWSB get!

The prediction of this mechanism? [This talk]

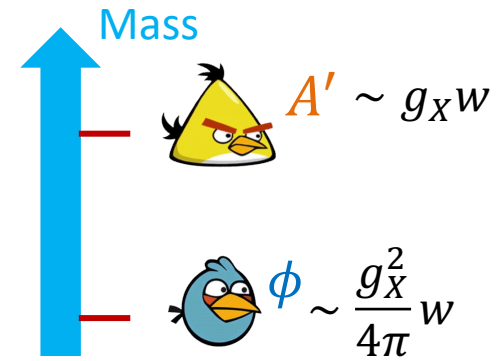
The distinctive feature

A **light** scalar compared to the NP scale

λ' portal -- h - ϕ **mixing**

Vacuum structure $(h, \phi) = (v_{ew}, w)$.

Only **two** free parameters!



The distinctive feature

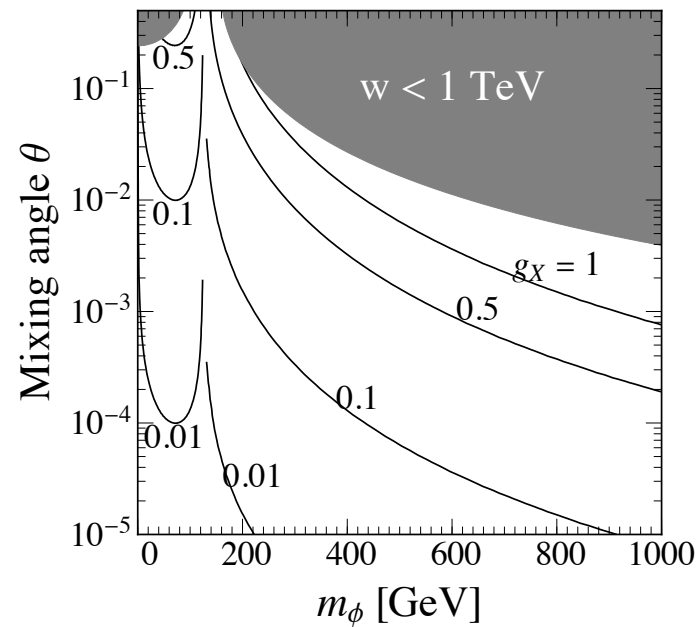
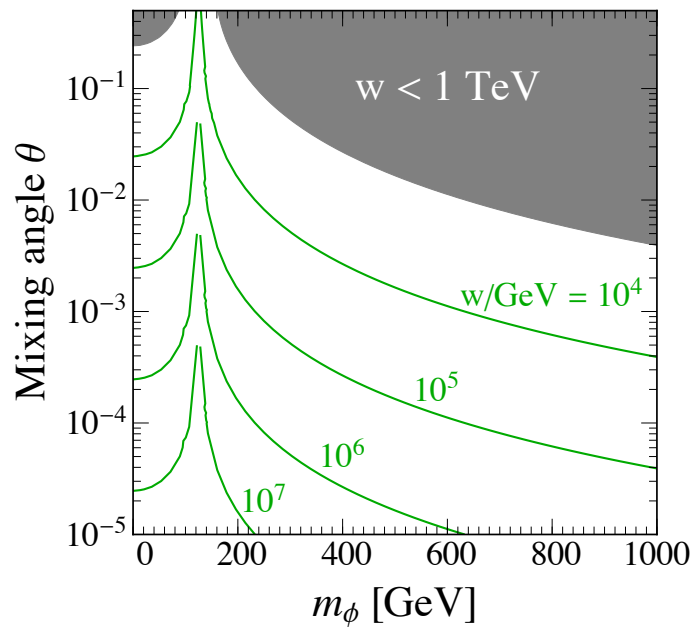
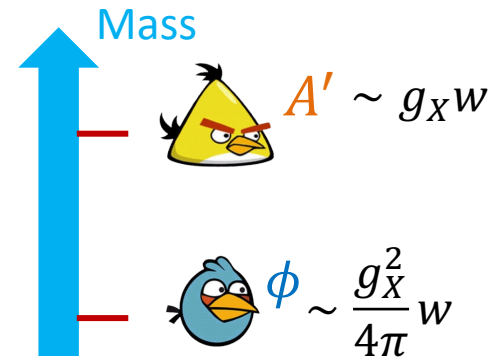
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Full expressions in our paper

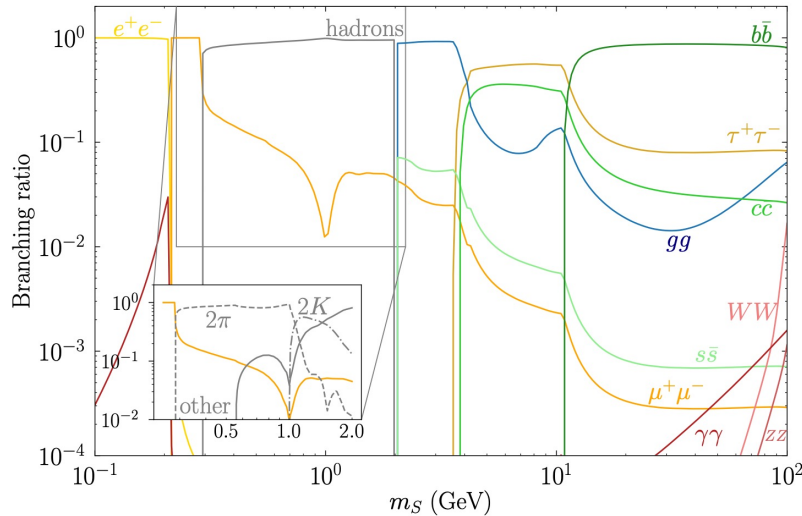


Searching for a singlet scalar mixing with the Higgs boson!

