Power corrections in Heavy meson DA within the LaMET procedure

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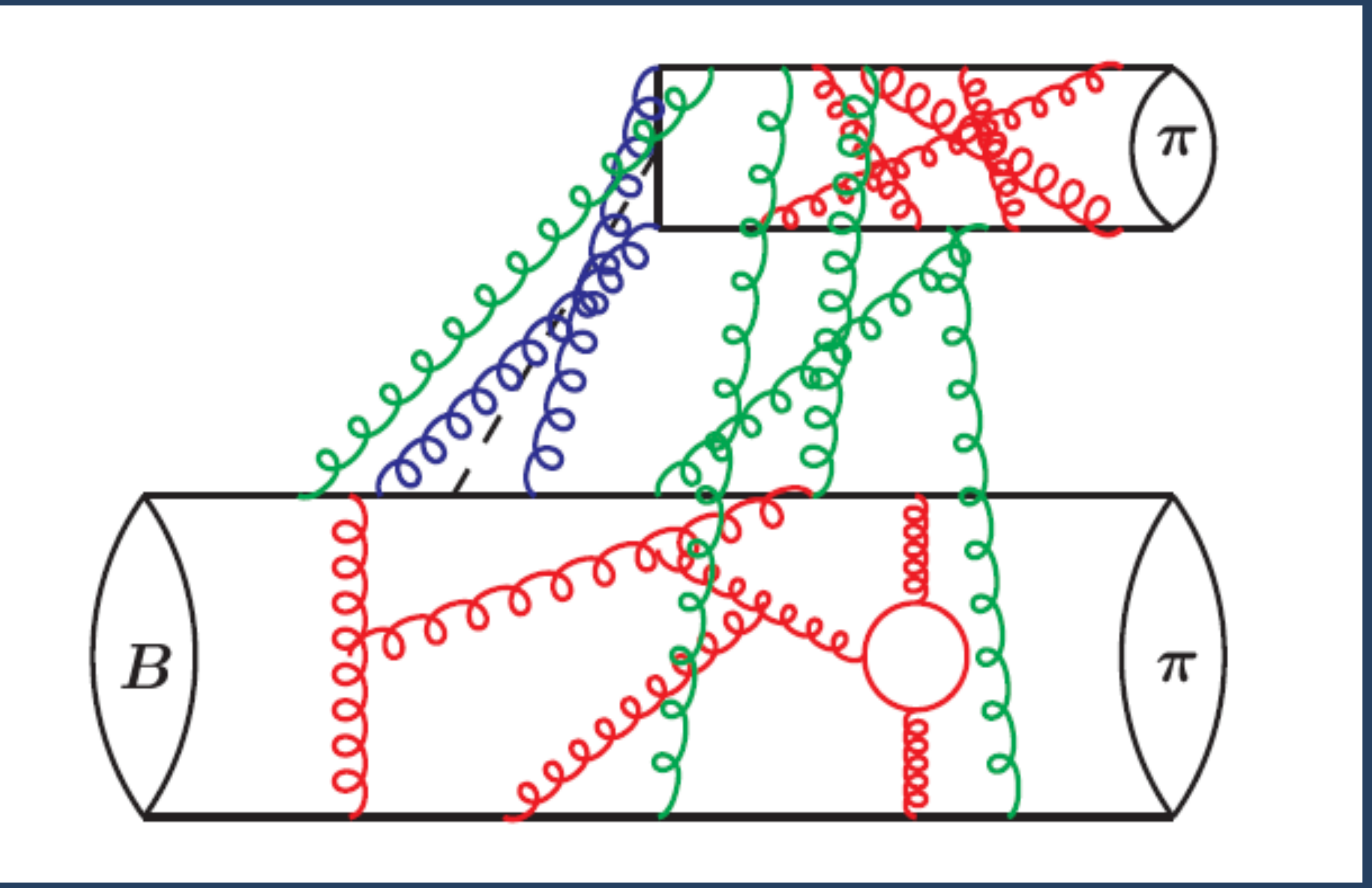
Why is LCDA important?



