QED corrections to $B_u \to \tau\nu$ at Subleading Power

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27 Oct. 2024 HFCPV@ Nanhua University, Hengyang

Based on:

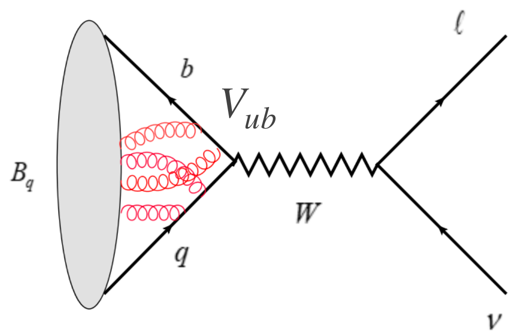
QED corrections to $B \to \tau\,\nu$ at Subleading Power

in preparation with **Y.L. Shen (沈月龙) and C. Wang（王超）**

Why $B_u \to \ell\nu$?

be interesting for several reasons:

- **Determination of $|V_{ub}|$** largely unaffected by hadronic uncertainties



$$\Gamma \sim m_\ell^2 \, f_{B_u}^2 \, |V_{ub}|^2$$

- Helicity suppression offers sensitive **probe of (pseudo)scalar new interactions**

$$\bar{\ell}\,\gamma^\mu P_L \nu \to \frac{m_\ell}{m_b} \bar{\ell}\,P_L \nu$$

- Testing Lepton Flavor Universality in charged currents

$$B_u \to \ell\nu, \quad \ell = \mu, \tau$$

Why do we need to know the QED corrections?

- QCD matrix element is known with **<1% accuracy**

$$\langle 0|\,\bar{u}\gamma^\mu\gamma_5 b\,|B^-(p)\rangle = i f_{B_u} p^\mu \quad \text{with} \quad f_{B_u} = (189.4 \pm 1.4)\,\text{MeV}$$

[FNAL/MILC 2017]

QED corrections can be of similar magnitude or even larger, due to presence of **large logarithms** $\alpha \ln(m_b^2/m_\ell^2)$ and $\alpha \ln(m_\ell/E_\gamma) \ln(m_b/m_\ell)$

$\longrightarrow$ **compete with QCD uncertainties**

- Belle II will measure the τ, μ channels with **5 − 7 % uncertainty**

[Belle II Physics Book]

QED large logarithmic $\alpha \ln^2(m_b/m_\ell)$ enhancements can mimic **lepton-flavor universality violation**

A muti-scale process

focus on $B_u \to \tau\,\nu$ new scales appear in the present of QED effects

m_W

$m_b \sim \mu_h$

$m_\tau \sim \mu_{hc}$

$\Lambda_{\rm QCD} \sim \mu_s$

$E_\gamma \sim \mu_{us}$

$\frac{m_\tau}{m_b} E_\gamma \sim \mu_{usc}$

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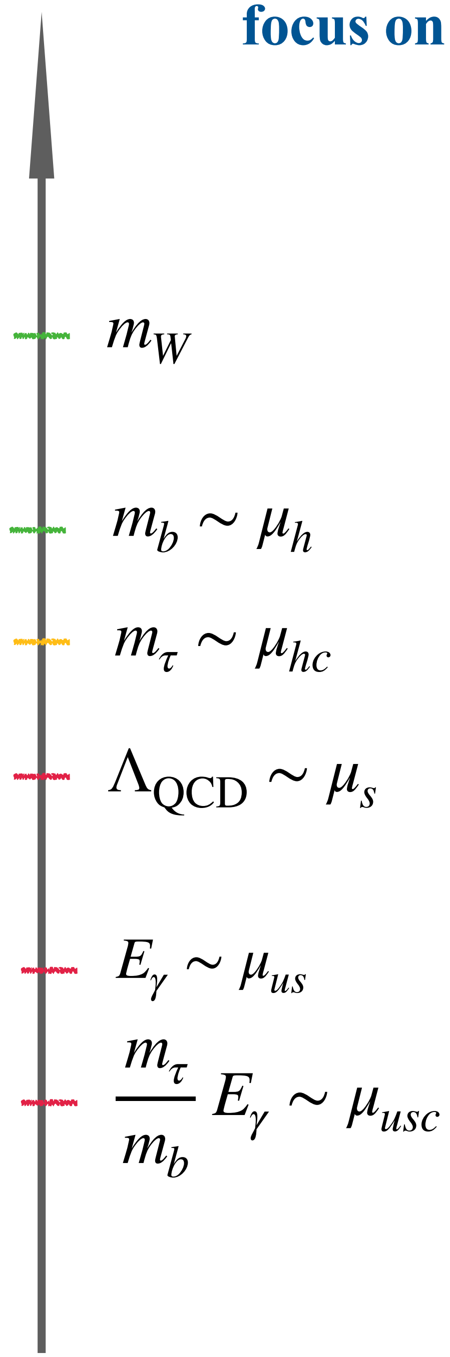
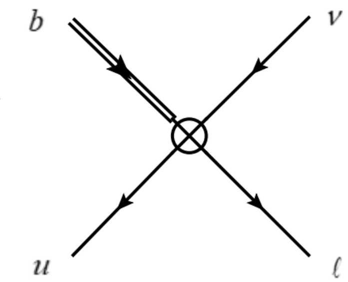


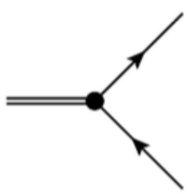
- QED for $\mu > m_b$ included in Effective weak Hamiltonian

$$\mathcal{L}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} K_{\rm EW}(\mu) V_{ub}\,(\bar{u}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu_\ell)$$



- Ultrasoft photons $\mu \ll \Lambda_{\rm QCD}$ see B meson as point-like particle

[Isidori, Nabeebaccus, Zwicky 2020; Zwicky 2021; Dai, Kim, Leibovich 2021]



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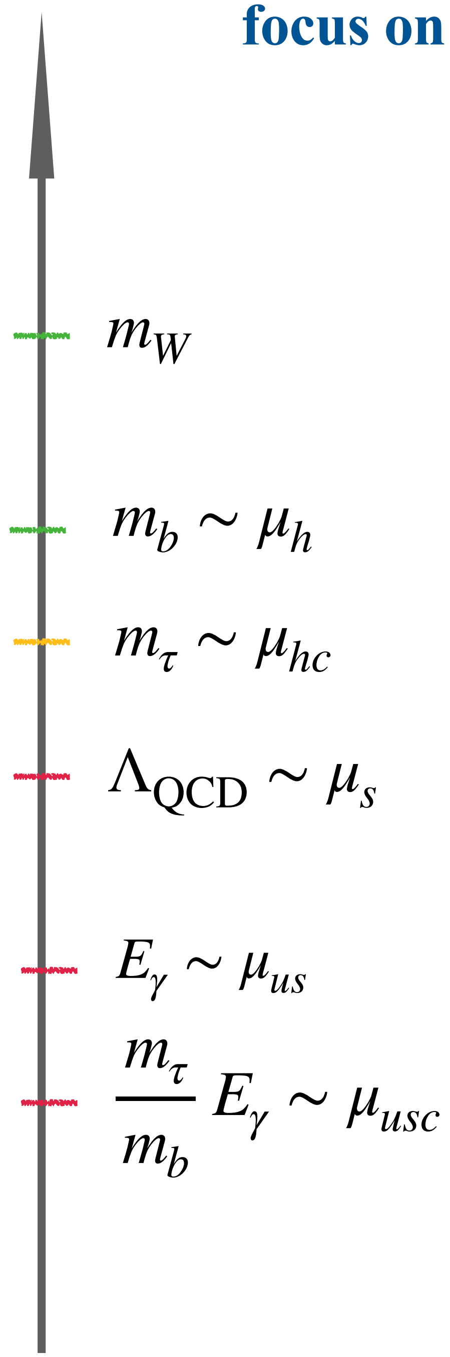

