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$K_1(1270)-K_1(1400)$ mixing in QCD sum rule

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1. The flavor SU(3) symmetry plays an important role in the conventional quark model, which classifies hadrons into various of irreducible representations.

2. Although the assumption of perfect flavor SU(3) symmetry succeeds in most of phenomenological analysis on hadron decays and spectrums, its breaking effect still cannot be neglected.

3. One of the physical effect due to the flavor SU(3) breaking is the hadron mixing.







There are two sources of the flavor SU(3) breaking.

- 1. The electric charge difference among the u, d, s quarks, which involves the QED effect. the QED effect is expected to be tiny!
- 2. The mass difference between u, d and s quarks, which only provides QCD contribution to hardon mixing.



Flavor SU(3) breaking:

$$\begin{pmatrix} |K_1(1270)\rangle \\ |K_1(1400)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{K_1} & \sin\theta_{K_1} \\ -\sin\theta_{K_1} & \cos\theta_{K_1} \end{pmatrix} \begin{pmatrix} |K_{1B}\rangle \\ |K_{1A}\rangle \end{pmatrix} {}^{1}P_1 \ 1^{+-} \\ {}^{3}P_1 \ 1^{++} \end{pmatrix}$$
Physical states Mixing matrix Flavor states

Mixing angle is very important!







Real process only happen for hadrons: b s B q

Mixing in axial vector kaons

$$K, K^*, K_1$$

 $0^-, 1^-, 1^+$

 \checkmark

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1. Introduction

- (1). Using early experimental information on masses and the partial rates of $K_1(1270)$ and $K_1(1400)$. *Phys.Rev.D* 47 (1993) 1252-1255
- $K_1(1270)$ - $K_1(1400)$ mixing angle in literatures



- (2). Phenomenologically analyzing on the τ weak decays: $\tau \rightarrow K_1(1270)\nu_{\tau}$ and $\tau \rightarrow K_1(1400)\nu_{\tau}$, Phys.Rev.D 67 (2003) 094007, Phys.Rev.D 79 (2009) 034014
- (3). Analyzing the $f_1(1285) f_1(1420)$ mixing angle and its correlation to θ_{K_1} , Z.Phys.C 76 (1997) 469-474
- (4). Analyzing both the mixing angle of $f_1(1285) f_1(1420)$ and $h_1(1175) h_1(1380)$, PoS Hadron2013 (2013) 090
- (5). Non-relativistic constituent quark model, Phys.Rev.D 56 (1997) R1368-R1372
- (6). Extract θ_{K_1} from $B \to J/\psi K_1(1270)$ and $B \to J/\psi K_1(1400)$ in PQCD, Eur.Phys.J.C 78 (2018) 3, 219
- (7). QCD sum rules with two-point correlation function, J.Phys.Conf.Ser. 348 (2012) 012012



2. mixing from SU(3) breaking

$$\mathcal{L}_{\rm QCD} = \mathcal{L}_0 + \Delta \mathcal{L}$$

SU(3) conserving term: The same mass for u, d, s:

$$\mathcal{L}_0 = \sum \bar{q}(i\mathcal{D} - m_u)q$$

SU(3) breaking terms: The different mass for u, d, s:

$$\Delta \mathcal{L} = \bar{s}(m_u - m_s)s$$

$$\Delta H = \int d^3x \Delta \mathcal{H}(x) = -\int d^3x \Delta \mathcal{L}(x)$$

the mass eigenstates of the full Hamiltonian H:

 $H|K_1(1270)\rangle = m_{1270}|K_1(1270)\rangle,$ $H|K_1(1400)\rangle = m_{1400}|K_1(1270)\rangle.$

the mass eigenstates of the SU(3) conserved Hamiltonian H0:

$$H_0|K_{1B}\rangle = m_{1B}|K_{1B}\rangle,$$

$$H_0|K_{1A}\rangle = m_{1A}|K_{1A}\rangle\rangle.$$

Full Hamiltonian: $H = H_0 + \Delta H_0$



This method was firstly proposed in the study of $\Xi_c - \Xi_c'$ mixing :