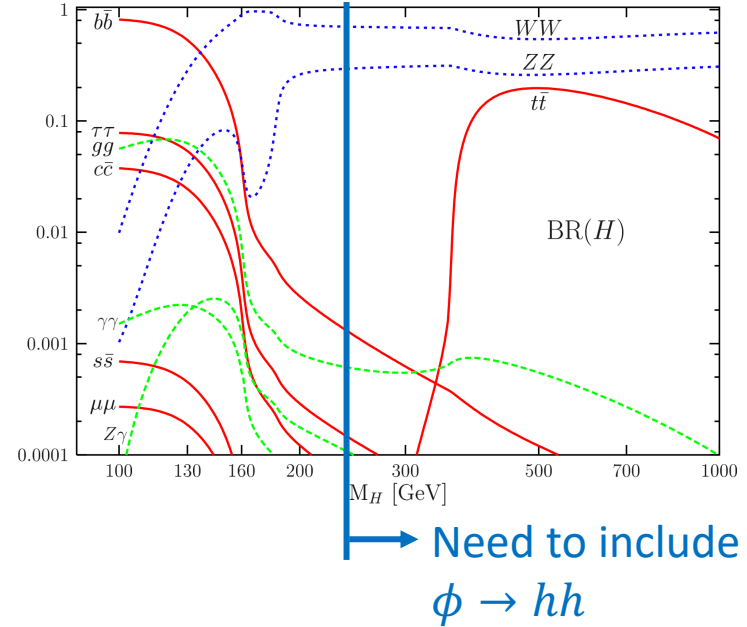
The particle experiment search

The decay branching ratios of ϕ

Gershtein et al, Phys.Lett.B 823 (2021) 136758



Djouadi, Phys.Rept. 457 (2008) 1-216

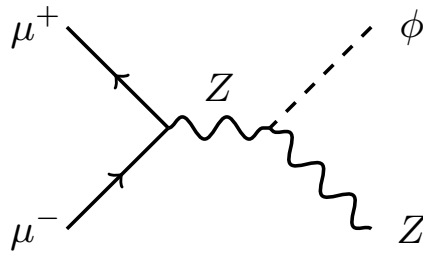


Landscape:

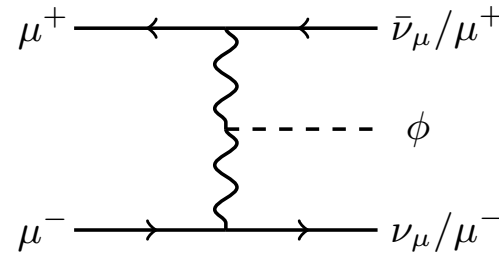
- $m_\phi \lesssim 4$ GeV: mesons, gg
- 4 GeV $\lesssim m_\phi \lesssim 10$ GeV: $\phi \rightarrow \tau^+\tau^-$
- 10 GeV $\lesssim m_\phi \lesssim 160$ GeV: $\phi \rightarrow b\bar{b}$
- 160 GeV $\lesssim m_\phi \lesssim 250$ GeV: $\phi \rightarrow VV$, with $V = W^\pm, Z$
- $m_\phi \gtrsim 250$ GeV: $\phi \rightarrow VV, hh$, or even $t\bar{t}$

Simulations at a 10 TeV muon collider

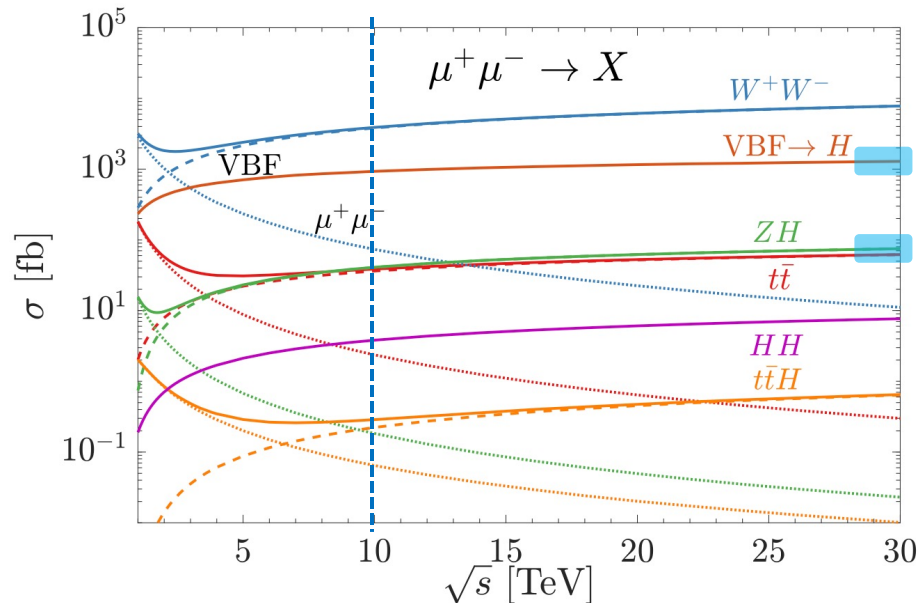
$Z\phi$ associated production



Vector boson fusion (VBF)



Cross section $\sigma = \sin^2 \theta \times \sigma_h(m_\phi)$ ← SM-like calculation



VBF dominates for multi-TeV colliders

Han *et al*, Phys.Rev.D 103 (2021) 3, L031301

The $\phi \rightarrow b\bar{b}$ channel

Parton-level simulation =
FeynRules model file +
MadGraph5 event generator +
10% jet energy smearing

Backgrounds

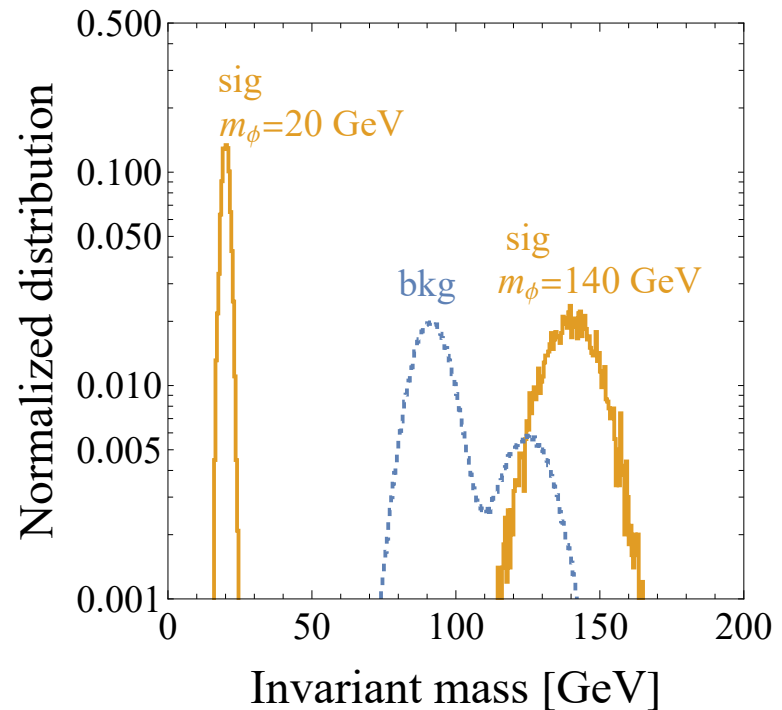
- VBF $\mu^+\mu^- \rightarrow jj$
- $\mu^+\mu^- \rightarrow Z/\gamma \rightarrow jj$ easily removed using $m(jj)$

