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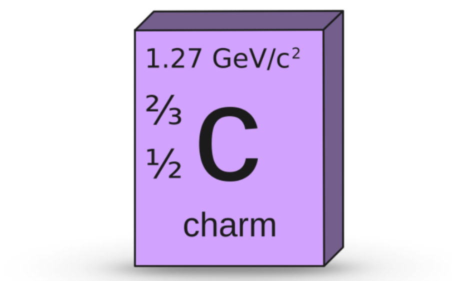


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**Heng Yang, China**

# Content

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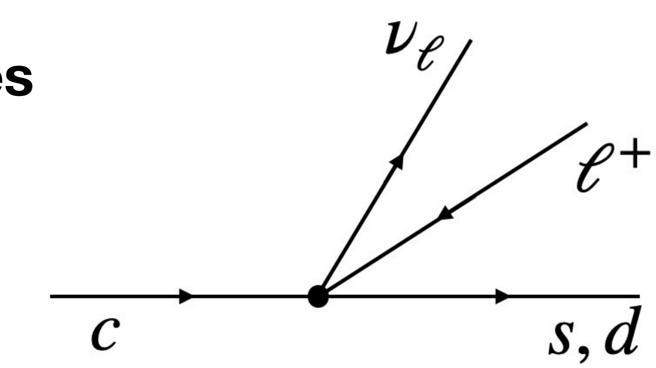
# 1. What and Why Electronic Semi-inclusive Charm decay?

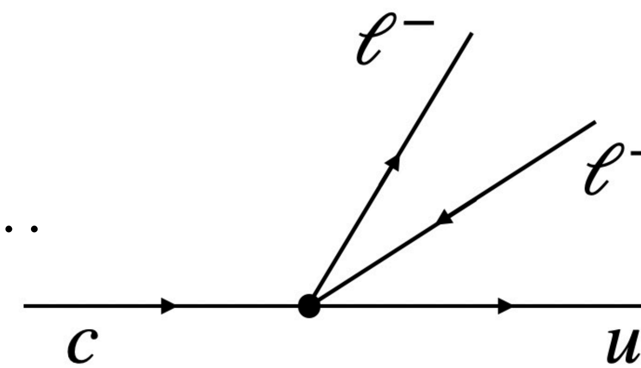
# What and Why Electronic Semi-inclusive Charm decay


- **Experimental detection of partial final state particles**

  ➡ $D \to e^+X$ ($D \to e^+\nu_e X$, only $e^+$ is detected)




- **Sum of a group of exclusive channels**




  ➡ $D^0 \to e^+X_s = D \to e^+\nu_e K^-, e^+\nu_e K^-\pi^0, e^+\nu_e \bar{K}^0\pi^-, \ldots$

  ➡ $D^0 \to e^+X_d = D \to e^+\nu_e \pi^-, e^+\nu_e \pi^-\pi^0, e^+\nu_e \pi^-\pi^+\pi^-, \ldots$

- **Compared to exclusive decays: Better theoretical control**
- **Compared to beauty decays: More sensitive to power corrections**

# What and Why Electronic Semi-inclusive Charm decay

**Charmed hadron lifetimes: theory vs experiment**

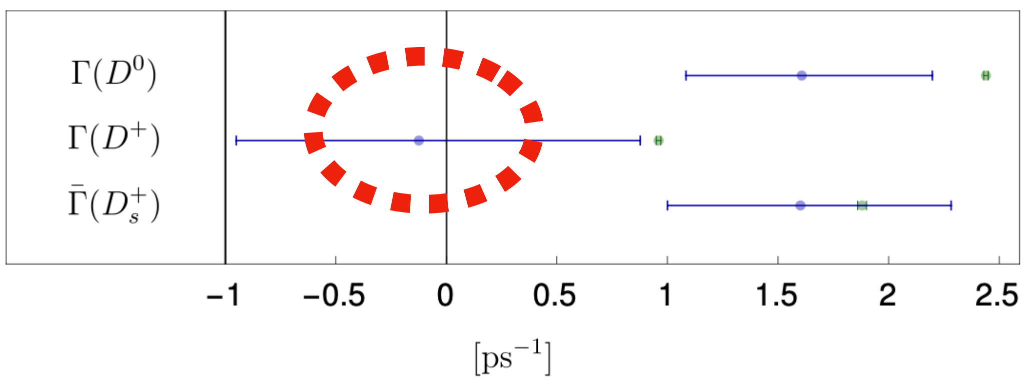



Fig 1 [Lenz et al, '22]

