Doubly charmed $\Lambda_c\Lambda_c$ scattering from Lattice QCD

Yiqi Geng（耿乙淇）

Institute of Morden Physics, and Nanjing Normal U.

Collaborators: Liuming Liu, Peng Sun, Jia-Jun Wu, Hanyang Xing, Ruilin Zhu

yqgeng@njnu.edu.cn

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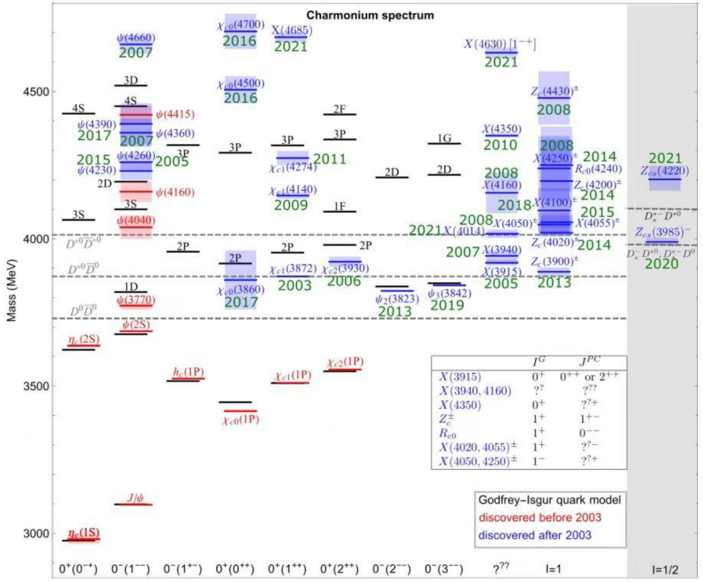
Outline

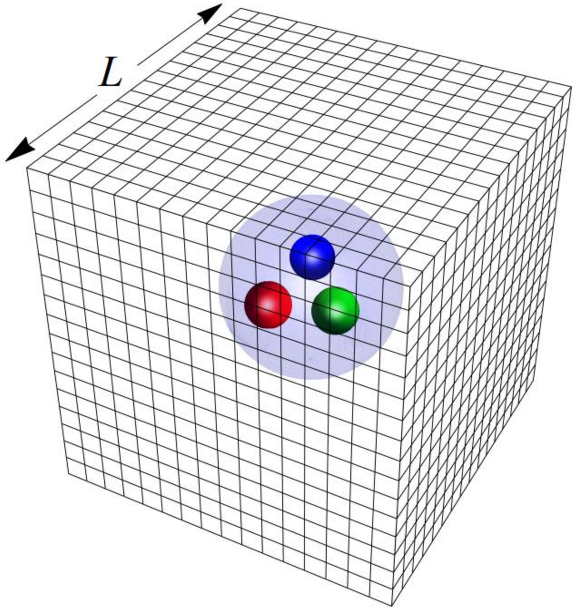
- Motivation
- Lattice Setup : CLQCD ensembles
- Formalism : Finite-volume scattering
- Analyses and Result
- Summary

- Since X(3872) was found in 2003, many exotic states beyond conventional quark model were found in experiments one after another.
- Recently, many new exotic states were found, such as $T_{cc}^{+}(3875)$, $P_c(4440\&4457)$, $X(6900)$ and so on.



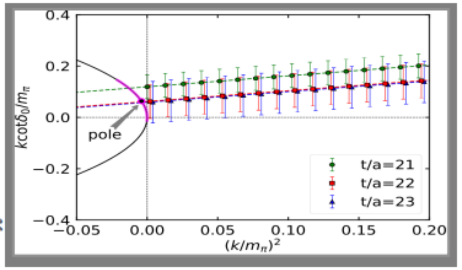
[F.-K. Guo et al, PoS LATTICE2022(2023)232]

- Unquenched Lattice QCD pushes ahead many lattice study on hadronic spectrum.



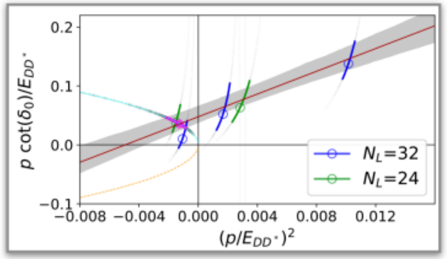
- Many lattice works on $T_{cc}^{+}(3875)$ are trying to explain its structure.

$m_\pi \approx 146$ MeV



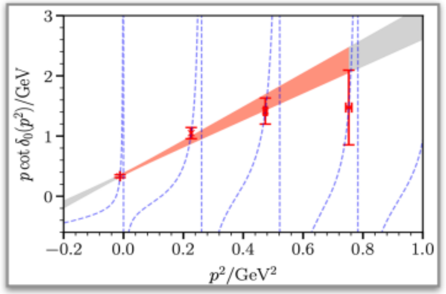
Lyu et al., PRL 131, 161901 (2023)