$$\langle \pi(p')\,\pi(q)\,|Q_i|\,\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx T_i^{\mathrm{I}}(x)\phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{\mathrm{II}}(\xi,x,y)\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$

$B \to \pi$ form factor

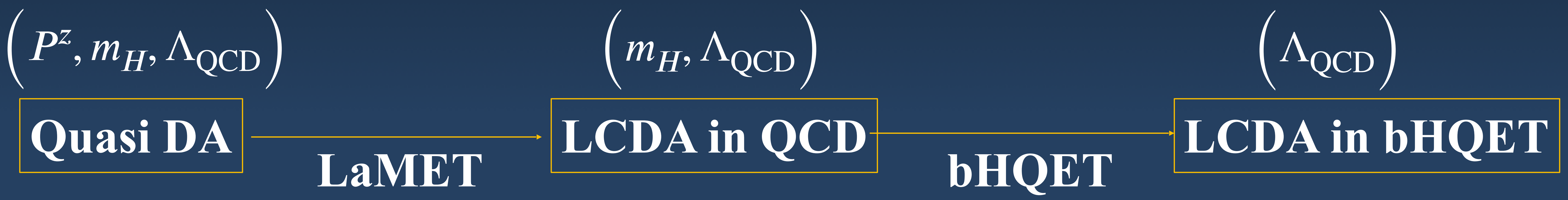
Hard kernel

B-meson LCDA

QCD Factorization: BBNS, PRL 83, 1914 (1999)
For PQCD, See: Keum, Li, Sanda PRD 63,054008 (2001)

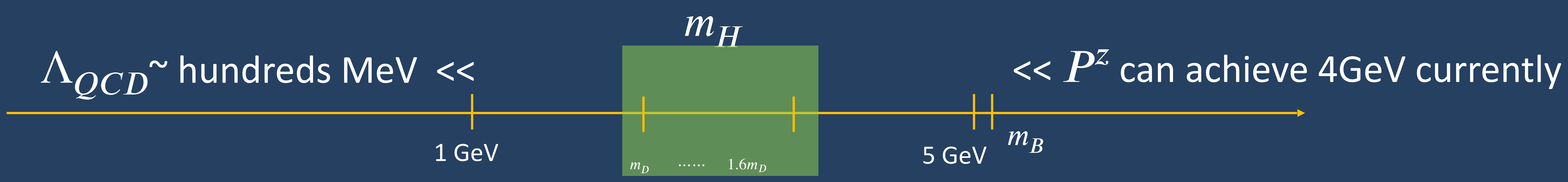
A two-step matching method

➢ Start from Quasi DA, calculable from LQCD



- A multi-scale processes:
  1. LaMET requires $\Lambda_{\rm QCD},\ m_H \ll P^z$ and finally integrate out P^z;
  2. bHQET requires $\Lambda_{\rm QCD} \ll m_H$ and integrate out m_H;

$\Rightarrow$ **Hierarchy $\boldsymbol{\Lambda_{\rm QCD} \ll m_H \ll P^z}$:** A big challenge for lattice simulation but still calculable on the lattice



Power correction in LaMET

$$\tilde{\phi}_R(x, P^z) = \int_0^1 dy C(x, y, \frac{\mu}{p^z})\phi(y, \mu)_R + O(\frac{M^2}{P^{z2}}, \frac{\Lambda_{QCD}}{(P^z)^2})$$

$m_c \sim 1.76 GeV$

$P^z \sim \{3,5\} GeV$

quasi-DA 格点可算　匹配核：微扰可算　待求LCDA

Power correction should be taken seriously when P^z is not large enough

Power correction in LaMET

The definition of LCDA

$$\langle 0 | \bar{q}(\frac{zn_+}{2}) \not{n}_+ \gamma_5 W_c(\frac{zn_+}{2}, -\frac{zn_+}{2}) Q(-\frac{zn_+}{2}) | H(P) \rangle = i f_H P^+ \int_0^1 dx e^{-i(x-\frac{1}{2})zP^+} \underline{\phi(x,\mu)}$$