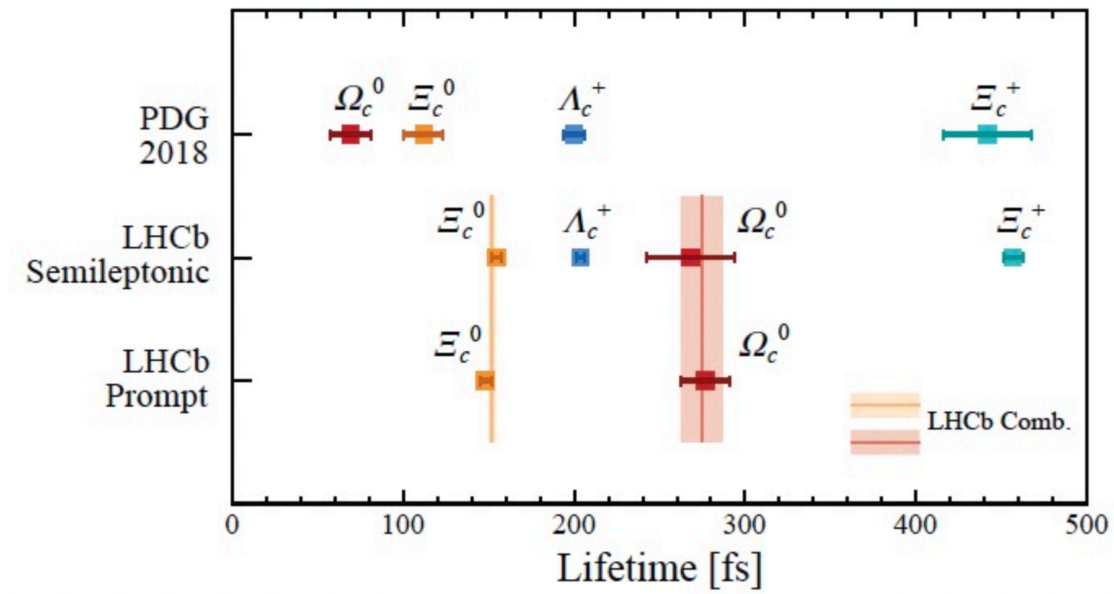



Fig 2

$$\mathcal{O}(1/m_c^3) \Rightarrow \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\Omega_c^0),$$
$$\mathcal{O}(1/m_c^4) \Rightarrow \tau(\Omega_c^0) > \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0),$$
$$\mathcal{O}(1/m_c^4) \text{ with } \alpha \Rightarrow \tau(\Xi_c^+) > \tau(\Omega_c^0) > \tau(\Lambda_c^+) > \tau(\Xi_c^0).$$

[Cheng, '21]