Cuts:

- Two jets with $p_T > 30$ GeV and $|\eta| < 2.43$
- $m_{\text{recoil}} > 200$ GeV
- $|m(jj) - m_\phi| < 0.2 \times m_\phi$

Assuming a 70% b -tagging rate yields similar sensitivity

Similar treatment for $\phi \rightarrow \tau^+\tau^-$ with 90% τ -tagging rate



The $\phi \rightarrow VV$ channel

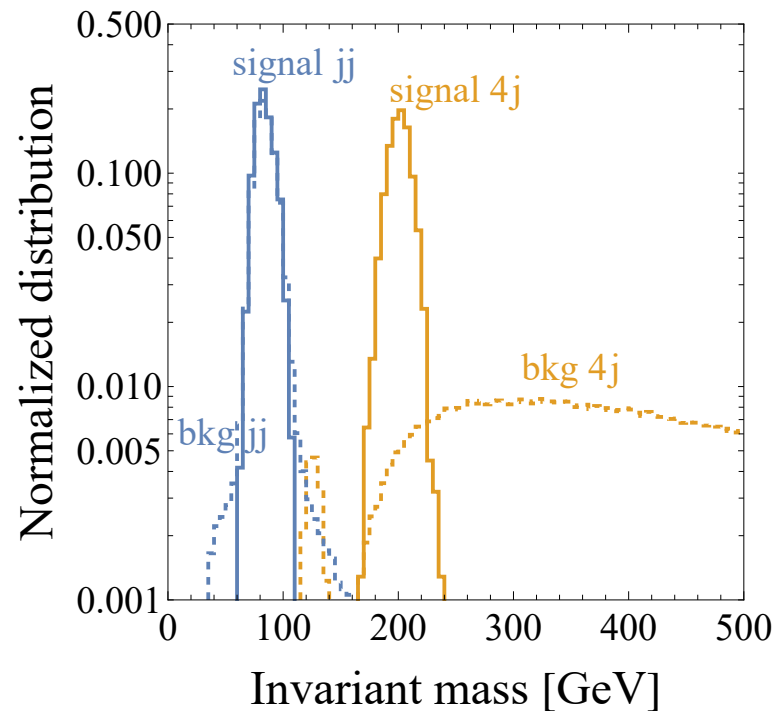
Backgrounds

- VBF $\mu^+ \mu^- \rightarrow VV \rightarrow jjjj$
- $\mu^+ \mu^- \rightarrow jjjj$ easily removed using $m(jjjj)$
- VBF $\mu^+ \mu^- \rightarrow jjjj$ via QCD splitting negligible after $m(jj)$ selection

Cuts:

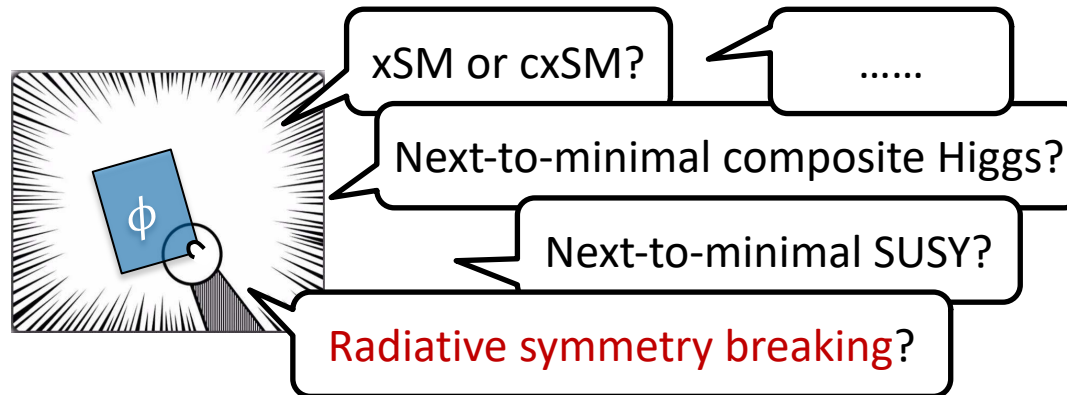
- Four jets with $p_T > 30$ GeV and $|\eta| < 2.43$
- $m_{\text{recoil}} > 200$ GeV
- Pairing jets to have V candidates with $|m(jj) - m_V| < 15$ GeV
- $|m(jjjj) - m_\phi| < 30$ GeV

Similar treatment for $\phi \rightarrow hh \rightarrow b\bar{b}b\bar{b}$ 70% b -tagging rate

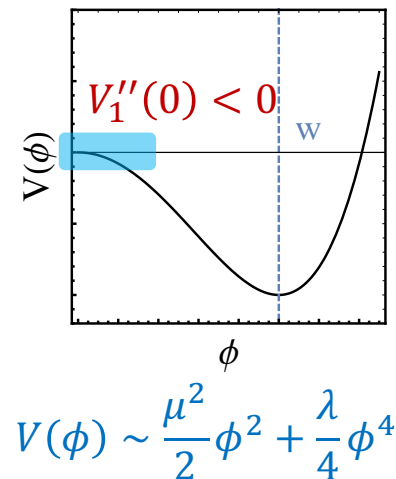
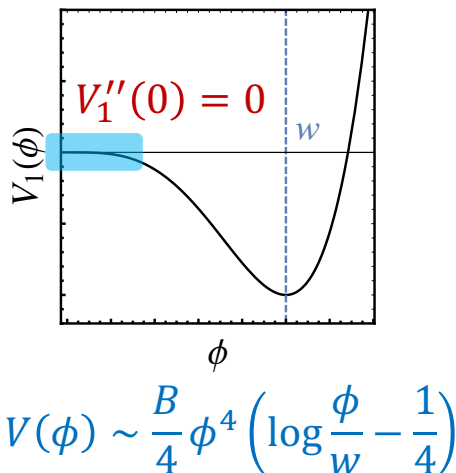


Another distinctive feature

A **new scalar** can be the consequence of **any** new physics model!

