Categorizing $SU(3)_f$ representations of scalar mesons by J/ψ decays

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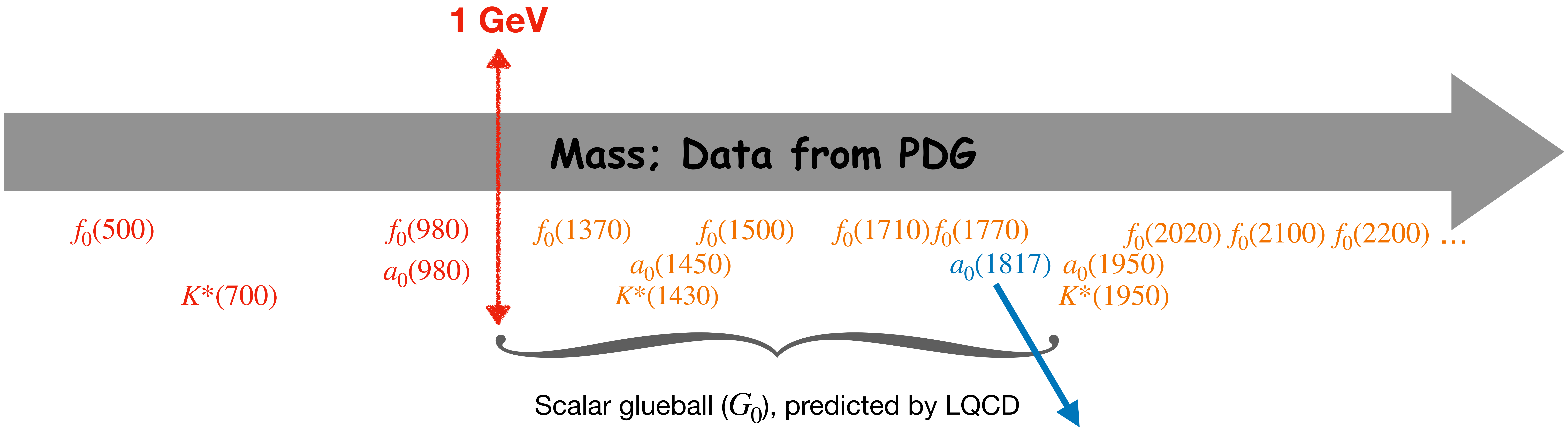
Background



1 GeV

Mass: Data from PDG

$f_0(500)$ $K^*(700)$ $f_0(980)$ $a_0(980)$

$f_0(1370)$ $f_0(1500)$ $a_0(1450)$ $K^*(1430)$ $f_0(1710)$ $f_0(1770)$ $a_0(1817)$ $a_0(1950)$ $K^*(1950)$ $f_0(2020)$ $f_0(2100)$ $f_0(2200)$ …

Scalar glueball (G_0), predicted by LQCD

Discovered by BESIII, see in 董燎原's slides.

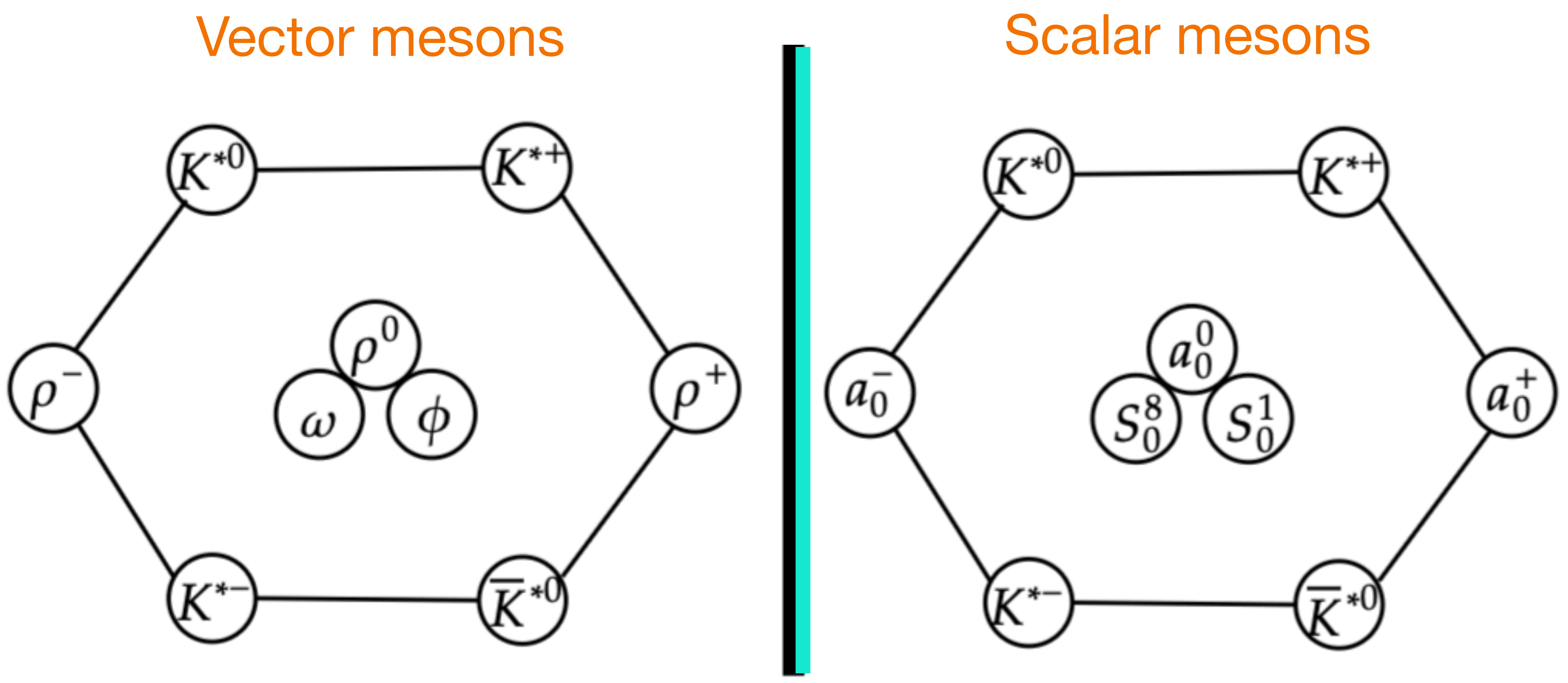
Problems:

- $q\bar{q}$, **glueball (**G_0**), or hybrids (**$q\bar{q}g$**)?**
- **The mixing between quarkonia and glueball?**
- **The number of** $SU(3)$ **nonet?**

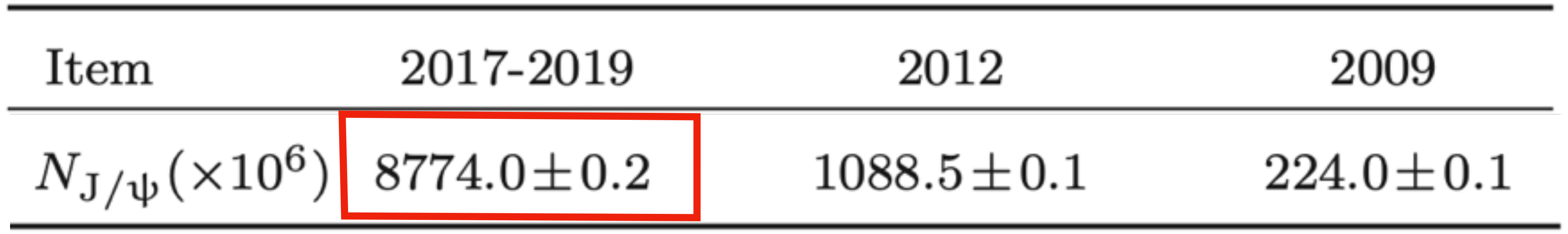
J/ψ is $SU(3)_f$ singlet, the final state should be the $SU(3)_f$ invariant form:

$$\mathscr{A}(J/\psi \to SV) \propto \mathrm{Tr}(\mathbf{SV})$$

$J/\psi \to$



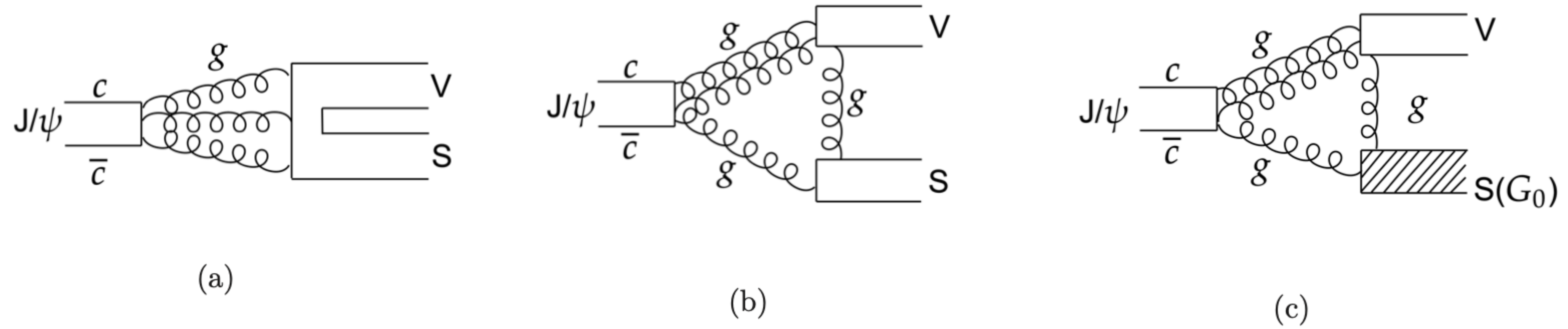
[(BESIII), CPC 46, 074001 (2022)]



Item	2017-2019	2012	2009
$N_{J/\psi}(\times 10^6)$	8774.0 ± 0.2	1088.5 ± 0.1	224.0 ± 0.1

The rich data enable us to categorize $SU(3)_f$ Rep. of ***S***

$J/\psi \to SV$



SOZI-suppressed

DOZI-suppressed
(with mesons)

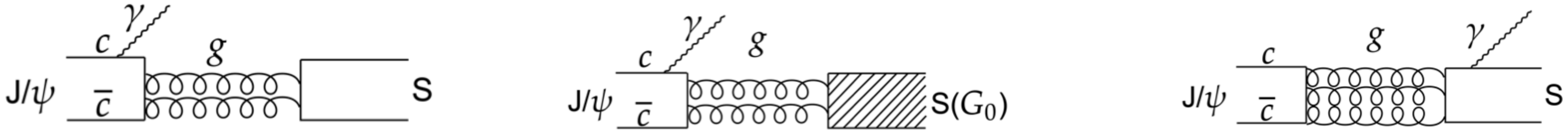
DOZI-suppressed
(with glueball)

$$\mathscr{L}^0_{SV} = \frac{g}{2}\,\mathrm{Tr}(\boldsymbol{S}\{\boldsymbol{V},\boldsymbol{S_g}\}) + rg\,\mathrm{Tr}(\boldsymbol{S}\boldsymbol{S_g})\,\mathrm{Tr}(\boldsymbol{V}\boldsymbol{S_g}) + r'g\,\boldsymbol{G_0}\,\mathrm{Tr}(\boldsymbol{V}\boldsymbol{S_g}),$$