$m_\pi \approx 280$ MeV



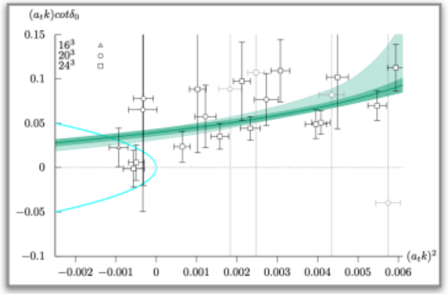
Padmanath, Prelovsek, PRL 129, 032002 (2022)
Collins et al., PRD 109 (2024) 9, 094509

$m_\pi \approx 350$ MeV



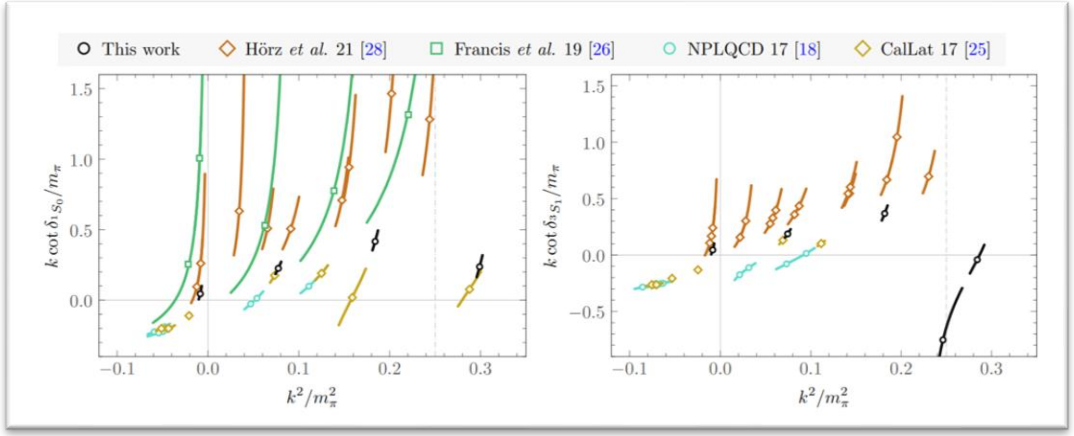
Chen et al. PLB 833, 137391 (2022)

