Charmed Baryon Decays from the Perspective of Topological Diagrams

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arXiv:2401.15926v3
arXiv:2404.01350v2
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CONTENTS

Introduction

Experimental Progress in 2024

- **BESIII**

$$\alpha(\Lambda_c^+ \to \Xi^0 K^+) = 0.01 \pm 0.16 \pm 0.03,$$

Phys. Rev. Lett. 132, 031801(2024)

$$\mathcal{B}(\Lambda_c^+ \to p\pi^0) = (1.56^{+0.72}_{-0.58} \pm 0.20) \times 10^{-4},$$
$$\mathcal{B}(\Lambda_c^+ \to n\pi^+) = (6.6 \pm 1.3) \times 10^{-4},$$
$$\mathcal{B}(\Lambda_c^+ \to p\eta) = (1.63 \pm 0.31 \pm 0.11) \times 10^{-3},$$

Phys. Rev. D 109, L091101(2024)

- **Belle**

$$\mathcal{B}(\Xi_c^0 \to \Xi^0\pi^0) = (6.9 \pm 0.3(\text{stat}) \pm 0.5(\text{syst}) \pm 1.5(\text{norm})) \times 10^{-3},$$
$$\mathcal{B}(\Xi_c^0 \to \Xi^0\eta) = (1.6 \pm 0.2(\text{stat}) \pm 0.2(\text{syst}) \pm 0.4(\text{norm})) \times 10^{-3},$$
$$\mathcal{B}(\Xi_c^0 \to \Xi^0\eta') = (1.2 \pm 0.3(\text{stat}) \pm 0.1(\text{syst}) \pm 0.3(\text{norm})) \times 10^{-3},$$
$$\alpha(\Xi_c^0 \to \Xi^0\pi^0) = -0.90 \pm 0.15(\text{stat}) \pm 0.23(\text{syst}).$$

arXiv:2406.04642 [hep-ex]

- **LHCb**

$$\alpha(\Lambda_c^+ \to \Lambda\pi^+) = -0.782 \pm 0.009 \pm 0.004,$$
$$\alpha(\Lambda_c^+ \to \Lambda K^+) = -0.569 \pm 0.059 \pm 0.028,$$
$$\alpha(\Lambda_c^+ \to pK_S^0) = -0.744 \pm 0.012 \pm 0.009,$$
$$\beta(\Lambda_c^+ \to \Lambda\pi^+) = 0.368 \pm 0.019 \pm 0.008,$$
$$\beta(\Lambda_c^+ \to \Lambda K^+) = 0.35 \pm 0.12 \pm 0.04,$$
$$\gamma(\Lambda_c^+ \to \Lambda\pi^+) = 0.502 \pm 0.016 \pm 0.006,$$
$$\gamma(\Lambda_c^+ \to \Lambda K^+) = -0.743 \pm 0.067 \pm 0.024,$$
$$\Delta(\Lambda_c^+ \to \Lambda\pi^+) = 0.633 \pm 0.036 \pm 0.013,$$
$$\Delta(\Lambda_c^+ \to \Lambda K^+) = 2.70 \pm 0.17 \pm 0.04,$$

arXiv:2409.02759 [hep-ex]

Introduction

Theoretical progress

- Dynamical model calculations
  - Quark model
  - Pole model
  - Rescattering
  - QCD sum rules
  - …
- Fit
  - The irreducible SU(3) approach (IRA)
  - The topological diagram approach (TDA)

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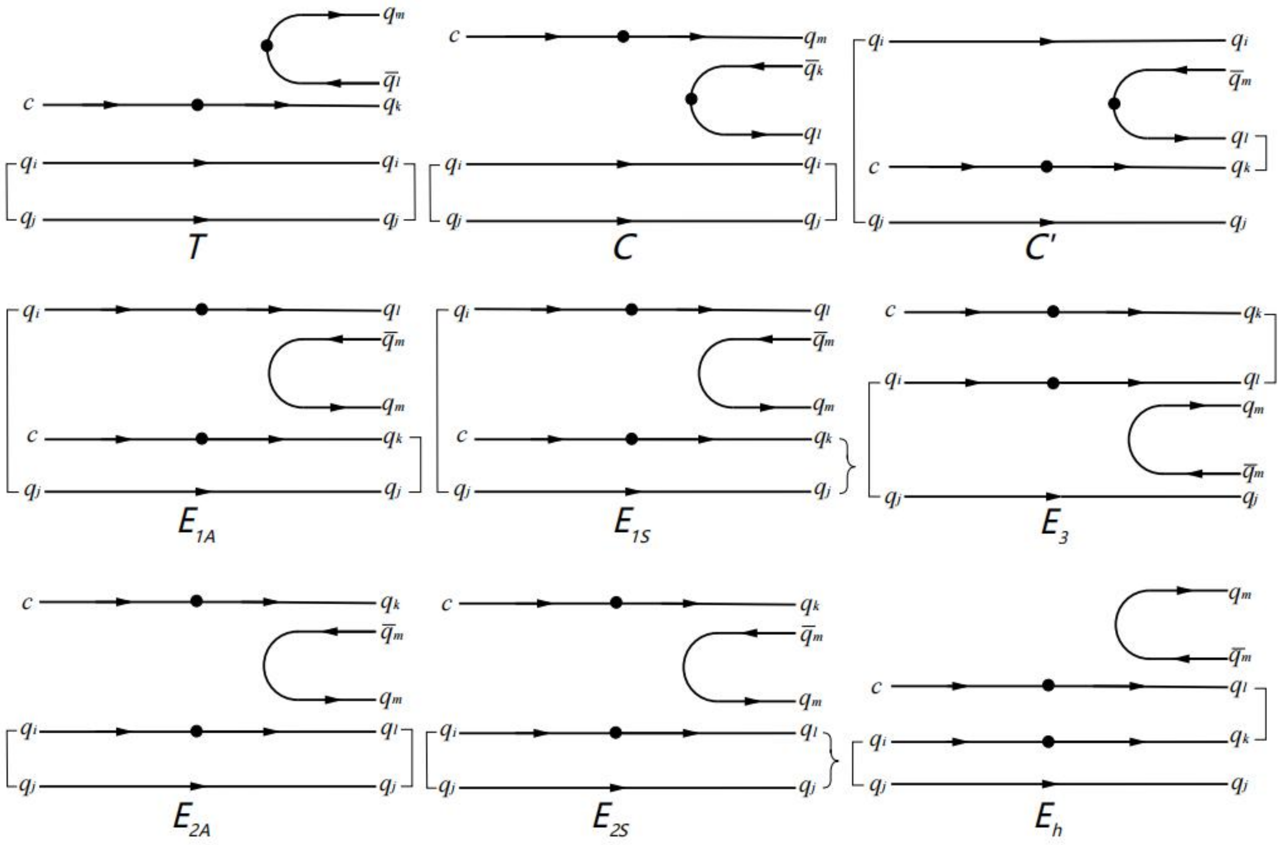
Formalism

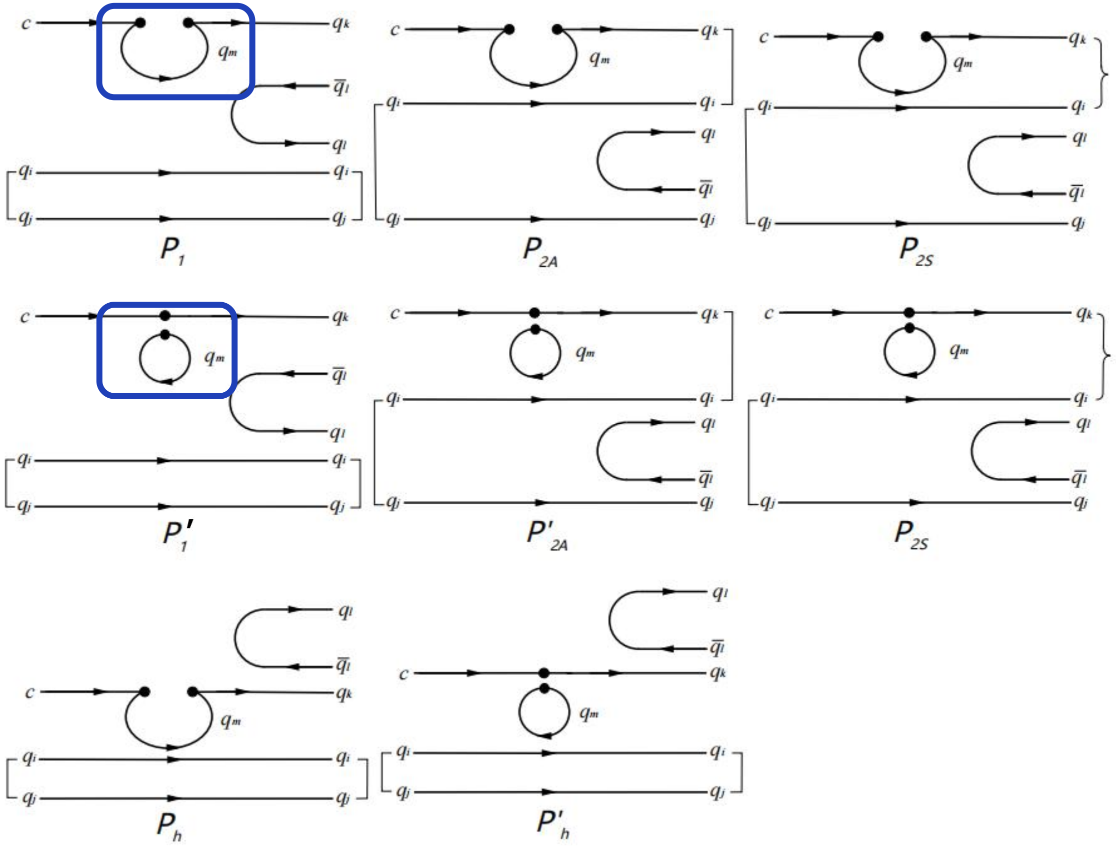
Topological diagrams

$$\mathcal{B}_c(\bar{3}) \to \mathcal{B}(8)P(8+1)$$

- **Tree**

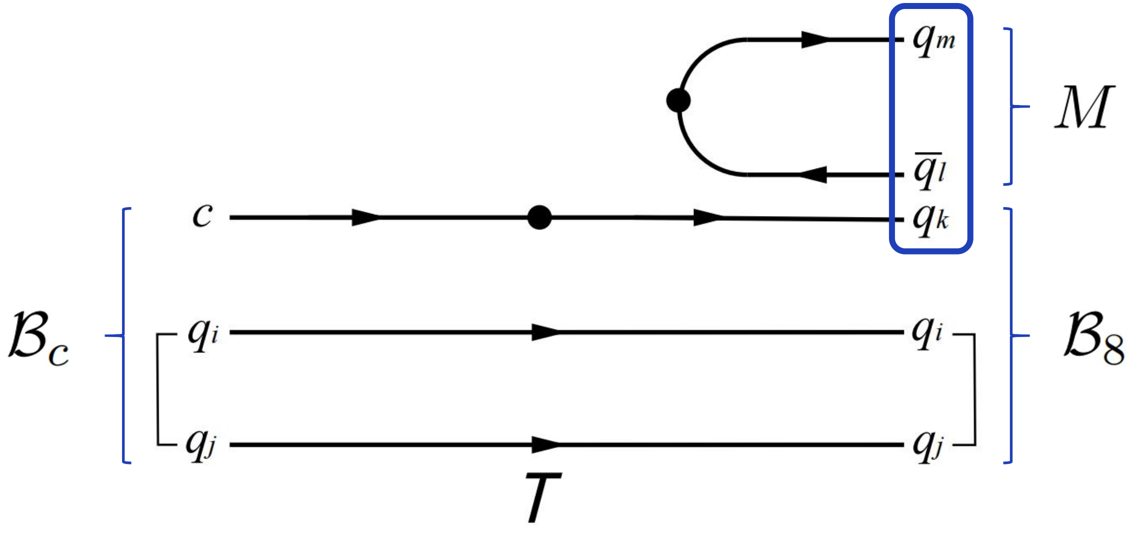


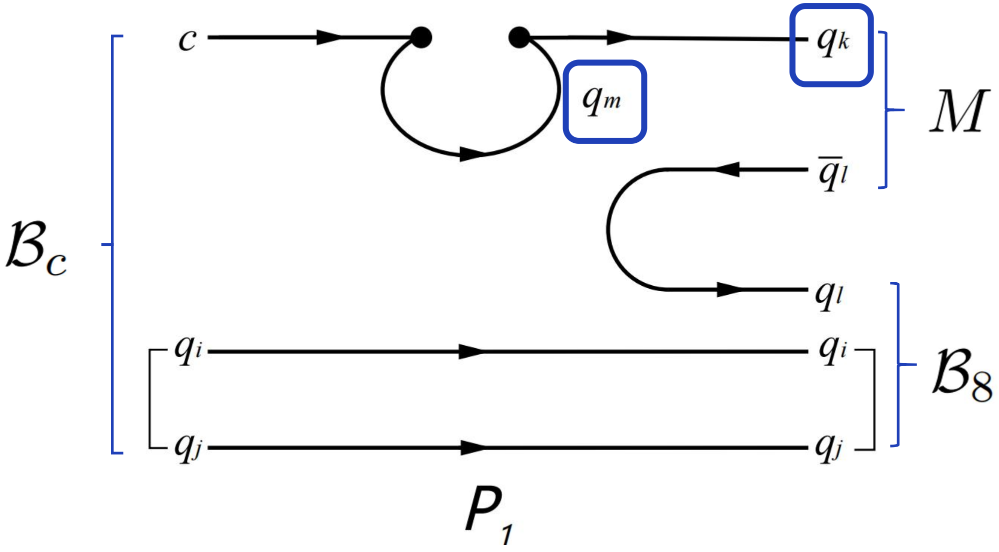
- **Penguin**



Formalism

From Topological diagrams to decay amplitudes





$$T(\mathcal{B}_c)^{ij} H_l^{km} (\mathcal{B}_8)_{ijk} (M)_m^l \qquad P_1(\mathcal{B}_c)^{ij} H_m^{mk} (\mathcal{B}_8)_{ijl} (M)_k^l$$

Formalism

From Topological diagrams to decay amplitudes

$$\mathcal{B}_c(\bar{3}) \to \mathcal{B}(8)P(8+1)$$

The SU(3) flavor representations:

$$(\mathcal{B}_c)_i = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+)$$

$$(\mathcal{B}_8)^i_j = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}$$

$$M^i_j = \begin{pmatrix} \frac{\pi^0+\eta_q}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0+\eta_q}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_s \end{pmatrix}$$

$$(\mathcal{B}_c)^{ij} = \epsilon^{ijk}(\mathcal{B}_c)_k$$

$$(\mathcal{B}_8)_{ijk} = \epsilon_{ijl}(\mathcal{B}_8^T)^l_k$$

$$(H_{\bar{6}})^{31}_3 = -(H_{\bar{6}})^{13}_3 = (H_{\bar{6}})^{12}_2 = -(H_{\bar{6}})^{21}_2 = \frac{1}{2}(\lambda_s - \lambda_d),$$

$$(H_{15})^{31}_3 = (H_{15})^{13}_3 = -(H_{15})^{12}_2 = -(H_{15})^{21}_2 = \frac{1}{2}(\lambda_s - \lambda_d),$$

$$(H_{3p})^1 = (H_{3t})^1 = -\frac{1}{4}\lambda_b.$$

The coefficient tensor H :

$$H^{ij}_k = \frac{1}{2}\left[(H_{15})^{ij}_k + (H_{\bar{6}})^{ij}_k\right] + \delta^j_k\left(\frac{3}{8}(H_{3p})^i - \frac{1}{8}(H_{3t})^i\right) + \delta^i_k\left(\frac{3}{8}(H_{3t})^j - \frac{1}{8}(H_{3p})^j\right)$$

Formalism

From Topological diagrams to decay amplitudes

before

$$
\begin{aligned}
\mathcal{A}_{\rm TDA} = \; & T(\mathcal{B}_c)^{ij} H^{km}_l M^l_m \left[b_1 (\mathcal{B}_8)_{ijk} + \cancel{b_2 (\mathcal{B}_8)_{ikj} + b_3 (\mathcal{B}_8)_{jki}} \right] \\
& + C(\mathcal{B}_c)^{ij} H^{ml}_k M^k_m \left[b_4 (\mathcal{B}_8)_{ijl} + \cancel{b_5 (\mathcal{B}_8)_{ilj} + b_6 (\mathcal{B}_8)_{jli}} \right] \\
& + C'(\mathcal{B}_c)^{ij} H^{kl}_m M^m_i \left[b_7 (\mathcal{B}_8)_{klj} + \cancel{b_8 (\mathcal{B}_8)_{kjl} + b_9 (\mathcal{B}_8)_{ljk}} \right] \\
& + E_1(\mathcal{B}_c)^{ij} H^{kl}_i M^m_l \left[b_{10} (\mathcal{B}_8)_{jkm} + b_{11} (\mathcal{B}_8)_{jmk} + b_{12} (\mathcal{B}_8)_{kmj} \right] \\
& + E_2(\mathcal{B}_c)^{ij} H^{kl}_i M^m_k \left[b_{13} (\mathcal{B}_8)_{jlm} + b_{14} (\mathcal{B}_8)_{jml} + b_{15} (\mathcal{B}_8)_{lmj} \right] \\
& + E_3(\mathcal{B}_c)^{ij} H^{kl}_i M^m_j \left[b_{16} (\mathcal{B}_8)_{klm} + \cancel{b_{17} (\mathcal{B}_8)_{kml} + b_{18} (\mathcal{B}_8)_{lmk}} \right] \\
& + E_h(\mathcal{B}_c)^{ij} H^{kl}_i M^m_m \left[b_{19} (\mathcal{B}_8)_{jkl} + \cancel{b_{20} (\mathcal{B}_8)_{jlk} + b_{21} (\mathcal{B}_8)_{klj}} \right],
\end{aligned}
$$

flavor symmetry

KPW theorem

11 independent TDA amplitudes

Formalism

From Topological diagrams to decay amplitudes

now

$E_{2A} = -E_{1A}, \qquad E_{2S} = -E_{1S}.$

$$\begin{aligned}
\mathcal{A}_{\rm TDA} = \;& T(\mathcal{B}_c)^{ij} H_l^{km} (\mathcal{B}_8)_{ijk} (M)_m^l \\
&+ C(\mathcal{B}_c)^{ij} H_k^{ml} (\mathcal{B}_8)_{ijl} (M)_m^k + C'(\mathcal{B}_c)^{ij} H_m^{kl} (\mathcal{B}_8)_{klj} (M)_i^m \\
&+ E_{1A}(\mathcal{B}_c)^{ij} H_i^{kl} (\mathcal{B}_8)_{jkm} (M)_l^m + E_{1S}(\mathcal{B}_c)^{ij} H_i^{kl} (M)_l^m \left[(\mathcal{B}_8)_{jmk} + (\mathcal{B}_8)_{kmj}\right] \\
&+ E_{2A}(\mathcal{B}_c)^{ij} H_i^{kl} (\mathcal{B}_8)_{jlm} (M)_k^m + E_{2S}(\mathcal{B}_c)^{ij} H_i^{kl} (M)_k^m \left[(\mathcal{B}_8)_{jml} + (\mathcal{B}_8)_{lmj}\right] \\
&+ E_3(\mathcal{B}_c)^{ij} H_i^{kl} (\mathcal{B}_8)_{klm} (M)_j^m + E_h(\mathcal{B}_c)^{ij} H_i^{kl} (\mathcal{B}_8)_{klj} (M)_m^m \\
&+ P_h(\mathcal{B}_c)^{ij} H_m^{mk} (\mathcal{B}_8)_{ijk} (M)_l^l + P_1(\mathcal{B}_c)^{ij} H_m^{mk} (\mathcal{B}_8)_{ijl} (M)_k^l \\
&+ P_{2A}(\mathcal{B}_c)^{ij} H_m^{mk} (\mathcal{B}_8)_{kil} (M)_j^l + P_{2S}(\mathcal{B}_c)^{ij} H_m^{mk} (M)_j^l \left[(\mathcal{B}_8)_{kli} + (\mathcal{B}_8)_{ilk}\right] \\
&+ P'_h(\mathcal{B}_c)^{ij} H_m^{km} (\mathcal{B}_8)_{ijk} (M)_l^l + P'_1(\mathcal{B}_c)^{ij} H_m^{km} (\mathcal{B}_8)_{ijl} (M)_k^l \\
&+ P'_{2A}(\mathcal{B}_c)^{ij} H_m^{km} (\mathcal{B}_8)_{kil} (M)_j^l + P'_{2S}(\mathcal{B}_c)^{ij} H_m^{km} (M)_j^l \left[(\mathcal{B}_8)_{kli} + (\mathcal{B}_8)_{ilk}\right],
\end{aligned}$$

Redefinition:

$$\tilde{T} = T - E_{1S}, \quad \tilde{C} = C + E_{1S}, \quad \tilde{C}' = C' - 2E_{1S},$$
$$\tilde{E}_1 = E_{1A} + E_{1S} - E_3, \quad \tilde{E}_h = E_h + 2E_{1S}.$$

5 tilde TDA amplitudes

Formalism

Decay amplitudes (tree)

- **CF**

Channel	$\widetilde{\mathrm{TDA}}$
$\Lambda_c^+ \to \Lambda\pi^+$	$\frac{1}{\sqrt{6}}(-4\tilde{T} + \tilde{C}' + \tilde{E}_1)$
$\Lambda_c^+ \to \Sigma^0\pi^+$	$\frac{1}{\sqrt{2}}(-\tilde{C}' - \tilde{E}_1)$
$\Lambda_c^+ \to \Sigma^+\pi^0$	$\frac{1}{\sqrt{2}}(\tilde{C}' + \tilde{E}_1)$
$\Lambda_c^+ \to \Sigma^+\eta_8$	$\frac{1}{\sqrt{6}}(-\tilde{C}' + \tilde{E}_1)$
$\Lambda_c^+ \to \Sigma^+\eta_1$	$\frac{1}{\sqrt{3}}(-\tilde{C}' + \tilde{E}_1 - 3\tilde{E}_h)$
$\Lambda_c^+ \to \Xi^0 K^+$	$\tilde{E}_1$
$\Lambda_c^+ \to p\bar{K}^0$	$2\tilde{C}$
$\Xi_c^0 \to \Lambda\bar{K}^0$	$\frac{1}{\sqrt{6}}(2\tilde{C} - \tilde{C}' - \tilde{E}_1)$
$\Xi_c^0 \to \Sigma^0\bar{K}^0$	$\frac{1}{\sqrt{2}}(2\tilde{C} + \tilde{C}' + \tilde{E}_1)$
$\Xi_c^0 \to \Sigma^+ K^-$	$-\tilde{E}_1$
$\Xi_c^0 \to \Xi^0\pi^0$	$\frac{1}{\sqrt{2}}(-\tilde{C}')$
$\Xi_c^0 \to \Xi^0\eta_8$	$\frac{1}{\sqrt{6}}(\tilde{C}' + 2\tilde{E}_1)$
$\Xi_c^0 \to \Xi^0\eta_1$	$\frac{1}{\sqrt{3}}(\tilde{C}' - \tilde{E}_1 + 3\widetilde{E}_h)$
$\Xi_c^0 \to \Xi^-\pi^+$	$2\tilde{T}$
$\Xi_c^+ \to \Sigma^+\bar{K}^0$	$-2\tilde{C} - \tilde{C}'$
$\Xi_c^+ \to \Xi^0\pi^+$	$-2\tilde{T} + \tilde{C}'$

- **SCS**

Channel	$\widetilde{\mathrm{TDA}}$
$\Lambda_c^+ \to \Lambda K^+$	$\frac{1}{\sqrt{6}}(-4\tilde{T} + \tilde{C}' - 2\tilde{E}_1)$
$\Lambda_c^+ \to \Sigma^0 K^+$	$\frac{1}{\sqrt{2}}(-\tilde{C}')$
$\Lambda_c^+ \to \Sigma^+ K^0$	$-\tilde{C}'$
$\Lambda_c^+ \to p\pi^0$	$\frac{1}{\sqrt{2}}(2\tilde{C} + \tilde{C}' + \tilde{E}_1)$
$\Lambda_c^+ \to p\eta_8$	$\frac{1}{\sqrt{6}}(-6\tilde{C} - \tilde{C}' + \tilde{E}_1)$
$\Lambda_c^+ \to p\eta_1$	$\frac{1}{\sqrt{3}}(-\tilde{C}' + \tilde{E}_1 - 3\tilde{E}_h)$
$\Lambda_c^+ \to n\pi^+$	$-2\tilde{T} + \tilde{C}' + \tilde{E}_1$
$\Xi_c^0 \to \Lambda\pi^0$	$\frac{1}{2\sqrt{3}}(2\tilde{C} + 2\tilde{C}' - \tilde{E}_1)$
$\Xi_c^0 \to \Lambda\eta_8$	$\frac{1}{2}(-2\tilde{C} - \tilde{E}_1)$
$\Xi_c^0 \to \Lambda\eta_1$	$\frac{1}{\sqrt{2}}(-\tilde{C}' + \tilde{E}_1 - 3\tilde{E}_h)$
$\Xi_c^0 \to \Sigma^0\pi^0$	$\frac{1}{2}(2\tilde{C} + \tilde{E}_1)$
$\Xi_c^0 \to \Sigma^0\eta_8$	$\frac{1}{2\sqrt{3}}(-6\tilde{C} - 2\tilde{C}' - \tilde{E}_1)$
$\Xi_c^0 \to \Sigma^0\eta_1$	$\frac{1}{\sqrt{6}}(\tilde{C}' - \tilde{E}_1 + 3\tilde{E}_h)$
$\Xi_c^0 \to \Sigma^+\pi^-$	$\tilde{E}_1$
$\Xi_c^0 \to \Sigma^-\pi^+$	$-2\tilde{T}$
$\Xi_c^0 \to \Xi^0 K^0$	$\tilde{C}' + \tilde{E}_1$
$\Xi_c^0 \to \Xi^- K^+$	$2\tilde{T}$
$\Xi_c^0 \to pK^-$	$-\tilde{E}_1$
$\Xi_c^0 \to n\bar{K}^0$	$-\tilde{C}' - \tilde{E}_1$
$\Xi_c^+ \to \Lambda\pi^+$	$\frac{1}{\sqrt{6}}(2\tilde{T} - 2\tilde{C}' + \tilde{E}_1)$
$\Xi_c^+ \to \Sigma^0\pi^+$	$\frac{1}{\sqrt{2}}(-2\tilde{T} - \tilde{E}_1)$
$\Xi_c^+ \to \Sigma^+\pi^0$	$\frac{1}{\sqrt{2}}(-2\tilde{C} + \tilde{E}_1)$
$\Xi_c^+ \to \Sigma^+\eta_8$	$\frac{1}{\sqrt{6}}(6\tilde{C} + 2\tilde{C}' + \tilde{E}_1)$
$\Xi_c^+ \to \Sigma^+\eta_1$	$\frac{1}{\sqrt{3}}(-\tilde{C}' + \tilde{E}_1 - 3\tilde{E}_h)$
$\Xi_c^+ \to \Xi^0 K^+$	$-2\tilde{T} + \tilde{C}' + \tilde{E}_1$
$\Xi_c^+ \to p\bar{K}^0$	$-\tilde{C}'$

- **DCS**

Channel	$\widetilde{\mathrm{TDA}}$
$\Lambda_c^+ \to pK^0$	$2\tilde{C} + \tilde{C}'$
$\Lambda_c^+ \to nK^+$	$2\tilde{T} - \tilde{C}'$
$\Xi_c^0 \to \Lambda K^0$	$\frac{1}{\sqrt{6}}(2\tilde{C} + 2\tilde{C}' + 2\tilde{E}_1)$
$\Xi_c^0 \to \Sigma^0 K^0$	$\frac{1}{\sqrt{2}}(2\tilde{C})$
$\Xi_c^0 \to \Sigma^- K^+$	$2\tilde{T}$
$\Xi_c^0 \to p\pi^-$	$-\tilde{E}_1$
$\Xi_c^0 \to n\pi^0$	$\frac{1}{\sqrt{2}}(\tilde{E}_1)$
$\Xi_c^0 \to n\eta_8$	$\frac{1}{\sqrt{6}}(-2\tilde{C}' - \tilde{E}_1)$
$\Xi_c^0 \to n\eta_1$	$\frac{1}{\sqrt{3}}(\tilde{C}' - \tilde{E}_1 + 3\tilde{E}_h)$
$\Xi_c^+ \to \Lambda K^+$	$\frac{1}{\sqrt{6}}(-2\tilde{T} + 2\tilde{C}' + 2\tilde{E}_1)$
$\Xi_c^+ \to \Sigma^0 K^+$	$\frac{1}{\sqrt{2}}(2\tilde{T})$
$\Xi_c^+ \to \Sigma^+ K^0$	$-2\tilde{C}$
$\Xi_c^+ \to p\pi^0$	$\frac{1}{\sqrt{2}}(-\tilde{E}_1)$
$\Xi_c^+ \to p\eta_8$	$\frac{1}{\sqrt{6}}(-2\tilde{C}' - \tilde{E}_1)$
$\Xi_c^+ \to p\eta_1$	$\frac{1}{\sqrt{3}}(\tilde{C}' - \tilde{E}_1 + 3\tilde{E}_h)$
$\Xi_c^+ \to n\pi^+$	$-\tilde{E}_1$

Formalism

Equivalence between TDA and IRA

(1) Writing down the TDA and IRA amplitudes and then comparing them to sort out their relations

$$\begin{aligned}\mathcal{A}_{\rm IRAa} = \; & a_1 (\mathcal{B}_c)_i (H_{\bar 6})^{ik}_j (\mathcal{B}_8)^j_k M^l_l + a_2 (\mathcal{B}_c)_i (H_{\bar 6})^{ik}_j (\mathcal{B}_8)^l_k M^j_l + a_3 (\mathcal{B}_c)_i (H_{\bar 6})^{ik}_j (\mathcal{B}_8)^j_l M^l_k \\ & + a_4 (\mathcal{B}_c)_i (H_{\bar 6})^{jk}_l (\mathcal{B}_8)^i_j M^l_k + a_5 (\mathcal{B}_c)_i (H_{\bar 6})^{jk}_l (\mathcal{B}_8)^l_j M^i_k + \cancel{a_6 (\mathcal{B}_c)_i (H_{15})^{ik}_j (\mathcal{B}_8)^j_k M^l_l} \\ & + \cancel{a_7 (\mathcal{B}_c)_i (H_{15})^{ik}_j (\mathcal{B}_8)^l_k M^j_l} + \cancel{a_8 (\mathcal{B}_c)_i (H_{15})^{ik}_j (\mathcal{B}_8)^j_l M^l_k} + a_9 (\mathcal{B}_c)_i (H_{15})^{jk}_l (\mathcal{B}_8)^i_j M^l_k \\ & + \cancel{a_{10} (\mathcal{B}_c)_i (H_{15})^{jk}_l (\mathcal{B}_8)^l_j M^i_k} + b_1 (\mathcal{B}_c)_i (H_3)^j (\mathcal{B}_8)^i_j M^l_l + b_2 (\mathcal{B}_c)_i (H_3)^j (\mathcal{B}_8)^l_j M^i_l \\ & + b_3 (\mathcal{B}_c)_i (H_3)^i (\mathcal{B}_8)^l_j M^j_l + b_4 (\mathcal{B}_c)_i (H_3)^l (\mathcal{B}_8)^i_j M^j_l\end{aligned}$$

$$\tilde T = \frac{1}{2}(-a_2 + a_4 + a_9), \qquad \tilde C = \frac{1}{2}(a_2 - a_4 + a_9),$$
$$\tilde C' = -a_2 - a_5, \qquad \tilde E_1 = a_3 + a_5, \qquad \tilde E_h = -a_1 + a_5.$$

(2) Make a direct transformation from the TDA to IRA

$$\begin{aligned}\mathcal{A}^{\rm tree}_{\rm TDA} = \; & (T+C)(\mathcal{B}_c)_i (H_{15})^{jl}_m (\mathcal{B}_8)^i_j M^m_l - E_h (\mathcal{B}_c)_i (H_{\bar 6})^{ij}_l (\mathcal{B}_8)^l_j M^m_m \\ & + (T - C - C' - 2E_{1S})(\mathcal{B}_c)_i (H_{\bar 6})^{jl}_m (\mathcal{B}_8)^i_j M^m_l - C'(\mathcal{B}_c)_i (H_{\bar 6})^{ij}_m (\mathcal{B}_8)^l_j M^m_l \\ & + (E_{1A} - E_{1S} - E_3)(\mathcal{B}_c)_i (H_{\bar 6})^{il}_j (\mathcal{B}_8)^j_m M^m_l + 2E_{1S}(\mathcal{B}_c)_i (H_{\bar 6})^{jl}_m (\mathcal{B}_8)^m_j M^i_l\end{aligned}$$

$$a_1 = -E_h, \qquad a_2 = -C', \qquad a_3 = E_{1A} - E_{1S} - E_3,$$
$$a_4 = T - C - C' - 2E_{1S}, \qquad a_5 = 2E_{1S}, \qquad a_9 = T + C$$

$$(\mathcal{B}_c)^{ij} = \epsilon^{ijk}(\mathcal{B}_c)_k$$
$$(\mathcal{B}_8)_{ijk} = \epsilon_{ijl}(\mathcal{B}_8^T)^l_k$$
$$H^{ij}_k$$

Numerical Results

Fitting scheme

The tidle TDA parameters:

$$|\tilde{T}|_S e^{i\delta_S^{\tilde{T}}},\quad |\tilde{C}|_S e^{i\delta_S^{\tilde{C}}},\quad |\tilde{C}'|_S e^{i\delta_S^{\tilde{C}'}},\quad |\tilde{E}_1|_S e^{i\delta_S^{\tilde{E}_1}},\quad |\tilde{E}_h|_S e^{i\delta_S^{\tilde{E}_h}},$$
$$|\tilde{T}|_P e^{i\delta_P^{\tilde{T}}},\quad |\tilde{C}|_P e^{i\delta_P^{\tilde{C}}},\quad |\tilde{C}'|_P e^{i\delta_P^{\tilde{C}'}},\quad |\tilde{E}_1|_P e^{i\delta_P^{\tilde{E}_1}},\quad |\tilde{E}_h|_P e^{i\delta_P^{\tilde{E}_h}},$$

The tidle IRA parameters:

$$|\tilde{f}^a|_S e^{i\delta_S^{\tilde{f}^a}},\quad |\tilde{f}^b|_S e^{i\delta_S^{\tilde{f}^b}},\quad |\tilde{f}^c|_S e^{i\delta_S^{\tilde{f}^c}},\quad |\tilde{f}^d|_S e^{i\delta_S^{\tilde{f}^d}},\quad |\tilde{f}^e|_S e^{i\delta_S^{\tilde{f}^e}},$$
$$|\tilde{f}^a|_P e^{i\delta_P^{\tilde{f}^a}},\quad |\tilde{f}^b|_P e^{i\delta_P^{\tilde{f}^b}},\quad |\tilde{f}^c|_P e^{i\delta_P^{\tilde{f}^c}},\quad |\tilde{f}^d|_P e^{i\delta_P^{\tilde{f}^d}},\quad |\tilde{f}^e|_P e^{i\delta_P^{\tilde{f}^e}}.$$

19 parameters were left after an overall phase shift, then we conducted an minimum χ^2 fit under:

Scheme I: Without the recent Belle data on $\Xi_c^0 \to \Xi^0\pi^0, \eta, \eta'$ (3 Br + 1 α)

Scheme II: All the currently available data included (38 observables in total)

The χ^2 is defined as $\chi^2 = [\mathcal{O}_{\text{theor}}(c_i) - \mathcal{O}_{\text{expt}}]^{\mathrm{T}} \Sigma^{-1} [\mathcal{O}_{\text{theor}}(c_i) - \mathcal{O}_{\text{expt}}]$

diagonal!

Numerical Results

The best fit points

	Scheme I		Scheme II	
	TDA	IRA	TDA	IRA
χ^2	34.31	33.21	59.17	57.08
$\chi^2/d.o.f.$	2.29	2.21	3.11	3.00

4 sets!

- **Scheme I**

	$\lvert X_i\rvert_S$ ($10^{-2}G_F$ GeV^2)	$\lvert X_i\rvert_P$	$\delta_S^{X_i}$ (in radian)	$\delta_P^{X_i}$
$\tilde{T}$	4.25 ± 0.11	12.43 ± 0.30	–	2.40 ± 0.04
$\tilde{C}$	3.08 ± 0.52	11.57 ± 1.00	3.02 ± 0.12	-0.77 ± 0.20
$\tilde{C}'$	5.39 ± 0.38	18.79 ± 0.86	-0.03 ± 0.05	2.23 ± 0.11
$\tilde{E}_1$	2.90 ± 0.19	10.22 ± 0.50	-2.80 ± 0.05	1.86 ± 0.10
$\tilde{E}_h$	4.06 ± 0.53	13.82 ± 1.93	2.66 ± 0.12	-1.90 ± 0.20
$\tilde{f}^a$	4.10 ± 0.52	16.18 ± 2.34	–	1.72 ± 0.12
$\tilde{f}^b$	7.00 ± 1.36	24.52 ± 1.72	-2.78 ± 0.11	-0.30 ± 0.19
$\tilde{f}^c$	2.91 ± 0.19	10.26 ± 0.49	-2.36 ± 0.11	2.29 ± 0.14
$\tilde{f}^d$	1.59 ± 1.19	7.50 ± 3.87	-2.89 ± 0.40	0.25 ± 0.40
$\tilde{f}^e$	1.57 ± 1.39	0.71 ± 2.92	-2.44 ± 0.22	-1.64 ± 3.55

- **Scheme II**

	$\lvert X_i\rvert_S$ ($10^{-2}G_F$ GeV^2)	$\lvert X_i\rvert_P$	$\delta_S^{X_i}$ (in radian)	$\delta_P^{X_i}$
$\tilde{T}$	4.22 ± 0.10	12.50 ± 0.28	–	2.42 ± 0.04
$\tilde{C}$	2.40 ± 0.66	12.70 ± 0.71	2.88 ± 0.59	-0.57 ± 0.15
$\tilde{C}'$	5.26 ± 0.35	19.04 ± 0.85	-0.02 ± 0.05	2.32 ± 0.11
$\tilde{E}_1$	2.86 ± 0.19	10.20 ± 0.50	-2.80 ± 0.05	1.83 ± 0.09
$\tilde{E}_h$	3.07 ± 0.47	11.80 ± 1.42	2.87 ± 0.09	-1.75 ± 0.19
$\tilde{f}^a$	3.16 ± 0.43	10.74 ± 1.73	–	1.68 ± 0.15
$\tilde{f}^b$	7.52 ± 0.30	23.27 ± 0.69	-2.98 ± 0.09	-0.56 ± 0.10
$\tilde{f}^c$	2.86 ± 0.19	10.19 ± 0.49	-2.51 ± 0.09	2.13 ± 0.12
$\tilde{f}^d$	2.34 ± 0.20	4.30 ± 0.65	3.02 ± 0.21	-0.75 ± 0.39
$\tilde{f}^e$	1.48 ± 0.32	3.82 ± 1.04	-2.05 ± 0.17	0.60 ± 0.15

Numerical Results

Predictions and comparison (Scheme I)

- **CF channels**

Channel	$10^2\mathcal{B}$	α	β	γ	$\lvert A\rvert$	$\lvert B\rvert$	$\delta_P-\delta_S$	$10^2\mathcal{B}_{\rm exp}$	$\alpha_{\rm exp}$	$\beta_{\rm exp}$ / $\gamma_{\rm exp}$
$\Lambda_c^+\to\Lambda^0\pi^+$	1.30 ± 0.04	-0.76 ± 0.01	0.39 ± 0.02	0.51 ± 0.01	5.57 ± 0.10	9.24 ± 0.20	2.67 ± 0.19	1.29 ± 0.05	-0.762 ± 0.006	0.368 ± 0.021
	1.30 ± 0.05	-0.76 ± 0.01	0.39 ± 0.02	0.51 ± 0.01	5.57 ± 0.10	9.23 ± 0.20	2.67 ± 0.02			0.502 ± 0.017
$\Lambda_c^+\to\Sigma^0\pi^+$	1.25 ± 0.05	-0.47 ± 0.01	0.35 ± 0.10	-0.81 ± 0.04	1.94 ± 0.22	19.20 ± 0.46	2.50 ± 0.14	1.27 ± 0.06	-0.466 ± 0.018	
	1.25 ± 0.05	-0.47 ± 0.01	0.36 ± 0.10	-0.80 ± 0.05	1.97 ± 0.23	19.14 ± 0.47	2.48 ± 0.13			
$\Lambda_c^+\to\Sigma^+\pi^0$	1.26 ± 0.05	-0.47 ± 0.01	0.35 ± 0.10	-0.81 ± 0.04	1.94 ± 0.22	19.20 ± 0.46	2.50 ± 0.14	1.24 ± 0.09	-0.484 ± 0.027	
	1.26 ± 0.05	-0.47 ± 0.01	0.36 ± 0.10	-0.81 ± 0.05	1.97 ± 0.23	19.14 ± 0.47	2.48 ± 0.13			
$\Lambda_c^+\to\Sigma^+\eta$	0.33 ± 0.04	-0.92 ± 0.04	-0.01 ± 0.15	0.40 ± 0.10	2.94 ± 0.21	6.98 ± 0.73	-3.13 ± 0.21	0.32 ± 0.05	-0.99 ± 0.06	
	0.32 ± 0.04	-0.92 ± 0.04	-0.15 ± 0.16	0.36 ± 0.12	2.87 ± 0.20	7.15 ± 0.83	-2.98 ± 0.17			
$\Lambda_c^+\to\Sigma^+\eta'$	0.39 ± 0.07	-0.44 ± 0.07	0.88 ± 0.06	0.16 ± 0.28	4.03 ± 0.78	21.52 ± 2.63	2.03 ± 0.08	0.41 ± 0.08	-0.46 ± 0.07	
	0.43 ± 0.07	-0.45 ± 0.07	0.90 ± 0.03	-0.02 ± 0.3	3.91 ± 0.80	24.80 ± 3.35	2.03 ± 0.08			
$\Lambda_c^+\to\Xi^0K^+$	0.34 ± 0.03	-0.04 ± 0.12	-0.98 ± 0.02	0.19 ± 0.09	2.76 ± 0.18	9.71 ± 0.47	-1.61 ± 0.12	0.55 ± 0.07	0.01 ± 0.16	
	0.34 ± 0.03	-0.06 ± 0.12	-0.98 ± 0.02	0.19 ± 0.09	2.77 ± 0.18	9.75 ± 0.46	-1.63 ± 0.12			
$\Lambda_c^+\to pK_S$	1.56 ± 0.06	-0.74 ± 0.03	0.56 ± 0.16	-0.37 ± 0.23	4.17 ± 0.74	15.70 ± 1.43	2.50 ± 0.14	1.59 ± 0.07	-0.743 ± 0.028	
	1.59 ± 0.06	-0.74 ± 0.03	0.44 ± 0.54	-0.51 ± 0.47	3.73 ± 1.76	16.60 ± 2.62	2.61 ± 0.49			
$\Xi_c^0\to\Xi^-\pi^+$	2.97 ± 0.09	-0.73 ± 0.03	0.67 ± 0.03	0.13 ± 0.04	8.07 ± 0.21	23.63 ± 0.57	2.40 ± 0.04	1.80 ± 0.52	-0.64 ± 0.05	
	2.96 ± 0.09	-0.72 ± 0.03	0.68 ± 0.03	0.13 ± 0.04	8.08 ± 0.21	23.47 ± 0.58	2.38 ± 0.04			
$\Xi_c^0\to\Xi^0\pi^0$	0.72 ± 0.04	-0.64 ± 0.07	0.77 ± 0.07	-0.06 ± 0.10	3.62 ± 0.26	12.63 ± 0.58	2.26 ± 0.10	$0.69\pm0.16^*$	$-0.90\pm0.27^*$	
	0.71 ± 0.04	-0.61 ± 0.08	0.79 ± 0.06	-0.04 ± 0.10	3.65 ± 0.25	12.49 ± 0.58	2.22 ± 0.10			
$\Xi_c^0\to\Xi^0\eta$	0.26 ± 0.04	0.23 ± 0.15	-0.09 ± 0.15	-0.97 ± 0.04	0.43 ± 0.27	12.58 ± 1.07	-0.36 ± 0.56	$0.16\pm0.05^*$		
	0.23 ± 0.04	0.23 ± 0.15	-0.15 ± 0.15	-0.96 ± 0.05	0.45 ± 0.26	11.75 ± 0.97	-0.57 ± 0.35			
$\Xi_c^0\to\Xi^0\eta'$	0.43 ± 0.06	-0.70 ± 0.06	0.71 ± 0.06	-0.07 ± 0.27	3.91 ± 0.75	23.72 ± 2.55	2.35 ± 0.08	$0.12\pm0.04^*$		
	0.49 ± 0.07	-0.67 ± 0.06	0.70 ± 0.09	-0.25 ± 0.28	3.73 ± 0.77	27.17 ± 3.32	2.33 ± 0.09			
$\Xi_c^+\to\Xi^0\pi^+$	1.0 ± 0.1	-0.88 ± 0.08	0.31 ± 0.10	0.36 ± 0.13	2.95 ± 0.23	6.67 ± 0.86	2.80 ± 0.45	1.6 ± 0.8		
	1.0 ± 0.1	-0.90 ± 0.07	0.29 ± 0.10	0.32 ± 0.13	2.93 ± 0.22	6.94 ± 0.85	2.83 ± 0.12			

Scheme II

Channel	$10^2\mathcal{B}$
$\Lambda_c^+\to\Sigma^+\eta'$	0.18 ± 0.03
	0.17 ± 0.04

Channel	$10^2\mathcal{B}$
$\Xi_c^0\to\Xi^0\eta$	0.25 ± 0.03
	0.26 ± 0.03
$\Xi_c^0\to\Xi^0\eta'$	0.23 ± 0.03
	0.22 ± 0.03

Numerical Results

Predictions and comparison (Scheme I)

- **SCS channels**

Channel	$10^2\mathcal{B}$	α	β	γ	$\lvert A\rvert$	$\lvert B\rvert$	$\delta_P-\delta_S$	$10^2\mathcal{B}_{\rm exp}$	$\alpha_{\rm exp}$	$\beta_{\rm exp}$ / $\gamma_{\rm exp}$
$\Lambda_c^+\to\Lambda^0K^+$	0.0635 ± 0.0030	-0.58 ± 0.04	0.40 ± 0.07	-0.71 ± 0.04	0.57 ± 0.04	4.41 ± 0.11	2.53 ± 0.10	0.0642 ± 0.0031	-0.579 ± 0.041	0.35 ± 0.13
	0.0636 ± 0.0030	-0.58 ± 0.04	0.40 ± 0.07	-0.71 ± 0.04	0.57 ± 0.04	4.42 ± 0.11	2.54 ± 0.10			-0.743 ± 0.071
$\Lambda_c^+\to\Sigma^0K^+$	0.0380 ± 0.0023	-0.64 ± 0.08	0.77 ± 0.06	0.01 ± 0.10	0.83 ± 0.06	2.91 ± 0.13	2.26 ± 0.10	0.0370 ± 0.0031	-0.54 ± 0.20	
	0.0379 ± 0.0022	-0.61 ± 0.08	0.79 ± 0.06	0.03 ± 0.10	0.84 ± 0.06	2.88 ± 0.13	2.22 ± 0.10			
$\Lambda_c^+\to\Sigma^+K_S$	0.0381 ± 0.0023	-0.64 ± 0.08	-0.77 ± 0.06	0.01 ± 0.10	0.83 ± 0.06	2.91 ± 0.13	2.26 ± 0.10	0.047 ± 0.014		
	0.0379 ± 0.0022	-0.61 ± 0.08	0.79 ± 0.06	0.03 ± 0.10	0.84 ± 0.06	2.88 ± 0.13	2.22 ± 0.10			
$\Lambda_c^+\to n\pi^+$	0.071 ± 0.007	-0.52 ± 0.11	-0.71 ± 0.04	0.47 ± 0.10	1.30 ± 0.06	1.87 ± 0.24	-2.21 ± 0.11	0.066 ± 0.013		
	0.073 ± 0.007	-0.56 ± 0.10	-0.70 ± 0.04	0.44 ± 0.10	1.30 ± 0.06	1.95 ± 0.23	-2.24 ± 0.10			
$\Lambda_c^+\to p\pi^0$	0.0186 ± 0.0034	-0.33 ± 0.58	-0.94 ± 0.23	-0.03 ± 0.57	0.54 ± 0.14	1.34 ± 0.44	-1.91 ± 0.62	$0.0156^{+0.0075}_{-0.0061}$		
	0.0196 ± 0.0060	-0.51 ± 0.16	-0.76 ± 0.62	-0.40 ± 1.29	0.43 ± 0.41	1.59 ± 0.95	-2.16 ± 0.34			
$\Lambda_c^+\to p\eta$	0.163 ± 0.010	-0.66 ± 0.06	0.44 ± 0.22	-0.61 ± 0.18	1.08 ± 0.26	5.63 ± 0.32	2.55 ± 0.23	0.158 ± 0.011		
	0.156 ± 0.009	-0.66 ± 0.08	0.24 ± 0.88	-0.71 ± 0.34	0.91 ± 0.54	5.70 ± 0.60	2.79 ± 1.16			
$\Lambda_c^+\to p\eta'$	0.052 ± 0.008	-0.44 ± 0.11	0.65 ± 0.19	-0.61 ± 0.20	0.73 ± 0.17	4.82 ± 0.55	2.17 ± 0.20	0.048 ± 0.009		
	0.050 ± 0.008	-0.46 ± 0.42	0.61 ± 0.26	-0.65 ± 0.24	0.68 ± 0.22	4.76 ± 0.59	2.22 ± 0.59			

- **Ratios of branching fractions**

Channel	$10^2\mathcal{R}_X$	α	β	γ	$\lvert A\rvert$	$\lvert B\rvert$	$\delta_P-\delta_S$	$10^2(\mathcal{R}_X)_{\rm exp}$
$\Xi_c^0\to\Lambda K_S^0$	23.0 ± 0.8	-0.62 ± 0.03	0.53 ± 0.11	-0.58 ± 0.11	2.47 ± 0.34	14.07 ± 0.50	2.43 ± 0.10	22.5 ± 1.3
	23.1 ± 0.8	-0.61 ± 0.03	0.47 ± 0.32	-0.64 ± 0.22	2.31 ± 0.72	14.28 ± 0.71	2.49 ± 0.35	
$\Xi_c^0\to\Sigma^0K_S^0$	3.8 ± 0.6	-0.40 ± 0.61	-0.92 ± 0.05	0.35 ± 0.58	1.80 ± 0.46	3.84 ± 1.65	-2.02 ± 0.62	3.8 ± 0.7
	3.6 ± 0.7	-0.63 ± 0.17	-0.86 ± 0.26	-0.06 ± 1.59	1.47 ± 1.33	4.79 ± 3.35	-2.26 ± 0.28	
$\Xi_c^0\to\Sigma^+K^-$	13.9 ± 1.0	-0.04 ± 0.12	-0.99 ± 0.01	-0.14 ± 0.09	2.76 ± 0.18	9.71 ± 0.47	-1.61 ± 0.12	12.3 ± 1.2
	14.0 ± 0.9	-0.06 ± 0.12	-0.99 ± 0.02	-0.14 ± 0.09	2.77 ± 0.18	9.75 ± 0.46	-1.63 ± 0.12	
$\Xi_c^0\to\Xi^-K^+$	4.39 ± 0.02	-0.72 ± 0.03	0.66 ± 0.03	0.21 ± 0.04	1.86 ± 0.05	5.44 ± 0.13	2.40 ± 0.04	2.75 ± 0.57
	4.39 ± 0.02	-0.71 ± 0.03	0.67 ± 0.03	0.22 ± 0.04	1.86 ± 0.05	5.41 ± 0.13	2.38 ± 0.04	

Numerical Results

Discussion

M. Ablikim et al. [BESIII], Phys. Rev. Lett. 132, 031801 (2024).

For the decay $\Lambda_c^+ \to \Xi^0 K^+$, BESIII found two sets of solutions for the magnitudes of S- and P-wave amplitudes in units of $10^{-2} G_F \mathrm{GeV}$:

$$\text{I.} \begin{cases} |A| = 1.6^{+1.9}_{-1.6} \pm 0.4\,, \\ |B| = 18.3 \pm 2.8 \pm 0.7\,, \end{cases} \qquad \text{II.} \begin{cases} |A| = 4.3^{+0.7}_{-0.2} \pm 0.4\,, \\ |B| = 6.7^{+8.3}_{-6.7} \pm 1.6\,, \end{cases}$$

$$\alpha = \frac{2\kappa|A^*B|\cos(\delta_P - \delta_S)}{|A|^2 + \kappa^2|B|^2}, \quad \beta = \frac{2\kappa|A^*B|\sin(\delta_P - \delta_S)}{|A|^2 + \kappa^2|B|^2}, \quad \gamma = \frac{|A|^2 - \kappa^2|B|^2}{|A|^2 + \kappa^2|B|^2},$$

and two solutions for the phase shift,

$$\delta_P - \delta_S = -1.55 \pm 0.25 \pm 0.05 \quad \text{or} \quad 1.59 \pm 0.25 \pm 0.05 \text{ rad.}$$

Our fits in Scheme I TDA give predictions:

$$|A| = 2.76 \pm 0.18,\ |B| = 9.71 \pm 0.47,\ \delta_P - \delta_S = -1.61 \pm 0.12 \text{ rad}$$

β and γ measured by LHCb provided help!

