Next-to-Leading-Order Weak Annihilation Correction to Rare $B \to \{K, \pi\}\, \ell^+ \ell^-$

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第二十一届全国重味物理与CP破坏研讨会

2024.10.27　　衡阳

Outline

- Necessity of study on the weak annihilation contribution in $B \to \{K, \pi\} \ell^+ \ell^-$ decays
- QCD factorization of the weak annihilation amplitudes of $B \to \{K, \pi\} \ell^+ \ell^-$ decays
- The phenomenological impact from weak annihilation contribution
- The power suppressed long-distance quark loops

Based on： Y. K. Huang(黄勇康), YLS, C. Wang(王超) and Y. M. Wang(王玉明), arXiv: 2403.11258

$b \to (d,s)\ell^+\ell^-$: prominent tools in searching for new physics

- Flavor-Changing Neutral Current processes: standard model contribution is loop suppressed
- Rich angular distribution observable in $B \to V\ell^+\ell^-$ decays

$$\frac{1}{d(\Gamma+\bar{\Gamma})/dq^2}\frac{d^4(\Gamma+\bar{\Gamma})}{dq^2\,d\vec{\Omega}}\Big|_{\mathrm{P}} = \frac{9}{32\pi}\Big[\frac{3}{4}\bar{F}_L\sin^2\theta_K + F_L\cos^2\theta_K + \frac{1}{4}\bar{F}_L\sin^2\theta_K\cos 2\theta_\ell$$
$$-F_L\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi$$
$$+S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + S_5\sin 2\theta_K\sin\theta_\ell\cos\phi$$
$$+\tfrac{4}{3}A_{FB}\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi$$
$$+S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big],$$

The "clean " observables

$$P_i'(i=4,5,6,8) = \frac{S_i(i=4,5,7,8)}{\sqrt{F_L(1-F_L)}}$$

Anomalies related to $b \to (d,s)\ell^+\ell^-$

- Branching ratio of $B^+ \to K^+\ell^+\ell^-$ [B. Capdevila, etc., arXiv: 2309.01311]

$$\mathcal{B}^{[1.1,2.0],\text{SM}}_{B^+\to K^+\mu^+\mu^-} = (0.33 \pm 0.03)\times 10^{-7},$$
$$\mathcal{B}^{[4.0,5.0],\text{SM}}_{B^+\to K^+\mu^+\mu^-} = (0.37 \pm 0.03)\times 10^{-7},$$
$$\mathcal{B}^{[5.0,6.0],\text{SM}}_{B^+\to K^+\mu^+\mu^-} = (0.37 \pm 0.03)\times 10^{-7},$$
$$\mathcal{B}^{[1.1,2.0],\text{LHCb}}_{B^+\to K^+\mu^+\mu^-} = (0.21 \pm 0.02)\times 10^{-7}\ (4.0\sigma),$$
$$\mathcal{B}^{[4.0,5.0],\text{LHCb}}_{B^+\to K^+\mu^+\mu^-} = (0.22 \pm 0.02)\times 10^{-7}\ (4.4\sigma),$$
$$\mathcal{B}^{[5.0,6.0],\text{LHCb}}_{B^+\to K^+\mu^+\mu^-} = (0.23 \pm 0.02)\times 10^{-7}\ (4.0\sigma).$$

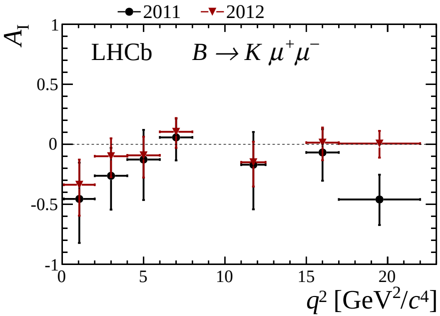
- the angular observable P_5'[B. Capdevila, etc., arXiv: 2309.01311]

$$P_5'{}^{[4.0,6.0]}_{\text{SM}} = -0.72 \pm 0.08; \qquad P_5'{}^{[6.0,8.0]}_{\text{SM}} = -0.81 \pm 0.08$$
$$P_5'{}^{[4.0,6.0]}_{\text{LHCb}} = -0.439 \pm 0.111 \pm 0.036\ \ (1.9\sigma), \qquad P_5'{}^{[6.0,8.0]}_{\text{LHCb}} = -0.583 \pm 0.090 \pm 0.030\ \ (1.9\sigma).$$

