Next-to-Leading-Order QCD Corrections to Nucleon Dirac Form Factor

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based on arXiv:2406.19994
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第十七届粒子物理、核物理和宇宙学交叉学科前沿问题研讨会
贵阳 2024年7月12日—16日

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- Introduction & Motivation
- Sketch of the Calculation
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Introduction & Motivation

As the composite particles made of strongly-coupled, relativistic quarks and gluons, proton and neutron are the building block of the atomic nuclei, consequently, of our planet and all its inhabitants, and even more, of our visible universe. Understanding the internal structure of nucleons from quantum chromodynamics (QCD), the underlying theory of strong interaction, is the fundamental challenge faced by the contemporary hadron and nuclear physics.

Eur. Phys. J. C**83**, 1125 (2023)

- Electromagnetic Form Factors (EMFFs)

$$\langle P'|j^{\mu}_{\mathrm{em}}|P\rangle = \bar{u}(P')\left[F_1(Q^2)\gamma^{\mu} - F_2(Q^2)\frac{\sigma^{\mu\nu}q_{\nu}}{2m_N}\right]u(P)$$

where $Q^2 \equiv -q^2 = -(P-P')^2$ SL: $q^2 < 0$ TL: $q^2 > 0$

- ◇ $F_1(Q^2)$: Dirac form factor (helicity-conserving) $\sim 1/Q^4$
- ◇ $F_2(Q^2)$: Pauli form factor (helicity-flipped) $\sim 1/Q^6$

EMFFs gauge the distributions of the electric charge and magnetization inside the nucleon, are the fundamental probes to the internal structure of nucleon.

Introduction & Motivation

- Experimental side on EMFFs

$$G_{\mathrm{E}}(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2} F_2(Q^2)$$

$$G_{\mathrm{M}}(Q^2) = F_1(Q^2) + F_2(Q^2)$$

◇ Hofstadter's pioneering work from mid-50s

Phys. Rev. **98**, 217 (1955); Phys. Rev. **102**, 851 (1956)

◇ SL EMFFs: Q^2 upto $30\mathrm{GeV}^2$ at `SLAC` in early 90s

Phys. Rev. D**48**, 29 (1993)

◇ TL EMFFs: Q^2 upto $40\mathrm{GeV}^2$ at `BaBar`

Phys. Rev. D**88**, 072009 (2013)

◇ Q^2 upto $40 - 50\mathrm{GeV}^2$ at `EIC` and `EicC`

arXiv:2207.04378 (2022)

Introduction & Motivation

- Theoretical side on EMFFs
  - ◇ vector meson dominance model

    PLB**43**,191 (1973); PRD**69**,054022 (2004); NCA**109**, 241 (1996); CPL**41**, 021302 (2024); ……
  - ◇ constitute quark model

    PRD**44**, 229 (1991); PRD**73**, 114021 (2006); PLB**671**, 153 (2009); ……
  - ◇ cloudy bag model

    PRD**24**, 216 (1981); PRD**28**, 2848 (1983); PRC**46**, 1077 (1992); ……
  - ◇ Dyson-Schwinger equation

    CTP**58**, 79 (2012); PRL**111**, 101803 (2013); FBS**55**, 1185(2014); ……
  - ◇ light-front Hamiltonian

    PRD**102**, 016008 (2020); ......
  - ◇ chiral perturbation theory

    EPJA**26**, 1 (2005); PRC**86**, 065206 (2012); ……
  - ◇ light-cone sum rule

    PRD**65**, 074011 (2002); PRD**78**, 033009 (2008); PRD**88**, 114021 (2013); ......
  - ◇ lattice QCD

    PRD**84**, 074507 (2011); PRD**83**, 094502 (2011); PRD**96**, 114509 (2017);
    PRD**92**, 054511 (2015); PRD**100**, 014509 (2019); PRD**101**, 014507 (2020); ......
  - ◇ dispersive analysis

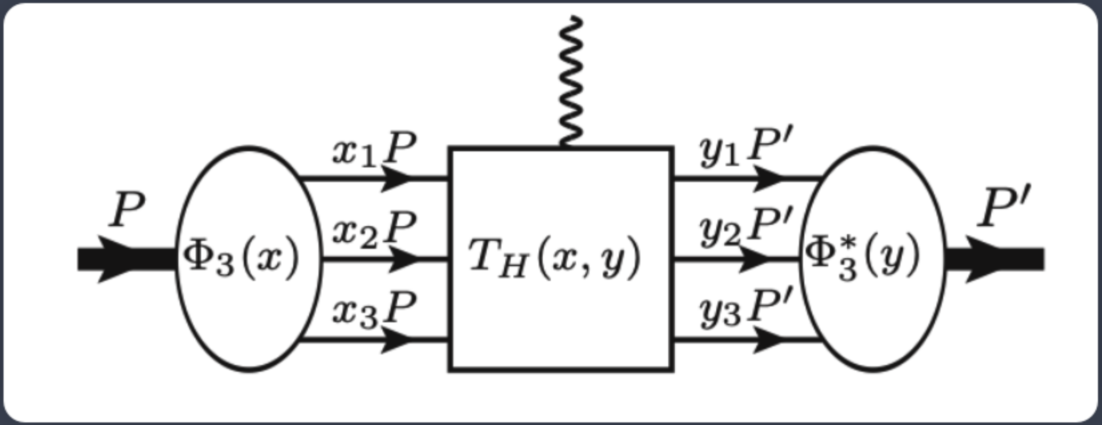
    NPA**596**, 367 (1996); PRC**75**, 035202 (2007); ......

Introduction & Motivation

- Collinear Factorization

Phys. Rev. D**22**, 2157 (1980)
Phys. Rept. **112**, 173 (1984)

Thanks to asymptotic freedom, the nucleon EMFFs with large momentum transfer are generally believed to be adequately accounted by perturbative QCD (pQCD).



$$F_1(Q^2) = \Phi_3(x,\mu_F) \underset{x}{\otimes} T_H\left(x,y,Q^2,\mu_R^2,\mu_F^2\right) \underset{y}{\otimes} \Phi_3^*(y,\mu_F) + \mathcal{O}(1/Q^2)$$

$$A(x) \underset{x}{\otimes} B(x) = \int_0^1 dx_1 dx_2 dx_3 \delta(1 - \sum_i x_i) A(x) B(x)$$

Introduction & Motivation

- Light-Cone-Distribution Amplitude (LCDA)

Phys. Rept. **112**, 173 (1984)
Nucl. Phys. B**589**, 381 (2000)

$$\langle 0|\epsilon_{ijk}(u_+^{i\,T}(a_1 n)\mathcal{C}\ \not{n}u_-^j(a_2 n))\ \not{n}d_+^k(a_3 n)|p^\uparrow(P)\rangle$$
$$= -\frac{1}{2}P\cdot n\ \not{n}u_p^\uparrow(P)\int[dx]e^{-iP\cdot n\sum x_i a_i}\Phi_3(x)$$

- ERBL Evolution Equation

Phys. Rev. D**22**, 2157 (1980)
Phys. Rev. D**78**, 033009 (2008)

$$\frac{d\Phi_3(x,\mu_F)}{d\ln\mu_F^2} = V(x,y)\underset{y}{\otimes}\Phi_3(y,\mu_F)$$

$$V(x,y) = \frac{\alpha_s}{\pi}V^{(1)}(x,y) + \frac{\alpha_s^2}{\pi^2}V^{(2)}(x,y) + \cdots$$

Introduction & Motivation

- Status of EMFF in pQCD

  - Leading Order

    G. P. Lepage and S. J. Brodsky, Phys. Rev. Lett. **43**, 545-549 (1979)

    G. P. Lepage and S. J. Brodsky, Phys. Rev. D**22**, 2157 (1980)

    V. L. Chernyak, A. R. Zhitnitsky and V. G. Serbo, JETP Lett. **26**, 594 (1977)

    V. L. Chernyak and A. R. Zhitnitsky, Phys. Rept. **112**, 173 (1984)

    ... ...