LCDA

Power correction in LaMET

The definition of LCDA

$$\langle 0|\bar{q}(\frac{zn_+}{2})\not{n}_+\gamma_5 W_c(\frac{zn_+}{2},-\frac{zn_+}{2})Q(-\frac{zn_+}{2})|H(P)\rangle = if_H P^+\int_0^1 dx e^{-i(x-\frac{1}{2})zP^+}\phi(x,\mu)$$

$$\langle 0|(\bar{q}(\frac{zn_z}{2})\not{n}_z\gamma_5 W_c(\frac{zn_z}{2},-\frac{zn_z}{2})Q(-\frac{zn_z}{2}))_{lt}|H(P)\rangle_{non-sing} = if_H P^+\int_0^1 dx (e^{-i(x-\frac{1}{2})zP_z})_{lt}\phi(x,\mu)$$

Nucl.Phys.B 361 (1991)   Ian I. Balitsky, Vladimir M. Braun

Power correction in LaMET

The definition of LCDA

$$\langle 0|\bar{q}(\frac{zn_+}{2})\not{n}_+\gamma_5 W_c(\frac{zn_+}{2},-\frac{zn_+}{2})Q(-\frac{zn_+}{2})|H(P)\rangle = if_H P^+\int_0^1 dx e^{-i(x-\frac{1}{2})zP^+}\phi(x,\mu)$$

$$\langle 0|(\bar{q}(\frac{zn_z}{2})\not{n}_z\gamma_5 W_c(\frac{zn_z}{2},-\frac{zn_z}{2})Q(-\frac{zn_z}{2}))_{lt}|H(P)\rangle_{non-sing} = if_H P^+\int_0^1 dx(e^{-i(x-\frac{1}{2})zP_z})_{lt}\phi(x,\mu)$$

The definition of Quasi-DA

$$\langle 0|(\bar{q}(\frac{zn_z}{2})\not{n}_z\gamma_5 W_c(\frac{zn_z}{2},-\frac{zn_z}{2})Q(-\frac{zn_z}{2}))|H(P)\rangle = if_H P^+\int_0^1 dx e^{-i(x-\frac{1}{2}zP_z)}\tilde{\phi}(x,\mu)$$

Leading matching procedure within LaMET discards the singular logarithms, and left the Leading-twist projection as it’s power corrections

Power correction in LaMET

Leading twist projection:

$$[\bar{q}(-\frac{zn_z}{2})\not{n}\gamma_5 W_c Q(\frac{zn_z}{2})]_{lt} = \sum_n \frac{z^n}{n!} n_z^{\mu_1}\ldots n_z^{\mu_n}[\bar{q}(0)\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\ldots\overleftrightarrow{D}_{\mu_n}\gamma_5 Q(0) - Trace]$$

Higher-twist operator correction $(\frac{\Lambda_{QCD}^2}{P^{z2}})^n$

$$[e^{ip\cdot z}]_{lt} = \sum_{n=0}^{\infty}\frac{1}{n!} z^{\mu_1}\ldots z^{\mu_n}(p_{\mu_1}\ldots p_{\mu_n} - Trace)$$

Target mass correction $(\frac{m_H^2}{P^{z2}})^n$

Mass correction

$$[e^{ip\cdot z}]_{lt} = \sum_{n=0}^{\infty} \frac{1}{n!} z^{\mu_1} \dots z^{\mu_n} (P_{\mu_1} \dots P_{\mu_n} - Trace)$$