# What and Why Electronic Semi-inclusive Charm decay

## Charmed hadron lifetimes: theory vs experiment

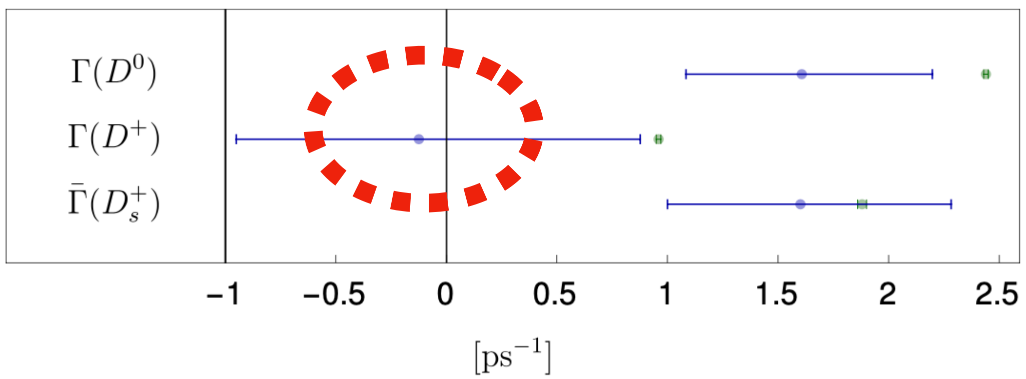



**Fig 1** [Lenz et al, '22]

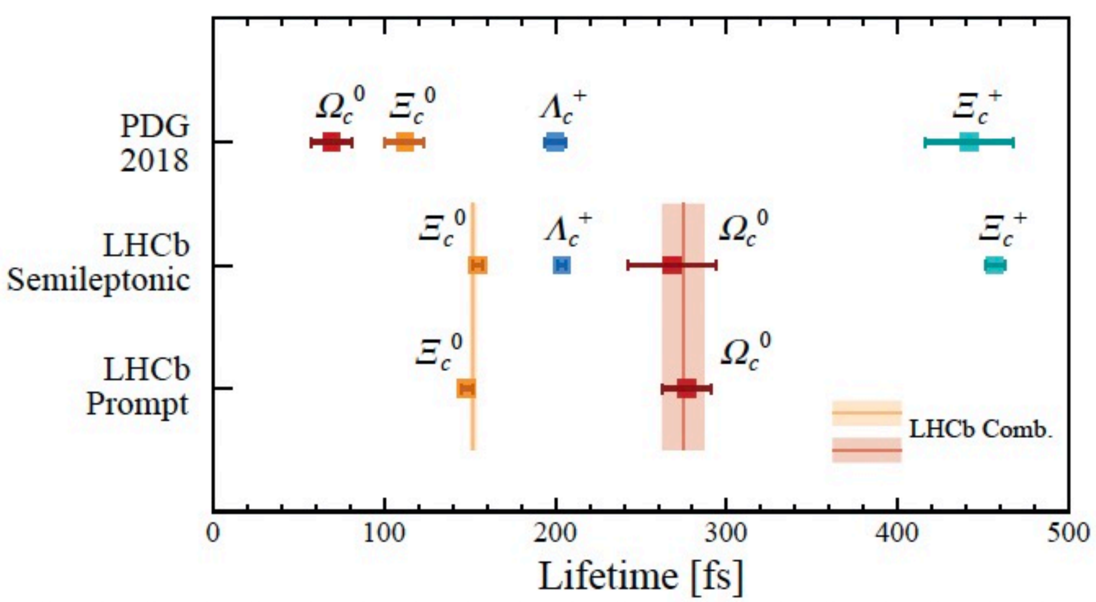



**Fig 2**

$$\mathcal{O}(1/m_c^3) \Rightarrow \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\boldsymbol{\Omega_c^0}),$$
$$\mathcal{O}(1/m_c^4) \Rightarrow \tau(\boldsymbol{\Omega_c^0}) > \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0),$$
$$\mathcal{O}(1/m_c^4) \text{ with } \alpha \Rightarrow \tau(\Xi_c^+) > \tau(\boldsymbol{\Omega_c^0}) > \tau(\Lambda_c^+) > \tau(\Xi_c^0).$$

[Cheng, '21]

## Solutions/hints

- Dependence on identical hadronic parameters in HQET, $\langle H_c | O_i | H_c \rangle$
- Extraction in the inclusive decay spectrum and application to lifetime

**" Again a more precise experimental determination of $\mu_\pi^2$ from fits to semi-leptonic $D^+, D^0$ and $D_s^+$meson decays — as it was done for the $B^+$ and $B^0$ decays — would be very desirable. "** [Lenz et al, '22]

