Optimized QCD two-loop correction to exclusive double J/ψ production at B factories

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based on arXiv:2306.11538 (PRL2023)

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Outline：

1. Background
2. Outline of calculation
3. Discussion
4. Summary

1. Background

- Exclusive double charmonium production at e^+e^- collider is among the simplest hard exclusive reactions in perturbative QCD, which can be used to testify the factorization

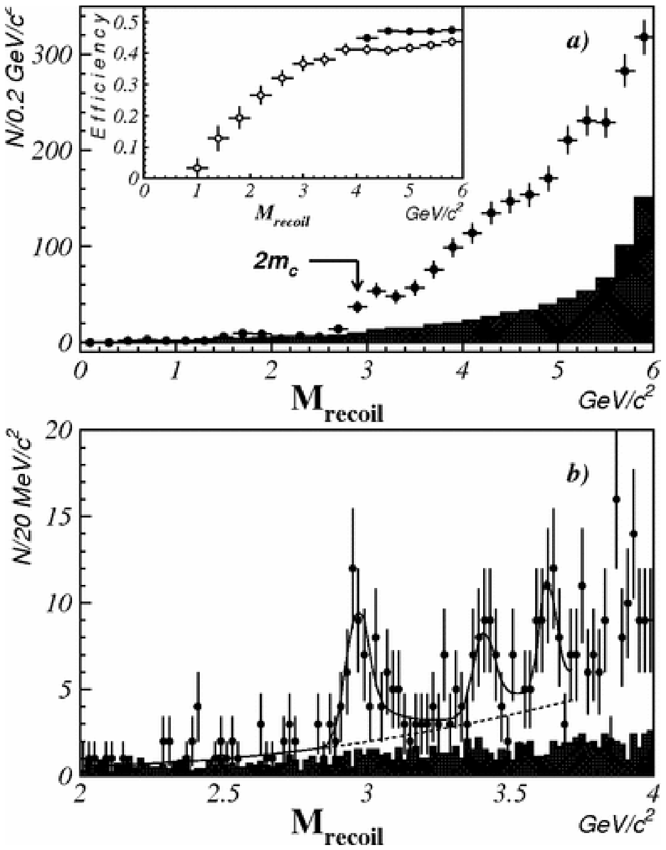
- **Significant attention** has been devoted to the study of double charmonium production at B factories at beginning of this century.

- Considerable effort has been paid to **reduce the discrepancy between experimental measurements and theoretical predictions** for the exclusive double charmonium production processes $e^+e^- \to J/\psi + \eta_c(\chi_{cJ})$

1. Background

$$e^+e^- \to J/\psi + \eta_c$$

On experiment side:



$$\sigma(e^+e^- \to J/\psi + \eta_c) \times \mathcal{B}_{>4} \quad = \quad 33^{+7}_{-6} \pm 9 \text{ fb} \quad \text{@BELLE(2002)},$$

$$\sigma(e^+e^- \to J/\psi + \eta_c) \times \mathcal{B}_{>2} \quad = \quad 25.6 \pm 2.8 \pm 3.4 \text{ fb} \quad \text{@BELLE(2004)},$$

$$\sigma(e^+e^- \to J/\psi + \eta_c) \times \mathcal{B}_{>2} \quad = \quad 17.6 \pm 2.8^{+1.5}_{-2.1} \text{ fb} \quad \text{@BABAR(2005)},$$

where $\mathcal{B}_{>2}$ signifies that branching fraction of η_c decay into the final states with more than 2 charged tracks

1. Background

$$e^+e^- \to J/\psi + \eta_c$$

There is an abundance of theoretical work on this process, based on various approaches. Below are several studies conduced within the framework of **NRQCD**

Braaten, Lee, PRD(2003)
Liu, He, Chao, PLB(2003)
Zhang, Gao, Chao, PRL(2006) ---- NLO QCD corrections
He, Fan, Chao, PRD(2007)
Bodwin, Lee, Yu, PRD(2008)
Gong, Wang, PRD(2008) ---- NLO QCD corrections
Dong, Feng, Jia, PRD(2012) ---- Mixed QCD and relativistic corrections
Li, Wang, CPC(2014) ---- Mixed QCD and relativistic corrections
Sun, Wu, Ma, Brodsky, PRD(2018)
… …

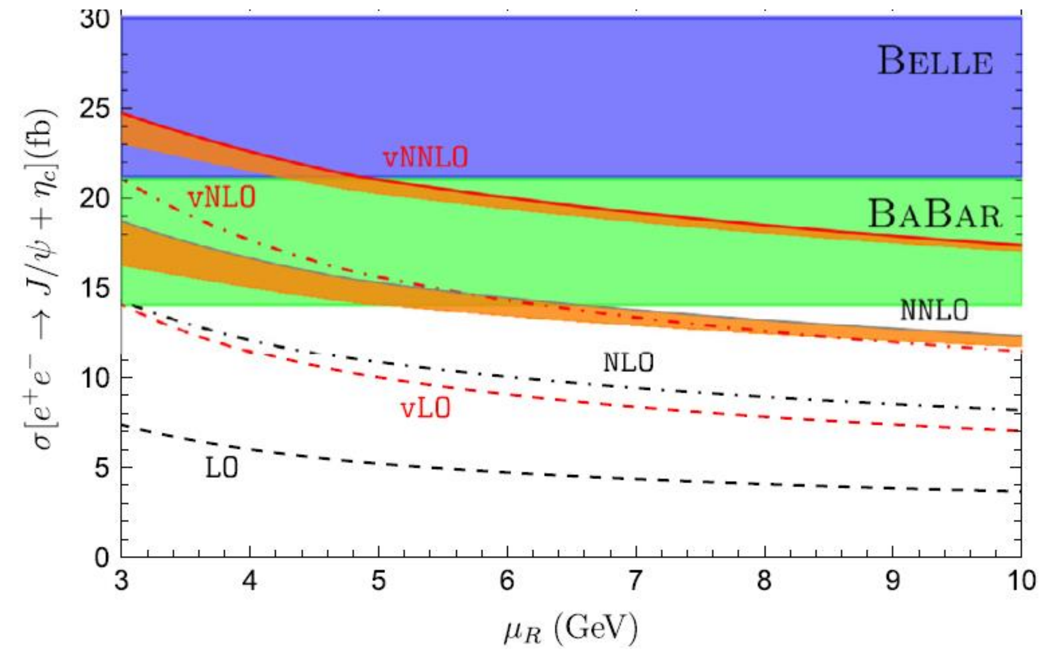
1. Background $e^+e^- \to J/\psi + \eta_c$