  - Next-to-Leading Order

Curiously, notwithstanding nearly half a centaury has elapsed, the next-to-leading order (NLO) perturbative correction to this fundamental observable of nucleon remains unknown until today!

Introduction & Motivation

- Status of EMFF in pQCD

Promotionsgesuch eingereicht am: 18.06.2014
Die Arbeit wurde angeleitet von: Prof. Dr. V. Braun

Nucleon Electromagnetic Form Factors in perturbative QCD

Dissertation
zur Erlangung des Doktorgrades
der Naturwissenschaften (Dr. rer. nat.)

Fakultät für Physik
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Mai 2015

CHAPTER 4. RESULTS AND CHECKS

corresponding Z-factors:

$$Z_q = 1 + \frac{\alpha_S}{3\pi}\frac{1}{\epsilon} \qquad \text{quark field renormalization} \tag{4.3}$$

$$Z_{\alpha_S} = 1 + \frac{\alpha_S}{4\pi}\frac{1}{\epsilon}\left(11 - \frac{2}{3}n_f\right) \qquad \text{gauge field renormalization} \tag{4.4}$$

$$Z_{f_N} = 1 - \frac{\alpha_S}{3\pi}\frac{1}{\epsilon} \qquad \text{renormalization of the distribution amplitude} \tag{4.5}$$

Note that the difference in signs from the renormalization constants from Eq. (2.51),...,(2.55) is due to the shift from $D = 4 - 2\epsilon$ to $D = 4 + 2\epsilon$. The renormalization constant for the leading twist distribution amplitude Z_{f_N} was taken from [53]. So adding up the next-to-leading and leading order times the appropriate renormalization constants we obtain for the term proportional to $\frac{1}{\epsilon}$:

$$\frac{1600}{9}\frac{1}{\epsilon}\pi\alpha_S^3(2n_f - 35)(2e_d + e_u) + \left(\frac{\alpha_S}{\pi\epsilon} + \frac{\alpha_S}{2\pi\epsilon}(11 - \frac{2}{3}n_f) - \frac{2\alpha_S}{3\epsilon\pi}\right)\frac{3200}{3}\pi^2\alpha_S^2(2e_d + e_u) \tag{4.6}$$

It is easy to check that this is zero. Up to now everything was calculated completely analytically. For the finite part in Eq. (4.1) this does not hold anymore. More than this: We were not able to even compute some parts of the finite results numerically. The occurring denominators have overlapping poles (between a_i and b_i) all over the integration path. We could calculate all terms which do not have such mixing between the incoming and outgoing momentum fractions. Since the final result is only given for the asymptotic wave function there was no need to calculate the mixed cases vector-axial amplitude. But we have calculated all leading twist combination in an completely general form up to the point that one has to insert a certain model for the wave function and integrate over the corresponding momentum fractions:

Sketch of the Calculation

- Perturbative Matching

$$F_1(Q^2) = \Phi_3(x,\mu_F) \underset{x}{\otimes} T_H\left(x,y,Q^2,\mu_R^2,\mu_F^2\right) \underset{y}{\otimes} \Phi_3^*(y,\mu_F) + \mathcal{O}(1/Q^2)$$

$$Q^4 T_H = T^{(0)} + \frac{\alpha_s}{\pi} T^{(1)} + \cdots$$

Yad. Fiz. **48**, 1410 (1988)
Nucl. Phys. B**311**, 585 (1989)

replacing

$$|p^\uparrow(P)\rangle = \frac{1}{4\sqrt{6}} \frac{\varepsilon^{ijk}}{\sqrt{6}} \frac{1}{\sqrt{u_1 u_2 u_3}} \left\{ |u^{i\uparrow}(u_1 P) u^{j\downarrow}(u_2 P) d^{k\uparrow}(u_3 P)\rangle - |u^{i\uparrow}(u_1 P) d^{j\downarrow}(u_2 P) u^{k\uparrow}(u_3 P)\rangle \right\}$$

$$\Longrightarrow \begin{cases} Q^4 F_1(u,v) = F_1^{(0)}(u,v) + \frac{\alpha_s}{\pi} F_1^{(1)}(u,v) + \cdots \\ \Phi_3(x|u) = Z(x,y) \underset{y}{\otimes} \Phi_3^{\text{Bare}}(y|u) \\ \qquad = \Phi_3^{(0)}(x) + \frac{\alpha_s}{\pi} \Phi_3^{(1)}(x|u) + \cdots \end{cases}$$

- Matching Condition

L. B. Chen, W. Chen, F. Feng and Y. Jia, Phys. Rev. Lett. **132**, 201901 (2024)

$$T^{(0)}(x,y) = F_1^{(0)}(x,y)$$

$$T^{(1)}(u,v) = F_1^{(1)}(u,v) - \Phi_3^{(1)}(x|u) \underset{x}{\otimes} T^{(0)}(x,v) - \Phi_3^{(1)}(y|v) \underset{y}{\otimes} T^{(0)}(u,y)$$

Sketch of the Calculation

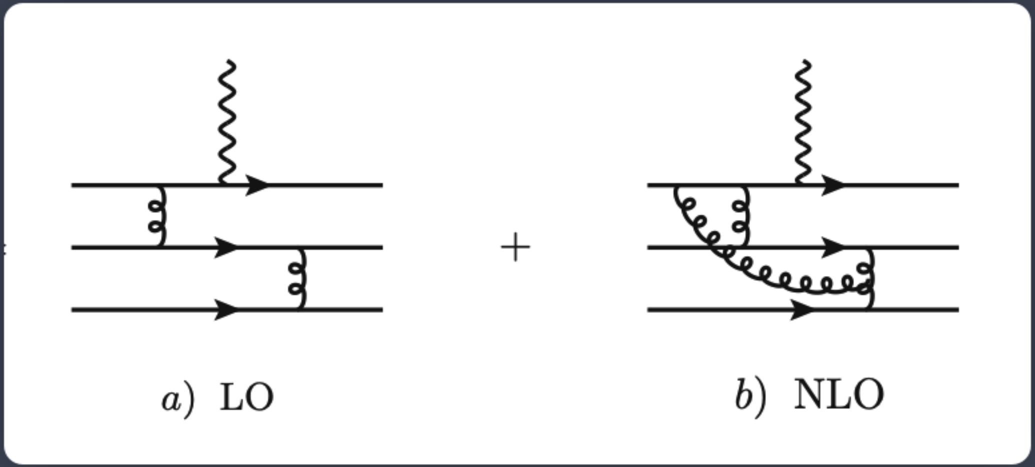
- Building Block

$$|\Lambda^{\uparrow}(P)\rangle = \frac{\varepsilon^{ijk}}{\sqrt{6}} \frac{1}{\sqrt{u_1 u_2 u_3}} |u^{i\uparrow}(u_1 P) d^{j\downarrow}(u_2 P) s^{k\uparrow}(u_3 P)\rangle$$

