Determining heavy meson LCDAs from lattice QCD

Based on 2403.17492 and 2410.18654

In collaboration with LPC members and C.D. Lü, J. Xu, S. Zhao, et al.

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Lattice Parton Collaboration

Outline

- **Motivation**
- **Theoretical framework for the two-step factorization method**
- **Lattice verification**
  - Matching from quasi DAs to QCD LCDAs
  - Determination of HQET LCDA
- **Phenomenological discussions**
  - Comparison with phenomenological models
  - Determination of the inverse and inverse-logarithmic moments
  - Impact on $B \to V$ form factors
- **Summary and prospect**

Motivation

Main tasks of heavy flavor physics:

- Precisely testing the standard model
- Indirect search for new physics
- Study on CP violation

- $B \to \pi\pi$: ***Beneke, Buchalla, Neubert, Sachrajda, 1999; 1452 citations***
- $B \to \pi K$: ***Beneke, Buchalla, Neubert, Sachrajda, 2001; 1205 citations***

- $B \to \pi\ell\nu$: ***Becher, Hill, 2005; 221 citations***
  Khodjamirian, Mannel, Offen, Wang, 2011; 201 citations
- $B \to K^{(*)}\ell\ell$: ***Khodjamirian, Mannel, Pivavorov, Wang, 2010; 505 citations***
- $B \to D\ell\nu$: ***HPQCD Collaboration, 2015; 400 citations***

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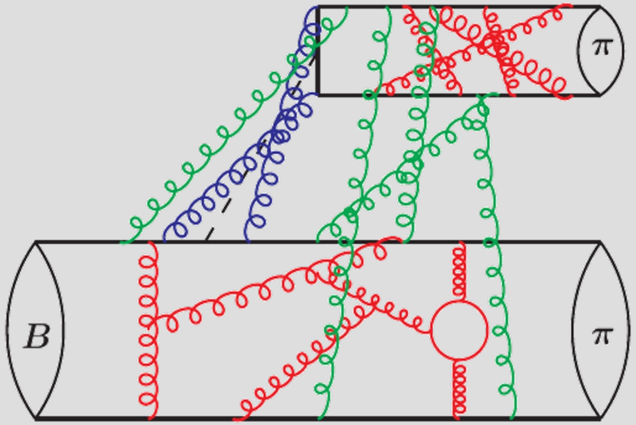
A multi-scale problem: factorization

$$\langle \pi(p')\pi(q)|Q_i|\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx T_i^{\mathrm{I}}(x)\phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{\mathrm{II}}(\xi,x,y)\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$



- Perturbative： matching, resummation, evolution
- Nonperturbative： Lattice QCD, sum rules, SU(3) symmetry, Quark model

Error analysis for B meson weak decay form factors

➢ The uncertainty of $B \to \pi,\ K^*$ form factors from LCSRs:

[Gao, Lu, Shen, Wang, Wei, 2020; Cui, Huang, Shen, Wang, 2023]

$$\mathcal{V}_{B\to K^*}(0) = 0.359\,{}^{+0.141}_{-0.085}\Big|_{\lambda_B}\,{}^{+0.019}_{-0.019}\Big|_{\sigma_1}\,{}^{+0.001}_{-0.062}\Big|_{\mu}\,{}^{+0.010}_{-0.004}\Big|_{M^2}\,{}^{+0.016}_{-0.017}\Big|_{s_0}\,{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

$$f^+_{B\to\pi}(0) = 0.122\times\Big[1\pm 0.07\Big|_{S_0^\pi}\pm 0.11\Big|_{\Lambda_q}\pm 0.02\Big|_{\lambda_E^2/\lambda_H^2}\,{}^{+0.05}_{-0.06}\Big|_{M^2}\pm 0.05\Big|_{2\lambda_E^2+\lambda_H^2}$$
$${}^{+0.06}_{-0.10}\Big|_{\mu_h}\pm 0.04\Big|_{\mu}\,{}^{+1.36}_{-0.56}\Big|_{\lambda_B}\,{}^{+0.25}_{-0.43}\Big|_{\sigma_1,\sigma_2}\Big].$$

λ_B and σ_1: the first inverse and inverse-log moments,

$\varphi_B^\pm$: uncertainties from different parameterizations of the B meson LCDA.

Without reliable B LCDA, it is impossible to discuss precision calculation!

Model dependence of heavy meson LCDAs

- Models for heavy meson LCDAs

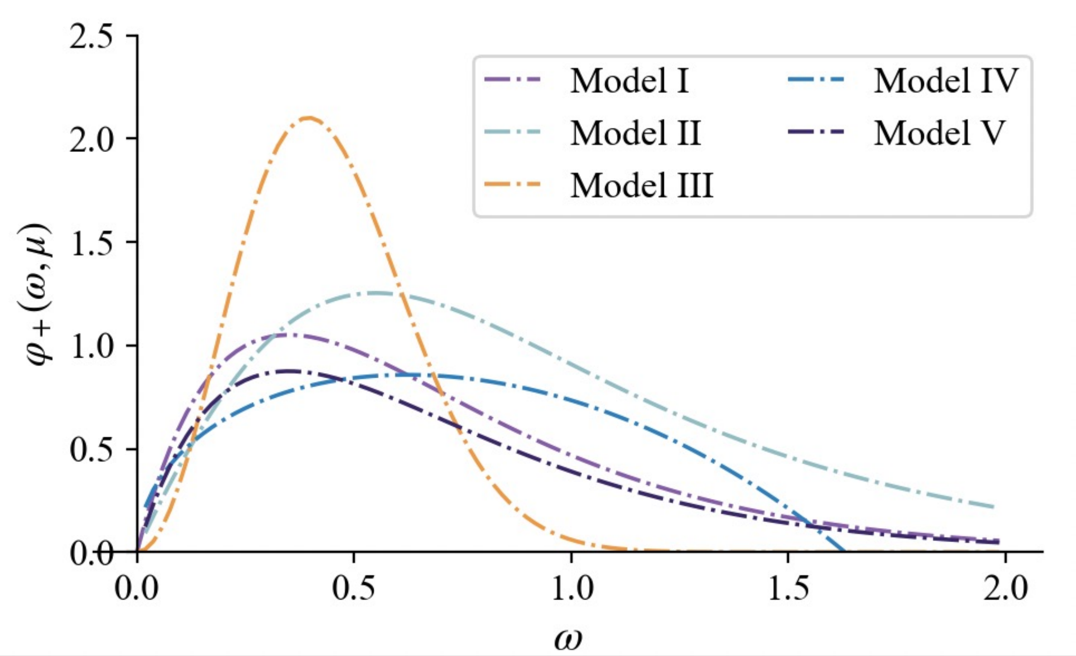
$$\varphi_{\mathrm{I}}^{+}(\omega,\mu_0)=\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0},$$

$$\varphi_{\mathrm{II}}^{+}(\omega,\mu_0)=\frac{4}{\pi\omega_0}\frac{k}{k^2+1}\left[\frac{1}{k^2+1}-\frac{2\left(\sigma_B^{(1)}-1\right)}{\pi^2}\ln k\right],$$

$$\varphi_{\mathrm{III}}^{+}(\omega,\mu_0)=\frac{2\omega^2}{\omega_0\omega_1^2}e^{-(\omega/\omega_1)^2},$$

$$\varphi_{\mathrm{IV}}^{+}(\omega,\mu_0)=\frac{\omega}{\omega_0\omega_2}\frac{\omega_2-\omega}{\sqrt{\omega(2\omega_2-\omega)}}\theta(\omega_2-\omega),$$

$$\varphi_{\mathrm{V}}^{+}(\omega,\mu_0)=\frac{\Gamma(\beta)}{\Gamma(\alpha)}\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0}U(\beta-\alpha,3-\alpha,\omega/\omega_0).$$

