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# Electromagnetic form factor of proton from the perturbative QCD

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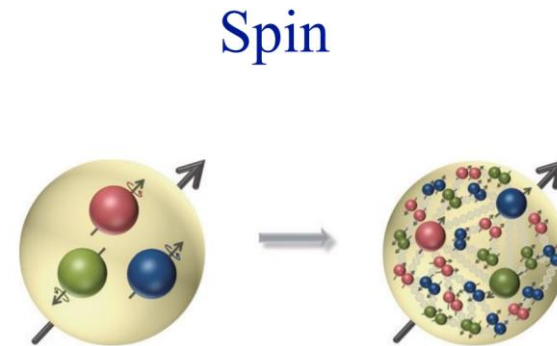
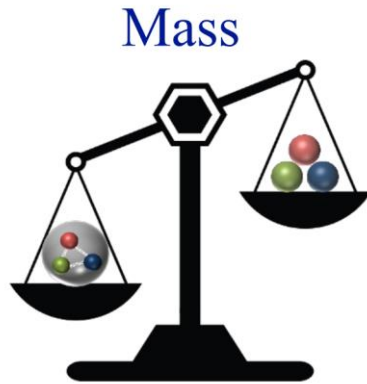
In collaboration with 韩佳杰, 程山, 于福升  
To be appear

第二十一届全国重味物理和CP破坏研讨会@衡阳

- Introduction
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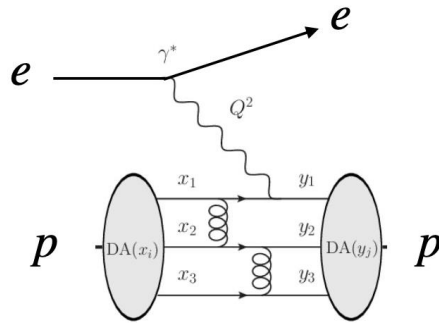
# Introduction

- Baryons play an important role in the evolution of the Universe, big-bang nucleosynthesis, matter-antimatter asymmetry, ...
- However, our [knowledge of the baryons are limited, even the simplest baryon](#)—proton, such as mass and spin of proton.
- The inner structure of baryon is highly related to perturbative and non-perturbative predictions, understanding of QCD dynamics.



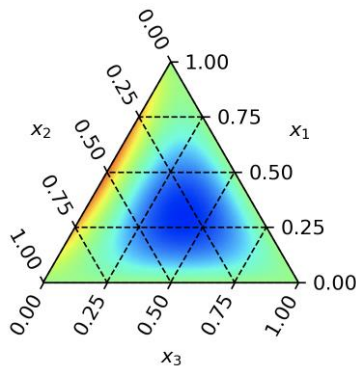
- Light-cone distribution amplitudes (LCDAs) are the fundamental structure of hadrons, describe hadron inner structure in exclusive processes.

## LCDAs: exclusive processes



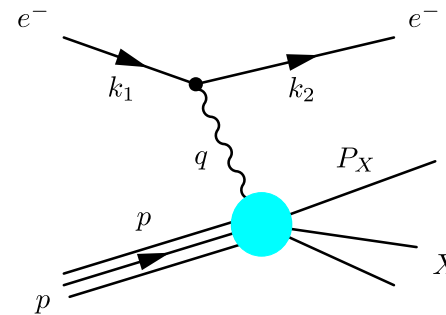
### 3-parton distribution

$$V_1 = 120x_1x_2x_3(\phi_3^0 + \phi_3^+(1 - 3x_3))$$

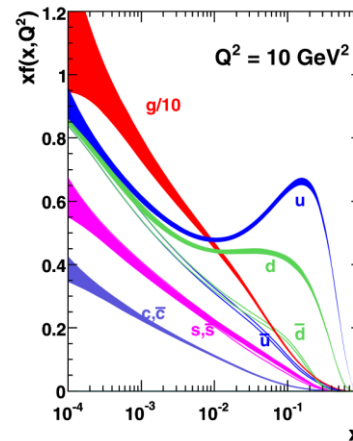


V.Braun, R.J.Fries, N.Mahnke, E.Stein, 2001

## PDF: inclusive processes



### 1-parton distribution



Eur. Phys. J. C (2009) 63

- Light-cone distribution amplitudes (LCDAs) are the fundamental structure of hadrons, describe hadron inner structure in exclusive processes.
- LCDAs are important inputs in the theoretical approaches—QCDF, PQCD, Sum rules, SCET,...
- LCDAs are **non-perturbative quantities**, thus difficult for predictions.
- **Baryon LCDAs are less known.** They dominate the theoretical uncertainties.

## proton LCDAs

$$\begin{aligned}
 & 4 \langle 0 | \varepsilon^{ijk} u_\alpha^i(a_1 z) u_\beta^j(a_2 z) d_\gamma^k(a_3 z) | P \rangle = \\
 & = S_1 M C_{\alpha\beta} (\gamma_5 N^+)_\gamma + S_2 M C_{\alpha\beta} (\gamma_5 N^-)_\gamma + P_1 M (\gamma_5 C)_{\alpha\beta} N_\gamma^+ + P_2 M (\gamma_5 C)_{\alpha\beta} N_\gamma^- \\
 & + V_1 (\not{p} C)_{\alpha\beta} (\gamma_5 N^+)_\gamma + V_2 (\not{p} C)_{\alpha\beta} (\gamma_5 N^-)_\gamma + \frac{V_3}{2} M (\gamma_\perp C)_{\alpha\beta} (\gamma^\perp \gamma_5 N^+)_\gamma \\
 & + \frac{V_4}{2} M (\gamma_\perp C)_{\alpha\beta} (\gamma^\perp \gamma_5 N^-)_\gamma + V_5 \frac{M^2}{2pz} (\not{z} C)_{\alpha\beta} (\gamma_5 N^+)_\gamma + \frac{M^2}{2pz} V_6 (\not{z} C)_{\alpha\beta} (\gamma_5 N^-)_\gamma \\
 & + A_1 (\not{p} \gamma_5 C)_{\alpha\beta} N_\gamma^+ + A_2 (\not{p} \gamma_5 C)_{\alpha\beta} N_\gamma^- + \frac{A_3}{2} M (\gamma_\perp \gamma_5 C)_{\alpha\beta} (\gamma^\perp N^+)_\gamma \\
 & + \frac{A_4}{2} M (\gamma_\perp \gamma_5 C)_{\alpha\beta} (\gamma^\perp N^-)_\gamma + A_5 \frac{M^2}{2pz} (\not{z} \gamma_5 C)_{\alpha\beta} N_\gamma^+ + \frac{M^2}{2pz} A_6 (\not{z} \gamma_5 C)_{\alpha\beta} N_\gamma^- \\
 & + T_1 (i\sigma_{\perp p} C)_{\alpha\beta} (\gamma^\perp \gamma_5 N^+)_\gamma + T_2 (i\sigma_{\perp p} C)_{\alpha\beta} (\gamma^\perp \gamma_5 N^-)_\gamma + T_3 \frac{M}{pz} (i\sigma_{pz} C)_{\alpha\beta} (\gamma_5 N^+)_\gamma \\
 & + T_4 \frac{M}{pz} (i\sigma_{zp} C)_{\alpha\beta} (\gamma_5 N^-)_\gamma + T_5 \frac{M^2}{2pz} (i\sigma_{\perp z} C)_{\alpha\beta} (\gamma^\perp \gamma_5 N^+)_\gamma + \frac{M^2}{2pz} T_6 (i\sigma_{\perp z} C)_{\alpha\beta} (\gamma^\perp \gamma_5 N^-)_\gamma \\
 & + M \frac{T_7}{2} (\sigma_{\perp\perp'} C)_{\alpha\beta} (\sigma^{\perp\perp'} \gamma_5 N^+)_\gamma + M \frac{T_8}{2} (\sigma_{\perp\perp'} C)_{\alpha\beta} (\sigma^{\perp\perp'} \gamma_5 N^-)_\gamma,
 \end{aligned}$$

V.Braun, R.J.Fries, N.Mahnke, E.Stein, 2001

- Light-cone distribution amplitudes (LCDAs) are the fundamental structure of hadrons, describe hadron inner structure in exclusive processes.

**Table 1** Twist classification of the proton LCDAs in Eq. (16)

	Twist-3	Twist-4	Twist-5	Twist-6
Vector	$V_1$	$V_2, V_3$	$V_4, V_5$	$V_6$
Pseudo-vector	$A_1$	$A_2, A_3$	$A_4, A_5$	$A_6$
Tensor	$T_1$	$T_2, T_3, T_7$	$T_4, T_5, T_8$	$T_6$
Scalar		$S_1$	$S_2$	
Pesudoscalar		$P_1$	$P_2$	

Model	Method	$f_N \cdot 10^3$ Gev <sup>2</sup>	$\lambda_1 \cdot 10^3$ Gev <sup>2</sup>	$\lambda_2 \cdot 10^3$ Gev <sup>2</sup>	$A_1^u$	$V_1^d$	$f_1^u$	$f_1^d$	$f_2^d$	Ref.
	QCDSR	5.0(5)	-27(9)	54(19)						
ASY		-	-	-	0	1/3	1/10	3/10	4/15	
CZ	QCDSR	5.3(5)	-	-	0.47	0.22	-	-	-	[1]
KS	QCDSR	5.1(3)	-	-	0.34	0.24	-	-	-	[2]
COZ	QCDSR	5.0(3)	-	-	0.39	0.23	-	-	-	[3]
SB	QCDSR	-	-	-	0.38	0.24	-	-	-	[4]
BK	PQCD	6.64	-	-	0.08	0.31	-	-	-	[5]
BLW	QCDSR	-	-	-	0.38(15)	0.23(3)	0.07(5)	0.40(20)	0.22(5)	[6]
BLW	LCSR (LO)	-	-	-	0.13	0.30	0.09	0.33	0.25	[6]
ABO1	LCSR (NLO)	-	-	-	0.11	0.30	0.11	0.27	-	[7]
ABO2	LCSR (NLO)				0.11	0.30	0.11	0.29	-	[7]
LAT09	LATTICE	3.23 (63)	-35.57 (65)	70.02 (13)	0.19 (2)	0.20 (1)	-	-	-	[8]
LAT14	LATTICE	3.07 (36)	-38.77 (18)	77.64 (37)	0.07 (4)	0.31 (2)	-	-	-	[9]
LAT19	LATTICE	3.54 (6)	-44.9 (42)	93.4 (48)	0.30 (32)	0.192 (22)	-	-	-	[10]

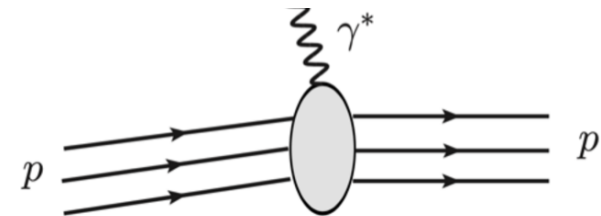
Slide from Ke-Sheng Huang

# Electromagnetic Form factors

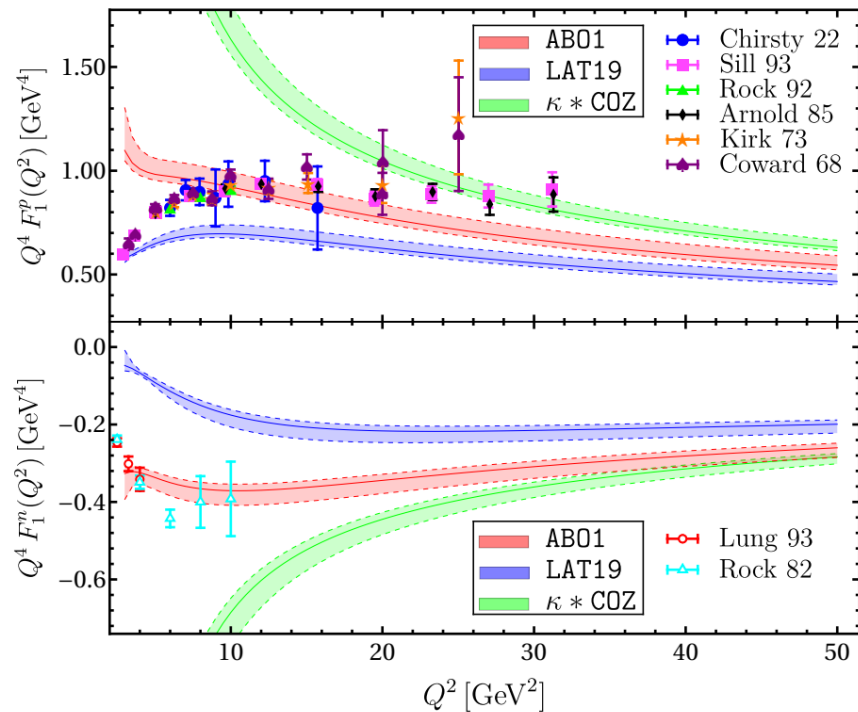
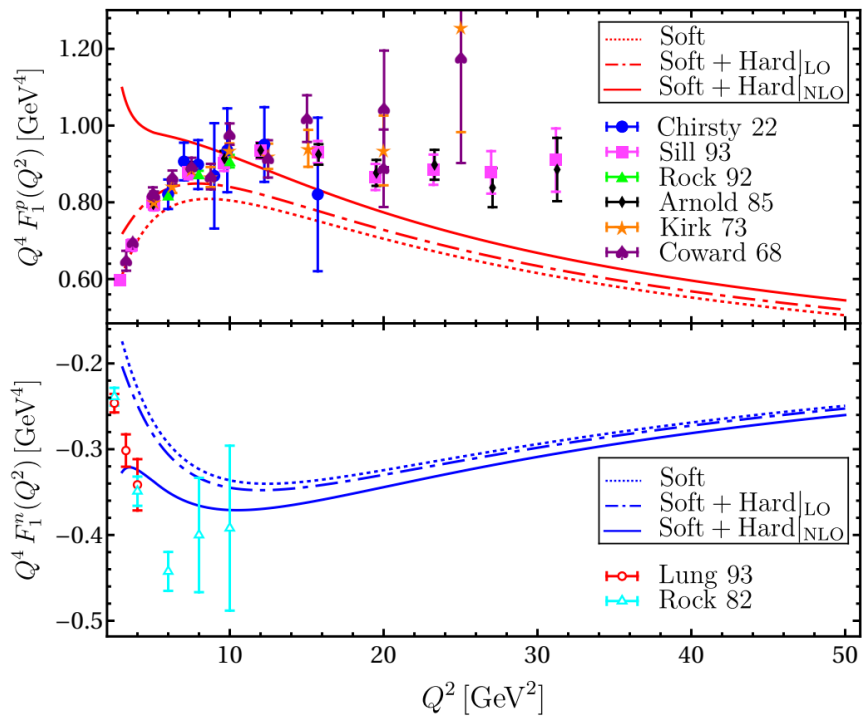
- Understanding **the internal structure of nucleons** from quantum chromodynamics , the underlying theory of strong interaction, is **the fundamental challenge** faced by the contemporary hadron and nuclear physics.
- Nucleon electromagnetic form factors , which gauge the distributions of the electric charge and magnetization inside the nucleon, are the fundamental probes to the internal structure of nucleon. F. Gross, E. Klempt, S. J. Brodsky, A. J. Buras, et al. Eur. Phys. J. C 83, 1125 (2023)
- Nucleon electromagnetic form factors are among the most fundamental quantities for exploring diverse facets of the **non-perturbative QCD dynamics** and for advancing our understanding towards the **perturbative factorization formalism**.  
arXiv:2407.18724v1 [hep-ph]
- The nucleon electromagnetic form factors can be defined as follows.

$$\mathcal{M} = \bar{u}(p') \left[ F_1(Q^2) \gamma^\mu - i \frac{F_2(Q^2)}{2m_p} \sigma^{\mu\nu} q_\nu \right] u(p)$$

$$Q^2 \rightarrow \infty, F_1(Q^2) \sim O\left(\frac{1}{Q^4}\right), F_2(Q^2) \sim O\left(\frac{1}{Q^6}\right)$$



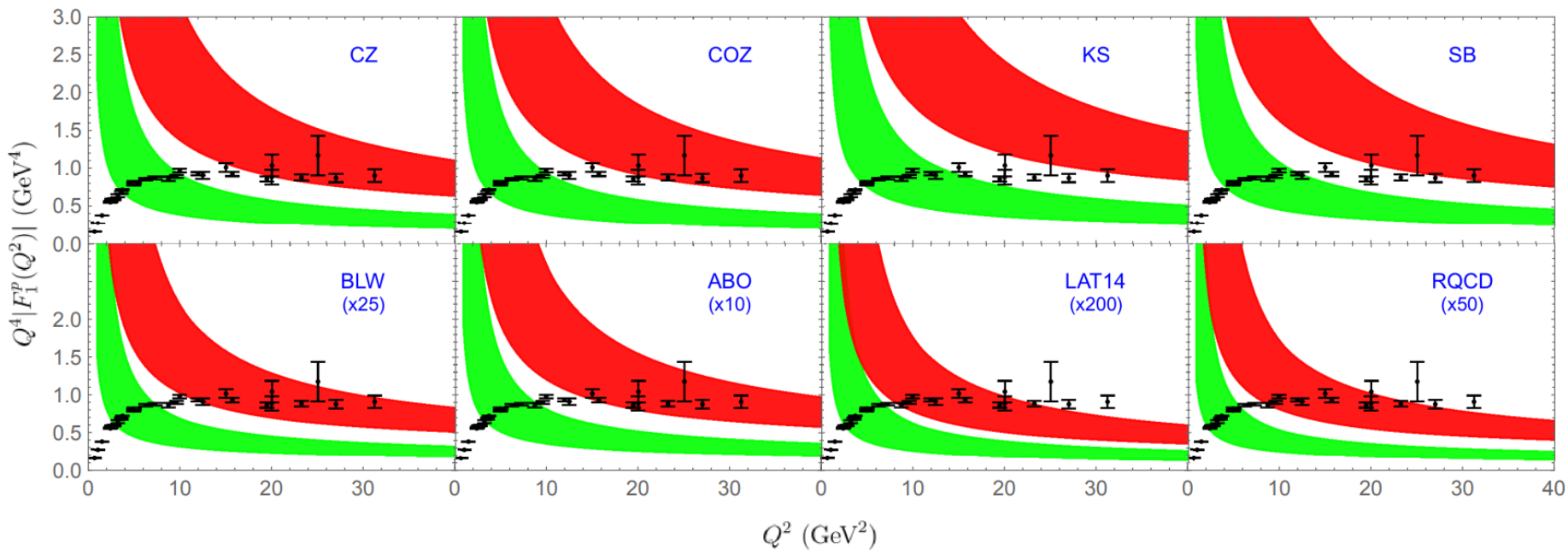
# Electromagnetic Form factors



arXiv:2407.18724v1 [hep-ph]



# Electromagnetic Form factors



arXiv:2406.19994v1 [hep-ph]

- Based on  $k_T$  factorization, the PQCD approach provides a framework applied to hard exclusive processes.

$$\mathcal{A} = \int_0^1 [dx][dx'] \int [d^2\mathbf{k}_T] \int [d^2\mathbf{k}'_T] \psi'_p(x', \mathbf{k}'_T, p', \mu) H(x, x', \mathbf{k}_T, \mathbf{k}'_T, \mu) \psi_p(x, \mathbf{k}_T, p, \mu)$$

- PQCD approach is powerful and helpful.

直接CP破坏 (%)	GFA	QCDF	PQCD	Exp.
$B \rightarrow \pi^+\pi^-$	$-5 \pm 3$	$-6 \pm 12$	$+30 \pm 20$	$+32 \pm 4$
$B \rightarrow K^+\pi^-$	$+10 \pm 3$	$+5 \pm 9$	$-17 \pm 5$	$-8.3 \pm 0.4$

YY.Keum, H.n.Li, A.I.Sanda., et.al., 2001  
C.D.Lü, K.Ukai, M.Z.Yang, et.al., 2001

- PQCD approach can calculate high-twist contributions. The high-twist contributions of the proton's LCDAs are significant and cannot be neglected.

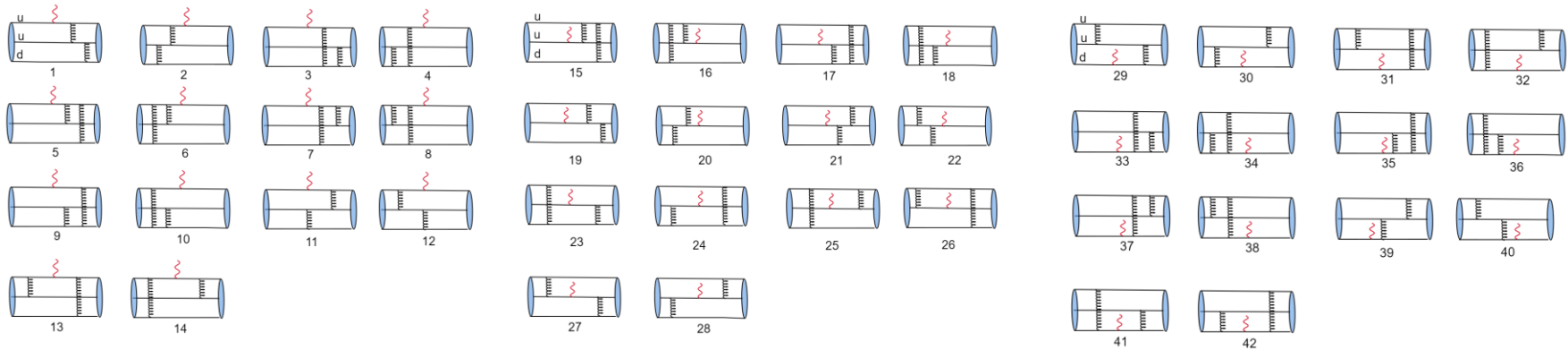
**Table 4** Form factor  $f_1(0)$  from various twist combinations of the  $\Lambda_b$  baryon and proton LCDAs. The first (second) theoretical errors of the total results come from the variations of the relevant parameters in the  $\Lambda_b$  baryon (proton) LCDAs

	Twist-3	Twist-4	Twist-5	Twist-6	Total
Exponential					
Twist-2	0.0007	-0.00007	-0.0005	-0.000003	0.0001
Twist-3 <sup>+-</sup>	-0.0001	0.002	0.0004	-0.000004	0.002
Twist-3 <sup>-+</sup>	-0.0002	0.0060	0.000004	0.00007	0.006
Twist-4	0.01	0.00009	0.25	0.000007	0.26
Total	0.01	0.008	0.25	0.00007	0.27±0.09±0.07

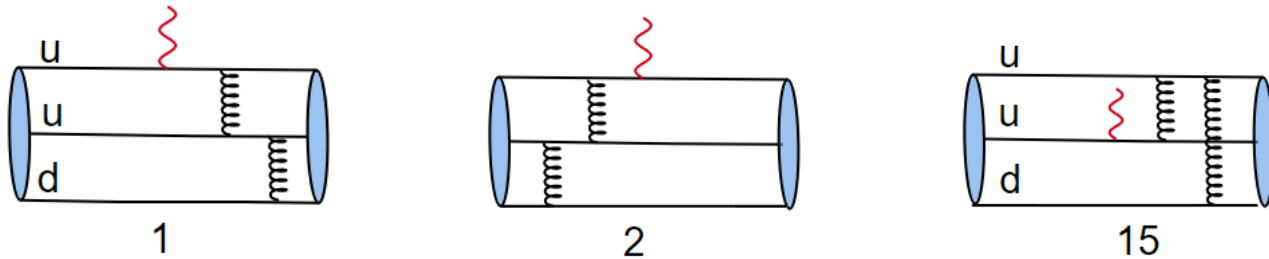
J.J.Han, Y.Li, H.n.Li, Y.L.Shen, Z.J.Xiao, F.S.Yu, Eur.Phys.J.C82,686 (2022)

# Preliminary result

- The hard-scattering  $H(x, x', k_T, k'_T, \mu)$  can be calculated perturbatively. To the lowest order of  $\alpha_s$  with two hard exchanged gluons, 42 diagrams can be drawn.

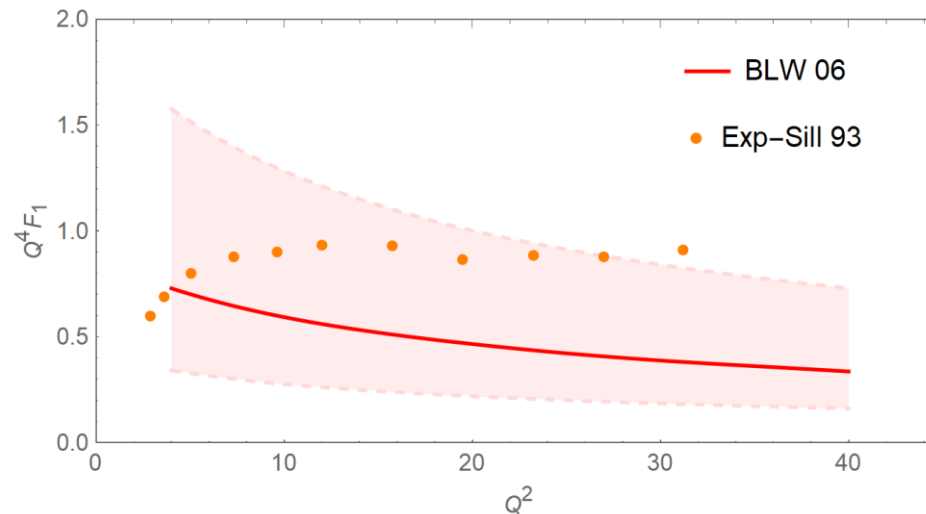


- If only the leading twist is considered, the number of diagrams can be reduced to just 11 by symmetries. However, high twists break these symmetries.



# Preliminary result

- This is the result after considering the high twists of the proton's LCDA. The uncertainties stem from the errors in the 8 non-perturbative parameters of the proton's LCDA.



- Within the margin of error, the theoretical and experimental results are in agreement, demonstrating the reliability of the PQCD method.
- The uncertainties in the non-perturbative parameters are relatively large, and the precision needs to be further improved.

# Preliminary result

- The high-twist contributions of the proton's LCDA are significant and cannot be neglected.

$Q^4 F_1^p$	twist-3	twist-4	twist-5	twist-6
twist-3	0.0035	-0.0510	0.2151	0.0002
twist-4	0.0010	0.2192	0.0429	0.0047
twist-5	0.2154	-0.1558	0.0877	0.0118
twist-6	-0.0001	0.0049	-0.0119	0.0050

$$Q^2 = 10\text{GeV}^2$$

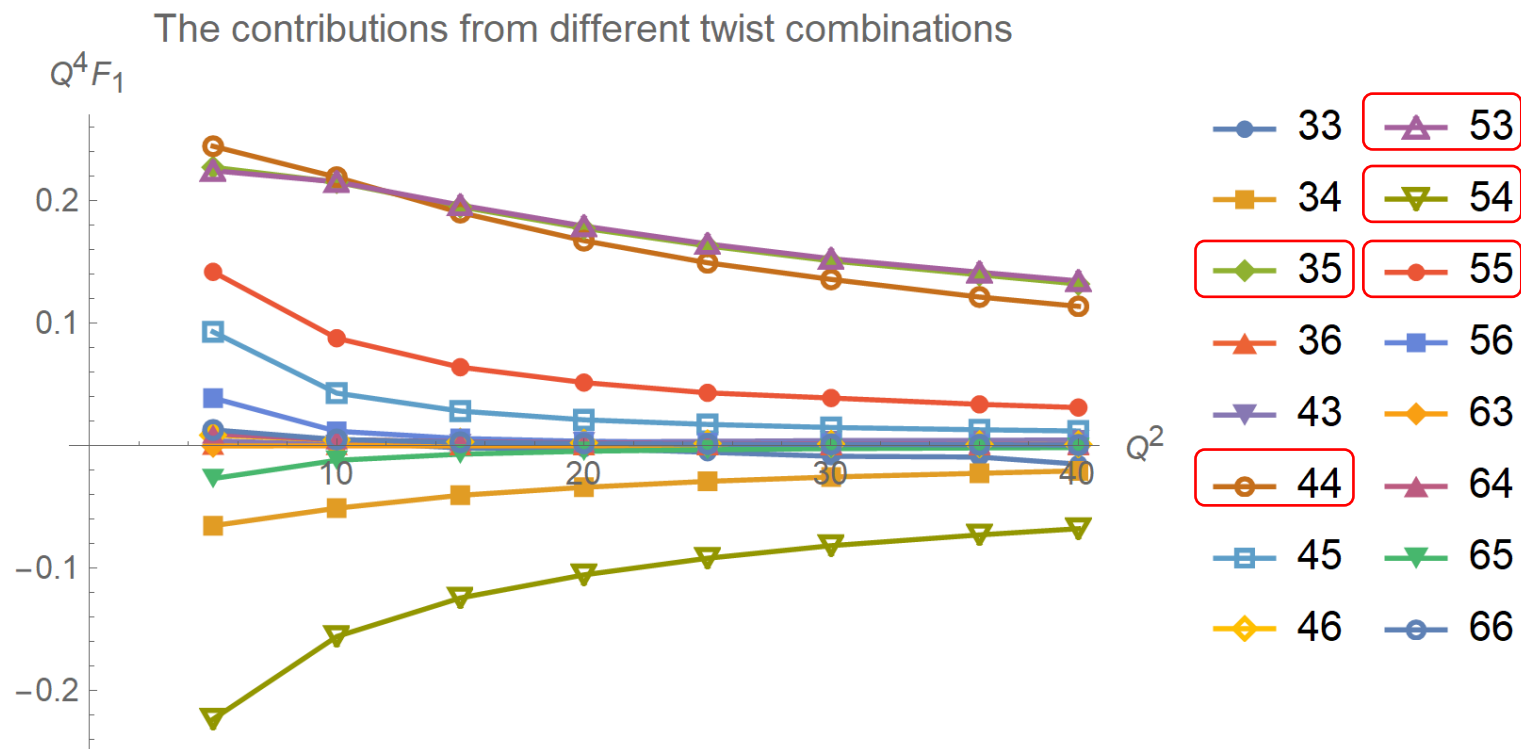
- This conclusion is consistent with the findings in the  $\Lambda_b \rightarrow p$  form factor, suggesting that it may be a general conclusion.

**Table 4** Form factor  $f_1(0)$  from various twist combinations of the  $\Lambda_b$  baryon and proton LCDAs. The first (second) theoretical errors of the total results come from the variations of the relevant parameters in the  $\Lambda_b$  baryon (proton) LCDAs

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Twist-4	0.01	0.00009	0.25	0.000007	0.26
Total	0.01	0.008	0.25	0.00007	0.27±0.09±0.07

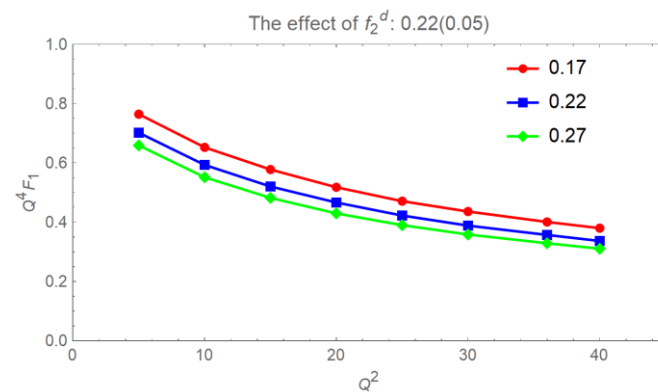
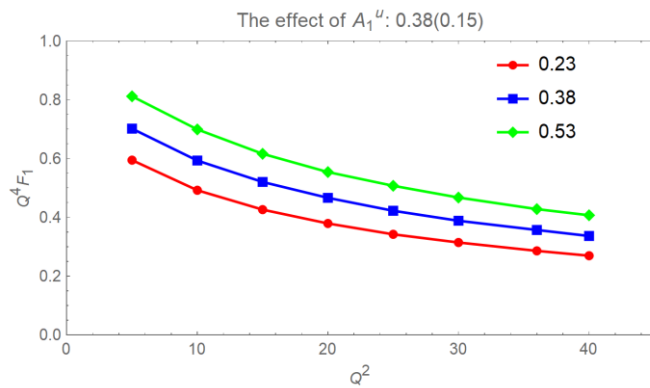
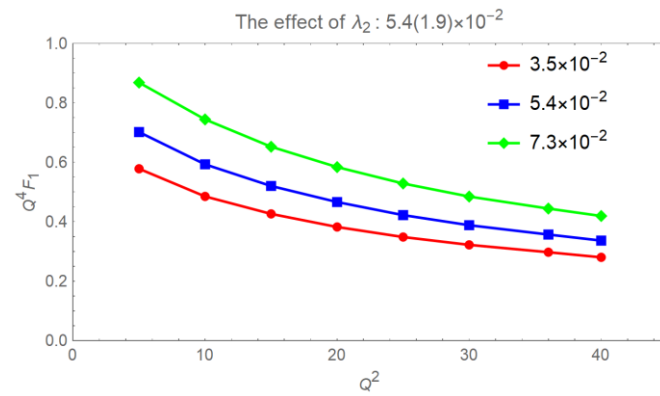
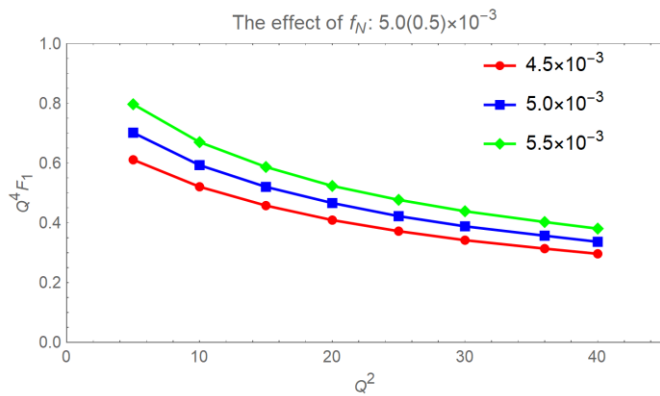
- We should gain new insights into the internal structure of the proton.
- At not high energy scales, the proton's LCDA is expected to be dominated by high-twist contributions.

# Preliminary result



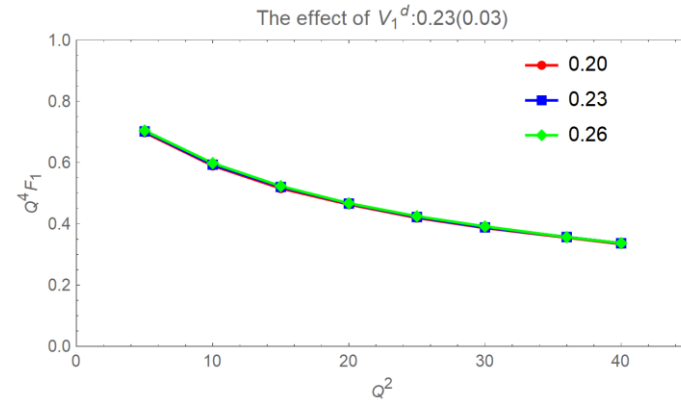
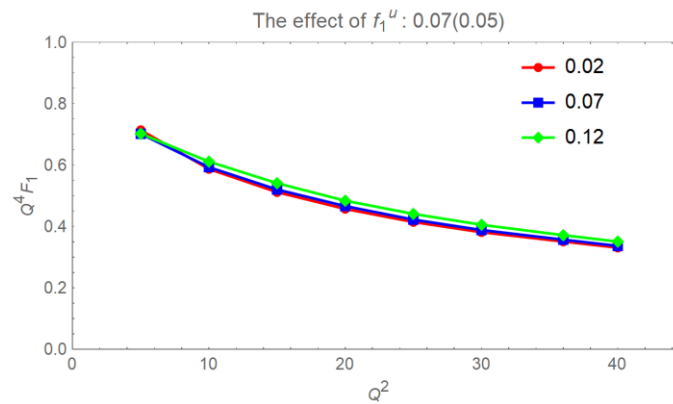
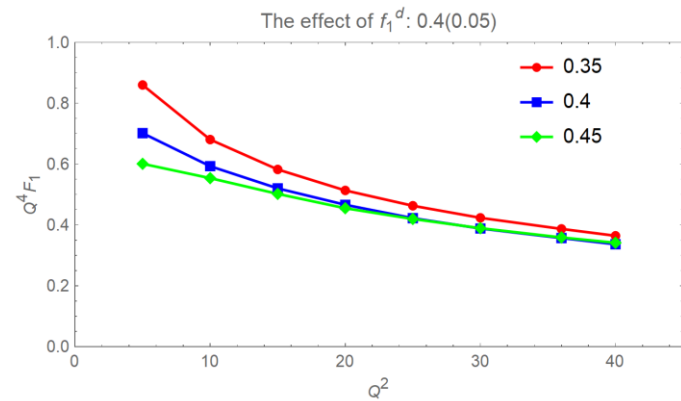
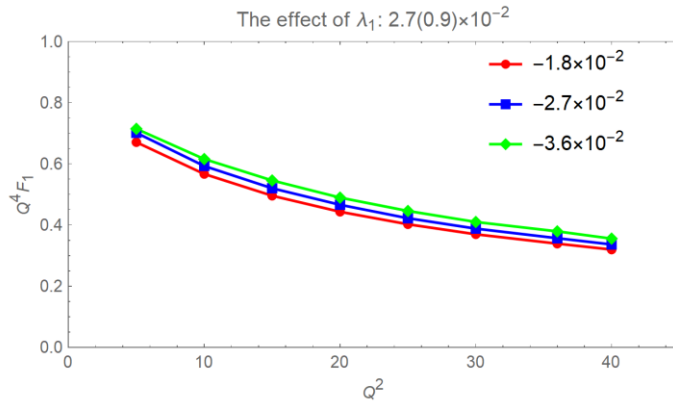
# Outlook

- The  $ep \rightarrow ep$  process or  $e^+e^- \rightarrow p\bar{p}$  process are very clean channels for constraining the proton's LCDA.
- Four non-perturbative parameters have a significant impact on F1.



# Outlook

- In future high-precision and high-energy measurements of electron-proton scattering or electron-positron annihilation, we expect to extract non-perturbative parameters with greater precision using the PQCD approach.





- **Baryon LCDAs are crucial inputs** for baryonic exclusive research in QCD methods, and play a very important role in understanding the inner structure of baryons
- We should gain new insights into the internal structure of the proton. The **high-twist** contributions of the proton's LCDA are significant and cannot be neglected.
- Determine LCDAs from experimental data in PQCD is a good **complement for PDF**, also can be a good **motivation for STCF, EIC,...**