$m_\pi \approx 391$ MeV



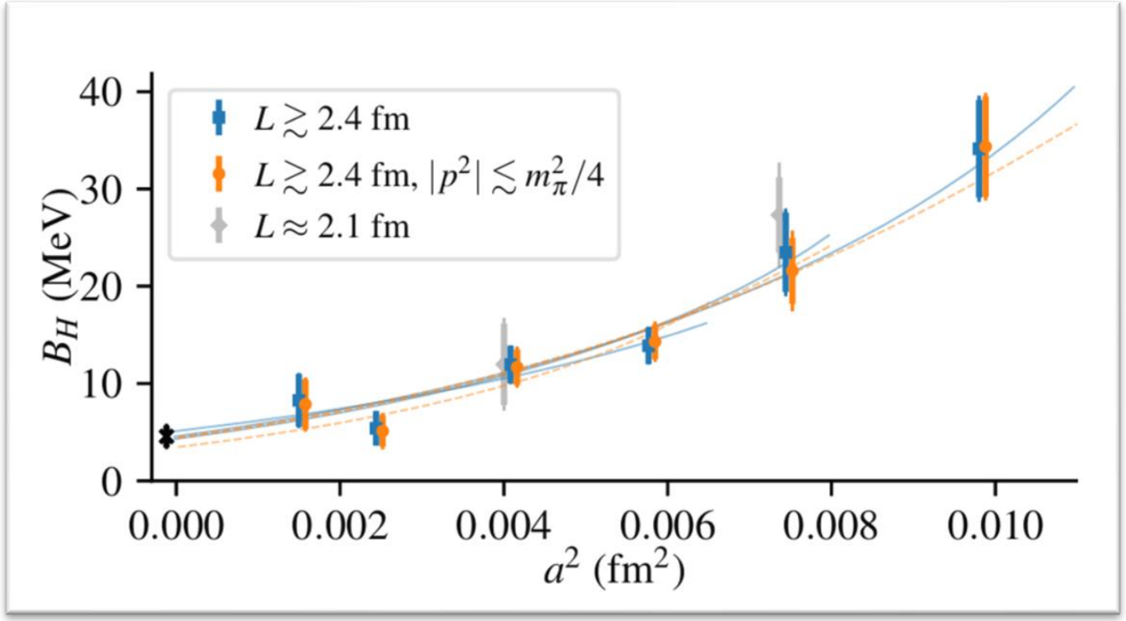
Whyte, Wilson, Thomas, arXiv:2405.15741

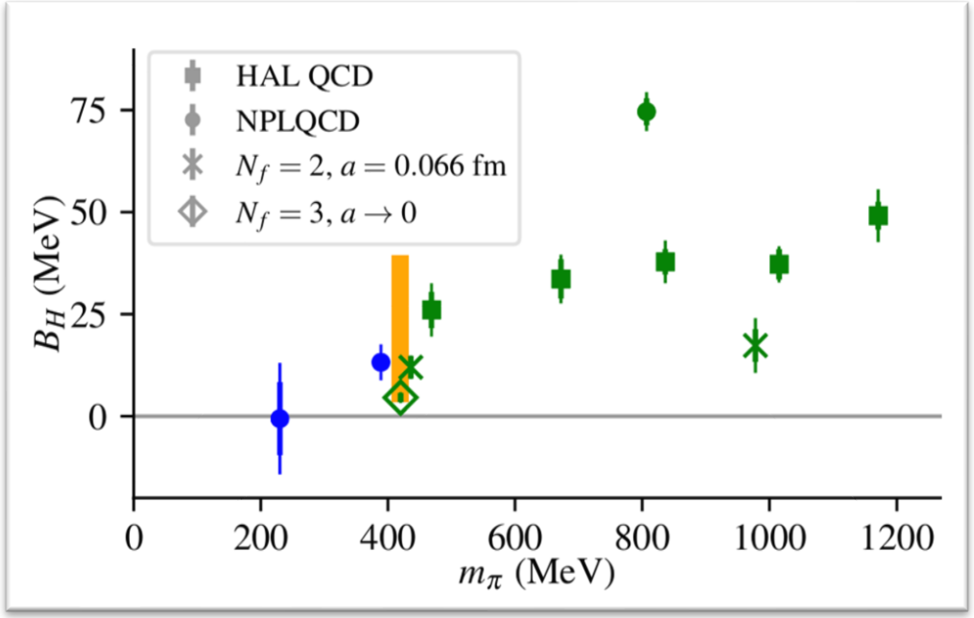
- Study of Baryon-Baryon interaction have many challenges on Lattice.
  - Deuteron bound state hasn't been confirmed in the lattice calculation.



[Saman Amarasinghe et al. PRD 107 (2023) 9, 094508]

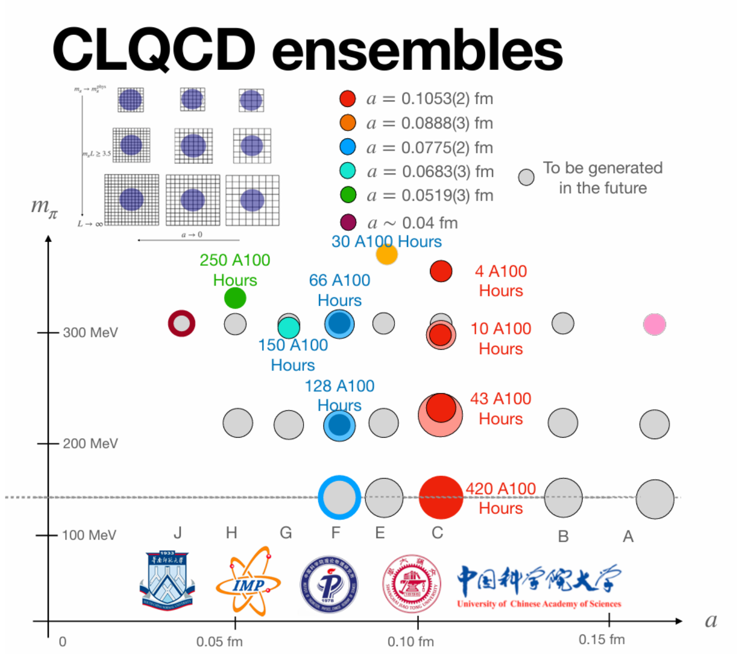
- A weekly bound state $\Lambda\Lambda$. However, it hasn't been found in experiment.





[Jeremy R. Green et al. Phys.Rev.Lett. 127 (2021) 24, 242003]

What about $\Lambda_c\Lambda_c$?



[Z.C. Hu et al. PRD 109 (2024) 5, 054507]

In this work,

- Two Wilson-Clover lattice ensembles are used.
- Space-time symmetrical and 2+1 flavor.
- Same m_π and lattice spacing, but different volume.

ensemble	$(L/a)^3 \times T/a$	β	a(fm)	m_π(MeV)	m_K(MeV)	$m_\pi \times L$	N_{conf}
F32P30	$32^3 \times 96$	6.41	0.07746(18)	303.2(1.3)	524.6(1.8)	3.81	567
F48P30	$48^3 \times 96$	6.41	0.07746(18)	303.4(0.9)	523.6(1.4)	5.72	201

☐ Operator construction [S. Prelovsek et al, JHEP 01 (2017) 129]

$$\vec{k} = \frac{2\pi}{L}\vec{n}$$

- Single baryon $\mathrm{B}(k, x^0) = \sum_{\vec{x}} P_+ \, \varepsilon_{abc} \, r_{ax} \left[s_{bx}^T \, (C\gamma_5) t_{cx} \right] e^{-i\vec{k}\cdot\vec{x}}$
- Two baryons $\mathcal{O}_{B_1 B_2}^{\Lambda}(|\vec{k}|) = \sum_j c_j^{\Lambda} B_1^T(|\vec{k_j}|) \, C\gamma_5 B_2(-|\vec{k_j}|)$

☐ Correlation function $C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle \approx Ae^{-E\,t}$