Partonic Process:

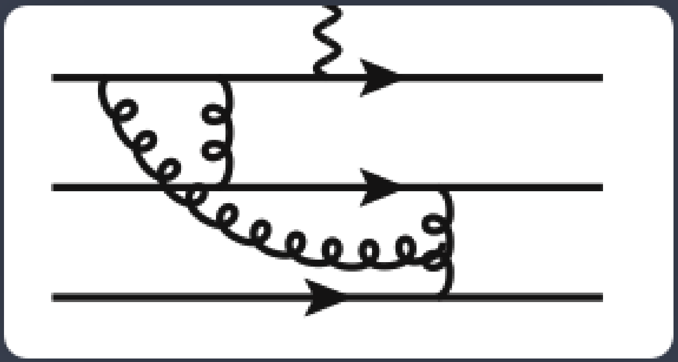
$$u^{\uparrow}(u_1 P) d^{\downarrow}(u_2 P) s^{\uparrow}(u_3 P) + \gamma^* \to u^{\uparrow}(v_1 P') d^{\downarrow}(v_2 P') s^{\uparrow}(v_3 P')$$

LO: **48** Feynman diagrams NLO: **2040** Feynman diagrams



Sketch of the Calculation

- Dirac Matrix Structures



$$\bar{u}^{\uparrow}(v_1 P')\Gamma_1 u^{\uparrow}(u_1 P)$$

$$\bar{u}^{\downarrow}(v_2 P')\Gamma_2 u^{\downarrow}(u_2 P)$$

$$\bar{u}^{\uparrow}(v_3 P')\Gamma_3 u^{\uparrow}(u_3 P)$$

- Reverse 2nd Line

$$\bar{u}^{\downarrow}(v_2 P')\Gamma_2 u^{\downarrow}(u_2 P) = -\bar{v}^{\downarrow}(u_2 P)\, \mathcal{C}\Gamma_2^T \mathcal{C}^{-1}\, v^{\downarrow}(v_2 P')$$

$$\left[\bar{u}^{\uparrow}(P')\Gamma_1 \not{P}\,\frac{1-\gamma_5}{2}\,\mathcal{C}\Gamma_2^T\mathcal{C}^{-1}\not{P}'\,\frac{1-\gamma_5}{2}\,\Gamma_3 u^{\uparrow}(P)\right]_{\mu} \propto F_1 \bar{u}^{\uparrow}(P')\gamma_{\mu} u^{\uparrow}(P)$$

- Final Formula

$$F_1^{\Lambda} = \frac{1}{2Q^4}\mathrm{Tr}\left\{[\Gamma_1 \not{P}\,\Gamma_2^C \not{P}'\Gamma_3]_{\mu}\not{P}\,\gamma^{\mu}\not{P}'\,\frac{1-\gamma_5}{2}\right\}$$

Sketch of the Calculation

- Proton EMFF from Building Block

$$|p^{\uparrow}(P)\rangle = \frac{1}{4\sqrt{6}} \frac{\varepsilon^{ijk}}{\sqrt{6}} \frac{1}{\sqrt{u_1 u_2 u_3}} \Big\{ |u^{i\uparrow}(u_1 P) u^{j\downarrow}(u_2 P) d^{k\uparrow}(u_3 P)\rangle - |u^{i\uparrow}(u_1 P) d^{j\downarrow}(u_2 P) u^{k\uparrow}(u_3 P)\rangle \Big\}$$

$\diamond$ $u^{\uparrow}(u_1 P) u^{\downarrow}(u_2 P) d^{\uparrow}(u_3 P) + \gamma^* \to u^{\uparrow}(v_1 P') u^{\downarrow}(v_2 P') d^{\uparrow}(v_3 P')$

$$\Longrightarrow \quad F_1^{\Lambda}(e_u, e_d, e_u; u, v)$$

$\diamond$ $u^{\uparrow}(u_1 P) d^{\downarrow}(u_2 P) u^{\uparrow}(u_3 P) + \gamma^* \to u^{\uparrow}(v_1 P') d^{\downarrow}(v_2 P') u^{\uparrow}(v_3 P')$

$$\Longrightarrow \quad F_1^{\Lambda}(e_u, e_d, e_u; u, v) + (v_1 \leftrightarrow v_3)$$

- Final EMFF for Proton

$$F_1^p(u,v) = \left[\frac{1}{4\sqrt{6}}\right]^2 \Big\{ F_1^{\Lambda}(e_u, e_u, e_d; u, v) + \big[F_1^{\Lambda}(e_u, e_d, e_u; u, v) + (v_1 \leftrightarrow v_3)\big] \Big\}$$

Sketch of the Calculation

- Calculations are done using HepLib, 100 MIs: DE
- Fully reproduce the well-known LO results
- Both double pole and single pole cancel out, respectively

$$T^{(1)}(u,v) = F_1^{(1)}(u,v) - \Phi_3^{(1)}(x|u) \underset{x}{\otimes} T^{(0)}(x,v) - \Phi_3^{(1)}(y|v) \underset{y}{\otimes} T^{(0)}(u,y)$$

- Analytical Hard Kernel T_H

$$F_1(Q^2) = \Phi_3(x,\mu_F) \underset{x}{\otimes} T_H\left(x,y,Q^2,\mu_R^2,\mu_F^2\right) \underset{y}{\otimes} \Phi_3^*(y,\mu_F) + \mathcal{O}(1/Q^2)$$

Phenomenology

- LCDA Expansions

$$\Phi_3(x, \mu_F) = f_N(\mu_F) \sum_{0 \le n \le m} a_{mn}(\mu_F) \phi_{mn}(x)$$

$$\phi_{mn}(x) = 120 x_1 x_2 x_3 \mathcal{P}_{mn}(x)$$

Orthogonal polynomials $\mathcal{P}_{mn}(x)$:
eigenfunctions of one-loop evolution equation

$$\mathcal{P}_{00} = 1,$$
$$\mathcal{P}_{10} = 21\,(x_1 - x_3)\,,$$
$$\mathcal{P}_{11} = 7\,(x_1 - 2x_2 + x_3)\,,$$
$$\mathcal{P}_{20} = \frac{63}{10}\left[3\,(x_1 - x_3)^2 - 3x_2\,(x_1 + x_3) + 2x_2^2\right],$$
$$\mathcal{P}_{21} = \frac{63}{2}\,(x_1 - 3x_2 + x_3)\,(x_1 - x_3)\,,$$
$$\mathcal{P}_{22} = \frac{9}{5}\left[x_1^2 + 9x_2\,(x_1 + x_3) - 12x_1 x_3 - 6x_2^2 + x_3^2\right].$$

Phys. Rev. D**89**, 094511 (2014)