# What and Why Electronic Semi-inclusive Charm decay

## $V_{cb}$, $V_{ub}$ puzzles: inclusive vs exclusive

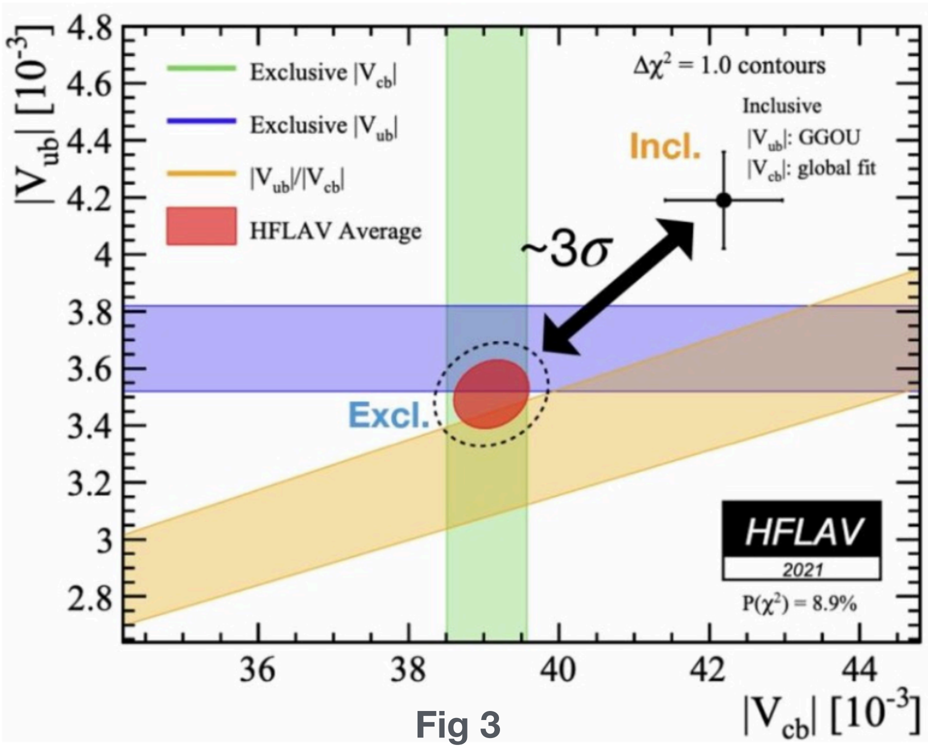



Fig 3

- $V_{cd}$, $V_{cs}$ test: Inclusive vs exclusive

## $b \to s$ anomalies: $P_5'$ in $B \to K^* \ell \ell$

- Test the $c \to u$ inclusive transition

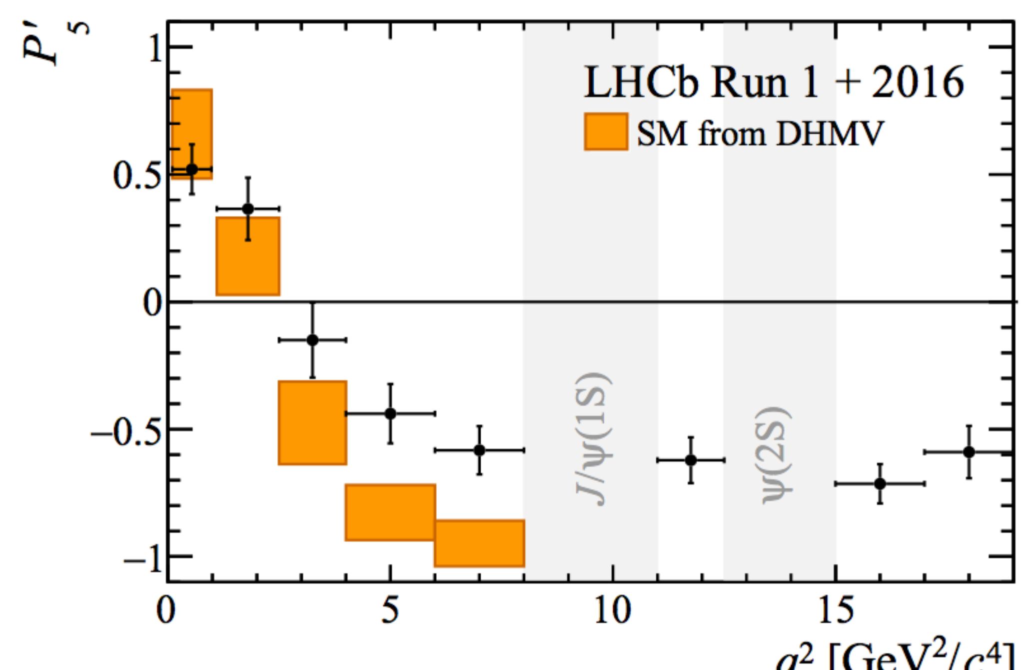



Fig 4

# 2. Theoretical Framework

# Theoretical Framework

- **Optical theorem**

$$\sum \langle D|H|X\rangle\langle X|H|D\rangle \propto \mathrm{Im}\int \mathrm{d}^4x\langle \mathrm{D}|\mathrm{T}\{\mathrm{H(x)H(0)}\}|\mathrm{D}\rangle$$