Anomalies related to $b \to (d, s)\ell^+\ell^-$

- Isospin asymmetry $A_I(B \to K\ell^+\ell^-)$

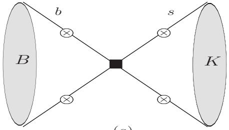
$$A_I = \frac{\mathcal{B}(B^0 \to K^{(*)0}\mu^+\mu^-) - (\tau_0/\tau_+)\cdot\mathcal{B}(B^+ \to K^{(*)+}\mu^+\mu^-)}{\mathcal{B}(B^0 \to K^{(*)0}\mu^+\mu^-) + (\tau_0/\tau_+)\cdot\mathcal{B}(B^+ \to K^{(*)+}\mu^+\mu^-)},$$



- Including 2012 data, $A_I(B \to K\ell^+\ell^-)$ deviates from zero at the level of 1.5σ (4σ from 2011 data) [LHCb, JHEP06,133(2024) arXiv:1403.8044]

The importance of annihilation contribution

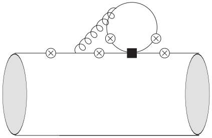
- Sensitive to isospin asymmetry and CP asymmetry in $B \to (K, \pi)\ell^+\ell^-$ decays

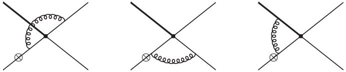


(a)

Charged B meson decays: (color allowed) tree operators contribute

- Necessary blocks for scale independence of the $\mathcal{A}(B \to (K, \pi)\ell^+\ell^-)$ at leading power
  - two-loop matrix elements for penguin operators$(q^2 \neq 0)$





(a) (b) (c)

  - one-loop weak annihilation contribution in the longitudinal amplitude

The decay amplitudes of $B \to (K, \pi)\ell^+\ell^-$ at large recoil

- the decay amplitude

$$\mathcal{A}(B \to P\ell^+\ell^-) = -\langle P(p')\ell^+\ell^- | \mathcal{H}_{\text{eff}} | B(p)\rangle$$

we will concentrate on the large recoil region($1\text{GeV}^2 < q^2 < 6\text{GeV}^2$)

- The effective weak Hamiltonian for the semileptonic $b \to D\ell^+\ell^-$ (with $D = d,\ s$)

$$\mathcal{H}_{\text{eff}} \;=\; -\frac{G_F}{\sqrt{2}}\left[V_{tb}V_{tD}^*\,\mathcal{H}_{\text{eff}}^{(t)} + V_{ub}V_{uD}^*\,\mathcal{H}_{\text{eff}}^{(u)}\right] + \text{h.c.}$$

where

$$\mathcal{H}_{\text{eff}}^{(t)} = C_1\,\mathcal{Q}_1^c + C_2\,\mathcal{Q}_2^c + \sum_{i=3-10} C_i\,\mathcal{Q}_i\,,\mathcal{H}_{\text{eff}}^{(u)} = C_1\,(\mathcal{Q}_1^c - \mathcal{Q}_1^u) + C_2\,(\mathcal{Q}_2^c - \mathcal{Q}_2^u)$$

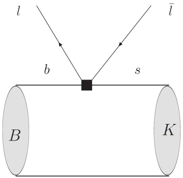
- the semileptonic operator

$$\mathcal{O}_{9,10} = \frac{\alpha_{\text{em}}}{2\pi}\,(\bar{\ell}\ell)_{V,A}\,(\bar{s}b)_{\text{V}-\text{A}},$$

The leading power decay amplitudes at large recoil

- Contribution from semileptonic operators $\mathcal{O}_{9,10}$

$$\mathcal{A}(B \to K\ell^+\ell^-)|_{\mathcal{O}_{9,10}}$$
$$= \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{\pi} V_{tb} V_{ts}^* \left[\bar{\ell}\gamma_\mu \ell \, p^\mu C_9 f_{BK}^+(q^2) + \bar{\ell}\gamma_\mu\gamma_5 \ell \, p^\mu C_{10} f_{BK}^+(q^2) \right],$$



- $B \to P$ form factors

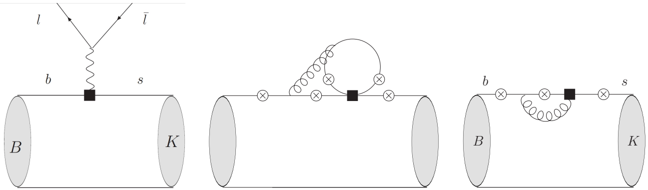
$$\langle P(p')|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}(p)\rangle = f_{BP}^+(q^2) \left[p^\mu + p'^\mu - \frac{m_B^2 - m_P^2}{q^2} q^\mu \right] + f_{BP}^+(q^2) \frac{m_B^2 - m_P^2}{q^2} q^\mu$$