Phenomenology

- Master Formula for EMFF

$$\begin{cases} F_1(Q^2) = \Phi_3(x) \underset{x}{\otimes} T_H(x, y, Q^2, \mu_F^2, \mu_R^2) \underset{y}{\otimes} \Phi_3^*(y) \\ \Phi_3(x, \mu_F) = f_N(\mu_F) \sum_{0 \leq n \leq m} a_{mn}(\mu_F) \phi_{mn}(x) \end{cases}$$

$$Q^4 F_1^p(Q^2) = f_N^2(\mu_F) \sum_{mnm'n'} a_{mn}(\mu_F) \mathcal{T}_{m'n'}^{mn} a_{m'n'}(\mu_F)$$

$$\begin{aligned} \mathcal{T}_{m'n'}^{mn} &= \phi_{mn}(x) \underset{x}{\otimes} Q^4 T_H(x, y) \underset{y}{\otimes} \phi_{m'n'}(y) \\ &= 10^3 \pi^2 \alpha_s^2 \Big\{ e_u\, c_0 \left[1 + \frac{\alpha_s}{\pi} (c_1 L_\mu + c_2) \right] \\ &\quad + e_d\, d_0 \left[1 + \frac{\alpha_s}{\pi} (d_1 L_\mu + d_2) \right] + \mathcal{O}(\alpha_s^2) \Big\} \end{aligned}$$

$\mu_R = \mu_F = \mu$

$L_\mu \equiv \ln \frac{\mu^2}{Q^2}$

$\mathrm{TL}: L_\mu \to L_\mu + i\pi$

$\mathrm{neutron}: e_u \leftrightarrow e_d$

Solution: Multiple-Precision Floating-Point (MPFR)
Parallel Numeric Integrator (Cubature)

$\mathcal{T}^{mn}_{m'n'}$

$(mn, m'n')$	c_0	c_2	d_0	d_2	$c_1 = d_1$
$(00, 00)$	0.5333	21.833 (7)	1.067	17.989 (7)	4.833
$(10, 00)$	4.356	24.882 (8)	-4.356	24.882 (8)	5.389
$(10, 10)$	90.01	29.192 (3)	92.92	28.761 (2)	5.944
$(11, 00)$	1.867	28.838 (6)	-1.867	18.372 (1)	5.500
$(11, 10)$	20.33	30.131 (8)	-20.33	30.131 (8)	6.056
$(11, 11)$	31.94	26.864 (5)	11.61	24.472 (3)	6.167
$(20, 00)$	11.39	26.768 (7)	4.293	24.764 (4)	5.722
$(20, 10)$	47.91	31.894 (6)	-47.91	31.894 (6)	6.278
$(20, 11)$	49.65	32.757 (4)	5.227	40.732 (9)	6.389
$(20, 20)$	125.2	34.896 (8)	29.27	33.713 (7)	6.611
$(21, 00)$	4.667	28.055 (3)	-4.667	28.055 (3)	5.944
$(21, 10)$	84.93	33.184 (3)	84.93	33.373 (3)	6.500
$(21, 11)$	23.96	33.066 (3)	-23.96	33.066 (3)	6.611
$(21, 20)$	45.08	36.366 (5)	-45.08	36.366 (5)	6.833
$(21, 21)$	84.93	38.068 (4)	84.93	38.028 (4)	7.056
$(22, 00)$	1.707	28.267 (1)	0.05333	59.977 (9)	6.000
$(22, 10)$	6.533	32.188 (2)	-6.533	32.188 (2)	6.556
$(22, 11)$	3.796	38.794 (4)	1.431	26.059 (3)	6.667
$(22, 20)$	16.18	36.875 (4)	2.632	40.787 (5)	6.889
$(22, 21)$	3.827	39.545 (5)	-3.827	39.545 (5)	7.111
$(22, 22)$	4.155	34.204 (1)	0.9173	25.254 (2)	7.167

Phenomenology

- Master Formula for EMFF

$$Q^4 F_1^p(Q^2) = f_N^2(\mu) \sum_{mnm'n'} a_{mn}(\mu) \mathcal{T}^{mn}_{m'n'} a_{m'n'}(\mu)$$

$$\begin{cases} f_N(\mu) = f_N(\mu_0) \left(\dfrac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{2/(3\beta_0)} \\ a_{mn}(\mu) = a_{mn}(\mu_0) \left(\dfrac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\gamma_{mn}/\beta_0} \end{cases}$$

anomalous dimensions:

Phys. Rev. D**89**, 094511 (2014)

$$\gamma_{00} = 0, \quad \gamma_{10} = \frac{20}{9}, \quad \gamma_{11} = \frac{8}{3}$$
$$\gamma_{20} = \frac{32}{9}, \quad \gamma_{21} = \frac{40}{9}, \quad \gamma_{22} = \frac{14}{3}$$

Phenomenology

- Input parameters

	CZ	COZ	KS	SB	BLW	ABO	LAT14	RQCD
a_{10}	0.191	0.163	0.144	0.152	0.0534	0.05	$0.030(7)$	0.051
a_{11}	0.252	0.194	0.169	0.205	0.0664	0.05	$0.031(4)$	0.033
a_{20}	0.32	0.41	0.56	0.65	0.000	$0.075(15)$	$-0.01(9)$	0.000
a_{21}	0.03	0.06	-0.01	-0.27	0.000	$-0.027(38)$	$-0.06(12)$	0.000
a_{22}	-0.003	-0.163	-0.163	0.020	0.000	$0.17(15)$	$-0.02(15)$	0.000
f_N	$5.3(5)$	$5.0(3)$	$5.1(3)$	$5.0(3)$	$5.0(5)$	$5.0(5)$	$2.84(1)(33)$	3.54^{+6}_{-4}

$f_N(\mu_0^2 = 1\mathrm{GeV}^2)(\times 10^{-3}\mathrm{GeV}^2)$ $\quad a_{mn}(\mu_0^2 = 2\mathrm{GeV}^2)$ $\quad \mu_0^2 = 4\mathrm{GeV}^2$

CZ: Nucl. Phys. B**246**, 52 (1984)

COZ: Yad. Fiz. **48**, 1410 (1988)

KS: Nucl. Phys. B**279**, 785 (1987)

SB: Phys. Rev. D**47**, R3685 (1993)

BLW: Phys. Rev. D**73**, 094019 (2006)

ABO: Phys. Rev. D**88**, 114021 (2013)