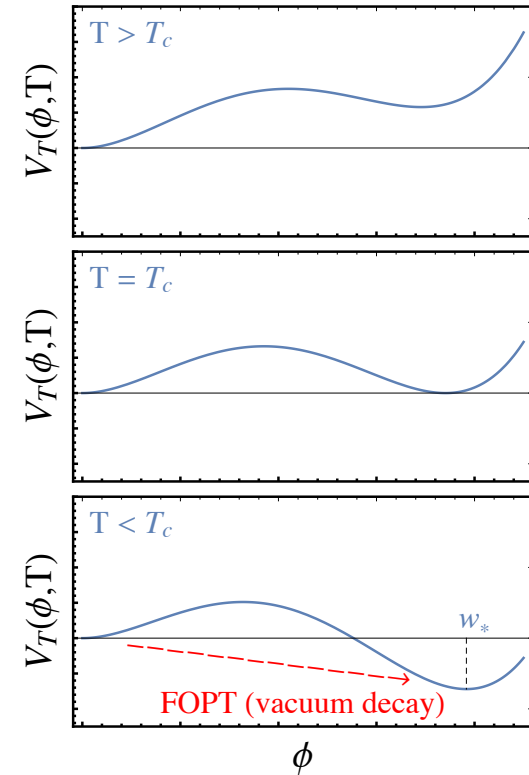
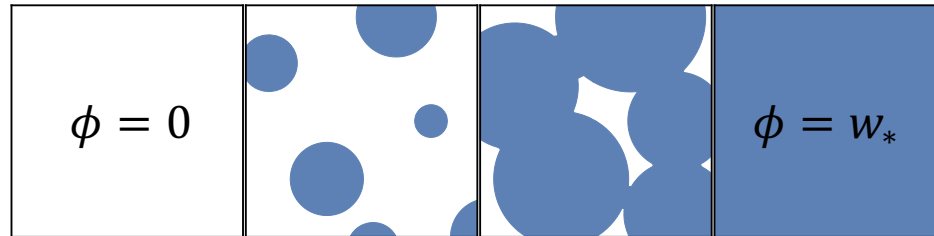


More evidences? The **special shape** of potential



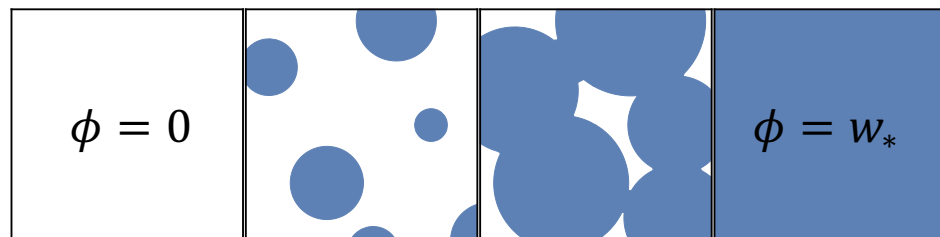
First-order phase transitions

Time evolution (boiling of the Universe) 

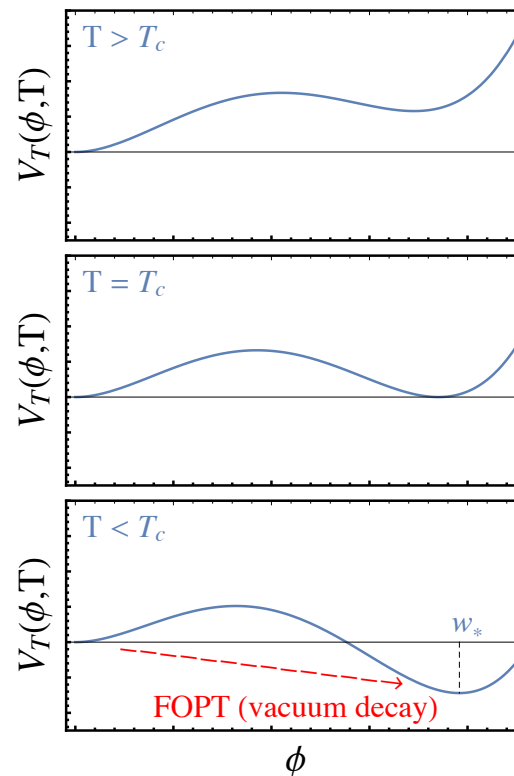
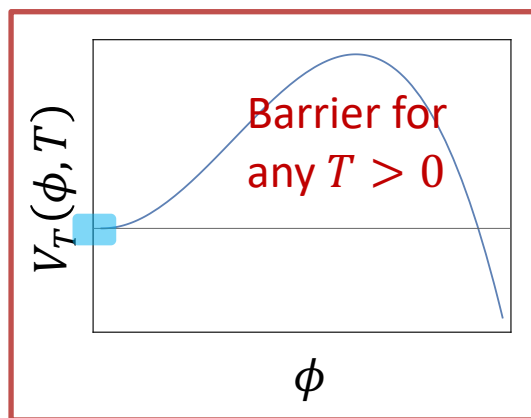


First-order phase transitions

Time evolution (boiling of the Universe) 



The necessary **barrier** is easily obtained in a classically conformal theory



$$V_T(\phi, T) \sim \frac{g_X T^2}{8} \phi^2 + V_1(\phi)$$

Vacuum decay rate $\sim e^{-1/g_X^3}$, when $g_X \downarrow$, FOPT temperature $T_p \downarrow$

Thermal history: normal pattern

$$V_T(h, \phi, T) \approx \frac{3g_X^4}{32\pi^2} \phi^4 \left(\log \frac{\phi}{w} - \frac{1}{4} \right) + \frac{g_X^2 T^2}{8} \phi^2 - \frac{\lambda'}{4} h^2 \phi^2 + \frac{\lambda_h}{4} h^4 + \frac{c_h T^2}{2} h^2$$