[PRD 44, 175-181 (1991)]

r and r' is the suppressed factor

$J/\psi \to \gamma S$



$$\mathscr{L}^0_{\gamma S} = d\,\mathrm{Tr}(\boldsymbol{S}\boldsymbol{S_g}) + \frac{r'}{r}d\,\boldsymbol{G_0} + \frac{f}{2}\,\mathrm{Tr}(\boldsymbol{S_g}\{\boldsymbol{S},\boldsymbol{S_e}\}).$$

$SU(3)_f$ representation

$$\mathscr{L}^0_{SV} = \frac{g}{2}\,\mathrm{Tr}(\boldsymbol{S}\{\boldsymbol{V},\boldsymbol{S_g}\}) + rg\mathrm{Tr}(\boldsymbol{S}\boldsymbol{S_g})\mathrm{Tr}(\boldsymbol{V}\boldsymbol{S_g}) + r'g\boldsymbol{G_0}\mathrm{Tr}(\boldsymbol{V}\boldsymbol{S_g}), \qquad \mathscr{L}^0_{\gamma S} = d\,\mathrm{Tr}(\boldsymbol{S}\boldsymbol{S_g}) + \frac{r'}{r}d\,\boldsymbol{G_0} + \frac{f}{2}\,\mathrm{Tr}(\boldsymbol{S_g}\{\boldsymbol{S},\boldsymbol{S_e}\})$$

$$\boldsymbol{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{-\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix} \qquad \boldsymbol{S} = \begin{pmatrix} \frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}S_0^8 + \frac{1}{\sqrt{3}}S_0^1 & a_0^+ & K_0^{*+} \\ a_0^- & -\frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}S_0^8 + \frac{1}{\sqrt{3}}S_0^1 & K_0^{*0} \\ K_0^{*-} & \bar{K}_0^{*0} & -\frac{2}{\sqrt{6}}S_0^8 + \frac{1}{\sqrt{3}}S_0^1 \end{pmatrix}$$

$$\mathbf{S_g} = \mathrm{diag}(\mathbf{1}, \mathbf{1}, \mathbf{1} - \mathrm{s})/\sqrt{\mathbf{3}}, \qquad \mathbf{S_e} = \mathrm{diag}(\mathbf{2}, -\mathbf{1}, -\mathbf{1})/\mathbf{3}$$

s is the symmetry-breaking term

$$\begin{pmatrix} f_0^1 \\ f_0^2 \\ f_0^3 \end{pmatrix}_i = U_{ij}\begin{pmatrix} S_0^8 \\ S_0^1 \\ G_0 \end{pmatrix}_j = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}_{ij}\begin{pmatrix} S_0^8 \\ S_0^1 \\ G_0 \end{pmatrix}_j,$$

	$S(980)$	$S(1450)$	$S(1710)$
	$a_0(980)$	$a_0(1450)$	$a_0(1817)$
	$K^*(700)$	$K^*(1430)$	$K^*(1950)$
f_0^1	$f_0(500)$	$f_0(1370)$	
f_0^2	$f_0(980)$	$f_0(1500)$	
f_0^3		$f_0(1710)$	

$c_{ij} = \cos\theta_{ij}, \qquad s_{ij} = \sin\theta_{ij} \qquad M_{f_0^3} > M_{f_0^2} > M_{f_0^1}$

Avoid assuming the nature of scalar mesons !!

The nonet below 1 GeV

Within exact $SU(3)_f$ symmetry:

$$\mathrm{Br}(J/\psi \to K^*(S)K^*(V)) = \mathrm{Br}(J/\psi \to \rho a_0)$$

$$g = (9.22 \pm 1.20) \times 10^{-3}, \quad r = -(0.166 \pm 0.083), \quad s = 0.02 \pm 0.15,$$

$$\theta_{12} = (82.9 \pm 4.4)^\circ, \quad d = (3.41 \pm 0.37) \times 10^{-3}, \quad f = -0.52 \times 10^{-3},$$

TABLE I: Amplitudes for the decays of $J/\psi \to SV$ and γS.

Channels	Amplitude
$K^*(S)K^*(V)$	$\frac{1}{\sqrt{3}}g$
ρa_0	$\frac{1}{\sqrt{3}}(1-\frac{1}{2}s)g$
ωf_0^i	$\left(\frac{1}{\sqrt{3}}g + \frac{2}{3\sqrt{3}}srg\right) \cdot U_{i1} + \left(\frac{\sqrt{2}}{3}g + \frac{\sqrt{2}}{\sqrt{3}}(1-s)rg\right) \cdot U_{i2} + \frac{\sqrt{2}}{\sqrt{3}}r'g \cdot U_{i3}$
ϕf_0^i	$\left(-\frac{\sqrt{2}}{3}g + \frac{\sqrt{2}}{3\sqrt{3}}srg\right)(1-s) \cdot U_{i1} + \left(\frac{1}{3}g + \frac{1}{\sqrt{3}}(1-\frac{1}{3}s)rg\right)(1-s) \cdot U_{i2} + \frac{1}{\sqrt{3}}r'g(1-s) \cdot U_{i3}$
γf_0^i	$\left(\frac{2}{\sqrt{3}}sd + \frac{\sqrt{2}}{6}(1-\frac{1}{3}s)f\right) \cdot U_{i1} + ((1-\frac{1}{3}s)d + \frac{1}{9}sf) \cdot U_{i2} + \frac{r'}{r}d \cdot U_{i3}$
γa_0	$\frac{1}{\sqrt{6}}f$