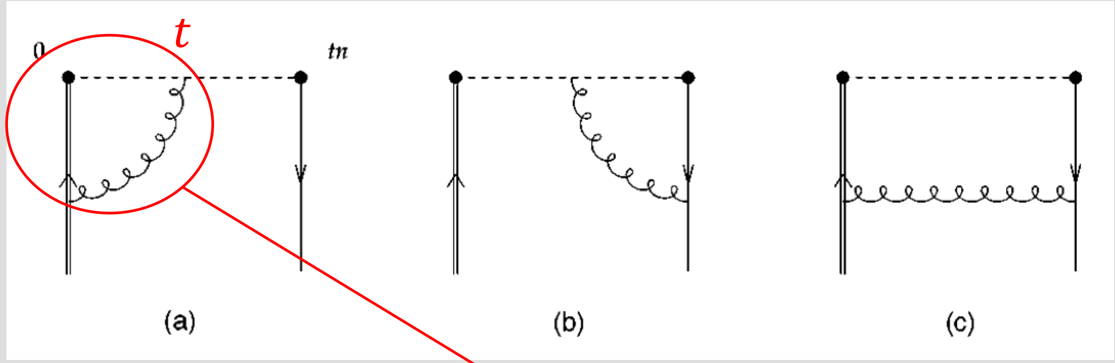


This leads to the largest systematic error in $B\to V$ form factors: [Gao, Lu, Shen, Wang, Wei, 2020]

$$\mathcal{V}_{B\to K^*}(0)=0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

Difficulties in first principle determinations

$$\langle H\,(p_H)|\,\bar{h}_v(0)\hbar_+\gamma_5\,[0,tn_+]\,q_s\,(tn_+)\,|0\rangle = -i\tilde{f}_H m_H n_+\cdot v\int_0^\infty d\omega e^{i\omega t n_+\cdot v}\varphi_+(\omega;\mu)$$



Diverge at $t \to 0$!

$$O_+^{\rm ren}(t,\mu) = O_+^{\rm bare}(t) + \frac{\alpha_s C_F}{4\pi}\left\{\left(\frac{4}{\hat{\epsilon}^2}+\frac{4}{\hat{\epsilon}}\ln(it\mu)\right)O_+^{\rm bare}(t) - \frac{4}{\hat{\epsilon}}\int_0^1 du\frac{u}{1-u}\left[O_+^{\rm bare}(ut)-O_+^{\rm bare}(t)\right]\right.$$

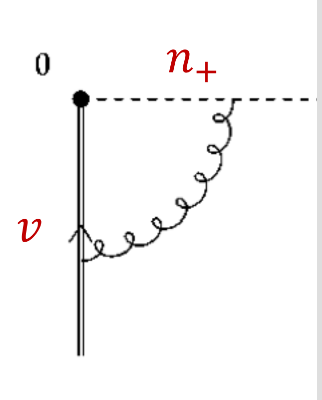
[Braun, Ivanov, Korchemsky, 2004]

Cusp divergence: No local limit!

- Non-negative moments are not related to OPE, and actually they diverge
- Cannot obtain φ_B from lattice QCD through their moments.

How to solve this problem?

Cusp divergence:



$$\cosh\theta = \frac{n_+ \cdot v}{\sqrt{n_+^2}\sqrt{v^2}}.$$

[Korchemskaya, Korchemsky, 1992]

Light cone $n_+^2 = 0 \Rightarrow$ divergence!

- ✓ Off light-cone Wilson line $n_+^2 \neq 0$, still heavy quark field h_v

  [Wang², Xu, Zhao, 2020; Xu, Zhang, 2022; Hu, Wang, Xu, Zhao, 2024]

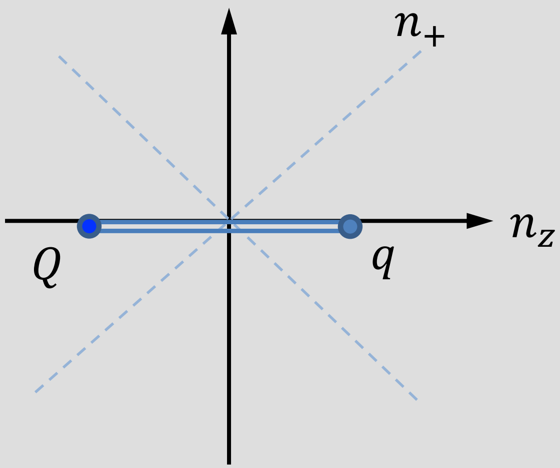
  🤔 Difficult to realize on lattice QCD.

- ✓ No h_v : QCD heavy quark. $\Rightarrow$ This work

  [Han, Wang, Zhang, et·al, 2403·17492; Han, Wang, Zhang, Zhang, 2408·13486; Deng, Wang, Wei, Zeng, 2409·00632]