↑
Dark A'
↑
SM particle

1. At T_p , the $U(1)_X$ FOPT occurs via $\phi: 0 \rightarrow w_*$

2. Then $V_T(h, \phi, T) \rightarrow V_T(h) \approx \frac{c_h T^2 - m_h^2/2}{2} h^2 + \frac{\lambda_h}{4} h^4$

Define $T_{ew} = \frac{m_h}{\sqrt{2c_h}} \sim 140$ GeV

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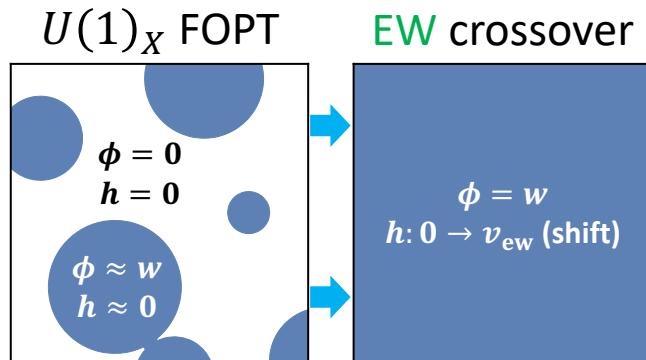
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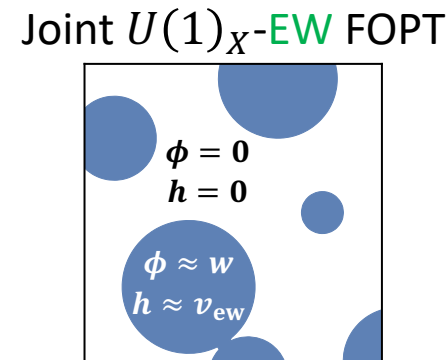
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Define $T_{ew} = \frac{m_h}{\sqrt{2c_h}} \sim 140$ GeV

- If $T_p > T_{ew}$



- If $T_p < T_{ew}$



Thermal history: inverted pattern

1. Even at $T_{\text{QCD}} \approx 85 \text{ MeV}$, $U(1)_X$ & EW still not transition
2. QCD FOPT occurs at T_{QCD} with 6 massless quarks
3. Top Yukawa $-\frac{y_t}{\sqrt{2}} \langle \bar{t}t \rangle h$ induces $\langle h \rangle = v_{\text{QCD}} \approx 100 \text{ MeV}$

$$4. V_T(h, \phi, T) \rightarrow V_T(\phi) \approx \frac{3g_X^4}{32\pi^2} \phi^4 \left(\log \frac{\phi}{w} - \frac{1}{4} \right) + \frac{g_X^2 T^2 - 2\lambda' v_{\text{QCD}}^2}{8} \phi^2$$

$$\text{Define } T_{\text{roll}} = \sqrt{2\lambda'} \frac{v_{\text{QCD}}}{g_X} \approx 1.8 \text{ MeV} \times \left(\frac{10^4 \text{ GeV}}{g_{XW}} \right)$$

Witten, NPB 177, 477 (1981); Iso *et al*, PRL 119 (2017) 14, 141301

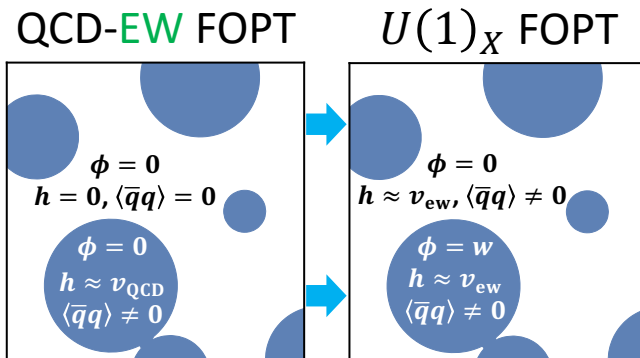
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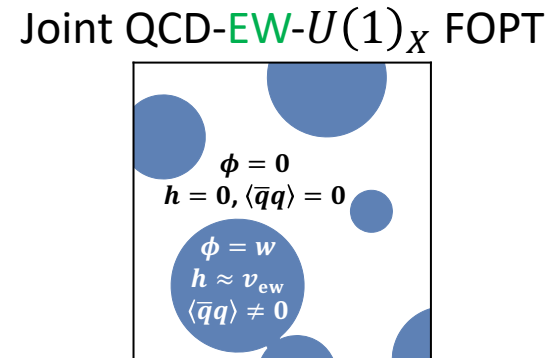
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- If $T_{\text{QCD}} > T_{\text{roll}}$



- If $T_{\text{QCD}} < T_{\text{roll}}$



The consequence of FOPTs

Stochastic gravitational waves [Caprini *et al*, JCAP 1604 (2016) 001]

1. Bubble collision

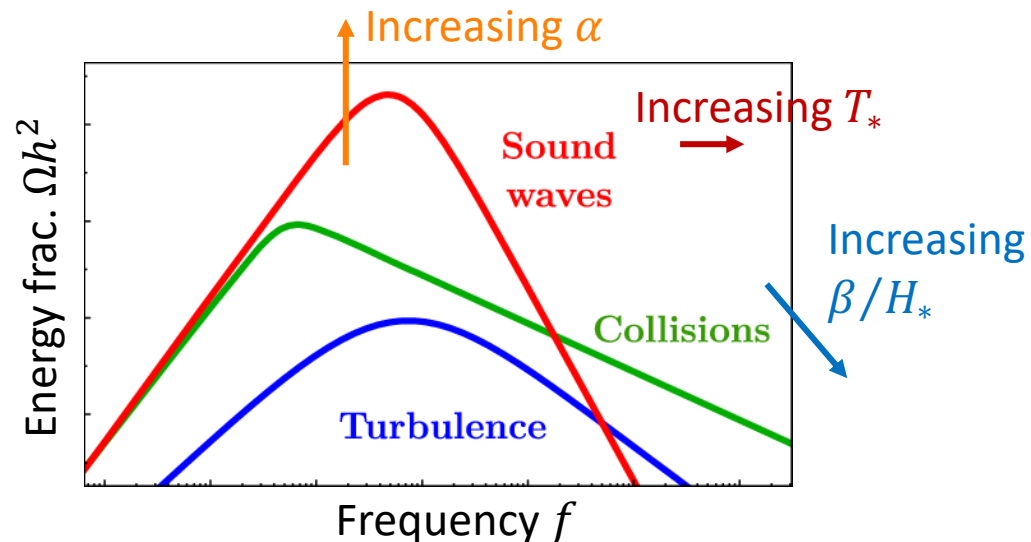
2. Sound waves in plasma

3. Turbulence in plasma

$$f_{\text{peak}} \sim 10^{-3} \text{ Hz} \times \left(\frac{1}{v_w}\right) \left(\frac{\beta/H_*}{10}\right) \left(\frac{T_*}{\text{TeV}}\right)$$

