第21届
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THE 21ST NATIONAL SYMPOSIUM ON HEAVY FLAVOR PHYSICS AND CP VIOLATION

Detecting true para-muonium via J/ψ decays at the electron-positron colliders

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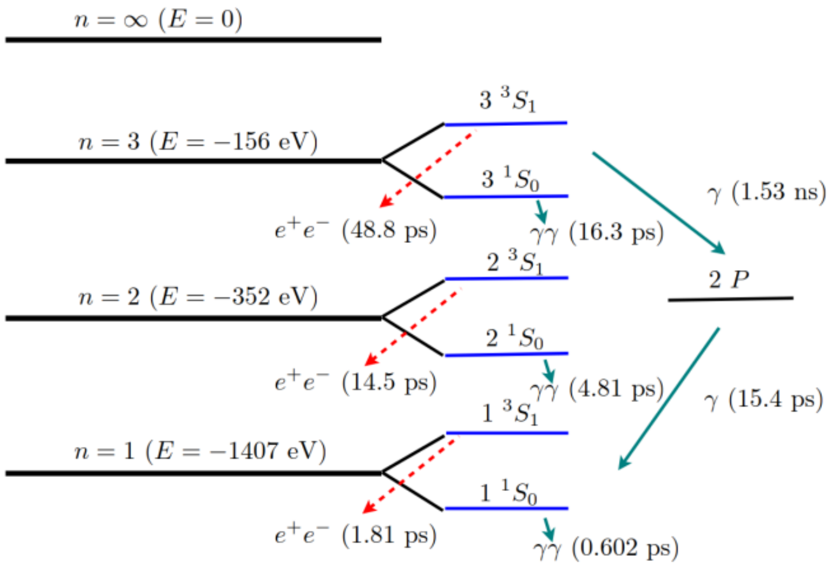
Lepton-antilepton bound states

• Positronium	(e^+e^-)	Deutsch in 1951
• Muonium	$(\mu^\pm e^\pm)$	V. W. Hughes, 1960
• Dipositronium	$(e^+e^-)(e^+e^-)$	D. B. Cassidy & A. P. Mills, 2007
• True muonium	$(\mu^+\mu^-)$	Predicted in 1969; To be discovered
• True tauonium	$(\tau^+\tau^-)$	To be discovered
• Mu-tauonium	$(\mu^\pm\tau^\pm)$	To be discovered
• E-tau atom	$(\tau^\pm e^\pm)$	To be discovered

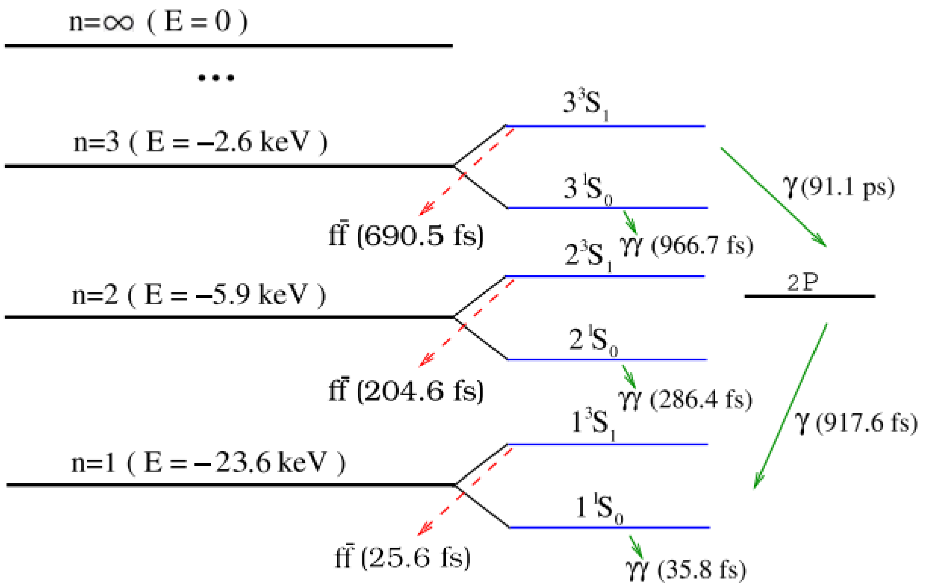
True muonium(TM) :the smallest pure QED atom