A two-step factorization method

- Start from Quasi DA, calculable from LQCD

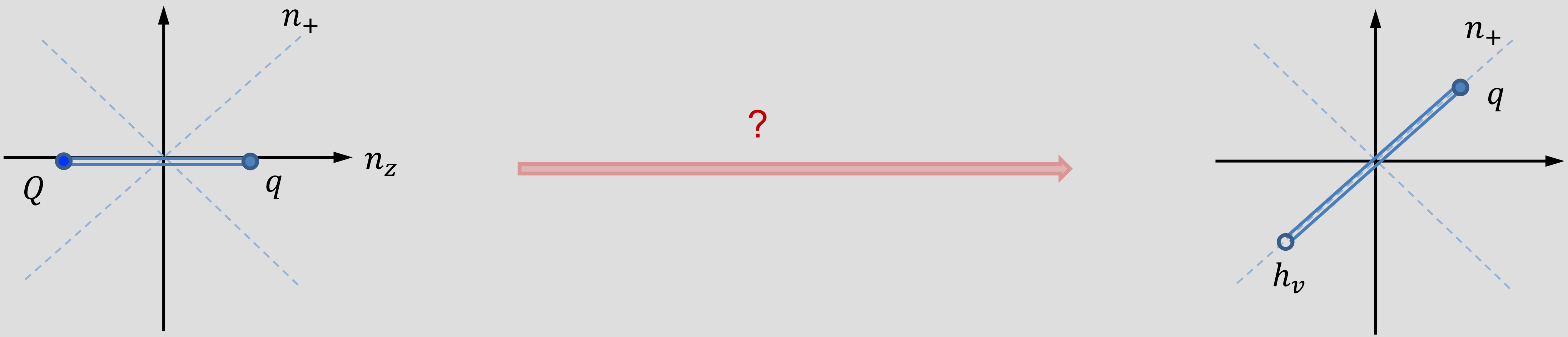


Quasi DA

$(P^z, m_H, \Lambda_{\text{QCD}})$

A two-step factorization method

- The target is HQET LCDA: contains HQET field and light-like correlation

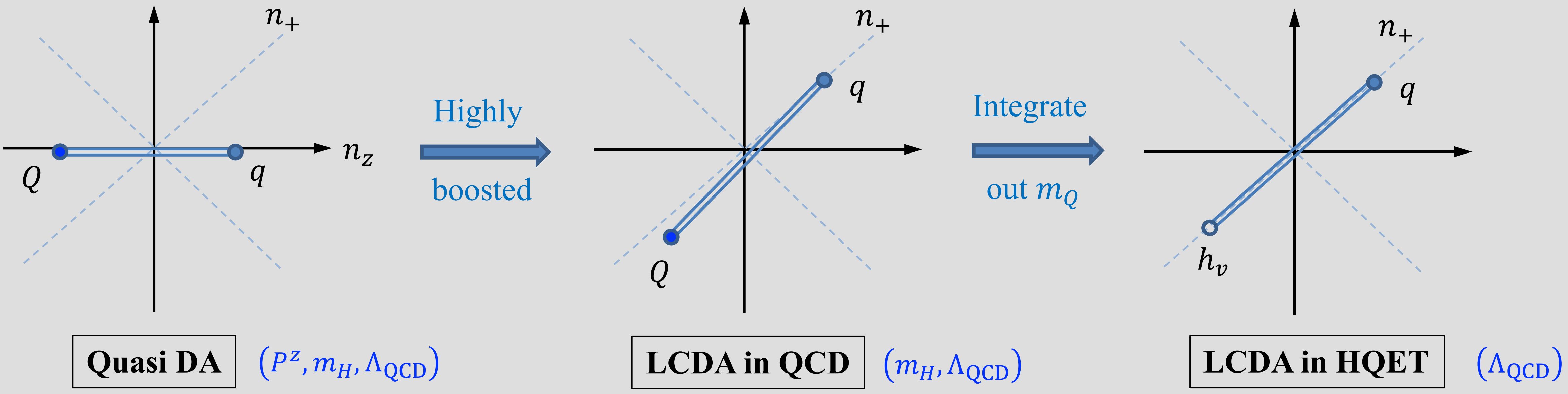


Quasi DA	LCDA in HQET
$(P^z, m_H, \Lambda_{\rm QCD})$	$(\Lambda_{\rm QCD})$

Need to integrate out P^z and m_H step by step ⇒ A two-step factorization

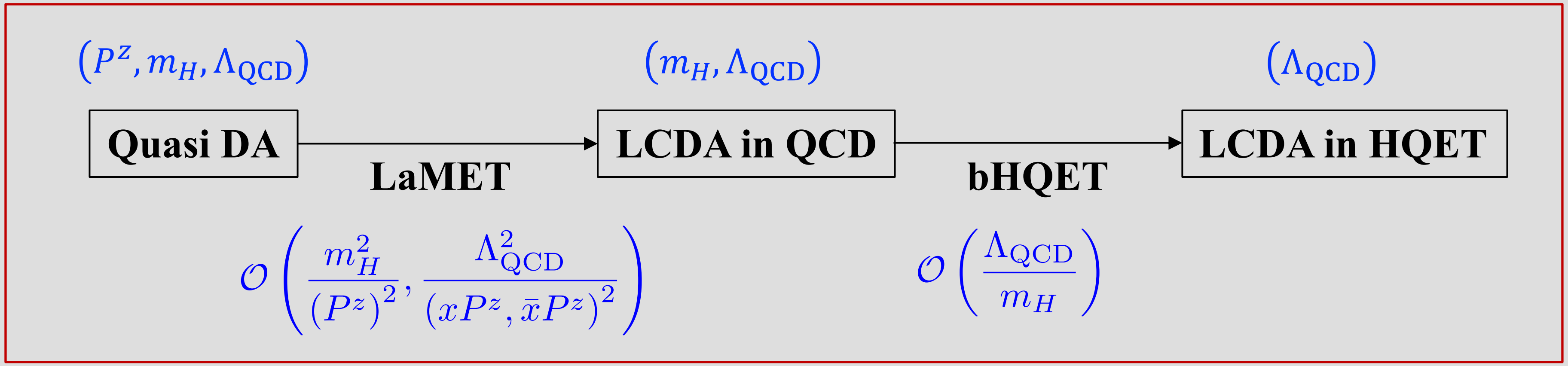
A two-step factorization method

- A multi-scale process: **hierarchy $\Lambda_{\rm QCD} \ll m_H \ll P^z$**

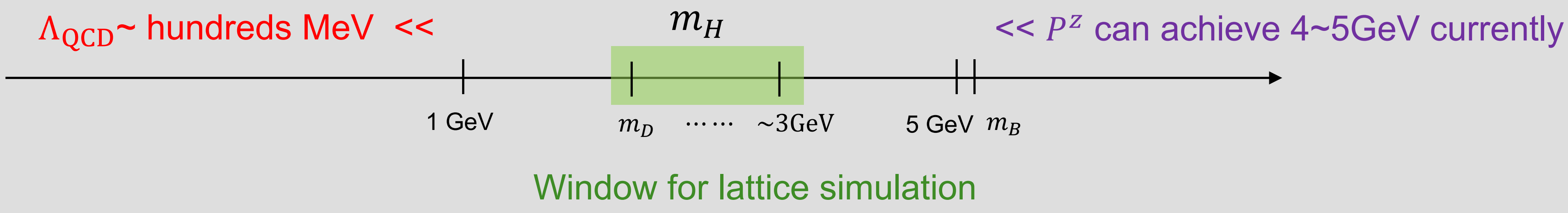


1. Assuming $\Lambda_{\rm QCD}, m_H \ll P^z$ and integrate out $P^z \Rightarrow$ LaMET [Ji, 2013; Ji, Liu[2], Zhang, Zhao, 2021]
2. Assuming $\Lambda_{\rm QCD} \ll m_H$ and integrate out $m_H \Rightarrow$ bHQET [Ishaq, Jia, Xiong, Yang, 2020; Beneke, Finauri, Vos, Wei, 2023]

A two-step factorization method



⇒ **Hierarchy $\boldsymbol{\Lambda_{\rm QCD} \ll m_H \ll P^z}$:** Still a big challenge for lattice simulation



Lattice QCD verification

- Simulating on the finest CLQCD ensemble:

  $n_s^3 \times n_t = 48^3 \times 144,\ a \simeq 0.052\text{fm};$

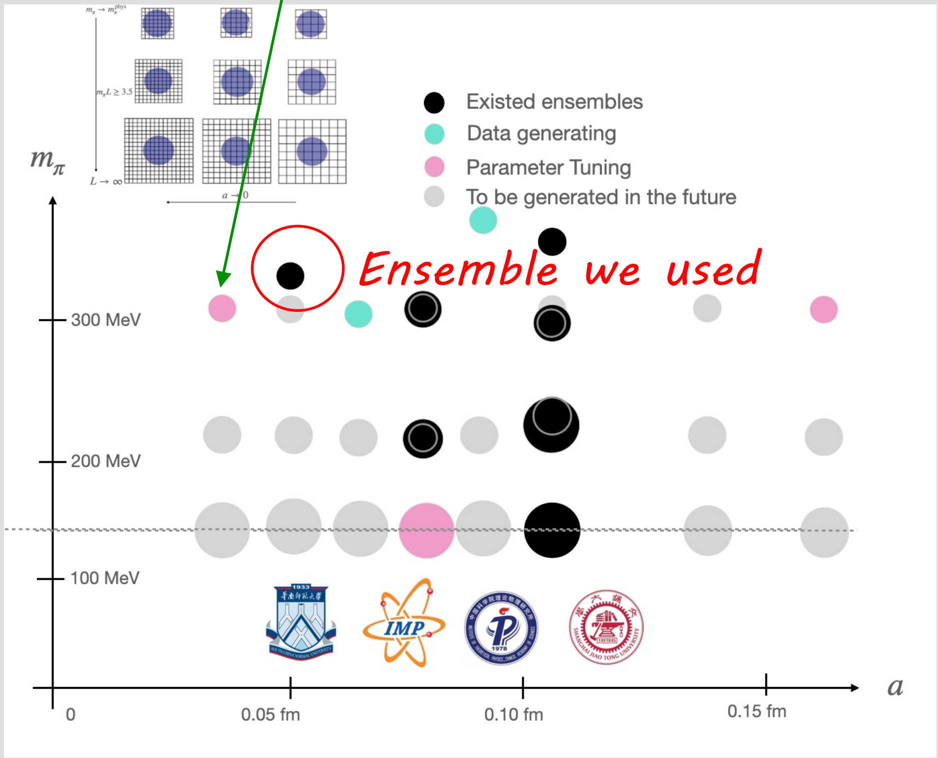
- $m_\pi \simeq 317\text{MeV}, m_D \simeq 1.92\text{GeV};$

- $P^z = \{2.99, 3.49, 3.98\}$GeV up to about 4GeV;

- Dispersion relation consistent with the relativistic one up to possible discretization error;

- The state-of-the-art techniques in renormalization and extrapolation on the lattice are adopted.

We are looking forward to this finer one!