Data are collected from *BESII* and *PDG:*

[PLB 603, 138-145 (2004)], [PLB 607, 243-253 (2005)], [HEPNP 28, 215-221 (2004)], [PLB 607, 243-253 (2005)], [PRD 94, no.7, 072005 (2016), BESIII]

TABLE III: The experimental and fitting branch ratios of $J/\psi \to S(980)V$ in units of 10^{-4}.

Channels	Data	This work
$\omega f_0(980)$	5.4 ± 1.8	4.3 ± 0.9
$\phi f_0(980)$	9.9 ± 1.7	10.0 ± 1.5
$\omega f_0(500)$	11.7 ± 7.3	10.0 ± 7.0
$\phi f_0(500)$	1.8 ± 0.7	1.8 ± 0.7
$K^*(892)^\pm K^*(700)^\mp$	11^{+10}_{-6}	17.5 ± 2.8
$\rho a_0(980)$	–	17.5 ± 4.6
$\gamma f_0(980)$	0.21 ± 0.04	0.21 ± 0.04
$\gamma f_0(500)$	11.4 ± 2.1	11.5 ± 2.1

χ^2/d.o.f $= 0.72/2$

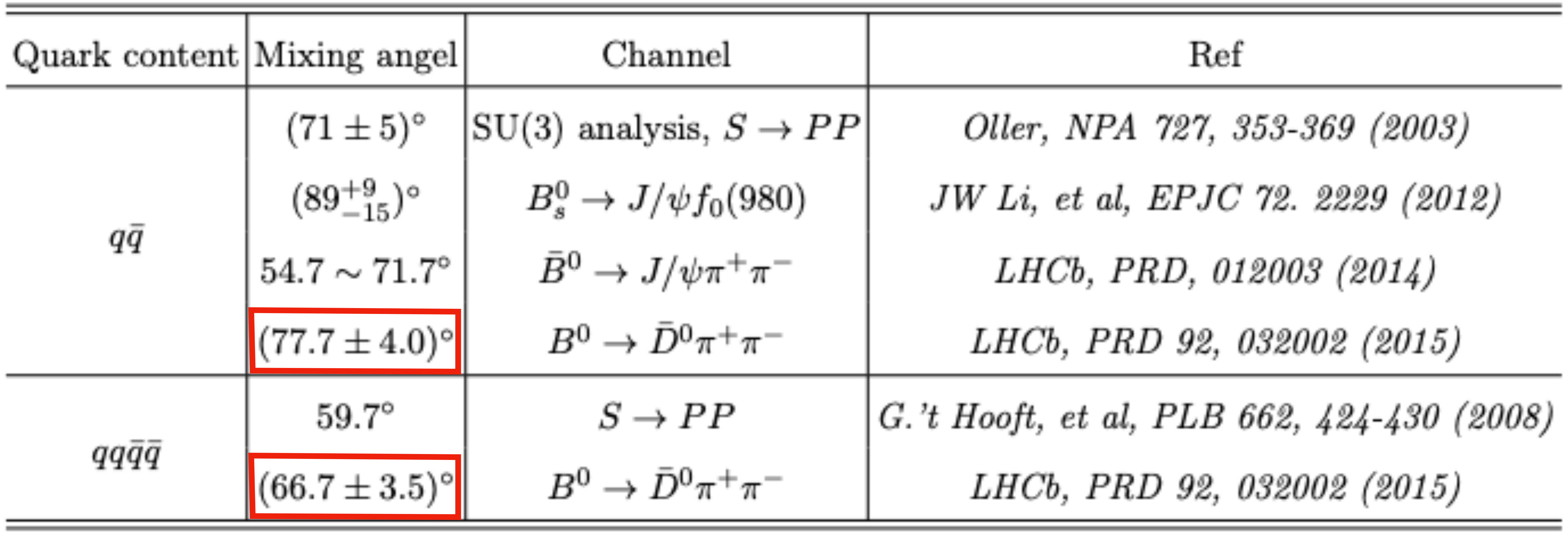
Large error!

The nonet below 1 GeV

$$g = (9.22 \pm 1.20) \times 10^{-3}, \quad r = -(0.166 \pm 0.083), \quad s = 0.02 \pm 0.15,$$

$$\boxed{\theta_{12} = (82.9 \pm 4.4)^\circ} \quad d = (3.41 \pm 0.37) \times 10^{-3}, \quad f = -0.52 \times 10^{-3},$$