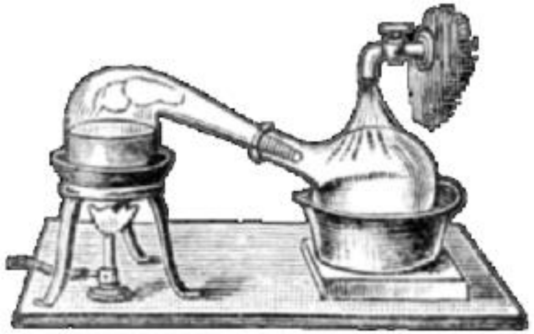
☐ Generalized eigenvalue problem(GEVP)

$$C(t)v_\alpha(t, t_0) = \lambda_\alpha(t, t_0)C(t_0)v_\alpha(t, t_0)$$

- eliminate the overlap between operators

□ Distillation quark smearing method

- Operate □(t)=$V(t)V^\dagger(t)$ on quarks → $P = VGV^\dagger$
  - G ~ propagator
  - P ~ perambulator
  - V ~ eigenvectors of ∇^2
  - Dimension of V is $[N_t, N_c \times N_x^3, N_{ev}]$
- Advantages
  - Improve precision
  - Efficient for large numbers of operations



[Hadron Spectrum Collaboration, Phys.Rev.D 80 (2009) 054506]

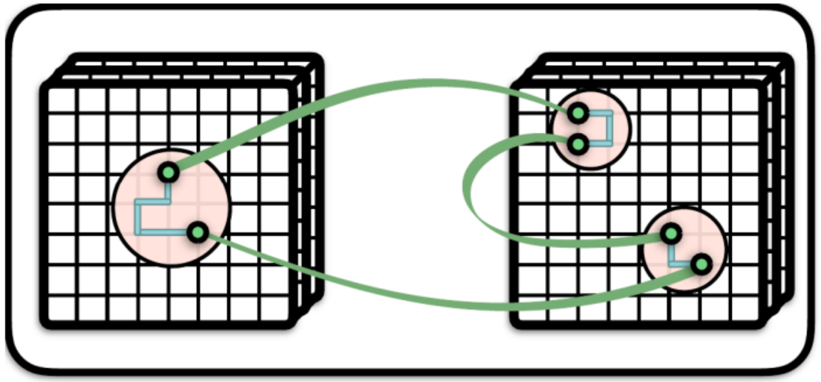
[Colin Morningstar, Phys.Rev.D 83 (2011) 114505]

☐ Lüscher's finite volume method [M. Lüscher, Nucl. Phys. B. 354 (1991) 531-578]

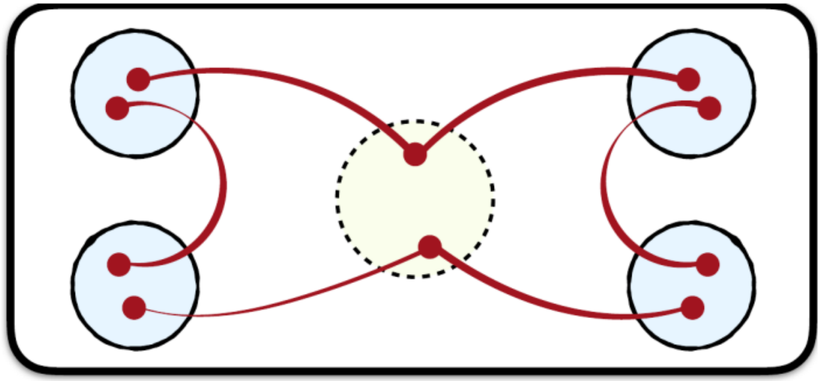
$$\mathbf{det}[\mathbf{1} + \boldsymbol{i\rho t}(\mathbf{1} + \boldsymbol{i}\,\mathcal{M})] = \mathbf{0}$$

- ρ ~ phase space
- t ~ scattering amplitude
- $\mathcal{M}$ ~ Lüscher matrix

- For S wave rest-frame case: $\mathcal{M}_{00}(\tilde{q}^2) = \frac{2}{\sqrt{\pi}L} Z_{00}(1;\tilde{q}^2)$



finite volume spectrum



scattering amplitude

- **A new modification on Lüscher equation is needed**

[Yan Li, Jia-Jun Wu at el, JHEP 08 (2024) 178]

- Caused by discretized effect with $Z_{1(2)} \neq 1$:

$$E(q) = \sqrt{m_1^2 + Z_1 q^2} + \sqrt{m_2^2 + Z_2 q^2}$$

q is the interacting-momentum.
$k^* = \frac{2\pi}{L} n$ is center-mass one.

- Consider a translation of the limit

$$\lim_{k^* \to q} \frac{1}{4\omega_1(k^*)\omega_2(k^*)} \frac{q - k^*}{E^*(q) - \omega_1(k^*) - \omega_2(k^*)} = \lim_{k^* \to q} \frac{1}{4q[Z_1\omega_2(q) + Z_2\omega_1(q)]}$$

$$\to \frac{1}{4\omega_1(k^*)\omega_2(k^*)} \frac{1}{E^*(q) - \omega_1(k^*) - \omega_2(k^*)} = \frac{1}{2[Z_1\omega_2(q) + Z_2\omega_1(q)]} \frac{1}{q^2 - k^{*2}}$$

$$\to \mathcal{M}_{lm,l'm'}(q,P) = \frac{16\pi^2}{q} \left(\frac{1}{L^3} \sum_k - P \int \frac{d^3 k^r}{(2\pi)^3} \right) \mathcal{J}^r \frac{Y_{lm}(\hat{k}^*) Y_{l'm'}(\hat{k}^*) \left(\frac{|k^*|}{q} \right)^{l+l'}}{q^2 - k^{*2}} \frac{\boldsymbol{E^*(q)}}{\boldsymbol{Z_1\omega_2(q) + Z_2\omega_1(q)}}$$