[Hu, et·al·, 2004]

Matching I: from quasi DAs to LCDAs in QCD

- D meson quasi DA $\tilde{\phi}(x, P^z)$, include the scales $\Lambda_{\rm QCD} \ll m_D \ll P^z$

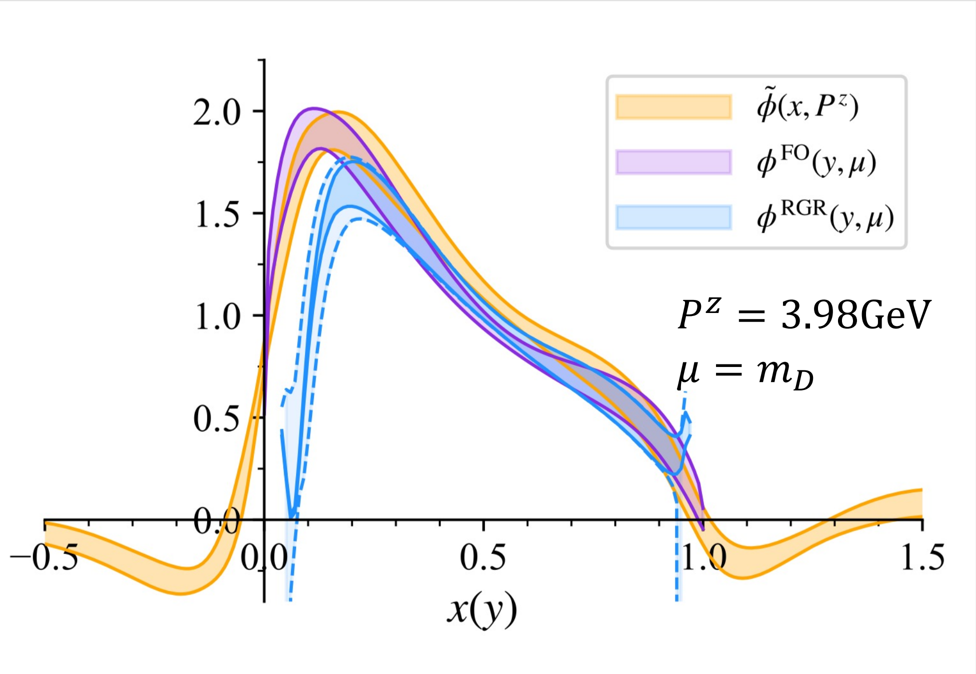
$$\tilde{\phi}(x, P^z) = \int \frac{dz}{2\pi} e^{-ixP^z z} \tilde{M}(z, P^z)$$

- Matching formula in LaMET:

$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

Liu, Wang, Xu, **QAZ**, Zhao, 2019;
Han, Hua, Ji, Lu, Wang, Xu, **QAZ**, Zhao, 2024

- FO: matching from fixed-order perturbation theory;
- RGR: resuming the large logs in C by using the ERBL evolution equation.



Systematic error from RGR: scale variation of $\mu_0 = 2yP^z$ with factor 0.8-1.2.

Matching I: from quasi DAs to LCDAs in QCD

➢ The power correction within the LaMET matching:

$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right)\phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{P^{z2}}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

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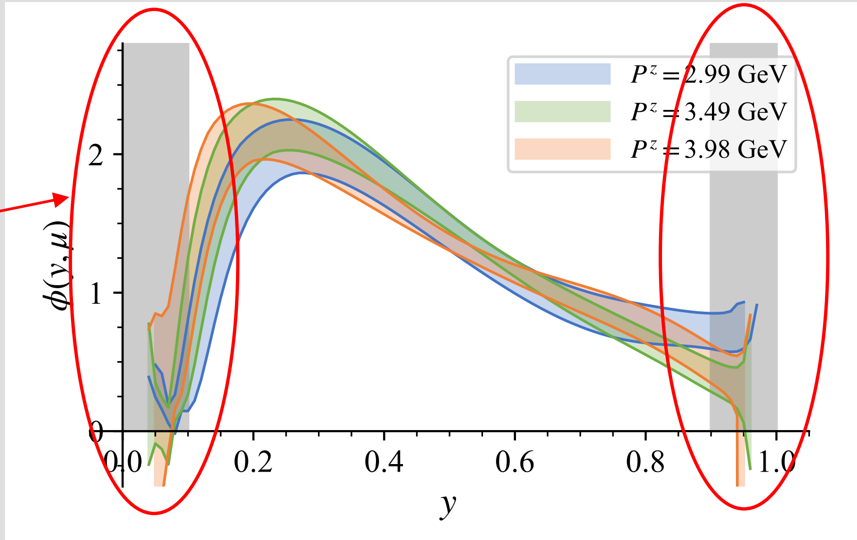

- Power correction $\Lambda_{\rm QCD}^2/(xP^z)^2$:

  Significant at end-point region

  Can be improved by considering the LRR, …

  [Su, Holligan, Ji, Yao, Zhang, Zhang, 2023]

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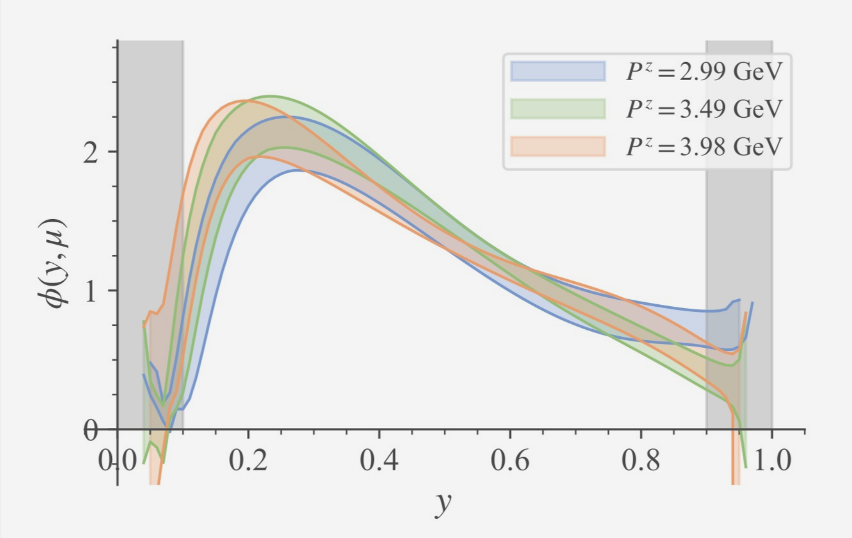
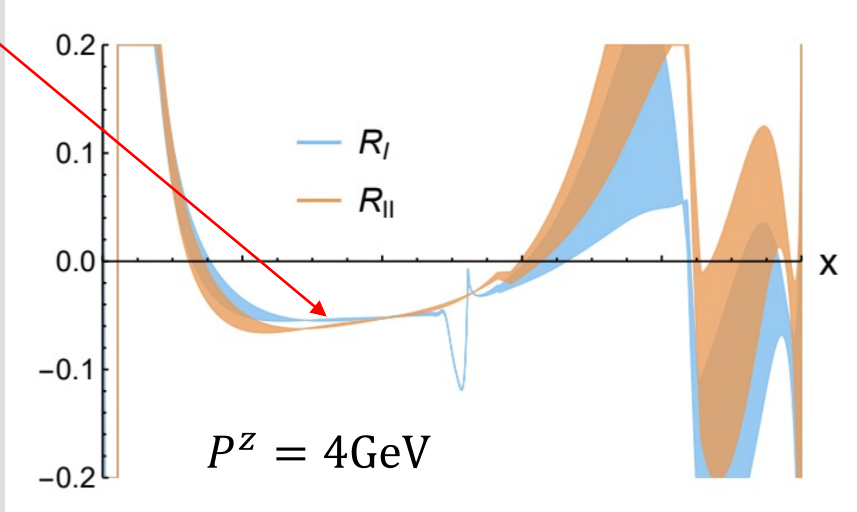
- Mass correction $m_H^2/(P^z)^2$:

  Smaller than 20% in most region, and smaller than 10% in the region $y \in [0.1, 0.5]$

  [Han, Wang, Zhang, Zhang, 2024]





Matching I: from quasi DAs to LCDAs in QCD

- The regions of QCD LCDA $\phi(y, \mu; m_H)$:
  - The shape of curves dominated by m_H and μ.