- **Operator product expansion (OPE)**

  - Short distance : $x \sim 1/m_c$

  - Fluctuation in D meson $\sim \Lambda_{\mathrm{QCD}}$

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0) \to 1 + \frac{\Lambda_{\mathrm{QCD}}}{m_c} + \frac{\Lambda^2_{\mathrm{QCD}}}{m_c^2} + \dots$$

Systematic OPE in HQET.

- **Heavy Quark Effective Theory**

$$h_v(x) \equiv e^{-im_c v\cdot x}\frac{1+\gamma\cdot v}{2}c(x) \qquad v = (1,0,0,0)$$

$$L \ni \bar{h}_v iv\cdot Dh_v - \bar{h}_v\frac{D_\perp^2}{2m_c}h_v - a(\mu)g\bar{h}_v\frac{\sigma\cdot G}{4m_c}h_v + \dots$$

Subtract the big intrinsic momentum,

Leave only ~$\Lambda_{\mathrm{QCD}}$ degrees of freedom.

# Theoretical Framework

- **OPE**

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0)$$

$C_n(x)$

- LO: $\alpha_s^0(m_c)$
- NLO: $\alpha_s(m_c)$
- **…**

$O_n(0)$

- Dim-3: $\bar{h}_v h_v \ (\bar{c}\gamma^\mu c) \rightarrow$ **partonic decay rate.**
- Dim-5: $\bar{h}_v D_\perp^2 h_v,\ g\bar{h}_v \sigma \cdot G h_v\,.$
- Dim-6: $\bar{h}_v D_\mu (v\cdot D) D^\mu h_v,\ (\bar{h}_v \Gamma_1 q)(\bar{q}\Gamma_2 h_v), \ldots$
- **…**

- **Contribute to inclusive decay rate and lifetime**

1. Matrix elements of the **same operators** (SL& NL)
2. Only different short-distance coefficients

$$\lambda_1 \equiv \frac{1}{4m_D}\langle D|\bar{h}_v (iD)^2 h_v|D\rangle = -\mu_\pi^2$$

$$\lambda_2 \equiv \frac{1}{16(s_c\cdot s_q)}\frac{1}{2m_D}\langle D|\bar{h}_v g\sigma\cdot G h_v|D\rangle = \frac{\mu_G^2}{3}$$

# Theoretical Framework

- **Structure functions**

$$\frac{d\Gamma}{d\hat{E}_\ell\, d\hat{q}^2\, d\hat{u}} = \frac{G_F^2 m_b^5 |V_{cb}|^2}{16\pi^3}\theta(\hat{u}_+ - \hat{u})\theta(\hat{E}_\ell)\theta(\hat{q}^2) \times$$
$$\times \left\{ \hat{q}^2\, W_1 - \left[ 2\hat{E}_\ell^2 - 2\hat{E}_\ell \hat{q}_0 + \frac{\hat{q}^2}{2} \right] W_2 + \hat{q}^2 (2\hat{E}_\ell - \hat{q}_0)\, W_3 \right\},$$

$$W_i = W_i^{(0)} + \frac{\mu_\pi^2}{2m_c^2} W_i^{(\pi,0)} + \frac{\mu_G^2}{2m_c^2} W_i^{(G,0)} + \frac{\alpha_s}{\pi}\left[ C_F W_i^{(1)} + C_F \frac{\mu_\pi^2}{2m_c^2} W_i^{(\pi,1)} + \frac{\mu_G^2}{2m_c^2} W_i^{(G,1)} \right] + \ldots$$

$$W_i^{(1)} = w_i^{(0)} \left\{ \mathcal{S}_i\, \delta(\hat{u}) - \left[\frac{\ln \hat{u}}{\hat{u}}\right]_+ - \left(\frac{7}{4} - 2\ln w\right)\left[\frac{1}{\hat{u}}\right]_+ + w\, B(\hat{q}^2, \hat{u})\, \theta(\hat{u}) \right\} + \mathcal{R}_i^{(1)}\, \theta(\hat{u}),$$