Feng, Jia, Mo, **SWL,** Zhang, arXiv: 1901.08447 (PLB 2024)

Table 2
Individual contributions to the predicted $\sigma[e^+e^- \to J/\psi + \eta_c]$ (in units of fb) at $\sqrt{s} = 10.58$ GeV. We take $\mu_R = \sqrt{s}/2$, and $\mu_\Lambda = 1$ GeV. The first error is obtained by varying m from $= 1.3$ to 1.7 GeV, and the second error is deduced by varying μ_R from $2m$ to $\sqrt{s}$.

LO	vLO	NLO	vNLO	NNLO	vNNLO	BELLE	BABAR
$5.05^{+0.92+2.31}_{-0.99-1.49}$	$9.70^{+2.73+4.45}_{-2.79-2.85}$	$10.60^{+2.87+3.74}_{-2.61-2.60}$	$15.25^{+4.69+5.87}_{-4.41-3.96}$	$15.09^{+5.03+3.68}_{-4.16-2.87}$	$20.74^{+8.84+4.00}_{-7.37-3.59}$	$25.6^{+2.8+3.4}_{-2.8-3.4}$	$17.6^{+2.8+1.5}_{-2.8-2.1}$

Despite of considerable uncertainty, the theoretical prediction is consistent with the experimental data.



Note: Huang, Gong, Wang, JHEP(2023)
The two-loop corrections are consistent with ours

1. Background

$e^+e^- \to J/\psi + \chi_{cJ}$

On experiment side:

$\sigma(e^+e^- \to J/\psi + \chi_{c0}) \times \mathcal{B}_{>2} = 6.4 \pm 1.7 \pm 1.0$ fb @BELLE, **PRD(2004), Belle**

$\sigma(e^+e^- \to J/\psi + \chi_{c0}) \times \mathcal{B}_{>2} = 10.3 \pm 2.5^{+1.4}_{-1.8}$ fb @BaBar, **PRD(2005), BaBar**

$[\sigma(e^+e^- \to J/\psi + \chi_{c1}) + \sigma(e^+e^- \to J/\psi + \chi_{c2})] \times \mathcal{B}_{>2} < 5.3$ fb @BELLE, **PRD(2004), Belle**

On theoretical side:

Braaten, Lee, **PRD2003**
Liu, He, Hagiwara, Kou, Qiao, **PLB2003**
He, Chao, **PLB 2003**
Zhang, Ma, Chao, **PRD2008** ---- NLO QCD corrections
Wang, Ma, Chao, **PRD2011** ---- NLO QCD corrections
Dong, Feng, Jia, **JHEP 2011** ---- NLO QCD corrections
Jiang, Sun, **EPJC 2018**
Sun, **JHEP 2021**
… …

1. Background

$e^+e^- \to J/\psi + \chi_{cJ}$

SWL, Feng, Jia, Mo, Zhang, arXiv:2202.11615v2

Comparison between our finest predictions to the **unpolarized cross sections** and the measurements in two B factories (in units of fb)

agree with experimental measurements

	LO	NLO	NNLO	BELLE $\sigma\times\mathcal{B}_{>2(0)}$ [53]	BABAR $\sigma\times\mathcal{B}_{>2}$ [2]
$\sigma(J/\psi+\chi_{c0})$	$4.80^{+2.47+2.20}_{-1.58-1.41}$	$9.20^{+5.81+2.45}_{-3.43-1.87}$	$9.22^{+7.08+0.09}_{-3.85-0.54}$	$6.4\pm1.7\pm1.0$	$10.3\pm2.5^{+1.4}_{-1.8}$
$\sigma(J/\psi+\chi_{c1})$	$0.807^{+0.528+0.370}_{-0.331-0.237}$	$1.11^{+0.93+0.21}_{-0.51-0.17}$	$0.965^{+1.039+0.002}_{-0.503-0.015}$	–	–
$\sigma(J/\psi+\chi_{c2})$	$1.16^{+1.05+0.53}_{-0.56-0.34}$	$1.36^{+1.35+0.11}_{-0.68-0.15}$	$0.832^{+0.898+0.157}_{-0.435-0.273}$	–	–
$\sigma(J/\psi+\chi_{c1})+\sigma(J/\psi+\chi_{c2})$	$1.97^{+1.58+0.90}_{-0.89-0.58}$	$2.46^{+2.27+0.31}_{-1.20-0.32}$	$1.80^{+1.94+0.16}_{-0.94-0.29}$	<5.3 at 90% C.L.	–
$\sigma(\psi(2S)+\chi_{c0})$	$3.13^{+1.61+1.44}_{-1.03-0.92}$	$6.01^{+3.80+1.60}_{-2.24-1.22}$	$6.02^{+4.62+0.06}_{-2.52-0.35}$	$12.5\pm3.8\pm3.1$	–
$\sigma(\psi(2S)+\chi_{c1})$	$0.527^{+0.345+0.241}_{-0.216-0.155}$	$0.722^{+0.605+0.134}_{-0.335-0.112}$	$0.630^{+0.678+0.001}_{-0.329-0.01}$	–	–
$\sigma(\psi(2S)+\chi_{c2})$	$0.759^{+0.688+0.348}_{-0.366-0.223}$	$0.886^{+0.880+0.069}_{-0.445-0.097}$	$0.543^{+0.586+0.102}_{-0.284-0.178}$	–	–
$\sigma(\psi(2S)+\chi_{c1})+\sigma(\psi(2S)+\chi_{c2})$	$1.29^{+1.03+0.59}_{-0.58-0.38}$	$1.61^{+1.48+0.20}_{-0.78-0.21}$	$1.17^{+1.26+0.10}_{-0.61-0.19}$	<8.6 at 90% C.L.	–