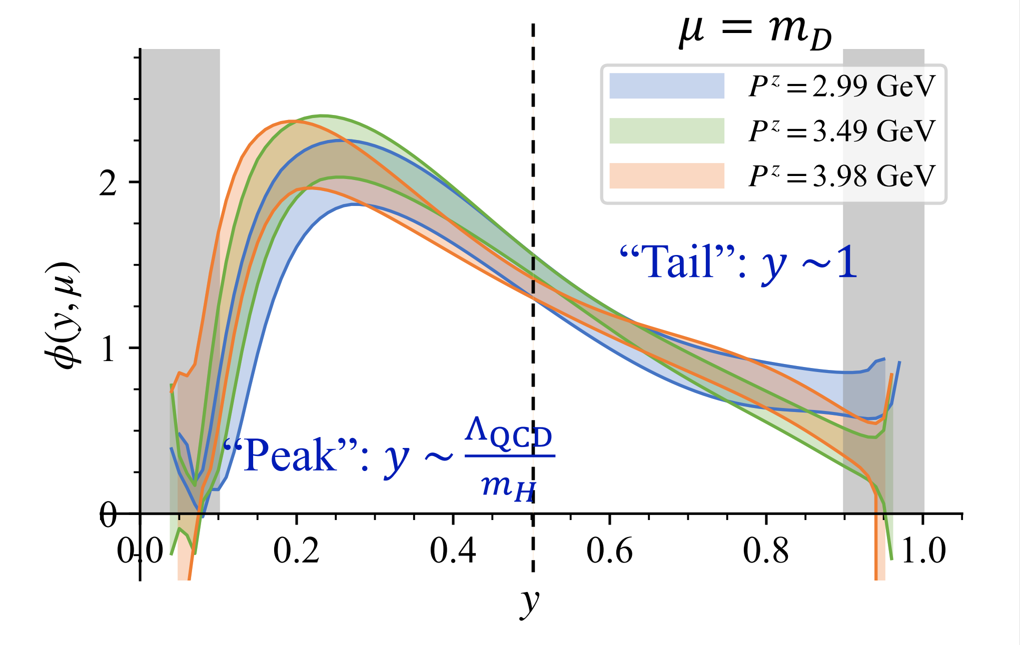
    At very large scale $\mu \gg m_H$, asymptotic form.
  - For the scale $\mu \lesssim m_Q$:

    Light quark carries small momentum fraction $y \sim \Lambda_{\text{QCD}}/m_H$

    $\Rightarrow$ peak region, related to the HQET LCDA;

    [Ishaq, Jia, Xiong, Yang, 2020; Beneke, Finauri, Vos, Wei, 2023]

    $y \sim O(1) \Rightarrow$ tail region, contain only hard-collinear physics, suppressed in LCDA.

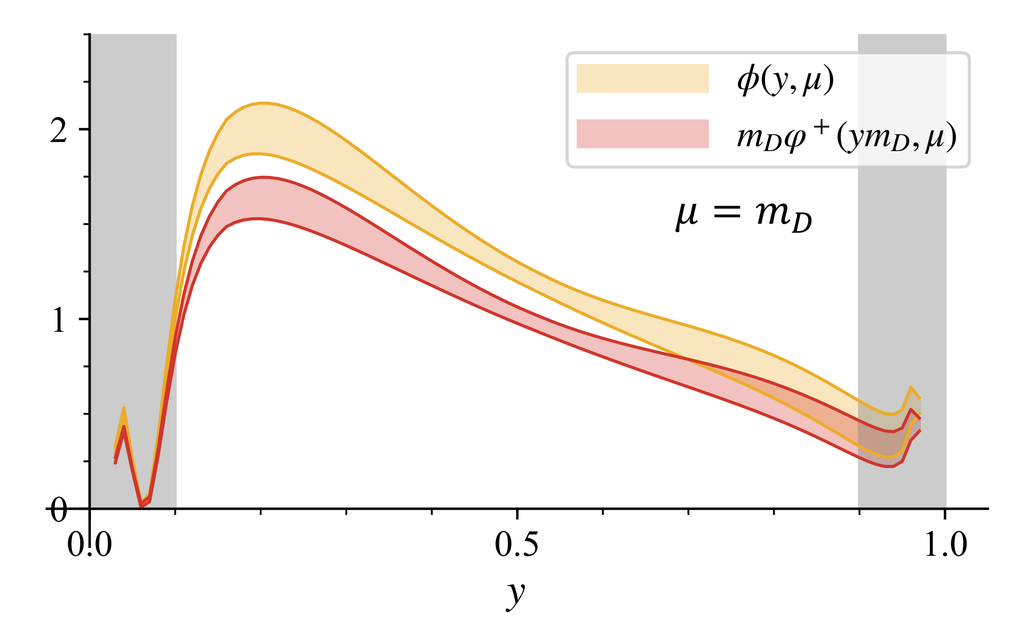


Matching II: connecting LCDAs in QCD and HQET

➢ In the peak region, HQET LCDA φ^+ connected with QCD LCDA ϕ through a multiplicative factorization:

[Beneke, Finauri, Vos, Wei, 2023]

$$\varphi^+_{\text{peak}}(\omega,\mu)=\frac{f_H}{\widetilde{f}_H}\frac{1}{\mathcal{J}_{\text{peak}}}\phi(y,\mu;m_H)+\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_H}\right)$$



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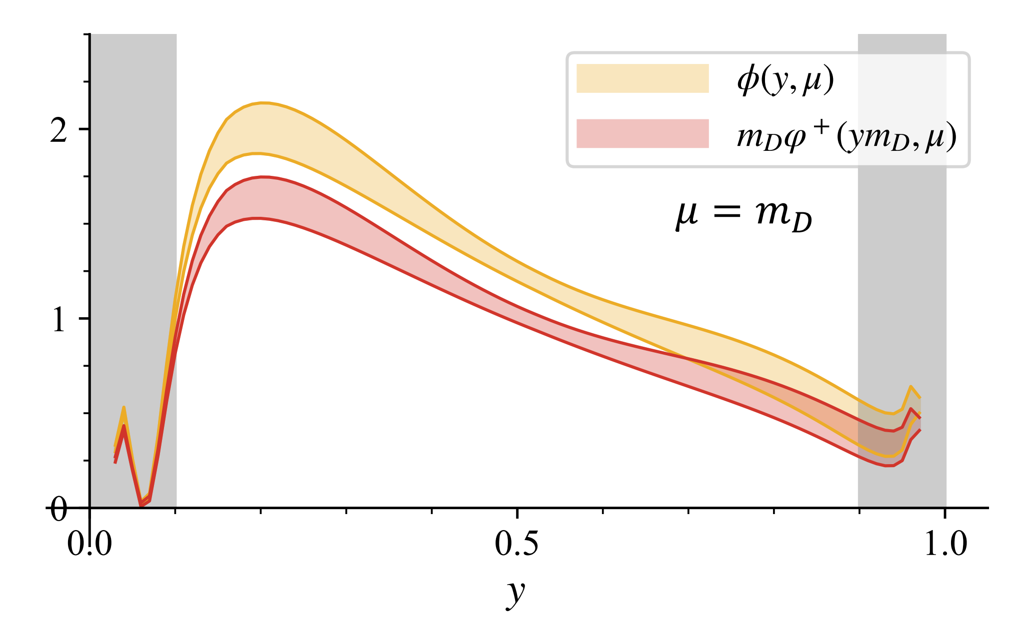
- HQET LCDA is independent on heavy quark mass, $m_Q = m_b$ or m_c are same at leading power.
- For simulating the D meson, power correction $\Lambda_{\rm QCD}/m_H$ still large:

A possible solution proposed in [Deng, Wang, Wei, Zeng, 2024]

