Theoretical Review on Heavy Flavor Physics

- selected topics & mainly on B physics -

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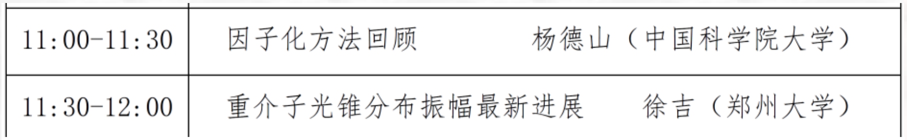
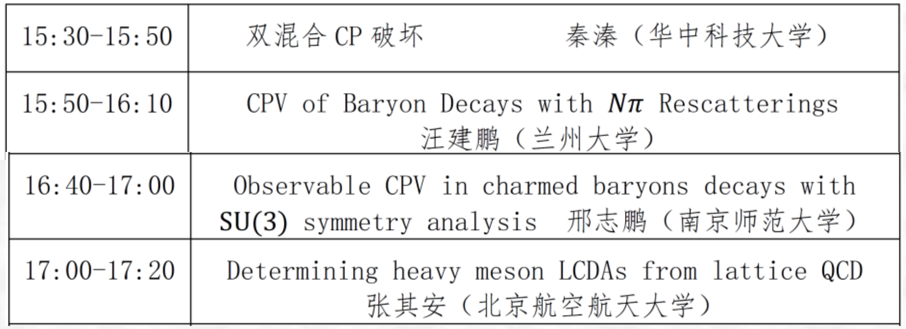
第21届全国重味物理与CP破坏研讨会，2024/10/26，衡阳

Outline

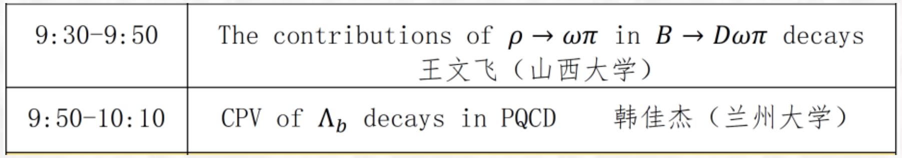
- □ **Introduction**
- □ **Purely leptonic decays**
- □ **Lifetime of b hadrons & neutral B-meson mixings**
- □ **Tree-level semi-leptonic decays**
- □ **Rare FCNC decays**
- □ **Hadronic B decays**
- □ **Summary**



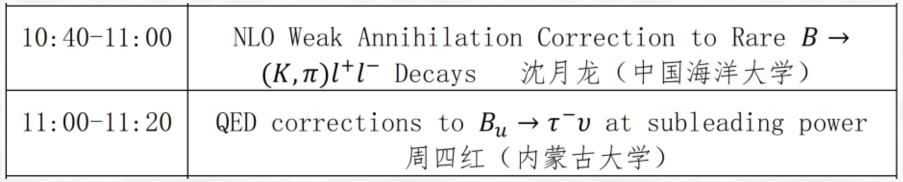
11:00-11:30	因子化方法回顾　　杨德山（中国科学院大学）
11:30-12:00	重介子光锥分布振幅最新进展　　徐吉（郑州大学）



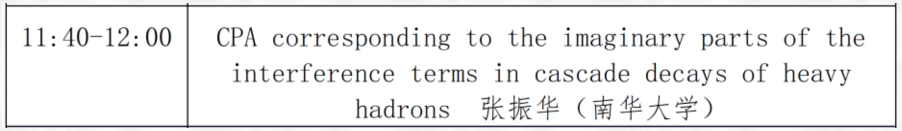
15:30-15:50	双混合 CP 破坏　　秦溱（华中科技大学）
15:50-16:10	CPV of Baryon Decays with $N\pi$ Rescatterings 汪建鹏（兰州大学）
16:40-17:00	Observable CPV in charmed baryons decays with SU(3) symmetry analysis　邢志鹏（南京师范大学）
17:00-17:20	Determining heavy meson LCDAs from lattice QCD 张其安（北京航空航天大学）



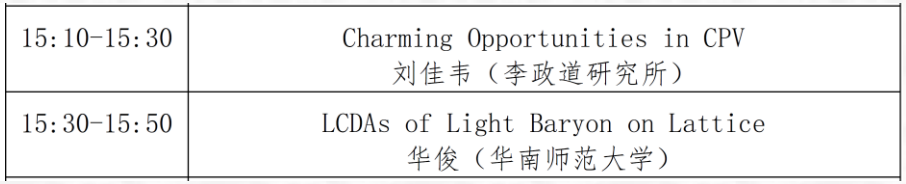
9:30-9:50	The contributions of $\rho \to \omega\pi$ in $B \to D\omega\pi$ decays 王文飞（山西大学）
9:50-10:10	CPV of Λ_b decays in PQCD　　韩佳杰（兰州大学）



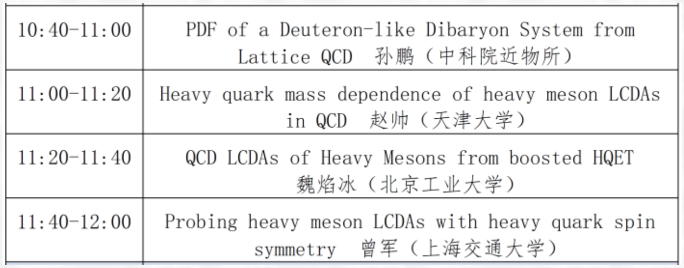
10:40-11:00	NLO Weak Annihilation Correction to Rare $B \to (K,\pi)l^+l^-$ Decays　沈月龙（中国海洋大学）
11:00-11:20	QED corrections to $B_u \to \tau^-\upsilon$ at subleading power 周四红（内蒙古大学）



11:40-12:00	CPA corresponding to the imaginary parts of the interference terms in cascade decays of heavy hadrons　张振华（南华大学）



15:10-15:30	Charming Opportunities in CPV 刘佳韦（李政道研究所）
15:30-15:50	LCDAs of Light Baryon on Lattice 华俊（华南师范大学）

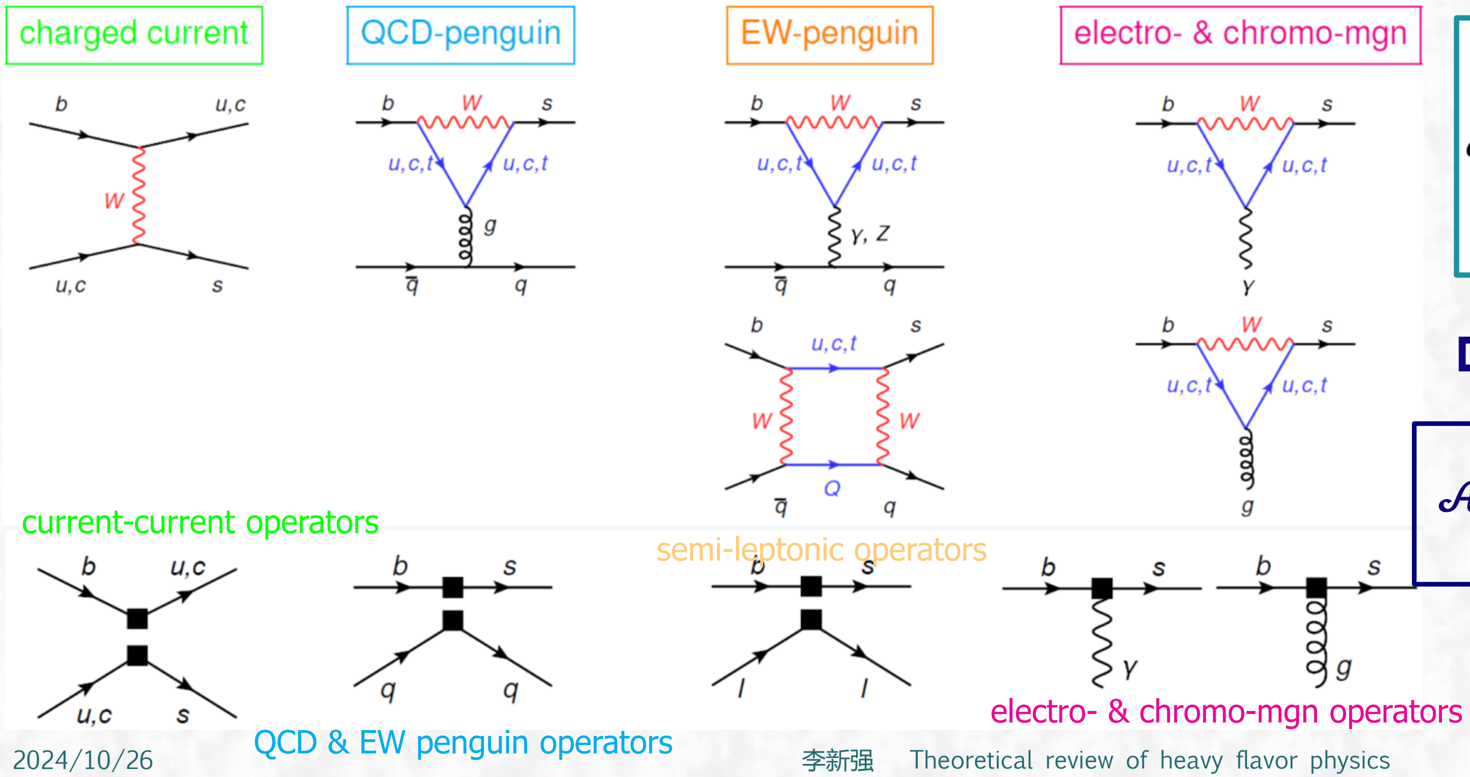


10:40-11:00	PDF of a Deuteron-like Dibaryon System from Lattice QCD　孙鹏（中科院近物所）
11:00-11:20	Heavy quark mass dependence of heavy meson LCDAs in QCD　赵帅（天津大学）
11:20-11:40	QCD LCDAs of Heavy Mesons from boosted HQET 魏焰冰（北京工业大学）
11:40-12:00	Probing heavy meson LCDAs with heavy quark spin symmetry　曾军（上海交通大学）

Effective weak Hamiltonian

☐ $\mathcal{H}_{\rm eff} = \sum_i C_i \mathcal{O}_i$: **standard starting point for any theoretical analysis**

- ➢ **Wilson coefficients C_i: all physics above the typical scale of a process (like $\mu_b \simeq m_b$); perturbatively calculable & NNLL program now complete** [Gorbahn, Haisch '04; Misiak, Steinhauser '04]
- ➢ **local operators $\mathcal{O}_i$: obtained after integrating out heavy d.o.f.** [Buras, Buchalla, Lautenbacher '96]



$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b), & i = 1, 2, & |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma_i' q), & i = 3, 4, 5, 6, & |C_i(m_b)| < 0.07 \\ \frac{e m_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, & C_7(m_b) \sim -0.3 \\ \frac{g m_b}{16\pi^2}\bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8, & C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2}(\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), & i = 9, 10 & |C_i(m_b)| \sim 4 \end{cases}$$

☐ **Amplitude of a given process:**

$$\mathcal{A}(\bar{B} \to f) = \sum_i [\lambda_{\rm CKM} \cdot C_i \cdot \langle f|\mathcal{O}_i|\bar{B}\rangle]$$

$$\Downarrow$$

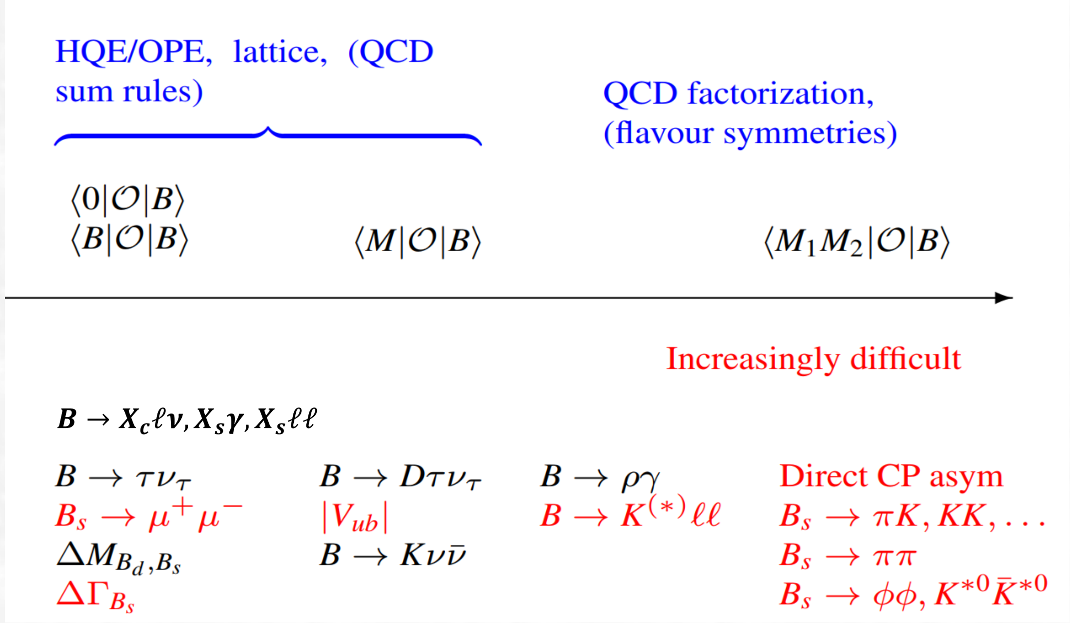
$$\langle f|\mathcal{O}_i|\bar{B}\rangle_{\rm QCD,QED}$$

Hadronic matrix elements

- $\langle f|\mathcal{O}_i|\bar{B}\rangle_{\mathrm{QCD,QED}}$: **how to reliably and precisely evaluate it? depending on the specific modes**

  - **Exclusive vs. (semi-)inclusive modes?**
  - **Hadronic decays** $\langle M_1M_2|\mathcal{O}_i|\bar{B}\rangle$: depends on spin & parity of $M_{1,2}$ and FSI introduces strong phases, and hence direct CPV

    a difficult, multi-scale, QCD & QED problem!



$\underbrace{\text{HQE/OPE, lattice, (QCD sum rules)}}$

QCD factorization, (flavour symmetries)

$\langle 0|\mathcal{O}|B\rangle$
$\langle B|\mathcal{O}|B\rangle$

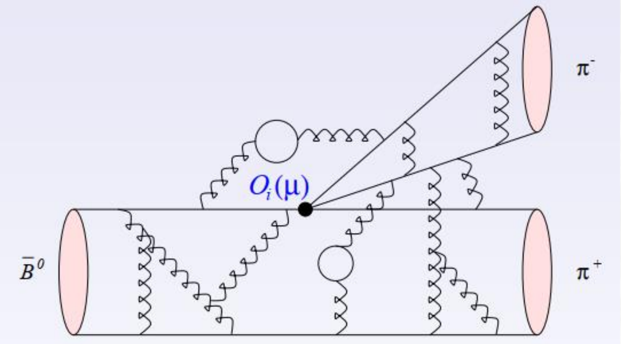
$\langle M|\mathcal{O}|B\rangle$

$\langle M_1M_2|\mathcal{O}|B\rangle$

Increasingly difficult

$B\to X_c\ell\nu, X_s\gamma, X_s\ell\ell$

$B\to\tau\nu_\tau$	$B\to D\tau\nu_\tau$	$B\to\rho\gamma$	Direct CP asym
$B_s\to\mu^+\mu^-$	$\lvert V_{ub}\rvert$	$B\to K^{(*)}\ell\ell$	$B_s\to\pi K, KK,\ldots$
$\Delta M_{B_d,B_s}$	$B\to K\nu\bar{\nu}$		$B_s\to\pi\pi$
$\Delta\Gamma_{B_s}$			$B_s\to\phi\phi, K^{*0}\bar{K}^{*0}$



$\langle M_1M_2|\mathcal{O}_{\rm i}|\bar{B}\rangle = \langle M_1|\bar{u}\ldots b|\bar{B}\rangle\langle M_2|\bar{d}\ldots u|0\rangle$

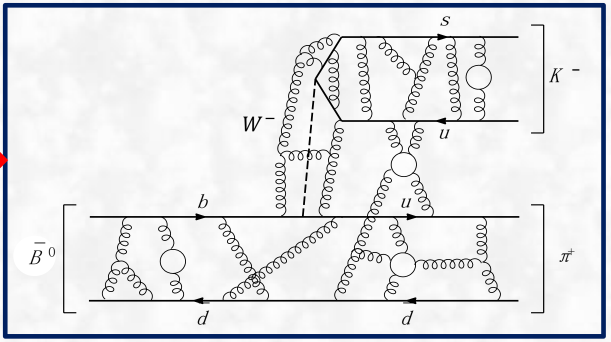
naïve fact. approach [Bauer, Stech, Wirbel '87]

- Dynamical approaches based on factorization theorems: PQCD, QCDF, SCET, · · ·
  [Keum, Li, Sanda, Lü, Yang '00;
  Beneke, Buchalla, Neubert, Sachrajda, '00;
  Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02]
- Symmetries of QCD: Isospin, U-Spin, V-Spin, and flavour SU(3) symmetries, · · ·
  [Zeppenfeld, '81;
  London, Gronau, Rosner, He, Chiang, Cheng *et al.*]
- Combination of dynamical approaches with flavor symmetries [FAT (Li, Lü et al.)…]

Example

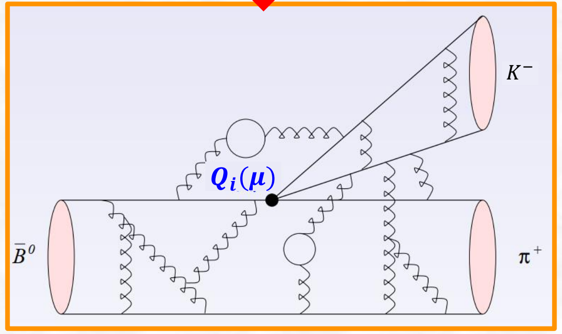
☐ **With $\langle f|\mathcal{O}_i|\bar B\rangle_{\rm QCD,QED}$ at hand, we can then do what we want to do**

$\bar B^0 \to \pi^+ K^-$



EW interaction scale	≫	ext. mom'a in B rest frame	≫	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	≫	$m_b \sim 5$ GeV	≫	$\Lambda_{\rm QCD} \sim 1$ GeV

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}}\sum_{p=u,c} V_{pD}^* V_{pb}\left(C_1Q_1^p + C_2Q_2^p + \sum_{i=3}^{10} C_iQ_i + C_{7\gamma}Q_{7\gamma} + C_{8g}Q_{8g}\right)$$



see talk by 杨德山

electroweak parameters

WCs due to NP

non-perp. parameters

$$\mathcal{A}(\bar B^0 \to \pi^+ K^-) = \frac{G_F}{\sqrt{2}}\sum_{ij} V_{\rm CKM}\,(\mathcal{C}_i^{\rm SM} + \mathcal{C}_i^{\rm NP})\Big[F_j^{B\to\pi}(m_K^2)\int_0^1 du\, T_{ij}^{\rm I}(u)\Phi_K(u) + (\pi \leftrightarrow K) + \int_0^1 d\xi\, du\, dv\, T_i^{\rm II}(\xi, u, v)\Phi_B(\xi)\Phi_\pi(v)\Phi_K(u)\Big]$$

related to exp. Br & CPV

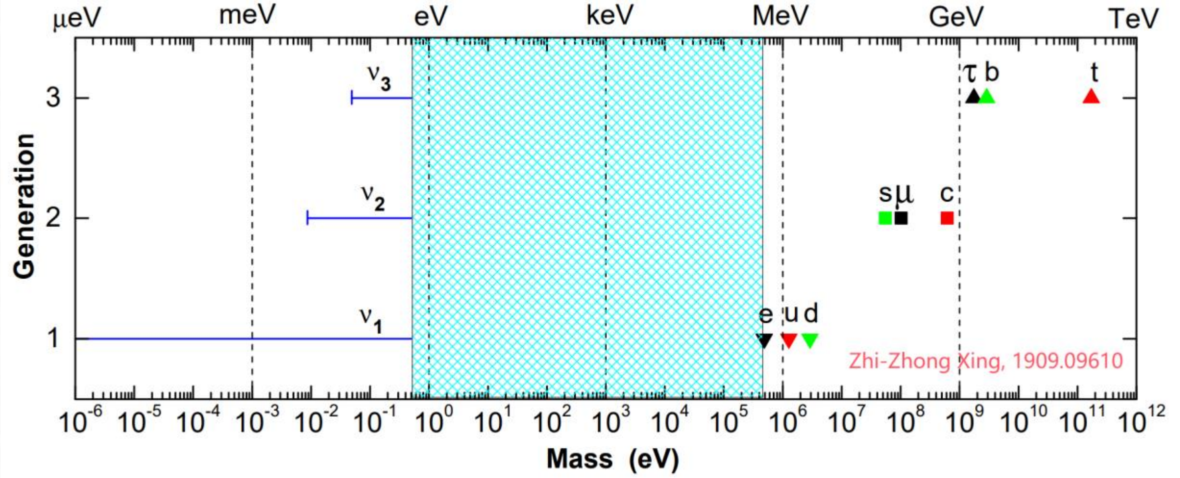
WCs from SM, also perp. calculable

perp. calculable in QCD & QED

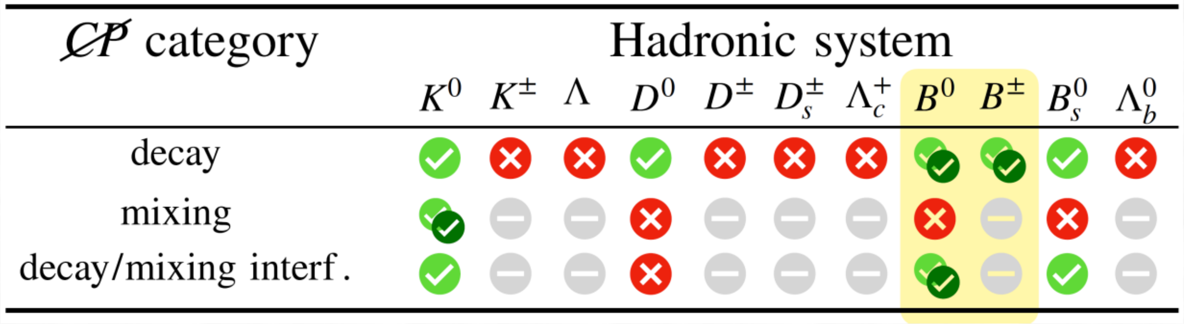
Why heavy flavor physics

☐ Main goals of dedicated exp. and theo. studies

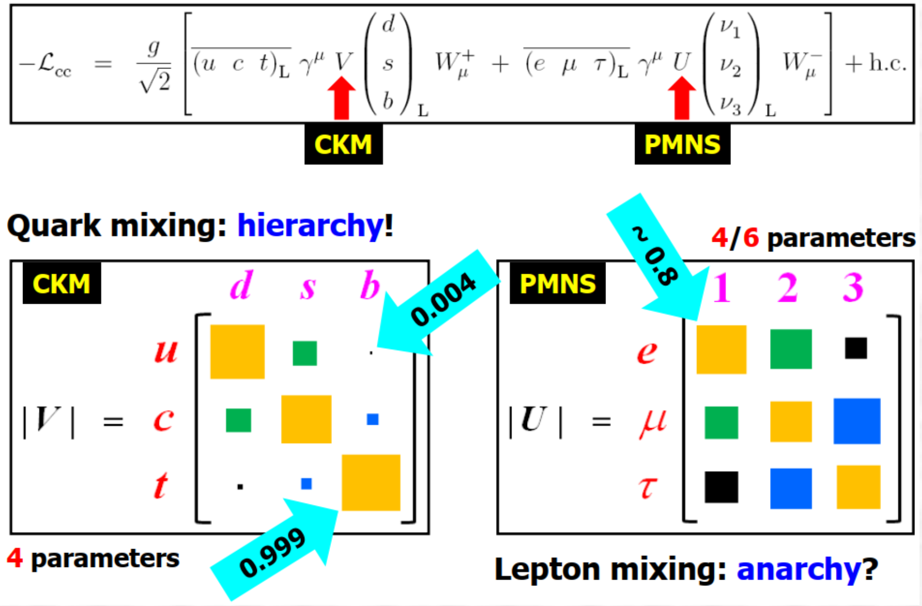
- what underlines the **hierarchical patterns** of **masses** & **mixings** of quarks & leptons?