- H. Liu, W. Wang, Q. Zhang, the $\Xi_c \Xi_c'$ mixing from lattice QCD, Phys.Lett.B 841 (2023) 137941.
- H. Liu, W. Wang, Q. Zhang, Improved method to determine the $\Xi_c \Xi_c'$ mixing, PRD 109(2024)036037.

and a further study including the QED effect:

Z. Deng, Y. Shi, W. Wang and J. Zeng, QED contributions to the $\Xi_c - \Xi_c'$ mixing, PRD109(2024)036014.

the QED effect is expected to be tiny!



 \times

Transformation between doublets:

$$\begin{split} |K_{P}\rangle &= (|K_{1}(1270)\rangle, |K_{1}(1400)\rangle)^{T} \qquad |K_{F}\rangle = (|K_{1B}\rangle, |K_{1A}\rangle)^{T} \\ |K_{P}\rangle &= \begin{pmatrix} \cos\theta_{K_{1}} & \sin\theta_{K_{1}} \\ -\sin\theta_{K_{1}} & \cos\theta_{K_{1}} \end{pmatrix} |K_{F}\rangle = U|K_{F}\rangle \\ \begin{pmatrix} \langle K_{1B}(\lambda')|H|K_{1B}(\lambda)\rangle & \langle K_{1B}(\lambda')|H|K_{1A}(\lambda)\rangle \\ \langle K_{1A}(\lambda')|H|K_{1B}(\lambda)\rangle & \langle K_{1A}(\lambda')|H|K_{1A}(\lambda)\rangle \end{pmatrix} \\ 2(2\pi)^{3}\delta^{(3)}(\vec{0})\delta_{\lambda\lambda'} &= \frac{\langle K_{1B}(\lambda')|H|K_{1A}(\lambda)\rangle}{(m_{1270}^{2} c_{k}^{2} + m_{1400}^{2} s_{k}^{2})} & s_{k} = \sin\theta_{K_{1}} \\ (m_{1270}^{2} c_{k}^{2} + m_{1400}^{2} s_{k}^{2}) & m_{1270}^{2} s_{k}^{2} + m_{1400}^{2} c_{k}^{2} \end{pmatrix} & s_{k} = \sin\theta_{K_{1}} \\ c_{k} = \cos\theta_{K_{1}} \end{split}$$



2. mixing from SU(3) breaking

$$\sin 2\theta_{K_1} = \frac{m_s - m_u}{m_{1270}^2 - m_{1400}^2} \langle K_{1B} | \bar{s}s(0) | K_{1A} \rangle$$

$$\langle K_{1B}(p_2) | \bar{s}s(0) | K_{1A}(p_1) \rangle$$

$$= \epsilon_{\mu}^*(p_2) \left[F_1 g^{\mu\nu} + \frac{F_2}{M^2} \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} + \frac{F_3}{M^2} p_1^{\mu} p_2^{\nu} \right] \epsilon_{\nu}(p_1) \text{ Only } F_1 \text{ is necessary!}$$

$$\sin 2\theta_{K_1} = \frac{m_s - m_u}{m_{1400}^2 - m_{1270}^2} F_1(0)$$



3. $K_1(1270)$ - $K_1(1400)$ mixing in QCDSR

$$\Pi_{\mu\nu\rho}(p_{1},p_{2}) = i^{2} \int d^{4}x d^{4}y \ e^{ip_{2} \cdot x} e^{-ip_{1} \cdot y} \langle 0|T\{J_{\mu\nu}^{1B}(x)\bar{s}s(0)J_{\rho}^{1A\dagger}(y)\}|0\rangle$$

