

# **Recent progress on heavy meson LCDA**

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### HQET and heavy meson LCDA

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#### **Heavy Flavor Physics**

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Flavor physics plays a very important role in particle physics

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- The bottom quark was first predicted in 1973 by Kobayashi and Maskawa to explain CP violation.



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Flavor physics plays a very important role in particle physics

#### **Heavy Flavor Physics**

- The charm quark was predicted by Glashow, Iliopoulos and Maiani in 1970.
- 1976

2008

The bottom quark was first predicted in 1973 by Kobayashi and Maskawa to explain CP violation.

> Heavy Flavor Physics: *b*, *c*,  $\tau$ .





Flavor physics plays a very important role in particle physics

#### **Heavy meson LCDA**



Heavy meson LCDA plays a very important role in Flavor physics

#### **Heavy Quark Effective Theory**

#### ≻ The Lagrangian of HQET.

$$\mathcal{L}_{\text{eff}} = \bar{h}_v i v \cdot D h_v + \frac{1}{2m_Q} \sum_{n=0}^{\infty} \bar{h}_v i \not\!\!D_\perp \left( -\frac{i v \cdot D}{2m_Q} \right)^n i \not\!\!D_\perp h_v \,.$$



Figure 5: Philosophy of the heavy-quark effective theory.

Neubert, Subnucl.Ser 34, 98-165 (1997)

#### HQET is constructed to describe heavy flavor physics

#### **Heavy Quark Effective Theory**

> Using HQET, observables can be written schematically as series.



LCDA is a crucial nonperturbative quantity

#### Why heavy meson LCDA important?



 $[B \rightarrow \pi \ell \nu$  Khodjamirian, Mannel, Offen, YMW, PRD 83, 094031 (2011); ZHL, ZGS, YW, NZ, NPB, 900, 198-211 (2015)]  $[B \rightarrow D \ell \nu$  HPQCD Collaboration, PRD 92, 054510 (2015)]

 $[B_s \rightarrow PP, PV, VV, Ali, Kramer, YL, CDL, YLS, et.al., PRD, 76, 074018 (2007)]$ 

LCDA is pivotal in determining  $V_{ub}$  and  $V_{cb}$ 

#### Why heavy meson LCDA important?



$$\left\langle K_{a}^{*}\ell^{+}\ell^{-}\right|H_{eff}|B\right\rangle = T_{a}^{I}\left(q^{2}\right)\zeta_{a}\left(q^{2}\right) + \sum_{\pm}\int_{0}^{\infty} \frac{d\omega}{\omega}\phi_{\pm}^{B}(\omega)\int_{0}^{1}du\phi_{K^{*}}^{a}(u)T_{a,\pm}^{II}\left(\omega,u,q^{2}\right).$$
  
**B-meson LCDA**

 $[B \rightarrow K^+ \ell^+ \ell^- \text{Ali, Kramer, GHZ, EPJC 47, 625 (2006)}]$  $[B \rightarrow K^* \ell^+ \ell^- \text{QC, XQL, YDY, JHEP 04, 052 (2010)}]$ 

Without precise knowledge on LCDAs, it is hard to probe NP

#### Why heavy meson LCDA important?



[QCD factorization: Beneke, Buchalla, Neubert, Sachrajda, PRL 83, 1914 (1999); YDY, XQL, Phys.Rev.D 73 (2006) 114027] [For PQCD, see: Keum, Li, Sanda, PRD 63, 054008 (2001); ZJX, CDL, et.al., PRD 73, 074002 (2006)] [For TMDF, see: JPM, QW, JHEP, 01, 067 (2006)]

LCDA is an indispensable part of factorization theory

#### **Definition on heavy meson LCDA**

> The light-ray HQET matrix element

[Grozin, Neubert, PRD 55, 272-290 (1997)]

$$\left\langle 0 \left| \bar{q}_{\beta}(z)[z,0] h_{v\alpha}(0) \right| \bar{B}(v) \right\rangle = -\frac{i\tilde{f}_B m_B}{4} \left[ \frac{1+\psi}{2} \left\{ 2\tilde{\varphi}_B^+(t,\mu) + \frac{\tilde{\varphi}_B^-(t,\mu) - \tilde{\varphi}_B^+(t,\mu)}{t} \not{\varphi} \right\} \gamma_5 \right]_{\alpha\beta} .$$
Leading twist Sub-leading twist

We assume that  $z^2 = 0$ , define  $t = v \cdot z$  and the path-ordered exponential

$$[z,0] = \operatorname{P} \exp\left(ig_s \int_{z_2}^{z_1} dz^{\mu} A_{\mu}(z)\right) \,.$$