Zhi-Zhong Xing, 1909.09610



$\not{C}\not{P}$ category	Hadronic system										
	K^0	$K^\pm$	Λ	D^0	$D^\pm$	$D_s^\pm$	Λ_c^+	B^0	$B^\pm$	B_s^0	Λ_b^0
decay	✓	✗	✗	✓	✗	✗	✗	✓	✓	✓	✗
mixing	✓	–	–	✗	–	–	–	✗	–	✗	–
decay/mixing interf.	✓	–	–	✗	–	–	–	✓	–	✓	–



$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\left[\overline{(u\ \ c\ \ t)_{\rm L}}\,\gamma^\mu\, V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\rm L} W_\mu^+ + \overline{(e\ \ \mu\ \ \tau)_{\rm L}}\,\gamma^\mu\, U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W_\mu^-\right] + {\rm h.c.}$$

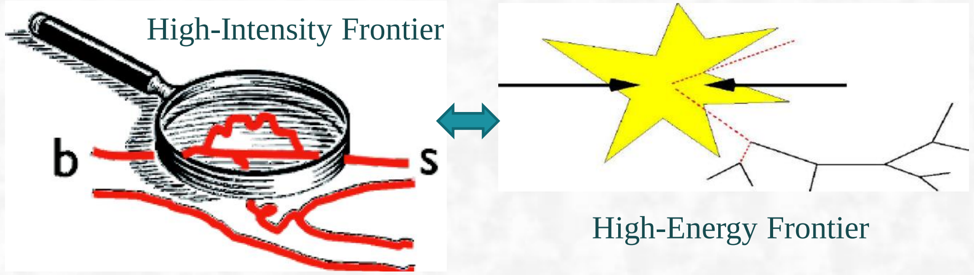
Quark mixing: hierarchy!

4/6 parameters

4 parameters

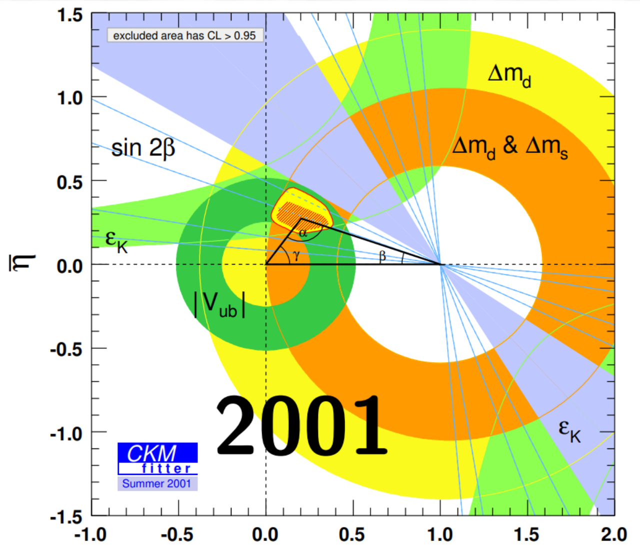
Lepton mixing: anarchy?

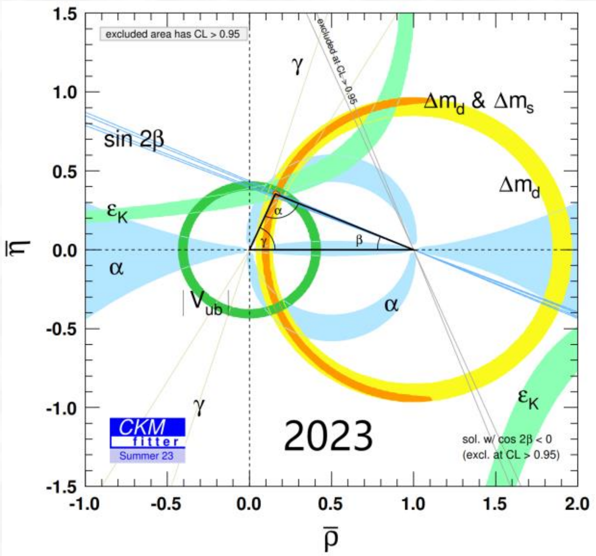
- any **new CP-violation mechanisms** beyond the KM of the SM?
- any **new particles/interactions** that are sensitive to **flavor structures**?

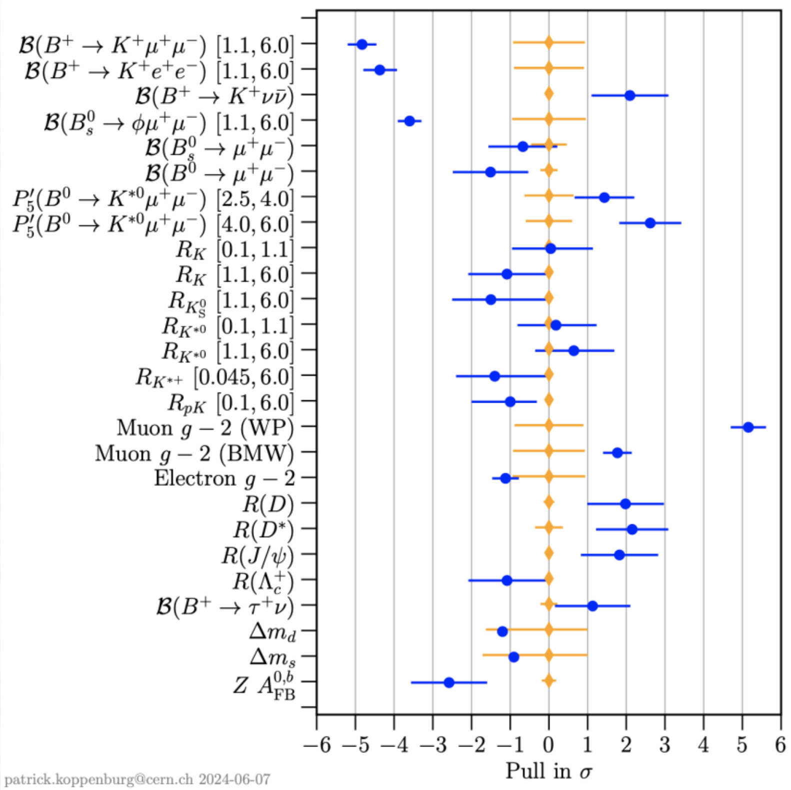


Flavor anomalies

- ☐ **Much progress achieved thanks to exp. & theor.:**







patrick.koppenburg@cern.ch 2024-06-07

https://www.nikhef.nl/~pkoppenb/anomalies.html

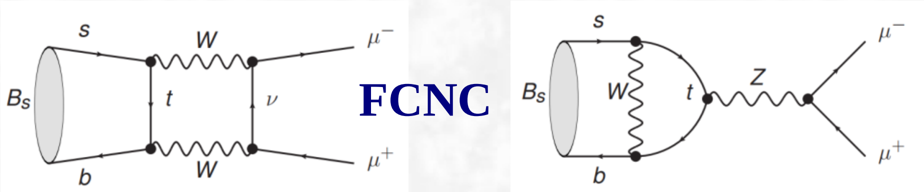
- ☐ **Anomalies in heavy flavor physics**
  - ✓ LFU violation in $R(D^{(*)}) = \frac{Br(B\to D^{(*)}\tau\nu_\tau)}{Br(B\to D^{(*)}l\nu_l)}$
  - ✓ $P_5'(B^0 \to K^{*0}\mu^+\mu^-)$?
  - ✓ deviations between $Br(B^+ \to K^+\mu^+\mu^-)$, $Br(B_s^0 \to \phi\mu^+\mu^-)$, and $Br(B^+ \to K^+\nu\bar{\nu})$?

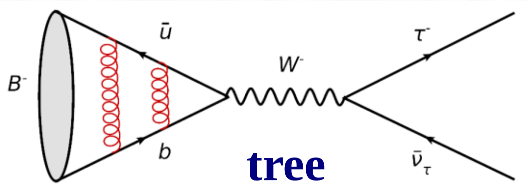
- ➢ $Br(B^0 \to \pi^0\pi^0) = (0.3 - 0.9)[(1.55 \pm 0.16)] \times 10^{-6}$
- ➢ $\Delta A_{CP}(\pi K) = A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-) = (11.3 \pm 1.2)\%$

- ☐ **Main tasks: more precise measurements & theoretical predictions; need collaboration!**

Purely leptonic decays

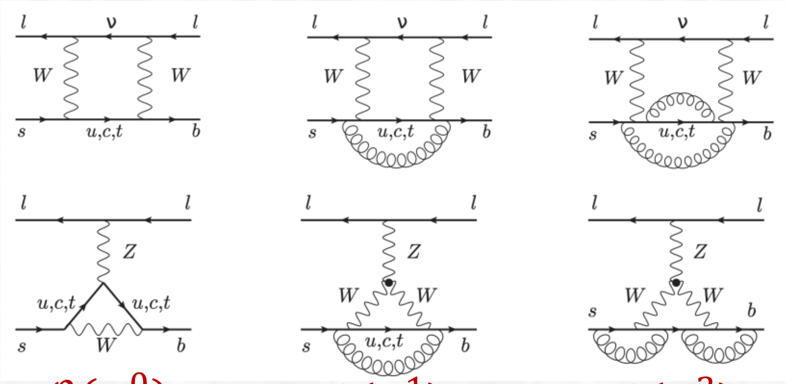
- ☐ **Have simplest hadronic structure; all QCD dynamics encoded in $f_{B_s} = (230.3 \pm 1.3)\text{MeV}$**





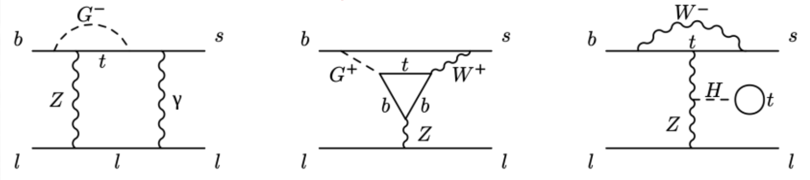
$$\langle 0|\bar{q}(0)\gamma^\mu\gamma_5 b(0)|\bar{B}(p)\rangle = i f_B p^\mu$$

- ☐ **Much progress achieved due to multi-loop techniques, EFTs , & LQCD, …**



$\mathcal{O}(\alpha_s^0)$ $\mathcal{O}(\alpha_s^1)$ $\mathcal{O}(\alpha_s^2)$

NNLO QCD correction

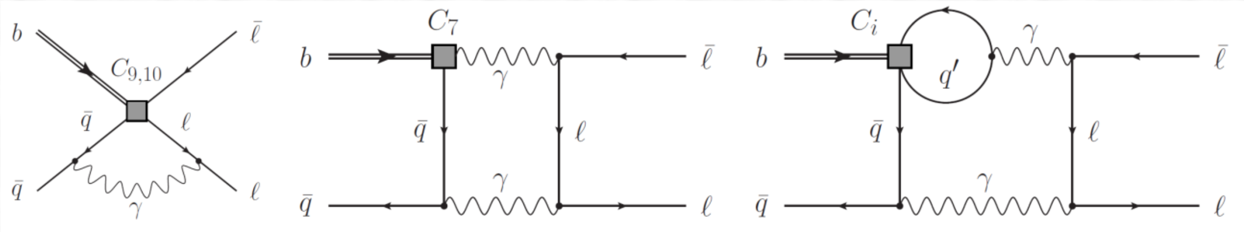


$\mathcal{O}(\alpha_e)$ NLO EW correction

$$\mathcal{L}_{\Delta B=1} = \mathcal{N}_{\Delta B=1}\left[\sum_{i=1}^{10} C_i(\mu_b)\,Q_i + \frac{V_{ub}V_{uq}^*}{V_{tb}V_{tq}^*}\sum_{i=1}^{2} C_i(\mu_b)\big(Q_i^u - Q_i^c\big)\right]$$

$$\begin{aligned} Q_9 &= \frac{\alpha_{\text{em}}}{4\pi}(\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu\ell) \\ Q_{10} &= \frac{\alpha_{\text{em}}}{4\pi}(\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu\gamma_5\ell) \\ Q_7 &= \frac{e}{16\pi^2}m_b\,(\bar{q}\sigma^{\mu\nu}P_R b)F_{\mu\nu} \end{aligned}$$

- ☐ **QED effects below m_b needed to match exp. precision**



when structure-dep. QED effects included, inner structure of B meson becomes relevant

For review, see P. Boer and T. Feldmann, 2312.12885

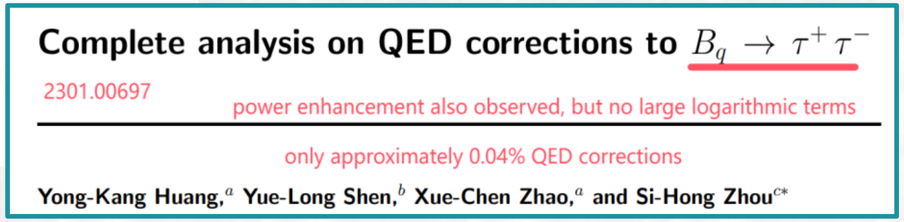
Purely leptonic decays

☐ **QED correction:** power-enhanced & helicity-suppression partially lifted [M. Beneke, C. Bobeth, R. Szafron, 1908.07011]

$$
\begin{aligned}
i\mathcal{A} \;=\; & \underbrace{m_\ell f_{B_q} \mathcal{N}\, C_{10}\, \bar{\ell}\gamma_5\ell}_{\text{tree-level amplitude}} + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q \underbrace{m_\ell m_B}_{\text{power \& helicity-suppression enhanced factors}} f_{B_q}\mathcal{N}\, \bar{\ell}(1+\gamma_5)\ell \\
& \times \Bigg\{ \int_0^1 du\,(1-u)\, C_9^{\rm eff}(u m_b^2) \underbrace{\int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega)}_{\text{convolution with the B-meson LCDA}} \left[\ln\frac{m_b\omega}{m_\ell^2} + \ln\frac{u}{1-u} \right] \\
& - Q_\ell C_7^{\rm eff} \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \Bigg[\underbrace{\ln^2\frac{m_b\omega}{m_\ell^2}}_{\text{double logarithmic enhancement due to endpoint singularity}} - 2\ln\frac{m_b\omega}{m_\ell^2} + \frac{2\pi^2}{3} \Bigg] \Bigg\}
\end{aligned}
$$

$$\eta_{BBS} = 0.995^{+0.003}_{-0.005}$$

only -0.5% effect due to partial cancellation between O_9 & O_7



Complete analysis on QED corrections to $B_q \to \tau^+\tau^-$

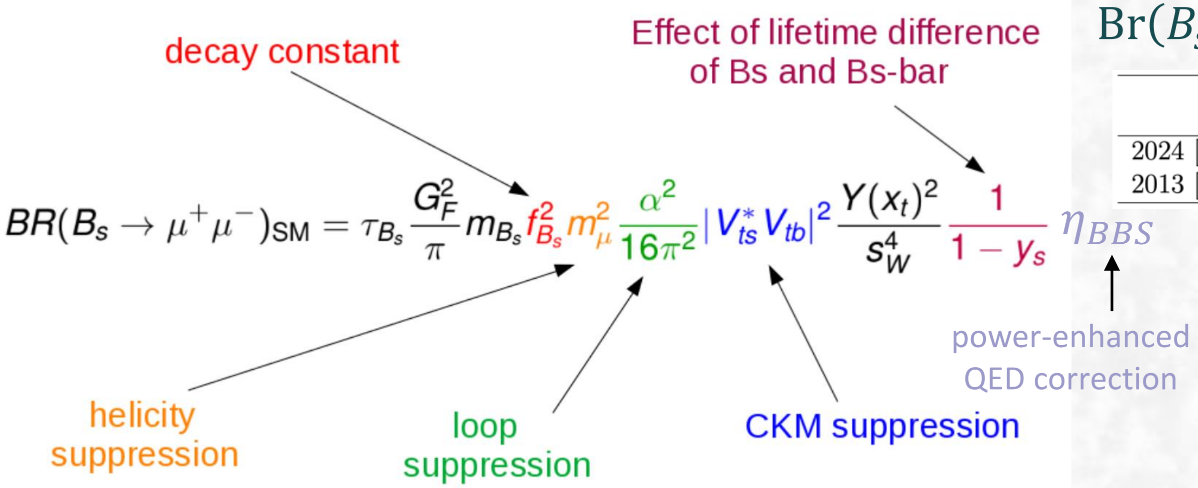
2301.00697

power enhancement also observed, but no large logarithmic terms

only approximately 0.04% QED corrections

Yong-Kang Huang,[a] Yue-Long Shen,[b] Xue-Chen Zhao,[a] and Si-Hong Zhou[c*]

☐ **Branching ratio** [M. Cjaza and M. Misiak, 2407.03810]



decay constant

Effect of lifetime difference of Bs and Bs-bar

$$BR(B_s \to \mu^+\mu^-)_{\rm SM} = \tau_{B_s} \frac{G_F^2}{\pi} m_{B_s} f_{B_s}^2 m_\mu^2 \frac{\alpha^2}{16\pi^2} |V_{ts}^* V_{tb}|^2 \frac{Y(x_t)^2}{s_W^4} \frac{1}{1-y_s}\; \eta_{BBS}$$

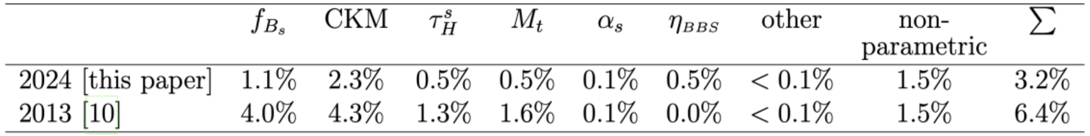
helicity suppression

loop suppression

CKM suppression

power-enhanced QED correction

$$\mathrm{Br}(B_s \to \mu^+\mu^-) = (3.64 \pm 0.12)[(3.34 \pm 0.27)] \times 10^{-9}$$

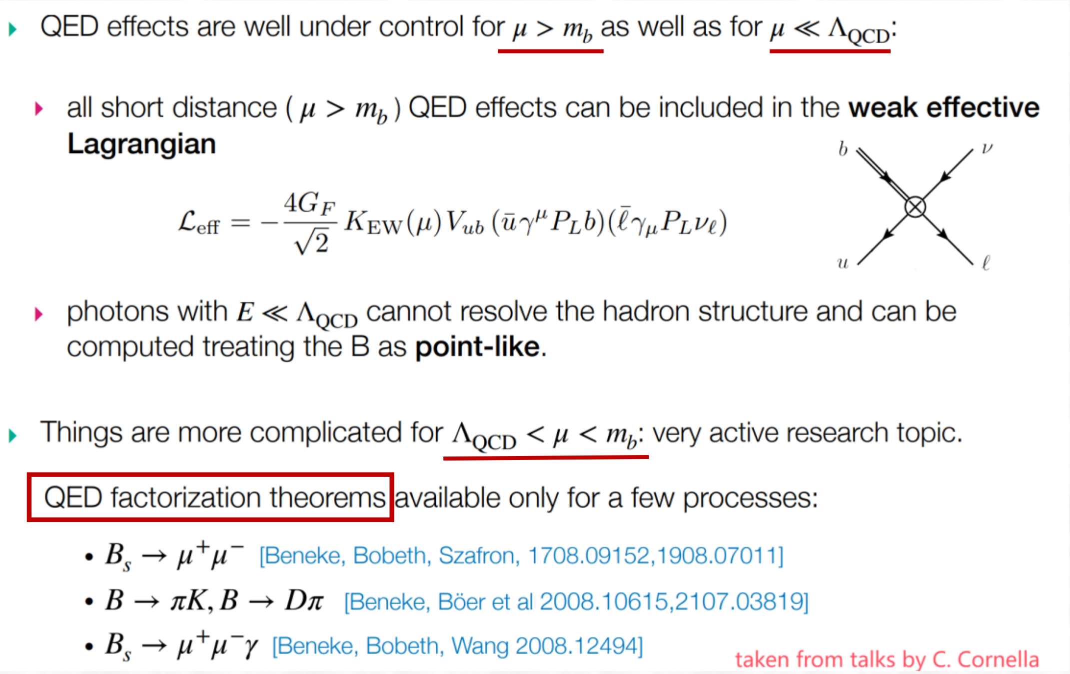


	f_{B_s}	CKM	τ_H^s	M_t	α_s	η_{BBS}	other	non-parametric	$\sum$
2024 [this paper]	1.1%	2.3%	0.5%	0.5%	0.1%	0.5%	< 0.1%	1.5%	3.2%
2013 [10]	4.0%	4.3%	1.3%	1.6%	0.1%	0.0%	< 0.1%	1.5%	6.4%

- ✓ parameter uncertainties dominated by $|V_{cb}|$
- ✓ non-parametric uncertainties mainly from treating m_t as the pole mass

Purely leptonic decays

☐ QED correction to $B \to \ell\nu$ decays [C. Cornella, M. König, M. Neubert 2212.14430; see talk by 周四红 on 10.27]



- QED effects are well under control for $\mu > m_b$ as well as for $\mu \ll \Lambda_{\rm QCD}$:
  - all short distance ($\mu > m_b$) QED effects can be included in the **weak effective Lagrangian**

$$\mathcal{L}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} K_{\rm EW}(\mu) V_{ub} \, (\bar{u}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu_\ell)$$

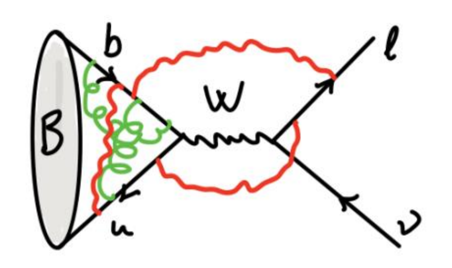
  - photons with $E \ll \Lambda_{\rm QCD}$ cannot resolve the hadron structure and can be computed treating the B as **point-like**.
- Things are more complicated for $\Lambda_{\rm QCD} < \mu < m_b$: very active research topic.