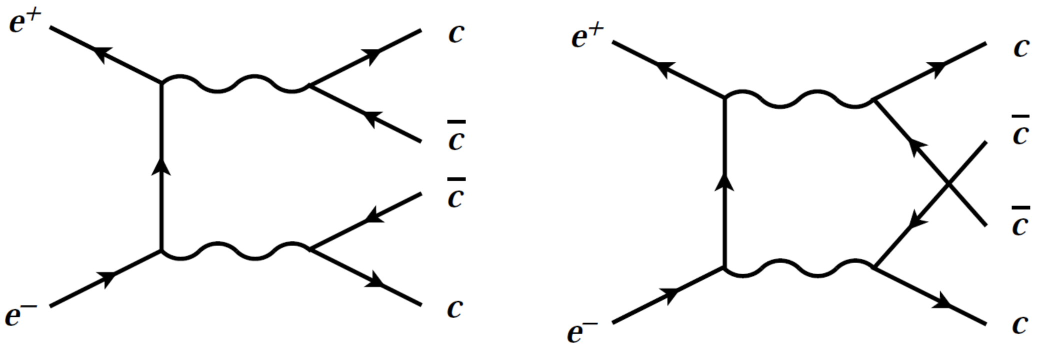
1. Background

How about double **vector** charmonia exclusive production at B factory, i.e.,

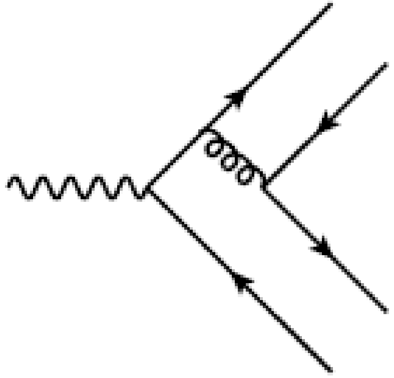
$$e^+e^- \to J/\psi + J/\psi$$

1) Such processes have to proceed via e^+e^- annihilating into **two** virtual photons

2) Naive expectation is that the production rate is more suppressed due to occurrence of the extra QED coupling constants



$$\sigma(e^+e^- \to J/\psi + J/\psi) \propto \alpha^4$$



$$\sigma(e^+e^- \to J/\psi + J/\psi) \propto \alpha^2\alpha_s^2$$

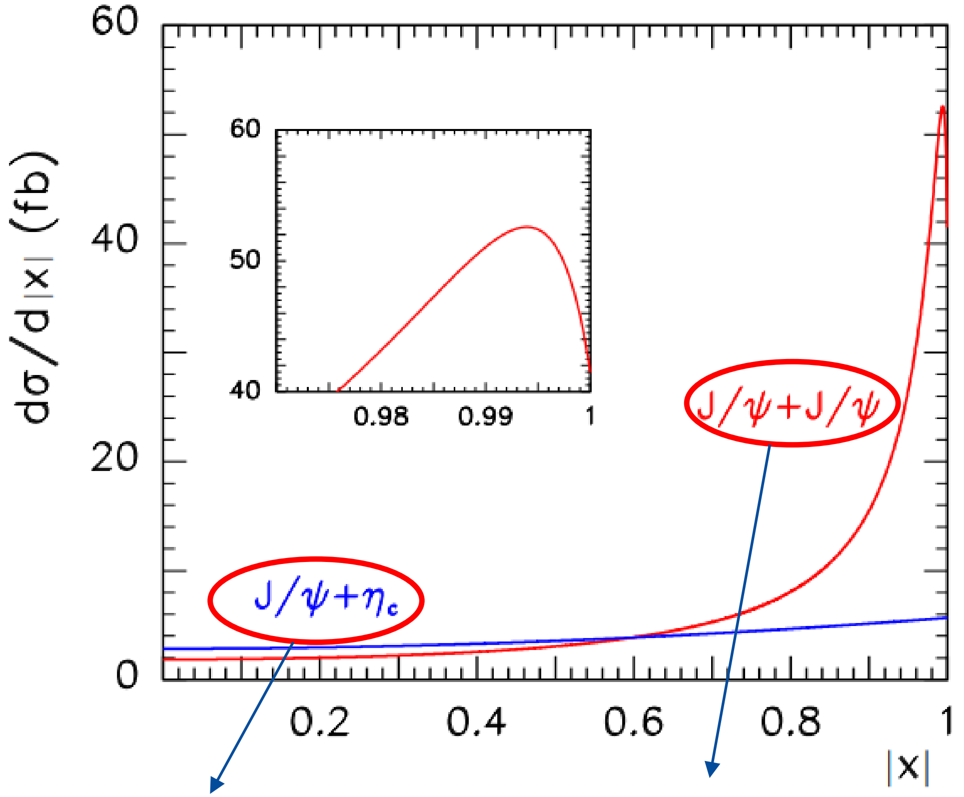
1. Background

Bodwin, Lee and Braaten PRL2003;
Bodwin, Lee and Braaten PRD, 2003 (E 2005)

Unpolarized Cross section (in units of **fb**)

$H_2 \setminus H_1$	J/ψ	$\psi(2S)$
J/ψ	6.65 ± 3.02	5.52 ± 2.50
$\psi(2S)$		1.15 ± 0.52

$H_2 \setminus H_1$	J/ψ	$\psi(2S)$
η_c	3.78 ± 1.26	1.57 ± 0.52
$\eta_c(2S)$	1.57 ± 0.52	0.65 ± 0.22