μ MEAN LIFE τ — $(2.1969811 \pm 0.0000022) \times 10^{-6}$ s

τ MEAN LIFE — $(2.903 \pm 0.005) \times 10^{-13}$ s



Brodsky, Lebed, 2009



D. d’Enterria, R.Perez-Ramos, H-S. Shao,2022

$(\mu^+\mu^-)$ 211.4 MeV mass and 512 fm Bohr radius

the heaviest and smallest purely leptonic QED atom

Production of true-muonium

$\pi^- p \rightarrow (\mu^+\mu^-)n$ S. Bilen'kii et al, Sov. J. Nucl. Phys. 10, 469 (1969)

$\gamma Z \rightarrow (\mu^+\mu^-)Z$ S. Bilen'kii et al, Sov. J. Nucl. Phys. 10, 469 (1969)

$e^+e^- \rightarrow (\mu^+\mu^-)$ Moffat, PRL 35,1605 (1975) Brodsky, Lebed, 2009

$\mu^+\mu^-$ collisions Hughes, Maglic, Bull. Am. Phys. Soc. 16, 65 (1971)

$\eta \rightarrow (\mu^+\mu^-)\gamma$ Nemenov, Sov.J.Nucl.Phys. 15,582(1972);
G.A.Kozlov, Sov.J.Nucl.Phys. 48,167 (1988)

$Z_1 Z_2 \rightarrow Z_1 Z_2(\mu^+\mu^-)$ I. F. Ginzburg et al Phys. Rev. C (1998) ······, Dai, Zhao, 2024

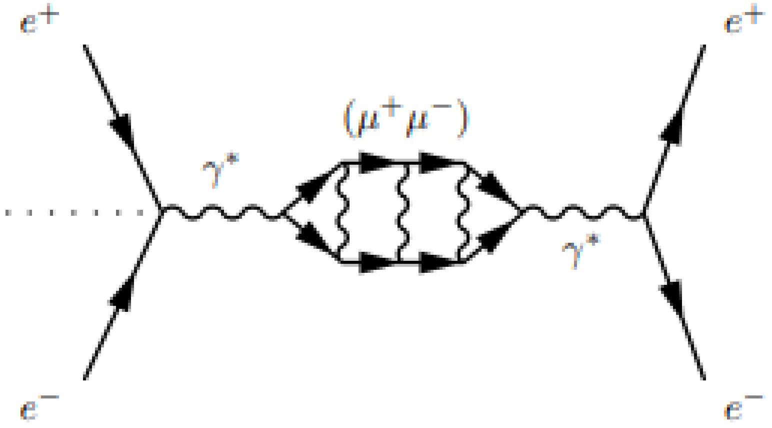
$K_L \rightarrow (\mu^+\mu^-)\gamma$ Ji, Lamm, PRD 98, 053008 (2018)

$B \rightarrow K\ (l^+l^-)$ Fael, Mannel, NPB 932,2018

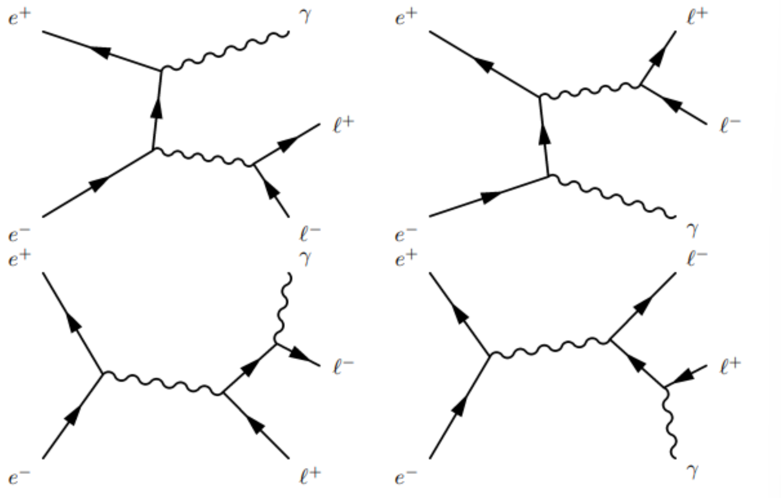
Production of true-muonium in electron-positron collisions