The leading power decay amplitudes at large recoil

- Contribution from electro-magnetic penguin and four-quark operators : [Beneke etc. , NPB 612 (2001) 25-58 arXiv: hep-ph/0106067] $B \to (K,\pi)\gamma^*, \gamma^* \to \ell^+\ell^-$

$$\langle P(p')\,\gamma^*(q,\mu)|\mathcal{H}_{\rm eff}^{(t,\,u)}|\bar{B}(p)\rangle = -\frac{g_{\rm em}\,m_b}{4\,\pi^2}\,\frac{\mathcal{T}_P^{(t,\,u)}(q^2)}{m_B}\,\left[q^2\,(p_\mu + p'_\mu) - (m_B^2 - m_P^2)\,q_\mu\right]$$

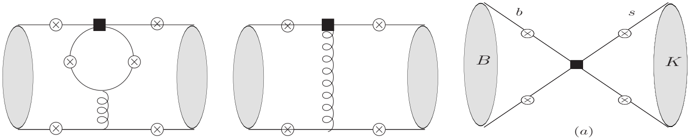
$$\begin{aligned}\mathcal{T}_P^{(t,\,u)} &= C_P^{(t,\,u)}(q^2)\,f_{BP}^+(q^2)\\ &-\frac{\pi^2}{N_c}\,\frac{\mathcal{F}_B\,f_P}{m_B}\,\sum_{m=\pm}\int_0^\infty \frac{d\omega}{\omega}\,\int_0^1 du\,T_{P,\,m}^{(t,\,u)}(\omega,u,\mu)\,\phi_{B,m}(\omega,\mu)\,\phi_P(u,\mu)\,,\end{aligned}$$



The leading power decay amplitudes at large recoil

- Hard spectator scattering and annihilation contribution

$$\mathcal{T}_P^{(t,\,u)} = C_P^{(t,\,u)}(q^2)\, f_{BP}^{+}(q^2) - \frac{\pi^2}{N_c}\,\frac{\mathcal{F}_B\, f_P}{m_B} \sum_{m=\pm} \int_0^\infty \frac{d\omega}{\omega} \int_0^1 du\, T_{P,\,m}^{(t,\,u)}(\omega, u, \mu)\, \phi_{B,m}(\omega,\mu)\, \phi_P(u,\mu)\,,$$



- weak annihilation diagrams contribute at leading power for $B \to P\ell^+\ell^-$ and the longitudinal polarization of $B \to V\ell^+\ell^-$

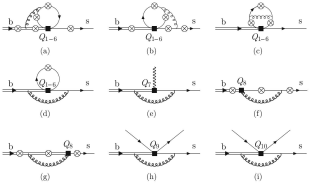
SCET analysis of $B \to (K,\pi)\ell^+\ell^-$

- Two step matching: QCD→SCET$_{\text{I}}$→SCET$_{\text{II}}$ [Ali etc. EPJC 47 (2006) 625-641 arXiv: hep-ph/0601034]

$$\mathcal{H}_{eff} \to -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left(\sum_{i=1}^{4} \int ds\, \widetilde{C}_i^A(s) J_i^A(s) + \sum_{j=1}^{4} \int ds \int dr\, \widetilde{C}_j^B(s,r) J_j^B(s,r) + \int ds \int dr \int dt\, \widetilde{C}^C(s,r,t) J^C(s,r,t) \right) + \text{Higher power operators}$$

- the A-type operators

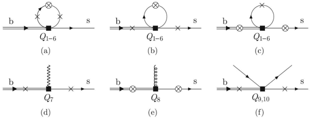
$$J_i^A = \bar{\mathfrak{X}}_{hc}(s\bar{n})(1+\gamma_5)\{\gamma_\perp^\mu, \frac{n^\mu}{n \cdot v}\} h(0)\, \bar{\ell}\gamma_\mu(\gamma_5)\ell \,,$$



SCET analysis of $B \to (K, \pi)\ell^+\ell^-$

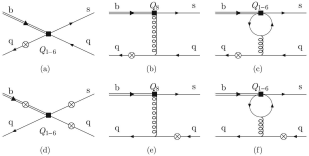
- the B-type operator

$$J_i^B = \bar{\mathfrak{X}}_{hc}(s\bar{n})(1+\gamma_5)\{\gamma_\perp^\mu, \frac{n^\mu}{n\cdot v}\}\not{\mathcal{A}}_{hc\perp}(r\bar{n})h(0)\ \bar{\ell}\gamma_\mu(\gamma_5)\ell\ ,$$