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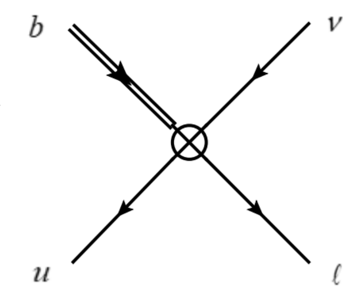

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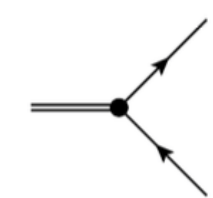
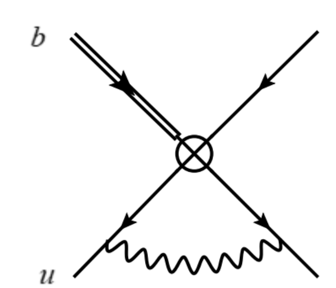


- Intermediate scale $\Lambda_{\rm QCD} < \mu < m_b$ gives rise to more intricate effect, as virtual photons can **resolve the structure of B meson**



$B_s \to \mu^+\mu^-$,$B \to \pi K, B \to D\pi$ *[M. Beneke, etc. 17 & 19, 20, 21*

$B_s \to \tau^+\tau^-$ *[Y.K.Huang, Y.L.Shen, X.C.Zhao, SHZ 23]*

$B_u \to \mu\nu$ *[M.Neubert, etc 2023]*

needs EFTs

Turning a muti-scale into a product of single scale

- Identifying the appropriate EFT description at each scale.
- Performing a step-by-step matching between each EFT.
- Deriving a factorization theorem to break this multi-scale problem into a convolution of single-scale objects.
- Using the renormalisation group to evaluate each object at its natural scale and run it to a common scale to resum logarithms.

From Fermi theory to HQET $\times$ SCET$_\text{I}$

m_W

$m_b \sim \mu_h$ ☑ Fermi Theory

$$\mathcal{L}_\text{eff} = -\frac{G_F}{\sqrt{2}} V_{ub} [\bar{q}\, \gamma_\mu (1-\gamma_5)\, b][\bar{\ell}\, \gamma^\mu (1-\gamma_5)\, \nu] \equiv -\frac{G_F}{\sqrt{2}} V_{ub}\, Q_1 \,,$$

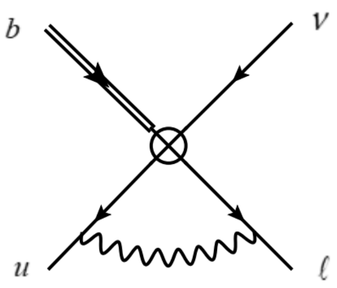
$m_\tau \sim \mu_{hc}$ ☐ …

$\Lambda_\text{QCD} \sim \mu_s$

$E_\gamma \sim \mu_{us}$

$\frac{m_\tau}{m_b} E_\gamma \sim \mu_{usc}$

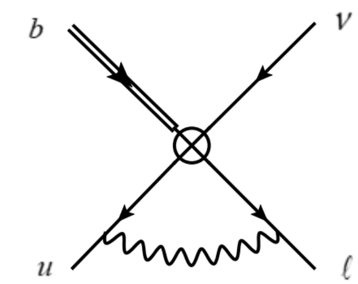
Fermi theory → HQET × SCET$_{\mathrm{I}}$



- The ***b* quark** can be described by a soft HQET field

$$b(x) \to \mathrm{e}^{-im_b v\cdot x}(1 + \mathcal{O}(\lambda^2))\, h_v(x)$$

Fermi theory $\to$ HQET $\times$ SCET$_{\text{I}}$



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$$b(x) \to e^{-im_b v\cdot x}(1 + \mathcal{O}(\lambda^2))\, h_v(x)$$

- Remaining fields can have large momenta, but small invariant mass, needs SCET

Relevant modes $p \sim (n_+p, n_-p, p_\perp)$

with expansion parameters:

$$\lambda^2 = \frac{\Lambda_{\text{QCD}}}{m_b}$$

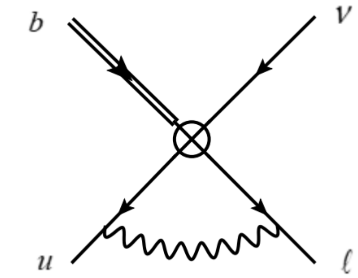
- Hard-collinear $p \sim (1, \lambda^2, \lambda)$

  $\rightarrow$given by the lepton virtuality

- Soft $p \sim (\lambda^2, \lambda^2, \lambda^2)$

  $\rightarrow$given by the spectator virtuality

Fermi theory → HQET × $\mathrm{SCET_I}$



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- Remaining fields can have large momenta, but small invariant mass, needs SCET

  In $\mathrm{SCET_I}$, subleading power description for the different modes of **the spectator and the lepton** :

$$q(x) \to \left(1 + \frac{i\ D_\perp}{in_+ D_C}\frac{\not{n}_+}{2}\right)\xi_C^{(q)} + \left((1 + \frac{1}{in_- D_s} Q_q\ \ A_{C\perp}\frac{\not{n}_-}{2}\right) q_s(x)$$

$$\ell(x) \to \left(1 + \frac{i\ D_\perp + m_\ell}{in_+ D_C}\frac{\not{n}_+}{2}\right)\xi_C^{(\ell)}(x) + \left((1 + \frac{1}{in_- D_s} Q_q\ \ A_{C\perp}\frac{\not{n}_-}{2}\right) \ell_s(x)$$

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  →given by the spectator virtuality

fields with power counting parameter

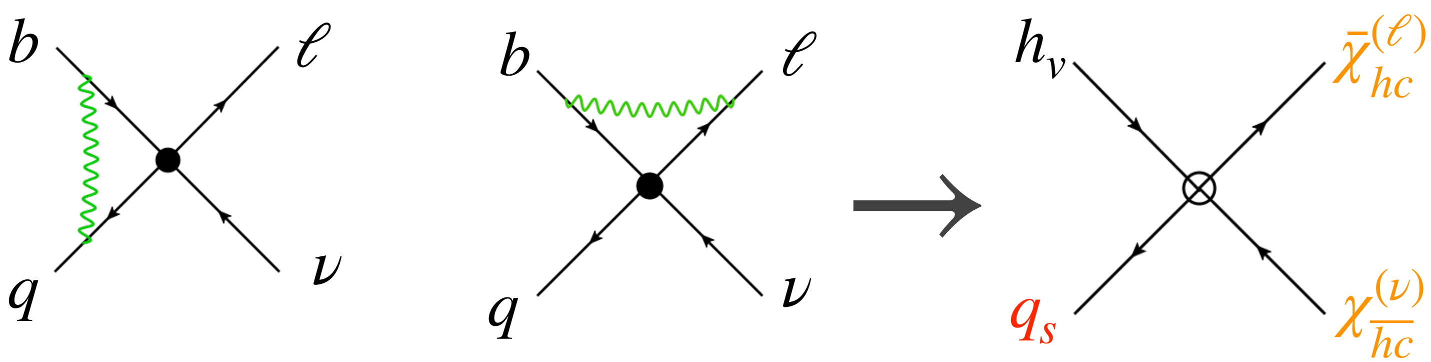
$$h_v, q_s \sim \lambda^3 \qquad \chi_{hc}^{(\ell)}, \chi_{\overline{hc}}^{(\nu)}, \chi_{hc}^{(q)} \sim \lambda$$