$$e^+e^- \to (\mu^+\mu^-)$$

$$e^+e^- \to (\mu^+\mu^-)\gamma$$



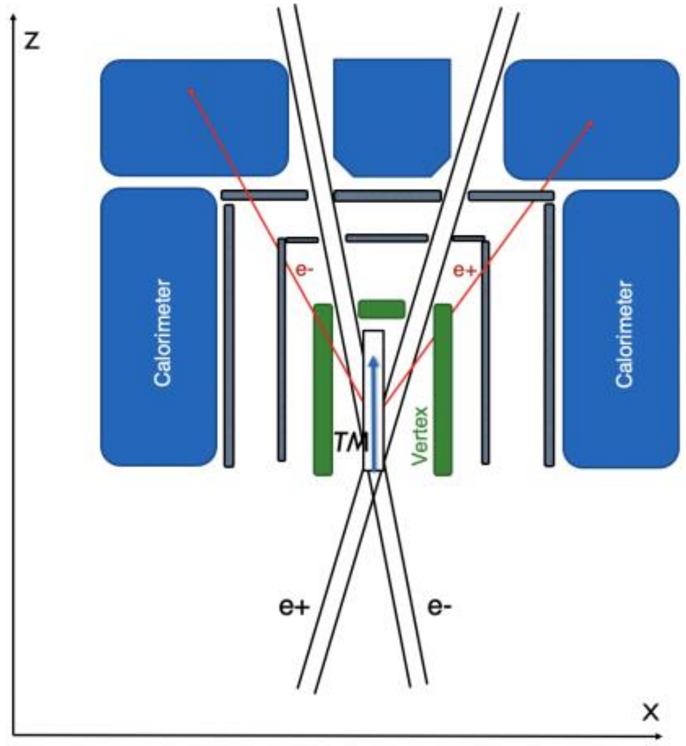
Brodsky and Lebed,2009

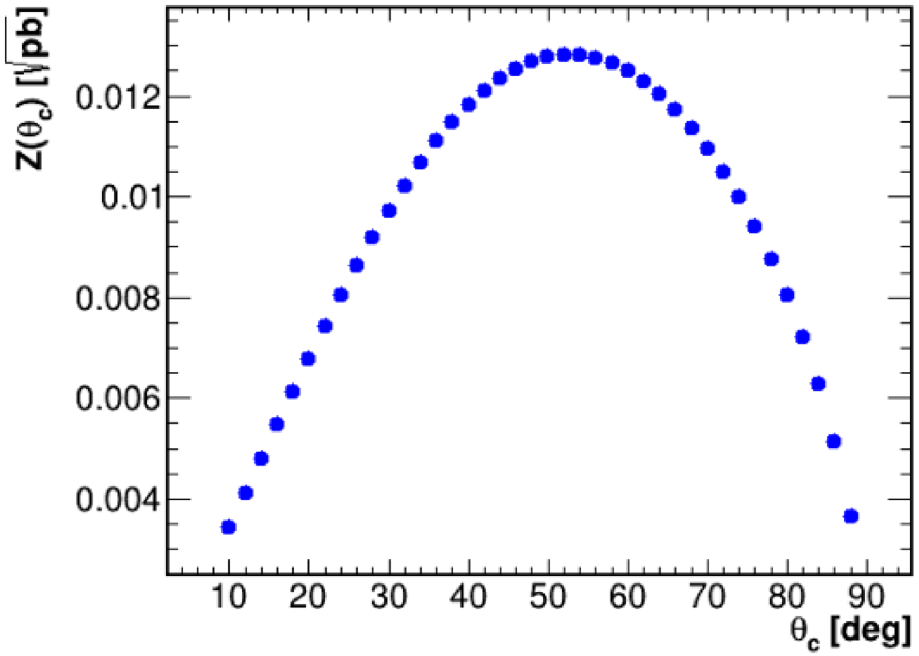


Production of true-muonium

Schematic of a detector for the DIMUS collider

$$Z(\theta_c) = \frac{\sigma_{\rm TM}(\theta_c < \theta < \pi - \theta_c)}{\sqrt{\sigma_{\rm Bhabha}(\theta_c < \theta < \pi - \theta_c)}}$$