### HQET and heavy meson LCDA

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### Accessing heavy meson LCDA

Equation of motion. [Kawamura, Kodaira, CFQ, Tanaka, PLB 523, 111 (2001)]

$$\tilde{\phi}'_{-}(t) - \frac{1}{t} \left( \tilde{\phi}_{+}(t) - \tilde{\phi}_{-}(t) \right)$$
$$= 2t \int_{0}^{1} duu \left( \tilde{\Psi}_{A}(t, u) - \tilde{\Psi}_{V}(t, u) \right) .$$

Equation of motion. [Kawamura, Kodaira, CFQ, Tanaka, PLB 523, 111 (2001)]

► B-LCDA with contribution from 3-particle Fock States. [TH, CFQ, XGW, PRD 73, 074004 (2006)]

$$\Psi_{+}(\omega, b) = \frac{\omega}{\omega_{0}^{2}} \exp\left(-\frac{\omega}{\omega_{0}}\right) \left(\Gamma[\delta] J_{\delta-1}[\kappa]\right) \\ + (1-\delta)\Gamma[2-\delta] J_{1-\delta}[\kappa]\right) \left(\frac{\kappa}{2}\right)^{1-\delta} .$$

Equation of motion. [Kawamura, Kodaira, CFQ, Tanaka, PLB 523, 111 (2001)]

► B-LCDA with contribution from 3-particle Fock States. [TH, CFQ, XGW, PRD 73, 074004 (2006)]

Evolution equations of  $\phi_+^B$ . [Lange, Neubert, PRL 91, 102001 (2003); Bell, Feldmann, JHEP 04, 061 (2008)]

$$\frac{d}{d\ln\mu}\phi_B^+(\omega,\mu) = -\frac{\alpha_s C_F}{4\pi} \int_0^\infty d\omega' \gamma_+^{(1)}(\omega,\omega',\mu) \phi_B^+(\omega',\mu) + \mathcal{O}\left(\alpha_s^2\right) \,.$$
  
With  
$$\gamma_+^{(1)}(\omega,\omega',\mu) = \left(\Gamma_{\rm cusp}^{(1)}\,\ln\frac{\mu}{\omega} - 2\right) \delta\left(\omega - \omega'\right) - \Gamma_{\rm cusp}^{(1)}\,\omega\left[\frac{\theta\left(\omega'-\omega\right)}{\omega'\left(\omega'-\omega\right)} + \frac{\theta\left(\omega-\omega'\right)}{\omega\left(\omega-\omega'\right)}\right]_+ \,.$$

- Equation of motion. [Kawamura, Kodaira, CFQ, Tanaka, PLB 523, 111 (2001)]
- ► B-LCDA with contribution from 3-particle Fock States. [TH, CFQ, XGW, PRD 73, 074004 (2006)]
- Evolution equations of  $\phi_+^B$ . [Lange, Neubert, PRL 91, 102001 (2003); Bell, Feldmann, JHEP 04, 061 (2008)]
- ► RG equations of  $\phi_B^+(\omega,\mu)$  at two-loops. [Braun, YJ, Manashov, PRD 100, 1, 014023 (2019); ZLL, Neubert, JHEP 06, 060 (2020)]

$$\begin{pmatrix} \mu \frac{\partial}{\partial \mu} + \beta(a) \frac{\partial}{\partial a} + \Gamma_{\text{cusp}}(a) \ln\left(\tilde{\mu}e^{\gamma_E}s\right) + \gamma_{\eta}(a) \end{pmatrix} \eta_+(s,\mu)$$
$$= 4C_F a^2 \int_0^1 du \frac{\bar{u}}{u} h(u) \eta_+(\bar{u}s,\mu) \,.$$

Solution of evolution equations. [Bell, Feldmann, YMW and Yip, JHEP 11, 191 (2013); Braun, Manashov, PLB 731, 316-319 (2014)]

$$\phi_B^+(\omega,\mu) = e^V \int_0^\infty \frac{d\omega'}{\omega'} \sqrt{\frac{\omega}{\omega'}} J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right) \left(\frac{\mu_0}{\hat{\omega}'}\right)^{-g} \rho_B^+(\omega',\mu_0)$$

- Solution of evolution equations. [Bell, Feldmann, YMW and Yip, JHEP 11, 191 (2013); Braun, Manashov, PLB 731, 316-319 (2014)]
- Factorization theorem connecting the LCDA in QCD and HQET. [Ishaq, YJ, XNX, DSY, PRL 125, 13, 132001 (2020); Beneke, Finauri, Vos, YBW, JHEP 09, 066 (2023)]

$$\Phi^{\text{QCD}}(x,\mu_Q) = \int_0^\infty d\omega Z(x,\omega,m_b;\mu_Q,\mu_H) \Phi^{\text{HQET}}_+(\omega,\mu_H)$$