QED factorization theorems available only for a few processes:

- $B_s \to \mu^+\mu^-$ [Beneke, Bobeth, Szafron, 1708.09152,1908.07011]
- $B \to \pi K, B \to D\pi$ [Beneke, Böer et al 2008.10615,2107.03819]
- $B_s \to \mu^+\mu^-\gamma$ [Beneke, Bobeth, Wang 2008.12494]

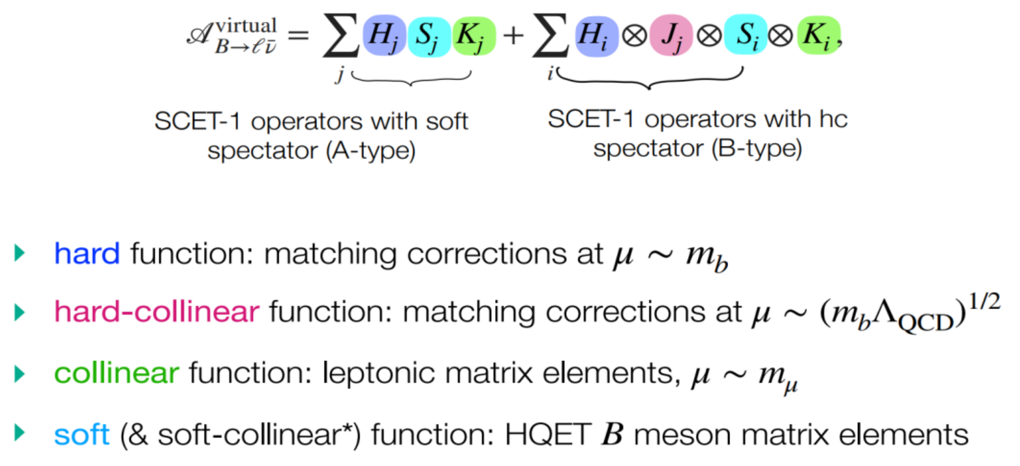
taken from talks by C. Cornella

☐ New features with QED effect:



- ➢ "universal" decay constants become process-dependent;
- ➢ sensitive to 2- and 3-particle LCDAs of B meson

☐ Factorization formula: QCD → SCET_I → SCET_II

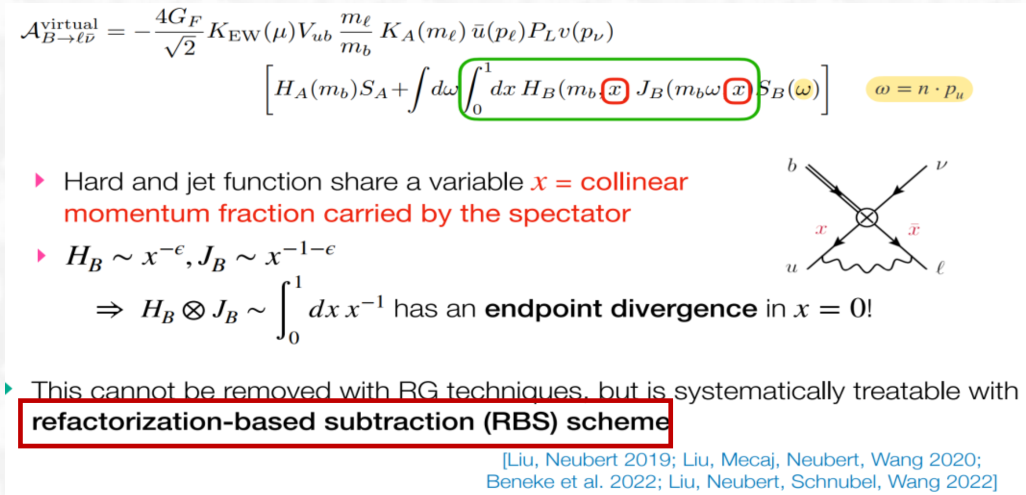


$$\mathscr{A}^{\rm virtual}_{B\to\ell\bar{\nu}} = \sum_j \underbrace{H_j\, S_j\, K_j}_{} + \sum_i \underbrace{H_i \otimes J_j \otimes S_i \otimes K_i}_{},$$

SCET-1 operators with soft spectator (A-type)

SCET-1 operators with hc spectator (B-type)

- hard function: matching corrections at $\mu \sim m_b$
- hard-collinear function: matching corrections at $\mu \sim (m_b \Lambda_{\rm QCD})^{1/2}$
- collinear function: leptonic matrix elements, $\mu \sim m_\mu$
- soft (& soft-collinear*) function: HQET B meson matrix elements



$$\mathcal{A}^{\rm virtual}_{B\to\ell\bar{\nu}} = -\frac{4G_F}{\sqrt{2}} K_{\rm EW}(\mu) V_{ub} \frac{m_\ell}{m_b} K_A(m_\ell)\, \bar{u}(p_\ell) P_L v(p_\nu)$$
$$\left[H_A(m_b) S_A + \int d\omega \boxed{\int_0^1 dx\, H_B(m_b, \boxed{x})\, J_B(m_b\omega, \boxed{x})} S_B(\omega)\right] \qquad \omega = n \cdot p_u$$

- Hard and jet function share a variable x = collinear momentum fraction carried by the spectator
- $H_B \sim x^{-\epsilon}, J_B \sim x^{-1-\epsilon}$

$$\Rightarrow \; H_B \otimes J_B \sim \int_0^1 dx\, x^{-1} \text{ has an **endpoint divergence** in } x = 0!$$

- This cannot be removed with RG techniques, but is systematically treatable with **refactorization-based subtraction (RBS) scheme**

[Liu, Neubert 2019; Liu, Mecaj, Neubert, Wang 2020; Beneke et al. 2022; Liu, Neubert, Schnubel, Wang 2022]

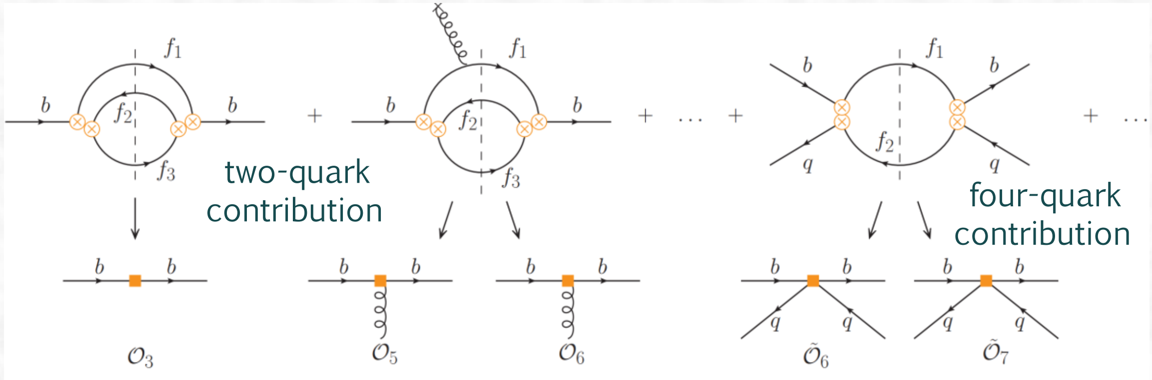
Lifetime of b-hadrons

☐ Lifetime based on HQE in $1/m_b$: [J. Albrecht, F. Bernlochner, A. Lenz, A. Rusov, 2402.04224]

$$\Gamma(\mathcal{B}) = \frac{1}{2m_{\mathcal{B}}} \sum_X \int_{\mathrm{PS}} (2\pi)^4 \delta^{(4)}(p_{\mathcal{B}} - p_X) \; |\langle X(p_X)|\mathcal{H}_{\mathrm{eff}}|\mathcal{B}(p_{\mathcal{B}})\rangle|^2$$

Optical Theorem

$$= \frac{1}{2m_{\mathcal{B}}} \mathrm{Im} \, \langle \mathcal{B}(p_{\mathcal{B}})| \, i \int d^4x \, T\left\{\mathcal{H}_{\mathrm{eff}}(x), \mathcal{H}_{\mathrm{eff}}(0)\right\} |\mathcal{B}(p_{\mathcal{B}})\rangle$$



$$\Gamma(\mathcal{B}) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right]$$

☐ status of SD coefficients:

	Semi-leptonic				Non-leptonic		
	LO	NLO	N²LO	N³LO	LO	NLO	N²LO
Γ_3	✓	✓	✓	✓★	✓	✓	✔
Γ_5	✓	✓			✓	✔●	
Γ_6	✓	✓			✓◇	🕒	
Γ_7	✓				🕒		
Γ_8	✓						
$\tilde{\Gamma}_6$	✓	✓	🕒		✓	✓	🕒
$\tilde{\Gamma}_7$	✓	🕒			✓	🕒	

◇ [Lenz, Piscopo, AR, 2004.09527], [Mannel, Moreno, Pivovarov, 2004.09485]

★ [Fael, Schönwald, Steinhauser, 2011.13654] ● [Mannel, Moreno, Pivovarov, 2304.08964 (for $m_c = 0$)]

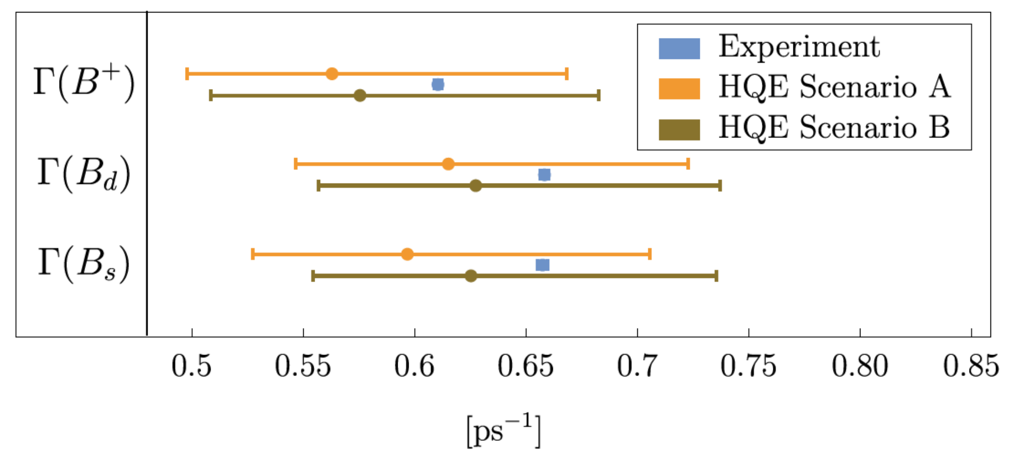
✓ - known ✔ - partly known 🕒 - in progress or planned [Karlsruhe, Siegen]

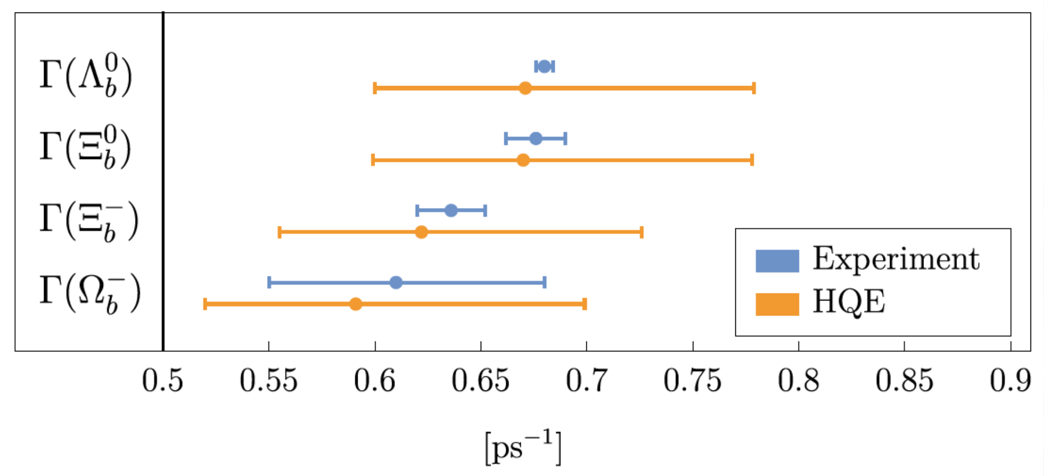
☐ status of hadronic matrix elements:

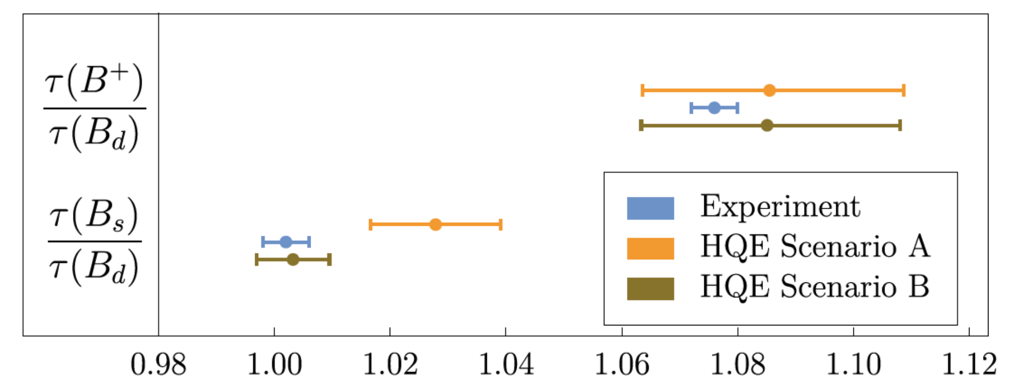
$\langle Q_5 \rangle_{B_d}$	1993/96 2013-2023 2017/18	QCD sum rule [234, 235] Fit of inclusive data [236–241] Lattice QCD [242, 243]
$\langle Q_5 \rangle_{B_s}$	2011	Spectroscopy relations [244]
$\langle Q_5 \rangle_{\mathcal{B}}$	2023	Spectroscopy relations [34]
$\langle Q_6 \rangle_{B_d}$	1994/2022 2013-2023	EOM relation [31, 245] Fit of inclusive data [236–241]
$\langle Q_6 \rangle_{B_s}$	1994/2022 2011	EOM relation [31, 245] Sum rule [244]
$\langle Q_6 \rangle_{\mathcal{B}}$	2023	EOM relation [34]
$\langle \tilde{Q}_6 \rangle_{B_d}$	2017	HQET sum rule [246]
$\langle \tilde{Q}_6 \rangle_{B_s}$	2022	HQET sum rule [247]
$\langle \tilde{Q}_6 \rangle_{\Lambda_b}$	1996	QCD sum rule [248]
$\langle \tilde{Q}_6 \rangle_{\mathcal{B}}$	2023	NRCQM [34]
$\langle \tilde{Q}_7 \rangle$		VIA

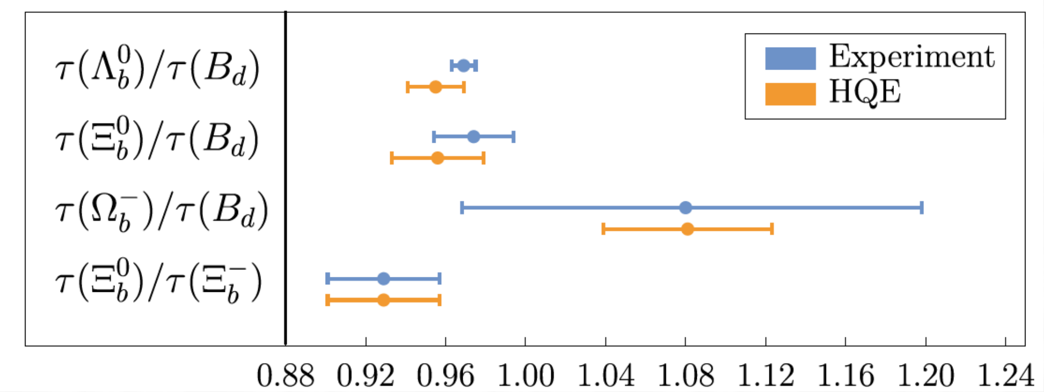
Lifetime of b-hadrons

☐ Exp. data & SM predictions: [J. Albrecht, F. Bernlochner, A. Lenz, A. Rusov, 2402.04224]









- ➢ excellent agreement between theory & data
- ➢ no indication of sizeable quark-hadron duality violation

- ➢ HQE also works for c-hadron lifetimes

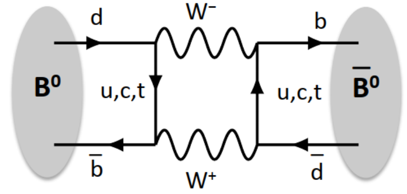
[H. Y. Cheng, C. W. Liu, 2305.00665]

Neutral B-meson mixings

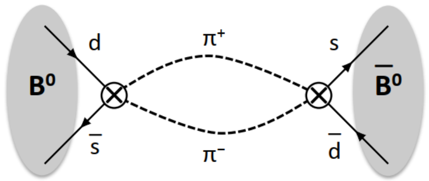
- ☐ **For B_q^0 meson:** flavor eigenstates ≠ mass eigenstates ⇒ mix with each other via box diagrams

- ☐ **Time evolution of a decaying particle**

$$i\frac{d}{dt}\begin{pmatrix}|B(t)\rangle\\|\bar{B}(t)\rangle\end{pmatrix} = \left(\hat{M} - \frac{i}{2}\hat{\Gamma}\right)\begin{pmatrix}|B(t)\rangle\\|\bar{B}(t)\rangle\end{pmatrix}$$



“short-distance”
(=virtual particle exchange)



“long-distance”
(=real particle exchange)

- ☐ **Three observables for B mixings**

  - **Mass difference:** $\Delta M := M_H - M_L \approx 2|M_{12}|$ **(off-shell)**
    $|M_{12}|$: heavy internal particles: t, SUSY, ...
  - **Decay rate difference:** $\Delta\Gamma := \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}|\cos\phi$ **(on-shell)**
    $|\Gamma_{12}|$: light internal particles: u, c, ... (almost) no NP!!!
  - **Flavor specific/semi-leptonic CP asymmetries:** e.g. $B_q \to Xl\nu$ (*semi-leptonic*)

$$a_{sl} \equiv a_{fs} = \frac{\Gamma(\bar{B}_q(t)\to f) - \Gamma(B_q(t)\to \bar{f})}{\Gamma(\bar{B}_q(t)\to f) + \Gamma(B_q(t)\to \bar{f})} = \left|\frac{\Gamma_{12}}{M_{12}}\right|\sin\phi$$

$$M_{12} = \frac{G_F^2}{12\pi^2}\left(V_{tq}^* V_{tb}\right)^2 M_W^2 S_0(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B$$