$$J_{\mu\nu}^{1B} = \bar{q}\sigma_{\mu\nu}s, \ J_{\rho}^{1A} = \bar{q}\gamma_{\rho}\gamma_{5}s.$$

$$\langle 0|J_{\mu\nu}^{1B}(0)|K_{1B}(p,\lambda)\rangle = if_{K_{1B}}\epsilon_{\mu\nu\alpha\beta}\epsilon^{\alpha}(p,\lambda)p^{\beta},$$

$$\langle 0|J_{\rho}^{1A}(0)|K_{1A}(p,\lambda)\rangle = -if_{K_{1A}}m_{1A}\epsilon_{\rho}(p,\lambda),$$

$$\langle 0|J_{\rho}^{1A}(0)|K(p)\rangle = if_{K}p_{\rho},$$
Hadron Level:
insert complete sets

$$\sum_{\lambda_{2}}\int \frac{d^{3}k_{2}}{(2\pi)^{3}}\frac{1}{2E_{k_{2}}}|K_{1B}(k_{2},\lambda_{2})\rangle\langle K_{1B}(k_{2},\lambda_{2})|$$

$$\sum_{\lambda_{1}}\int \frac{d^{3}k_{1}}{(2\pi)^{3}}\frac{1}{2E_{k_{1}}}|K_{1A}(k_{1},\lambda_{1})\rangle\langle K_{1A}(k_{1},\lambda_{1})|$$

$$\int d^{4}z e^{iq \cdot z}T[O_{a}(z)O_{b}(0)] = \sum_{k}C_{abk}(q)O_{k}(q)$$

))



Hadron Level: $\Pi^{H}_{\mu\nu\rho}(p_1,p_2)$

$$\begin{aligned} &= -m_A f_A f_B^{\perp} \epsilon_{\mu\nu\alpha\beta} p_2^{\beta} \left(-g^{\alpha\kappa} + \frac{p_2^{\alpha} p_2^{\kappa}}{m_B^2} \right) \frac{1}{p_2^2 - m_{1B}^2} \left[F_1 g_{\kappa\tau} + \frac{F_2}{M^2} \epsilon_{\kappa\tau\rho\sigma} p_1^{\rho} p_2^{\sigma} + \frac{F_3}{M^2} p_{1\kappa} p_{2\tau} \right] \left(-g_{\rho}^{\tau} + \frac{p_1^{\tau} p_{1\rho}}{m_{1A}^2} \right) \frac{1}{p_1^2 - m_{1A}^2} \\ &+ f_K f_{1B}^{\perp} \epsilon_{\mu\nu\alpha\beta} p_2^{\beta} p_{1\rho} \frac{1}{p_2^2 - m_{1B}^2} \left(-g^{\alpha\kappa} + \frac{p_2^{\alpha} p_2^{\kappa}}{m_{1B}^2} \right) p_{1\kappa} \frac{G(q^2)}{m_K} \frac{1}{p_1^2 - m_K^2} \\ &+ \int_{s_1^{\text{th}}}^{\infty} ds_1 \int_{s_2^{\text{th}}}^{\infty} ds_2 \frac{\rho_{\mu\nu\rho}^{\text{conti}}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} + \int_{s_1^{\text{th}}}^{\infty} ds_1 \frac{\rho_{1,\mu\nu\rho}^{\text{conti}}(s_1, p_2, q^2)}{s_1 - p_1^2} + \int_{s_2^{\text{th}}}^{\infty} ds_2 \frac{\rho_{2,\mu\nu\rho}^{\text{conti}}(p_1, s_2, q^2)}{s_2 - p_2^2}. \end{aligned}$$

 $\langle K_{1B}(p_2)|\bar{s}s(0)|K(p_1)\rangle = \epsilon^*_{\mu}(p_2)p_1^{\mu}\frac{G(q^2)}{m_K}$