$$\mathcal{S}_i = -\frac{5}{4} - \frac{\pi^2}{3} - \mathrm{Li}_2(1-w) - 2\ln^2 w - \frac{5w-4}{2(1-w)}\ln w + \frac{\ln w}{2(1-w)}\,\delta_{i2}$$

$$\mathcal{R}_1^{(1)} = \frac{3}{4} + \frac{\hat{u}(12 - w - \hat{u})}{2\tilde{\lambda}} + \left(w + \frac{\hat{u}}{2} - \frac{\hat{u}(2\hat{u}+3w)}{\tilde{\lambda}}\right)\mathcal{I}_1$$

$$\mathcal{R}_2^{(1)} = \frac{6\hat{u}(\hat{u}^2 - (3-w)\hat{u} - 12 + 13w)}{\tilde{\lambda}^2} + \frac{\hat{u} - 38 + 21w}{\tilde{\lambda}} - 4\frac{\frac{w}{2}\hat{u}^3 + (2w^2-6)\hat{u}^2 + (7 - 3w + \frac{5}{2}w^2)w\hat{u} + w^3(w-4)}{\tilde{\lambda}^2}\mathcal{I}_1$$

$$\mathcal{R}_3^{(1)} = \frac{3\hat{u} - 8 + 5w}{\tilde{\lambda}} + \frac{\hat{u}^2 - (6-w)\hat{u} + 4w}{\tilde{\lambda}}\mathcal{I}_1$$

$$\mathcal{I}_1 = \frac{1}{\sqrt{\tilde{\lambda}}}\ln\frac{\hat{u} + w + \sqrt{\tilde{\lambda}}}{\hat{u} + w - \sqrt{\tilde{\lambda}}}$$