† 1-loop calculation $S_0\left(x_t = m_t^2/M_W^2\right)$

† 2-loop perturbative QCD corrections $\hat{\eta}_B$

† $\frac{8}{3} B_{B_q} f_{B_q}^2 M_{B_q} = \left\langle \overline{B}_q \middle| (\bar{b}q)_{V-A}(\bar{b}q)_{V-A} \middle| B_q \right\rangle$

$$\Gamma_{12} = \left(\frac{\Lambda}{m_b}\right)^3\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right)$$
$$+\left(\frac{\Lambda}{m_b}\right)^4\left(\Gamma_4^{(0)} + \ldots\right) + \left(\frac{\Lambda}{m_b}\right)^5\left(\Gamma_5^{(0)} + \ldots\right) + \ldots$$

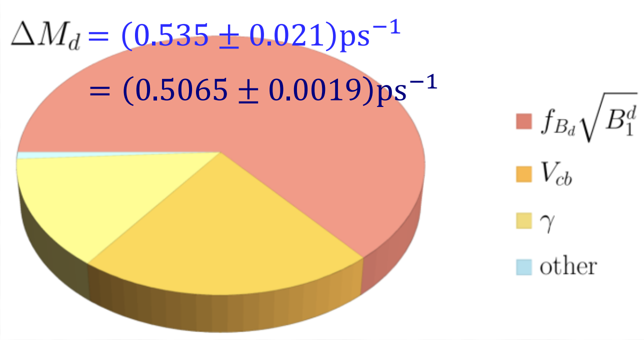
HQE

- ✓ $\boldsymbol{M_{12}}$**: dispersive (off-shell) part of the box diagram**
- ✓ $\boldsymbol{\Gamma_{12}}$**: absorptive (on-shell) part of the box diagram**
- ✓ $\boldsymbol{\phi = arg(-M12/\Gamma12)}$**: relative phase between them**

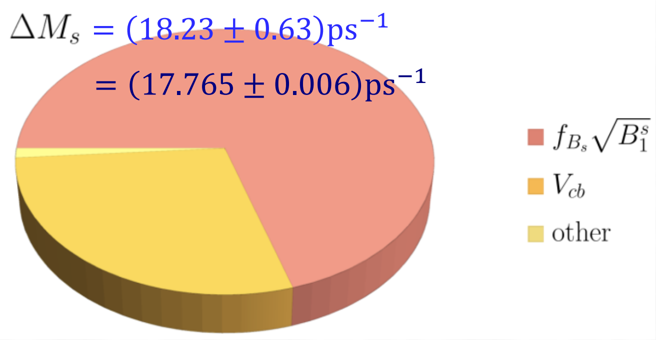
- ➢ **deepening our understanding of QCD**
- ➢ **indirect searches for BSM effects**

Neutral B-meson mixings

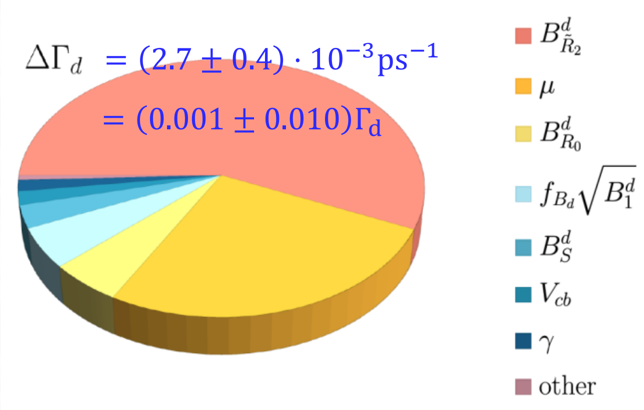
☐ Status of theo. predictions & exp. data [J. Albrecht, F. Bernlochner, A. Lenz, A. Rusov, 2402.04224]


$\Delta M_d = (0.535 \pm 0.021)\mathrm{ps}^{-1}$

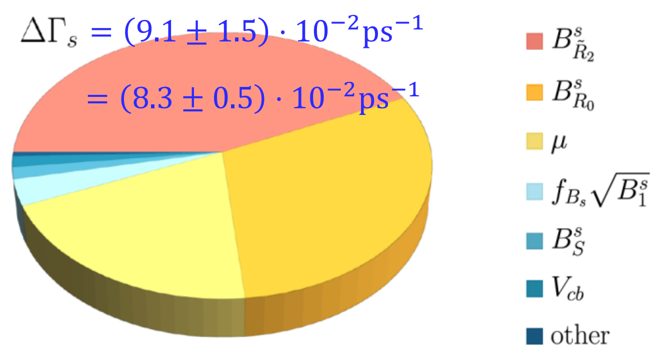
$= (0.5065 \pm 0.0019)\mathrm{ps}^{-1}$


$\Delta M_s = (18.23 \pm 0.63)\mathrm{ps}^{-1}$

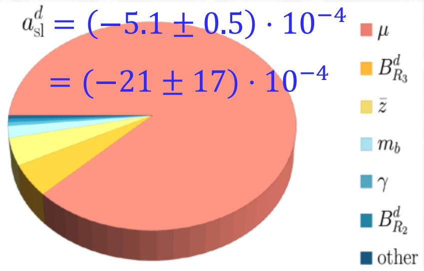
$= (17.765 \pm 0.006)\mathrm{ps}^{-1}$


$\Delta\Gamma_d = (2.7 \pm 0.4) \cdot 10^{-3}\mathrm{ps}^{-1}$

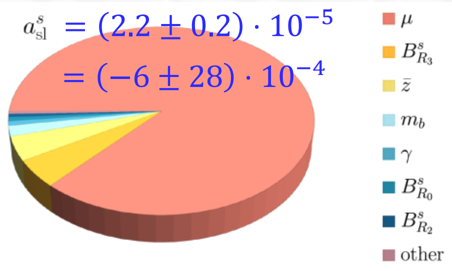
$= (0.001 \pm 0.010)\Gamma_d$


$\Delta\Gamma_s = (9.1 \pm 1.5) \cdot 10^{-2}\mathrm{ps}^{-1}$

$= (8.3 \pm 0.5) \cdot 10^{-2}\mathrm{ps}^{-1}$


$a^d_{\mathrm{sl}} = (-5.1 \pm 0.5) \cdot 10^{-4}$

$= (-21 \pm 17) \cdot 10^{-4}$


$a^s_{\mathrm{sl}} = (2.2 \pm 0.2) \cdot 10^{-5}$

$= (-6 \pm 28) \cdot 10^{-4}$

$$\boxed{\begin{aligned} \frac{\Delta M_d^{\mathrm{SM}}}{\Delta M_d^{\mathrm{Exp.}}} &= 1.056 \pm 0.042\,, & \frac{\delta\Delta M_d^{\mathrm{SM}}}{\delta\Delta M_d^{\mathrm{Exp.}}} &\simeq 11\,, \\ \frac{\Delta M_s^{\mathrm{SM}}}{\Delta M_s^{\mathrm{Exp.}}} &= 1.026 \pm 0.036\,, & \frac{\delta\Delta M_s^{\mathrm{SM}}}{\delta\Delta M_s^{\mathrm{Exp.}}} &\simeq 105\,, \end{aligned}}$$

- ✓ **theory can reproduce data, but theo. errors much larger than exp. ones**
- ✓ **main uncertainties still dominated by non-pert. bag parameters**
- ✓ **M_{12}: a 2nd-order weak interaction process and thus very sensitive to NP**

☐ Generic parametrization of NP contribution to B mixings:

$$\boxed{C_{B_q} e^{2i\phi_{B_q}} = \frac{\left\langle B_q^0 \middle| \mathcal{H}_{\mathrm{eff}}^{\mathrm{full}} \middle| \bar{B}_q^0 \right\rangle}{\left\langle B_q^0 \middle| \mathcal{H}_{\mathrm{eff}}^{\mathrm{SM}} \middle| \bar{B}_q^0 \right\rangle}}$$

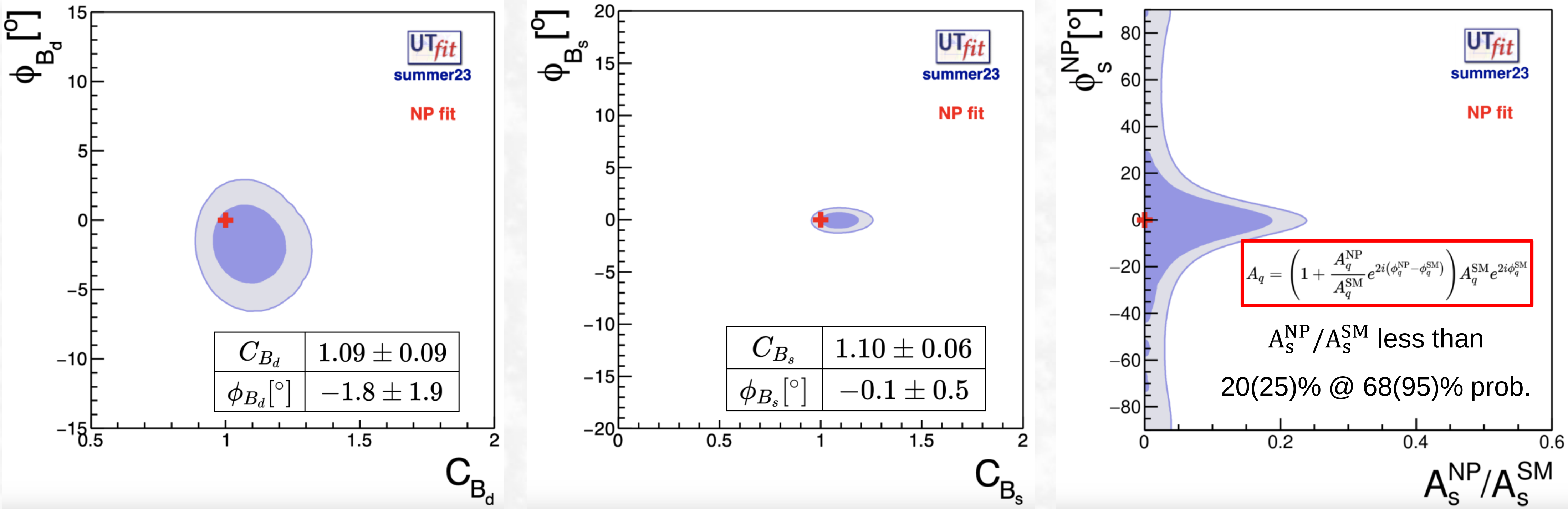
NP constraints from neutral B mixings

- ☐ **Exp. observables are related to the SM and NP parameters:**

$$\Delta M_d^{\rm exp} = C_{B_d}\Delta M_d^{\rm SM}\,, \qquad \sin 2\beta^{\rm exp} = \sin\left(2\beta^{\rm SM} + 2\phi_{B_d}\right)$$

$$\Delta M_s^{\rm exp} = C_{B_s}\Delta M_s^{\rm SM}\,, \qquad \phi_s^{\rm exp} = \left(\beta_s^{\rm SM} - \phi_{B_s}\right)$$

- ☐ **Latest fit results by UTfit group:**



C_{B_d}	1.09 ± 0.09
$\phi_{B_d}[^\circ]$	-1.8 ± 1.9

C_{B_s}	1.10 ± 0.06
$\phi_{B_s}[^\circ]$	-0.1 ± 0.5

$$A_q = \left(1 + \frac{A_q^{\rm NP}}{A_q^{\rm SM}} e^{2i(\phi_q^{\rm NP} - \phi_q^{\rm SM})}\right) A_q^{\rm SM} e^{2i\phi_q^{\rm SM}}$$

$A_s^{\rm NP}/A_s^{\rm SM}$ less than 20(25)% @ 68(95)% prob.

consistency between data & SM of B mixing observables puts stringent constraint on NP

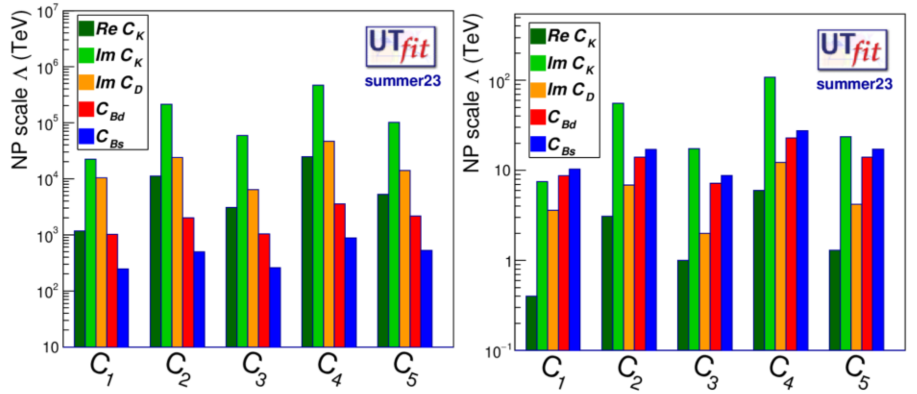
NP constraints from neutral B mixings

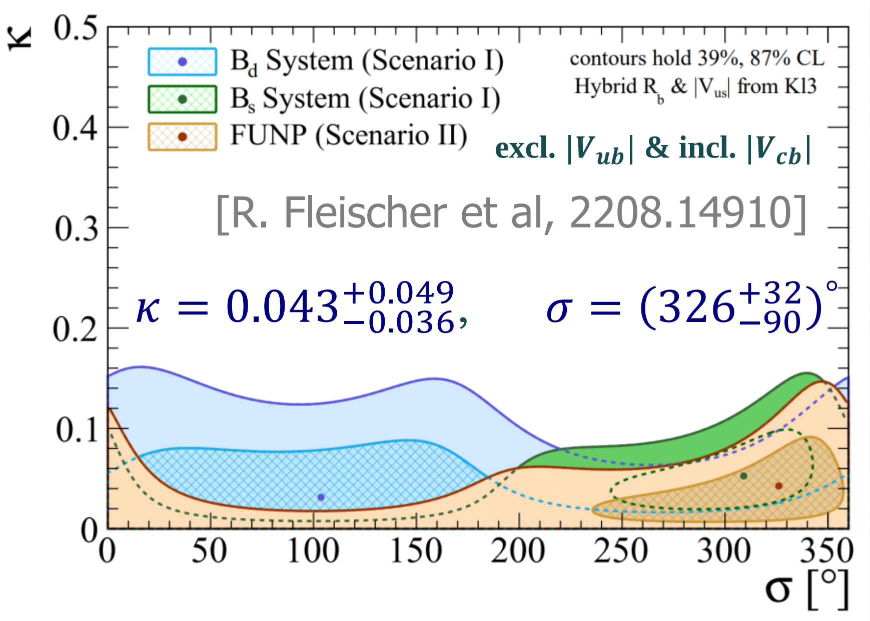
- ☐ **Very high scales probed by neutral meson mixings:**

$$\mathcal{H}_{\rm eff}^{\Delta B=2}=\sum_{i=1}^{5}C_i\mathcal{O}_i^{bq}+\sum_{i=1}^{3}\tilde{C}_i\tilde{\mathcal{O}}_i^{bq}\qquad \underline{C_i(\Lambda)=\frac{F_iL_i}{\Lambda^2}}\qquad \text{with } F_i\sim L_i\sim 1$$

$$\mathcal{O}_1=\left(\bar{b}^\alpha\gamma_\mu Lq^\alpha\right)\left(\bar{b}^\beta\gamma_\mu Lq^\beta\right),\qquad \mathcal{O}_2=\left(\bar{b}^\alpha Lq^\alpha\right)\left(\bar{b}^\beta Lq^\beta\right),\qquad \mathcal{O}_3=\left(\bar{b}^\alpha Lq^\beta\right)\left(\bar{b}^\beta Lq^\alpha\right)$$

$$\mathcal{O}_4=\left(\bar{b}^\alpha Lq^\alpha\right)\left(\bar{b}^\beta Rq^\beta\right),\qquad \mathcal{O}_5=\left(\bar{b}^\alpha Lq^\beta\right)\left(\bar{b}^\beta Rq^\alpha\right)$$





[R. Fleischer et al, 2208.14910]

$\kappa=0.043^{+0.049}_{-0.036},\qquad \sigma=(326^{+32}_{-90})^\circ$

- ☐ **Flavor universal NP scenario:** assuming NP couples predominantly to 3rd-generation quarks and leptons, and can be easily realized in NP models with a U(2)^5 symmetry

$$\rightarrow\ \boxed{\kappa_d=\kappa_s=\kappa,\qquad \sigma_d=\sigma_s=\sigma}$$

$$M_{12}^q=M_{12}^{q,\rm SM}\left(1+\kappa_q e^{i\sigma_q}\right)\qquad \begin{aligned}\Delta m_q&=\Delta m_q^{\rm SM}\left|1+\kappa_q e^{i\sigma_q}\right|,\\ \phi_q&=\phi_q^{\rm SM}+\phi_q^{\rm NP}=\phi_q^{\rm SM}+\arg\left(1+\kappa_q e^{i\sigma_q}\right).\end{aligned}$$

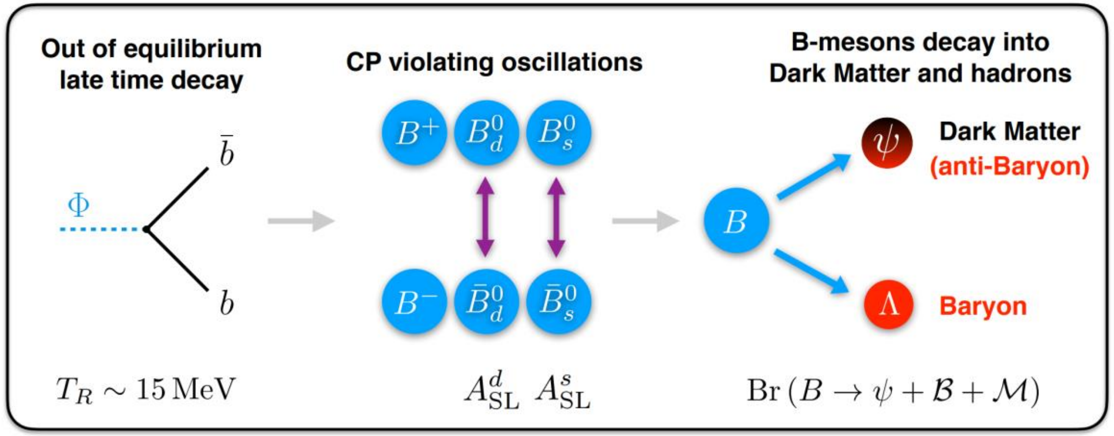
- ☐ **Analysis with dim-8 SMEFT operators:**

[Y. Liao, X. D. Ma and H. L. Wang, 2409.10305]

these $\Delta F=2$ processes probe up to tens of TeV, far beyond the sensitivity of other dim-8 operators to collider searches!