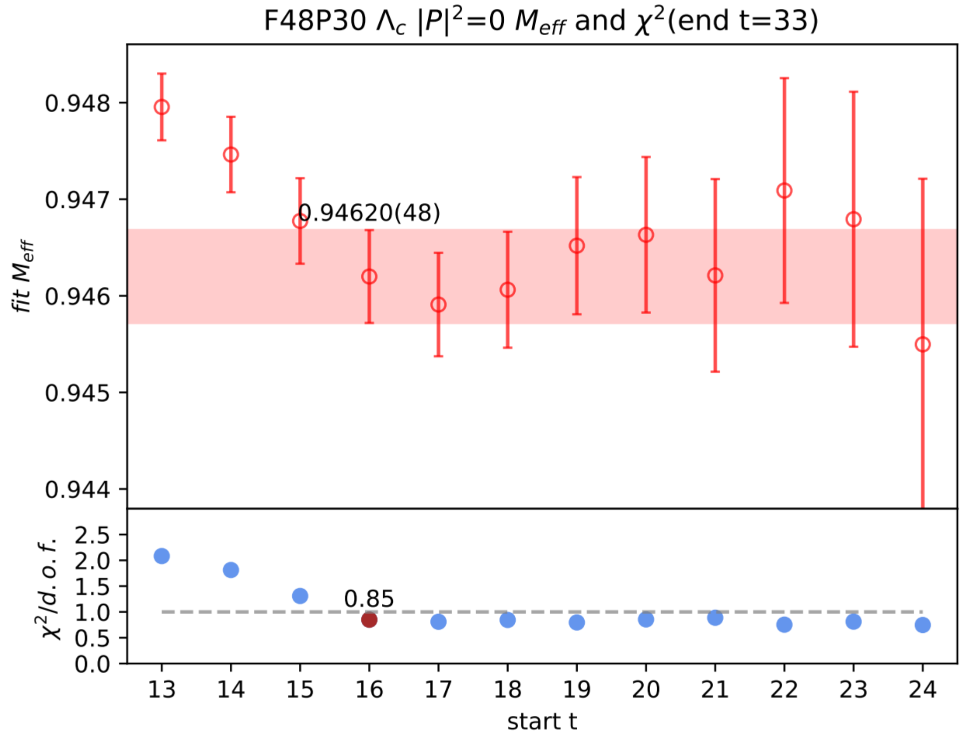
- For S wave rest-frame case: $\mathcal{M}_{00}(\tilde{q}^2) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; \tilde{q}^2) \times \frac{\boldsymbol{E^*(q)}}{\boldsymbol{Z_1\omega_2(q) + Z_2\omega_1(q)}}$

☐ Energy Fitting Method

➢ Fitting range selection

Single Λ_c correlation function is fitted with one exponential term:

$$C(t) = A\exp[-M_{eff}t]$$



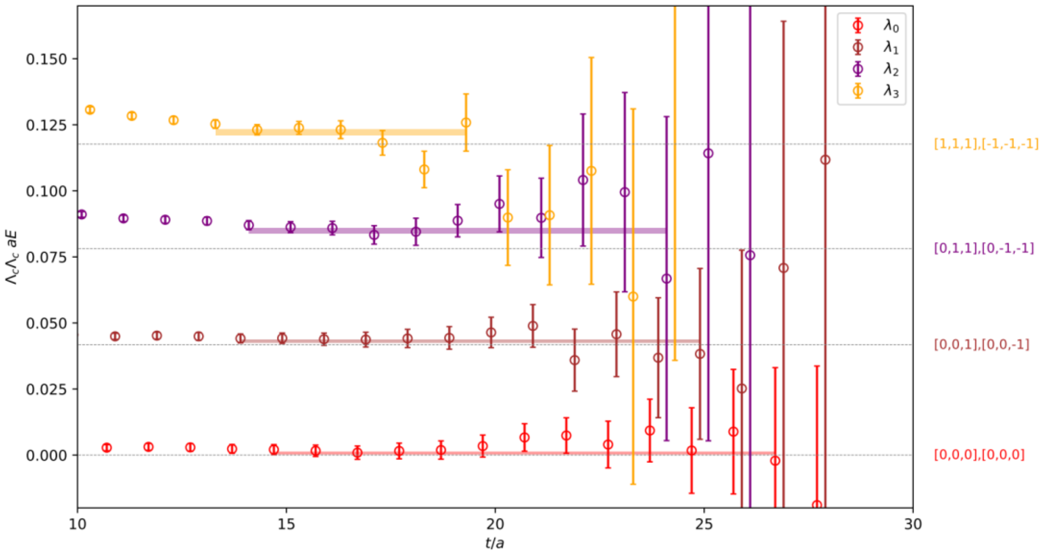
- Fitting window is $[t_{start}, t_{end}]$ while t_{end} is fixed.
- t_{start} should give a stable fit result and a reasonable χ^2.
- Fitting window should be as wide as possible.