To remove the irrelevant form factors we operate the following projection on the correlation function

$$\epsilon^{\mu\rho\alpha\beta}p_1^{\nu}\Pi_{\mu\nu\rho}(p_1,p_2) = \tilde{\Pi}(p_1,p_2)(p_1^{\beta}p_2^{\alpha} - p_1^{\alpha}p_2^{\beta})$$
$$\tilde{\Pi}^H(p_1,p_2) = \frac{2m_{1A}f_{1A}f_{1B}^{\perp}F_1(q^2)}{(p_1^2 - m_{1A}^2)(p_2^2 - m_{1B}^2)} + \cdots$$



3. $K_1(1270)$ - $K_1(1400)$ mixing in QCDSR











$$\begin{aligned} \Pi^{\text{pert}}_{\mu\nu\rho}(p_1, p_2, q^2) \\ = & \frac{iN_c}{(2\pi)^4} \int d^4k_1 d^4k_2 d^4k \ \delta^4(p_2 - k_2 - k) \delta^4(p_1 - k_1 - k) \\ & \times \frac{\text{tr}[\not\! k \sigma_{\mu\nu}(\not\! k_2 + m_s)(\not\! k_1 + m_s)\gamma_\rho\gamma_5]}{k^2(k_2^2 - m_s^2)(k_1^2 - m_s^2)}. \end{aligned}$$

 $\Pi^{\bar{q}q}_{\mu\nu\rho}(p_1, p_2, q^2) = i^2 \int d^4x d^4y \ e^{ip_2 \cdot x} e^{-ip_1 \cdot y}$ $\times [\sigma_{\mu\nu} D_s^{(0)}(x, 0) D_s^{(0)}(0, y) \gamma_\rho \gamma_5] \langle 0 | \bar{q}_a^i(x) q_b^i(y) | 0 \rangle$ $\mathbf{dim-3 + dim-5}$

$$\begin{split} \Pi^{GG(a)}_{\mu\nu\rho}(p_{1},p_{2},q^{2}) & \Pi^{GG(b)}_{\mu\nu\rho}(p_{1},p_{2},q^{2}) \\ = \int d^{4}xd^{4}y \ e^{ip_{2}\cdot x}e^{-ip_{1}\cdot y} \int \frac{d^{4}k_{1}}{(2\pi)^{4}} \frac{d^{4}k_{2}}{(2\pi)^{4}} \frac{d^{4}k_{2}}{(2\pi)^{4}} \ e^{ik_{1}\cdot y}e^{-ik_{2}\cdot x}e^{-ik\cdot(y-x)} \left(-\frac{i}{4}\right)^{2} \\ & \times \operatorname{tr}\left[\frac{-ik}{k^{2}}\sigma_{\mu\nu}\frac{\sigma^{\alpha\beta}(k_{2}+m_{s}) + (k_{2}+m_{s})\sigma^{\alpha\beta}}{(k_{2}^{2}-m_{s}^{2})^{2}} \frac{\sigma^{\kappa\tau}(k_{1}+m_{s}) + (k_{1}+m_{s})\sigma^{\kappa\tau}}{(k_{2}^{2}-m_{s}^{2})^{2}}\gamma_{\rho}\gamma_{5}\right] \\ & \times \operatorname{tr}\left[t^{A}t^{B}\right]g_{s}^{2}\langle 0|G_{\alpha\beta}^{A}(0)G_{\kappa\tau}^{B}(0)|0\rangle. \end{split} \\ \begin{aligned} \Pi^{GG(b)}_{\mu\nu\rho}(p_{1},p_{2},q^{2}) \\ & = \int d^{4}xd^{4}y \ e^{ip_{2}\cdot x}e^{-ip_{1}\cdot y} \int \frac{d^{4}k_{1}}{(2\pi)^{4}} \frac{d^{4}k_{2}}{(2\pi)^{4}} \ e^{ik_{1}\cdot y}e^{-ik_{2}\cdot x} \left(-\frac{i}{32\pi^{2}}\right) \left(-\frac{i}{4}\right)\operatorname{tr}[t^{A}t^{B}] \\ & = \int d^{4}xd^{4}y \ e^{ip_{2}\cdot x}e^{-ip_{1}\cdot y} \int \frac{d^{4}k_{1}}{(2\pi)^{4}} \frac{d^{4}k_{2}}{(2\pi)^{4}} \ e^{ik_{1}\cdot y}e^{-ik_{2}\cdot x} \left(-\frac{i}{4}\right)\operatorname{tr}[t^{A}t^{B}] \\ & \times \operatorname{tr}\left[\frac{(\psi-\psi)\sigma^{\kappa\tau}+\sigma^{\kappa\tau}(\psi-\psi)}{(y-x)^{2}}\sigma_{\mu\nu}\frac{i(k_{2}+m_{s})}{k_{2}^{2}-m_{s}^{2}} \frac{\sigma^{\alpha\beta}(k_{1}+m_{s}) + (k_{1}+m_{s})\sigma^{\alpha\beta}}{(k_{1}^{2}-m_{s}^{2})^{2}}\gamma_{\rho}\gamma_{5}\right] \\ & \times \operatorname{tr}\left[t^{A}t^{B}\right]g_{s}^{2}\langle 0|G_{\alpha\beta}^{A}(0)G_{\kappa\tau}^{B}(0)|0\rangle. \end{aligned}$$