Example

☐ B-mesogenesis mechanism: [G. Elor, M. Escudero, and A. Nelson, 1810.00880; 2101.02706]



Baryogenesis and **Dark Matter**

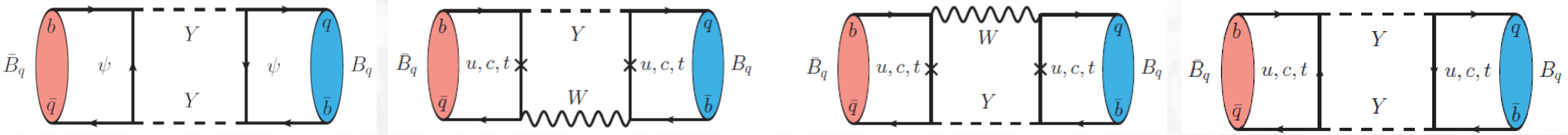
$$Y_B = 8.7 \times 10^{-11} \qquad \Omega_{\rm DM} h^2 = 0.12$$

With:

$$\boxed{Y_B \simeq 8.7 \times 10^{-11} \frac{\mathrm{Br}(B \to \psi + \mathcal{B} + \mathcal{M})}{10^{-2}} \sum_q \alpha_q \frac{A_{\rm SL}^q}{10^{-4}}}$$

→ $A_{\rm SL}^q > +10^{-4}$ [required by baryogenesis]

☐ Possible of the mechanism? [C. Miro, M. Escudero, and M. Nebot, 2410.13936]



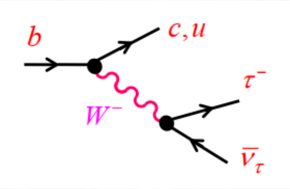
Y: color-triplet scalar
ψ: dark-sector antibaryon

→ $$\left|A_{\rm SL}^{q,\rm NP}(\psi)\right| < 4 \times 10^{-5} \left(\frac{500\ \mathrm{GeV}}{M_Y}\right)^2, \qquad |A_{\rm SL}^{q,\rm NP}(\not\psi)| \lesssim 10^{-4}$$

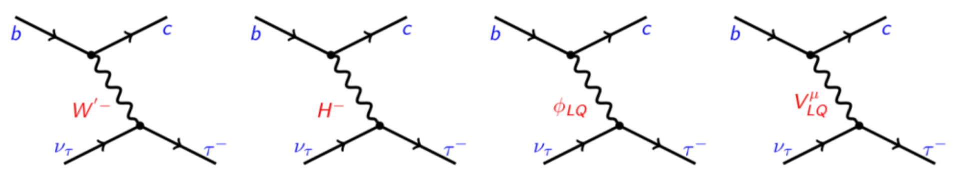
→ **fine tuning the maximum values of the semileptonic asymmetries in the minimal realization of the B-mesogenesis mechanism needed!**

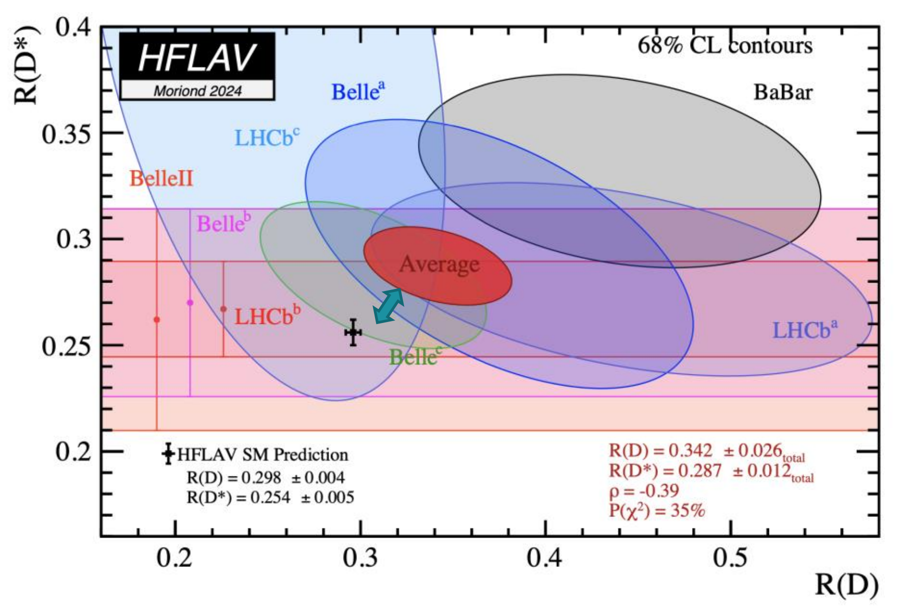
Semi-leptonic B decays

- ☐ $R(D^{(*)})$ **anomalies: first observed by BaBar in 2012; currently still having ~3.31σ deviation**



$$R\left(D^{(*)}\right) = \frac{Br(B \to D^{(*)}\tau\nu_\tau)}{Br(B \to D^{(*)} l\nu_l)}$$





- ☐ **Model-indep. result for $R(D)$, $R(D^*)$ & $R(\Lambda_c)$:** [Iguro, Kitahar, Watanabe, 2405.06062; Duan, Iguro, Li, Watanabe, Yang, to appear soon]

$$\mathcal{H}_{\rm eff} = 2\sqrt{2}G_F V_{cb}\Big[(1+C_{V_L})O_{V_L} + C_{V_R}O_{V_R} + C_{S_L}O_{S_L} + C_{S_R}O_{S_R} + C_T O_T\Big]$$

with

$$O_{V_L} = (\bar{c}\gamma^\mu P_L b)(\bar{\tau}\gamma_\mu P_L \nu_\tau)\,, \qquad O_{V_R} = (\bar{c}\gamma^\mu P_R b)(\bar{\tau}\gamma_\mu P_L \nu_\tau)\,,$$
$$O_{S_L} = (\bar{c}P_L b)(\bar{\tau}P_L \nu_\tau)\,, \qquad O_{S_R} = (\bar{c}P_R b)(\bar{\tau}P_L \nu_\tau)\,,$$
$$O_T = (\bar{c}\sigma^{\mu\nu}P_L b)(\bar{\tau}\sigma_{\mu\nu}P_L \nu_\tau)\,,$$

$$\begin{aligned}\frac{R_P}{R_P^{\rm SM}} &= |1 + C_{V_L}^{q\tau} + C_{V_R}^{q\tau}|^2 + a_P^{SS}|C_{S_L}^{q\tau} + C_{S_R}^{q\tau}|^2 + a_P^{TT}|C_T^{q\tau}|^2 \\ &+ a_P^{VS}{\rm Re}\left[\left(1 + C_{V_L}^{q\tau} + C_{V_R}^{q\tau}\right)\left(C_{S_L}^{q\tau*} + C_{S_R}^{q\tau*}\right)\right] + a_P^{VT}{\rm Re}\left[\left(1 + C_{V_L}^{q\tau} + C_{V_R}^{q\tau}\right)C_T^{q\tau*}\right]\,,\end{aligned}$$

$$\begin{aligned}\frac{R_V}{R_V^{\rm SM}} &= |1 + C_{V_L}^{q\tau}|^2 + |C_{V_R}^{q\tau}|^2 + a_V^{SS}|C_{S_L}^{q\tau} - C_{S_R}^{q\tau}|^2 + a_V^{TT}|C_T^{q\tau}|^2 \\ &+ a_V^{V_LV_R}{\rm Re}\left[\left(1 + C_{V_L}^{q\tau}\right)C_{V_R}^{q\tau*}\right] + a_V^{VS}{\rm Re}\left[\left(1 + C_{V_L}^{q\tau} - C_{V_R}^{q\tau}\right)\left(C_{S_L}^{q\tau*} - C_{S_R}^{q\tau*}\right)\right] \\ &+ a_V^{V_LT}{\rm Re}\left[\left(1 + C_{V_L}^{q\tau}\right)C_T^{q\tau*}\right] + a_V^{V_RT}{\rm Re}\left[C_{V_R}^{q\tau}C_T^{q\tau*}\right]\,,\end{aligned}$$

$$\begin{aligned}\frac{R_H}{R_H^{\rm SM}} &= |1 + C_{V_L}^{q\tau}|^2 + |C_{V_R}^{q\tau}|^2 + a_H^{SS}\left[|C_{S_L}^{q\tau}|^2 + |C_{S_R}^{q\tau}|^2\right] + a_H^{TT}|C_T^{q\tau}|^2 + a_H^{V_LV_R}{\rm Re}\left[\left(1 + C_{V_L}^{q\tau}\right)C_{V_R}^{q\tau*}\right] \\ &+ a_H^{VS_1}{\rm Re}\left[\left(1 + C_{V_L}^{q\tau}\right)C_{S_L}^{q\tau*} + C_{V_R}^{q\tau}C_{S_R}^{q\tau*}\right] + a_H^{VS_2}{\rm Re}\left[\left(1 + C_{V_L}^{q\tau}\right)C_{S_R}^{q\tau*} + C_{V_R}^{q\tau}C_{S_L}^{q\tau*}\right] \\ &+ a_H^{S_LS_R}{\rm Re}\left[C_{S_L}^{q\tau}C_{S_R}^{q\tau*}\right] + a_H^{V_LT}{\rm Re}\left[\left(1 + C_{V_L}^{q\tau}\right)C_T^{q\tau*}\right] + a_H^{V_RT}{\rm Re}\left[C_{V_R}^{q\tau}C_T^{q\tau*}\right]\,.\end{aligned}$$

Sum rule for $b \to c$ sector

$$\left|\cancel{1 + C_{V_L}^{q\tau}}\right|^2$$

$$\mathrm{Re}[(\cancel{1 + C_{V_L}^{q\tau}})C_{S_L}^{q\tau*}]$$

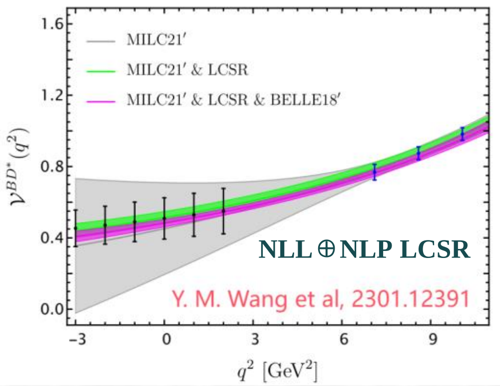
□ Sum rule for $R(D)$, $R(D^*)$ & $R(\Lambda_c) = Br(\Lambda_b \to \Lambda_c \tau \nu_\tau)/Br(\Lambda_b \to \Lambda_c \ell \nu_\ell)$:

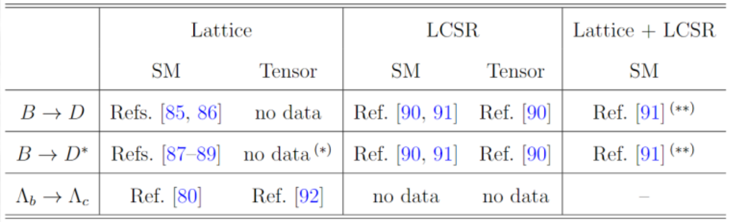
$$\boxed{\frac{R_H}{R_H^{\mathrm{SM}}} = b\frac{R_P}{R_P^{\mathrm{SM}}} + c\frac{R_V}{R_V^{\mathrm{SMM}}} + \delta_H(C_i)}$$

⟹ $b + c = 1$ & $a_P^{VS} b + a_V^{VS} c = a_H^{VS_1}$, so that $\delta_H(C_i)$ small

⟹ model-indep. & holds for any tau-philic NP!

□ State-of-the-art prediction: [Duan, Iguro, Li, Watanabe, Yang, to appear soon]





	Lattice		LCSR		Lattice + LCSR
	SM	Tensor	SM	Tensor	SM
$B \to D$	Refs. [85, 86]	no data	Ref. [90, 91]	Ref. [90]	Ref. [91] $^{(**)}$
$B \to D^*$	Refs. [87–89]	no data $^{(*)}$	Ref. [90, 91]	Ref. [90]	Ref. [91] $^{(**)}$
$\Lambda_b \to \Lambda_c$	Ref. [80]	Ref. [92]	no data	no data	–

- important to properly consider the correlations among FF parameters

⟹
$$\boxed{\frac{R_{\Lambda_c}}{R_{\Lambda_c}^{\mathrm{SM}}} = (0.272 \pm 0.015)\frac{R_D}{R_D^{\mathrm{SM}}} + (0.728 \mp 0.015)\frac{R_{D^*}}{R_{D^*}^{\mathrm{SM}}} + \delta_{\Lambda_c}}$$

- provide a unique prediction of $R(\Lambda_c)$ model-indep.ly

$$\begin{aligned}
\delta_{\Lambda_c} = & (-0.001 \pm 0.005)\ \left(|C_{S_L}^{c\tau}|^2 + |C_{S_R}^{c\tau}|^2\right) + (-0.007 \pm 0.005)\,\mathrm{Re}\left(C_{S_L}^{c\tau} C_{S_R}^{c\tau*}\right) \\
& + \underline{(-2.681 \pm 6.907)\,|C_T^{c\tau}|^2} + (-0.561 \pm 1.439)\,\mathrm{Re}\left(C_{V_R}^{c\tau} C_T^{c\tau*}\right) \\
& + \mathrm{Re}\left[\left(1 + C_{V_L}^{c\tau}\right)\left\{(0.041 \pm 0.034) C_{V_R}^{c\tau*} + (0.594 \pm 1.274) C_T^{c\tau*}\right\}\right] \\
& + (-0.002 \pm 0.009)\mathrm{Re}\left[\left(1 + C_{V_L}^{c\tau}\right) C_{S_R}^{c\tau*} + C_{S_L}^{c\tau} C_{V_R}^{c\tau*}\right]
\end{aligned}$$

$$R_{\Lambda_c}^{\mathrm{SR}} = 0.370 \pm 0.017|_{R_X^{\mathrm{SM,exp}}} \pm (< 0.001)|_{\mathrm{SR}}$$

$$\Updownarrow$$

$$R_{\Lambda_c}^{\mathrm{LHCb}} = 0.242 \pm 0.026 \pm 0.040 \pm 0.059$$

Sum rule for $b \to c$ sector

☐ Sum rule for $R(D)$, $R(D^*)$ & $R(X_c) = Br(B \to X_c \tau \nu_\tau)/Br(B \to X_c \ell \nu_\ell)$:

$$\boxed{\frac{R_{X_c}}{R_{X_c}^{\rm SM}} \simeq 0.288\,\frac{R_D}{R_D^{\rm SM}} + 0.712\,\frac{R_{D^*}}{R_{D^*}^{\rm SM}} + \delta_{X_c}}$$

$$\begin{aligned}\delta_{X_c} \simeq\, & 0.015\,\left(|C_{S_L}^{c\tau}|^2 + |C_{S_R}^{c\tau}|^2\right) - 0.003\,{\rm Re}\left(C_{S_L}^{c\tau} C_{S_R}^{c\tau *}\right) - 1.655\,|C_T^{c\tau}|^2 \\ & + {\rm Re}\left[\left(1 + C_{V_L}^{c\tau}\right)\left\{0.192 C_{V_R}^{c\tau *} + 0.896 C_T^{c\tau *}\right\}\right] - 3.405\,{\rm Re}\left(C_{V_R}^{c\tau} C_T^{c\tau *}\right) \\ & + 0.043{\rm Re}\left[\left(1 + C_{V_L}^{c\tau}\right) C_{S_R}^{c\tau *} + C_{S_L}^{c\tau} C_{V_R}^{c\tau *}\right]\end{aligned}$$

➡ $R_{X_c}^{\rm SR} \simeq\ 0.247 \pm 0.008\,|_{R_X^{\rm SM,exp}}$ vs $R_{X_c}^{\rm exp} = 0.228 \pm 0.039$ [Belle II, 2311.07248]

- $\Gamma(B \to X_c \ell \nu_\ell) = \sum \Gamma(B \to D \ell \nu_\ell) + \Gamma(B \to D^* \ell \nu_\ell) + \Gamma(B \to D^{**} \ell \nu_\ell)$, saturate already inclusive rate?
- the sum rule relation provides another complementary test of the dynamics behind the decays

☐ Sum rule for $R(D^*)$ & $R(J/\psi) = Br(B \to J/\psi \tau \nu_\tau)/Br(B \to J/\psi \ell \nu_\ell)$:

$$\frac{R_{J/\psi}}{R_{J/\psi}^{SM}} \simeq \frac{R_{D^*}}{R_{D^*}^{SM}}$$

➡ $$\frac{R_{J/\psi}}{R_{J/\psi}^{SM}} - \frac{R_{D^*}}{R_{D^*}^{SM}} = 1.2 \pm 0.7$$

- satisfied within the 2σ error bars; would be significant once $R_{J/\psi}$ measurement improved

Sum rule for $b \to u$ sector

☐ **Sum rule for $R(\pi)$, $R(\rho)$ & $R(p)$ in $b \to u$:**

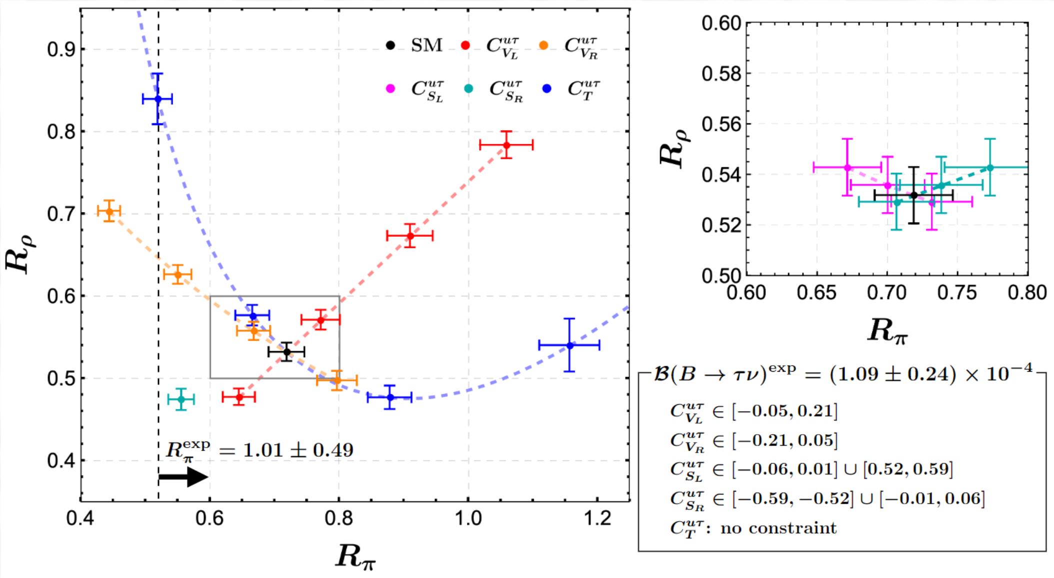
$$\boxed{\frac{R_p}{R_p^{\rm SM}} = (0.284 \pm 0.037)\,\frac{R_\pi}{R_\pi^{\rm SM}} + (0.716 \mp 0.037)\,\frac{R_\rho}{R_\rho^{\rm SM}} + \delta_p}$$

$$\begin{aligned}\delta_p =& \underline{(-0.090 \pm 0.059)}\,\left(|C_{S_L}^{u\tau}|^2 + |C_{S_R}^{u\tau}|^2\right) + (-0.185 \pm 0.038)\,{\rm Re}\left(C_{S_L}^{u\tau} C_{S_R}^{u\tau *}\right)\\ &+ \underline{(-0.913 \pm 2.403)}\,|C_T^{u\tau}|^2 + (-0.203 \pm 0.538)\,{\rm Re}\left(C_{V_R}^{u\tau} C_T^{u\tau *}\right)\\ &+ {\rm Re}\left[\left(1 + C_{V_L}^{u\tau}\right)\left\{(0.169 \pm 0.158) C_{V_R}^{u\tau *} + (0.370 \pm 0.632) C_T^{u\tau *}\right\}\right]\\ &+ (-0.079 \pm 0.056)\,{\rm Re}\left[\left(1 + C_{V_L}^{u\tau}\right) C_{S_R}^{u\tau *} + C_{S_L}^{u\tau} C_{V_R}^{u\tau *}\right]\,.\end{aligned}$$

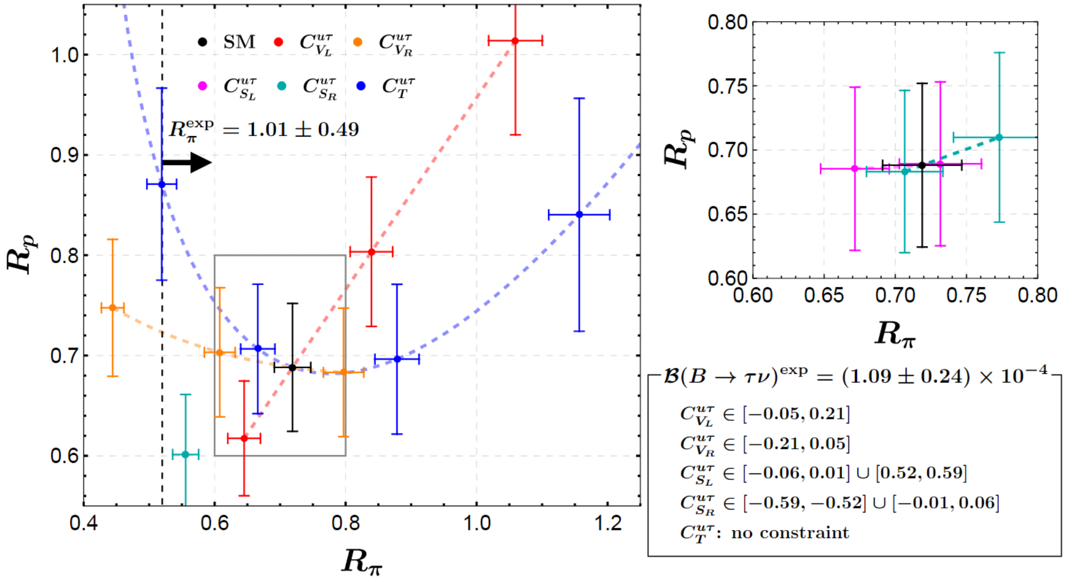
➡ the sum rule for $b \to u$ more (less) sensitive to the scalar (tensor) NP compared to $b \to c$

	Lattice		LCSR		Lattice + LCSR
	SM	Tensor	SM	Tensor	SM + Tensor
$B \to \pi$	Refs. [98–100]	Ref. [101]	Refs. [90, 103–105]		Ref. [106]
$B \to \rho$	no data	no data	Refs. [77, 90, 107]		**B.-Y. Cui et al., 2212.11624**
$\Lambda_b \to p$	Ref. [80]	no data	Ref. [108]	no data	–

☐ **Correlation among $R(\pi)$, $R(\rho)$ & $R(p)$:**



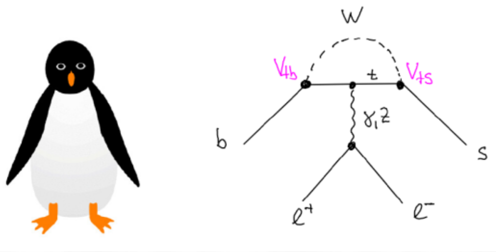
$\mathcal{B}(B \to \tau\nu)^{\rm exp} = (1.09 \pm 0.24) \times 10^{-4}$
$C_{V_L}^{u\tau} \in [-0.05, 0.21]$
$C_{V_R}^{u\tau} \in [-0.21, 0.05]$
$C_{S_L}^{u\tau} \in [-0.06, 0.01] \cup [0.52, 0.59]$
$C_{S_R}^{u\tau} \in [-0.59, -0.52] \cup [-0.01, 0.06]$
$C_T^{u\tau}$: no constraint