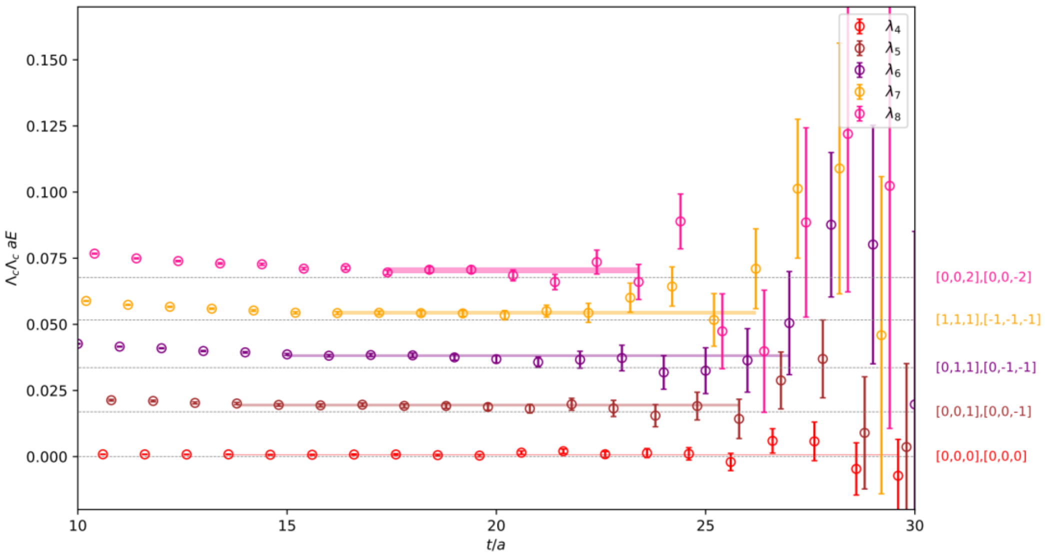
☐ Energy Fitting Method

➢ Two baryons correlation function

Since these energy levels are quite close to their corresponding free energies, a ratio form fitting could perform better

$$C_R(t) = \frac{C_{BB}(t)}{C_{thre.}(t)} \simeq Ae^{-(E_{BB}-2m_{\Lambda_c})t} = Ae^{-\Delta E t}$$





[Jozef J. Dudek et al, Phys.Rev.D 82 (2010) 034508]

V=$32^3 \times 96$ & $48^3 \times 96$
$m_\pi \approx 303$ Mev
a=0.07746 fm

- Operators

$$\Lambda_c\Lambda_c^{I=0} = [\Lambda_c\Lambda_c]$$

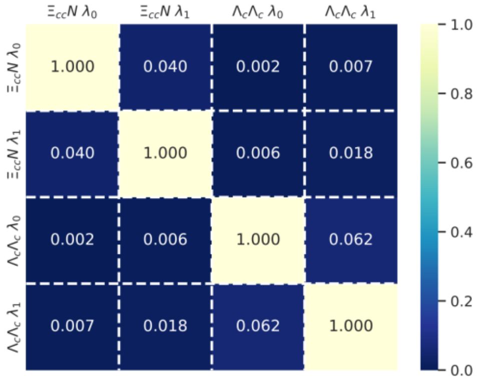
$$\Xi_{cc}N^{I=0} = \frac{1}{2}([p\Xi_{cc}^{+}] + [\Xi_{cc}^{+}p] - [n\Xi_{cc}^{++}] - [\Xi_{cc}^{++}n])$$

- Overlap factor

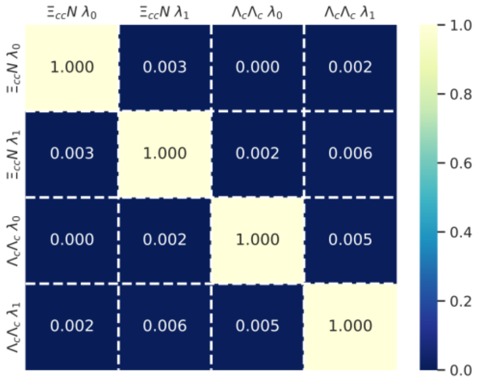
$$Z_i^\alpha = \sqrt{2E_\alpha} e^{E_\alpha t_0/2} v_j^{\alpha *} C_{ji}(t_0)$$

- Cross-correlation matrix

$$\tilde{C}_{ij}(\vec{P},t) = C_{ij}(\vec{P},t)/\sqrt{|C_{ii}(\vec{P},t)C_{jj}(\vec{P},t)|}$$