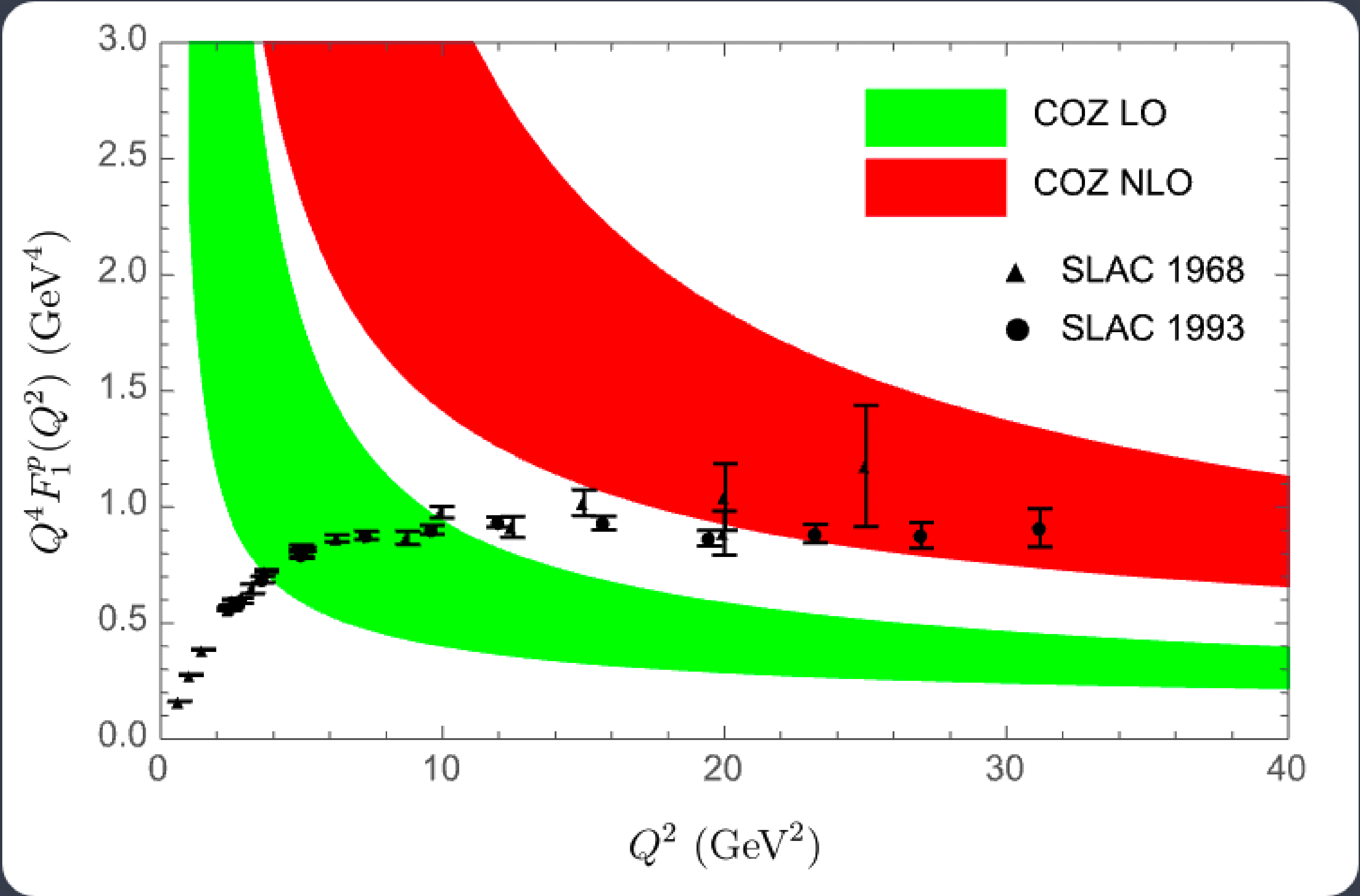
LAT14: Phys. Rev. D**89**, 094511 (2014)

RQCD: Eur. Phys. J. A **55**, 116 (2019)

Phenomenology

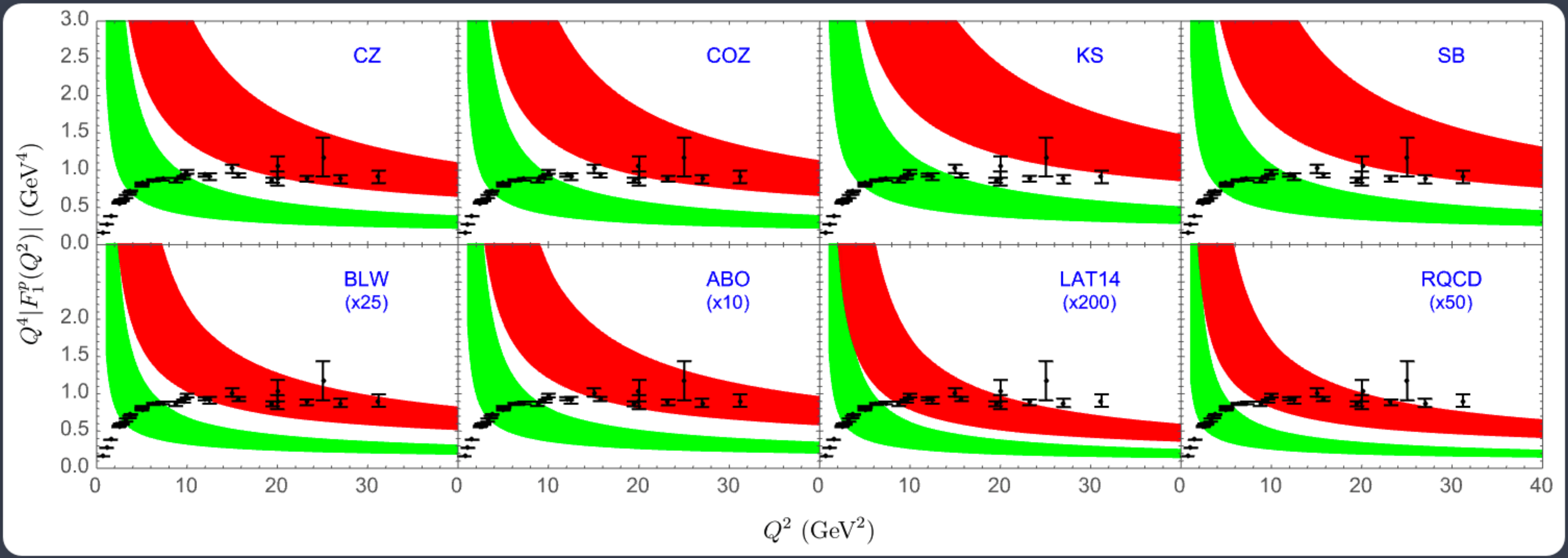
SLAC 1968: Phys. Rev. Lett. **20**, 292 (1968)
SLAC 1993: Phys. Rev. **D49**, 29 (1993)