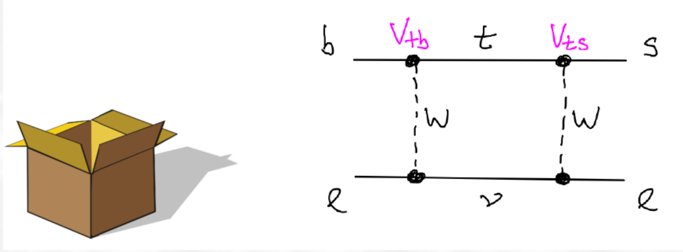


$\mathcal{B}(B \to \tau\nu)^{\rm exp} = (1.09 \pm 0.24) \times 10^{-4}$
$C_{V_L}^{u\tau} \in [-0.05, 0.21]$
$C_{V_R}^{u\tau} \in [-0.21, 0.05]$
$C_{S_L}^{u\tau} \in [-0.06, 0.01] \cup [0.52, 0.59]$
$C_{S_R}^{u\tau} \in [-0.59, -0.52] \cup [-0.01, 0.06]$
$C_T^{u\tau}$: no constraint

Rare FCNC decays

☐ Why $b \to s\ell^+\ell^-$ processes:

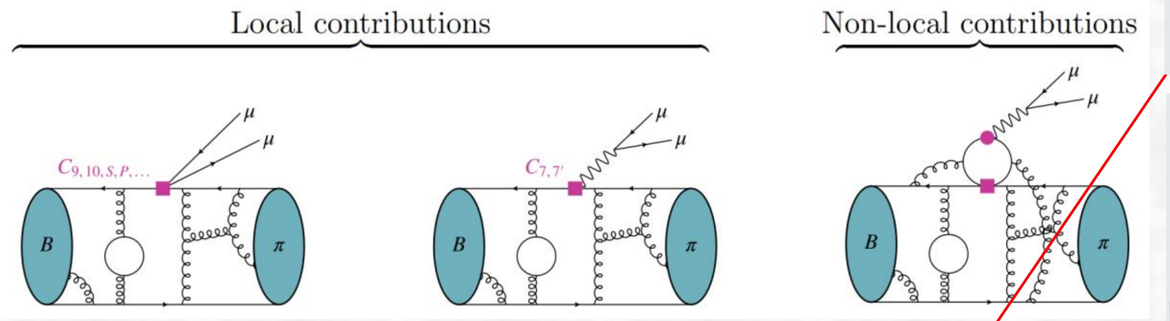




- occur firstly at 1-loop; suppressed by loop factor
- proportional to $|V_{tb}V_{ts}^*|$; $Br(b \to s\,\ell\ell) \sim 10^{-6}$
- sensitive to various NP

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_{i=7,9,10} C_i \mathcal{O}_i + \ldots$$

☐ Effective Hamiltonian & matrix elements:



$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

- B → K$^{(*)}$ μμ
- B$_s$ → φ μμ, …

Local form-factors,

involves e.g. $\mathcal{F}_\lambda(k,q) = \mathcal{P}_\lambda^\mu \langle \bar{M}(k) | \bar{s}\gamma_\mu b_L | \bar{B}(q+k) \rangle$

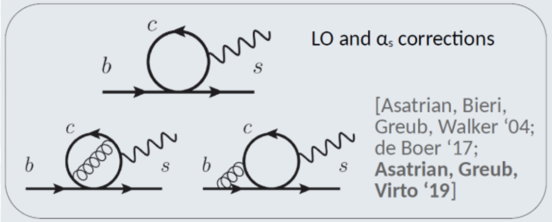
BSZ parametrization
LQCD & LCSR

Non-local form-factors:

$$\mathcal{H}_\lambda(k,q) = i \int d^4x\, e^{iq\cdot x} \mathcal{P}_\lambda^\mu \langle \bar{M}(k) | T\{ Q_c[\bar{c}\gamma_\mu c](x), \mathcal{C}_i \mathcal{O}_i \} | \bar{B}(q+k) \rangle$$

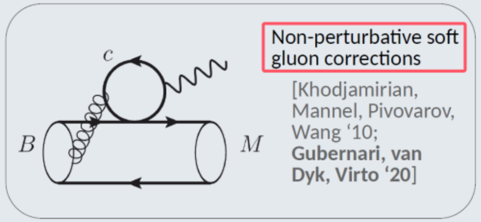
$\mathcal{H}_\lambda$ can be calculated in **two kinematics regions**:

- **Local** OPE $|q|^2 \gtrsim m_b^2$ [Grinstein, Piryol '04; Beylich, Buchalla, Feldmann '11]
- **Light Cone** OPE $q^2 \ll 4m_c^2$ [Khodjamirian, Mannel, Pivovarov, Wang '10]



LO and αₛ corrections

[Asatrian, Bieri, Greub, Walker '04; de Boer '17; **Asatrian, Greub, Virto '19**]



Non-perturbative soft gluon corrections

[Khodjamirian, Mannel, Pivovarov, Wang '10; **Gubernari, van Dyk, Virto '20**]

$$\mathcal{H}_\lambda(z) = \frac{1}{\phi(z)\mathcal{P}(z)} \sum_{k=0}^{N} a_{\lambda,k}\, p_k(z)$$

dispersive bound

Gubernari, van Dyk, Virto '20

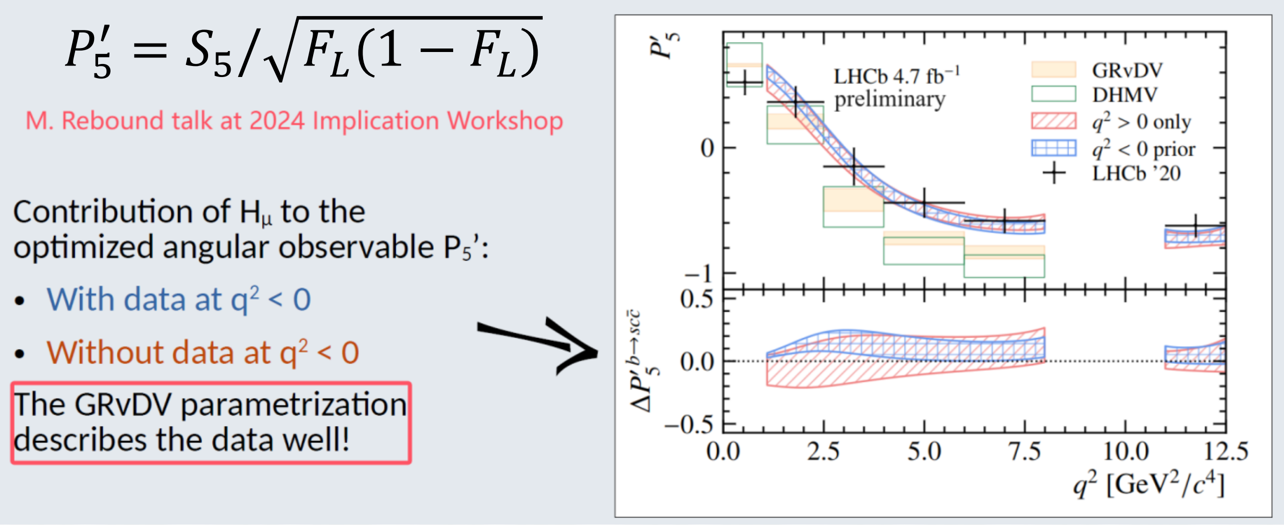
Rare FCNC decays

$$\mathcal{H}_\lambda(z) = \frac{1}{\phi(z)\mathcal{P}(z)} \sum_{k=0}^{N} a_{\lambda,k}\, p_k(z)$$

dispersive bound

Gubernari, van Dyk, Virto '20

☐ Test with the exp. data:



$$P_5' = S_5/\sqrt{F_L(1-F_L)}$$

M. Rebound talk at 2024 Implication Workshop

Contribution of H_μ to the optimized angular observable P_5':

- With data at $q^2 < 0$
- Without data at $q^2 < 0$

The GRvDV parametrization describes the data well!

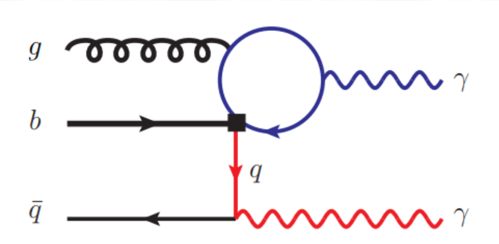
- ➢ The description of **non-local form factors** far more involved than expected
- ➢ Analyticity properties fully understood, constrained also by theory & experiment
- ➢ uncertainties still large, but controlled by **dispersive bounds & systematically improvable**

☐ Further detailed studies required:

➢ Novel soft-function introduced [Qin, Shen, Wang, Wang, 2023; Huang, Ji, Shen, Wang, Wang, Zhao,2023]

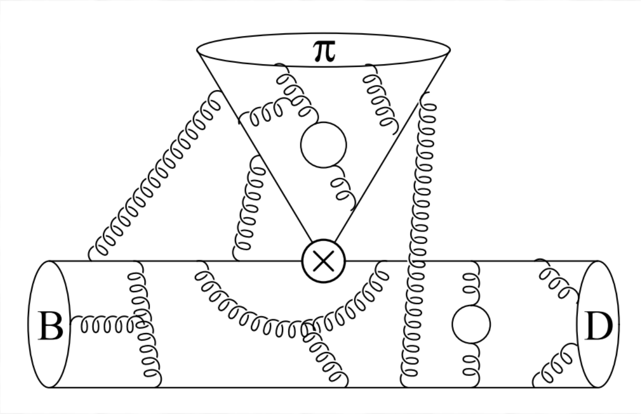
$$\langle 0|(\bar{q}_s S_n)(\tau_1 n) S_n^\dagger S_{\bar{n}}(0) S_{\bar{n}}^\dagger g_s G_{\mu\nu} S_{\bar{n}}(\tau_2 \bar{n}) \bar{n}^\nu n \cdot \gamma \gamma_\perp^\mu \gamma_5 S_{\bar{n}}^\dagger h_v(0)|\bar{B}_v\rangle$$

$$= 2F_{\rm B}(\mu) m_B \int_{-\infty}^{\infty} d\omega_1 d\omega_2\ {\rm e}^{-i(\omega_1\tau_1+\omega_2\tau_2)} \Phi_G(\omega_1,\omega_2,\mu)$$



$m_c^2 \approx m_b \Lambda_{QCD}$: hard-collinear

$B_q^0 \to D_q^{(*)-} L^+$ class-I decays



- ☐ **At the quark-level, these decays mediated by $b \to c\bar{u}d(s)$**

all four flavors different from each other,

no penguin operators & no penguin topologies!

$$\mathcal{Q}_2 = \bar{d}\gamma_\mu(1-\gamma_5)u\ \ \bar{c}\gamma^\mu(1-\gamma_5)b$$
$$\mathcal{Q}_1 = \bar{d}\gamma_\mu(1-\gamma_5)T^A u\ \ \bar{c}\gamma^\mu(1-\gamma_5)T^A b$$

- ☐ **For class-I decays:** QCDF formula much simpler;

only the form-factor term at leading power

[Beneke, Buchalla, Neubert, Sachrajda '99-'03; Bauer, Pirjol, Stewart '01]

$$\langle D_q^{(*)+} L^- |\, \mathcal{Q}_i \,| \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u) \phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

- i) only color-allowed tree topology $T = a_1$
- ii) spectator & annihilation power-suppressed
- iii) annihilation absent in $B_{d(s)}^0 \to D_{d(s)}^- K(\pi)^+$ etc.
- iv) they are theoretically simpler and cleaner

these decays used to test factorization theorems

- ☐ **Hard kernel T:** both NLO and NNLO results known;

[Beneke, Buchalla, Neubert, Sachrajda '01; Huber, Kränkl, Li '16]

$$T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)$$

Non-leptonic/semi-leptonic ratios

☐ Non-leptonic/semi-leptonic ratios : [Bjorken '89; Neubert, Stech '97; Beneke, Buchalla, Neubert, Sachrajda '01]

$$R^{(*)}_{(s)L} \equiv \frac{\Gamma(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} L^-)}{d\Gamma(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} \ell^- \bar{\nu}_\ell)/dq^2 \,|_{q^2=m_L^2}} = 6\pi^2 \,|V_{uq}|^2 \, f_L^2 \, |a_1(D^{(*)+}_{(s)} L^-)|^2 \, X^{(*)}_L$$

free from the uncertainties from V_{cb} & $B_{d,s} \to D^{(*)}_{d,s}$ form factors

☐ Updated predictions vs data: [Huber, Kränkl, Li '16; Cai, Deng, Li, Yang '21]

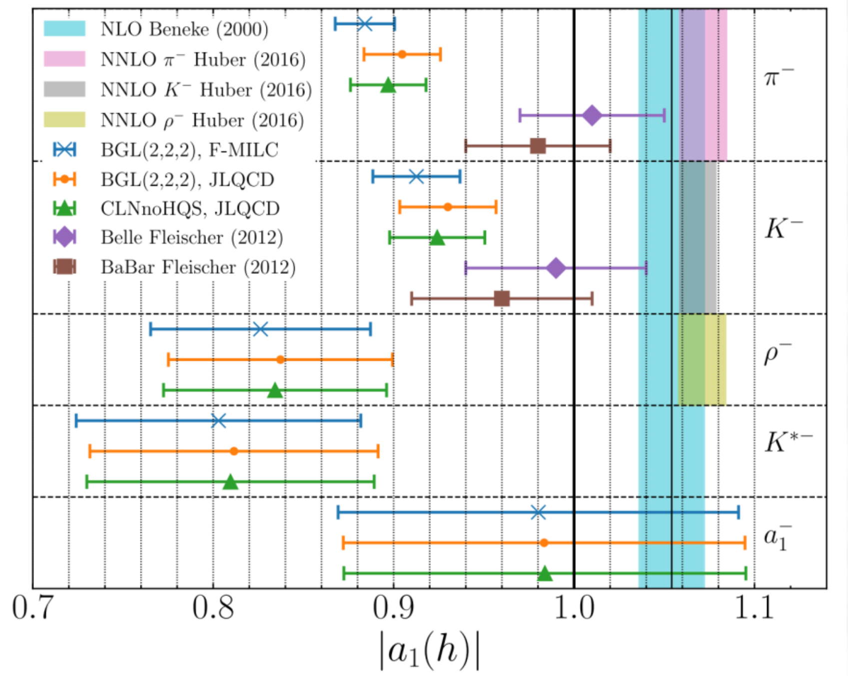
$R^{(*)}_{(s)L}$	LO	NLO	NNLO	Exp.	Deviation (σ)
R_π	1.01	$1.07^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.74 ± 0.06	5.4
R^*_π	1.00	$1.06^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.80 ± 0.06	4.5
R_ρ	2.77	$2.94^{+0.19}_{-0.19}$	$3.02^{+0.17}_{-0.18}$	2.23 ± 0.37	1.9
R_K	0.78	$0.83^{+0.03}_{-0.03}$	$0.85^{+0.01}_{-0.02}$	0.62 ± 0.05	4.4
R^*_K	0.72	$0.76^{+0.03}_{-0.03}$	$0.79^{+0.01}_{-0.02}$	0.60 ± 0.14	1.3
R_{K^*}	1.41	$1.50^{+0.11}_{-0.11}$	$1.53^{+0.10}_{-0.10}$	1.38 ± 0.25	0.6
$R_{s\pi}$	1.01	$1.07^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.72 ± 0.08	4.4
R_{sK}	0.78	$0.83^{+0.03}_{-0.03}$	$0.85^{+0.01}_{-0.02}$	0.46 ± 0.06	6.3

$\bar{B} \to D^+ K^-$

$\bar{B}_s \to D_s^+ \pi^-$

$|a_1(\bar{B} \to D^{*+}\pi^-)| = 0.884 \pm 0.004 \pm 0.003 \pm 0.016 \; [1.071^{+0.020}_{-0.016}]$

☐ Confirmed by Belle: 2207.00134



15% lower than SM

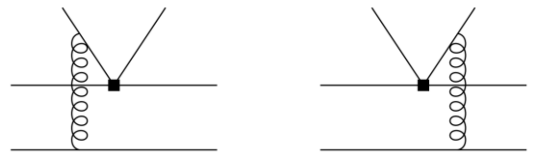
$|a_1(\bar{B} \to D^{*+}K^-)| = 0.913 \pm 0.019 \pm 0.008 \pm 0.013 \; [1.069^{+0.020}_{-0.016}]$

Large power corrections?

$$\langle D_q^{(*)+} L^- | \mathcal{Q}_i | \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u) \phi_L(u) + \boxed{\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)}$$

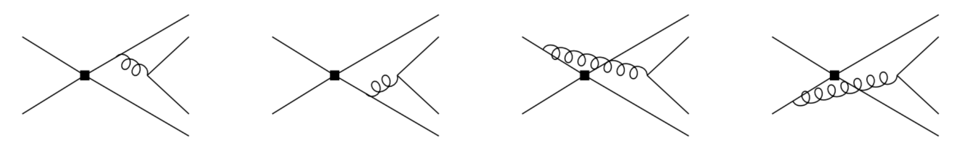
☐ Sources of sub-leading power corrections: [Beneke, Buchalla, Neubert, Sachrajda '01; Bordone, Gubernari, Huber, Jung, van Dyk '20]

- ➢ non-factorizable spectator interactions

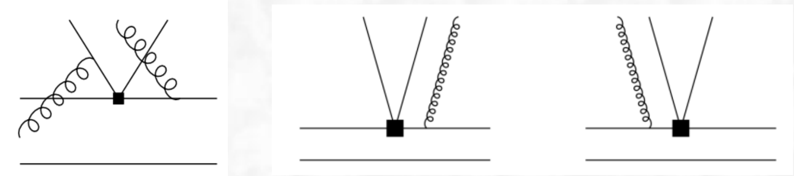


$$\frac{\Lambda_{\rm QCD}}{m_b}$$

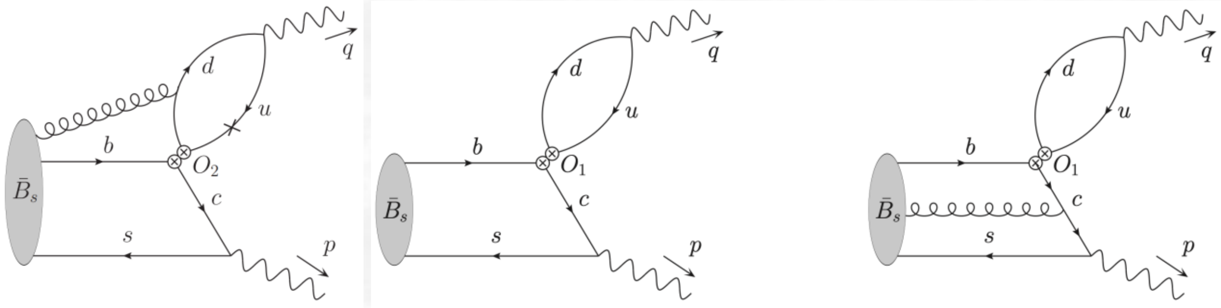
- ➢ annihilation topologies



- ➢ non-leading higher Fock-state contributions



- ➢ all are estimated to be power-suppressed, and no chirality-enhancement due to $(V-A)\otimes(V-A)$ structure
- ➢ very difficult to explain why the measured values of $|a_1(h)|$ several σ smaller than the SM predictions
- ➢ *must consider sub-leading power corrections more carefully*



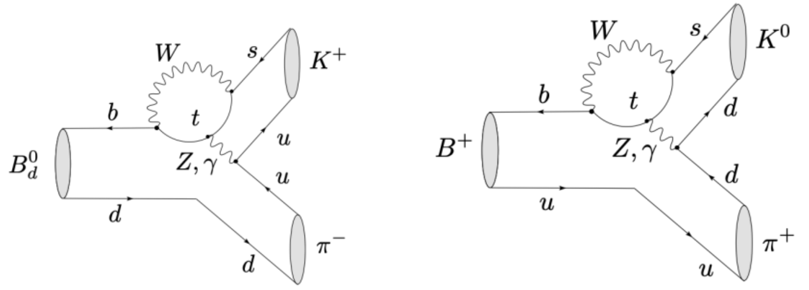
☐ Non-fact. soft-gluon contributions in LCSR with B-meson LCDA: [Maria Laura Piscopo, Aleksey V. Rusov, '23]

$$\mathrm{Br}(\bar{B}_s^0 \to D_s^+ \pi^-) = \left(2.15^{+2.14}_{-1.35}\right) [2.98 \pm 0.14] \times 10^{-3}$$
$$\mathrm{Br}(\bar{B}^0 \to D^+ K^-) = \left(2.04^{+2.39}_{-1.20}\right) [2.05 \pm 0.08] \times 10^{-4}$$

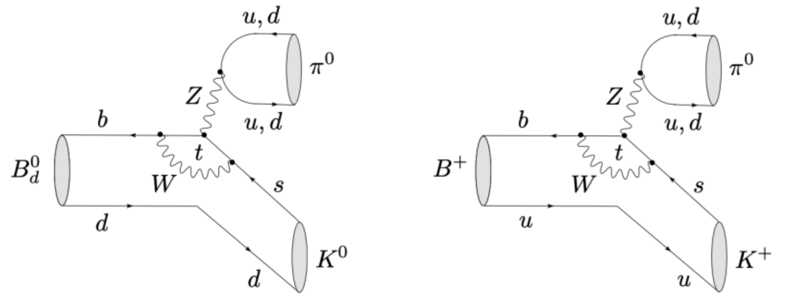
$B \to \pi K$ puzzle

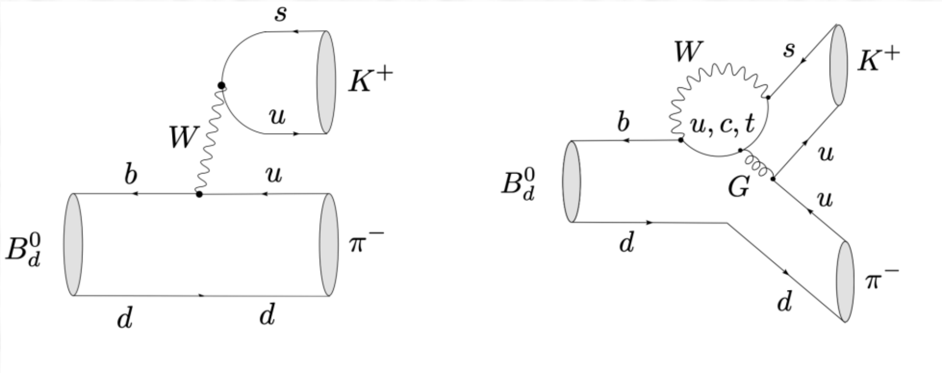
- ☐ **$B \to \pi K$ decays dominated by QCD penguin**
- ☐ **For direct CPV, tree & EW penguin also crucial**

- EW penguins are *colour-suppressed:* → *tiny* contributions ...