$$\Pi_{\mu\nu\rho}^{\bar{q}Gq(2)}(p_{1},p_{2},q^{2}) = i^{2} \int d^{4}x d^{4}y \ e^{ip_{2} \cdot x} e^{-ip_{1} \cdot y} \int \frac{d^{4}k_{1}}{(2\pi)^{4}} \frac{d^{4}k_{2}}{(2\pi)^{4}} \ e^{ik_{1} \cdot y} e^{-ik_{2} \cdot x} \\ \times \left[\sigma_{\mu\nu} \left(\frac{-i}{4} \right) \frac{\sigma^{\alpha\beta}(k_{2}+m_{s}) + (k_{2}+m_{s})\sigma^{\alpha\beta}}{(k_{2}^{2}-m_{s}^{2})^{2}} \frac{i(k_{1}+m_{s})}{k_{1}^{2}-m_{s}^{2}} \gamma_{\rho} \gamma_{5} \right]_{ab}$$

$$\times t_{ij}^{A} \langle 0|\bar{q}_{a}^{i}(x)g_{s}G_{\alpha\beta}^{A}(0)q_{b}^{i}(y)|0\rangle.$$
dim-5

Threshold:





The continuous spectrum contribution must be smaller than the pole contribution!



3. $K_1(1270)$ - $K_1(1400)$ mixing in QCDSR





Demand:

3. $K_1(1270)$ - $K_1(1400)$ mixing in QCDSR



The contribution from higher dimension less than lower dimension!



 $\begin{array}{l} 1.16 \ {\rm GeV}^2 < T_{\rm upper}^2 < 2.39 \ {\rm GeV}^2 \\ 1.17 \ {\rm GeV}^2 < T_{\rm lower}^2 < 2.64 \ {\rm GeV}^2 \end{array}$



$$1.17 \text{ GeV}^2 < T^2 < 2.39 \text{ GeV}^2$$

However, instead of the physical region, QCDSR calculation establish on the deep Euclidean region result: $F1(q2 \ll 0)$ is known.

$$F_1(q^2) = \frac{F_1(0)}{1 - q^2/m_{\text{pole}}^2}$$





3. $K_1(1270)$ - $K_1(1400)$ mixing in QCDSR











- Divide the Hamiltonian into the SU(3) conserving and breaking terms.
- > Relate $\sin \theta_{K_1}$ with kaon matrix element.
- QCD sum rules calculation for the kaon matrix element.
- Determine the appropriate Borel parameters.