$$\mathscr{A}_{hc}^{\perp} \sim \lambda$$

Construction of $\mathrm{HQET} \times \mathrm{SCET_I}$ operator

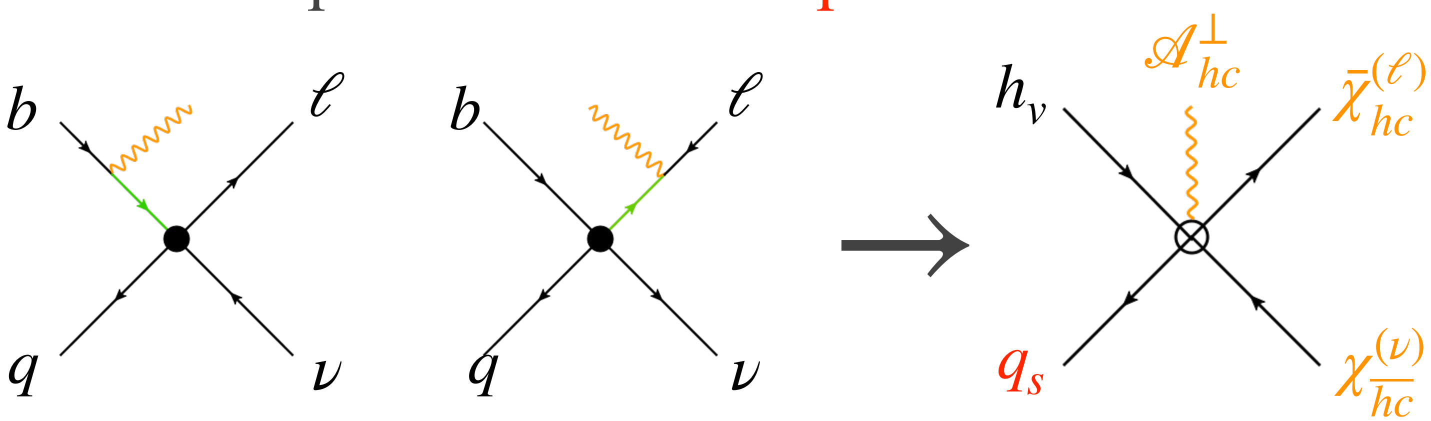
- **three classes operator are relevant**

1. local operator with soft spectator $\mathcal{O}_A$



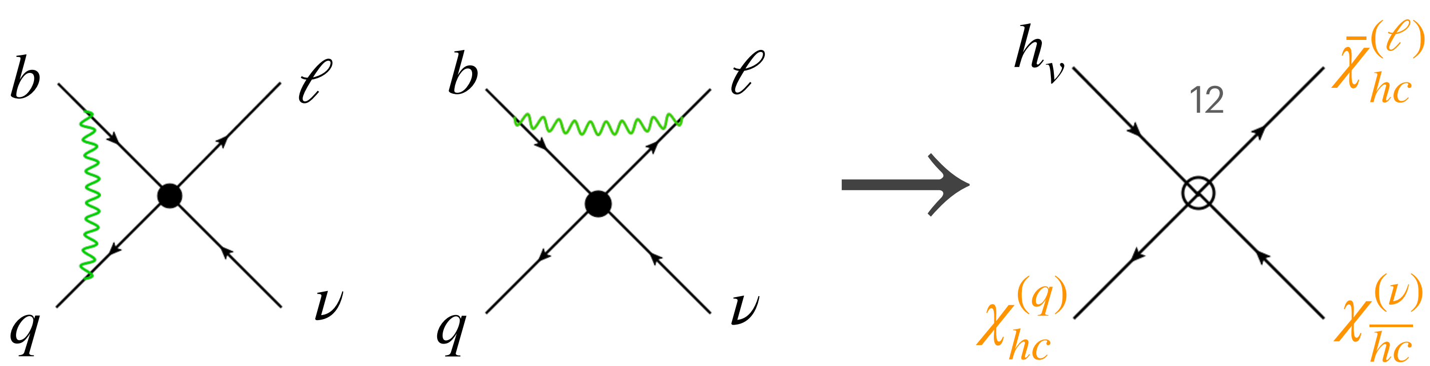
$$\mathcal{O}_A = [\bar{q}_s \ldots h_v]\,[\bar{\chi}_{hc}^{(\ell)} \ldots \chi_{\overline{hc}}^{(\nu)}]$$

2. local operator with soft spectator and hard-collinear photon $\mathcal{O}_B$



$$\mathcal{O}_B = [\bar{q}_s \ldots h_v]\,[\bar{\chi}_{hc}^{(\ell)} \ldots \chi_{\overline{hc}}^{(\nu)}]\;\; \mathcal{A}_{hc}^{\perp}$$

3. nonlocal operator with hard-collinear spectator $\mathcal{O}_C$

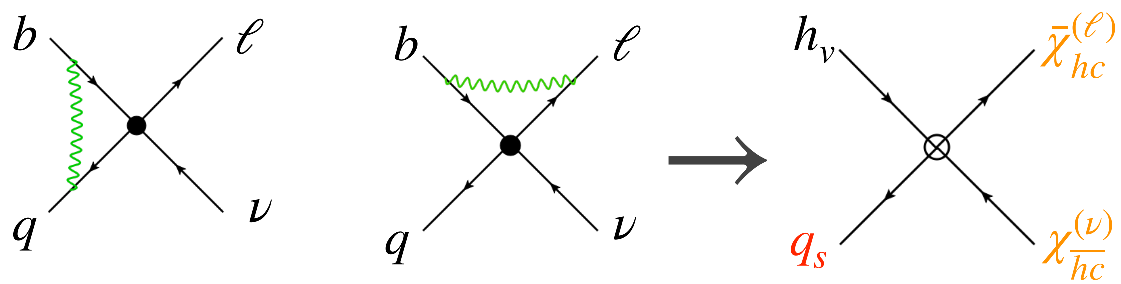


$$\mathcal{O}_C = [\;\chi_{hc}^{(q)}\;\ldots h_v]\,[\bar{\chi}_{hc}^{(\ell)} \ldots \chi_{\overline{hc}}^{(\nu)}]$$

Construction of $\mathrm{HQET}\times\mathrm{SCET_I}$ operator

- **three classes operator are relevant**

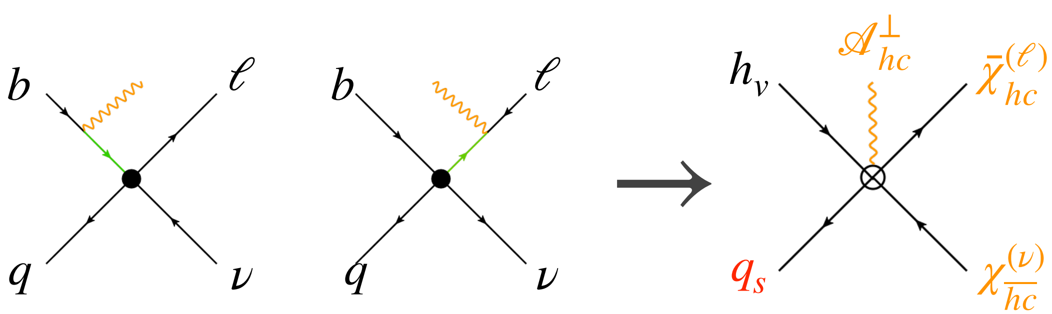
1. local operator with soft spectator $\mathcal{O}_A^{(\lambda)}$



$$\mathcal{O}_A^{(9)} = m_\ell\,[\bar q_s \frac{\not n_+}{2} P_L h_v]\,[\bar\chi_{hc}^{(\ell)} \frac{1}{i\,n_+\overleftarrow{\partial}_{hc}} P_L \chi_{\overline{hc}}^{(\nu)}]$$

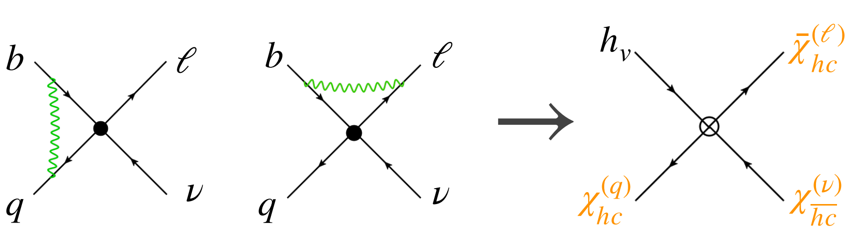
$$\mathcal{O}_{A'}^{(8)} = [\bar q_s \gamma_{\mu\perp} P_L h_v]\,[\bar\chi_{hc}^{\ell}\gamma_\perp^\mu P_L \chi_{\overline{hc}}^{\nu}] \quad \times$$