- Solution of evolution equations. [Bell, Feldmann, YMW and Yip, JHEP 11, 191 (2013); Braun, Manashov, PLB 731, 316-319 (2014)]
- Factorization theorem connecting the LCDA in QCD and HQET. [Ishaq, YJ, XNX, DSY, PRL 125, 13, 132001 (2020); Beneke, Finauri, Vos, YBW, JHEP 09, 066 (2023)]
- $\geq$  Perturbative constraint for large  $\omega$ . [Lee, Neubert, PRD 72, 094028 (2005)]

$$\phi_{+}^{B}(\omega,\mu) = \frac{C_{F}\alpha_{s}}{\pi\omega} \left[ \left( \frac{1}{2} - \ln\frac{\omega}{\mu} \right) + \frac{4\bar{\Lambda}}{3\omega} \left( 2 - \ln\frac{\omega}{\mu} \right) + \dots \right]$$

#### But...

> Compare with several phenomenological models.



There are considerable differences between various models.

#### But...

 $\triangleright$  Recent studies have utilized these models to calculate the form factors for *B* → *K*<sup>\*</sup> and *B* → *π*.

[JG, CDL, YLS, YMW, YBW, PRD 101,7, 074035 (2020); BYC, YKH, YLS, CW, YMW, JHEP 03, 014 (2023)]

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085} \Big|_{\lambda_B} \Big|_{0.019} \Big|_{\sigma_1} \Big|_{\sigma_1 - 0.062} \Big|_{\mu} \\ + 0.010_{-0.004} \Big|_{M^2 - 0.017} \Big|_{s_0} \Big|_{\sigma_1 - 0.079} \Big|_{\varphi_{\pm}(\omega)},$$

$$f_{B\to\pi}^{+}(0) = 0.122 \times \left[ 1 \pm 0.07 \Big|_{S_0^{\pi}} \pm 0.11 \Big|_{\Lambda_q} \\ \pm 0.02 \Big|_{\lambda_E^2/\lambda_H^2} \stackrel{+0.05}{_{-0.06}} \Big|_{M^2} \pm 0.05 \Big|_{2\lambda_E^2+\lambda_H^2} \\ \stackrel{+0.06}{_{-0.10}} \Big|_{\mu_h} \pm 0.04 \Big|_{\mu} \stackrel{+1.36}{_{-0.56}} \Big|_{\lambda_B} \stackrel{+0.25}{_{-0.43}} \Big|_{\sigma_1,\sigma_2} \right].$$

**Uncertainties from heavy meson LCDAs are dominant** 

#### **Difficulties in first-principle determinations**

The LCDAs are defined on the light-cone. [Grozin, Neubert, PRD 55, 272-290 (1997)]



They cannot be directly simulated on the lattice

#### **Difficulties in first-principle determinations**

- The LCDAs are defined on the light-cone. [Grozin, Neubert, PRD 55, 272-290 (1997)]
- Non-negative moments  $\int dk \, k^n \varphi_B^+(k)$  for n = 0, 1, 2 ... are not related to matrix elements of local operators.



[Braun, Ivanov, Korchemsky, PRD 69, 034014 (2004)]

$$\left[\bar{q}(tn)\not[tn,0]\Gamma h_v(0)\right]_R \neq \sum_{p=0}^{\infty} \frac{t^p}{p!} \left[\bar{q}(0)(\overleftarrow{D}\cdot n)^p h_v(0)\right]_R$$

Cannot get  $\varphi_B^+$  by their moments

HQET and heavy meson LCDA

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$$\langle H(p_H)|\,\bar{h}_v(0)\not\!\!\!/_+\gamma_5\,[0,tn_+]\,q_s\,(tn_+)\,|0\rangle = -i\tilde{f}_H m_H n_+ \cdot v \int_0^\infty d\omega e^{i\omega tn_+ \cdot v}\varphi_+(\omega;\mu)\,.$$

Cusp divergence:



✓  $n^2 \neq 0$ , still heavy quark field  $h_v$ 

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Cusp divergence:



- ✓  $n^2 \neq 0$ , still heavy quark field  $h_v$
- ✓  $n^2 \neq 0$ , and No  $h_v$ : QCD heavy quark





Light-cone can be accessed by simulating correlation functions with a large but finite P<sup>z</sup>. [XDJ, PRL 110 (2013)] HQET can be accessed by simulating correlation functions with a large but finite m<sub>0</sub>. [Isgur, Wise, PLB 232, 113-117 (1989)]