χ^2/d.o.f $= 0.72/2$



Quark content	Mixing angel	Channel	Ref
$q\bar{q}$	$(71 \pm 5)^\circ$	SU(3) analysis, $S \to PP$	*Oller, NPA 727, 353-369 (2003)*
	$(89^{+9}_{-15})^\circ$	$B_s^0 \to J/\psi f_0(980)$	*JW Li, et al, EPJC 72. 2229 (2012)*
	$54.7 \sim 71.7^\circ$	$\bar{B}^0 \to J/\psi \pi^+ \pi^-$	*LHCb, PRD, 012003 (2014)*
	$\boxed{(77.7 \pm 4.0)^\circ}$	$B^0 \to \bar{D}^0 \pi^+ \pi^-$	*LHCb, PRD 92, 032002 (2015)*
$qq\bar{q}\bar{q}$	59.7°	$S \to PP$	*G.'t Hooft, et al, PLB 662, 424-430 (2008)*
	$\boxed{(66.7 \pm 3.5)^\circ}$	$B^0 \to \bar{D}^0 \pi^+ \pi^-$	*LHCb, PRD 92, 032002 (2015)*

This shows $\theta_{q\bar{q}} > \theta_{qq\bar{q}\bar{q}}$

[Hsiao, et al, arXiv:2306.06091]

Our result is closed to the $q\bar{q}$ hypothesis!

The nonet in range of 1-2 GeV

Key: $f_0(1370,1500,1710)$ ***are mixtures of*** $S_0^8, S_0^1,$ ***and*** G_0

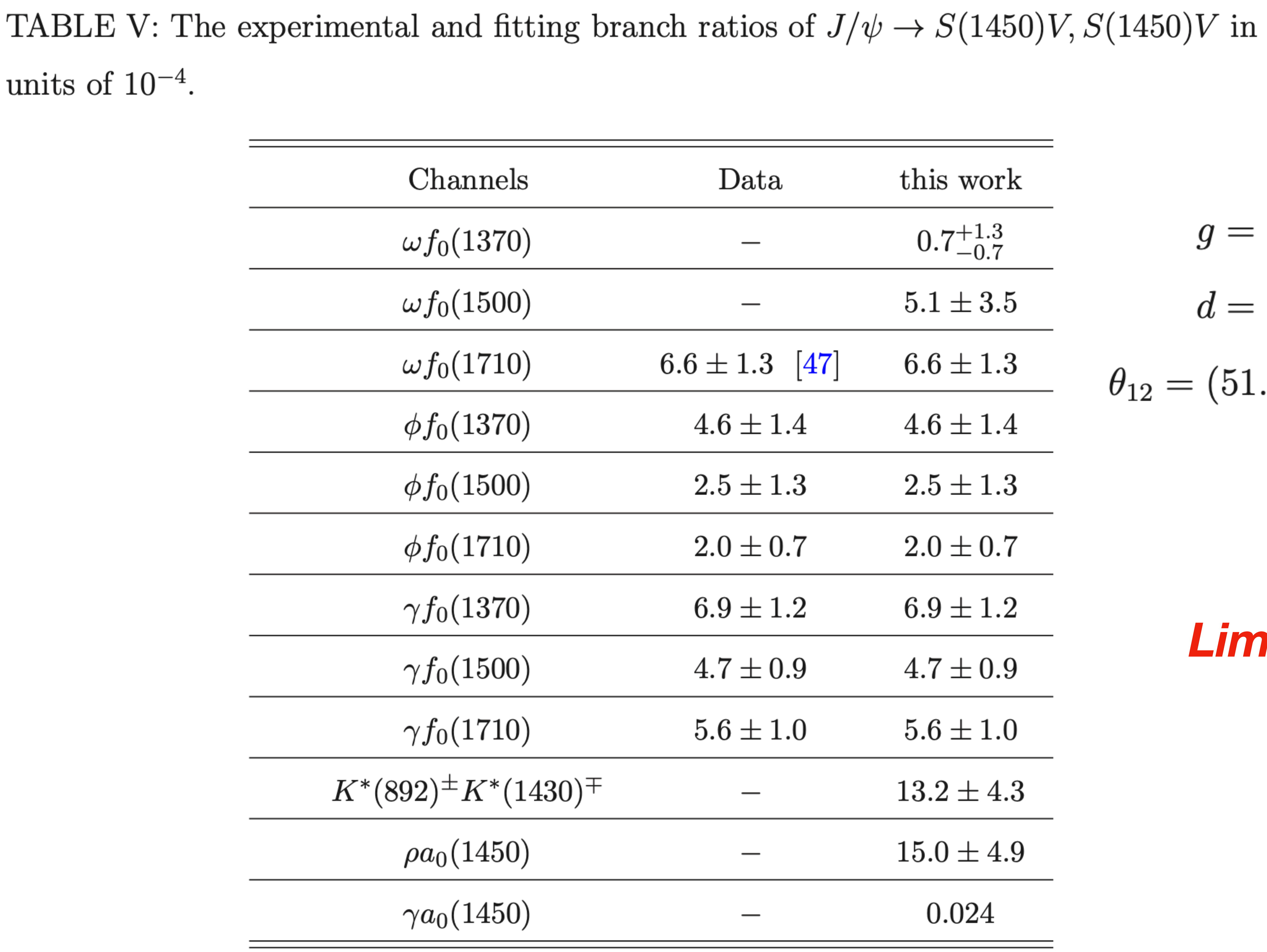


TABLE V: The experimental and fitting branch ratios of $J/\psi \to S(1450)V, S(1450)V$ in units of 10^{-4}.