2. local operator with soft spectator and and hard-collinear photon $\mathcal{O}_B^{(\lambda)}$



$$\mathcal{O}_B^{(9)} = \frac{1}{i\,n_+\partial_{hc}}[\bar q_s \frac{\not n_-}{2}\gamma_{\mu\perp}\ A_{hc\perp} P_L h_v]\,[\bar\ell_{hc}\gamma_\perp^\mu P_L \nu_{\overline{hc}}]$$

3. nonlocal operator with hard-collinear spectator $\mathcal{O}_C^{(\lambda)}$



$$\mathcal{O}_{C,1}^{(6)}(s,t) = [\bar\chi_{hc}^{(q)}(sn_+)\gamma_{\mu\perp} P_L h_v(0)]\,[\bar\chi_{hc}^{(\ell)}(tn_+)\gamma_\perp^\mu P_L \chi_{\overline{hc}}^{(\nu)}(0)]$$

$$\mathcal{O}_{C,2}^{(7)}(s,t) = m_\ell\,[\bar\chi_{hc}^{(q)}(sn_+)\frac{\not n_+}{2} P_L h_v(0)]\,[\bar\chi_{hc}^{(\ell)}(tn_+)\frac{1}{i\,n_+\overleftarrow{\partial}_{hc}} P_L \chi_{\overline{hc}}^{(\nu)}(0)]$$

Hard function at $\mu \sim m_b$

m_W

$m_b \sim \mu_h$ ☑ Fermi Theory

$m_\tau \sim \mu_{hc}$ ☑ $\mathrm{HQET} \times \mathrm{SCET_I}$

Hard function $H_A^{(1)}, H_C^{(1)}$

$\Lambda_{\rm QCD} \sim \mu_s$ ☐ …

$E_\gamma \sim \mu_{us}$

$\frac{m_\tau}{m_b} E_\gamma \sim \mu_{usc}$

$$H_A^{(1)}(\mu_b) = -2\,\mathcal{N}\,\frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_b \left(\frac{1}{2}L^2 + \frac{\pi^2}{12} + \frac{7}{8}\right) \qquad L = \ln(\mu^2/m_b^2)$$

$$H_C^{(1)}(u,\mu_b) = -\,\mathcal{N}\,\frac{\alpha_{\rm em}}{4\pi} Q_u Q_b \left[\frac{1}{2} L\,(L - 4\ln\bar{u} + 2) + \ln\bar{u}\left(2\ln\bar{u} - 3 + \frac{1}{u} + 2\,{\rm Li}_2(u) + \frac{\pi^2}{12} + 4\right)\right] + \mathcal{N}\,\frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_b \left[2\left(-\ln u + \frac{1}{\bar{u}} + L\right)\ln u - \frac{1}{2}(L-4)\,L - 2\,{\rm Li}_2(\bar{u}) + 4 - \frac{\pi^2}{12}\right]$$

$\mathrm{SCET_I} \to$ bHLET

$\mu \sim m_b \Lambda_{\mathrm{QCD}}$

lower the virtuality to remove the hard-collinear mode to reach to bHLET

- heavy tau filed become to a soft-collinear (sc) field in boosted HLET after integrating m_τ

$$\chi_{hc}^{(\ell)} \to \mathrm{e}^{-i m_\ell v_\ell \cdot x} (1 + b \frac{\not{n}_+}{2}) \; \chi_{sc}^{(\ell)}$$

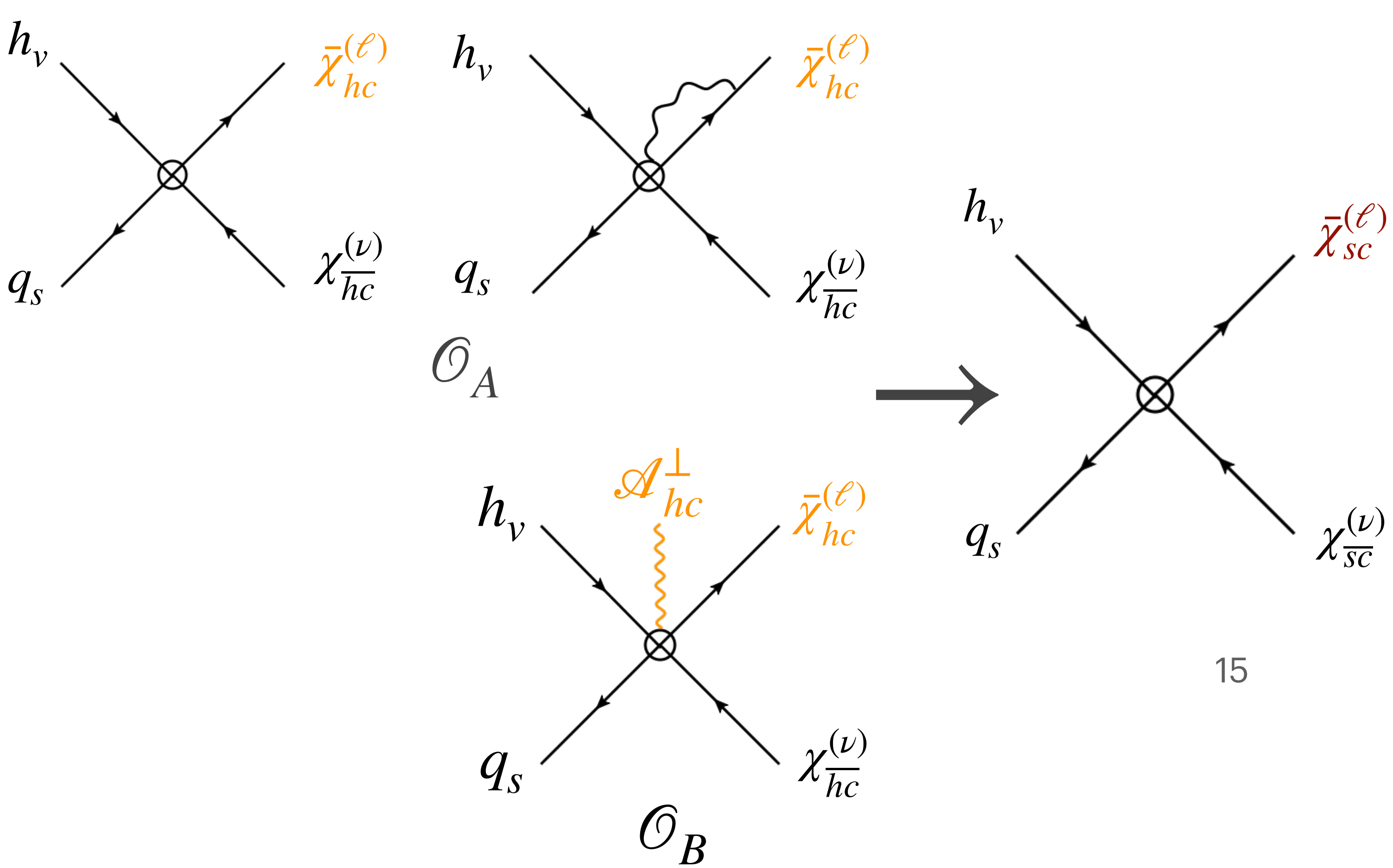
boosted parameters:

$$b = \frac{m_\tau}{m_b} \sim \lambda \qquad \lambda^2 = \frac{\Lambda_{\mathrm{QCD}}}{m_b}$$