- EW penguins are *colour-allowed:* → *sizeable*, competing with trees!





$\propto A\lambda^4 R_b e^{i\gamma}$ $\propto A\lambda^2$

$\lambda^2 R_b = \mathcal{O}(0.02) \Rightarrow$ QCD penguins *dominate*

$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} = A_{\pi\bar{K}}\big[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\big] + A_{\bar{K}\pi}\big[\delta_{pu}\alpha_2 + \delta_{pc}\tfrac{3}{2}\alpha_{3,\rm EW}^c\big],$$
$$\mathcal{A}_{\bar{B}^0\to\pi^+K^-} = A_{\pi\bar{K}}\big[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\big],$$

$$A_{\rm CP}(\pi^0K^\pm) - A_{\rm CP}(\pi^\mp K^\pm) = -2\sin\gamma\Big(\mathrm{Im}(r_C) - \mathrm{Im}(r_T\, r_{\rm EW})\Big) + \ldots$$

$$\Delta A_{CP}(\pi K) = A_{CP}(\pi^0K^-) - A_{CP}(\pi^+K^-)$$

$= (11.3 \pm 1.2)\%$ **differs from 0 by ~9σ**

$\Delta A_{CP}(\pi K)$ puzzle

some mechanism or sub-leading power corr.s or even

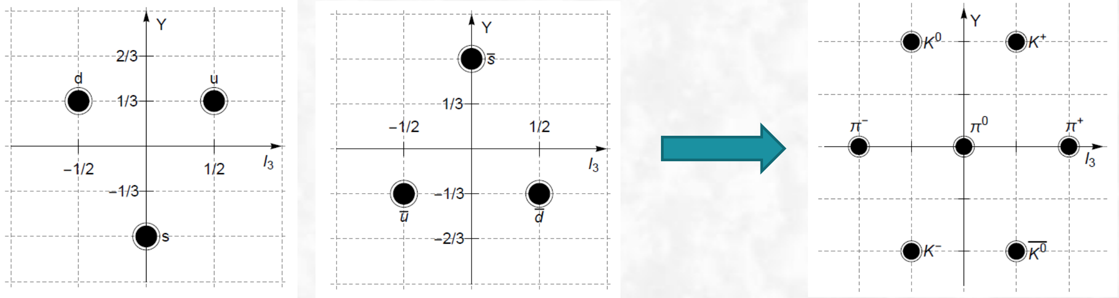
NP to enhance $C = \alpha_2$ or $P_{\rm EW} = \alpha_{3,\rm EW}^p$?

[R. X. Wang and M. Z. Yang, 2212.09054; 2401.03670]

$B \to PP$ based on $\mathrm{SU(3)_F}$ symmetry

☐ Analysis based on $\mathrm{SU(3)_F}$ symmetry:

- 3 light quarks, u, d, s, much lighter than b quark
- $u, d, s = \mathrm{SU(3)}_F$ triplet; State → $|\mathbf{irrep}, Y, I, I_3\rangle$
- $|u\rangle = \left|\mathbf{3}, \frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right\rangle$, $|d\rangle = \left|\mathbf{3}, \frac{1}{3}, \frac{1}{2}, -\frac{1}{2}\right\rangle$, $|s\rangle = \left|\mathbf{3}, -\frac{2}{3}, 0, 0\right\rangle$
- $|\bar{d}\rangle = \left|\mathbf{3}^*, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right\rangle$; Y = hypercharge, I = Isospin



- $\mathbf{3} \times \mathbf{3}^* = \mathbf{1} + \mathbf{8}$: *These are the 3 pions, 4 kaons, η, η'*
- $|\pi^+\rangle = |u\bar{d}\rangle = |\mathbf{8}, 0, 1, 1\rangle$ Similarly other pions and kaons are also octets
- Apply to two-body final states

  $|PP\rangle_{\mathrm{sym}} = (\mathbf{8} \times \mathbf{8})_{\mathrm{sym}} = \mathbf{1} + \mathbf{8} + \mathbf{27} = 36$

$$\boxed{|\pi^+\pi^-\rangle = \frac{1}{2}\,|\mathbf{1}\rangle_{0,0,0} - \sqrt{\frac{2}{5}}\,|\mathbf{8}\rangle_{0,0,0} - \frac{1}{2\sqrt{15}}\,|\mathbf{27}\rangle_{0,0,0} + \frac{1}{\sqrt{3}}\,|\mathbf{27}\rangle_{2,0,0}}$$

$$\boxed{\bar{B}^0 = \left|\bar{d}b\right\rangle = \left|\bar{\mathbf{3}}\right\rangle_{\frac{1}{2},\frac{1}{2},-\frac{1}{3}}\,, \quad \bar{B}_s = |\bar{s}b\rangle = \left|\bar{\mathbf{3}}\right\rangle_{0,0,\frac{2}{3}}\,, \quad B^- = -\,|\bar{u}b\rangle = \left|\bar{\mathbf{3}}\right\rangle_{\frac{1}{2},-\frac{1}{2},-\frac{1}{3}}}$$

	$\overline{15}_{I=1}$	$\overline{15}_{I=0}$	$6_{I=1}$	$\overline{3}^{(a)}_{I=0}$	$\overline{3}^{(s)}_{I=0}$	$\overline{15}_{I=3/2}$	$\overline{15}_{I=1/2}$	$6_{I=1/2}$	$\overline{3}^{(a)}_{I=1/2}$	$\overline{3}^{(s)}_{I=1/2}$
$\overline{d}\,\overline{d}\,d$						$\sqrt{1/3}$	$-\sqrt{1/6}$			$\sqrt{1/2}$
$\overline{d}\,\overline{u}\,u$						$-\sqrt{1/3}$	$-\sqrt{1/24}$	$-1/2$	$1/2$	$\sqrt{1/8}$
$\overline{d}\,\overline{s}\,d$	$1/2$	$-\sqrt{1/8}$	$1/2$	$-1/2$	$\sqrt{1/8}$					

☐ Effective weak Hamiltonian:

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}}\left[\lambda_u^{(s)}(C_1Q_1^{(u)} + C_2Q_2^{(u)}) + \lambda_c^{(s)}(C_1Q_1^{(c)} + C_2Q_2^{(c)}) - \lambda_t^{(s)}\sum_{i=3}^{10} C_iQ_i\right]$$

☐ Physical amplitudes:

$$\boxed{\langle PP|\mathcal{H}_{\mathrm{eff}}|B\rangle = \langle \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27}|\mathbf{3}^* \oplus \mathbf{6} \oplus \mathbf{15}^*|\mathbf{3}\rangle = \sum_i C_i\langle \mathbf{1}, \mathbf{8}, \mathbf{27}|\mathbf{3}^*, \mathbf{6}, \mathbf{15}^*|\mathbf{3}\rangle_i}$$

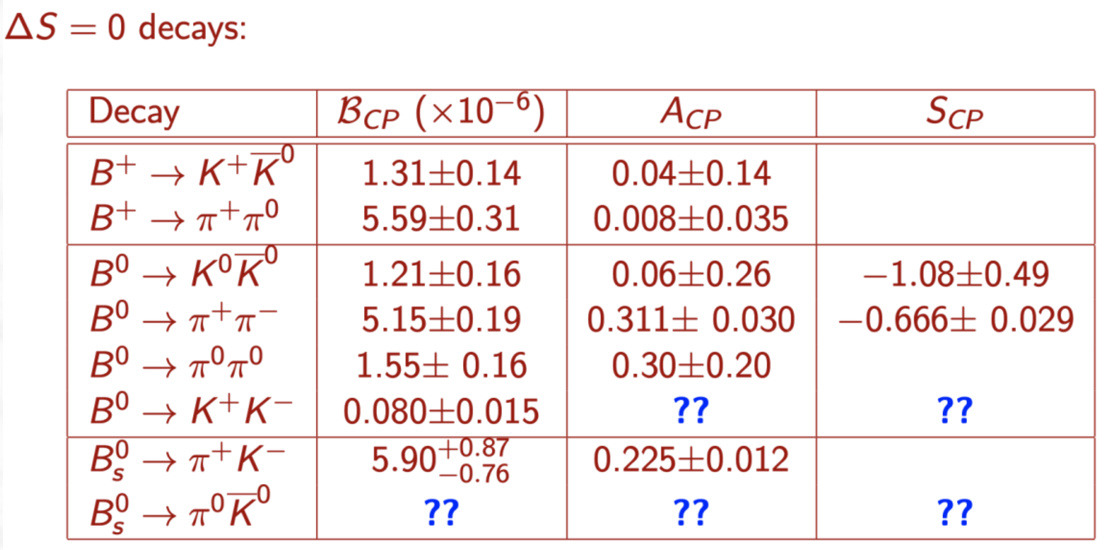
$B \to PP$ based on $SU(3)$ flavor symmetry

- ☐ **Physical amplitudes:**

$$\boxed{\langle PP|\mathcal{H}_{\text{eff}}|B\rangle = \langle \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27}|\mathbf{3}^* \oplus \mathbf{6} \oplus \mathbf{15}^*|\mathbf{3}\rangle = \sum_i C_i \langle \mathbf{1}, \mathbf{8}, \mathbf{27}|\mathbf{3}^*, \mathbf{6}, \mathbf{15}^*|\mathbf{3}\rangle_i}$$

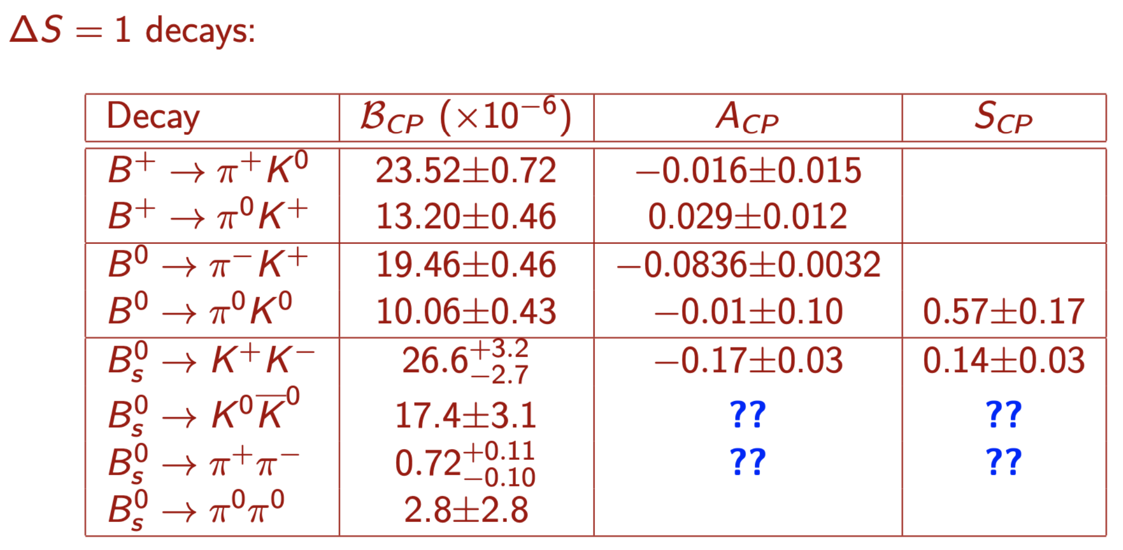
- ➢ $B \to PP$ decay amplitudes expressed in terms of $\mathrm{SU(3)_F}$ RMEs & C-G coefficients, and then fit to all the data
- ➢ **Key point:** no any theoretical assumptions on RMEs ⇒ completely rigorous on group-theoretical side
- ➢ Indep. RMEs: $V_{ub}V_{us}^* \quad \to \quad 5, \qquad V_{tb}V_{ts}^* \quad \to \quad 2;$ ➡ 7 indep. RMEs = 13 real parameters

- ☐ **Enough data for the fit with only 7 RMEs in exact $SU(3)_F$**



$\Delta S = 0$ decays:

Decay	$\mathcal{B}_{CP}$ $(\times 10^{-6})$	A_{CP}	S_{CP}
$B^+ \to K^+\overline{K}^0$	1.31±0.14	0.04±0.14	
$B^+ \to \pi^+\pi^0$	5.59±0.31	0.008±0.035	
$B^0 \to K^0\overline{K}^0$	1.21±0.16	0.06±0.26	−1.08±0.49
$B^0 \to \pi^+\pi^-$	5.15±0.19	0.311± 0.030	−0.666± 0.029
$B^0 \to \pi^0\pi^0$	1.55± 0.16	0.30±0.20	
$B^0 \to K^+K^-$	0.080±0.015	??	??
$B_s^0 \to \pi^+K^-$	$5.90^{+0.87}_{-0.76}$	0.225±0.012	
$B_s^0 \to \pi^0\overline{K}^0$	??	??	??



$\Delta S = 1$ decays:

Decay	$\mathcal{B}_{CP}$ $(\times 10^{-6})$	A_{CP}	S_{CP}
$B^+ \to \pi^+K^0$	23.52±0.72	−0.016±0.015	
$B^+ \to \pi^0K^+$	13.20±0.46	0.029±0.012	
$B^0 \to \pi^-K^+$	19.46±0.46	−0.0836±0.0032	
$B^0 \to \pi^0K^0$	10.06±0.43	−0.01±0.10	0.57±0.17
$B_s^0 \to K^+K^-$	$26.6^{+3.2}_{-2.7}$	−0.17±0.03	0.14±0.03
$B_s^0 \to K^0\overline{K}^0$	17.4±3.1	??	??
$B_s^0 \to \pi^+\pi^-$	$0.72^{+0.11}_{-0.10}$	??	??
$B_s^0 \to \pi^0\pi^0$	2.8±2.8		

$B \to PP$ based on $SU(3)$ flavor symmetry

☐ State-of-the-art $SU(3)_F$ fit [Huber, Li, Malami, Tetlalmatzi-Xolocotzi, w.i.p; D. London et al., 2311.18011]

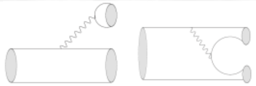
$$\begin{aligned}
\lambda_u^{(q)} &: A_1 = \langle \mathbf{1} || \mathbf{3}_1^* || \mathbf{3} \rangle \ , \ A_8 = \langle \mathbf{8} || \mathbf{3}_1^* || \mathbf{3} \rangle \ , \\
\lambda_t^{(q)} &: B_1 = \langle \mathbf{1} || \mathbf{3}_2^* || \mathbf{3} \rangle \ , \ B_8 = \langle \mathbf{8} || \mathbf{3}_2^* || \mathbf{3} \rangle \ , \\
\lambda_u^{(q)} \ \& \ \lambda_t^{(q)} &: R_8 = \langle \mathbf{8} || \mathbf{6} || \mathbf{3} \rangle \ , \ P_8 = \langle \mathbf{8} || \mathbf{15}^* || \mathbf{3} \rangle \ , \\
& P_{27} = \langle \mathbf{27} || \mathbf{15}^* || \mathbf{3} \rangle \ .
\end{aligned}$$

$$B_1 = -\frac{4}{\sqrt{3}} \left(\frac{3}{2} PA_{tc} + P_{tc} \right) \ , \quad B_8 = -\sqrt{\frac{5}{3}} P_{tc} \ .$$

$$\begin{aligned}
A_1 &= \frac{1}{2\sqrt{3}} \left(-3\widetilde{T} + \widetilde{C} - 8\widetilde{P}_{uc} - 12\widetilde{PA}_{uc} \right) \ , \\
A_8 &= \frac{1}{8}\sqrt{\frac{5}{3}} \left(-3\widetilde{T} + \widetilde{C} - 8\widetilde{P}_{uc} - 3\widetilde{A} \right) \ , \\
R_8 &= \frac{\sqrt{5}}{4} \left(\widetilde{T} - \widetilde{C} - \widetilde{A} \right) \ , \\
P_8 &= \frac{1}{8\sqrt{3}} \left(\widetilde{T} + \widetilde{C} + 5\widetilde{A} \right) \ , \\
P_{27} &= -\frac{1}{2\sqrt{3}} \left(\widetilde{T} + \widetilde{C} \right) \ .
\end{aligned}$$

$\lvert\widetilde{T}'/\widetilde{T}\rvert$	$\lvert\widetilde{C}'/\widetilde{C}\rvert$	$\lvert\widetilde{P}'_{uc}/\widetilde{P}_{uc}\rvert$	$\lvert\widetilde{A}'/\widetilde{A}\rvert$	$\lvert P\widetilde{A}'_{uc}/P\widetilde{A}_{uc}\rvert$
12±4	6.6±2.2	16±22	14±13	10±13

$\Delta S = 0$ fit:



$\lvert\widetilde{T}\rvert$	$\lvert\widetilde{C}\rvert$	$\lvert\widetilde{P}_{uc}\rvert$	$\lvert\widetilde{A}\rvert$	$\lvert P_{tc}\rvert$
4.0 ± 0.5	6.6 ± 0.7	3 ± 4	6 ± 5	0.8 ± 0.4

$\Delta S = 1$ fit:

$\lvert\widetilde{T}'\rvert$	$\lvert\widetilde{C}'\rvert$	$\lvert\widetilde{P}'_{uc}\rvert$	$\lvert\widetilde{A}'\rvert$	$\lvert P'_{tc}\rvert$
48 ± 14	41 ± 14	48 ± 15	81 ± 28	0.78 ± 0.16

- ✓ $\left|\frac{\tilde{C}}{\tilde{T}}\right| = 1.65$ (ΔS=0), 0.85 (ΔS = 1), 1.23 $(SU(3)_F)$ vs $0.13 \le \left|\frac{\tilde{C}}{\tilde{T}}\right| = 0.23 \le 0.43$ based on QCDF
- ✓ for combined ΔS = 0 & ΔS = 1 decays: very poor fit, with 3.6σ disagreement with the $SU(3)_F$ limit
- ✓ a 1000% $SU(3)_F$-breaking effect required, much large than naive expectation of $f_K/f_\pi - 1 \sim 20\%$

☐ More precise measurements, especially of the missing observables (e.g. $B_s^0 \to K^0\bar{K}^0$ and $B_s^0 \to \pi^0\bar{K}^0$) may help to figure out true dynamical mechanism behind charmless B decays

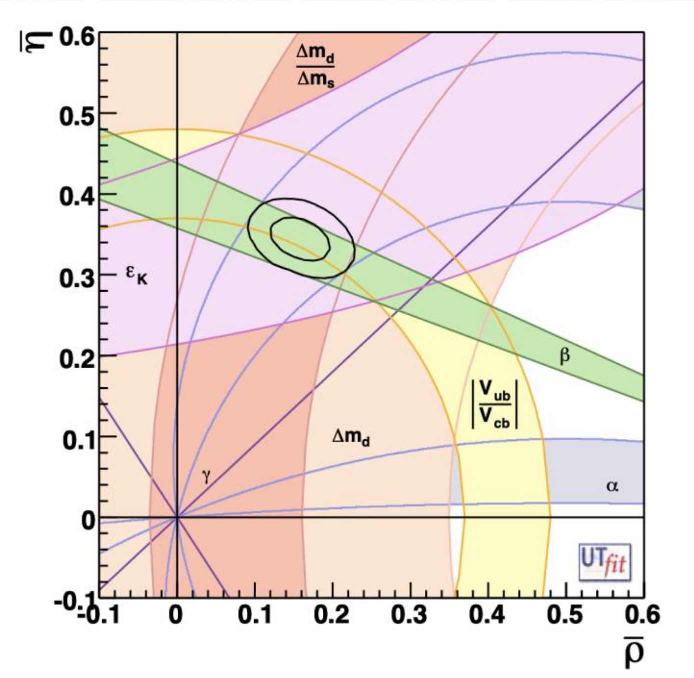
Summary

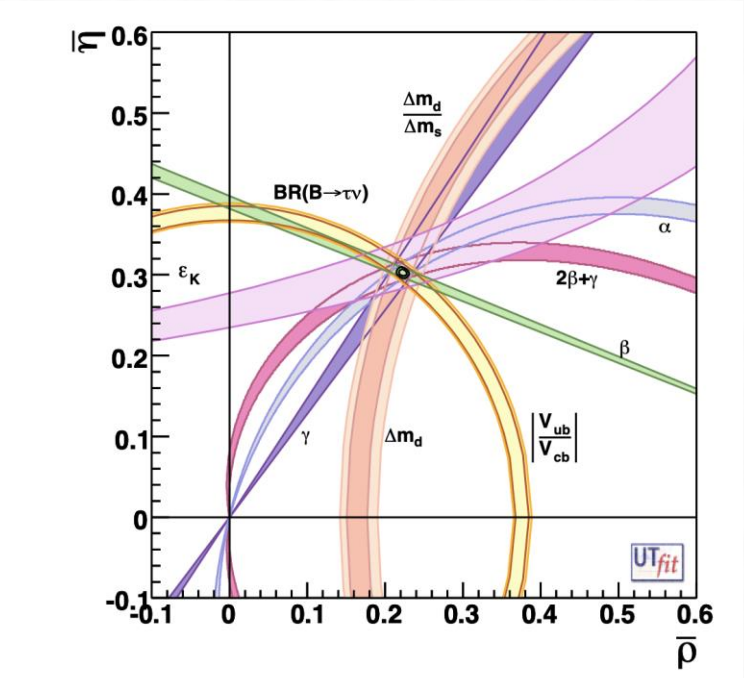
☐ **With exp. and theor. progress, we are now entering a precision era for flavor physics**