- the C-type operator(incomplete)

$$J^C = \bar{\mathfrak{X}}_{hc}(s\bar{n})(1+\gamma_5)\frac{\not{\bar{n}}}{2}\mathfrak{X}_{hc}(r\bar{n})\ \bar{\mathfrak{X}}_{\overline{hc}}(an)(1+\gamma_5)\frac{\not{n}}{2}h(0)\ ,$$



SCET analysis of $B \to (K,\pi)\ell^+\ell^-$

- Mathching from $\mathrm{SCET_I}$ to $\mathrm{SCET_{II}}$

$$\langle \int d^4x T\{J_i^B(s,t), \mathcal{L}^{(1)}_{\xi q_s}(x)\}\rangle_{\mathrm{FT}} = \mathbb{J}^B_{ij} \otimes \langle O^B_j(s,t)\rangle_{FT}$$

- B-type SCET_I operators are matched onto the following SCET_{II} operators

$$O_i^B = \bar{\mathfrak{X}}_c(s\bar{n})(1+\gamma_5)\{\gamma_\perp^\mu, \frac{n^\mu}{n\cdot v}\}\frac{\not{\bar{n}}}{2}\mathfrak{X}_c(0)\ \bar{\mathfrak{Q}}_s(tn)(1-\gamma_5)\frac{\not{n}}{2}\mathcal{H}_s(0)\ \bar{\ell}\gamma_\mu(\gamma_5)\ell\ ,$$

- Definition of LCDAs

$$\langle P(p')|\,(\bar{\chi}W_{\bar{c}})\,(t\bar{n})\,\frac{\not{\bar{n}}}{2}\gamma_5\left(W^\dagger_{\bar{c}}\chi\right)(0)|0\rangle = -i\,f_P\,\frac{\bar{n}\cdot p'}{2}\int_0^1 du\, e^{i\,(\bar{n}\cdot p')\,u\,t}\,\phi_P(u,\mu),$$

$$\langle 0|\,(\bar{q}_s Y_s)\,(t\bar{n})\ \not{\bar{n}}(\not{n})\,\gamma_5\,(Y_s^\dagger h_v)(0)|0\rangle = -\frac{if_B(\mu)m_B}{4}\int_0^\infty d\omega\, e^{i\omega t}\ \phi_B^{+(-)}(u,\mu),$$

The NLO annihilation contribution

- An additional operator $\mathcal{O}^{(\mathrm{B1})}$ also contributes at leading power

$$\mathcal{Q}_i = H_i^{\mathrm{I}} \star \mathcal{O}^{(\mathrm{A0})} + H_i^{\mathrm{II}} \star \mathcal{O}^{(\mathrm{B1})} \,,$$

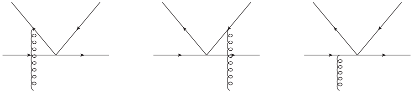
$$\begin{aligned}
\mathcal{O}^{(\mathrm{A0})} &= \left[(\bar{\chi} W_{\bar{c}})\,(t\bar{n})\, \frac{\not{\bar{n}}}{2}\,(1-\gamma_5)\,\left(W_{\bar{c}}^{\dagger}\chi\right)(0)\right] \left[\left(\bar{\xi} W_c\right)(0)\ \not{n}\,(1-\gamma_5)\,h_v(0)\right], \\
\mathcal{O}^{(\mathrm{B1})} &= \frac{1}{m_b}\left[(\bar{\chi} W_{\bar{c}})\,(t\bar{n})\, \frac{\not{\bar{n}}}{2}\,(1-\gamma_5)\,\left(W_{\bar{c}}^{\dagger}\chi\right)(0)\right] \left[\left(\bar{\xi} W_c\right)(0)\, \frac{\not{n}}{2}\left[W_c^{\dagger} i\, \not{D}_{\perp c} W_c\right](sn)\,(1+\gamma_5)\,h_v(0)\right],
\end{aligned}$$

- Hard functions $H_i^{\mathrm{I,II}}$ have been calculated up to two-loop order.
  - tree operator [Beneke etc. , NPB 832 (2010) 109-151 arXiv: 0911.3655]
  - penguin operator [Bell etc. , JHEP 04 (2020) 055 arXiv: 2002.03262]