- Soft-collinear $p \sim \lambda^2 (\frac{1}{b}, b, 1)$

$\rightarrow$soft scale to τ boosted in the B frame

$$p' \sim m_\tau (\lambda^2, \lambda^2, \lambda^2)$$



$$\mathcal{J}_{AB} = [\bar{q}_s \frac{\not{n}_+}{2} P_L h_v] \, Y_{v_\ell}^{(\ell)\dagger} \, [\bar{\chi}_{sc}^{(\ell)} P_L \chi_{\overline{sc}}^{(\nu)}]$$

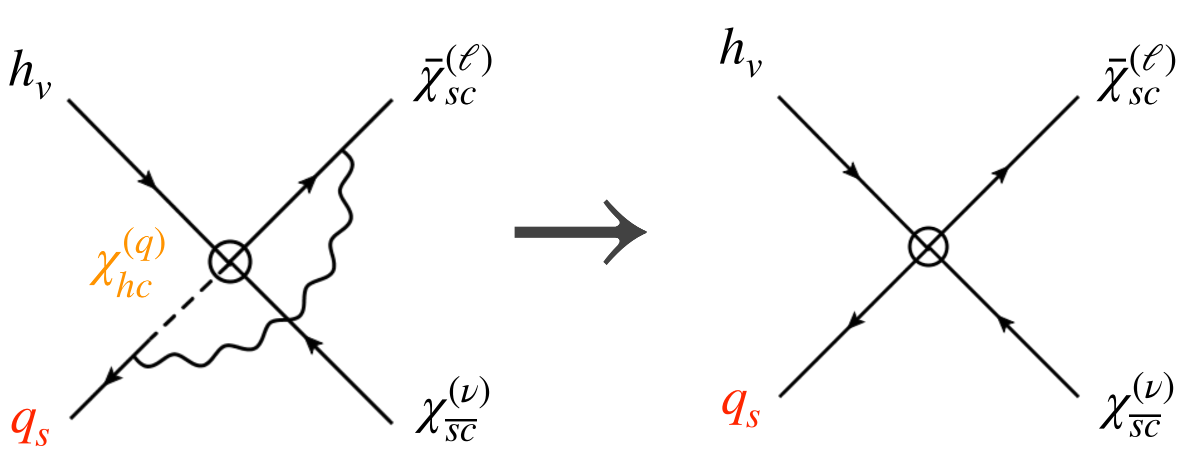
$\text{SCET}_{\text{I}} \to \text{HQET} \times \text{bHLET}$

- $\chi_{hc}^{(q)} \to q_s$ by inserting the following NLP and NNLP interactions

$$\mathcal{O}_C = [\ \chi_{hc}^{(q)} \ \ldots h_v]\,[\bar{\chi}_{hc}^{(\ell)} \ldots \chi_{\overline{hc}}^{(\nu)}]$$

$$L_{\xi q}^{(1)}(x) = \bar{q}_s(x_-)[W_{\xi C} W_C]^\dagger(x)\, i \not{D}_{C\perp}\, \xi_C(x) + \text{h.c.}$$

$$L_{\xi q}^{(2)}(x) = \bar{q}_s(x_-)\left[W_{\xi,hc} W_{hc}\right]^\dagger(x)\,\left(i n_- D_{hc} + i \not{D}_{hc\perp}(i n_+ D_{hc})^{-1} i \not{D}_{hc\perp}\right) \frac{\not{n}_+}{2}\, \xi_{hc}(x) + \ldots$$



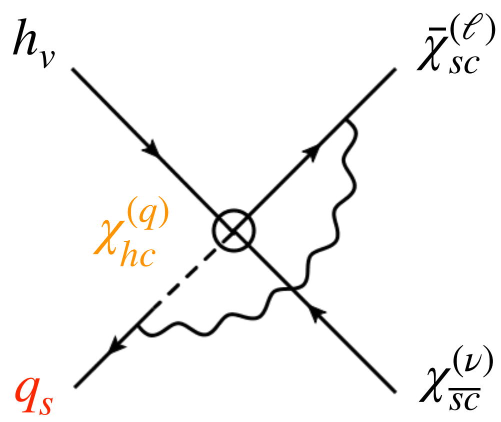
intermediate propagators
introduce **non-local operators**

$$\mathcal{J}_C(v) = [\bar{q}_s(v\,n_-)\, Y(v\,n_-,0)\, \frac{\not{n}_+}{2}\, P_L\, h_v(0)]\, Y_{v_\ell}^{(\ell)\dagger}(0)\, [\bar{\chi}_{sc}^{(\ell)}(0)\, P_L\, \chi_{\overline{sc}}^{(\nu)}(0)]$$

The helicity suppression be relaxed or not

$$\mathcal{J}_C(v) = [\bar{q}_s(v\,n_-)\,Y(v\,n_-,0)\,\frac{\not{n}_+}{2}\,P_L\,h_v(0)]\,Y_{v_\ell}^{(\ell)\dagger}(0)\,[\bar{\chi}_{sc}^{(\ell)}(0)\,P_L\,\chi_{\overline{sc}}^{(\nu)}(0)]$$

Nonlocal annihilation can **probe the meson structure**, and **possibly overcome** the **helicity suppression**



$$< 0|\,q_s\,\frac{1}{in_-\partial_s}\ldots h_v\,|B> \sim \frac{1}{\omega} \sim \frac{1}{\Lambda_{\mathrm{QCD}}}$$

Happens for $B_s \to \ell^+\ell^-$, but not for $B_u \to \ell\,\nu$

[Beneke, Bobeth, Szafron 2017 & 2019, Y.K.Huang, Y.L.Shen, X.C.Zhao, SHZ 2023]

$\longrightarrow$ No power enhancement in $B_u \to \ell\,\nu$!

$\mathrm{HQET} \times \mathrm{SCET_I}$ **matching onto** $\mathrm{HQET} \times \mathrm{bHLET}$

$$\mu \sim m_b \Lambda_{\mathrm{QCD}}$$

- $m_b \sim \mu_h$ ☑ Fermi Theory
- $m_\tau \sim \mu_{hc}$ ☑ $\mathrm{HQET} \times \mathrm{SCET_I}$
- $\Lambda_{\mathrm{QCD}} \sim \mu_s$ ☑ $\mathrm{HQET} \times \mathrm{bHLET}$
- $E_\gamma \sim \mu_{us}$ ☐ …
- $\frac{m_\tau}{m_b} E_\gamma \sim \mu_{usc}$

Hard function $H_A^{(1)}, H_C^{(1)}$

Hard-collinear function $J_{A,B}, J_C$

$$J_{C,1} = -\frac{\alpha}{4\pi} Q_\ell Q_u \frac{m_\ell}{m_b} (u\,\bar{u}) \times$$

$$\left[\frac{\bar{u}\, m_\ell^2}{u\,\omega} \ln\left(1 + \frac{u\, n \cdot p_\ell\, \omega}{\bar{u}\, m_\ell^2}\right) + m_b \ln\left(\frac{\bar{u}^2 m_b\, \bar{n} \cdot p_\ell + u\,\bar{u}\, m_b\, \omega}{\mu^2}\right)\right] \theta(u)\theta(\bar{u})$$

$\rightarrow$ No endpoint div. ($1/u \rightarrow \infty$, when u$\rightarrow$ 0) in $B_u \rightarrow \tau \nu$!

subtractions scheme independence

Factorization Formula

- **After two-step matching starting from QED onto** $\mathrm{SCET_I}$ **, and successively onto** $\mathrm{HQET} \times \mathrm{bHLET}$

$$A^{\text{virtual}}_{B\to\tau\nu} \sim H_A\, J_{AB}\, \langle \tau^-\nu \,|\, \mathcal{J}_{AB} \,|\, \bar{B}_u \rangle + \int_0^1 du\, H_C(u) \int_0^\infty d\omega\, J_C(u;\omega)\, \langle \tau^-\nu \,|\, \mathcal{J}_C \,|\, \bar{B}_u \rangle$$