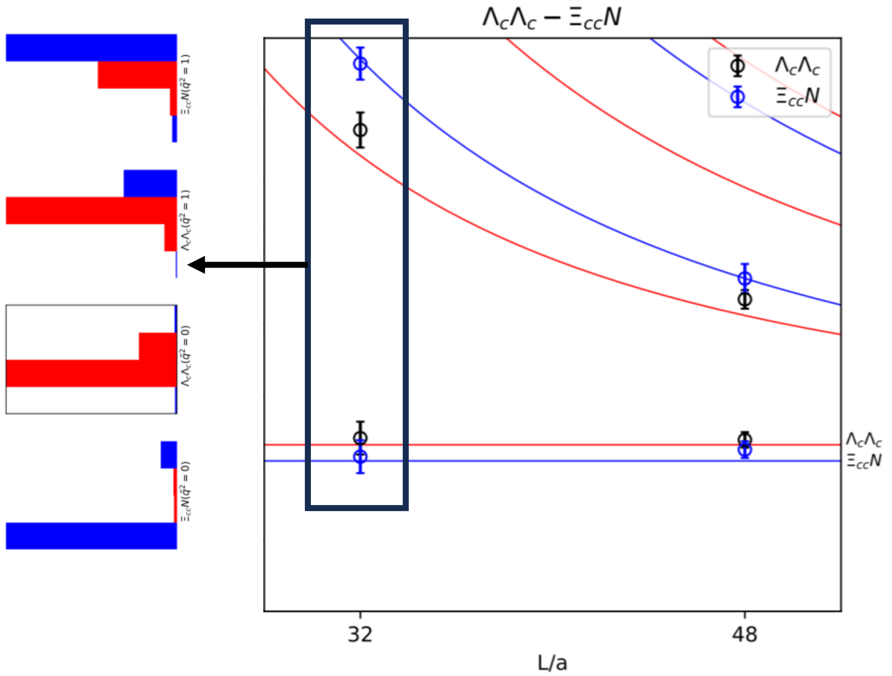


(a)F32P30 $\Xi_{cc}N-\Lambda_c\Lambda_c$



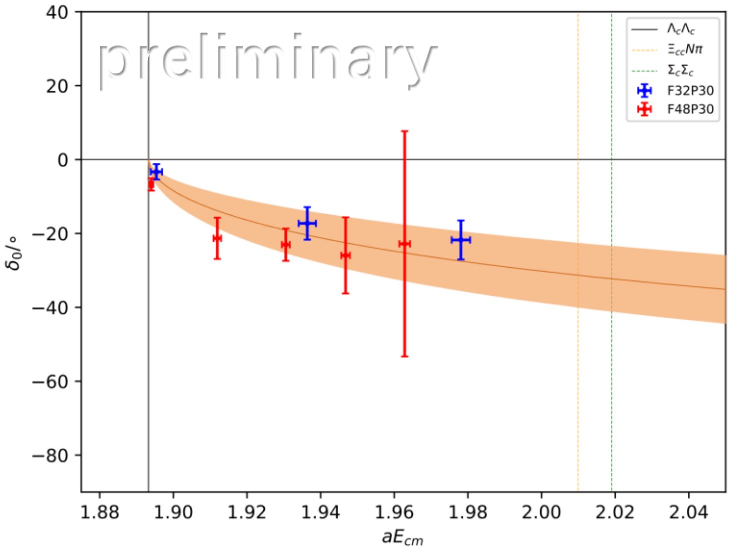
(b)F48P30 $\Xi_{cc}N-\Lambda_c\Lambda_c$

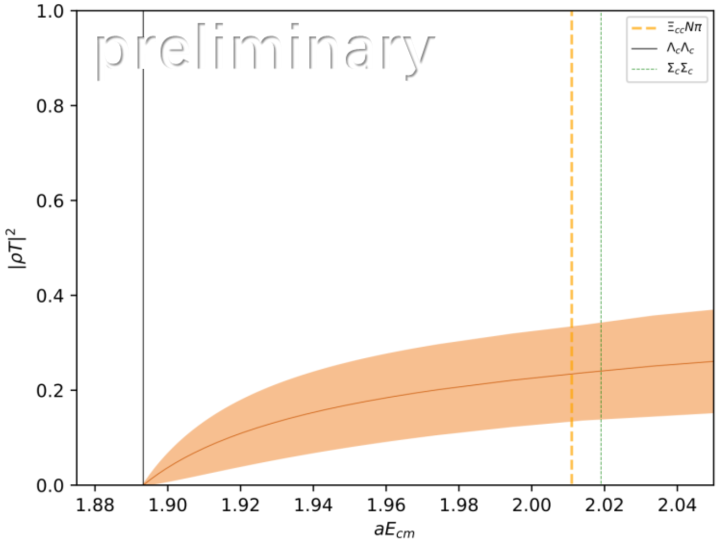
[C. Liu, L. Liu et al, Phys.Rev.D 101 (2020) 5, 054502]