The annihilation contribution in SCET

- Matching to $\mathcal{O}^{(\mathrm{B1})}$:

$$\langle u(l')\bar{u}(\bar{u}p)s(up)g(l)|T\{\int d^4x\mathcal{L}_{\mathrm{QCD}}(x), P_i\}|b(p_b)\rangle^{(0)}$$
$$= \mathbb{H}^{\mathrm{II}(0)}_{ia}\langle u(l')\bar{u}(\bar{u}p)s(up)g(l)|O^{\mathrm{II}}_a|b(p_b)\rangle^{(0)}_{\mathrm{FT}}$$

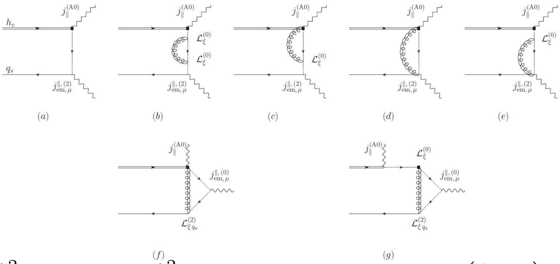


$$\mathbb{H}^{\mathrm{II}(0)}_1 = \frac{1}{N_c}\frac{1}{\bar{u}}, \qquad \mathbb{H}^{\mathrm{II}(0)}_2 = 2,$$
$$\tilde{\mathbb{H}}^{\mathrm{II}(0)}_{1,4} = \frac{1}{N_c}\left[2\,C_F - \frac{1}{N_c}\frac{1}{\bar{u}}\right], \qquad \tilde{\mathbb{H}}^{\mathrm{II}(0)}_{2,3} = \frac{2}{N_c}\left[1 + \frac{1}{\bar{u}}\right],$$
$$\tilde{\mathbb{H}}^{\mathrm{II}(0)}_5 = 16\,\tilde{\mathbb{H}}^{\mathrm{II}(0)}_2, \qquad \tilde{\mathbb{H}}^{\mathrm{II}(0)}_6 = 16\,\tilde{\mathbb{H}}^{\mathrm{II}(0)}_1,$$

calculation of jet function

- Match $\mathcal{O}^{(\mathrm{A0})}$ to $\mathrm{SCET_{II}}$

$$\mathcal{J}_P^{(\mathrm{A0})}(n\cdot q,\bar n\cdot q)\,\bar n_\mu = \int d^4x\, e^{iq\cdot x}\langle 0|\mathrm{T}\left\{j^{\|,(2)}_{\mathrm{em},\,\mu}(x),\left(\bar\xi W_c\right)(0)\gamma_5 h_v(0)\right\}|\bar B(p)\rangle + \int d^4y\langle 0|\mathrm{T}\left\{j^{\|,(0)}_{\mathrm{em},\,\mu}(x), i\mathcal{L}^{(2)}_{\xi q_s}(y)\left(\bar\xi W_c\right)(0)\gamma_5 h_v(0)\right\}|\bar B(p)\rangle,$$

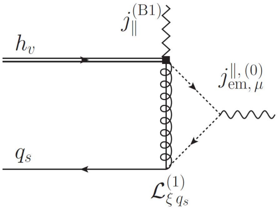


$$\mathbb{J}^{(1)}_{\|,-} = 1 + C_F\left[\ln^2\frac{\hat\mu^2}{\omega-\bar n\cdot q} - 2\ln\frac{\hat\mu^2}{\omega-\bar n\cdot q}\ln(1+\eta) - \ln^2(1+\eta) + \left(\frac{2}{\eta}-1\right)\ln(1+\eta) - \frac{\pi^2}{6} - 1\right],$$

Calculation of jet function

- jet function from B-type operators

$$\frac{n\cdot q}{2\,\pi}\int d^4x\, e^{iq\cdot x}\int ds\, e^{i(n\cdot q)\tau s}$$
$$\int d^4y\langle 0|\mathrm{T}\left\{j^{\parallel,(0)}_{\mathrm{em},\,\mu}(x), i\mathcal{L}^{(1)}_{\xi q_s}(y), \left(\bar{\xi}W_c\right)(0)\gamma_5\left[W_c^\dagger i\,\not{D}_{\perp c}W_c\right](sn)\,h_v(0)\right\}|\bar{B}(p)\rangle$$
$$=\mathcal{J}_P^{(\mathrm{B1})}(n\cdot q,\bar{n}\cdot q,\tau)\,\bar{n}_\mu.$$



$$\mathbb{J}^{(1)}_{\parallel,+}=2\,C_F\,\frac{n\cdot q}{\omega}\,\ln(1+\eta)\,(1-\tau)\,\theta(\tau)\,\theta(1-\tau),$$