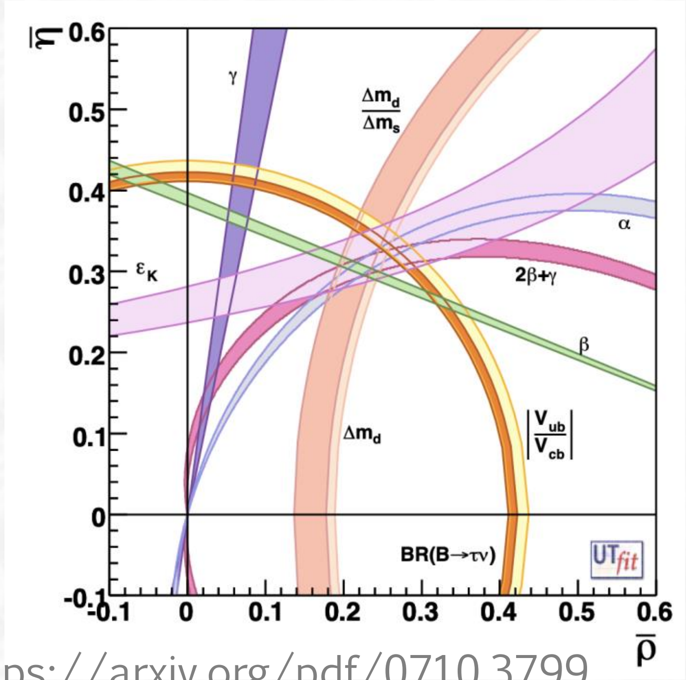
☐ **Several deviations between data & SM observed** ⟶ **NP signals beyond the SM?**

☐ **More precise exp. measurements, theor. predictions, & LQCD inputs needed**

many opportunities to explore SM & BSM physics in heavy flavor physics





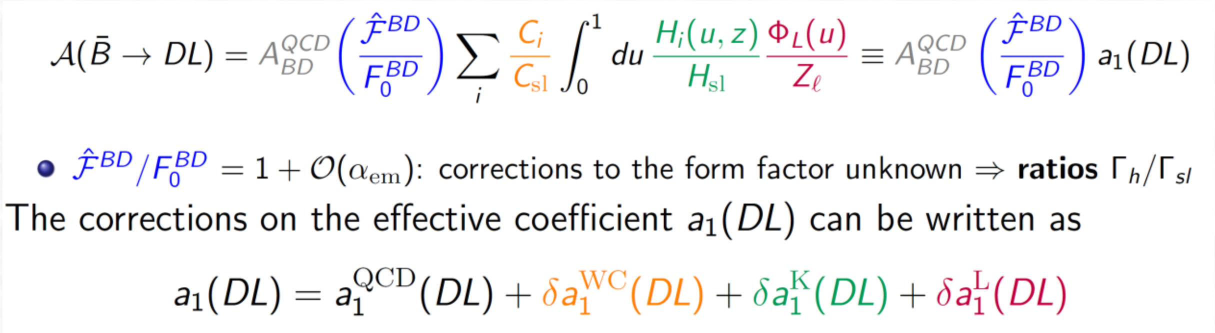


https://arxiv.org/pdf/0710.3799

back-up

QED corrections in other processes

☐ QED corrections to semi-leptonic & hadronic B decays [M. Beneke et al, 2008.10615; 107.03819]

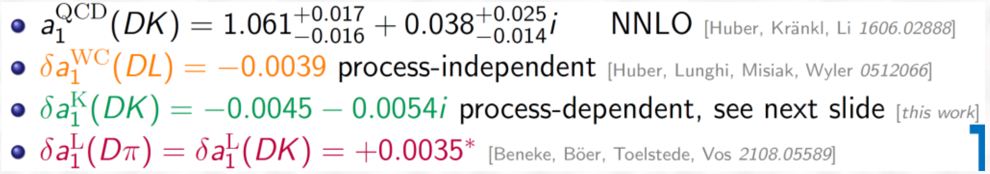


$$\mathcal{A}(\bar{B} \to DL) = A_{BD}^{QCD} \left(\frac{\hat{\mathcal{F}}^{BD}}{F_0^{BD}} \right) \sum_i \frac{C_i}{C_{\rm sl}} \int_0^1 du \, \frac{H_i(u,z)}{H_{\rm sl}} \frac{\Phi_L(u)}{Z_\ell} \equiv A_{BD}^{QCD} \left(\frac{\hat{\mathcal{F}}^{BD}}{F_0^{BD}} \right) a_1(DL)$$

- $\hat{\mathcal{F}}^{BD}/F_0^{BD} = 1 + \mathcal{O}(\alpha_{\rm em})$: corrections to the form factor unknown ⇒ **ratios** Γ_h/Γ_{sl}

The corrections on the effective coefficient $a_1(DL)$ can be written as

$$a_1(DL) = a_1^{\rm QCD}(DL) + \delta a_1^{\rm WC}(DL) + \delta a_1^{\rm K}(DL) + \delta a_1^{\rm L}(DL)$$



- $a_1^{\rm QCD}(DK) = 1.061^{+0.017}_{-0.016} + 0.038^{+0.025}_{-0.014} i$ NNLO [Huber, Kränkl, Li *1606.02888*]
- $\delta a_1^{\rm WC}(DL) = -0.0039$ process-independent [Huber, Lunghi, Misiak, Wyler *0512066*]
- $\delta a_1^{\rm K}(DK) = -0.0045 - 0.0054i$ process-dependent, see next slide [*this work*]
- $\delta a_1^{\rm L}(D\pi) = \delta a_1^{\rm L}(DK) = +0.0035^*$ [Beneke, Böer, Toelstede, Vos *2108.05589*]

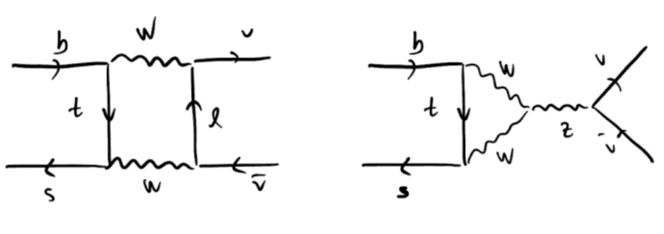
QED corrections turn out to be 1 order of magnitude smaller than NNLO QCD uncertainties

☐ Why QED corrections? [P. Boer and T. Feldmann, 2312.12885]

- ➢ **confronted with precision exp. data, QED effects need included in a systematic manner**
- ➢ **some virtual QED radiations lead to qualitatively new effect: logarithmic dependence on m_ℓ violates LFU, new isospin violation sources from different quark charges Q_q**
- ➢ **virtual photons with $\Lambda_{\rm QCD} < \mu < m_b$ resolve the inner hadronic structure, and generally not included in typical Monte Carlo implementations like PHOTOS**
- ➢ **QED factorization theorems for many processes still unknown; we need a consistent treatment of QED effects between theoretical & exp. analyses**

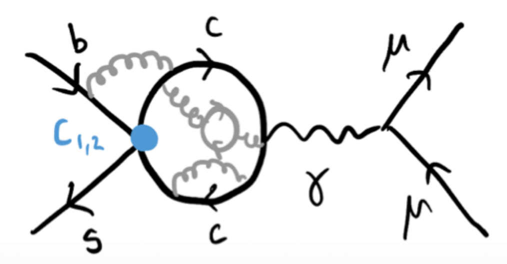
Rare FCNC B decays

☐ Another interesting FCNC decays: $B \to K^{(*)}\nu\bar{\nu}$



- there are no **photon-penguin diagrams**
- theoretically cleaner than $b \to s\ell\ell$ decays

  due to absence of **LD $c\bar{c}$-loop contributions**



☐ State-of-the-art SM prediction:

- **Effective Hamiltonian** in the SM:

see e.g. [Buras et al. '14]

$$\mathcal{L}_{\text{eff}}^{\text{b}\to\text{s}\nu\nu} = \frac{4G_F\lambda_t}{\sqrt{2}}\frac{\alpha_{\text{em}}}{2\pi}\sum_i C_L^{\text{SM}}\,\underline{(\bar{s}_L\gamma_\mu b_L)(\bar{\nu}_{Li}\gamma^\mu\nu_{Li})} + \text{h.c.}\,,$$

$\lambda_t = V_{tb}V_{ts}^*$

- **Short-distance** contributions known to **good precision**:

$$C_L^{\text{SM}} = -X_t/\sin^2\theta_W$$
$$= -6.32(7)$$

Including NLO QCD and two-loop EW contributions:

$$X_t = 1.462(17)(2)$$

[Buchala et al. '93, '99], [Misiak et al. '99], [Brod et al. '10]

$$\langle K^{(*)}|\bar{s}_L\gamma^\mu b_L|B\rangle = \sum_a K_a^\mu\,\mathcal{F}_a(q^2)$$

Form-factors (e.g., LQCD)

$$|V_{tb}V_{ts}^*| = |V_{cb}|\,(1+\mathcal{O}(\lambda^2))$$

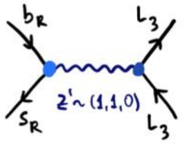
$$\mathcal{B}(B\to K\nu\bar{\nu})^{\text{SM}}/|\lambda_t|^2 = \begin{cases}(1.33\pm0.04)_{K_S}\times10^{-3}\\(2.87\pm0.10)_{K^+}\times10^{-3}\end{cases}$$

≃ 3% uncertainty

$$\mathcal{B}(B\to K^*\nu\bar{\nu})^{\text{SM}}/|\lambda_t|^2 = \begin{cases}(5.9\pm0.8)_{K^{*0}}\times10^{-3}\\(6.4\pm0.9)_{K^{*+}}\times10^{-3}\end{cases}$$

≃ 15% uncertainty

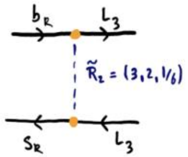
- Z':



$\mathcal{L}_{Z'} \supset g_{ij}^{\psi}\,(\bar{\psi}_i\gamma^\mu\psi_j)Z'_\mu$

sensitive to NP

- LQs:

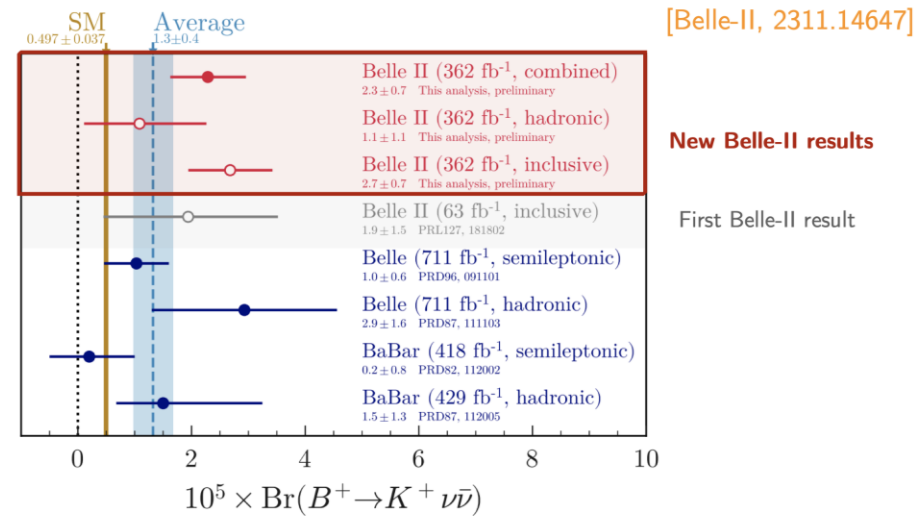


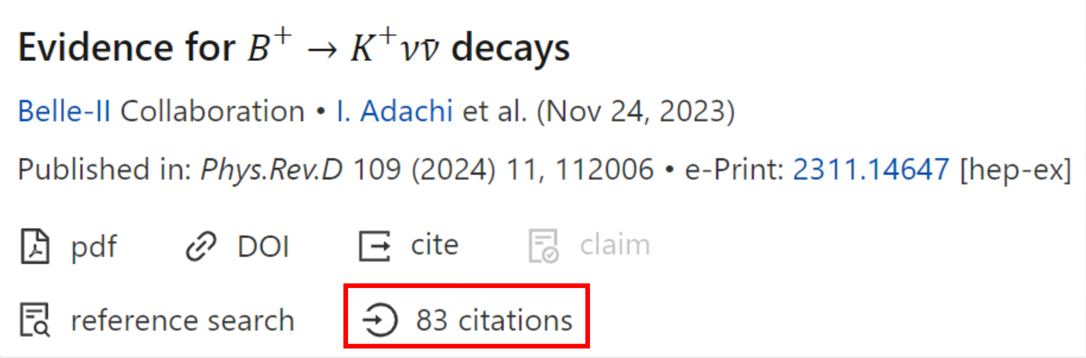
$\mathcal{L}_{\widetilde{R}_2} \supset y_{ij}^R\,(\bar{d}_{Ri}\widetilde{R}_2 i\tau_2 L_j) + \text{h.c.}$

Rare FCNC B decays

- **Belle-II first evidence of $B^+ \to K^+\nu\bar{\nu}$:**

  exp. data ≃ 2.7σ above the SM prediction



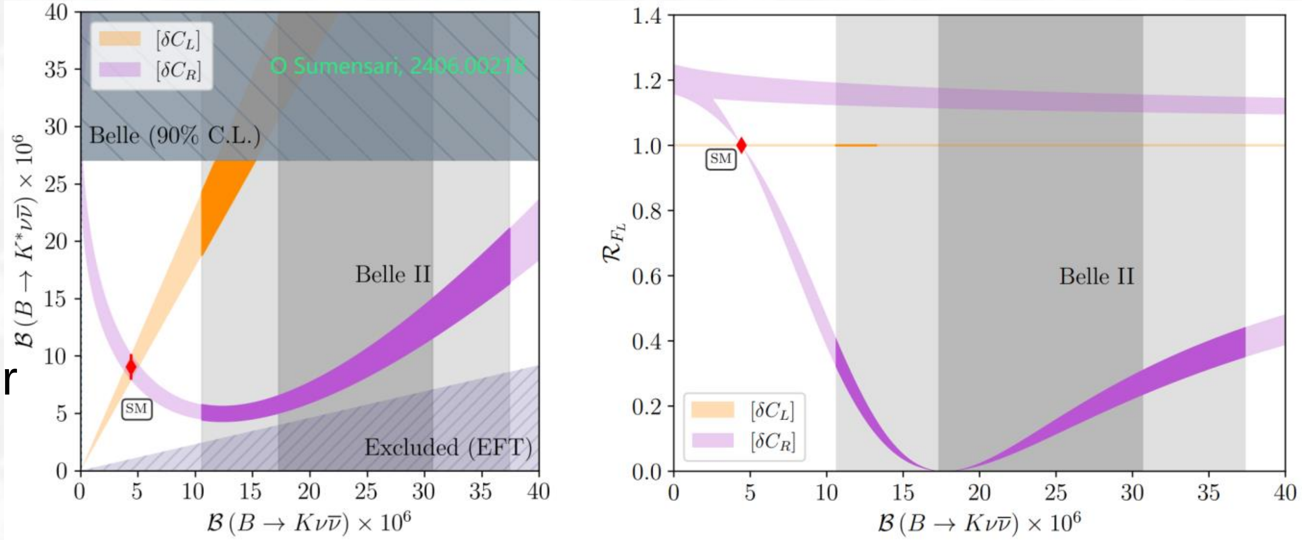


Evidence for $B^+ \to K^+\nu\bar{\nu}$ decays

Belle-II Collaboration • I. Adachi et al. (Nov 24, 2023)

Published in: *Phys.Rev.D* 109 (2024) 11, 112006 • e-Print: 2311.14647 [hep-ex]

$$\mathcal{L}_{\rm eff}^{b\to s\nu\nu} = \frac{4G_F\lambda_t}{\sqrt{2}}\frac{\alpha_{\rm em}}{2\pi}\sum_{ij}\left[C_L^{\nu_i\nu_j}(\bar{s}_L\gamma_\mu b_L)(\bar{\nu}_{Li}\gamma^\mu\nu_{Lj}) + C_R^{\nu_i\nu_j}(\bar{s}_R\gamma_\mu b_R)(\bar{\nu}_{Li}\gamma^\mu\nu_{Lj})\right]$$



- **NP signal:** new heavy mediator in loop or

  new light invisible particle in final state?

  [X. G. He, X. D. Ma, M. A. Schmidt, G. Valencia and R. R. Volkas, 2403.12485;

  B. F. Hou, X. Q. Li, M. Shen, Y. D. Yang and X. B. Yuan, 2402.19208; [see talk by 袁兴博 on 10.28]

  F. Z. Chen, Q. Y. Wen and F. R. Xu, 2401.11552]

- the deviation can easily accommodated by an EFT with operators coupled to 3-generation leptons
- verify the excess via $Br(B^0 \to K_s\nu\bar{\nu})$, $Br(B \to K^*\nu\bar{\nu})$ & $F_L^{K^*}$ @ Belle II

Analysis in SMEFT with flavor symmetry

☐ Combined analysis of $b \to u$ & $b \to c$ sectors in SMEFT:

$$\sum_i c_i^{(6)} \mathcal{Q}_i^{(6)}\Big|_{b\to ql\nu} = c_{H_\ell}^{ij}\left(H^\dagger i \overleftrightarrow{D}_\mu^I H\right)\left(\bar{L}^i\gamma^\mu\tau^I L^j\right) + c_{H_q}^{mn}\left(H^\dagger i \overleftrightarrow{D}_\mu^I H\right)\left(\bar{Q}^m\gamma^\mu\tau^I Q^n\right)$$
$$+ \underline{c_V^{mnij}\left(\bar{Q}^m\gamma^\mu\tau^I Q^n\right)\left(\bar{L}^i\gamma_\mu\tau^I L^j\right)} + \Big\{ c_{H_{\tilde q}}^{mn}\left(\tilde{H}^\dagger i D_\mu H\right)\left(\bar{U}^m\gamma^\mu D^n\right)$$
$$+ \underline{c_{S_d}^{mnij}\left(\bar{L}^i E^j\right)\left(\bar{D}^m Q^n\right)} + \underline{c_{S_u}^{mnij}\left(\bar{L}^{a,i} E^j\right)\epsilon_{ab}\left(\bar{Q}^{b,m} U^n\right)}$$
$$+ \underline{c_T^{mnij}\left(\bar{L}^{a,i}\sigma_{\mu\nu} E^j\right)\epsilon_{ab}\left(\bar{Q}^{b,m}\sigma^{\mu\nu} U^n\right)} + \mathrm{h.c.}\Big\}$$

- ➢ $U(2)^5$ flavor symmetry:

$$U(2)^5 = U(2)_Q \otimes U(2)_U \otimes U(2)_D \otimes U(2)_L \otimes U(2)_E$$

$$c_V'^0\left(\Gamma_L^\dagger\right)^{in}\left(\Gamma_L\right)^{mj}\left(\bar{Q}^m\gamma^\mu\tau^I Q^n\right)\left(\bar{L}^i\gamma_\mu\tau^I L^j\right),$$
$$c_{S_d}'^0\left(\Gamma_L^\dagger\right)^{in}\left(\Gamma_R\right)^{mj}\left(\bar{L}^i E^j\right)\left(\bar{D}^m Q^n\right),$$
$$c_{S_u}'^0\left(\Gamma_L^\dagger\right)^{im}\left(\Gamma_R\right)^{nj}\left(\bar{L}^{a,i} E^j\right)\epsilon_{ab}\left(\bar{Q}^{b,m} U^n\right),$$
$$c_T'^0\left(\Gamma_L^\dagger\right)^{im}\left(\Gamma_R\right)^{nj}\left(\bar{L}^{a,i}\sigma_{\mu\nu} E^j\right)\epsilon_{ab}\left(\bar{Q}^{b,m}\sigma^{\mu\nu} U^n\right).$$

☐ LEFT WC at EW scale:

$$C_{V_L}^{q\tau} = -\frac{v^2}{\Lambda^2} c_V'^0\left[1 + \lambda_Q^s\left(\frac{V_{qs}}{V_{qb}} + \frac{V_{qd}}{V_{qb}}\frac{V_{td}^*}{V_{ts}^*}\right)\right]$$
$$= -\frac{v^2}{\Lambda^2} c_V'^0\left(1 - \lambda_Q^s\frac{V_{tb}^*}{V_{ts}^*}\right),$$

$$C_{S_R}^{q\tau} = -\frac{v^2}{2\Lambda^2} c_{S_u}'^0\left(1 - \lambda_Q^s\frac{V_{tb}^*}{V_{ts}^*}\right),$$

$$C_{S_L}^{u\tau} = C_T^{u\tau} \simeq 0, \qquad C_{S_L}^{c\tau} = C_T^{c\tau} \propto m_c/m_t\,,$$

- ➢ left-handed vector & right-handed scalar NP have same sizes in $b \to u$ & $b \to c$

☐ Projections in $R(D) - R(\pi)$ & $R(D^*) - R(\rho)$ planes: