Precision Calculation of Heavy Meson and Light Vector Meson Couplings Based on LCSRs

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Motivation

Coupling constant $g_{H^{(*)}HV}$, where $H = (B_{(s)}, D_{(s)})$, $V = (\rho, K^*, \omega, \phi)$

- It enable us to obtain the fundamental parameter of the effective Lagrangian of heavy meson chiral perturbative theory (**HMχPT**).

$$\mathcal{L}_V = i\,\lambda\,\mathrm{Tr}[\mathcal{H}_b\,\sigma^{\mu\nu}\,F_{\mu\nu}(\rho)_{ba}\,\bar{\mathcal{H}}_a]\,,\,\lambda = \frac{\sqrt{2}}{4}\,\frac{1}{g_V}\,g_{H^*H\rho}\,.$$

- It relates the pole residue of the $H \to V$ transition form factors V, A_0 and T_1 , which are key to extracting the **CKM** and the **LFU** parameters. [X.Q.Li, et al.2011.0269]

$$g_{H_1 H_2 V} \;=\; \frac{2m_V}{c_V f_{H_1}} \lim_{q^2 \to m_{H_1}^2} \left[\left(1 - \frac{q^2}{m_{H_1}^2}\right) A_0(q^2)\right],$$

- **Phenomenologically**, it describes the strength of the final state interactions (FSIs), which significantly influence the **strong phases** and **CP violation**.

- Currently, the overall accuracy in coupling constant needs improvement.

By including NLO corrections and higher-twist/power contributions, we accurately determined these **coupling constants** via **LCSRs**

LCSRs for the $H^*_{(s)} H_{(s)} V$ couplings

This work is an extension and generalization based on [Chao Wang and Hua-Dong Li,2001.05112]

- Def. with the eff. Lagrangian[Casalbuoni,et al.,9605342][Cheng,et al.,0409317][Yan,et al.,92']

$$\langle V(p,\eta^*) H^*(p+q,\varepsilon) | \mathcal{L}_{eff} | H(q) \rangle \equiv -g_{H^*HV} \epsilon_{\alpha\beta\rho\sigma} \eta^{*\alpha} \varepsilon^{*\beta} p^\rho q^\sigma ,$$

- Vacuum-V correlation function [V-meson LCDAs]

$$F_\mu(p,q) = i \int d^4x e^{-i(p+q)\cdot x} \langle \mathbf{V}(p,\eta^*) | T\{ j_\mu(x), j_5(0) \} | 0 \rangle = \epsilon_{\mu\nu\rho\sigma} \eta^{*\nu} p^\rho q^\sigma F((p+q)^2, q^2),$$

where, $j_\mu = \bar{q_1} \gamma_\mu Q$ and $j_5 = (m_Q + m_{q_2}) \bar{Q}\, i\, \gamma_5\, q_2$,

- Double dispersion relation

$$F((p+q)^2, q^2) = -\frac{g_{H^*HV} f_{H^*} f_H m_H^2 m_{H^*}}{\left[m_{H^*}^2 - (p+q)^2\right]\left[m_H^2 - q^2\right]} + \iint_\Sigma \frac{\rho_{cont}(s_1, s_2)\, ds_1\, ds_2}{[s_1 - (p+q)^2]\,(s_2 - q^2)} + \cdots .$$

The relevant matrix elements used to define the heavy meson decay constants

$$\langle 0 | j_5 | H(p) \rangle = f_H m_H^2, \qquad \langle H^*(p+q,\varepsilon) | j_\mu | 0 \rangle = f_{H^*} m_{H^*} \varepsilon_\mu .$$

$\rho_{cont}(s_1, s_2)$ captures the combined spectral density of excited and continuum states
Σ denotes the duality region occupied by these states in the (s_1, s_2)-plane

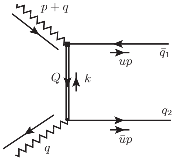
LCSRs for the $H^*_{(s)} H_{(s)} V$ couplings

- light-cone OPE expressed as a double dispersion relation

$$F^{(\mathrm{OPE})}((p+q)^2, q^2) = \iint ds_1\, ds_2 \frac{\rho^{(\mathrm{OPE})}(s_1, s_2)}{(s_1 - (p+q)^2)(s_2 - q^2)} ,$$

where the involved dual spectral density $\rho^{(\mathrm{OPE})}(s_1, s_2)$ refers to

$$\rho^{(\mathrm{OPE})}(s_1, s_2) \equiv \frac{1}{\pi^2} \mathrm{Im}_{s_1} \mathrm{Im}_{s_2} F^{(\mathrm{OPE})}(s_1, s_2) ,$$



- performing the double Borel transformation with respect to the variables $(p+q)^2 \to M_1^2$ and $q^2 \to M_2^2$

$$f_H f_{H^*} g_{H^* HV} = -\frac{1}{m_H^2 m_{H^*}} \iint^{\widetilde{\Sigma}} ds_1\, ds_s \exp\left(\frac{m_{H^*}^2 - s_1}{M_1^2} + \frac{m_H^2 - s_2}{M_2^2}\right) \rho^{(\mathrm{OPE})}(s_1, s_2) .$$

Boundary $\widetilde{\Sigma}$ dual to ground state, arises from subtracting continuum contributions with the parton-hadron duality ansatz

Double spectral density at LO

- Sum contributions from individual twists up to twist-5

$$F^{(\rm LO)} = F^{(\rm LO)}_{2p,tw2} + F^{(\rm LO)}_{2p,tw3} + F^{(\rm LO)}_{2p,tw4} + F^{(\rm LO)}_{2p,tw5} + F^{(\rm LO)}_{3p,tw4}$$

- Compact form through the power expansion

$$F^{(\rm LO)} = \frac{1+\hat{m}_{q_2}}{m_Q} \sum_{i=0}^{4} \sum_{j=1}^{4} \Big\{ \int_0^1 du\, \delta_V{}^i\, \mathcal{C}(r_1, r_2, u)^j\, \mathcal{A}^{\rm LO}_{\rm 2p,\, ij}(u) + \int_0^1 dv \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_3\, \delta_V{}^i\, \mathcal{C}(r_1, r_2, \alpha)^j\, \mathcal{A}^{\rm LO}_{\rm 3p,\, ij}(v, \underline{\alpha}) \Big\}$$

where $\delta_V = m_V/m_Q$, $\underline{\alpha} = (\alpha_1, \alpha_3)$, $\hat{m}_{q_2} = m^2_{q_2}/m^2_Q$

$$\mathcal{C}(r_1, r_2, u) = \frac{1}{\bar{u} r_1 + u r_2 - 1}\,,$$

and $r_1 = (p+q)^2/m^2_Q$, $r_2 = q^2/m^2_Q$

Double spectral density at LO

- The part involving soft functions is denoted by V-meson LCDAs

$$\mathcal{A}^{(\rm LO)}_{\rm 2p,\,01}(u) = f_V^{\perp}\,\phi^{\perp}_{2;V}(u)\,, \quad \mathcal{A}^{(\rm LO)}_{\rm 2p,\,12}(u) = -\frac{1}{2} f_V^{\parallel}\,\psi^{\perp}_{3;V}(u)\,,$$

$$\mathcal{A}^{(\rm LO)}_{\rm 2p,\,22}(u) = \frac{f_V^{\perp}}{4}\left[4u\bar{u}\phi^{\perp}_{2;V}(u) + \phi^{\perp}_{4;V}(u)\right], \quad \mathcal{A}^{(\rm LO)}_{\rm 2p,\,23}(u) = -\frac{1}{2} f_V^{\perp}\,\phi^{\perp}_{4;V}(u)\,,$$

$$\mathcal{A}^{(\rm LO)}_{\rm 2p,\,33}(u) = -f_V^{\parallel}\,u\bar{u}\psi^{\perp}_{3;V}(u)\,, \quad \mathcal{A}^{(\rm LO)}_{\rm 2p,\,34}(u) = \frac{3}{4} f_V^{\parallel}\,\psi^{\perp}_{5;V}(u),$$

$$\mathcal{A}^{(\rm LO)}_{\rm 2p,\,43}(u) = \frac{f_V^{\perp}}{2}\,u\bar{u}\left[2u\bar{u}\phi^{\perp}_{2;V}(u) + \phi^{\perp}_{4;V}(u)\right], \quad \mathcal{A}^{(\rm LO)}_{\rm 2p,\,44}(u) = -\frac{3}{2} f_V^{\perp}\,u\bar{u}\phi^{\perp}_{4;V}(u)\,,$$

$$\begin{aligned}\mathcal{A}^{(\rm LO)}_{\rm 3p,\,22}(v,\underline{\alpha}) = f_V^{\perp}\Big\{&2\bar{v}\left[\Phi^{\perp(2)}_{4;V}(\underline{\alpha}) - \Phi^{\perp(1)}_{4;V}(\underline{\alpha})\right] - \Psi^{\perp}_{4;V}(\underline{\alpha}) - (v-\bar{v})\widetilde{\Psi}^{\perp}_{4;V}(\underline{\alpha})\\ &- \frac{2(v-\bar{v})}{\bar{\alpha}}\left[\widehat{\Phi}^{\perp(2)}_{4;V}(\underline{\alpha}) - \widehat{\Phi}^{\perp(1)}_{4;V}(\underline{\alpha}) + \widehat{\Phi}^{\perp(3)}_{4;V}(\underline{\alpha}) - \widehat{\Phi}^{\perp(4)}_{4;V}(\underline{\alpha})\right]\Big\},\end{aligned}$$

$$\mathcal{A}^{(\rm LO)}_{\rm 3p,\,23}(v,\underline{\alpha}) = f_V^{\perp}\,\frac{2(v-\bar{v})(r_2-1)}{\bar{\alpha}}\left[\widehat{\Phi}^{\perp(2)}_{4;V}(\underline{\alpha}) - \widehat{\Phi}^{\perp(1)}_{4;V}(\underline{\alpha}) + \widehat{\Phi}^{\perp(3)}_{4;V}(\underline{\alpha}) - \widehat{\Phi}^{\perp(4)}_{4;V}(\underline{\alpha})\right].$$

- the LO double spectral density

$$\rho^{\rm LO}(s_1, s_2) = \frac{1}{\pi^2}\,{\rm Im}_{s_1}\,{\rm Im}_{s_2}\,F^{(\rm LO)}((p+q)^2, q^2).$$

Double spectral density at LO

- The derivation of the double spectral density in the two-particle case

$$\rho_{j,2p}(s_1, s_2) \equiv \frac{1}{\pi^2}\,\mathrm{Im}_{s_1}\,\mathrm{Im}_{s_2}\int_0^1 du\,\frac{\phi_{2p}(u)}{[\bar{u}\, s_1 + u\, s_2 - m_Q^2 + i\,0]^j}$$
$$= \frac{1}{\Gamma(j)}\,\frac{d^{j-1}}{(d\,m_Q^2)^{j-1}}\sum_k \frac{(-1)^{k+1}\, c_k^{(\phi_{2p})}}{\Gamma(k+1)}\,(s_1 - m_Q^2)^k\,\delta^{(k)}(s_1 - s_2)$$

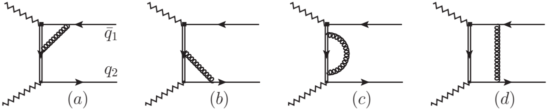
- For the three-particle scenario

$$\{\rho_{j,3p}(s_1, s_2), \widehat{\rho}_{j,3p}(s_1, s_2)\}$$
$$= \frac{1}{\pi^2}\,\mathrm{Im}_{s_1}\,\mathrm{Im}_{s_2}\int_0^1 dv\int \mathcal{D}\alpha\,\frac{v^\ell\,\{\Phi_{3p}(\underline{\alpha}), \widehat{\Phi}_{3p}(\underline{\alpha})/\bar{\alpha}\}}{\left[\bar{\alpha}\, s_1 + \alpha\, s_2 - m_Q^2 + i\,0\right]^j}$$
$$= \frac{1}{\Gamma(j)}\,\frac{d^{j-1}}{(d\,m_Q^2)^{j-1}}\,\frac{1}{\pi^2}\,\mathrm{Im}_{s_1}\,\mathrm{Im}_{s_2}\int_0^1 d\alpha\,\frac{\{\overline{\Phi}_{3p}(\alpha,\ell), \overline{\widehat{\Phi}}_{3p}(\alpha,\ell)\}}{\bar{\alpha}\, s_1 + \alpha\, s_2 - m_Q^2 + i\,0}$$
$$= \frac{1}{\Gamma(j)}\,\frac{d^{j-1}}{(d\,m_Q^2)^{j-1}}\sum_k \frac{(-1)^{k+1}\,\{c_{\ell,k}^{\Phi_{3p}}, c_{\ell,k}^{\widehat{\Phi}_{3p}}\}}{\Gamma(k+1)}\,(s_1 - m_Q^2)^k\,\delta^{(k)}(s_1 - s_2)$$

Double spectral density at NLO

- To achieve NLO precision, we express the invariant amplitude

$$F^{(\mathrm{OPE})}((p+q)^2, q^2) = F^{(\mathrm{LO})}((p+q)^2, q^2) + \frac{\alpha_s C_F}{4\pi} F^{(\mathrm{NLO})}((p+q)^2, q^2),$$



- factorization formula for twist-2 contributions at NLO

$$F^{(\mathrm{NLO})}((p+q)^2, q^2) = (1 + \hat{m}_{q_2}) f_V^{\perp}(\mu) \int_0^1 du\, \mathcal{C}(r_1, r_2, u) \cdot \mathcal{T}^{(1)}(r_1, r_2, \mu) \cdot \phi^{\perp}_{2;V}(u, \mu),$$

- the NLO double spectral density:

$$\rho^{\mathrm{NLO}}(s_1, s_2) = \frac{1}{\pi^2} \mathrm{Im}_{s_1} \mathrm{Im}_{s_2} F^{(\mathrm{NLO})}((p+q)^2, q^2).$$

Double spectral density at NLO

- Combining all one-loop contrib., the hard amplitude reads [H.D. Li,et al.,2002.03825]

$$
\begin{aligned}
&\mathcal{T}^{(1)}(r_1, r_2, \mu) \\
&= \frac{\alpha_s C_F}{4\pi}\left\{(-2)\left[\frac{1-r_1}{r_3-r_1}\ln\frac{1-r_3}{1-r_1}+\frac{1-r_2}{r_3-r_2}\ln\frac{1-r_3}{1-r_2}+\frac{3}{1-r_3}\right]\ln\frac{\mu^2}{m_Q^2}\right. \\
&+2\left[\left(\frac{1-r_1}{r_3-r_1}+\frac{1-r_2}{r_3-r_2}\right)\mathrm{Li}_2(r_3)-\frac{1-r_1}{r_3-r_1}\mathrm{Li}_2(r_1)-\frac{1-r_2}{r_3-r_2}\mathrm{Li}_2(r_2)\right] \\
&+2\left[\left(\frac{1-r_1}{r_3-r_1}+\frac{1-r_2}{r_3-r_2}\right)\ln^2(1-r_3)-\frac{1-r_1}{r_3-r_1}\ln^2(1-r_1)-\frac{1-r_2}{r_3-r_2}\ln^2(1-r_2)\right] \\
&+\left[\frac{1-r_3}{r_3}\left(\frac{1-r_3-2r_1}{r_3-r_1}-\frac{2(1-r_3+r_2)}{r_3-r_2}\right)+\frac{1-6r_3}{r_3^2}+1\right]\ln(1-r_3)+\frac{1-9r_3}{r_3(1-r_3)} \\
&\left.-\frac{(1-r_1)(1-3r_1)}{r_1(r_3-r_1)}\ln(1-r_1)+\frac{2(1-r_2)}{r_2(r_3-r_2)}\ln(1-r_2)-3\right\},
\end{aligned}
$$

- Double spectral density

$$
\rho^{(\mathrm{OPE})}(s_1, s_2) = \rho^{(\mathrm{LO})}(s_1, s_2) + \frac{\alpha_s C_F}{4\pi}\rho^{(\mathrm{NLO})}(s_1, s_2)
$$

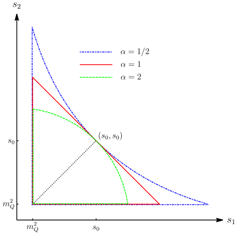
Quark-hadron duality and sum rules

- parameterization of the boundaries [Khodjamirian,Melić,Wang, Wei, 2011.11275]

$$\left(\frac{s_1}{s_*}\right)^\alpha + \left(\frac{s_2}{s_*}\right)^\alpha \leq 1\,, \qquad s_1\,,\, s_2 \geq m_Q^2\,.$$

probe the three duality regions, generated at

$$\alpha = 1\,, \qquad s_* = 2s_0\,, \qquad \text{(triangle)};$$
$$\alpha = \frac{1}{2}\,, \qquad s_* = 4s_0\,, \qquad \text{(concave)};$$
$$\alpha = 2\,, \qquad s_* = \sqrt{2}s_0\,, \qquad \text{(convex)};$$



- Assuming equal Borel parameters $M_1^2 = M_2^2 = 2M^2$ allows us to rewrite the sum rule

$$f_H f_{H^*}\, g_{H^* HV} = -\frac{1}{m_H^2 m_{H^*}} \exp\left(\frac{m_H^2 + m_{H^*}^2}{2M^2}\right) \left[\mathcal{F}^{(\mathrm{LO})}(M^2, s_0) + \frac{\alpha_s\, C_F}{4\pi}\, \mathcal{F}^{(\mathrm{NLO})}(M^2, s_0)\right].$$

- define the integral over the triangular region

$$\mathcal{F}(M^2, s_0) \equiv \int_{-\infty}^{\infty} ds_1 \int_{-\infty}^{\infty} ds_2\, \theta(2s_0 - s_1 - s_2)\, \exp\left(-\frac{s_1 + s_2}{2M^2}\right) \rho(s_1\,, s_2)\,.$$

LCSRs for the $H_{(s)} H_{(s)} V$ couplings

This is a brand new piece of work

- Def. with the eff. Lagrangian[Casalbuoni,et al.,9605342][Cheng,et al.,0409317][Yan,et al.,92']

$$\langle V(p,\varepsilon^*) H_2(p+q) | \mathcal{L}_{eff} | H_1(q) \rangle \equiv g_{H_1 H_2 V} (q \cdot \varepsilon^*),$$

- Vacuum-V correlation function[V-meson LCDAs]

$$\Pi((p+q)^2, q^2) = i \int d^4x\, e^{iq\cdot x} \langle V(p,\varepsilon^*) | T\{ j_5^\dagger(x), j_5(0) \} | 0 \rangle,$$

where, $j_5 = (m_Q + m_{q_2}) \bar{Q}\, i\, \gamma_5\, q$

- Double dispersion relation

$$\Pi((p+q)^2, q^2) = \frac{(q\cdot\varepsilon^*) g_{H_1 H_2 V} f_{H_1} f_{H_2} m_{H_1}^2 m_{H_2}^2}{[m_{H_1}^2 - q^2][m_{H_2}^2 - (p+q)^2]} + \iint_\Sigma \frac{\rho_{cont}(s_1, s_2)\, ds_1\, ds_2}{[s_1 - q^2]\,[s_2 - (p+q)^2]} + \cdots$$
$$\equiv (q\cdot\varepsilon^*)\, F((p+q)^2, q^2)$$

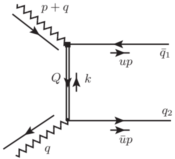
LCSRs for the $H_{(s)}H_{(s)}V$ couplings

- light-cone OPE expressed as a double dispersion relation

$$\Pi^{\rm OPE}((p+q)^2, q^2) = (q\cdot\varepsilon^*)\iint ds_1\, ds_2\, \frac{\rho^{\rm OPE}(s_1, s_2)}{(s_1-(p+q)^2)(s_2-q^2)}$$

where the involved dual spectral density $\rho^{\rm (OPE)}(s_1, s_2)$ refers to

$$\rho^{\rm (OPE)}(s_1, s_2) \equiv \frac{1}{\pi^2}{\rm Im}_{s_1}{\rm Im}_{s_2}F^{\rm (OPE)}(s_1, s_2)\,,$$



- performing the double Borel transformation with respect to the variables $q^2 \to M_1^2$ and $(p+q)^2 \to M_2^2$

$$f_{H_1} f_{H_2} g_{H_1 H_2 V} = \frac{1}{m^2_{H_1} m^2_{H_2}}\int_{-\infty}^{+\infty} ds_1 \int_{-\infty}^{2s_0 - s_2} ds_2 \exp\left[\frac{m^2_{H_1}-s_1}{M_1^2}+\frac{m^2_{H_2}-s_2}{M_2^2}\right]\rho^{\rm OPE}(s_1, s_2)\,.$$

confined to the triangular duality region with $\alpha = 1$ and $s_* = 2s_0$, wherein $s_2 \le 2s_0 - s_1$

Hard-collinear factorization for the correlation function at LP

- factorization form:

$$\Pi^{\rm OPE}_{\rm LP}((p+q)^2, q^2) = q^\mu \left(\mathcal{T}_V^{(0)} + \mathcal{T}_V^{(1)}\right) \otimes \langle O_\mu^V \rangle$$

- matrix element of the vector light-cone operator

$$\langle O_\mu^V \rangle = \langle V(p,\varepsilon^*)|\bar{q}'(x)\gamma_\mu q(0)|0\rangle \,, O_\mu^V = \bar{q}'(x)\gamma_\mu q(0) \stackrel{\rm LP}{=} n_\mu \bar{\chi}(x)\frac{\not{\bar{n}}}{2}\chi(0),$$

$$O_\mu^V = n_\mu \int d\omega_1 d\omega_2\, e^{\frac{i}{2}\omega_1 (n\cdot x)} \mathcal{J}_V(\vec{\omega}) \equiv n_\mu \mathcal{O}_V \,, \mathcal{J}_V(\vec{\omega}) \equiv \bar{\chi}_{n,\,\omega_1}\frac{\not{\bar{n}}}{2}\chi_{n,\,\omega_2}$$

- Therefore, rewrite the factorization form

$$\Pi^{\rm OPE}_{\rm LP}((p+q)^2, q^2) = (n\cdot q)\int_0^1 du \left(\mathcal{T}_V^{(0)} + \mathcal{T}_V^{(1)}\right) \langle \mathcal{J}_V(\vec{\omega})\rangle \,.$$

where

$$\langle \mathcal{O}_V \rangle = \langle V(p,\varepsilon^*)| \int d\omega_1 d\omega_2\, e^{\frac{i}{2}\omega_1 (n\cdot x)} \mathcal{J}_V(\vec{\omega})|0\rangle \,, \langle \mathcal{J}_V(\vec{\omega})\rangle = \frac{\bar{n}\cdot\varepsilon^*}{2} m_V f_V^{\parallel} \phi^{\parallel}_{2;V}(u) \,,$$

Hard-collinear factorization for the correlation function at LP

- NLO hard function

$$\mathcal{T}_V^{(1)}(r_1, r_2, u) = \frac{\alpha_s C_F}{4\pi} \left[\mathcal{T}_V^{(1),\,a} + \mathcal{T}_V^{(1),\,b} + \mathcal{T}_V^{(1),\,c} + \mathcal{T}_V^{(1),\,d} \right],$$

- the NLO renormalized hard function New

$$\begin{aligned}
\mathcal{T}_V^{(1)}(r_1, r_2, u) &= \frac{\alpha_s C_F}{4\pi} \frac{1}{r_3 - 1} \Bigg\{ \Bigg[2 \left(2 - \frac{1 - r_1}{r_3 - r_1} \ln \frac{1 - r_3}{1 - r_1} - \frac{1 - r_2}{r_3 - r_2} \ln \frac{1 - r_3}{1 - r_2} \right) \\
&- 2(r_3 - 1) \left(\frac{(r_3 - 1)\ln(1 - r_3)}{(r_3 - r_1)(r_3 - r_2)} + \frac{(1 - r_1)\ln(1 - r_1)}{(r_3 - r_1)(r_1 - r_2)} + \frac{(1 - r_2)\ln(1 - r_2)}{(r_3 - r_2)(r_2 - r_1)} \right) - \frac{r_3 - 7}{r_3 - 1} \Bigg] \ln \frac{\mu^2}{m_Q^2} \\
&+ \frac{2\left[r_3^2 - r_3(r_1 + r_2) + 2 r_1 r_2 - r_1 - r_2 + 1 \right]}{(r_3 - r_1)(r_3 - r_2)} \left[\mathrm{Li}_2(r_3) + \ln^2(1 - r_3) \right] \\
&- \frac{2(r_1 - 1)(r_3 - r_1 + r_2 - 1)}{(r_3 - r_1)(r_1 - r_2)} \left[\mathrm{Li}_2(r_1) + \ln^2(1 - r_1) \right] - \frac{2(r_2 - 1)(r_3 + r_1 - r_2 - 1)}{(r_3 - r_2)(r_2 - r_1)} \left[\mathrm{Li}_2(r_2) + \ln^2(1 - r_2) \right] \\
&- \left[\frac{r_1^2 + r_3(2 - 3r_1 - 3r_2) + 3 r_1 r_2 + 1}{(r_3 - r_1)(r_3 - r_2)} + \frac{r_3(2 r_1 r_2 - r_1 - r_2) - r_1 r_2}{r_3^2 (r_3 - r_1)(r_3 - r_2)} \right] \ln(1 - r_3) \\
&- \frac{2(r_1 - 1)(r_3 r_1 - r_2)}{r_1 (r_3 - r_1)(r_1 - r_2)} \ln(1 - r_1) - \frac{2(r_2 - 1)(r_3 r_2 - r_1)}{r_2 (r_3 - r_2)(r_2 - r_1)} \ln(1 - r_2) + \frac{3 r_1^2 + 6 r_3 - 1}{r_3 (r_3 - 1)} \Bigg\},
\end{aligned}$$

subtracting the UV and IR divergence in the $\overline{\mathrm{MS}}$ scheme

The factorization scale independence for the correlation function

- The derivative of the twist-two NLO correlation function on the scale μ

$$\frac{d}{d\ln\mu}\Pi^{\rm OPE}_{\rm LP}(r_1,r_2,\mu) = -m_V f_V^{\parallel}(q\cdot\varepsilon^*)\int_0^1 du\left\{\left(\frac{d}{d\ln\mu}\phi^{\parallel}_{2;V}(u,\mu)\right)\left[\mathcal{T}_V^{(0)}(r_1,r_2,u,\mu)+\mathcal{T}_V^{(1)}(r_1,r_2,u,\mu)\right]\right.$$
$$\left. + \phi^{\parallel}_{2;V}(u,\mu)\frac{d}{d\ln\mu}\left[\mathcal{T}_V^{(0)}(r_1,r_2,u,\mu)+\mathcal{T}_V^{(1)}(r_1,r_2,u,\mu)\right]\right\},$$

- the RG evolution equations of the twist-2 LCDA and the heavy quark mass

$$\frac{d}{d\ln\mu}\phi^{\parallel}_{2;V}(u,\mu) = 2\int_0^1 du'\,V(u,u')\phi^{\parallel}_{2;V}(u',\mu)\,,$$
$$\frac{d}{d\ln\mu}m_Q(\mu) = -6\frac{\alpha_s C_F}{4\pi}m_Q(\mu)\,,$$

where the evolution kernel is given by[Efremov and Radyushkin, Phys.Lett.B 94 (1980)245]

$$V(u,u') = \frac{\alpha_s C_F}{4\pi}\left[\frac{2\bar{u}}{\bar{u}'}\left(1+\frac{1}{u-u'}\right)\theta(u-u')\Big|_+ + \frac{2u}{u'}\left(1+\frac{1}{u'-u}\right)\theta(u'-u)\Big|_+\right],$$

the plus distribution function is defined as

$$V(u,u')\Big|_+ = V(u,u') - \delta(u-u')\int_0^1 dt V(t,u')\,.$$

The factorization scale independence for the correlation function

- Inserting these evolution equations, then we obtain the first term

$$\int_0^1 du \left(\frac{d}{d\ln\mu} \phi^{\|}_{2;V}(u,\mu) \right) \left[\mathcal{T}_V^{(0)}(r_1,r_2,u,\mu) + \mathcal{T}_V^{(1)}(r_1,r_2,u,\mu) \right]$$
$$= 4 \cdot \frac{\alpha_s C_F}{4\pi} \int_0^1 du \, \frac{\phi^{\|}_{2;V}(u,\mu)}{r_3 - 1} \left\{ \left[\frac{1-r_1}{r_3-r_1} \ln\frac{1-r_3}{1-r_1} + \frac{1-r_2}{r_3-r_2} \ln\frac{1-r_3}{1-r_2} + 2 \right] \right.$$
$$\left. + \left[-\frac{1}{2} + \frac{(r_3-1)^2 \ln(1-r_3)}{(r_3-r_1)(r_3-r_2)} - \frac{(r_3-1)(r_1-1)\ln(1-r_1)}{(r_3-r_1)(r_1-r_2)} + \frac{(r_3-1)(r_2-1)\ln(1-r_2)}{(r_3-r_2)(r_1-r_2)} \right] \right\},$$

and the second term reads

$$\int_0^1 du \, \phi^{\|}_{2;V}(u,\mu) \frac{d}{d\ln\mu} \left[\mathcal{T}_V^{(0)}(r_1,r_2,u,\mu) + \mathcal{T}_V^{(1)}(r_1,r_2,u,\mu) \right]$$
$$= 4 \cdot \frac{\alpha_s C_F}{4\pi} \int_0^1 du \, \frac{\phi^{\|}_{2;V}(u,\mu)}{r_3 - 1} \left\{ \left[-\frac{1-r_1}{r_3-r_1} \ln\frac{1-r_3}{1-r_1} - \frac{1-r_2}{r_3-r_2} \ln\frac{1-r_3}{1-r_2} - 2 \right] \right.$$
$$\left. + \left[-\frac{(r_3-1)^2 \ln(1-r_3)}{(r_3-r_1)(r_3-r_2)} + \frac{(r_3-1)(r_1-1)\ln(1-r_1)}{(r_3-r_1)(r_1-r_2)} - \frac{(r_3-1)(r_2-1)\ln(1-r_2)}{(r_3-r_2)(r_1-r_2)} \right] + \frac{1}{2} \right\}.$$

- Put them together, we obtain Exactly cancel!

$$\frac{d}{d\ln\mu} \Pi^{\rm OPE}_{\rm LP}(r_1,r_2,\mu) = O(\alpha_s^2),$$

The LCSRs for the $H_{(s)} H_{(s)} V$ at LP

- Inserting the double spectral density into the sum rules, we finally get the LP LCSRs

$$f_{H_1} f_{H_2} g^{\rm LP}_{H_1 H_2 V} = \frac{1}{m^2_{H_1}\, m^2_{H_2}}\, e^{\frac{m^2_{H_1}+m^2_{H_2}}{2M^2}} \left[\mathcal{F}^{\rm LO}_{\rm LP}(M^2, s_0) + \mathcal{F}^{\rm NLO,\,as}_{\rm LP}(M^2, s_0)\right],$$

where [introduce two auxiliary function]

$$\mathcal{F}^{\rm LO}_{\rm LP}(M^2, s_0) = f^{\parallel}_V m_V m^4_Q \hat{M}^2 \left(e^{-\frac{1}{\hat{M}^2}} - e^{-\frac{\hat{s}_0}{\hat{M}^2}}\right) \phi^{\parallel}_{2;V}\left(\frac{1}{2}\right),$$

$$\mathcal{F}^{\rm NLO,\,as}_{\rm LP}(M^2, s_0) = -\frac{\alpha_s C_F}{4\pi} f^{\parallel}_V m_V m^4_Q \left[\int_0^{2\hat{s}_0 - 2} d\sigma\, e^{-\frac{\sigma+2}{2\hat{M}^2}} g(\sigma) + \Delta g(M^2, m^2_Q)\right],$$

$$\begin{aligned} g(\sigma) = {} & \frac{3}{4}\left[-2\mathrm{Li}_2(-1-\sigma) - 2\mathrm{Li}_2(-\sigma) - 5\mathrm{Li}_2\left(-\frac{\sigma}{2}\right)\right] - 6\ln\left(\frac{\sigma}{2}\right)\ln\left(\frac{\sigma+2}{2}\right) \\ & - \frac{3}{2}\ln(2+\sigma)\ln(\sigma+1) + \frac{3[\sigma(\sigma+2)(\sigma+8)+8]}{(\sigma+2)^3}\ln(\sigma) + \frac{3}{2}\ln(\sigma) \\ & + \frac{3(\sigma+1)(\sigma+6)}{(\sigma+2)^3}\ln(\sigma+1) - \frac{33}{8}\ln(\sigma+2) - \frac{3(3\sigma+20)}{8(\sigma+2)} \\ & - \frac{12\sigma(\sigma+1)}{(\sigma+2)^3}\ln 2 - \frac{3}{8}\ln 2 - \frac{5\pi^2}{8} - \frac{9}{2}\ln\left(\frac{\mu^2}{m^2_Q}\right), \end{aligned}$$

$$\Delta g(M^2, m^2_Q) = 3\left[4 + 3\ln\left(\frac{\mu^2}{m^2_Q}\right)\right] e^{-\frac{1}{\hat{M}^2}}.$$

The LCSRs for the $H_{(s)} H_{(s)} V$ at NLP

- Subleading power corrections and higher-twist contributions

$$\Pi^{\rm LO}_{\rm NLP}(r_1, r_2) = m_Q(1+\hat{m}_{q_1}+\hat{m}_{q_2})\,(q\cdot\varepsilon^*) \times \sum_{i=2}^{4}\sum_{j=1}^{4}\Big\{\int_0^1 du\,\delta_V{}^i\,(-1)^j H_V^{(0)\,j}(r_1, r_2, u)\mathcal{A}_{2{\rm p},\,ij}(u) + \int_0^1 dv\int_0^1 d\alpha_2\int_0^{1-\alpha_2} d\alpha_g\,\delta_V{}^i\,(-1)^j H_V^{(0)\,j}(r_1, r_2, \alpha)\mathcal{A}_{3{\rm p},\,ij}(v, \underline{\alpha})\Big\},$$

The part involving soft functions is denoted by V-meson LCDAs

$$\mathcal{A}_{2{\rm p},\,22}(u) = f_V^\perp\,\psi^\parallel_{3;V}(u)\,,\quad \mathcal{A}_{2{\rm p},\,31}(u) = -f_V^\parallel\,\Delta\phi^\parallel_{2;V}(u)\,,$$

$$\mathcal{A}_{2{\rm p},\,32}(u) = -f_V^\parallel\Big[u\bar{u}\phi^\parallel_{2;V}(u) + \frac{1}{4}\phi^\parallel_{4;V}(u) + 2\hat{\hat{\mathbb{C}}}(u)\Big],$$

$$\mathcal{A}_{2{\rm p},\,33}(u) = \frac{1}{2}f_V^\parallel\Big[\phi^\parallel_{4;V}(u) + 8\hat{\hat{\mathbb{C}}}(u)\Big],\quad \mathcal{A}_{2{\rm p},\,43}(u) = 2f_V^\perp\,u\bar{u}\psi^\parallel_{3;V}(u)\,,$$

$$\mathcal{A}_{3{\rm p},\,33}(v, \underline{\alpha}) = \frac{2f_V^\parallel}{\bar{\alpha}}\Big[(v-\bar{v})\,\hat{\Psi}^\parallel_{4;V}(\underline{\alpha}) + \hat{\hat{\Psi}}^\parallel_{4;V}(\underline{\alpha}) + 2\,(v-\bar{v})\,\hat{\Phi}^\parallel_{4;V}(\underline{\alpha}) + 2\hat{\hat{\Phi}}^\parallel_{4;V}(\underline{\alpha})\Big],$$

$$\mathcal{A}_{3{\rm p},\,34}(v, \underline{\alpha}) = -\frac{2f_V^\parallel}{\bar{\alpha}}(r_2-1)\Big[(v-\bar{v})\,\hat{\Psi}^\parallel_{4;V}(\underline{\alpha}) + \hat{\hat{\Psi}}^\parallel_{4;V}(\underline{\alpha}) + 2\,(v-\bar{v})\,\hat{\Phi}^\parallel_{4;V}(\underline{\alpha}) + 2\hat{\hat{\Phi}}^\parallel_{4;V}(\underline{\alpha})\Big],$$

- and we also introduce the following notations:

$$\mathbb{C}(u) = \phi^\parallel_{2;V}(u) - 2\phi^\perp_{3;V}(u) + \psi^\parallel_{4;V}(u)\,,\quad \Delta\phi^\parallel_{2;V}(u) = \frac{2u\phi^\parallel_{2;V}(u)}{r_1-r_2}\,,$$

1. Motivation
2. LCSRs for the $H^*_{(s)} H_{(s)} V$ couplings
3. LCSRs for the $H_{(s)} H_{(s)} V$ couplings
4. **Numerical analysis**
5. Conclusions

Numerical analysis

- Decay constant from two-point QCDSRs[Gelhausen,et al.1305.5432]

$$f_D = 190.4^{+8.9}_{-7.9}\ \text{MeV}, \qquad f_B = 201.1^{+21.2}_{-17.6}\ \text{MeV},$$

$$f_{D_s} = 200.7^{+10.4}_{-8.7}\ \text{MeV}, \qquad f_{B_s} = 223.2^{+22.2}_{-18.0}\ \text{MeV},$$

$$f_{D^*} = 245.5^{+24.1}_{-21.3}\ \text{MeV}, \qquad f_{B^*} = 214.2^{+10.1}_{-17.8}\ \text{MeV}.$$

$$f_{D_s^*} = 281.1^{+24.6}_{-20.9}\ \text{MeV}, \qquad f_{B_s^*} = 245.2^{+18.9}_{-25.3}\ \text{MeV}.$$

parameter	input value	[Ref.]	rescaled values
$\alpha_s(m_Z)$	0.1179 ± 0.0010		$\alpha_s(1.5\,\text{GeV}) = 0.3487^{+0.0102}_{-0.0097}$ $\alpha_s(3.0\,\text{GeV}) = 0.2527^{+0.0050}_{-0.0048}$
$\overline{m}_c(\overline{m}_c)$	1.280 ± 0.025 GeV	[48]	$\overline{m}_c(1.5\,\text{GeV}) = 1.205 \pm 0.034$ GeV
$\overline{m}_b(\overline{m}_b)$	4.18 ± 0.03 GeV		$\overline{m}_b(3.0\,\text{GeV}) = 4.473 \pm 0.04$ GeV
$(\overline{m}_u + \overline{m}_d)(2\ \text{GeV})$	6.78 ± 0.08 MeV	[48, 49]	$(\overline{m}_u + \overline{m}_d)(1.5\ \text{MeV}) = 7.305 \pm 0.09$ MeV $(\overline{m}_u + \overline{m}_d)(3.0\ \text{GeV}) = 6.331 \pm 0.07$ MeV
$(\overline{m}_s)(2\ \text{GeV})$	93.1 ± 0.6 MeV	[48, 49]	$(\overline{m}_s)(1.5\ \text{MeV}) = 100.305 \pm 0.65$ MeV $(\overline{m}_s)(3.0\ \text{GeV}) = 86.936 \pm 0.56$ MeV

- Decay constant from Lattice[FLAG:2021][Lubicz,Melis,Simula(ETM),1707.04529]

$$f_D = 212.0 \pm 0.7\,\text{MeV}, \qquad f_B = 190.0 \pm 1.3\,\text{MeV},$$

$$f_{D_s} = 249.9 \pm 0.5\,\text{MeV}, \qquad f_{B_s} = 230.3 \pm 1.3\,\text{MeV},$$

$$f_{D^*} = 228.5 \pm 7.7\,\text{MeV}, \qquad f_{B^*} = 182.02 \pm 4.4\,\text{MeV},$$

$$f_{D_s^*} = 271.6 \pm 5.0\,\text{MeV}, \qquad f_{B_s^*} = 224.3 \pm 2.6\,\text{MeV}.$$

Methods	charmed meson		bottom meson	
	f_D [MeV]	f_{D_s} [MeV]	f_B [MeV]	f_{B_s} [MeV]
two-point QCDSRs	201^{+12}_{-13}	238^{+13}_{-23}	207^{+17}_{-09}	242^{+17}_{-12}
LQCD	212.0 ± 0.7	249.9 ± 0.5	190.0 ± 1.3	230.3 ± 1.3
Experiment	207.3 ± 6.2	249.5 ± 3.16	201.6 ± 21.2	$--$

Numerical analysis

- Renormalization scale μ equal to the factorization scale

$$\mu \sim \sqrt{m_H^2 - m_Q^2} \sim \sqrt{2 m_Q \bar{\Lambda}}\,, (\bar{\Lambda} = m_H - m_Q).$$

Parameter	default value (interval)	[Ref.]	Parameter	default value (interval)	[Ref.]
charmed meson sum rules			bottom meson sum rules		
μ (GeV)	1.5 (1.0 - 3.0)	[48]	μ (GeV)	3.0 (2.5 - 4.5)	[49]
M^2 (GeV^2)	4.5 (3.5 - 5.5)		M^2 (GeV^2)	16.0 (12.0 - 20.0)	
s_0 (GeV^2)	7.0 (6.5 - 7.5)		s_0 (GeV^2)	37.5 (35.0 - 40.0)	

- the validity of the power expansion

Power	δ^0_ϕ	δ^1_ϕ	δ^2_ϕ	δ^3_ϕ	δ^4_ϕ	total
$g_{D^*_s D_s \phi}$	2.41	0.63	-0.48	-0.045	0.020	2.54
$g_{B^*_s B_s \phi}$	2.47	0.53	-0.48	-0.016	0.020	2.86

$\delta_\phi = \delta_\phi^{(Q)} = m_\phi/m_Q$

$\delta_\phi^{(c)} = 0.85$

$\delta_\phi^{(b)} = 0.23$

Power	δ^1_V	$\delta^1_V \alpha_s$	δ^2_V	δ^3_V	δ^4_V	total
$g_{DD\rho}$	2.76	0.80	0.65	-0.34	-0.022	3.86
$g_{BB\rho}$	2.69	0.44	0.65	-0.10	-0.006	3.68
$g_{D_s D_s \phi}$	2.52	0.77	0.65	-0.59	-0.037	3.31
$g_{B_s B_s \phi}$	2.39	0.41	0.66	-0.16	-0.011	3.29

$\delta_V = \delta_V^{(Q)} = m_V/m_Q$

$\delta_\rho^{(c)} = 0.64$

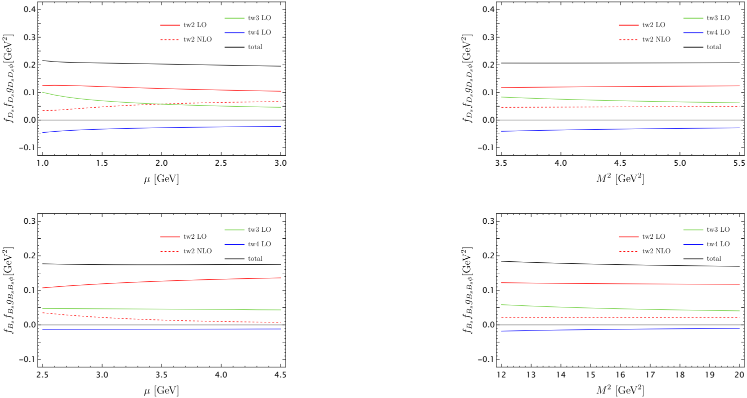
$\delta_\rho^{(b)} = 0.17$

$\delta_\phi^{(c)} = 0.85$

$\delta_\phi^{(b)} = 0.23$

Numerical analysis

- Scale and M^2 dependence of the products $f_{H_s} f_{H_s} g_{H_s H_s \phi}$



although each twist has its own dependence, the overall result exhibit no dependence when summed together![check!]

Numerical analysis

- Numerical results for the $H^*_{(s)}H_{(s)}V$ couplings[Errors have decreased, and precision has improved!]

Method	$g_{D^*D\rho}$	$g_{D^*D\omega}$	$g_{D^*_sDK^*}$	$g_{D^*D_sK^*}$	$g_{D^*_sD_s\phi}$	
LCSR[8]	$3.80^{+0.59}_{-0.45}$	–	–	–	–	[C.Wang,H-D.Li,20']
LCSR[19]	3.56 ± 0.60	–	–	4.04 ± 0.8	3.28 ± 0.64	[Z.-G.Wang,07']
LCSR[17]	4.17 ± 1.04	–	–	–	–	[Z.-H.Li,et al.02']
QCDSR[21]	–	–	3.74 ± 1.38	–	–	Decay Const. from
LCSR (this work)	$3.53^{+0.61}_{-0.57}$	$2.25^{+0.42}_{-0.41}$	$3.55^{+0.65}_{-0.61}$	$3.87^{+0.73}_{-0.68}$	$2.69^{+0.55}_{-0.52}$	QCDSR
	$3.40^{+0.49}_{-0.43}$	$2.17^{+0.34}_{-0.32}$	$3.30^{+0.52}_{-0.48}$	$3.34^{+0.54}_{-0.53}$	$2.54^{+0.40}_{-0.39}$	Lattice QCD

Method	$g_{B^*B\rho}$	$g_{B^*B\omega}$	$g_{B^*_sBK^*}$	$g_{B^*B_sK^*}$	$g_{B^*_sB_s\phi}$	
LCSR[8]	$3.89^{+0.52}_{-0.48}$	–	–	–	–	
LCSR[17]	5.70 ± 1.43	–	–	–	–	
QCDSR[21]	–	–	3.24 ± 1.08	–	–	Decay Const. from
LCSR (this work)	$3.00^{+0.49}_{-0.44}$	$1.88^{+0.34}_{-0.30}$	$3.12^{+0.60}_{-0.50}$	$3.16^{+0.52}_{-0.47}$	$2.70^{+0.49}_{-0.41}$	QCDSR
	$3.74^{+0.38}_{-0.38}$	$2.38^{+0.28}_{-0.28}$	$3.61^{+0.40}_{-0.39}$	$3.61^{+0.41}_{-0.40}$	$2.86^{+0.32}_{-0.30}$	Lattice QCD

Problem 1

- static coupling $\hat{g}$ in HQET

$$\hat{g}_{HHV} = \frac{e^{\overline{\Lambda}/\tau}}{\hat{f}^2}\left\{2\tau\left(1 - e^{-\omega_0/\tau}\right) m_V f_V^{\|}\phi_{2;V}^{\|}(1/2) + m_V^2 f_V^{\perp}\psi_{3;V}^{\|}(1/2) + m_V^3 f_V^{\|}\Delta\mathcal{F}_{\phi_{2;V}^{\|}}\right\} + \ldots,$$

where $f_H = \frac{\hat{f}}{\sqrt{m_Q}}$, $m_H = m_Q + \overline{\Lambda}$, $M^2 = 2m_Q\tau$, $s_0 = m_Q^2 + 2m_Q\omega_0$

- parameterize the LCSR result to estimate the $\hat{g}$ and its inverse mass corrections δ

$$g_{HHV} = \hat{g}_{HHV}\left(1 + \frac{\delta}{m_H}\right), \quad (H = D_{(s)}, B_{(s)})$$

- we encounter the two equations yielding the parameters $\hat{g}$ and δ.and predict the β

Parameter	$\hat{g}_{HHV}$	δ	β
QCDSRs	$2.44^{+0.73}_{-0.95}$	$1.42^{+2.29}_{-1.15}$	$0.30^{+0.09}_{-0.12}$
LQCD	$3.58^{+0.59}_{-0.60}$	$0.15^{+0.57}_{-0.46}$	0.44 ± 0.07
Experiment	$2.49^{+0.70}_{-0.89}$	$1.29^{+1.86}_{-0.97}$	$0.30^{+0.08}_{-0.11}$

Vector meson dominance (VMD) model: $\beta \simeq 0.9$?

$$\beta = \frac{\sqrt{2}}{2}\frac{\hat{g}_{HH\rho}}{g_V}$$

Possible explanations: 1.may be that the model predictions have potential larger errors? 3. It is actually not reliable?

2.besides the possible influences of excitation contributions, the values of NLO corrections to higher twists unknown?

Problem 2

- An alternative approach to extracting the coupling[X.Q.Li, et al.2011.0269]

$$g_{H_1 H_2 V} = \frac{2 m_V}{c_V f_{H_1}} \lim_{q^2 \to m_{H_1}^2} \left[\left(1 - \frac{q^2}{m_{H_1}^2} \right) A_0(q^2) \right] ,$$

[Bharucha,Straub.Zwicky,16'] [Gao, et al. 20']

Method	This work	$V(q^2)$[28]	$V(q^2)$[32]	$T_1(q^2)$[28]	$T_1(q^2)$[32]
$g_{B^*B\rho}$	$3.00^{+0.49}_{-0.44}$	$7.24^{+1.38}_{-1.26}$	$5.51^{+3.69}_{-2.34}$	$6.83^{+1.26}_{-1.15}$	$6.17^{+4.14}_{-2.61}$
	$3.74^{+0.38}_{-0.38}$	$8.53^{+1.44}_{-1.44}$	$6.49^{+4.31}_{-2.74}$	$8.04^{+1.31}_{-1.31}$	$7.25^{+4.83}_{-3.06}$
$g_{B^*B\omega}$	$1.88^{+0.34}_{-0.30}$	$6.78^{+1.73}_{-1.65}$	$6.00^{+4.00}_{-2.55}$	$6.50^{+1.58}_{-1.5}$	$6.69^{+4.52}_{-2.83}$
	$2.38^{+0.28}_{-0.28}$	$7.98^{+1.92}_{-1.92}$	$7.06^{+4.67}_{-2.99}$	$7.65^{+1.74}_{-1.74}$	$7.88^{+5.28}_{-3.31}$
$g_{B_s^*BK^*}$	$3.12^{+0.60}_{-0.50}$	$7.34^{+1.69}_{-1.55}$	$6.22^{+3.92}_{-2.39}$	$6.96^{+1.54}_{-1.41}$	$6.95^{+4.38}_{-2.67}$
	$3.61^{+0.40}_{-0.39}$	$8.02^{+1.60}_{-1.60}$	$6.80^{+4.21}_{-2.57}$	$7.61^{+1.44}_{-1.44}$	$7.60^{+4.71}_{-2.87}$
$g_{B^*B_sK^*}$	$3.16^{+0.52}_{-0.47}$	$6.75^{+1.42}_{-1.32}$	−−	$6.43^{+1.26}_{-1.16}$	−−
	$3.61^{+0.41}_{-0.40}$	$7.95^{+1.52}_{-1.52}$	−−	$7.57^{+1.33}_{-1.33}$	−−
$g_{B_s^*B_s\phi}$	$2.70^{+0.49}_{-0.41}$	$6.88^{+1.33}_{-1.18}$	−−	$6.54^{+1.24}_{-1.09}$	−−
	$2.86^{+0.32}_{-0.30}$	$7.51^{+1.18}_{-1.18}$	−−	$7.15^{+1.08}_{-1.08}$	−−

[Bharucha,Straub.Zwicky,16'] [Gao, et al. 20']

Coupling	decay constants	This work	$A_0(q^2)$ [24]	$A_0(q^2)$ [28]
$g_{BB\rho}$	two-point QCDSRs	$3.10^{+0.40}_{-0.54}$	$6.11^{+1.17}_{-1.23}$	$6.15^{+2.47}_{-2.05}$
	LQCD	$3.68^{+0.34}_{-0.35}$	$6.65^{+1.25}_{-1.25}$	$6.70^{+2.79}_{-2.18}$
	Experiment	$3.27^{+0.87}_{-0.67}$	$6.27^{+1.39}_{-1.32}$	$6.32^{+2.73}_{-2.14}$
$g_{BB\omega}$	two-point QCDSRs	$2.02^{+0.27}_{-0.36}$	$4.24^{+1.04}_{-1.07}$	$4.74^{+2.01}_{-1.53}$
	LQCD	$2.39^{+0.24}_{-0.24}$	$4.62^{+1.12}_{-1.12}$	$5.17^{+2.18}_{-1.63}$
	Experiment	$2.13^{+0.57}_{-0.44}$	$4.36^{+1.17}_{-1.13}$	$4.87^{+2.13}_{-1.60}$
$g_{B_sBK^*}$	two-point QCDSRs	$3.26^{+0.39}_{-0.46}$	$6.30^{+1.47}_{-1.49}$	$7.46^{+3.10}_{-2.22}$
	LQCD	$3.73^{+0.37}_{-0.37}$	$6.62^{+1.50}_{-1.50}$	$7.84^{+3.23}_{-2.27}$
$g_{B_sB_s\phi}$	two-point QCDSRs	$2.98^{+0.45}_{-0.49}$	$7.13^{+1.43}_{-1.46}$	−−
	LQCD	$3.29^{+0.36}_{-0.34}$	$7.49^{+1.45}_{-1.45}$	−−

Possible solution: 1.calculate NLO QCD corrections to the higher-twist contributions of the vector meson DAs?

2.incorporating high excited heavy-meson states in the hadronic components of the sum rules?

Conclusions

- We obtain **improved** values for the coupling $H^{(*)}_{(s)} H_{(s)} V$ beyond leading order, incorporating higher-twist/power corrections in **LCSRs**.
- The **NLO** corrections constituting nearly 20% of the leading twist, closely match the **LO** contributions at twist-three.
- Our **LCSR** predictions for the $H^{(*)}_{(s)} H_{(s)} V$ couplings show **satisfactory** agreement with values from previous studies within **theoretical** uncertainties.
- Our results are **systematically** smaller than those independently derived from form factors or VMD model.This discrepancy indicates that further investigation is needed to address the underlying issues.

Thanks!