The analytic result

- the explicit expressions of the NLO weak annihilation corrections to the short-distance matching functions $T_{P,\,m}^{(t,\,u)}$

$$\left\{T_{P,+}^{(u)},\ T_{P,+}^{(t)}\right\} \supset -Q_q N_c \frac{2m_B}{m_b} \int_0^1 d\tau \mathbb{J}_{\|,+}(n\cdot q, \bar{n}\cdot q, \omega, \tau) \left\{ -\sum_{i=1}^{2} C_i \left[\mathbb{H}_i^{\rm II}(u,\tau)\,\delta_{qu} - \tilde{\mathbb{H}}_i^{\rm II}(u,\tau)\,\delta_{qd}\,\delta_{P\pi}\right], \sum_{i=3}^{6} C_i\,\tilde{\mathbb{H}}_i^{\rm II}(u,\tau)\right\},$$

$$\left\{T_{P,-}^{(u)},\ T_{P,-}^{(t)}\right\} \supset Q_q N_c \frac{4m_B}{m_b} \frac{\mathbb{J}_{\|,-}(n\cdot q, \bar{n}\cdot q, \omega)}{\hat{q}^2 - 1 + i\,0} \left\{ -\sum_{i=1}^{2} C_i \left[\mathbb{H}_i^{\rm I}(u)\,\delta_{qu} - \tilde{\mathbb{H}}_i^{\rm I}(u)\,\delta_{qd}\,\delta_{P\pi}\right], \sum_{i=3}^{6} C_i\,\tilde{\mathbb{H}}_i^{\rm I}(u)\right\},$$

the effective Wilson coefficients

$$\mathcal{C}_{9,P}^{(i)}(q^2) = C_9\,\delta^{it} + \frac{2m_b}{m_B} \frac{\mathcal{T}_P^{(i)}(q^2)}{f_{BP}^{+}(q^2)}\,.$$

Convergence of the integral

- the asymptotic behavior of B meson LCDA

$$\phi_B^+(\omega) \to \omega, \quad \phi_B^-(\omega) \to 1, \qquad \omega \to 0$$

$$\phi_B^+(\omega) \to \frac{1}{\omega} \ln \frac{\mu}{\omega}, \quad \phi_B^-(\omega) \to \frac{1}{\omega} \ln \frac{\mu}{\omega} \qquad \omega \to \infty$$

- the asymptotic behavior of the jet function

$$\mathbb{J}_+(\omega) \to 1, \quad \mathbb{J}_-(\omega) \to 1, \qquad \omega \to 0$$

$$\mathbb{J}_+(\omega) \to \frac{1}{\omega} \ln \frac{\omega}{\bar{n} \cdot q}, \quad \mathbb{J}_-(\omega) \to \ln^2 \frac{\omega}{\bar{n} \cdot q}, \qquad \omega \to 0$$

- convergence of the convolution

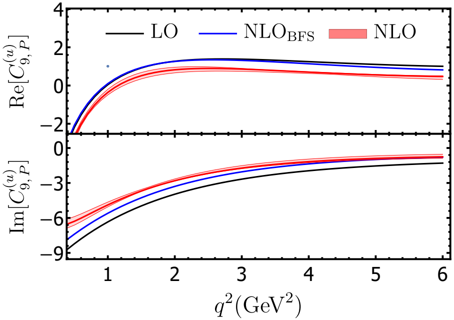
$$\int d\omega \mathbb{J}_+(\omega) \phi_B^+(\omega)$$

and

$$\int \frac{d\omega}{\bar{n} \cdot q - \omega} \mathbb{J}_-(\omega) \phi_B^-(\omega)$$

Phenomenology

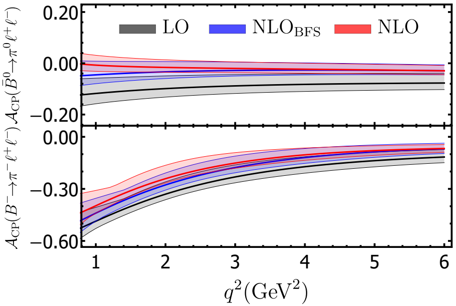
- impact on $\mathcal{C}_9^{(u)}$ in $b \to \pi \ell^+ \ell^-$: NLO WA contribution brings about 35% reduction for real part, and 15% reduction for imaginary part(comparable to other NLO contribution)