Two important parameters:

- α -- latent heat over radiation energy
- β/H_* -- inverse ratio of FOPT duration to Hubble time



Our calculations

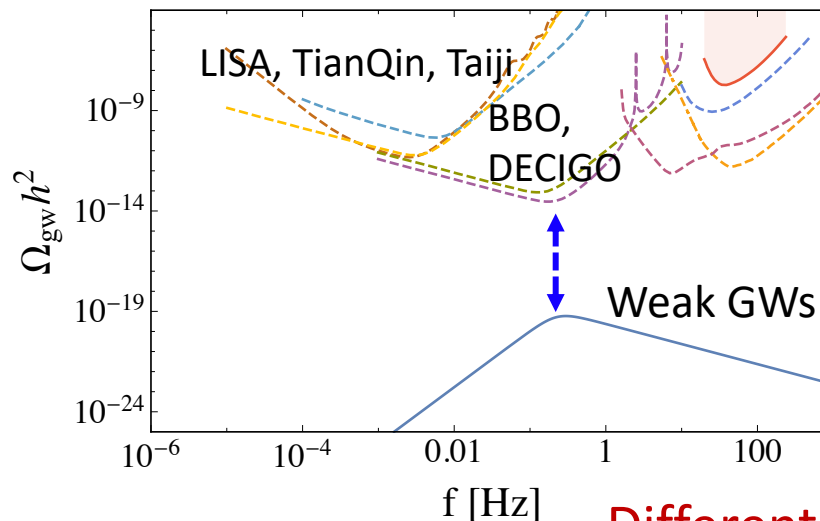
Resolve the FOPT dynamics using full one-loop finite temperature field theory, optimize calculation at **extremely low** temperatures



One example:

m_ϕ	θ	g_X	w	T_p	α	β/H_*	T_{rh}	T_Λ
0.79 GeV	10^{-6}	1.3×10^{-4}	2.5×10^8 GeV	0.56 MeV	5.2×10^{27}	4.9×10^6	1.7 GeV	4.7×10^3 GeV

Extremely strong! “Thermal inflation” below T_Λ
 But very *prompt*: a **strong** but **fast** FOPT

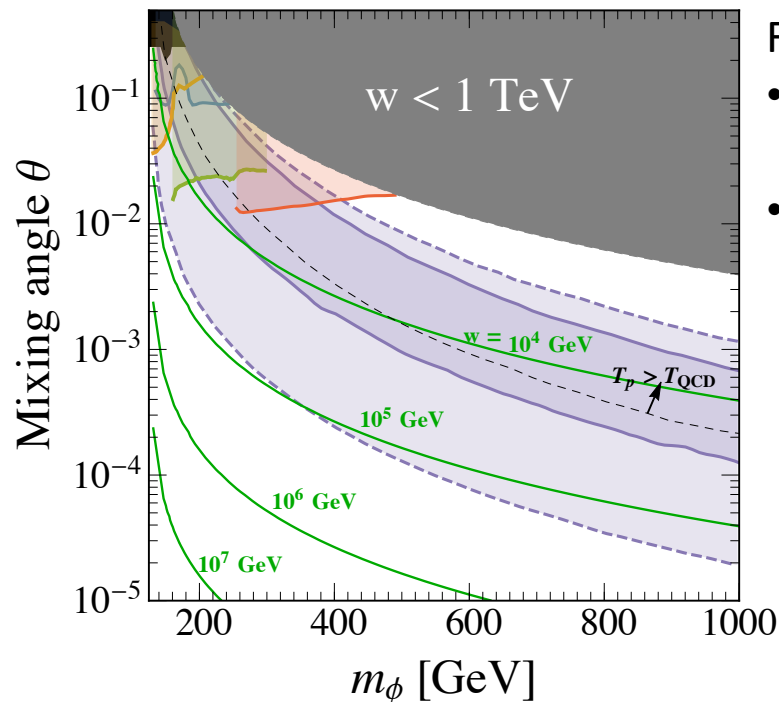


Different from naive estimate

Combined results (1)

The heavy singlet scenario $m_\phi > m_h$

- Collider: HL-LHC $pp \rightarrow \phi \rightarrow ZZ$; 10 TeV $\mu^+ \mu^- \rightarrow \phi \rightarrow b\bar{b}, VV, hh$
- GWs: [LISA, TianQin, Taiji] and [BBO, DECIGO]



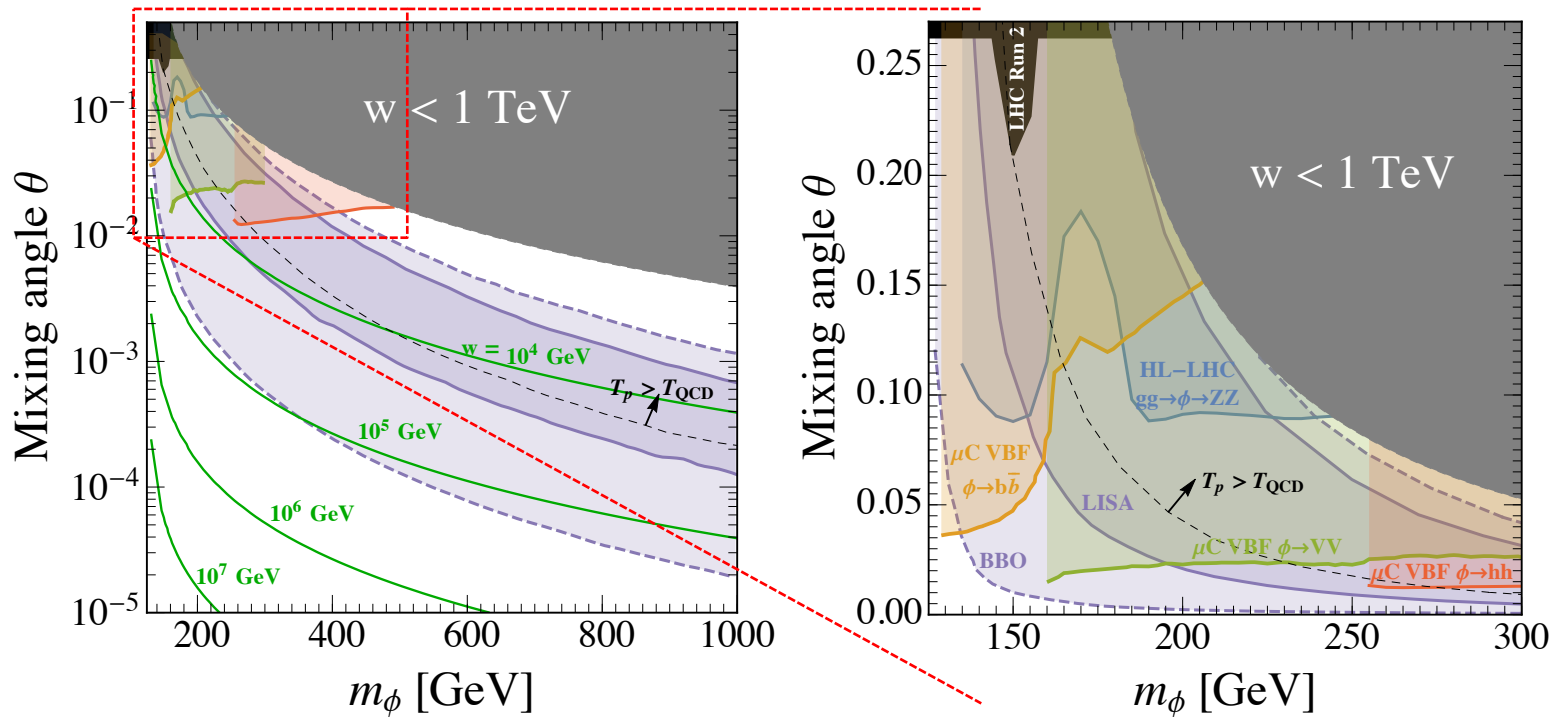
FOPT pattern:

- $U(1)_X$ FOPT + EW crossover or $U(1)_X$ -EW FOPT
- $U(1)_X$ transition after the QCD-EW transition

Combined results (1)

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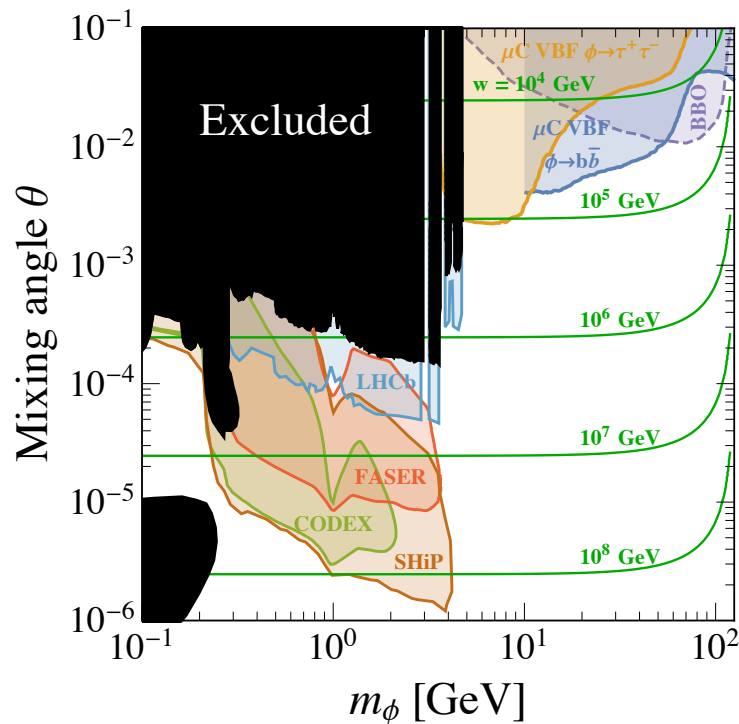
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Combined results (2)

The light singlet scenario $m_\phi < m_h$

- Collider: LLP searches; 10 TeV $\mu^+ \mu^- \rightarrow \phi \rightarrow \tau^+ \tau^-, b\bar{b}$
- GWs: [BBO, DECIGO]