- > Can we utilize the heavy meson quasi-DA in HQET to obtain LCDA in HQET ?
- Our first attempts.
   [WW, YMW, JX, SZ, PRD 102, 011502 (2020)]





**LCDA in HQET** 

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 [WW, YMW, JX, SZ, PRD 102, 011502 (2020)]
 [JX, XRZ, PRD 106, 114019 (2022)]
 [JX, XRZ, SZ, PRD 106, L011503 (2022)]

[SMH, WW, JX, SZ, PRD 109, 034001 (2024)]

**Quasi-DA in HQET** 



**LCDA in HQET** 

#### > Can we utilize the heavy meson quasi-DA in HQET to obtain LCDA in HQET ?

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**Quasi-DA in HQET** 



**LCDA in HQET** 



• Difficult to realize the boosted HQET field on lattice QCD.

#### The second road

> Two-step factorization to access heavy meson LCDA.



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### $\Rightarrow$ Hierarchy $\Lambda_{\rm QCD} \ll m_H \ll P_z$ :



A big challenge for lattice simulation but still calculable on the lattice

#### The second road

#### > Two-step factorization to access heavy meson LCDA.



Determining heavy meson LCDAs from lattice QCD

QCD LCDAs of heavy mesons from boosted HQET

#### **Final results for HQET LCDAs**

#### > Compare with several phenomenological models



The first step towards heavy meson LCDAs

#### First inverse moment (IM)

$$\lambda_B^{-1}(\mu) \equiv \int_{-\infty}^{\infty} d\omega \frac{\phi_B^+(\omega,\mu)}{\omega} \, .$$

 $\checkmark$  The IM is a crucial quantity in LCSR and QCD factorization theorems in heavy flavor physics.

 $\checkmark$  We determine the IM by employing a model-independent parametrization formula.

$\mu$		$\lambda_B ~({ m GeV})$	$\sigma_B^{(1)}$	
$1 \mathrm{GeV}$	N = 1	0.389(35)	1.63(8)	[e-Print: 2403.17492]
1GeV	Ref. [31]	> 0.24		[PRD 98, 112016 (2018)]
	Ref. [18]	0.383(153)		[JHEP 10, 043 (2020)]
	Ref. [7]	0.48(11)	1.6(2)	[PRD 72, 094082 (2005)]
	Ref. [15]	0.46(11)	1.4(4)	[PRD 69, 034014 (2004)]
	Ref. [1]	0.35(15)		[PRD 55, 272 (1997)]
	Ref. [21]	$0.343\substack{+0.064\\-0.079}$		[PRD 101, 7, 074035 (2020)]
	Ref. [73]	0.338(68)		[PLB 848, 138345 (2024)]

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2448 citations







### Summary

> Two-step effective theories to calculate the heavy meson LCDA in HQET.

 $\succ$  CLQCD ensembles (0.05 fm) to simulate heavy meson quai-DAs.

> The first (preliminary) results for LCDAs are consistent with models.







Power corrections to quasi-distribution amplitudes of a heavy meson

Chao Han,<sup>1</sup> Wei Wang,<sup>1,2,\*</sup> Jia-Lu Zhang,<sup>1,†</sup> and Jian-Hui Zhang<sup>3,‡</sup> <sup>1</sup>INPAC, Key Laboratory for Particle Astrophysics and Cosmology (MOE), Shanghai Key Laboratory for Particle Physics and Cosmology, School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China <sup>2</sup>Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern Physics, Chinese Academy of Sciences, Huizhou 516000, Guangdong Province, China <sup>3</sup>School of Science and Engineering, The Chinese University of Hong Kong, Shenzhen 518172, China

[CH, WW, JLZ, JHZ, e-Print: 2408.13486] Report: Power corrections to quasi-DA of a heavy meson







Leading Power Accuracy in Lattice Calculations of Parton Distributions

Rui Zhang,<sup>1,\*</sup> Jack Holligan,<sup>2,†</sup> Xiangdong Ji,<sup>1,‡</sup> and Yushan Su<sup>1,3,§</sup>

<sup>1</sup>Department of Physics, University of Maryland, College Park, MD 20742, USA <sup>2</sup>Biomedical and Physical Sciences Building, Michigan State University, East Lansing, MI, 48824, USA <sup>3</sup>Physics Division, Argonne National Laboratory, Lemont, Illinois 60439, USA

[RZ, Holligan, XDJ, YSS, PLB, 844, 138081 (2023)]









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- > Two-loop calculation;
- $> \Lambda/m_Q$  corrections;

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- > Other theoretical approaches;
- Need support of computational resources;
- > Need support of new minds;