- only 3% corrections to $\mathcal{T}_P^{(t)}$

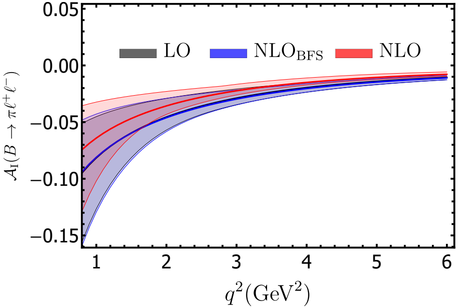
Phenomenology

- including the NLO weak annihilation correction in the improved theory predictions can lead to the negligible impacts on $\mathcal{BR}(B \to K\ell^+\ell^-)$ and $\mathcal{BR}(B \to \pi\ell^+\ell^-)$.
- $\mathcal{A}_{\rm CP} = \{1.02^{+0.05}_{-0.22},\, 0.64^{+0.06}_{-0.24},\, 0.42^{+0.10}_{-0.17},\, 0.31^{+0.10}_{-0.13}, 0.26^{+0.09}_{-0.10}\}\ B^- \to K^-\ell^+\ell^-$
  $\mathcal{A}_{\rm CP} = \{0.01^{+0.08}_{-0.12},\, 0.04^{+0.07}_{-0.11},\, 0.06^{+0.07}_{-0.10},\, 0.08^{+0.07}_{-0.10}, 0.09^{+0.07}_{-0.09}\}\ \bar{B}^0 \to \bar{K}^0\ell^+\ell^-$,
  $[q^2_{\rm min}, q^2_{\rm max}] = \{[1.1, 2.0],\, [2.0, 3.0],\, [3.0, 4.0],\, [4.0, 5.0],\, [5.0, 6.0]\}\ {\rm GeV}^2$.
- CP asymmetry in $B \to \pi\ell^+\ell^-$



Phenomenology

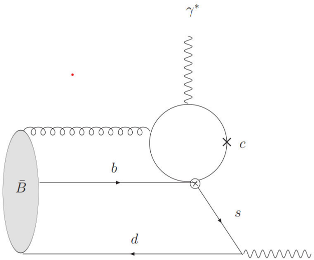
- the isospin asymmetry of $B \to K\ell^+\ell^-$ in the kinematic ranges $q^2 \in [2.0, 4.0]\,\mathrm{GeV}^2$ and $q^2 \in [4.0, 6.0]\,\mathrm{GeV}^2$ are given by $(-0.74)^{+0.21}_{-0.12}\,\%$ and $(-0.44)^{+0.08}_{-0.06}\,\%$, respectively,
- isospin asymmetry in $B \to \pi\ell^+\ell^-$



about 25% corrections to $\mathcal{A}_\mathrm{I}(B \to \pi\ell^+\ell^-)$

Power suppressed quark-loop contribution

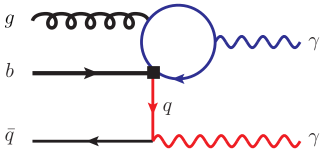
- the power supprssed contribution： long distance quark-loop



- Firstly calculated by A. Khodjamirian etc.using light-cone sum rules [JHEP09(2010)089 arXiv:1006.4945], the predicted contribution is about 5% for $B \to K\ell^+\ell^-$ and 20% for $B \to K^*\ell^+\ell^-$
- A more recent study by N. Gubernari etc. [JHEP02(2021)088 arXiv:2011.09813]obtains a much smaller result by employing the full set of 3-particle B meson LCDAs.

Power suppressed quark-loop contribution

- long distance quark-loop in $B_q \to \gamma\gamma$ [Qin etc. PRL131(2023)9, 091902 arXiv:2207.02691]



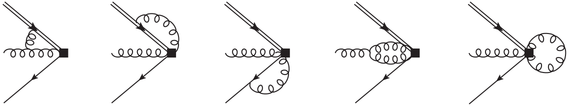
- power counting of the loop momentum: hard-collinear rather than hard $m_c^2 \sim m_b \Lambda$
- the soft light quank and soft gluon must be along opposite light-cone direction
- the soft function

$$\langle 0|(\bar{q}_s S_n)(\tau_1 n)\,(S_n^\dagger\, S_{\bar{n}})(0)\,(S_{\bar{n}}^\dagger\, g_s\, G_{\mu\nu}\, S_{\bar{n}})(\tau_2 \bar{n})\ \bar{n}^\nu\ \not{n}\gamma_\perp^\mu \gamma_5\,(S_{\bar{n}}^\dagger h_v)(0)|\bar{B}_v\rangle$$
$$= 2\,\tilde{f}_B(\mu)\, m_B \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \exp\left[-i(\omega_1\tau_1 + \omega_2\tau_2)\right]\ \Phi_G(\omega_1,\omega_2,\mu)\,,$$