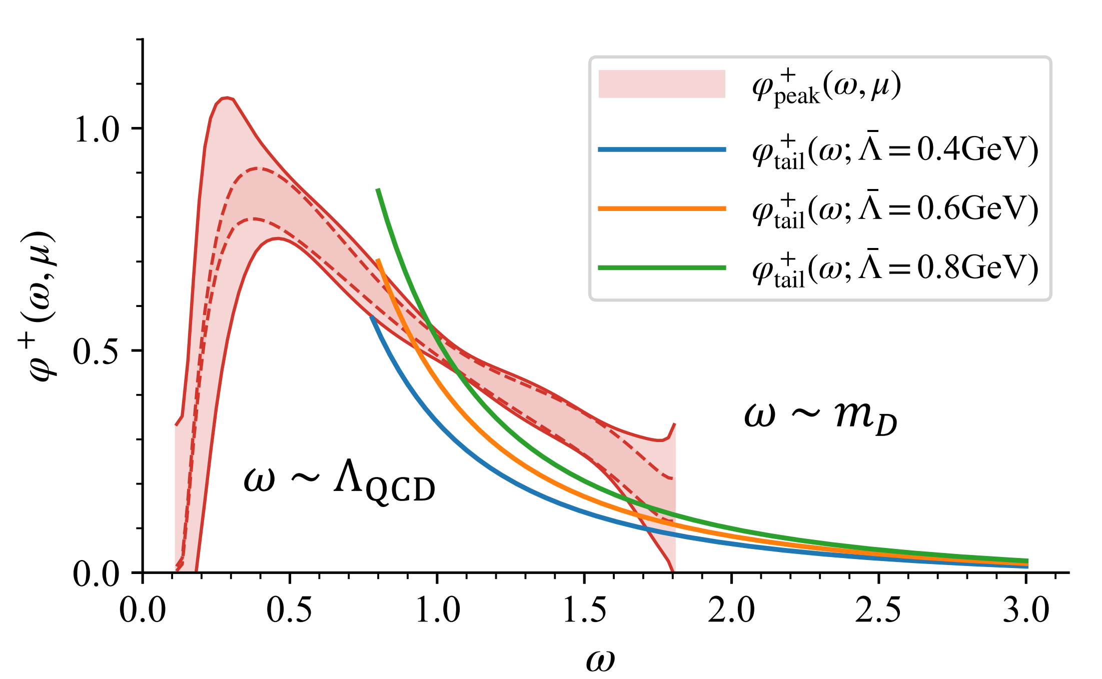


Matching II: connecting LCDAs in QCD and HQET

➢ The tail region of HQET LCDA is perturbative: [Lee, Neubert, 2005]

$$\varphi^{+}_{\rm tail}(\omega,\mu) = \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right]$$

where $\bar{\Lambda} \equiv m_H - m_Q^{\rm pole}$ reflect the power correction, and usually be chosen as hundreds of MeV.



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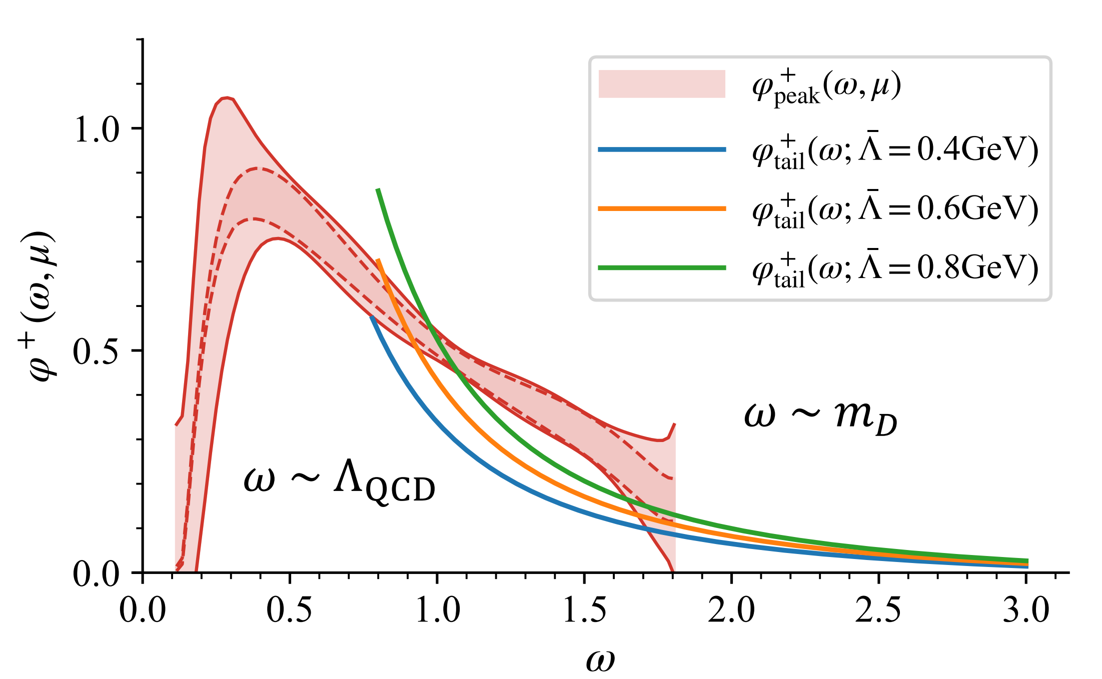


- Merging the peak and tail region:

$$\varphi^+(\omega,\mu) = \varphi^+_{\rm peak}(\omega,\mu)\theta(\omega_b - \omega) + \varphi^+_{\rm tail}(\omega,\mu)\theta(\omega - \omega_b)$$

We joint the peak and tail region, and use the Savitzky-Golay filter to smooth the data within a vicinity of $\delta = 0.05$GeV around the intersection position ω_b.

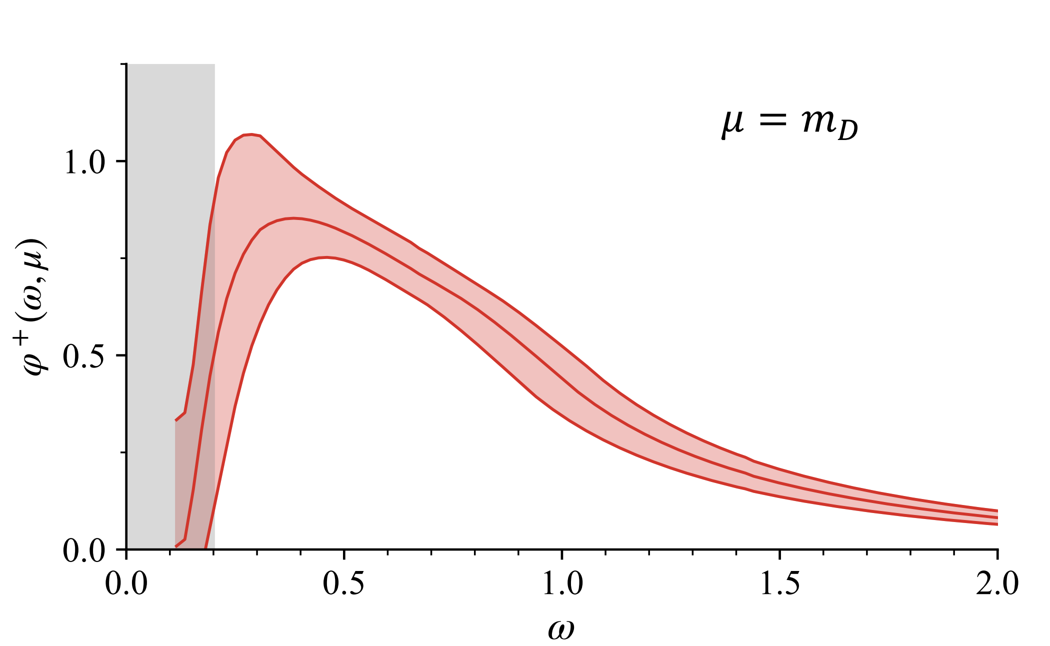
Final result of leading twist HQET LCDA

- Finally, we obtain the final result of HQET LCDA.
  - Just a verification of the two-step factorization method, the numerical result is still preliminary.
  - Considered the systematic errors in lattice analysis:

    From extrapolation, scale uncertainty in matching, large momentum limit, ……
  - Some key systematic errors are still absent:

    Only one lattice spacing,

    Power corrections within two matchings are still significant, ……



Although the current result is preliminary, it still warrants some phenomenological discussions...