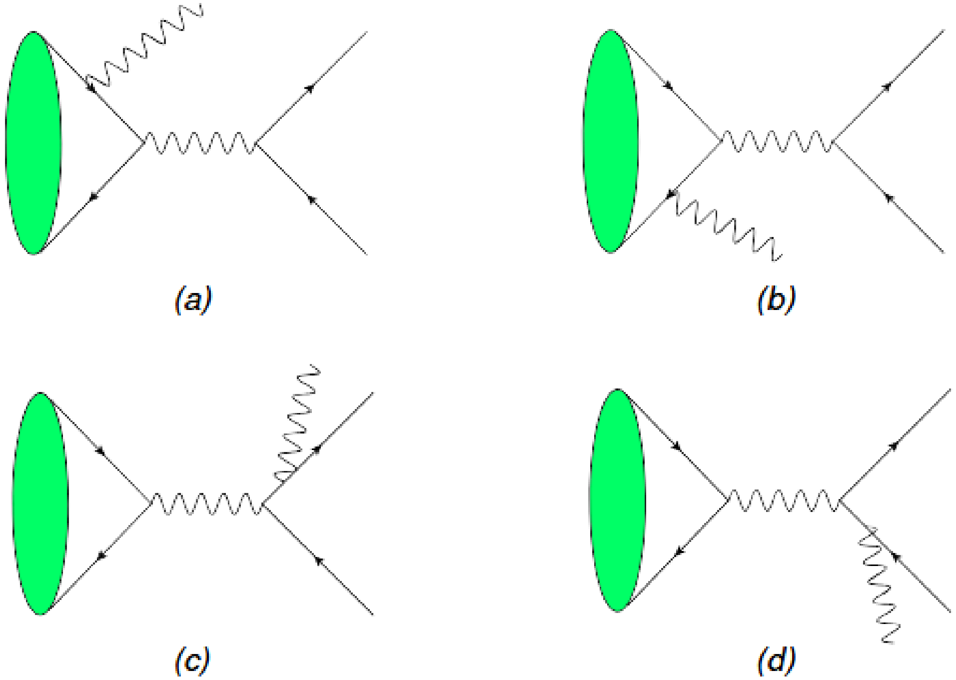


Journal of Instrumentation, Patrick J.Fox et al ,2022

J.Phys.G Ruben Gargiulo et al,2024

$J/\psi \to (\mu^+\mu^-)\gamma$

- Analogous to $e^+e^- \to (\mu^+\mu^-) + \gamma$
- Good for spin-singlet TM (para-TM)
- Cleaner channel than electron-positron collision.
- Large statistics, more events



Happy 50th birthday to J/Ψ!

Decay width

- The squared amplitude at the threshold region

$$\frac{1}{3}\sum_{\text{pols.}}|i\mathcal{M}|^2 \approx e_Q^2 e^6 \frac{1}{2m_c^3}\frac{R^2(0)}{4\pi}\frac{8}{3}\left(\frac{x_1}{x_2}+\frac{x_2}{x_1}\right)$$

$x_i = E_i/m_c$ R(0): radial wave function of J/Ψ at the origin

- Phase space at the threshold region $\sqrt{s_1}$: invariant mass of $\mu^+\mu^-$

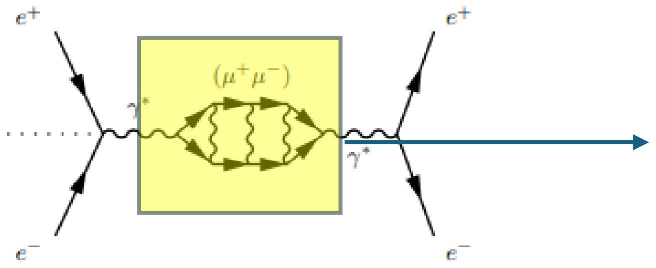
$$\frac{1}{(2\pi)^3}\frac{1}{16}\int ds_1 \int_{\frac{1}{2}(1-\beta)}^{\frac{1}{2}(1+\beta)} dx_1. \qquad \beta = \sqrt{1-\frac{4m_\mu^2}{s_1}}$$

Velocity of muon (in the center of mass frame)

- Coulomb rescattering at the threshold region

$\frac{\alpha}{\beta} \sim 1$, needs resummation

Sommerfeld, Schwinger, Sakharov



Coulomb rescattering

Decay width

- Coulomb resummation

$$\frac{d\Gamma(J/\psi \to \gamma(\mu^+\mu^-))}{ds_1} = \frac{d\Gamma(J/\psi \to \gamma\mu^+\mu^-)}{ds_1}\bigg|_{s_1 \ll 4m_c^2} \frac{4\pi}{m_\mu^2 \beta} \mathrm{Im}\, G_{E+i\Gamma}(0,0)$$