Power suppressed quark-loop contribution

- Renormalizaiton of the soft function[Huang etc. PRL133(2024)17, 171901 arXiv:2312.15439]

$$\mathcal{O}_G^{\rm ren}(\omega_1,\omega_2,\mu) = \int_{-\infty}^{+\infty} d\omega_1' \int_{-\infty}^{+\infty} d\omega_2' Z_G(\omega_1,\omega_2,\omega_1',\omega_2',\mu)\mathcal{O}_G^{\rm bare}(\omega_1',\omega_2'),$$



- the renormalization constant

$$Z_G^{(e)} \supset \frac{\alpha_s}{4\pi}\frac{C_A}{\epsilon}\left\{\left[\frac{1}{\epsilon}+\ln\frac{\mu^2}{\omega_1\omega_2-i0}+i\,\pi\,\theta(\omega_1\omega_2)\right]\delta(\omega_1-\omega_1')\,\delta(\omega_2-\omega_2') + \left(\frac{i}{2\,\pi}\right)\left[H_+(\omega_1,\omega_1')-H_-(\omega_1,\omega_1')-2i\pi\delta(\omega_1-\omega_1')\,\theta(\omega_2'-\omega_2)\right] \left[H_+(\omega_2,\omega_2')-H_-(\omega_2,\omega_2')-2i\pi\delta(\omega_2-\omega_2')\,\theta(\omega_1'-\omega_1)\right]\right\},$$

- the support region of $\Phi_G(\omega_1,\omega_2,\mu)$ must be extended to the entire real axes $-\infty<\omega_{1,2}<+\infty$.

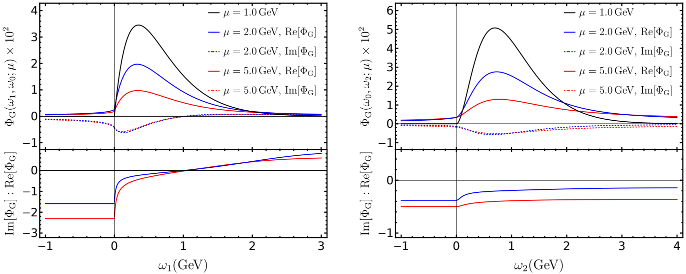
$$H_\pm(\omega_i,\omega_i') = \theta(\pm\omega_i)F^{>(<)}(\omega_i,\omega_i')+\theta(\mp\omega_i)G^{<(>)}(\omega_i,\omega_i')$$

Power suppressed quark-loop contribution

- the aysmtotic behavior

$$\Phi_{\rm G}^{\rm exp}(\omega_{1,2} \to 0, \mu) = \frac{\lambda_E^2 + \lambda_H^2}{6} \frac{1}{\omega_0^2} \exp\left[V + 2\,\gamma_E\,(a_1 + a_2)\right] \left(\frac{\mu_0}{\omega_0}\right)^{a_1+a_2} \frac{1 - e^{-i\pi a_2}}{4\pi^2} \Gamma(1+a_1)\Gamma(1+a_2)$$

$$\Phi_{\rm G}^{\rm exp}(\omega_{1,2} \to \pm\infty, \mu) \propto \frac{\lambda_E^2 + \lambda_H^2}{6} \frac{1}{\omega_0^2} \exp\left[V + 2\,\gamma_E\,(a_1 + a_2)\right] \left(\frac{\mu_0}{\omega_0}\right)^{a_1+a_2} \left(\frac{\omega_0}{\pm\omega_1}\right)^{1+a_1} \left(\frac{\omega_0}{\pm\omega_2}\right)^{1+a_2},$$



Summary

- The annihilation contribution is necessary for the scale independence of the $B \to (\pi, K)\ell^+\ell^-$ decay amplitudes
- The annihilation contribution is also has significant impact on the CP asymmetry and isospin asymmetry in the $B \to \pi\ell^+\ell^-$.
- The power suppressed contribution is also required for the precise study on the heavy meson decays.