Phenomenological Discussions I: comparison with models

- Models for heavy meson LCDAs

$$\varphi_{\mathrm{I}}^{+}(\omega,\mu_0)=\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0},$$

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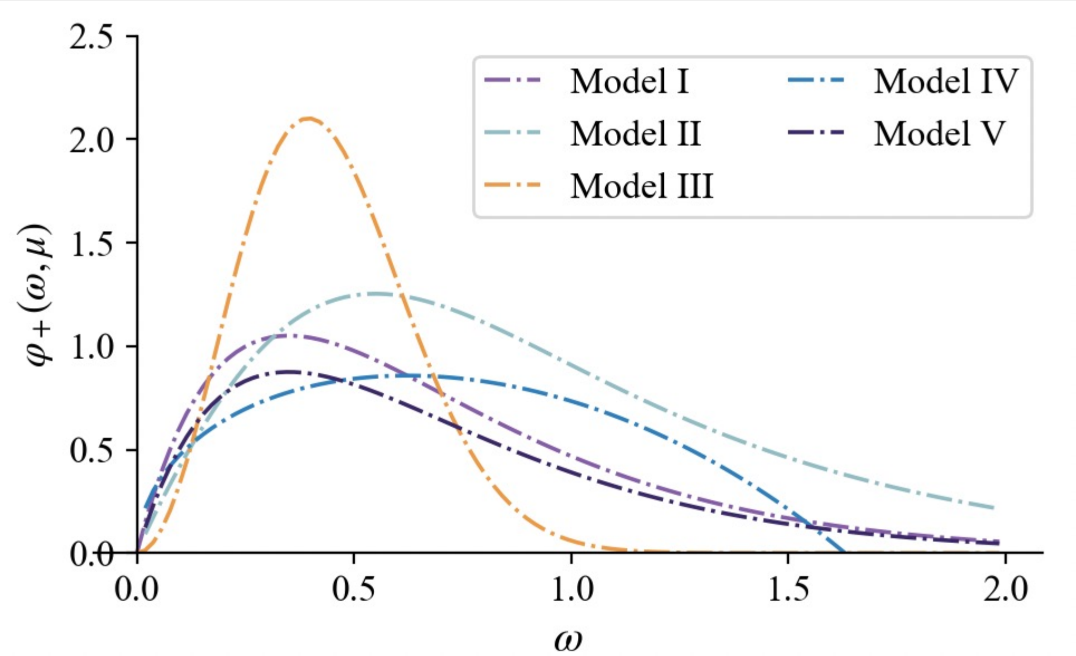


This leads to the largest systematic error:

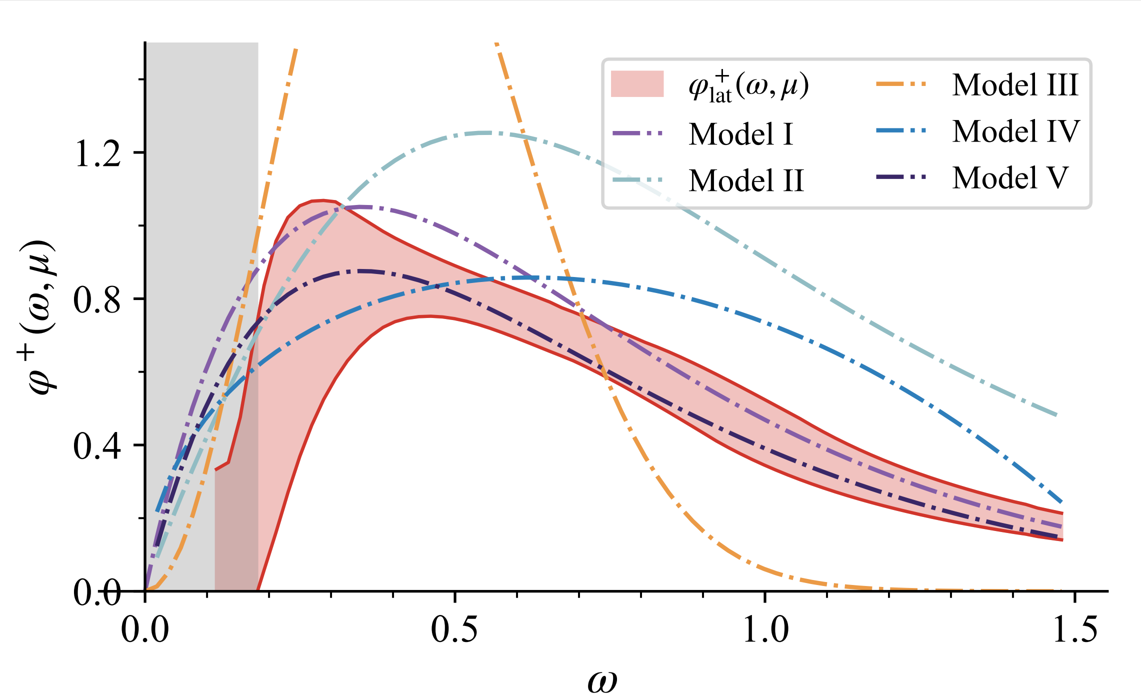
[Gao, Lu, Shen, Wang, Wei, 2020]

$$\mathcal{V}_{B\to K^*}(0)=0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

Pheno discussions I: Comparison with models

- Our results are consistent with the model estimates. Especially agree with model V, which constrained by the RG evolution of HQET LCDA.

- Result from first-principles of QCD will help to remove the primary uncertainties arising from the model parametrizations.



Pheno discussions II: Inverse and inverse-logarithmic moments

- Significant uncertainties from λ_B and σ_1: [Gao, Lu, Shen, Wang, Wei, 2020]

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- Definition of Inverse and inverse-logarithmic moments:

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega}\varphi^+(\omega,\mu),$$

$$\sigma_B^{(n)}(\mu) = \lambda_B(\mu)\int_0^\infty \frac{d\omega}{\omega}\ln\left(\frac{\mu}{\omega}\right)^{(n)}\varphi^+(\omega,\mu).$$

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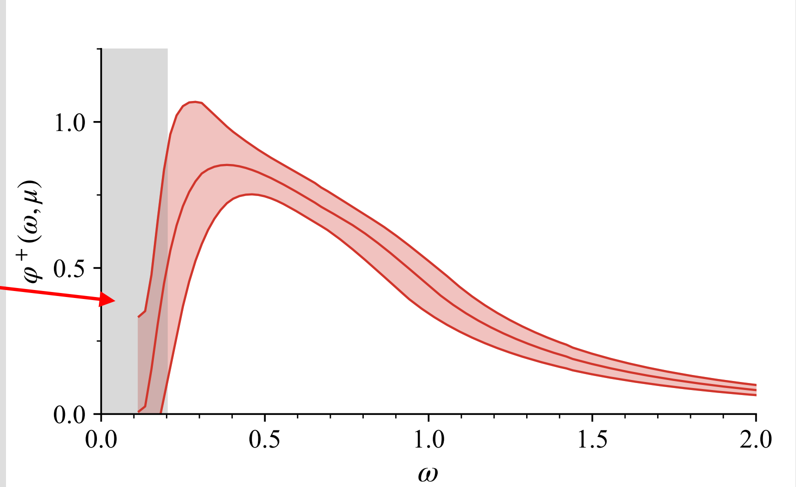
The power corrections at small ω makes the integral non-computable.