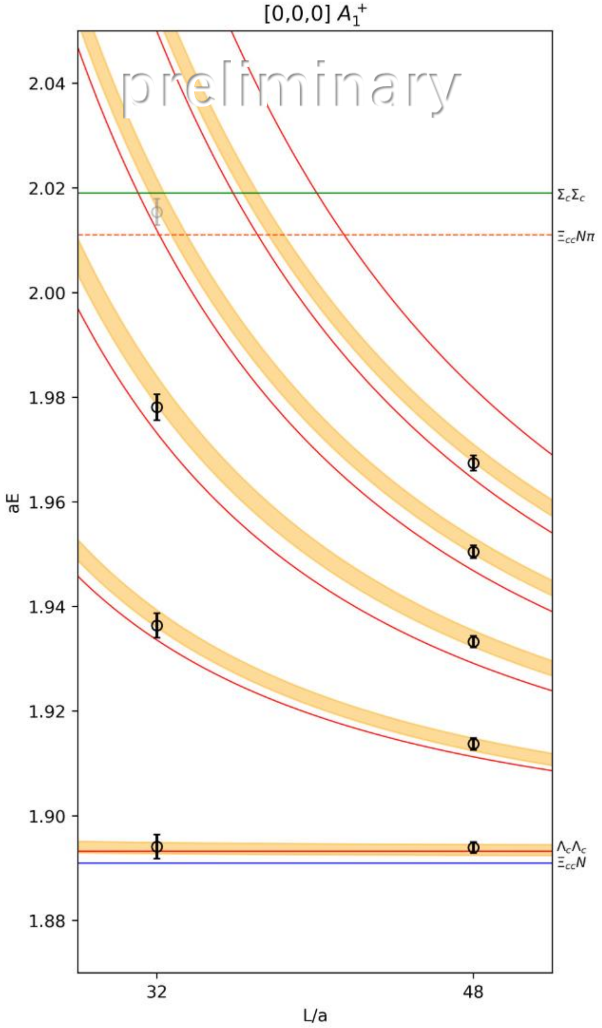


The coupling between Ξ_{cc}N and $\Lambda_c\Lambda_c$ is quite small. Single channel $\Lambda_c\Lambda_c$ is contained latter.

Analyses and Result







Parameterization: $k\cot\delta_0 = \frac{1}{a_0}$.

Fitting result: $a_0 = -0.143(49)$ fm, with $\frac{\chi^2}{dof} = 0.86$.

Conclusion: **No bound state** in S wave $\Lambda_c\Lambda_c$ system.

- Repulsive interaction

- Due to the **discretization** on lattice, a **modification on Lüscher equation** is proposed in this work.

- First calculation of $\Lambda_c\Lambda_c$ system. Coupling between Ξ_{cc}N and $\Lambda_c\Lambda_c$ is small. Therefore, **single channel** is contained.

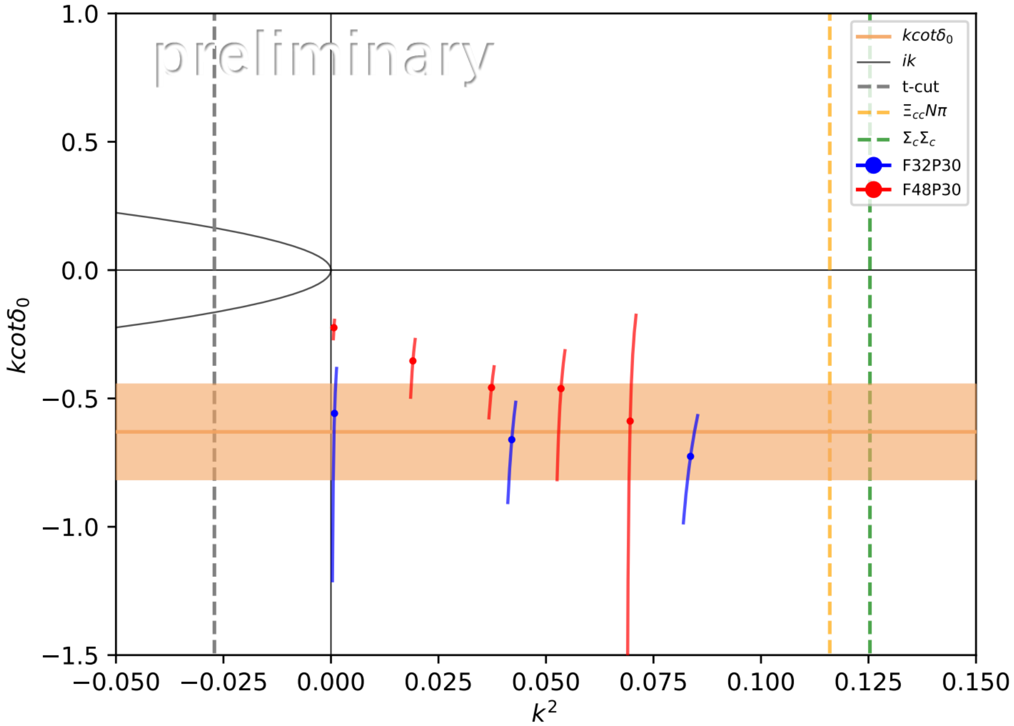
- Showing a **repulsive interaction**, there's **no bound state** in this system.

- Scattering length $\boldsymbol{a_0 = -0.143(49)}$ **fm.**

Thanks!

Backup

- Left-hand cut



The left-hand cut gives the limit of pole extraction with $k^2 = -\frac{m_\omega^2}{4}$ from the exchanging of omega boson.

[X.-K. Dong et al, Phys.Rev.D 105 (2022) 7, 074506]
[Also see talk in Aug 1st 11:30 and 11:50 in LT1]

- **Finite volume effect**
  - The scattering length are consistent in the two ensembles.
  - However, there's a little difference.