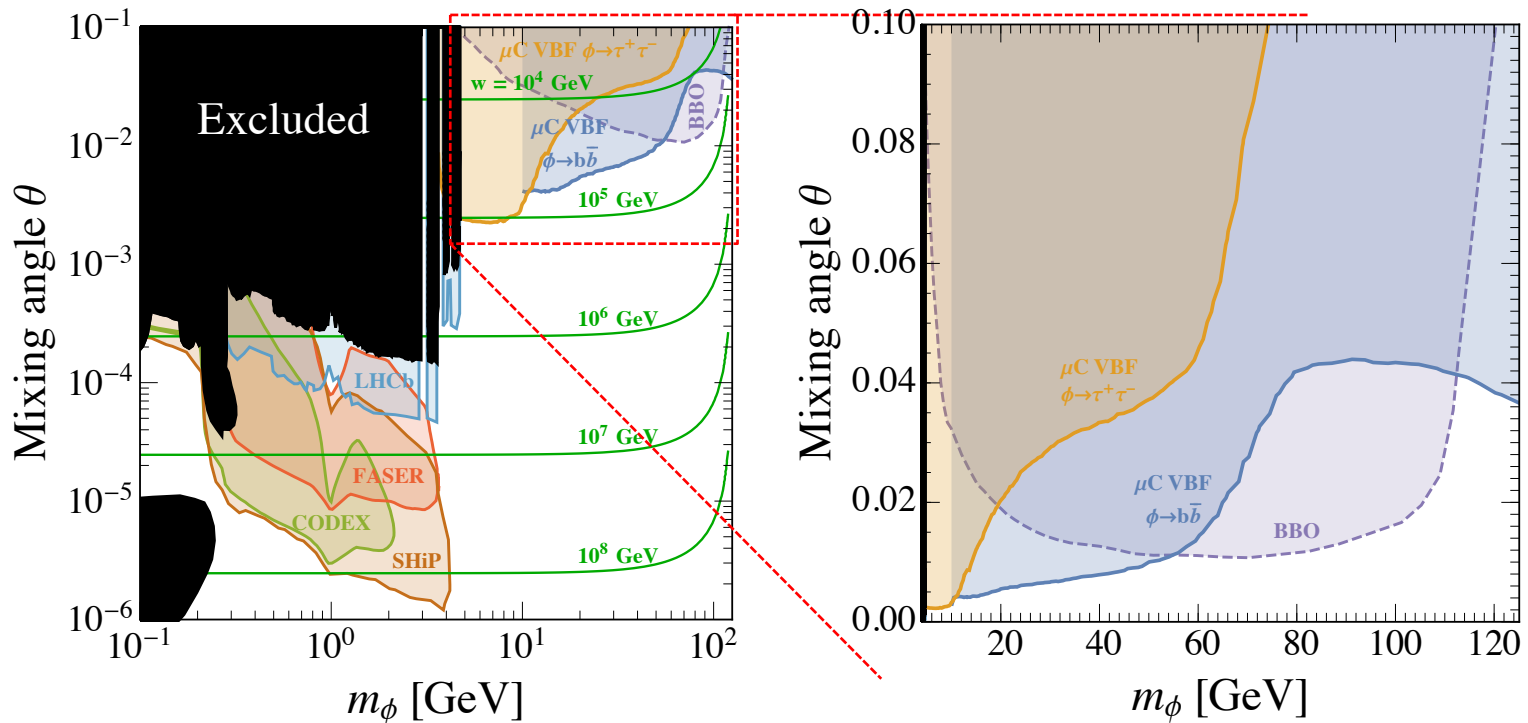


FOPT pattern:
 $U(1)_X$ transition after
the QCD-EW transition

Combined results (2)

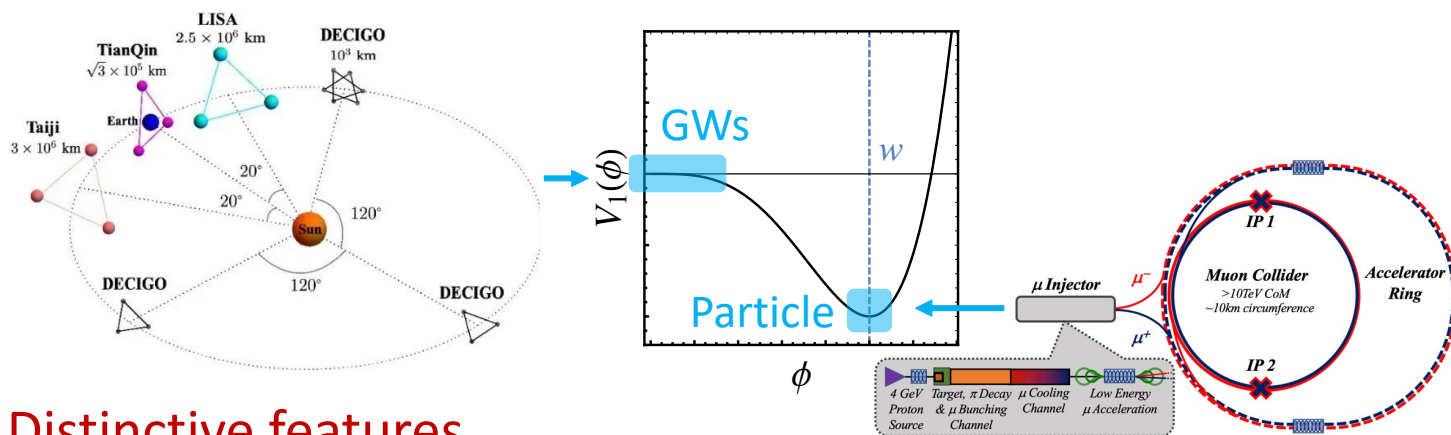
The light singlet scenario $m_\phi < m_h$

- Collider: LLP searches; 10 TeV $\mu^+ \mu^- \rightarrow \phi \rightarrow \tau^+ \tau^-, b\bar{b}$
- GWs: [BBO, DECIGO]



Closing remarks

Radiative symmetry breaking can be efficiently probed by combining the particle and GWs

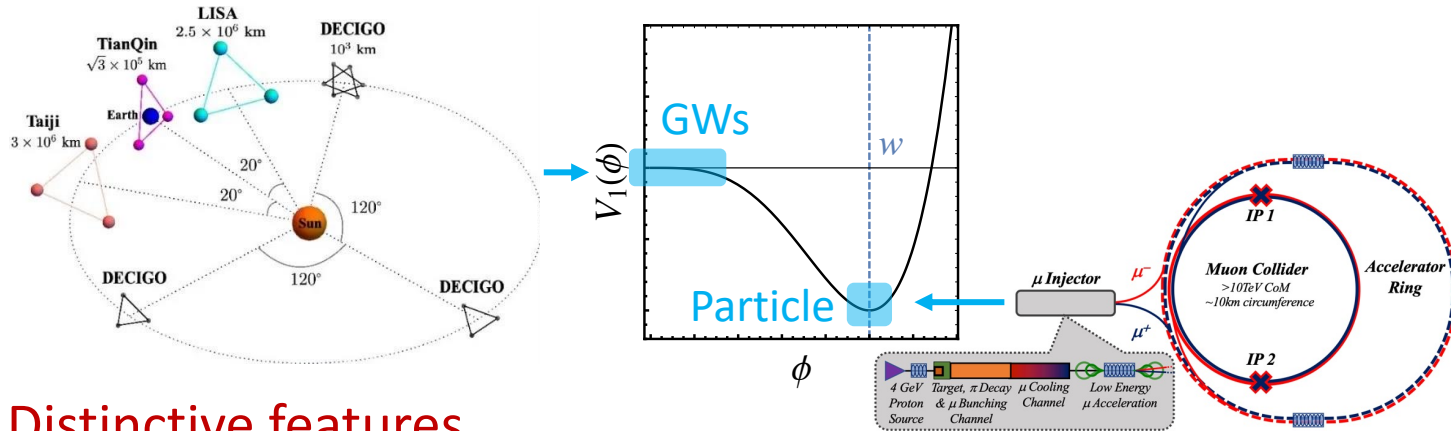


Distinctive features

- A relatively light singlet scalar $m_\phi \ll w$
- A supercooled FOPT $T_p \ll w$

Closing remarks

Radiative symmetry breaking can be efficiently probed by combining the particle and GWs



Distinctive features

- A relatively light singlet scalar $m_\phi \ll w$
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Future research

Exotic thermal history for dark matter, [\[1805.01473, 2304.00908\]](#)
primordial black holes, [\[2311.13640, 2312.04628, 2401.09411\]](#) and
baryogenesis. [\[2206.04691, 2305.10759\]](#)

Thank you!