- SL EMFF for Proton



Phenomenology

- SL EMFF for Proton



red band: NLO prediction μ from $Q/2$ to Q

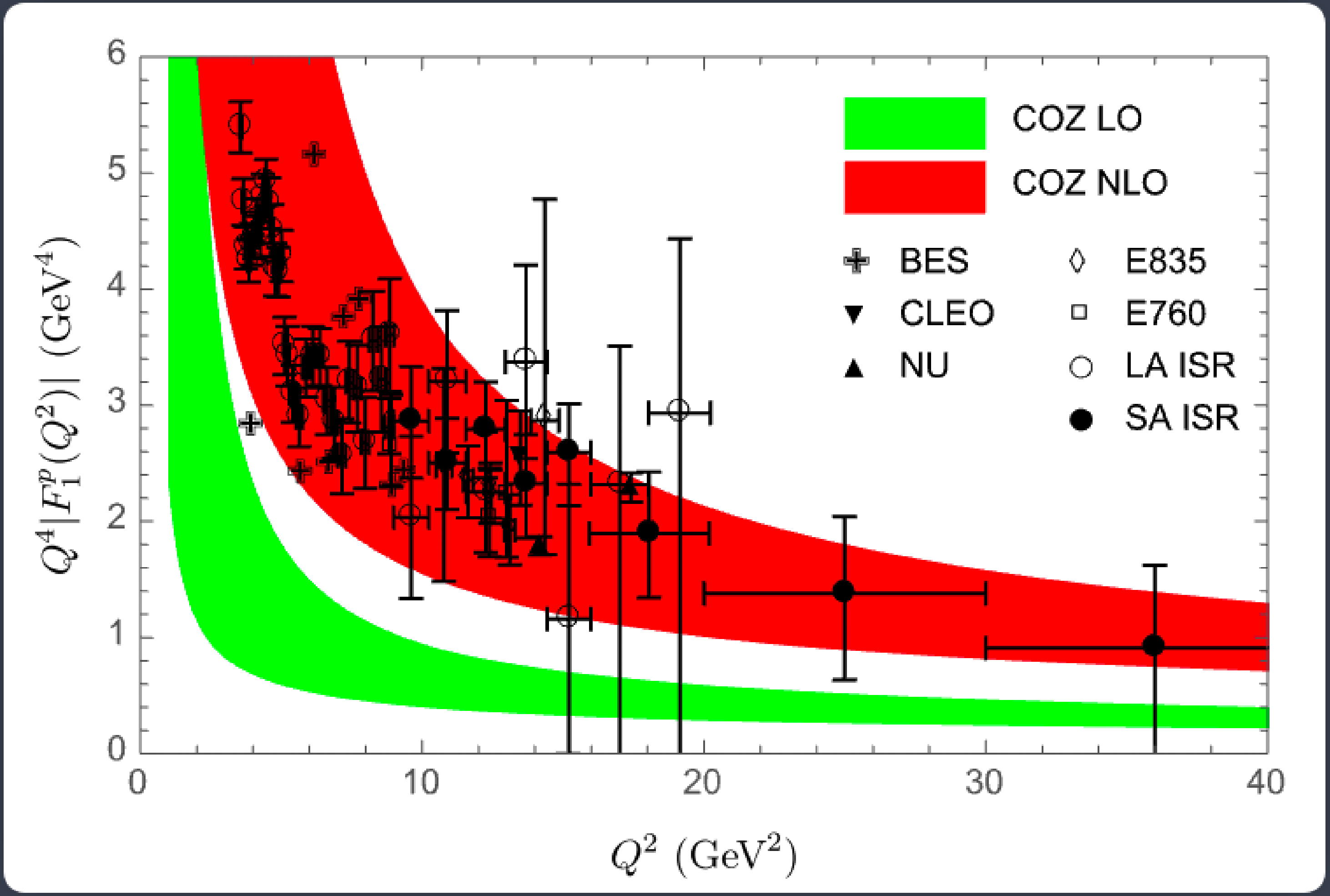
green band: LO prediction μ from $Q/2$ to Q

(x25): magnifying factor

Phenomenology

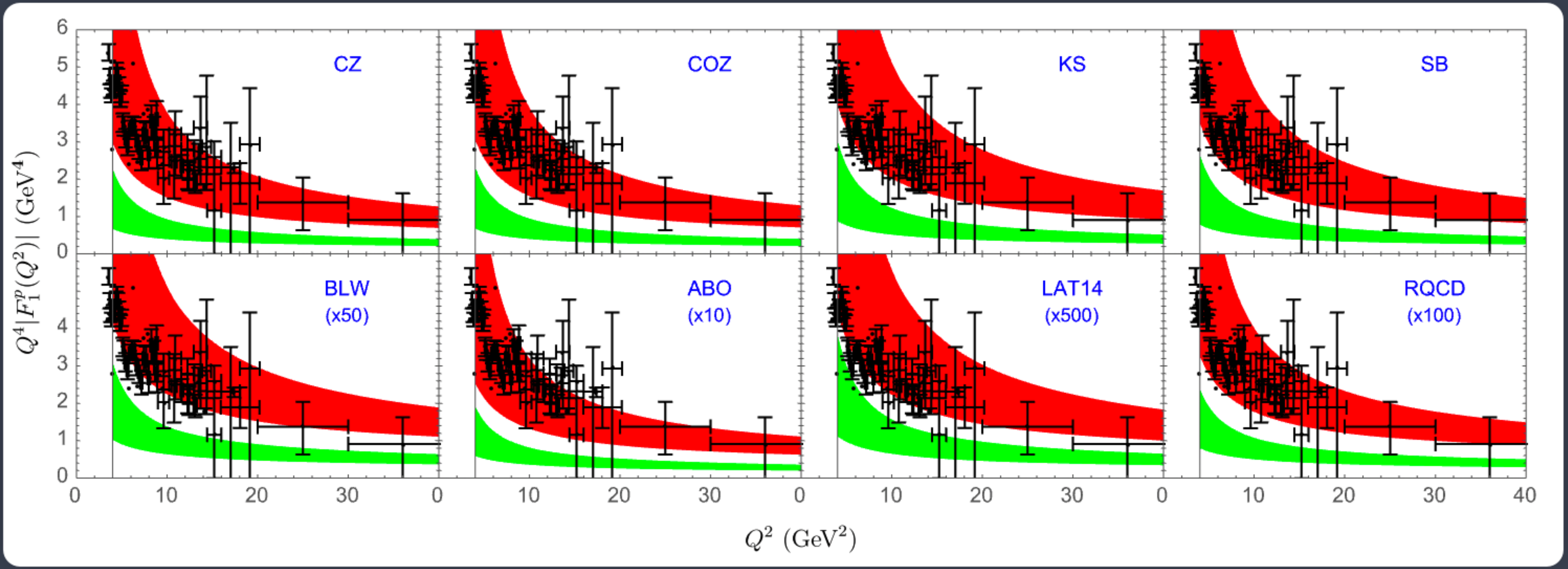
BES: Phys. Lett. B**630**, 14 (2005)
CLEO: Phys. Rev. Lett. **95**, 261803 (2005)
NU: Phys. Rev. Lett. **110**, 022002 (2013)
E835: Phys. Rev. D**60**, 032002 (1999)
E760: Phys. Rev. Lett. **70**, 1212 (1993)
LA ISR: Phys. Rev. D**87**, 092005 (2013)
SA ISR: Phys. Rev. D**88**, 072009 (2013)