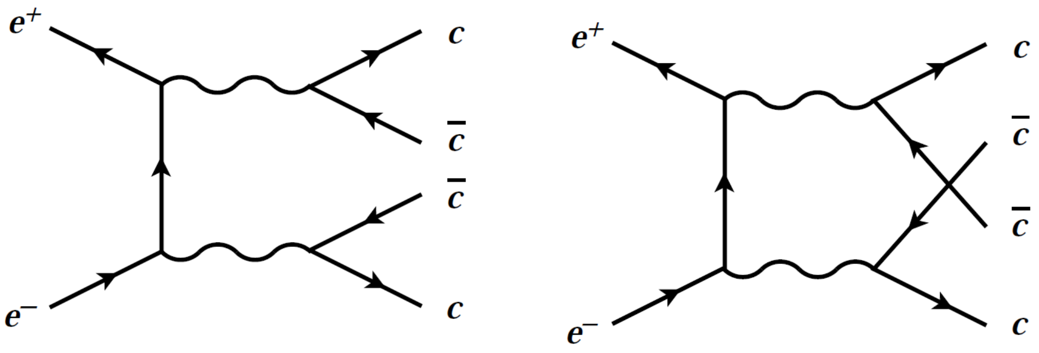


3.78 fb

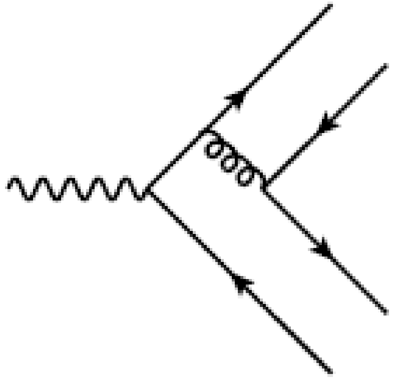
6.65 fb

Note the tree-level prediction for $\sigma(J/\psi + J/\psi)$ is even bigger than $\sigma(J/\psi + \eta_c)$

1. Background



$$\sigma(e^+e^- \to J/\psi + J/\psi) \propto \alpha^4$$



$$\sigma(e^+e^- \to J/\psi + J/\psi) \propto \alpha^2\alpha_s^2$$

How to explain?

1) Enhanced by fragmentation factor $(s/m_{J/\psi}^2)^4$

2) Enhanced by the propagator of the electron at small azimuth angle θ

1. Background

PRD(RC) (2004), BELLE   K. Abe et al. PRD 2008

Unfortunately, double J/ψ production has not yet been observed at B factories

TABLE I. Summary of the signal yields (N), charmonium masses (M), significances, and cross sections ($\sigma_{\rm Born} \times \mathcal{B}_{>2}[(c\bar{c})_{\rm res}]$) for $e^+e^- \to J/\psi(c\bar{c})_{\rm res}$; $\mathcal{B}_{>2}$ denotes the branching fraction for final states with more than two charged tracks.

$(c\bar{c})_{\rm res}$	N	M [GeV/c^2]	Signif.	$\sigma_{\rm Born} \times \mathcal{B}_{>2}$ [fb]
η_c	235 ± 26	2.972 ± 0.007	10.7	$25.6 \pm 2.8 \pm 3.4$
J/ψ	-14 ± 20	fixed	···	<9.1 at 90% CL
χ_{c0}	89 ± 24	3.407 ± 0.011	3.8	$6.4 \pm 1.7 \pm 1.0$
$\chi_{c1} + \chi_{c2}$	10 ± 27	fixed	···	<5.3 at 90% CL
$\eta_c(2S)$	164 ± 30	3.630 ± 0.008	6.0	$16.5 \pm 3.0 \pm 2.4$
$\psi(2S)$	-26 ± 29	fixed	···	<13.3 at 90% CL

TABLE III. Summary of the signal yields (N), significances, and cross sections ($\sigma_{\rm Born} \times \mathcal{B}_{>0}[(c\bar{c})_{\rm res}]$) for $e^+e^- \to \psi(2S) \times (c\bar{c})_{\rm res}$; $\mathcal{B}_{>0}$ denotes the branching fraction for final states containing charged tracks.

$(c\bar{c})_{\rm res}$	N	Signif.	$\sigma_{\rm Born} \times \mathcal{B}_{>0}$ [fb]
η_c	36.7 ± 10.4	4.2	$16.3 \pm 4.6 \pm 3.9$
J/ψ	6.9 ± 8.9	···	<16.9 at 90% CL
χ_{c0}	35.4 ± 10.7	3.5	$12.5 \pm 3.8 \pm 3.1$
$\chi_{c1} + \chi_{c2}$	6.6 ± 8.0	···	<8.6 at 90% CL
$\eta_c(2S)$	36.0 ± 11.4	3.4	$16.0 \pm 5.1 \pm 3.8$
$\psi(2S)$	-8.3 ± 8.5	···	<5.2 at 90% CL

Belle finds no evidence for $e^+e^- \to J/\psi + J/\psi$

Why have the experiment found the signal for the process J/ψ+η_c, but not for the process J/ψ+J/ψ

1. Background

We have known that both the radiative and relativistic corrections can considerably alter the LO cross section for J/ψ+η_c

Table 2
Individual contributions to the predicted $\sigma[e^+e^- \to J/\psi + \eta_c]$ (in units of fb) at $\sqrt{s} = 10.58$ GeV. We take $\mu_R = \sqrt{s}/2$, and $\mu_\Lambda = 1$ GeV. The first error is obtained by varying m from $= 1.3$ to 1.7 GeV, and the second error is deduced by varying μ_R from $2m$ to $\sqrt{s}$.

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How about the process J/ψ+J/ψ ?

1. Background

$e^+e^- \to J/\psi + J/\psi$

Current Research Progress

2002:	Bodwin, Lee, Braaten	NRQCD LO	**8.7** fb
2003:	Bodwin, Lee, Braaten	NRQCD LO	**6.65** fb
2006:	Davier, Peskin, Snyder	VMD	**2.38** fb
2006:	Bodwin, Braaten, Lee, Yu	fragmentation+nonfragmentation	**1.69±0.35** fb
2008:	Gong, Wang	**NRQCD NLO in α_s**	**-3.4—2.3 fb**
2013:	Fan, Lee, Yu	NRQCD NLO in α_s and v^2	**1—1.5** fb

The order-α_s correction is negative and significant!