$\mathrm{SCET_I}$ operators with soft spectator (A-type and B-type)

$\mathrm{SCET_I}$ operators with hc spectator (C-type)

Factorization Formula

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$\mathrm{SCET_I}$ operators with soft spectator (A-type and B-type)

$\mathrm{SCET_I}$ operators with hc spectator (C-type)

- **Modified** B-meson decay constant and LCDA

$$\mathcal{J}_{AB} = [\bar{q}_s \frac{\not{n}_+}{2} P_L\, h_v]\, S^{(\ell)\dagger}_{n_-} [\bar{\chi}^{(\ell)}_{sc}\, P_L\, \chi^{(\nu)}_{\overline{sc}}]$$

Soft photon decoupling from lepton

$$\mathcal{J}_C(v) = [\bar{q}_s(v\, n_-)\, Y(v\, n_-, 0) \frac{\not{n}_+}{2} P_L\, h_v(0)]\, S^{(\ell)\dagger}_{n_-}(0)\, [\bar{\chi}^{(\ell)}_{sc}(0)\, P_L\, \chi^{(\nu)}_{\overline{sc}}(0)]$$

Additional QED soft Wilson lines

$$S^{(i)}_r(x) = \exp\left[-i\, e\, Q_i \int_0^\infty ds\, r \cdot A_s(x + s \cdot r) \right]$$

Generalized decay constant and LCDA

- Factorization anomaly

$$\mathcal{J}^S_{AB} = [\bar{q}_s \frac{\not{n}_+}{2} P_L h_v]\, S^{(\ell)\dagger}_{n_-}$$

$$\mathcal{J}^S_C(v) = [\bar{q}_s(v\, n_-)\, Y(v\, n_-,0) \frac{\not{n}_+}{2} P_L h_v(0)]\, S^{(\ell)\dagger}_{n_-}(0)$$

- Refactorization

$$\mathcal{F}_B \equiv \frac{\langle 0 | \mathcal{J}^S_{AB} | B \rangle}{\langle 0 | [\, S^{(B)}_{v_B}(0)\, S^{(\ell)\dagger}_{n_-}(0)\,] | 0 \rangle} \qquad \mathcal{F}_B \Phi_B(v) \equiv \frac{\langle 0 | \mathcal{J}^S_C(v) | B \rangle}{\langle 0 | [\, S^{(B)}_{v_B}(0)\, S^{(\ell)\dagger}_{n_-}(0)\,] | 0 \rangle}$$

- For $\alpha \to 0$, $\mathcal{F}_B$ and $\mathcal{F}_B \Phi_B$ reduces to the standard HQET decay constant and LCDA

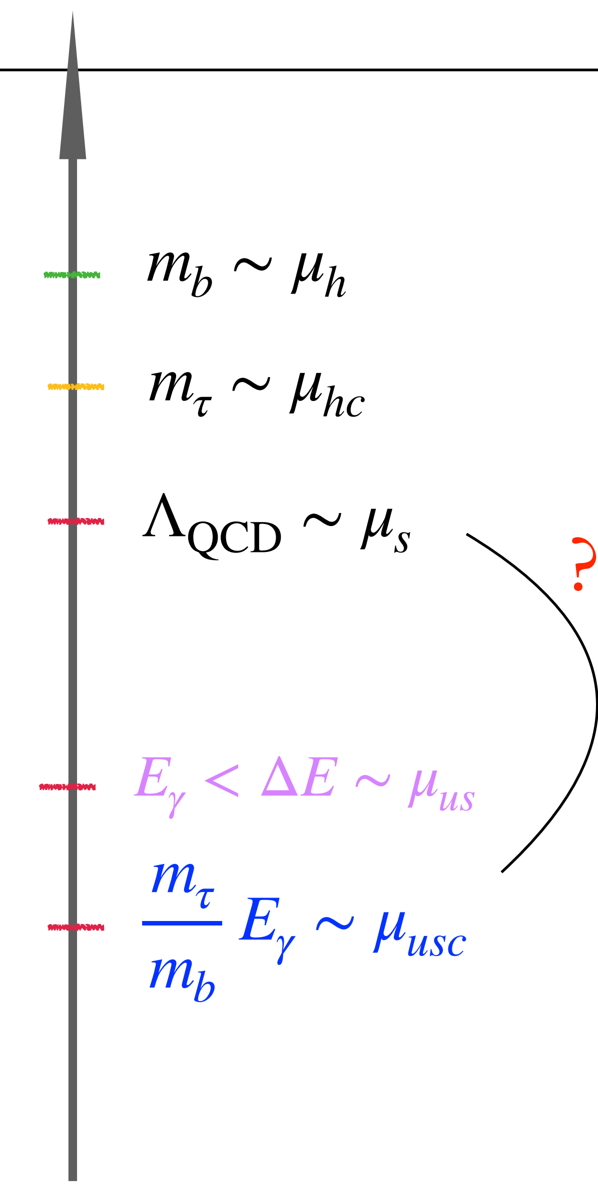
- For $\alpha \neq 0$, high order $\mathcal{F}_B$ and $\mathcal{F}_B \Phi_B$ are **new nonperturbative hadronic parameters**. Lattice determination ? QCD SR estimate ?

HQET $\times$ bHLET $\to$ **Low-energy theory** ($\mu < \mu_s, \mu_{sc}$)

power parameters: $\lambda_E^2 = \dfrac{E_\gamma}{m_b} \sim \lambda^4$

- Ultra-soft $\quad p \sim (\lambda_E^2, \lambda_E^2, \lambda_E^2)$
- Ultra-soft-collinear $\quad p \sim \lambda_E^2 (1, b^2, b)$

$\to$ultrosoft scale to τ boosted in the B frame



HQET × bHLET → **Low-energy theory ($\mu < \mu_s, \mu_{sc}$)**

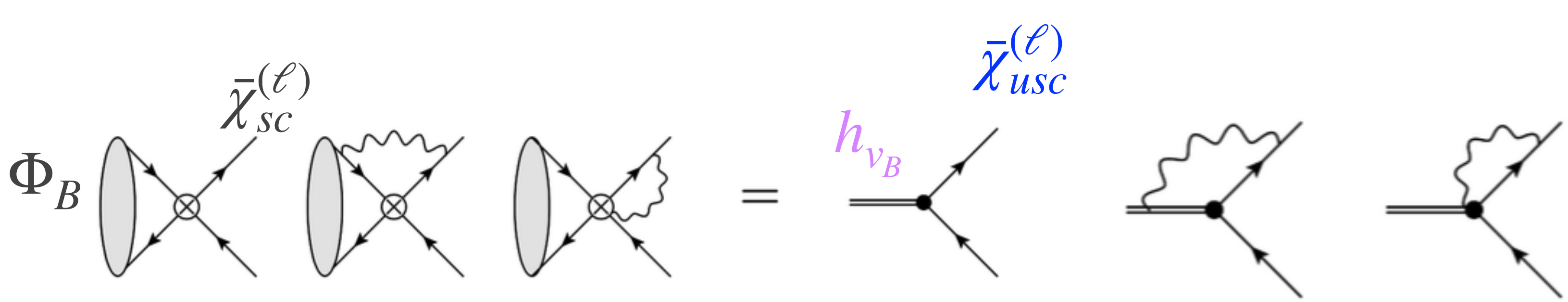
- $\mu < \Lambda_{\mathrm{QCD}}$, the hadronic B meson can be described as a heavy scalar effective theory (HSET)

$$\Phi_B(x) \to \mathrm{e}^{-i m_B v_B \cdot x}\, h_{v_B}(x) \qquad m_B v_B \sim \mu_s$$