- TL EMFF for Proton



Phenomenology

- TL EMFF for Proton



red band: NLO prediction μ from $Q/2$ to Q

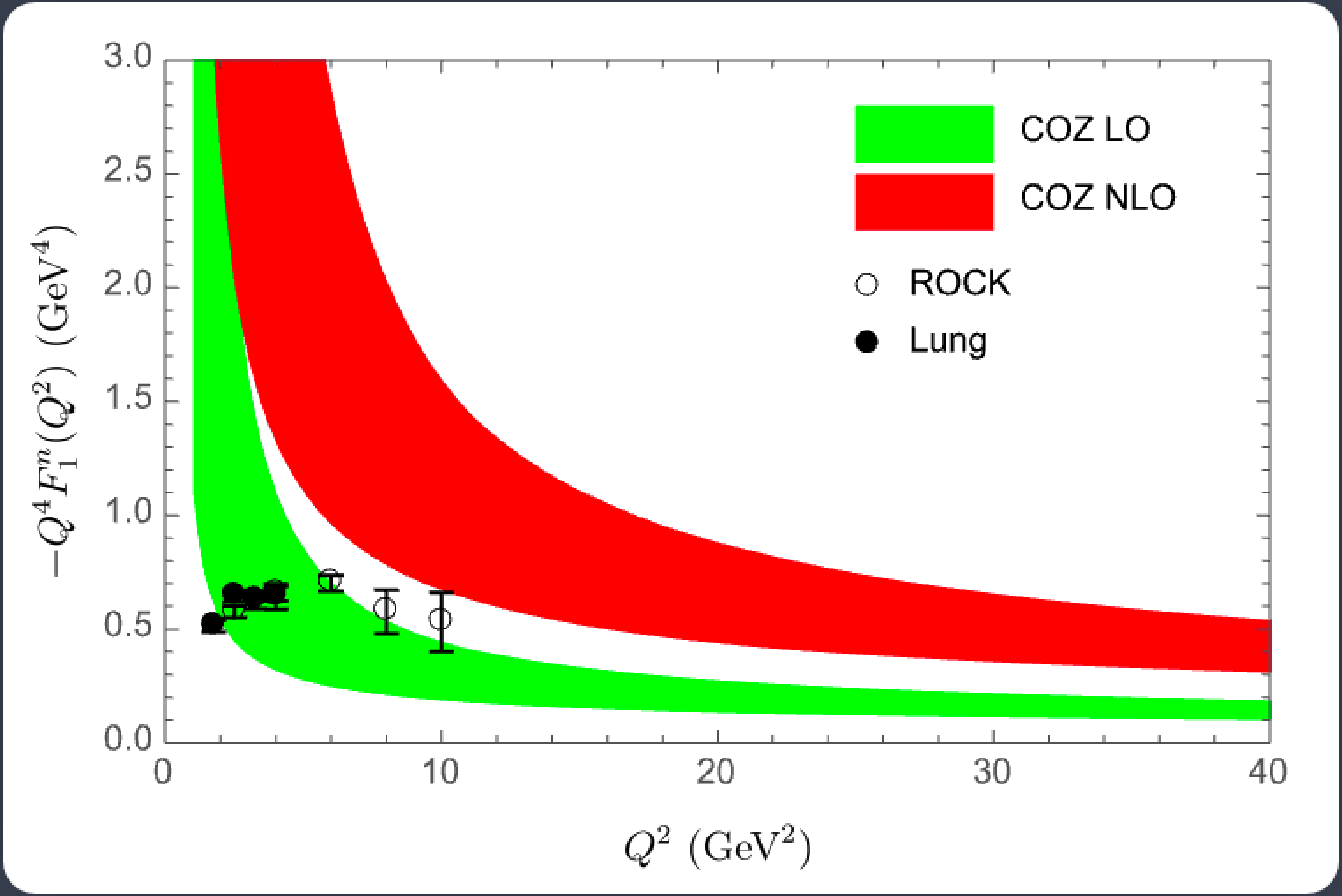
green band: LO prediction μ from $Q/2$ to Q

(x50): magnifying factor

Phenomenology

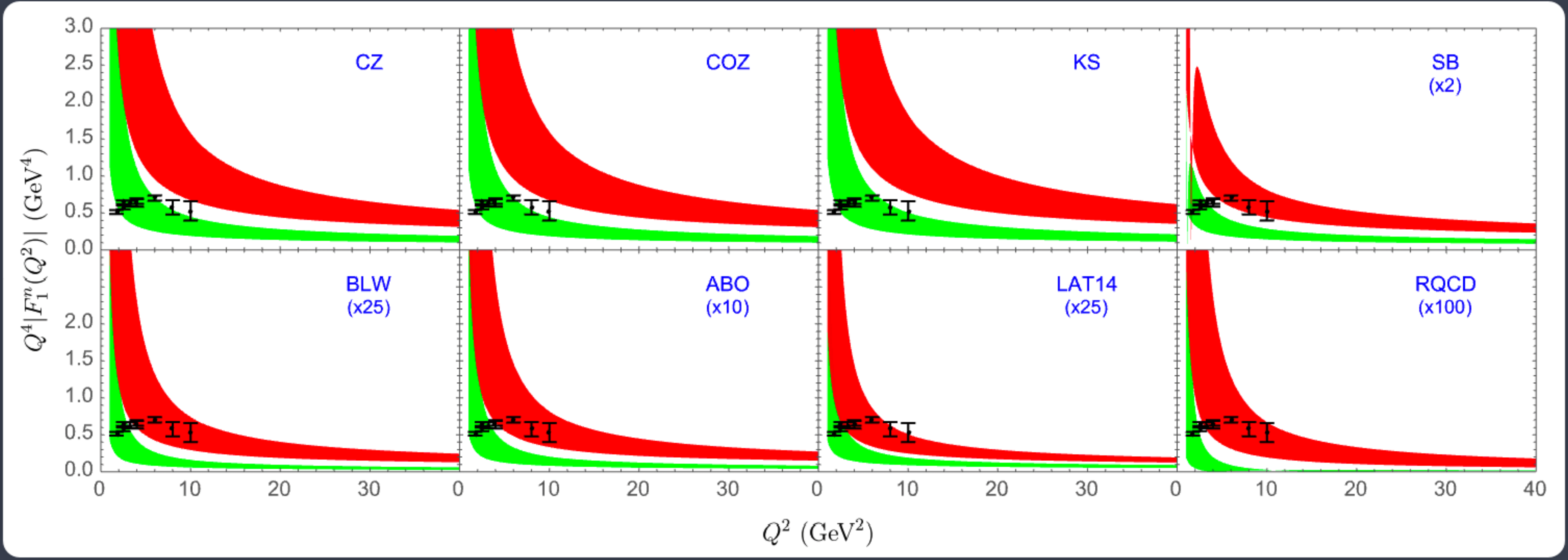
- SL EMFF for Neutron

ROCK: Phys. Rev. Lett. **49**, 1139 (1982)
Lung: Phys. Rev. Lett. **70**, 718 (1993)