1) Negative total cross section in some range of renormalization scale
2) How about the **perturbative convergence**? NNLO correction?

To provide useful guidance for experimentalists to search for this channel, it is crucial to present the **precise theoretical prediction**

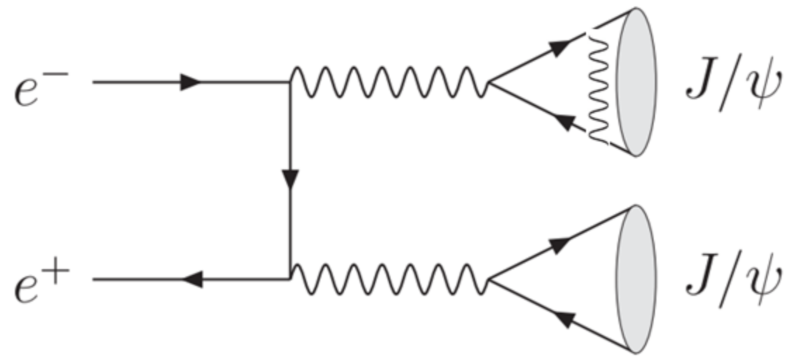
1. Background

$e^+e^- \to J/\psi + J/\psi$

Gong, Wang PRL(2008)

m_c (GeV)	μ	$\alpha_s(\mu)$	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NLO}/\sigma_{\rm LO}$
1.5	m_c	0.369	7.409	−2.327	−0.314
1.5	$2m_c$	0.259	7.409	0.570	0.077
1.5	$\sqrt{s}/2$	0.211	7.409	1.836	0.248
1.4	m_c	0.386	9.137	−3.350	−0.367
1.4	$2m_c$	0.267	9.137	0.517	0.057
1.4	$\sqrt{s}/2$	0.211	9.137	2.312	0.253

The main contribution comes from the fragmentation diagrams.



$$\sigma_{\rm NLO}/\sigma_{\rm LO} \approx (1 + f^{(1)}\frac{\alpha_s}{\pi})^4$$
$$\approx 1 - 11\frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \approx 1 - 0.88$$

$$\langle J/\psi|\bar{c}\gamma^\mu c|0\rangle = -f_{J/\psi} M_{J/\psi} \varepsilon^{*\mu}_{J/\psi}$$

$$f_{J/\psi} = \sqrt{\frac{2\langle\mathcal{O}\rangle_{J/\psi}}{M_{J/\psi}}}\left(1 + f^{(1)}\frac{\alpha_s}{\pi} + f^{(2)}\frac{\alpha_s^2}{\pi^2} + f^{(3)}\frac{\alpha_s^3}{\pi^3}\cdots\right)$$

$$f^{(1)} = -2C_F \qquad f^{(2)} \approx -43$$

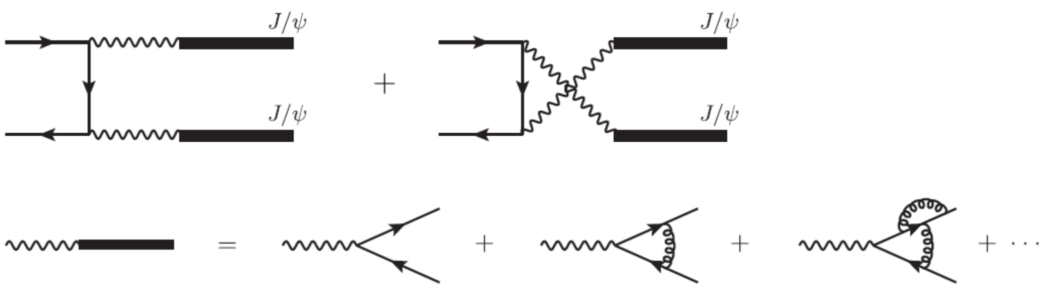
Czarnecki, etc. PRL(98); Beneke, etc. PRL(98)

$$f^{(3)} \approx -1736$$

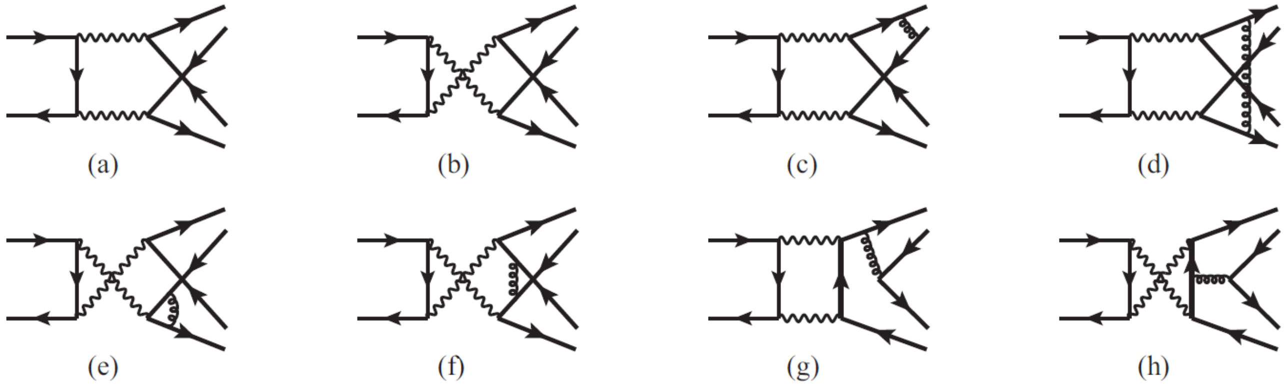
Marquard, etc. PRD(2014); Feng, etc. arXiv:2207.14259