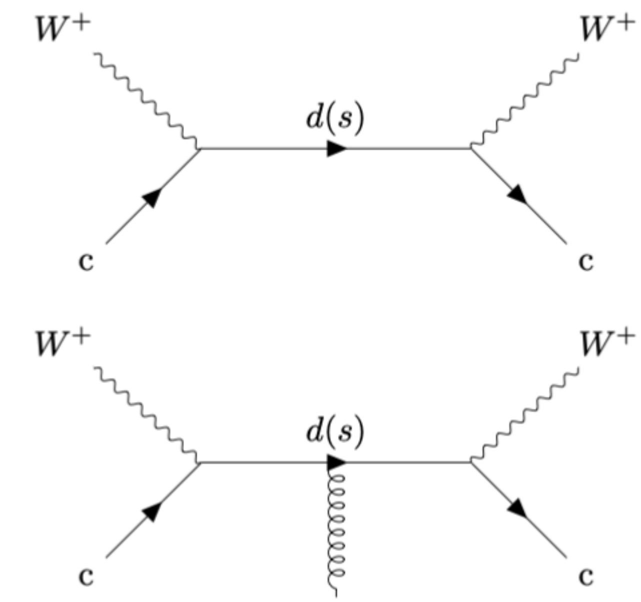



**Fig 5.** Leading Order

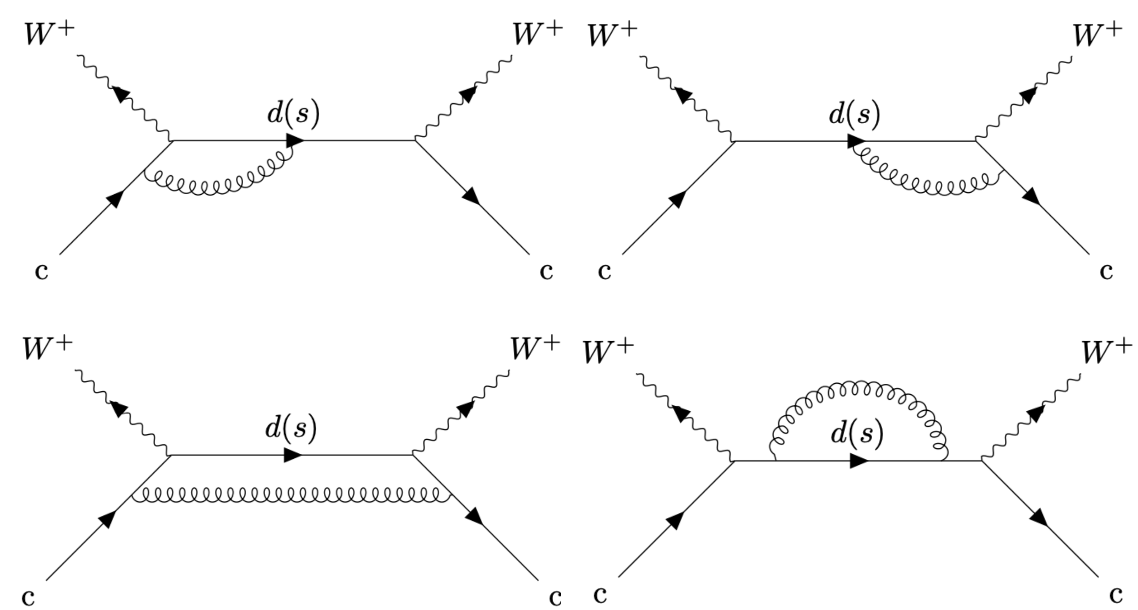



**Fig 6.** Next to Leading Order

**NLO** dim-3

[Capdevila et al. '22]

# Theoretical Framework

- **Structure functions**

$$
\begin{aligned}
W_1^{(\pi,1)}=&w\Big[B_1-\frac{C}{2}+\frac{5w-2}{12}\Big[\frac{1}{\hat u^2}\Big]_+ +\Big(\frac{16+3w-10w^2}{12}-\frac{8w^3-w^2-14w+8}{6(1-w)}\ln w\Big)\delta'(\hat u)\Big]\\
&-\frac{4}{3}(2-w)\Big(\Big[\frac{\ln\hat u}{\hat u}\Big]_+ +L_w\delta(\hat u)\Big)+\Big(\frac{8}{3}(2-w)\ln w-\frac{4+18w-13w^2}{6w}\Big)\Big[\frac{1}{\hat u}\Big]_+ +\mathcal{R}_1^{(\pi)}\theta(\hat u)\\
&+\Big(\frac{13w}{12}-\frac{1}{6}-\frac{1}{3w}-\frac{w^2}{12}+\frac{w^3}{4}+\frac{4+6w-13w^2+3w^3+2w^5}{3w(1-w)}\ln w\Big)\delta(\hat u) \qquad \text{(A.1)}
\end{aligned}
$$

$$
\begin{aligned}
W_2^{(\pi,1)}=&4B_2+6C+\frac{9w-10}{3}\Big[\frac{1}{\hat u^2}\Big]_+ +\Big(\frac{4+6w+16w^2}{3}\ln w-\frac{22-21w+10w^2}{3}\Big)\delta'(\hat u)\\
&+\Big(w^2+\frac{116}{3w^2}-7w-\frac{50}{w}+\frac{88}{3}-4\,\frac{42-34w+17w^2-6w^3+2w^4}{3w^2}\ln w\Big)\delta(\hat u)\\
&+\Big(\frac{10}{3}-\frac{68}{3w}+\frac{28}{w^2}\Big)\Big[\frac{1}{\hat u}\Big]_+ +\mathcal{R}_2^{(\pi)}\theta(\hat u) \qquad \text{(A.2)}
\end{aligned}
$$

$$
\begin{aligned}
W_3^{(\pi,1)}=&2B_3+C+\Big(\frac{7w}{6}-1\Big)\Big[\frac{1}{\hat u^2}\Big]_+ +\Big(\frac{5}{3}(1-w)w+\frac{w(6+3w-8w^2)}{3(1-w)}\ln w\Big)\delta'(\hat u)\\
&+2\Big[\frac{\ln\hat u}{\hat u}\Big]_+ +\Big(\frac{19}{6}-\frac{2}{w}+\frac{4}{w^2}-4\ln w\Big)\Big[\frac{1}{\hat u}\Big]_+ +\Big[2L_w+\frac{w^2}{2}+\frac{14}{3w^2}-\frac{11w}{6}-\frac{20}{3w}\\
&+\frac{41}{6}+\Big(\frac{7w-6}{1-w}+\frac{4}{3}w-\frac{4}{3}w^2-\frac{8}{w^2}+\frac{4}{w}\Big)\ln w\Big]\delta(\hat u)+\mathcal{R}_3^{(\pi)}\theta(\hat u) \qquad \text{(A.3)}
\end{aligned}
$$