- $\mu < \mu_{sc}$, soft-coll. (sc) field in bHLET turned into ultra-soft-coll. one (usc) in bHLET-2

$$\chi_{sc}^{(\ell)} \to \mathrm{e}^{-i m_\ell v'_\ell \cdot x}\, \chi_{usc}^{(\ell)} \qquad m_\ell v'_\ell \sim \mu_{sc}$$

- HQET × bHLET → HSET × $\mathrm{bHLET_{II}}$ $\quad \mu \sim \Lambda_{\mathrm{QCD}}$



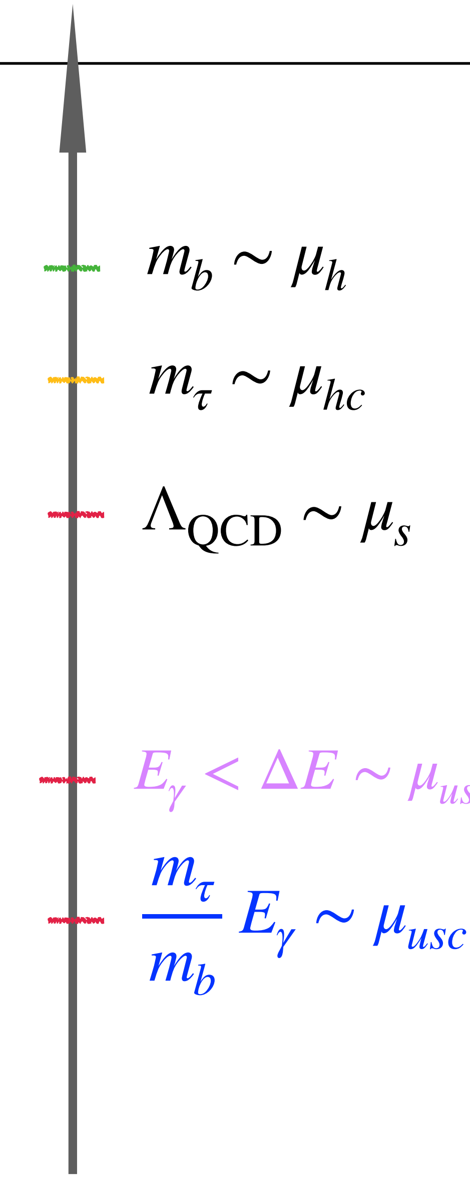
nonperturbative hadronic matrix element before decoupling

$$= y_B\, h_{v_B} [\bar{\chi}_{usc}^{(\ell)} P_L \chi_{usc}^{(\nu)}]$$

power parameters: $\lambda_E^2 = \dfrac{E_\gamma}{m_b} \sim \lambda^4$

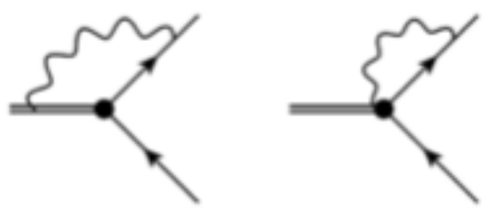
- Ultra-soft $\quad p \sim (\lambda_E^2, \lambda_E^2, \lambda_E^2)$
- Ultra-soft-collinear $\quad p \sim \lambda_E^2 (1, b^2, b)$

→ultrosoft scale to τ boosted in the B frame



Real correction to $\mathrm{HSET} \times \mathrm{bHLET_{II}}$

- all interactions of the B and the tauon with ultra-soft **and** ultra-soft-collinear photons can be decoupled into **Wilson lines** via field redefinitions



$$h_{v_B}(x) \to S_{v_B}^{(B)}\; C_{n_+}^{(B)}\; h_{v_B}^{(0)}(x)$$

$$\chi_{usc}^{(\ell)}(x) \to S_{n_-}^{(\ell)}\; C_{v_\ell'}^{(\ell)}\; \chi_{usc}^{(\ell,0)}(x)$$

- **Real corrections** are matrix elements of these Wilson lines

$$S(E_\gamma,\mu) = \int_0^\infty d\,\omega_{us} \int_0^\infty d\,\omega_{usc}\, \theta(\frac{E_\gamma}{2} - \omega_{us} - \omega_{usc})\;\; W_{us}(\omega_{us},\mu)\;\; W_{usc}(\omega_{usc},\mu)$$

- Real emissions are factorized at the level of the decay rate

$$\Gamma[B_u \to \tau\nu] \sim |A^{\text{virtual}}_{B\to\tau\nu}|^2 \otimes S(E_\gamma,\mu)$$

$$A^{\text{virtual}}_{B\to\tau\nu} \sim H_i \otimes J_i \otimes S \otimes SC$$

Summary

- Subleading power factorization formula for QED corrections to $B_u \to \tau\nu$ derived in SCET, HQET and bHLET

  no endpoint divergences in this subleading power process

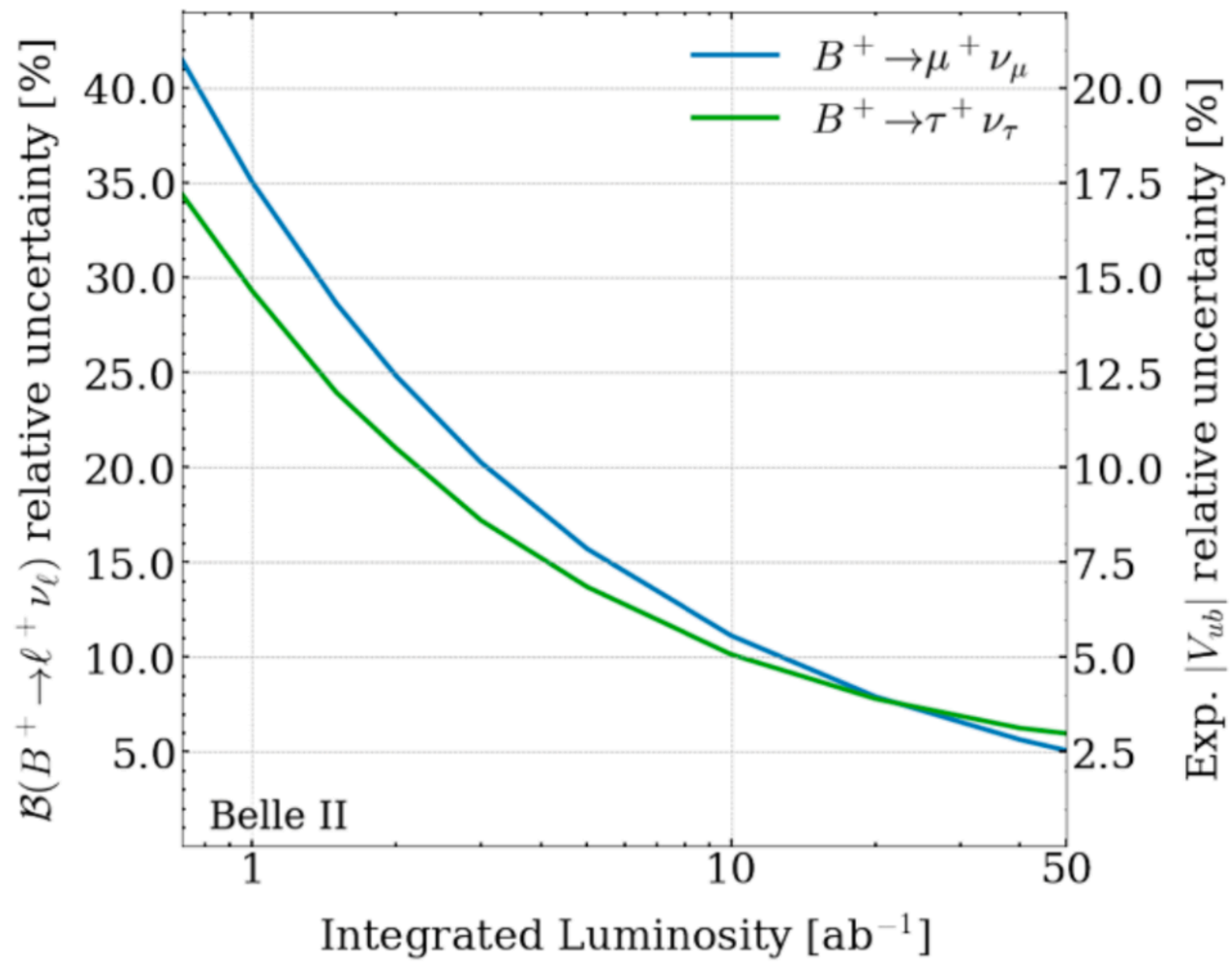
  $\longrightarrow$ **subtractions scheme independence**