2. Outline of calculation

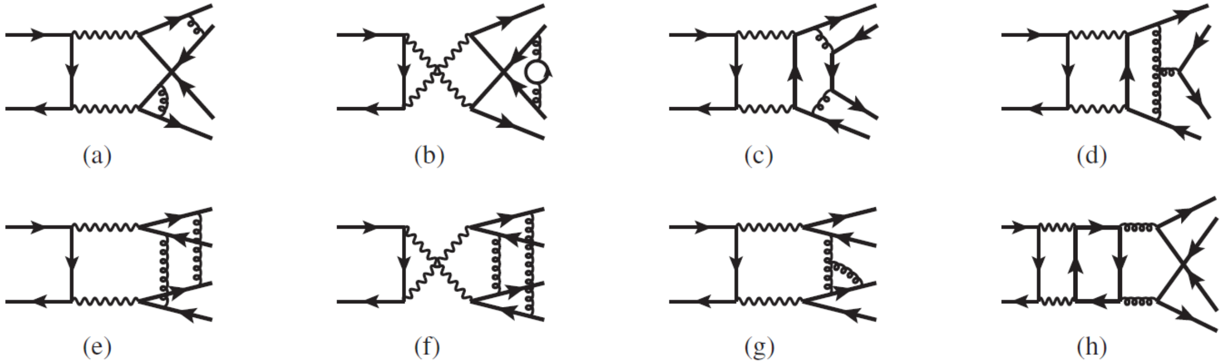
We split the Feynman diagrams into the fragmentation and non-fragmentation pieces



Fragmentation part



LO+NLO



NNLO

$$\mathcal{M} = \mathcal{M}_{\rm fr} + \mathcal{M}_{\rm nfr}$$

Total amplitude

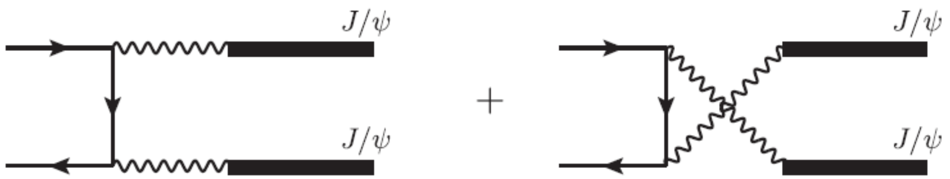
Note both amplitudes are gauge-invariant

Non-fragmentation part

2. Outline of calculation

$e^+e^- \to J/\psi + J/\psi$

Special treatment for the fragmentation part



The $\gamma^* \to J/\psi$ can be calculated using the VMD method

$$\langle J/\psi|\bar{c}\gamma^\mu c|0\rangle = -f_{J/\psi} M_{J/\psi} \varepsilon^{*\mu}_{J/\psi} \quad \Rightarrow \quad g_{\gamma^* \to J/\psi} = e_c M_{J/\psi} f_{J/\psi}$$

$f_{J/\psi}$ can be derived through $\quad f_{J/\psi} = \left(\frac{3M_{J/\psi}}{4\pi e_c^2 \alpha^2} \Gamma[J/\psi \to l^+ l^-] \right)^{1/2}$

Through this treatment, it implied that **we have resummed an infinite towers of perturbative and relativistic corrections** to **all orders**.

2. Outline of calculation

The differential cross section reads:

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{2s}\frac{\beta}{16\pi}\frac{1}{4}\sum_{\text{spin}} |\mathcal{M}_{\text{fr}} + \mathcal{M}_{\text{nfr}}|^2 \,.$$

Short-Distance coefficients

So we get

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{2s}\frac{\beta}{16\pi}\frac{e^8 e_c^4}{4}\left[\mathcal{C}_{\text{fr}} f_{J/\psi}^4 + \mathcal{C}_{\text{int}} f_{J/\psi}^2 \frac{\langle\mathcal{O}\rangle_{J/\psi}}{m_c} + \mathcal{C}_{\text{nfr}}\left(\frac{\langle\mathcal{O}\rangle_{J/\psi}}{m_c}\right)^2\right]$$

Fragmentation interference Non-fragmentation

Optimized NRQCD

2. Outline of calculation

$$\mathcal{C}_{\rm fr} = \frac{8\left(\left(t^2+u^2\right)\left(tu - M_{J/\psi}^4\right) + 4stuM_{J/\psi}^2\right)}{t^2u^2M_{J/\psi}^4}$$

Davier, Peskin, Snyder, hep-ph/0606155; Bodwin, Lee, Braaten and Yu, PRD(2006)

According to NRQCD, the other two SDCs can be parameterized **in LO in *v* but through α_s^2** as

$$\begin{aligned}
\mathcal{C}_{\rm int} &= \mathcal{C}_{\rm int}^{(0)}\left[1 + \frac{\alpha_s}{\pi}\hat{c}_{\rm int}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{\beta_0}{4}\ln\frac{\mu_R^2}{m_c^2}\hat{c}_{\rm int}^{(1)} + 2\gamma_{J/\psi}\ln\frac{\mu_\Lambda^2}{m_c^2} + \hat{c}_{\rm int}^{(2)}\right) + \cdots\right] \\
\mathcal{C}_{\rm nfr} &= \mathcal{C}_{\rm nfr}^{(0)}\left[1 + \frac{\alpha_s}{\pi}\hat{c}_{\rm nfr}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{\beta_0}{4}\ln\frac{\mu_R^2}{m_c^2}\hat{c}_{\rm nfr}^{(1)} + 4\gamma_{J/\psi}\ln\frac{\mu_\Lambda^2}{m_c^2} + \hat{c}_{\rm nfr}^{(2)}\right) + \cdots\right]
\end{aligned}$$

μ_R : renormalization scale

β_0 : one-loop coefficient of the QCD β function

The occurrence of the $\boldsymbol{\beta_0 \ln\mu_R}$ term is constrained by the renormalization group invariance