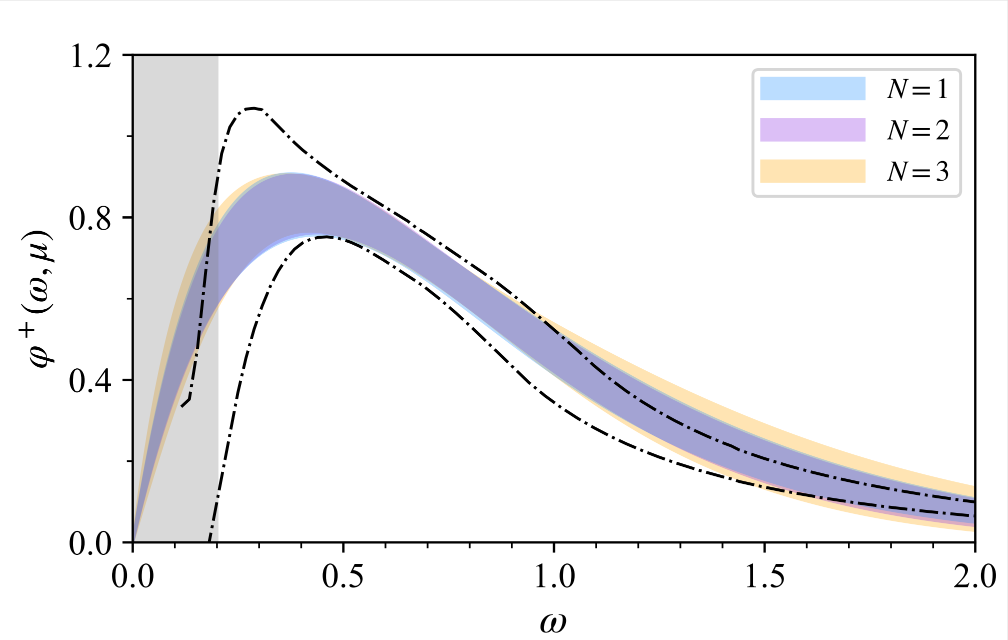
Pheno discussions II: Inverse and inverse-logarithmic moments

- A model-independent parametrization form:

$$\varphi^{+}(\omega,\mu)=\sum_{n=1}^{N} c_n \frac{\omega^n}{\omega_0^{n+1}} e^{-\omega/\omega_0}$$
$$=\frac{c_1\omega}{\omega_0^2}\left[1+c_2'\frac{\omega}{\omega_0}+c_3'\left(\frac{\omega}{\omega_0}\right)^2+\cdots\right]e^{-\omega/\omega_0},$$

Fit results of the N-th order:

$$N=1:\ \omega_0=0.403(44),\ c_1=0.932(73);$$
$$N=2:\ \omega_0=0.352(82),\ c_1=0.69(37),\ c_2'=0.17(32);$$
$$N=3:\ \omega_0=0.32(15),\ c_1=0.63(44),\ c_2'=0.12(37), c_3'=0.04(19).$$



Pheno discussions II: Inverse and inverse-logarithmic moments

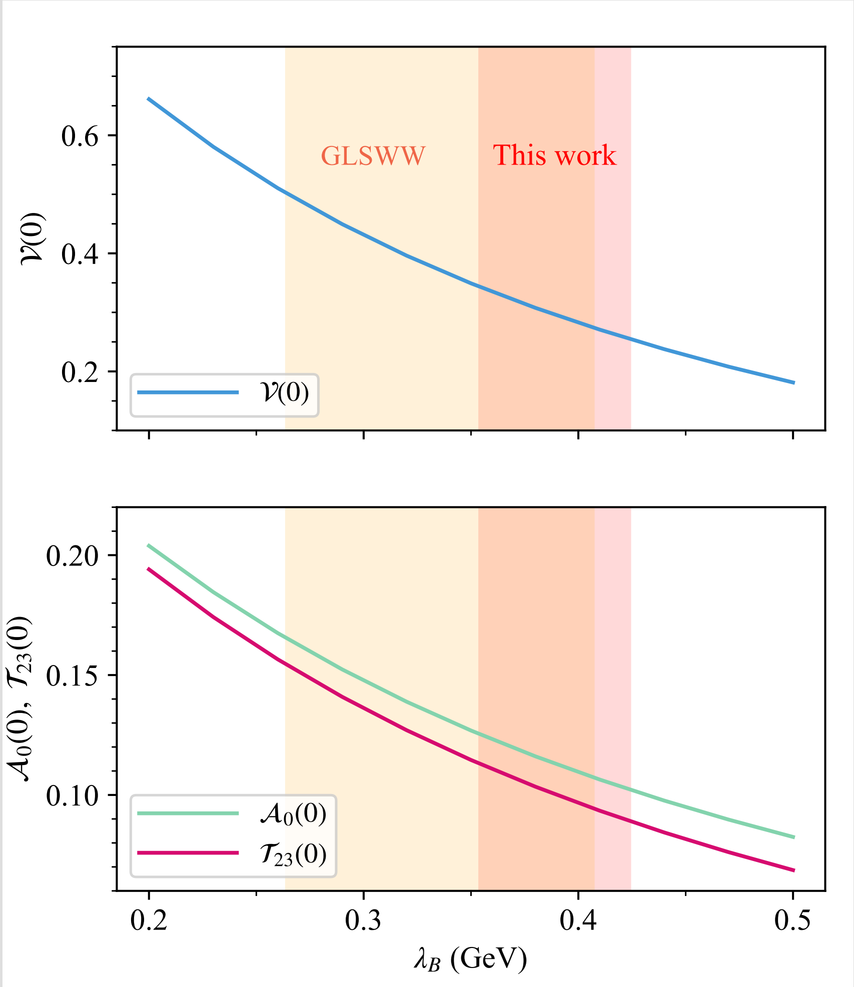
➢ Numerical results of λ_B and $\sigma_1^{(1)}$ at $\mu = 1\text{GeV}$:

		λ_B (GeV)	$\sigma_B^{(1)}$
Our results	*N=1*	0.389(35)	1.63(8)
	N=2	0.393(37)	1.62(7)
	N=3	0.381(59)	1.63(12)
Experiment	*Belle 2018*	> 0.24	
Other theoretical approach	*Khodjamirian, Mandal, Mannel, 2020*	0.383(153)	
	Gao, Lu, Shen, Wang, Wei, 2020	$0.343^{+0.064}_{-0.079}$	
	Lee, Neubert, 2005	0.48(11)	1.6(2)
	Braun, Ivanov, Korchemsky, 2004	0.46(11)	1.4(4)
	Grozin, Neubert, 1997	0.35(15)	
	Mandal, Nandi, Ray, 2024	0.338(68)	

Pheno discussions III: Impact on $B \to V$ form factors

- An accurate λ_B will significantly improve the prediction for the $B \to K^*$ form factors: [Gao, Lu, Shen, Wang, Wei, 2020]

	GLSWW		Our result
λ_B:	0.343^{+64}_{-79}	$\to$	0.389(35)
Error of $\mathcal{V}(0)$:	0.23	$\to$	0.11



We greatly thank Yuming Wang's code for this form factor calculation.

Pheno discussions III: Impact on $B \to V$ form factors

- An accurate λ_B will significantly improve the prediction for the $B \to K^*$ form factors: [Gao, Lu, Shen, Wang, Wei, 2020]

	GLSWW		Our result
λ_B:	0.343^{+64}_{-79}	→	0.389(35)
Error of $\mathcal{V}(0)$:	0.23	→	0.11

- We are looking forward to a more precise analysis of the form factors and accordingly physical observables.

$$\mathcal{V}_{B\to K^*}(0) = 0.359\,{}^{+0.141}_{-0.085}\Big|_{\lambda_B}\;{}^{+0.019}_{-0.019}\Big|_{\sigma_1}\;{}^{+0.001}_{-0.062}\Big|_{\mu}\;{}^{+0.010}_{-0.004}\Big|_{M^2}\;{}^{+0.016}_{-0.017}\Big|_{s_0}\;{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

- REDUCE the errors from λ_B and $\sigma_B^{(n)}$;
- REMOVE the errors from model dependence.

Summary and outlook

- ✓ We present a first lattice-implementable method to extract the heavy meson LCDA, and implement it on a CLQCD ensemble.
- ✓ Although the results are preliminary, they can be continually improved.
- ✓ The phenomenological implications demonstrate that our results will significantly advance the theoretical studies towards the frontier of high precision.

More importantly, improving the reliability of our results for the next stage:

- How to properly control the power corrections within two step factorization?
- More systematic lattice QCD calculations: more a, larger P^z, …

Thanks for your attention!