Numerical Results

Discussion

Y. B. Li et al. [Belle], Phys. Rev. Lett. 122, no.8, 082001 (2019) .
S. Navas et al. [Particle Data Group], Phys. Rev. D 110, 030001 (2024).

The predicted $Br(\Xi_c^0 \to \Xi^-\pi^+) = (2.97 \pm 0.09)\%$ is noticeably higher than the measured value of $(1.80 \pm 0.52)\%$ by Belle and two times larger than the PDG value of $(1.47 \pm 0.27)\%$. Howerver, from sum rule derived in both TDA and IRA:

$$\frac{\tau_{\Lambda_c^+}}{\tau_{\Xi_c^0}}\mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+) \cong 3\mathcal{B}(\Lambda_c^+ \to \Lambda\pi^+) + \mathcal{B}(\Lambda_c^+ \to \Sigma^0\pi^+) - \frac{1}{\sin^2\theta_C}\mathcal{B}(\Lambda_c^+ \to n\pi^+).$$

and the measured data, we find $Br(\Xi_c^0 \to \Xi^-\pi^+) = (2.85 \pm 0.30)\%$ in good agreement with the aforementioned prediction.

$$\begin{cases} \mathcal{A}\left(\Xi_c^0 \to \Xi^-\pi^+\right) = 3\mathcal{A}\left(\Lambda_c^+ \to \Lambda\pi^+\right) + \mathcal{A}\left(\Lambda_c^+ \to \Sigma^0\pi^+\right) - \frac{1}{\sin^2\theta_C}\mathcal{A}\left(\Lambda_c^+ \to n\pi^+\right) \\ \Gamma = \frac{p_c}{8\pi}\frac{(m_i+m_f)^2 - m_P^2}{m_i^2}\left(|A|^2 + \kappa^2|B|^2\right) \end{cases}$$

$$\kappa = p_c/(E_f + m_f) = \sqrt{(E_f - m_f)/(E_f + m_f)}$$

Numerical Results

Discussion

M. Ablikim et al. [BESIII], JHEP09(2024)007.

First measurement of the $K_S^0 - K_L^0$ asymmetries from BESIII

$$R(\Lambda_c^+, K_{S,L}^0 X) = \frac{\mathcal{B}(\Lambda_c^+ \to K_S^0 X) - \mathcal{B}(\Lambda_c^+ \to K_L^0 X)}{\mathcal{B}(\Lambda_c^+ \to K_S^0 X) + \mathcal{B}(\Lambda_c^+ \to K_L^0 X)}$$

$$R(\Lambda_c^+, pK_{S,L}^0) = -0.025 \pm 0.031$$

and the absolute branching fractions:

$$\mathcal{B}(\Lambda_c^+ \to pK_L^0) = (1.67 \pm 0.06 \pm 0.04)\%$$

	Exp.	S1 (TDA)	S1 + pK_L (TDA)
$Br(\Lambda_c^+ \to pK_S)$	$(1.59 \pm 0.07)\%$	$(1.56 \pm 0.06)\%$	$(1.63 \pm 0.05)\%$
$Br(\Lambda_c^+ \to pK_L)$	$(1.67 \pm 0.07)\%$	$(1.50 \pm 0.06)\%$	$(1.57 \pm 0.04)\%$

Summary

- In tree level, the number of the minimum set of tensor invariants in the IRA and the topological amplitudes in the TDA is the same, namely, five in the tree-induced amplitudes.
- The recent measurements of the decay parameters β and γ by LHCb enable to fix the sign ambiguity of β and pick up the solution for S- and P-wave amplitudes.
- There is an issue of how to accommodate both $\Lambda_c^+ \to \Sigma^+\eta'$ and $\Xi_c^0 \to \Xi^0\eta'$ simultaneously, which needs to be clarified in the future.
- The predicted branching fraction of $(2.97 \pm 0.09)\%$ for $\Xi_c^0 \to \Xi^-\pi^+$ is higher than its current value, which should be tested soon.

Thank you !

Formalism

From Topological diagrams to decay amplitudes

$$\mathcal{B}_c(\bar{3}) \to \mathcal{B}(8)P(8+1)$$

The effective Hamiltonian:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\left[\sum_{q_1,q_2}^{d,s} V^*_{cq_1}V_{uq_2}(c_1O_1^{q_1q_2}+c_2O_2^{q_1q_2}) - \lambda_b\sum_{i=3}^{6}c_iO_i\right] + h.c.$$

$$O_1^{q_1q_2} = (\bar{u}q_2)(\bar{q}_1c),\ O_2^{q_1q_2} = (\bar{q}_1q_2)(\bar{u}c)$$

O_{3-6} are QCD-penguin operators

For the SCS processes:

$$\begin{aligned}
\mathcal{H}_{\rm eff} \propto\ & \lambda_d\left[c_1(\bar{u}d)(\bar{d}c)+c_2(\bar{d}d)(\bar{u}c)\right]+\lambda_s\left[(c_1(\bar{u}s)(\bar{s}c)+c_2(\bar{s}s)(\bar{u}c)\right] \\
=\ & \frac{1}{2}(\lambda_s-\lambda_d)\Big\{c_+\left[(\bar{u}s)(\bar{s}c)+(\bar{s}s)(\bar{u}c)-(\bar{u}d)(\bar{d}c)-(\bar{d}d)(\bar{u}c)\right]_{\mathbf{15}} \\
& + c_-\left[(\bar{u}s)(\bar{s}c)-(\bar{s}s)(\bar{u}c)-(\bar{u}d)(\bar{d}c)+(\bar{d}d)(\bar{u}c)\right]_{\bar{\mathbf{6}}}\Big\} \\
& -\frac{\lambda_b}{4}\Big\{c_+\left[(\bar{u}s)(\bar{s}c)+(\bar{s}s)(\bar{u}c)+(\bar{u}d)(\bar{d}c)+(\bar{d}d)(\bar{u}c)-2(\bar{u}u)(\bar{u}c)\right]_{\mathbf{15^b}} \\
& +(c_++2c_-)(O_{3t})_{\mathbf{3t}}+(c_+-2c_-)(O_{3p})_{\mathbf{3p}}\Big\},
\end{aligned}$$

$$O_{3t} \equiv (\bar{u}u)(\bar{u}c)+(\bar{u}d)(\bar{d}c)+(\bar{u}s)(\bar{s}c) = \sum_{q=u,d,s}(\bar{u}q)(\bar{q}c),$$

$$O_{3p} \equiv (\bar{u}u)(\bar{u}c)+(\bar{d}d)(\bar{u}c)+(\bar{s}s)(\bar{u}c) = \sum_{q=u,d,s}(\bar{q}q)(\bar{u}c),$$

The weak Hamiltonian can be decomposed as:

$$\mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3} = \mathbf{3_p}\oplus\mathbf{3_t}\oplus\bar{\mathbf{6}}\oplus\mathbf{15}.$$