Coulomb Green function

$$\mathrm{Im}\, G_{E+i\Gamma}(0,0) = \frac{m_\mu^2}{4\pi}\left[\frac{p_2}{m_\mu} + \frac{2p_s}{m_\mu}\arctan\frac{p_2}{p_1} + \frac{2p_s^2}{m_\mu^2}\sum_{n=1}^{\infty}\frac{1}{n^4}\frac{\Gamma p_s n + p_2(n^2\sqrt{E^2+\Gamma^2} + p_s^2/m_\mu)}{\left(E + \frac{p_s^2}{m_\mu n^2}\right)^2 + \Gamma^2}\right]$$

$$p_s = \frac{1}{2} m_\mu \alpha, \quad p_{1,2} = \sqrt{\frac{m_\mu}{2}(\sqrt{E^2+\Gamma^2} \mp E)}.$$

Fadin, Khoze, Sjostrand, 1990

- Introducing ratio R

$$R = \frac{\Gamma(J/\psi \to \gamma(\mu^+\mu^-))}{\Gamma(J/\psi \to \mu^-\mu^+)}$$

Cancel the wave function at the origin

Decay width

$$E = \sqrt{s_1} - 2m_\mu$$

- E<0: below the threshold, bound state contribution

$$\left.\frac{d\Gamma(J/\psi \to \gamma(\mu^+\mu^-))}{dE}\right|_{E<0} = \frac{e_Q^2\alpha^3}{12\pi m_c^4} R^2(0) 2(E+2m_\mu)\left(\ln\frac{1+\beta}{1-\beta} - \beta\right) \frac{4\pi}{m_\mu^2\beta}\sum_{n=1}^{\infty}\frac{\alpha^3 m_\mu^3}{8n^3}\delta\left(E + \frac{\alpha^2 m_\mu}{4n^2}\right)$$

$$R|_{E<0} \approx \frac{\alpha^4 m_\mu^2}{2m_c^2}\zeta(3)$$

$$|\psi_n^2(0)| = \frac{\alpha^3 m_\mu^3}{8\pi n^3}.$$

Binding energies

- E>0: above the threshold

$$\left.\frac{d\Gamma(J/\psi \to \gamma(\mu^+\mu^-))}{dE}\right|_{E>0} = \frac{e_Q^2\alpha^3}{12\pi m_c^4} R^2(0) 2(E+2m_\mu)\left(\ln\frac{1+\beta}{1-\beta} - \beta\right) \frac{4\pi}{m_\mu^2\beta}\frac{\alpha m_\mu^2}{4\left(1-e^{-\frac{\alpha\pi m_\mu}{\sqrt{E m_\mu}}}\right)}.$$

Sommerfeld enhancement

$$R|_{E>0} \approx \alpha^2\frac{m_\mu}{m_c^2}\int_0^\Lambda \frac{dE}{1-e^{-\pi\alpha\sqrt{\frac{m_\mu}{E}}}}$$

Λ: depends on the energy resolution

Numerical Result

- Input: $m_\mu = 105.66 \text{ MeV}, \quad m_c = 1.27 \text{ GeV}, \quad \alpha = \frac{1}{137}.$

  $\text{Br}(J/\psi \to \mu^- \mu^+) = 5.961\%$

- Below the threshold:

  $R|_{E<0} \approx 1.18 \times 10^{-11}.$ $\quad \text{Br}(J/\psi \to \gamma(\mu^- \mu^+)) \approx 7.03 \times 10^{-13}.$

- Above the threshold: if Λ~MeV,

  $R|_{E>0} \sim 10^{-8}$, Br$\sim 10^{-10}$

- Hard to measure at BESIII

- Super tau-charm facility: O(1) events for bound state, $O(10^3)$ for above the threshold events (per year)

Summary

- QED has been exactly tested, but true muonium—the smallest QED atom, has not been observed yet

- J/Ψ radiative decay offers a promising channel to detect true muonium

- The branching ratio is of the order $\mathbf{10^{-13}}$ for TM bound states

- Had to detect true muonium via J/Ψ radiative decay at BESIII; STCF (and its updates) offer promising avenues to discovering the true muonium.

Thank you!