Phenomenology

- SL EMFF for Neutron



red band: NLO prediction μ from $Q/2$ to Q

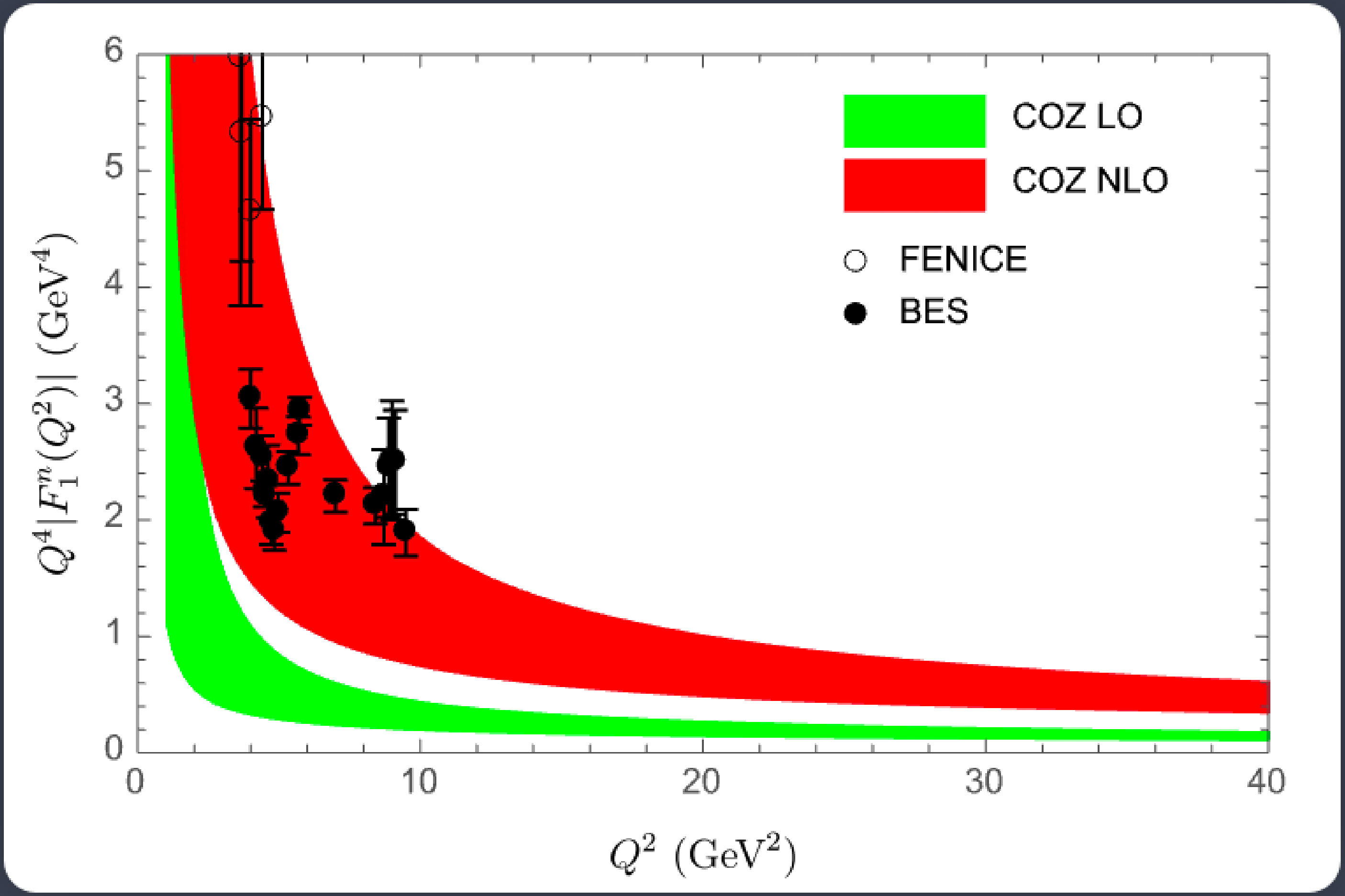
green band: LO prediction μ from $Q/2$ to Q

(x25): magnifying factor

Phenomenology

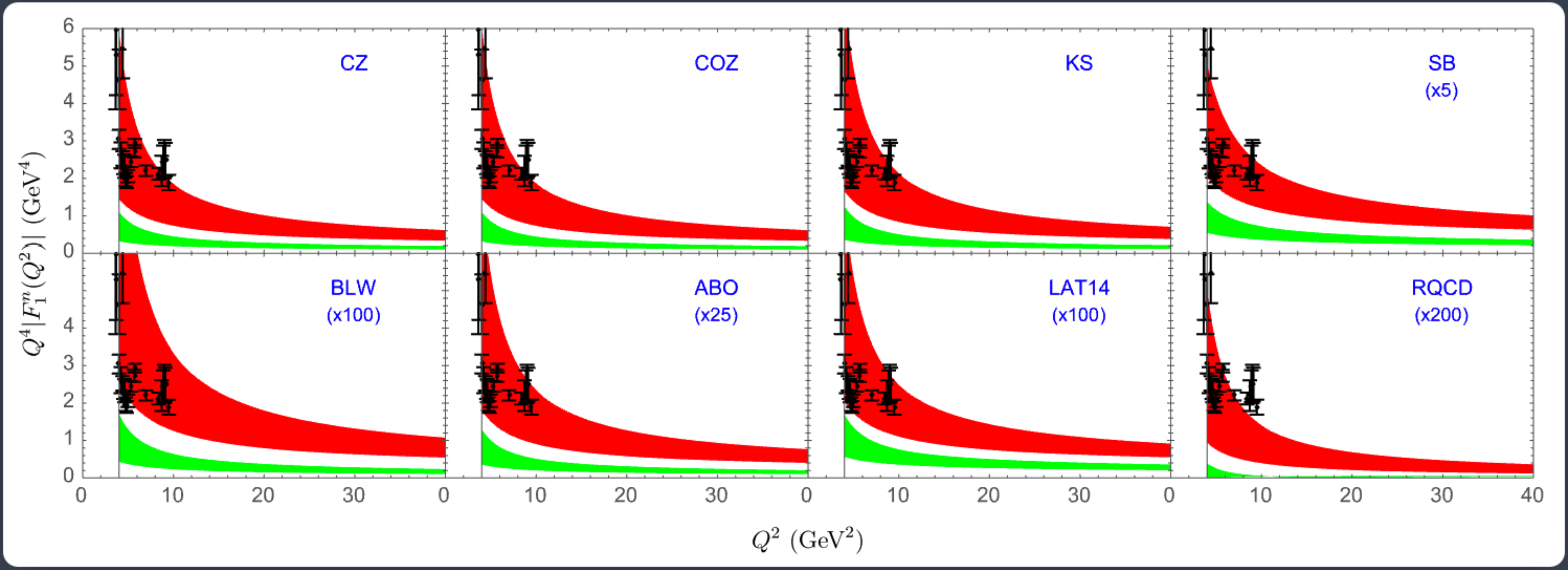
- TL EMFF for Neutron

FENICE: Nucl. Phys. B**517**, 3 (1998)
BES: Nature Phys. **17**, no.11, 1200 (2021)



Phenomenology

- TL EMFF for Neutron



red band: NLO prediction μ from $Q/2$ to Q

green band: LO prediction μ from $Q/2$ to Q

(x100): magnifying factor

Summary & Outlook:

- Nearly half a century later, we accomplish the calculation of the NLO QCD corrections to proton and neutron's Dirac form factors in collinear factorization.

- Remarkably, the effect of NLO perturbative corrections turns to be positive and significant. NLO predictions can decently describe the data for both proton and neutron's Dirac form factors, in both space-like and time-like regions, by taking LCDAs from a class of QCD sum rules-based models.

- In a sense, this work may herald the coming of a new phase of perturbative QCD, where NLO QCD corrections start to be systematically explored for a class of important hard exclusive reactions in baryon sector.

Pauli (axial, gravitational) form factors, nucleon Compton scattering, $\gamma\gamma \to N\overline{N}$, quarkonium decay into $N\overline{N}$, and so on.

Thanks for your attention!