Channels	Data	this work
$\omega f_0(1370)$	–	$0.7^{+1.3}_{-0.7}$
$\omega f_0(1500)$	–	5.1 ± 3.5
$\omega f_0(1710)$	6.6 ± 1.3 [47]	6.6 ± 1.3
$\phi f_0(1370)$	4.6 ± 1.4	4.6 ± 1.4
$\phi f_0(1500)$	2.5 ± 1.3	2.5 ± 1.3
$\phi f_0(1710)$	2.0 ± 0.7	2.0 ± 0.7
$\gamma f_0(1370)$	6.9 ± 1.2	6.9 ± 1.2
$\gamma f_0(1500)$	4.7 ± 0.9	4.7 ± 0.9
$\gamma f_0(1710)$	5.6 ± 1.0	5.6 ± 1.0
$K^*(892)^\pm K^*(1430)^\mp$	–	13.2 ± 4.3
$\rho a_0(1450)$	–	15.0 ± 4.9
$\gamma a_0(1450)$	–	0.024

$$g = (1.14 \pm 0.18) \times 10^{-2}, \qquad r = -(0.716 \pm 0.291),$$

$$d = (4.52 \pm 1.07) \times 10^{-3}, \qquad r' = -(0.633 \pm 0.132),$$

$$\theta_{12} = (51.4 \pm 6.4)^\circ, \qquad \theta_{13} = (-0.1 \pm 14.6)^\circ, \qquad \theta_{23} = (2.3 \pm 8.6)^\circ.$$

Limited available experimental data!

The nonet in range of 1-2 GeV

Our work:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.67(18) & -0.74(16) & 0.04(12) \\ 0.74(16) & 0.67(18) & 0.08(8) \\ 0.03(10) & -0.08(10) & 0.99(1) \end{pmatrix} \begin{pmatrix} S_0^8 \\ S_0^1 \\ G_0 \end{pmatrix}$$

$$= \begin{pmatrix} 0.98(2) & 0.12(24) & 0.04(12) \\ -0.12(241) & 0.98(2) & 0.08(8) \\ -0.05(12) & -0.07(8) & 0.99(1) \end{pmatrix} \begin{pmatrix} n\bar{n} \\ s\bar{s} \\ G_0 \end{pmatrix}$$

The nonet in range of 1-2 GeV

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$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.67(18) & -0.74(16) & 0.04(12) \\ 0.74(16) & 0.67(18) & 0.08(8) \\ 0.03(10) & -0.08(10) & 0.99(1) \end{pmatrix} \begin{pmatrix} S_0^8 \\ S_0^1 \\ G_0 \end{pmatrix}$$

$$= \begin{pmatrix} 0.98(2) & 0.12(24) & 0.04(12) \\ -0.12(241) & 0.98(2) & 0.08(8) \\ -0.05(12) & -0.07(8) & 0.99(1) \end{pmatrix} \begin{pmatrix} n\bar{n} \\ s\bar{s} \\ G_0 \end{pmatrix}$$

Lee and Weingarten:

$$\begin{pmatrix} |f_0(1370)\rangle \\ |f_0(1500)\rangle \\ |f_0(1710)\rangle \end{pmatrix} = \begin{pmatrix} 0.819(89) & 0.290(91) & -0.495(118) \\ -0.399(113) & 0.908(37) & -0.128(52) \\ 0.413(87) & 0.302(52) & 0.859(54) \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}.$$

[PRD 61, 014015 (1999)]

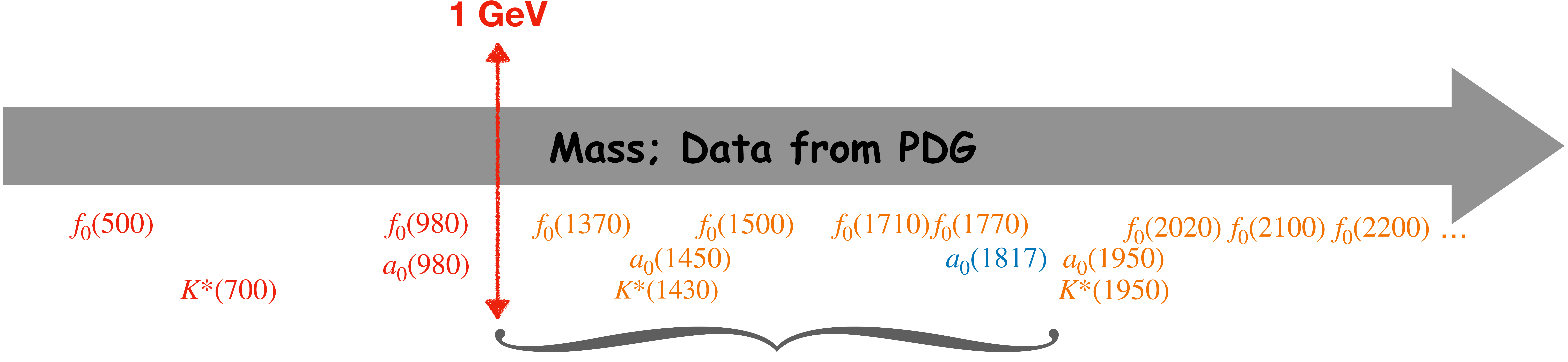
H. Y. Cheng, C. K. Chua and K. F. Liu:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.78 & 0.51 & -0.36 \\ -0.54 & 0.84 & 0.03 \\ 0.32 & 0.18 & 0.93 \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}.$$

[PRD 74, 094005 (2006)]

$f_0(1710)$ ***is a plausible glueball candidate !!***

Another possibility



1 GeV

Mass; Data from PDG

$f_0(500)$ $K^*(700)$ $f_0(980)$ $a_0(980)$ $f_0(1370)$ $a_0(1450)$ $K^*(1430)$ $f_0(1500)$ $f_0(1710)$ $f_0(1770)$ $a_0(1817)$ $a_0(1950)$ $K^*(1950)$ $f_0(2020)$ $f_0(2100)$ $f_0(2200)$ …

Scalar glueball (G_0), predicted by LQCD

	$S(980)$	$S(1450)$	$S(1710)$
	$a_0(980)$	$a_0(1450)$	$\boldsymbol{a_0(1817)}$
	$K^*(700)$	$K^*(1430)$	$K^*(1950)$
f_0^1	$f_0(500)$	$f_0(1370)$	
f_0^2	$f_0(980)$	$f_0(1500)$	
f_0^3		$f_0(1710)$	

?