2. Outline of calculation

$$
\begin{aligned}
\mathcal{C}_{\rm int} &= \mathcal{C}^{(0)}_{\rm int}\left[1+\frac{\alpha_s}{\pi}\hat{c}^{(1)}_{\rm int}+\left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{\beta_0}{4}\ln\frac{\mu_R^2}{m_c^2}\hat{c}^{(1)}_{\rm int}+2\gamma_{J/\psi}\ln\frac{\mu_\Lambda^2}{m_c^2}+\hat{c}^{(2)}_{\rm int}\right)+\cdots\right]\\
\mathcal{C}_{\rm nfr} &= \mathcal{C}^{(0)}_{\rm nfr}\left[1+\frac{\alpha_s}{\pi}\hat{c}^{(1)}_{\rm nfr}+\left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{\beta_0}{4}\ln\frac{\mu_R^2}{m_c^2}\hat{c}^{(1)}_{\rm nfr}+4\gamma_{J/\psi}\ln\frac{\mu_\Lambda^2}{m_c^2}+\hat{c}^{(2)}_{\rm nfr}\right)+\cdots\right]
\end{aligned}
$$

μ_Λ : factorization scale, the explicit expression is constrained by the NRQCD factorization

Where $\gamma_{J/\psi}$ is the **anomalous dimension of the NRQCD vector current** **(first arises at two loop!)**

At two-loop $\gamma_{J/\psi}=-\frac{\pi^2}{12}C_F(2C_F+3C_A)$

Czarnecki, Melnikov, PRL(1998)
Beneke, Signer, Smirnov, PRL(1998)

The occurrence of the $\gamma_{J/\psi}$ $\ln\mu_\Lambda$ term is demaded by the NRQCD factorization

It is crucial to carry out the nontrivial SDCs $\hat{c}^{(1)}_{\rm int}$, $\hat{c}^{(2)}_{\rm int}$, $\hat{c}^{(1)}_{\rm nfr}$, $\hat{c}^{(2)}_{\rm nfr}$

2. Outline of calculation

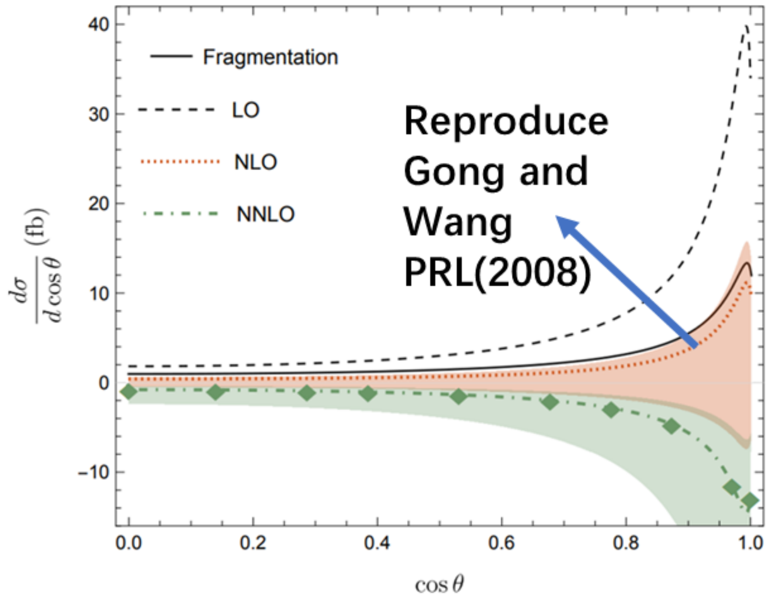
Our main results

$\cos\theta$	$\mathcal{C}_{\rm fr}$ (GeV^{-4})	$\mathcal{C}_{\rm int}^{(0)}$ (GeV^{-4})	$\hat{c}_{\rm int}^{(1)}$	$\hat{c}_{\rm int}^{(2)}$	$\mathcal{C}_{\rm nfr}^{(0)}$ (GeV^{-4})	$\hat{c}_{\rm nfr}^{(1)}$	$\hat{c}_{\rm nfr}^{(2)}$
0.999	4.163	-0.334	-3.62	-71.75	0.006	-7.42	$-143.174 + 42.974 = -100.20$
0.970	3.646	-0.242	-1.34	-76.57	0.007	-6.33	$-146.117 + 37.424 = -108.69$
0.872	1.573	-0.193	-0.73	-80.64	0.008	-5.07	$-152.144 + 25.321 = -126.82$
0.775	0.988	-0.176	-1.27	-81.77	0.010	-5.11	$-155.633 + 19.124 = -136.51$
0.677	0.722	-0.164	-1.85	-82.00	0.011	-5.49	$-157.716 + 15.969 = -141.75$
0.531	0.522	-0.152	-2.58	-81.67	0.012	-6.15	$-159.349 + 14.092 = -145.26$
0.384	0.422	-0.143	-3.12	-81.08	0.012	-6.73	$-160.032 + 13.777 = -146.26$
0.287	0.383	-0.139	-3.38	-80.71	0.012	-7.03	$-160.222 + 13.898 = -146.32$
0.140	0.350	-0.135	-3.63	-80.31	0.012	-7.32	$-160.324 + 14.160 = -146.16$
0	0.340	-0.133	-3.70	-80.17	0.012	-7.41	$-160.341 + 14.271 = -146.07$