Case n=2

$$n_z^{\mu_1} n_z^{\mu_2} P_{\mu_1} P_{\mu_2} = (n_z^{\mu_1} n_z^{\mu_2} - \frac{1}{4} n_z^2 g^{\mu_1\mu_2}) P_{\mu_1} P_{\mu_2} + \frac{1}{4} n_z^2 g^{\mu_1\mu_2} P_{\mu_1} P_{\mu_2}$$

$$\downarrow$$

$$[n_z^{\mu_1} n_z^{\mu_2} P_{\mu_1} P_{\mu_2}]_{lt}$$

$$[n_z^{\mu_1} n_z^{\mu_2} P_{\mu_1} P_{\mu_2}]_{lt} = n_z^{\mu_1} n_z^{\mu_2} P_{\mu_1} P_{\mu_2} \cdot (1 + \frac{1}{4} (\frac{M}{P^z})^2)$$

However the coefficients is different at different n.

$$[n_z^{\mu_1} \dots n_z^{\mu_n} P_{\mu_1} \dots P_{\mu_n}]_{lt} = n_z^{\mu_1} \dots n_z^{\mu_n} P_{\mu_1} \dots P_{\mu_n} \cdot (1 + ?(\frac{M}{P^z})^2 + ?(\frac{M}{P^z})^4 + \dots)$$

Mass correction

$$\psi(x) = \frac{1}{\sqrt{1+4c}}[\frac{f_+}{2}\psi_{lt}(\frac{1}{2} - \frac{1-2x}{f_+}) - \frac{f_-}{2}\psi_{lt}(\frac{1}{2} + \frac{1-2x}{f_-})]$$

$$c = m_H^2/(4P^z)^2 \qquad f_\pm = \sqrt{1+4c} \pm 1$$

Phys.Rev.D 95 (2017) 9, 094514   Jian-Hui Zhang, Jiunn-Wei Chen,…

$$\psi(x,\mu) = \psi(x,\mu)_{lt} - \frac{1}{8}\frac{m_H^2}{(P^z)^2} \times \frac{d}{dx}\left((2x-1)\psi(x,\mu)_{lt} + \mathcal{O}(\frac{m_H^4}{(P^z)^4}\right)$$

Nucl.Phys.B 361(1991) Ian I. Balitsky, Vladimir M. Braun

Λ_{QCD} correction

Quasi-DA also receives contribution from higher-twist operators

$$\mathcal{O}_{twist-4} = \int_0^z \bar{q}(0)\gamma^5\gamma^\nu W_c(0,z_1)D_\nu W_c(z_1,z)Q(z)dz_1$$

$$\mathcal{O}'_{twist-4} = \int_0^z dz_1 \int_0^{z_1} dz_2 \bar{q}(0)\gamma^5 \not{n} W_c(0,z_2)D_\nu W_c(z_2,z_1)D^\nu W_c(z_1,z)Q(z)$$

High dimensional operator would mix with low dimension operator on Lattice.

Covariant derivatives are realized through Wilson lines, increasing uncertainty.

Thus, hard to calculate using Lattice QCD.

Λ_{QCD} correction- Renormalon model

Cutoff regularization

$$C(x, y, \frac{\mu_F}{P^z}) = \delta(x-y) + \sum_{i=1}^{\infty} c_i(\mu_F)\alpha_s^i - \frac{\mu_F^2}{(P^z)^2} D_{\mathcal{Q}}(x) + \dots$$

Since the quasi-DA is independent of the factorization scale, so the $\frac{\mu_F^2}{(P^z)^2} D_{\mathcal{Q}}(x)$ *term must cancel with contribution from higher-twist operators*

We could estimate the higher-twist contribution from $\frac{\mu_F^2}{(P^z)^2} D_{\mathcal{Q}}(x)$.

Λ_{QCD} correction- Renormalon model

Dimensional regularization

$$C(x, y, \frac{\mu_F}{P^z}) = \delta(x - y) + \sum_{i=1}^{\infty} c_i \alpha_s^i$$

The series $c_i \alpha_s^i$ becomes divergent ,and the power dependence on μ_F disappears.

Renormalon model: Estimate the range of power dependence from divergent series.

Renormalons 1994 Beneke

Numerical calculation

Goal: Use DA models for D-meson to estimate the impact of power corrections $(\frac{M}{P^z})^{2n}, (\frac{\Lambda_{QCD}}{P^z})^2$ on Quasi-DA, when using LaMET matching formula.

Models of LCDA

$$\phi(x,\mu) = 6x(1-x)\left[1 + \sum_{n=1}^{\infty} a_n(\mu) C_n^{(3/2)}(2x-1)\right],$$

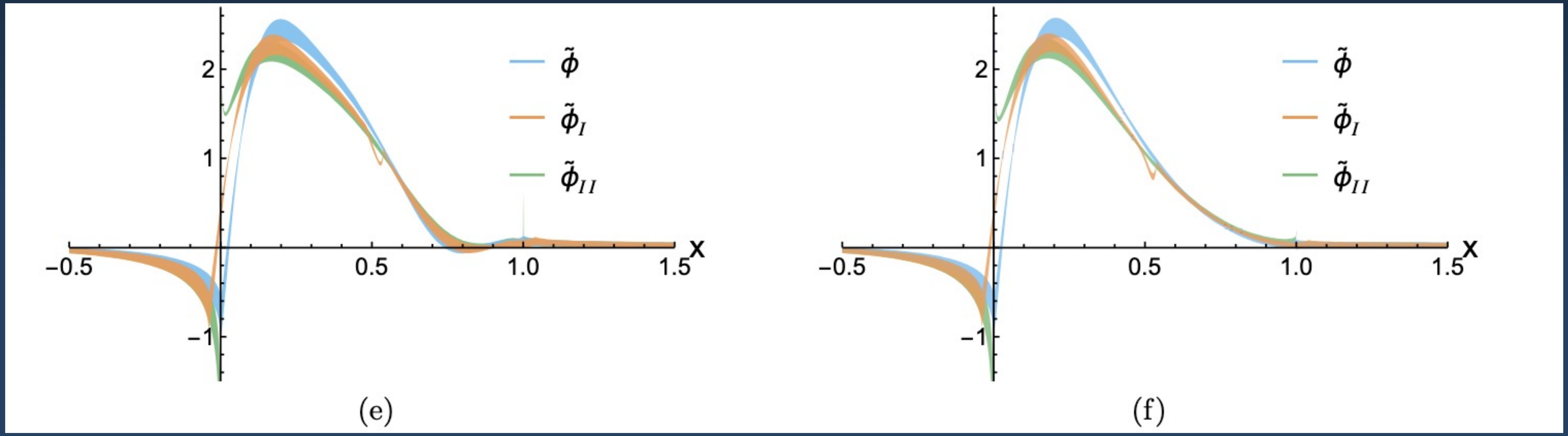
$$a_n^D(1.6GeV) = \{1, -0.659, 0.206, -0.057, 0.036, -0.004, -0.007, \ldots\}$$

$$\phi(x,\mu) = a(1-x)xe^{-bx}$$

The choice of parameters:

$m_H = 1.76\ \text{GeV}, \Lambda_{QCD} = 0.5\ \text{GeV}, z_s = 0.1672\ \text{fm}, \mu = 1.6 \sim 3\ \text{GeV}, P^z = \{3,4,5\}\ \text{GeV}$

Numerical calculation- Mass correction at Pz=3 GeV



Blue: Quasi-DA without mass correction;

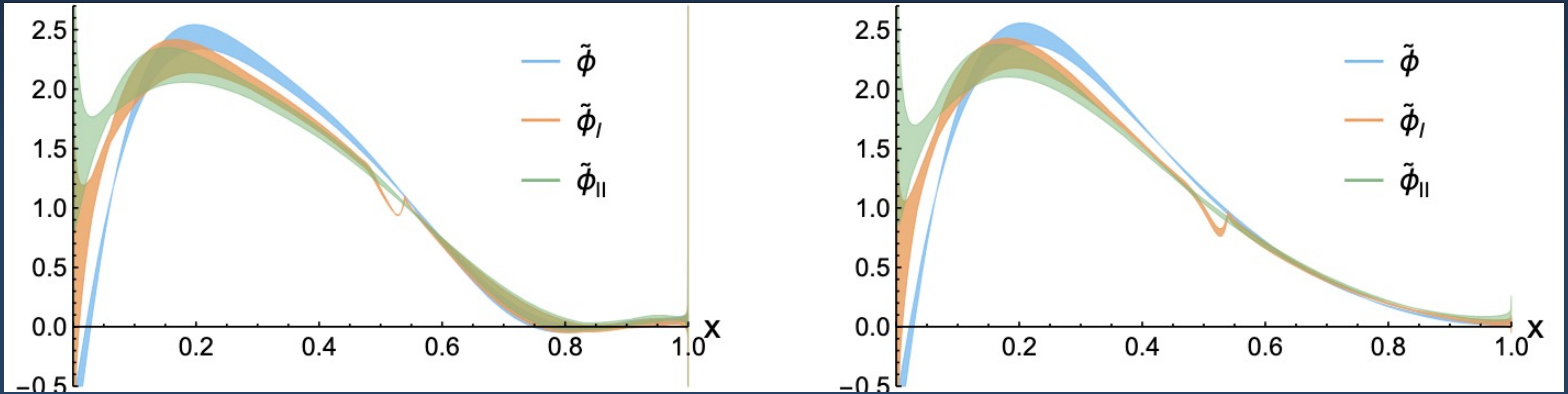
Error bands: factorization scale

Orange: Quasi-DA, $(\frac{M}{P^z})^{2n}$;

Green: Quasi-DA, $(\frac{M}{P^z})^2$.

1. ***Distribution is wilder. Peak region is shifted to left.***
2. ***There is a dent at x=1/2.***

Numerical calculation—Mass correction+ Λ_{QCD} correction at Pz=3 GeV



Error bands: factorization scale and renormalon ambiguity

Blue: Quasi-DA without correction;

Orange: Quasi-DA, $(\frac{M}{P^z})^{2n}$ and $(\frac{\Lambda_{QCD}}{P^z})^2$;

Green: Quasi-DA, $(\frac{M}{P^z})^2$ and $(\frac{\Lambda_{QCD}}{P^z})^2$

Numerical calculation—Mass correction+ Λ_{QCD} correction

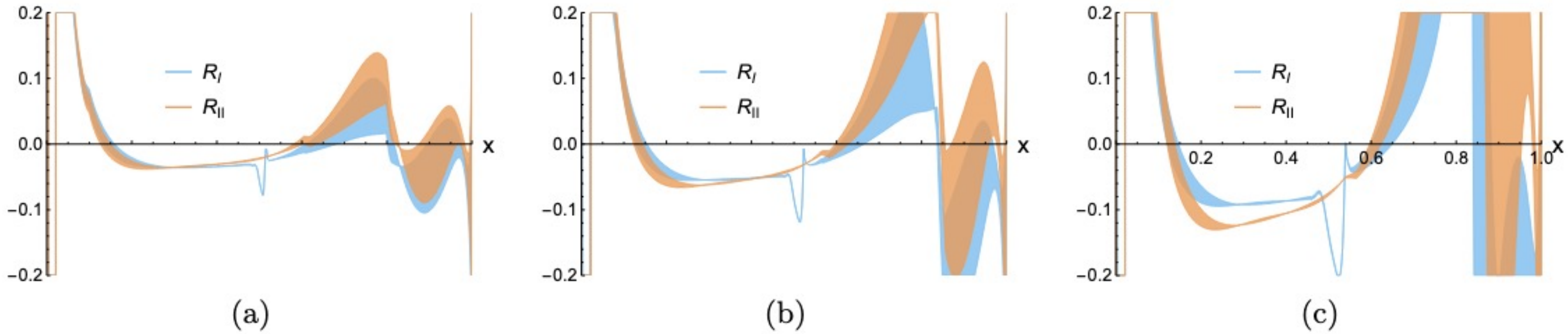


FIG. 7. Panel (a) corresponds to the ratio $R_{I/II}$ at $P^z = 5\,\text{GeV}$, panel (b) to the ratio $R_{I/II}$ at $P^z = 4\,\text{GeV}$, and panel (c) to the ratio $R_{I/II}$ at $P^z = 3\,\text{GeV}$.

$$R_{I/II} = \frac{\tilde{\phi}_{I/II} - \tilde{\phi}}{\tilde{\phi}}$$

Error bands: factorization scale and renormalon ambiguity

For the D-meson case, in the moderate x region $x \in [0.2, 0.8]$, the corrections from mass and power are typically smaller than 20%. At peak region, mass correction dominates.

Thank you for your attention!

Origin of the dent in mass correction

$$\tilde{\psi}_{lt}(x) = \frac{1}{\sqrt{1+4c}}[\frac{f_+}{2}\psi(\frac{1}{2} - \frac{1-2x}{f_+}) - \frac{f_-}{2}\psi(\frac{1}{2} + \frac{1-2x}{f_-})]$$

$c = m_H^2/(4P^z)^2$ $\quad$ $f_\pm = \sqrt{1+4c} \pm 1$

Taylor expansion in c of $\psi(\frac{1}{2} - \frac{1-2x}{f_+})$ gives:

$$\psi(x,\mu) - \frac{1}{8}\frac{m_H^2}{(P^z)^2} \times \frac{d}{dx}((2x-1)\psi(x,\mu) + \mathcal{O}(\frac{m_H^4}{(P^z)^4})$$

However, Taylor expansion in c of $\psi(\frac{1}{2} + \frac{1-2x}{f_-})$ is zero

This is the non-perturbative effect in mass correction,

which cannot be seen by fixed order correction.

Λ_{QCD} correction- Renormalon model

$$
\begin{aligned}
2\delta_R\Phi(x,\mu) = &\left[\frac{\Phi'(x,\mu)}{x-1}\right. \\
&- \frac{1}{(x-1)^2}\int_{-1}^{x} dy \left[\frac{x-1}{y-1} + \ln\left(1-\frac{x-1}{y-1}\right)\right] \Phi'(y,\mu)\Bigg] \\
&+ \left[\frac{\Phi'(x,\mu)}{x+1}\right. \\
&+ \frac{1}{(x+1)^2}\int_{x}^{1} dy \left[\frac{x+1}{y+1} + \ln\left(1-\frac{x+1}{y+1}\right)\right] \Phi'(y,\mu)\Bigg] \\
&+ \frac{1}{4}\Phi''(x,\mu),
\end{aligned}
$$

The range of power correction $\Phi' = \Phi \pm \delta_R\Phi$

Λ_{QCD} correction- Renormalon model

Borel summation

$$C(x, y, \frac{\mu_F}{P^z}) = \delta(x-y) + \sum_{i=1}^{\infty} c_i(\mu_F)\alpha_s^i$$

$$B[T](w) = \sum_{k=1}^{\infty} \frac{c_k}{k!}\left(\frac{w}{\beta_0}\right)^k.$$

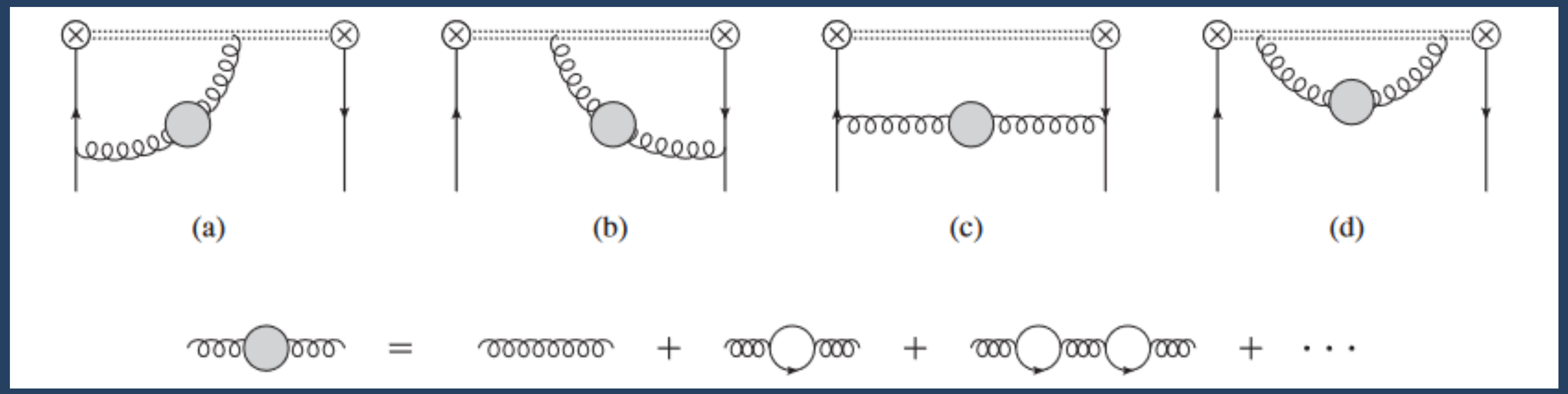
$$T(\alpha_s) = \frac{1}{\beta_0}\int_0^{\infty} dw e^{-w/(\beta_0 a_s)} B[T](w)$$

The asymptotic series of $T(\alpha_s)$ is exactly $\sum_{i=1}^{\infty} c_i \alpha_s^i$

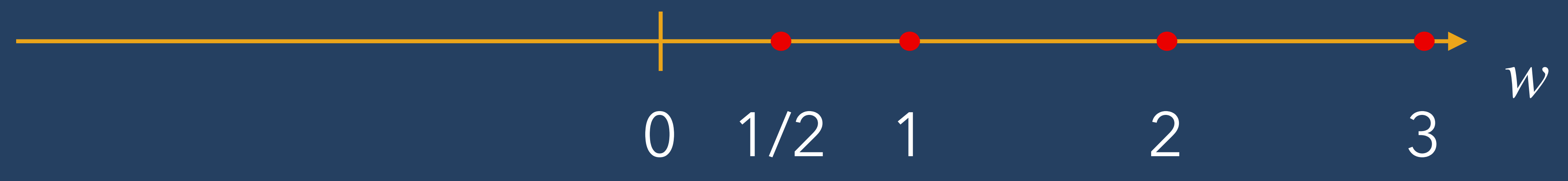
Λ_{QCD} correction- Renormalon model

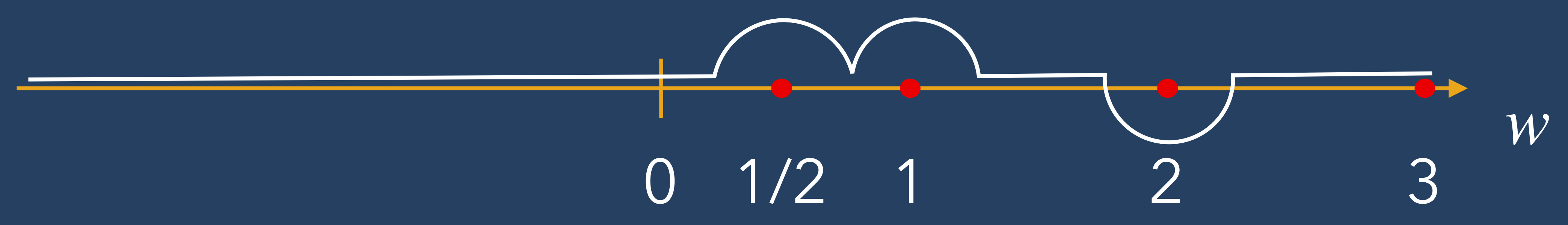
$$T(\alpha_s) = \frac{1}{\beta_0} \int_0^\infty dw e^{-w/(\beta_0 \alpha_s)} B[T](w)$$

Diagrams contribute to these poles



$B[T](w)$





Renormalon ambiguity: the choice of contour