The discovery of $a_0(1817)$, $a_0(1950)$ and $K^*(1950)$ indicates other $SU(3)_f$ nonets.

There is not enough experimental data to do the analysis!!

Summary

- J/ψ decays can provide a simple and effective way to understand the properties of scalar particles.

- Within $S(980)$, we determined the singlet-octet mixing angle $\theta = (82.9 \pm 4.4)^{\circ}$, which is consistent with the caculations of the $q\bar{q}$ hypothesis.

- For the nonet beyond 1 GeV, our fitting results show that $f_0(1710)$ is mainly composed of glueball.

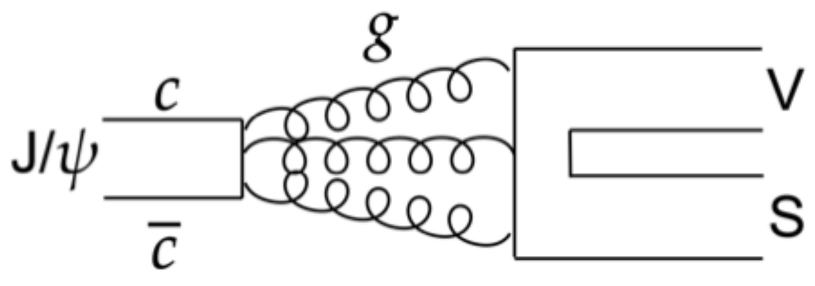
- We hope that our experimental group partners, such as BESIII, to provide more and more accurate experimental data.

Thanks!!

back-up

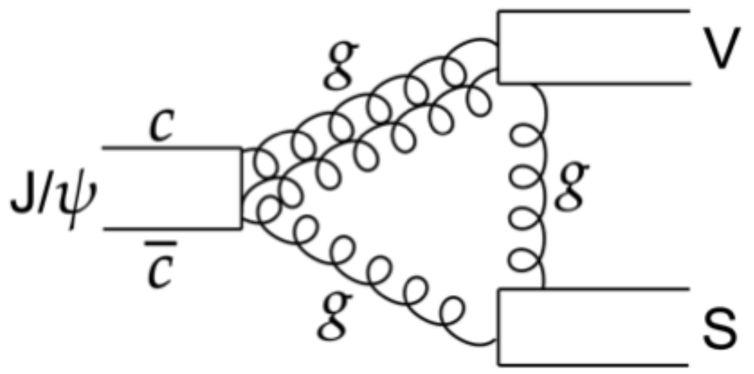
$J/\psi \to SV$

[PRD 38, 824 (1988)];
[EPJC 65, 467-473 (2010)]



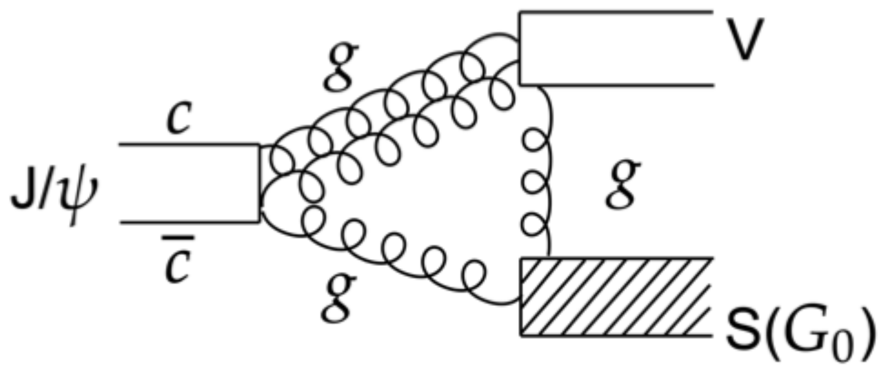
(a)

SOZI-suppressed



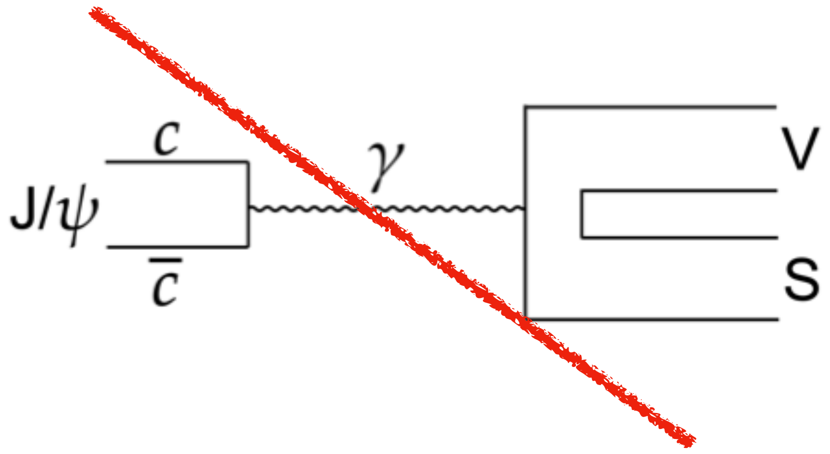
(b)