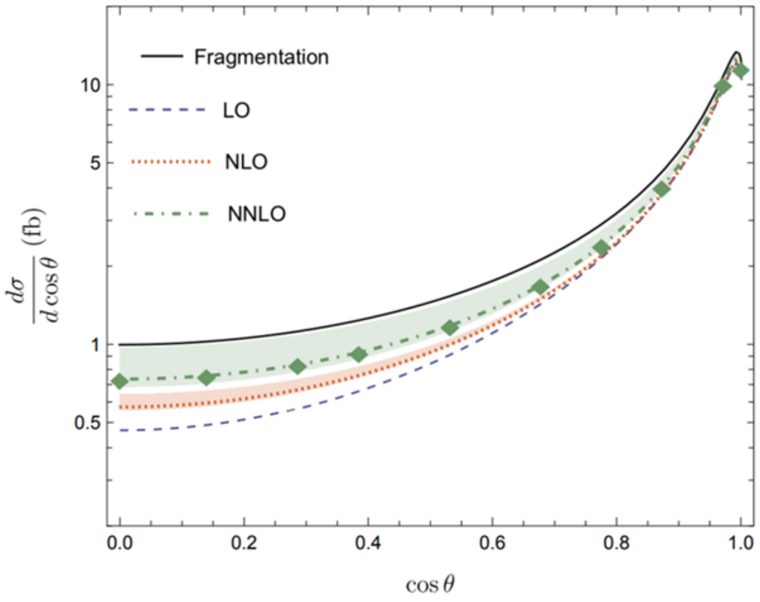
Table 1: Numerical values of various SDCs through $\mathcal{O}(\alpha_s^2)$ for ten different values of $\cos\theta$.

3. Discussion

Our main results



Traditional
NRQCD



Optimized
NRQCD

Differential cross sections for e+e−⟶J/ψ+J/ψ against cosθ at various perturbative accuracy from traditional NRQCD and our improved NRQCD approach.

- Both NLO and NNLO correction is positive!
- Exhibit decent convergence behavior!
- When J/ψ production plane is nearly collinear to e+e−($\theta \to 0$), fragmentation contribution dominates!
- As θ deviates from 0, corrections from non-fragmentation amplitude start to play non-negligible role!

3. Discussion

σ (fb)	Fragmentation	LO	NLO	NNLO
Optimized NRQCD	2.52	1.85	$1.93^{+0.05}_{-0.01}$	$2.13^{+0.30}_{-0.06}$
Traditional NRQCD		6.12	$1.56^{+0.73}_{-2.95}$	$-2.38^{+1.27}_{-5.35}$

Integrated cross section of $e^+e^- \to J/\psi + J/\psi$ at various perturbative accuracy. The uncertainties are estimated by varying μ_R from m_c to $\sqrt{s}$

- To date the Belle and the Belle2 experiments have accumulated about 1500 fb^{-1} data, so we expect about **3105~3645** exclusive double J/ψ events.
- Taking into account $Br(J/\psi \to l^+l^-)=12\%$, **about 45~52 four-lepton events from double J/ψ can be produced**.
- Assuming 40% reconstruction efficiency, we expect about **18~21** signal events may be reconstructed.
- With the designed **50 ab^{-1} integrated luminosity at Belle2**, it seems that the observation prospects of exclusive double J/ψ production **is promising in the foreseeable future**.

3. Discussion

A Very recent work prediction from distinct treatment $\sigma_{\text{NNLO}} = 1.76^{+2.42}_{-1.66}\text{fb}$

X.-D. Huang, B. Gong, R.-C. Niu, H.-M. Yu, J.-X. Wang, JHEP 2024

Next-to-next-to-leading-order QCD corrections to double J/ψ production at the B factories

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Abstract: In this paper, we study the next-to-next-to-leading-order (NNLO) QCD corrections for the process $e^+e^- \to J/\psi + J/\psi$ at the B factories. By including the NNLO corrections, the cross section turns negative due to the poor convergence of perturbative expansion. Consequently, to obtain a reasonable estimation for the cross section, the square of the amplitude up to NNLO is used. In addition, the contributions from the bottom quark and the light-by-light part, which are usually neglected, are also included. The final cross section is obtained as $1.76^{+2.42}_{-1.66}$ fb at a center-of-mass energy of $\sqrt{s} = 10.58\,\text{GeV}$. Our result for total cross section and differential cross section could be compared with precise experimental measurement in future at the B factories.

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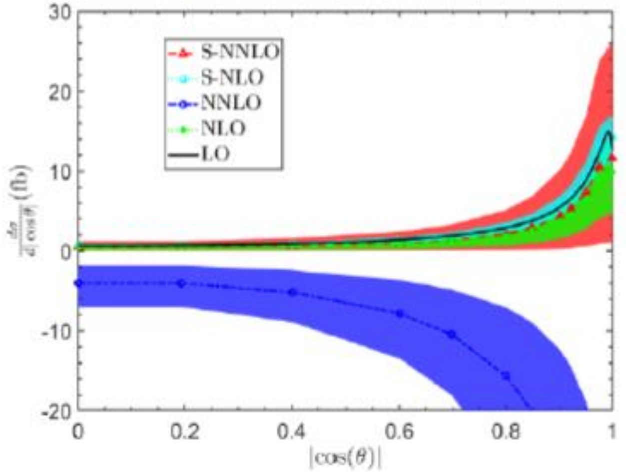

Figure 3. The differential cross section of $e^+e^- \to J/\psi + J/\psi$ as function of $|\cos\theta|$ at various perturbative order, and the bands are obtained by varying the renormalization scale μ_R within the range of $[2m_c, \sqrt{s}]$.

$\sigma(fb)$	LO	NLO	NNLO	S-NLO	S-NNLO
$\mu_R = 2m_c$	2.29	0.61	−21.10	1.83	0.12
$\mu_R = \sqrt{s}/2$	2.29	1.54	−11.97	2.37	1.76
$\mu_R = \sqrt{s}$	2.29	2.25	−5.27	2.84	4.17

4. Summary

- NNLO prediction for double J/ψ at B factories in conventional NRQCD approach exhibits poor perturbative convergence, leading to unphysical negative cross section.

- Useful to split the amplitude into photon fragmentation and non-fragmentation pieces. In the **optimized NRQCD** approach, both NLO and NNLO corrections are positive and exhibit a **reasonable convergence** pattern.

- The NNLO prediction for the total cross section is $2.13^{+0.30}_{-0.06}$ fb at CM energy 10.58 GeV in the optimized approach . With the projected integrated luminosity of 50 ab^{-1}, the **prospect to observe this exclusive process at Belle 2 experiment appears to be bright**.

Thank you for your attention！