- **Structure depended QED corrections** arising from hard-collinear photons exchange $\to$ important source of **large logarithmic corrections**

- Structure depended QED corrections from leptonic field decoupling produce **generalized *B* decay constant and LCDA** $\to$ new hadronic parameters

Thank you

Backup slides



[Belle II Physics Book]

Figure 1: Projection of uncertainties on the branching fractions $\mathcal{B}(B^+ \to \mu^+ + \nu_\mu)$ and $\mathcal{B}(B^+ \to \tau^+ + \nu_\tau)$. The corresponding uncertainty on the experimental value of $|V_{ub}|$ is shown on the right-hand vertical axis.

modes

- Relevant modes $k \sim (n_+k, n\_k, k_\perp)$ for **virtual** QED corrections:

Expansion parameters:

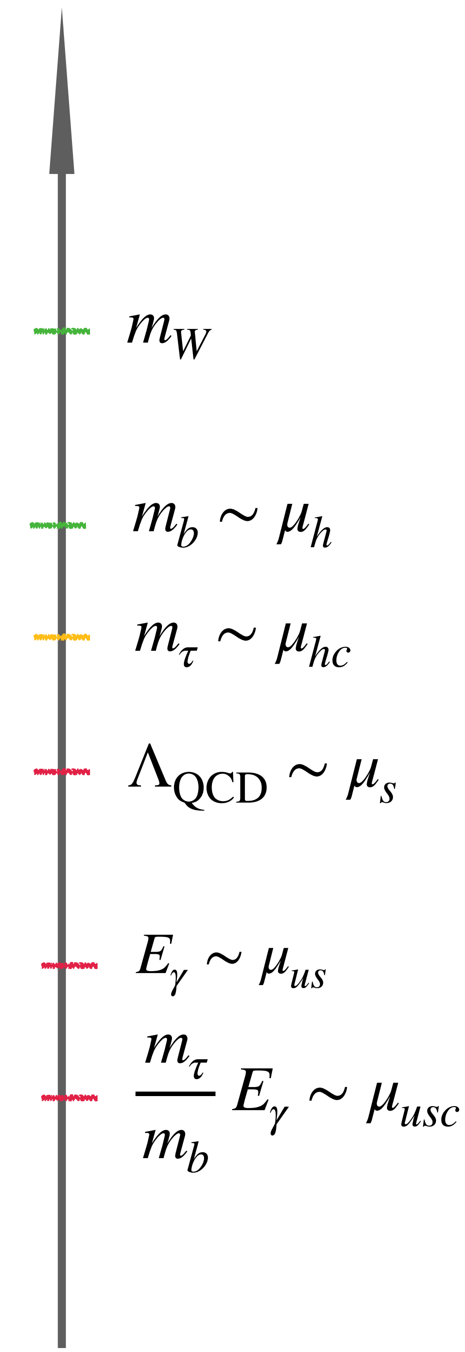
- Hard $(1, 1, 1)$
- Hard-collinear $(1, \lambda^2, \lambda)$
- Soft $(\lambda^2, \lambda^2, \lambda^2)$
- Soft-collinear $\lambda^2(1, b^2, b) \sim (\lambda^2, \lambda^4, \lambda^3)$

$$\lambda^2 = \frac{\Lambda_{\rm QCD}}{m_b}$$

$$b = \frac{m_\tau}{m_b} \sim \lambda$$

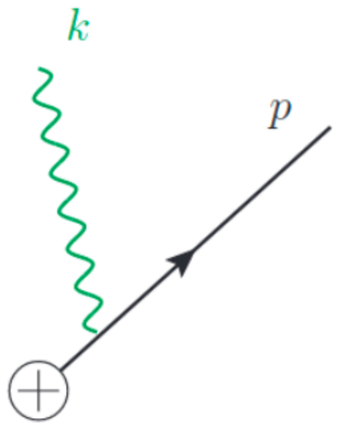
- Relevant modes for **real** QED corrections:
  - ultrasoft $(\lambda_E^2, \lambda_E^2, \lambda_E^2)$
  - ultrasoft soft-collinear $\lambda_E^2(1, b^2, b)$

$$\lambda_E^2 = \frac{E_\gamma}{m_b} \sim \lambda^4$$



- Ultra-soft photons (under the assumption that $\Delta E \ll \Lambda_{\text{QCD}}$)

Based on eikonal approximation,



$$\varepsilon_\mu(k)\bar{u}(p)\gamma^\mu \frac{\not{p}+\not{k}+m}{(k+p)^2-m^2} \to \frac{\varepsilon_\mu(k)p^\mu}{p\cdot k}\bar{u}(p)\,,$$

note $k^\mu \ll p^\mu, m$

$$\delta_{\text{QED}} \sim \frac{\alpha}{\pi}\ln^2\frac{m_B}{m_\ell}$$

Large logarithmic enhancements can mimic lepton-flavor universality violation

- Main challenges in formulating a factorization theorem:

1. Quark current $\bar{u}\,\gamma^\mu P_L\, b$ is not gauge invariant under QED

$$\bar{u}\,\gamma^\mu P_L\, b \longrightarrow \bar{u}\,\gamma^\mu P_L\, b\, Y^\dagger_{v_\ell}$$

add a **Wilson line** Y_{v_ℓ} to account for (ultra-)soft photon interactions with charged lepton → anomalous dimension sensitive to **IR regulators**

2. Beyond leading power convolutions have **endpoint divergences**

[Feldmann, Gubernari, Huber, Neubert, Seitz 2022; Hurth, Neubert, Szafron 2023]

cannot be dealt with using standard renormalization techniques and require appropriate subtractions.

e.g. "refactorization-based subtraction (RBS) scheme" in $B_u \to \mu\,\nu$

For $B_u \to \tau\,\nu$, no endpoint divergences, **subtractions scheme independence**

$$
\begin{aligned}
i\mathcal{M}^{(c),hc} = \frac{-i\tilde{f}_B m_B}{2} \frac{\alpha_{em}}{4\pi} Q_l Q_q \Big[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\Big(L' + 2\ln\eta - \frac{5}{2}\Big) + \frac{L'}{2}(L' - 5 + 4\ln\eta) + \ln\eta(\ln\eta - 1) + \frac{\pi^2}{12} \\
- \frac{11}{2} - 2\mathrm{Li}_2(1-\eta)\Big] \phi_B^-(\omega)\, \bar{u}\not{n}(1-\gamma_5)v \qquad (43)
\end{aligned}
$$

$$Y(x,y) = \exp\left[i\,e\,Q_q \int_y^x dz_\mu\, A_s^\mu(z)\right] \mathcal{P}\exp\left[i\,g_s \int_y^x dz_\mu\, G_s^\mu(z)\right],$$

$$Y_\pm(x) = \exp\left[-i\,e\,Q_\ell \int_0^\infty ds\, n_\mp A_s\,(x + s n_\mp)\right].$$

$$S_r^{\prime(i)}(x) \;=\; \exp\left[-i\,e\,Q_i \int_0^\infty ds\, r \cdot A_{us}\,(x + s \cdot r)\right]$$

$$C_r^{\prime(i)}(x) \;=\; \exp\left[-i\,e\,Q_i \int_0^\infty ds\, r \cdot A_{usc}\,(x + s \cdot r)\right]$$