DOZI-suppressed
(with mesons)



(c)

DOZI-suppressed
(with glueball)



(d)

EM

$$\mathscr{L}^0_{SV} = \frac{g}{2}\mathrm{Tr}(\boldsymbol{S}\{\boldsymbol{V},\boldsymbol{S_g}\}) + rg\mathrm{Tr}(\boldsymbol{S}\boldsymbol{S_g})\mathrm{Tr}(\boldsymbol{V}\boldsymbol{S_g}) + r'g\boldsymbol{G_0}\mathrm{Tr}(\boldsymbol{V}\boldsymbol{S_g}),$$

[PRD 44, 175-181 (1991)]

r and r' is the suppressed factor

The nonet below 1 GeV

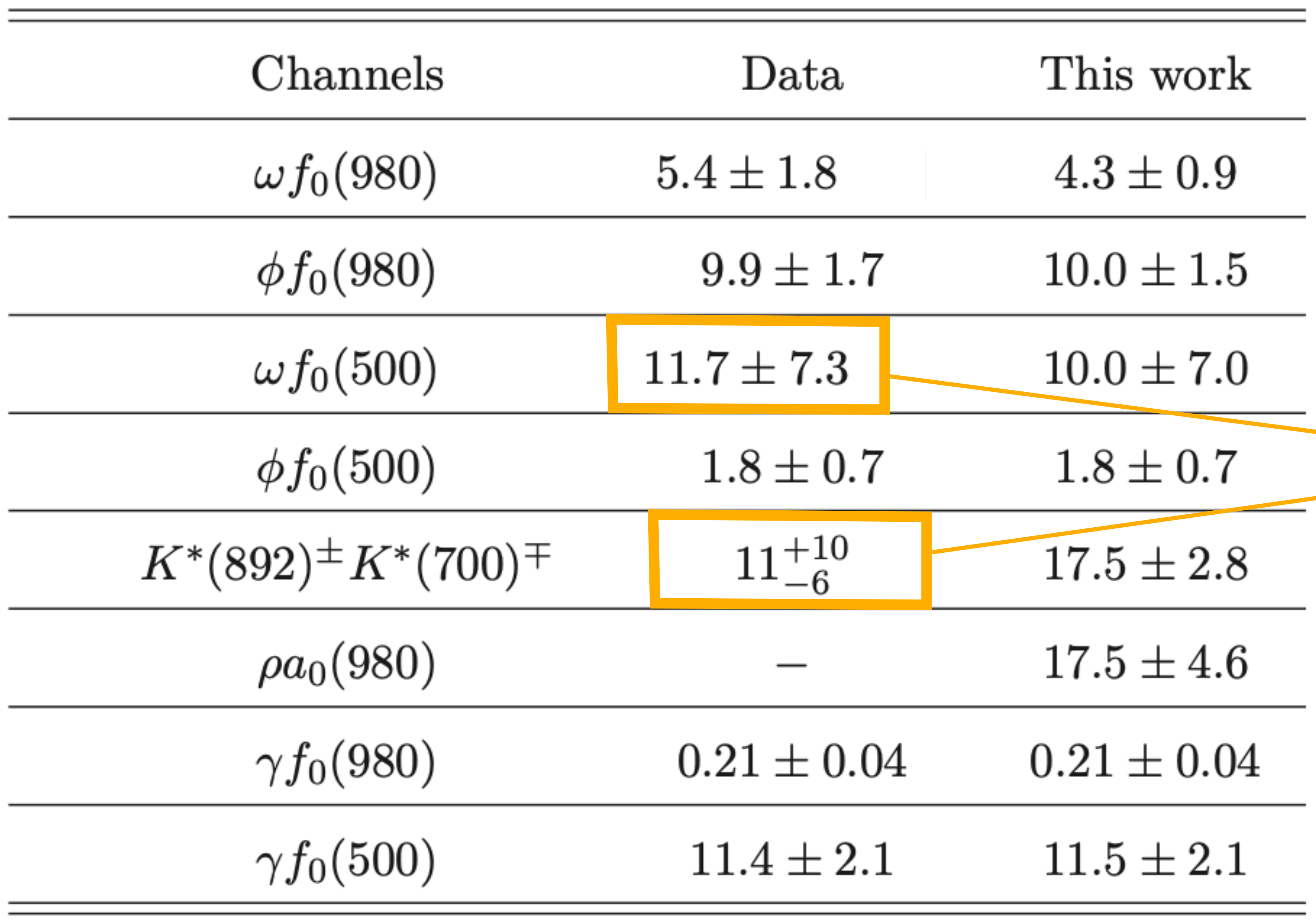
$$g = (9.22 \pm 1.20) \times 10^{-3}, \quad r = -(0.166 \pm 0.083), \quad s = 0.02 \pm 0.15,$$

$$\theta_{12} = (82.9 \pm 4.4)^\circ, \quad d = (3.41 \pm 0.37) \times 10^{-3}, \quad f = -0.52 \times 10^{-3},$$

Within the first order of the $SU(3)_f$ breaking, it's impact on the mixing angle of less than 3%.

χ^2/d.o.f $= 0.72/2$

TABLE III: The experimental and fitting branch ratios of $J/\psi \to S(980)V$ in units of 10^{-4}.



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Large error!

The small χ^2 due to the large error.