$$
\begin{aligned}
B_i=&\tfrac{w^2}{6}\Big(\Big[\tfrac{7}{4}-2L_w-\tfrac{2-w}{1-w}\ln w+\delta_{i2}\tfrac{\ln w}{1-w}\Big]\delta''(\hat u)-4\Big[\tfrac{\ln\hat u}{\hat u^3}\Big]_+ +(8\ln w-1)\Big[\tfrac{1}{\hat u^3}\Big]_+\Big),\\
C=&\tfrac{2(2-w)}{3}\Big(-\Big[\tfrac{\ln\hat u}{\hat u^2}\Big]_+ +2\ln w\Big[\tfrac{1}{\hat u^2}\Big]_+ +L_w\,\delta'(\hat u)\Big) \qquad \text{(A.4)}\\
L_w=&\mathrm{Li}_2(1-w)+2\ln^2 w+\tfrac{\pi^2}{3}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}_1^{(\pi)}=&\frac{(4\hat u-w)(2-w)\hat u+2w^3}{3\hat u^3}\ln\frac{\hat u}{w^2}+\Big[\frac{2w^6}{3\hat u^3}+\frac{7w^5}{3\hat u^2}-\frac{14-5\hat u}{3\hat u^2}w^4-\frac{13\hat u+32}{6\hat u}w^3\\
&-\frac{23\hat u^2-36\hat u-48}{6\hat u}w^2-(13\hat u^2-58\hat u+36)\frac{w}{6}-\frac{\hat u}{6}(3\hat u^2-26\hat u+8)\Big]\frac{\mathcal{I}_1}{\tilde\lambda} \qquad \text{(A.5)}\\
&-\frac{4w^2}{3\hat u^2}+\frac{2\hat u^2+2\hat uw-13\hat u+17w-28}{3\tilde\lambda}+\frac{4w}{3\hat u^2}+\frac{2}{3\hat uw}-\frac{7\hat u+8}{12\hat u}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}_2^{(\pi)}=&\frac{12(2-w)\hat u+8w^2}{3\hat u^3}\ln\frac{\hat u}{w^2}+\Big[w\Big(\frac{8w^4}{3\hat u^3}-\frac{40}{3}-\frac{14w}{3}-2\hat u+\frac{(4w-8)w^2}{\hat u^2}-4\frac{8-8w+w^2}{\hat u}\Big)\\
&+68+60\hat u-\frac{4}{\tilde\lambda}(15\hat u^3-35\hat u^2-76\hat u+14w+63w\hat u+19w\hat u^2)\Big]\frac{\mathcal{I}_1}{\tilde\lambda}+\frac{16(1+2\hat u)}{3\hat u^2}\\
&-\frac{16w}{3\hat u^2}-\frac{28}{\hat uw^2}+\frac{68}{3\hat uw}-\frac{2\left(9\hat u^2+50\hat uw-201\hat u+86w-78\right)}{3\hat u\tilde\lambda} \qquad \text{(A.6)}\\
&-\frac{4\left(2\hat u^2w+2\hat u^3+11\hat u^2+49\hat uw-81\hat u+45w-28\right)}{\tilde\lambda^2}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}_3^{(\pi)}=&-\frac{2(3\hat u^2-(2-w)\hat u-2w^2)}{3\hat u^3}\ln\frac{\hat u}{w^2}+\Big(8w-\frac{13}{3}w^2-4-\frac{10}{3}\hat u(w-2)-\hat u^2\\
&+\frac{10\hat u(w-2)w^3+4w^5}{3\hat u^3}\Big)\frac{\mathcal{I}_1}{\tilde\lambda}-\frac{2\left(7\hat u^2+11\hat uw-19\hat u+17w-16\right)}{3\hat u\tilde\lambda} \qquad \text{(A.7)}\\
&-\frac{8w}{3\hat u^2}+\frac{8(\hat u+1)}{3\hat u^2}-\frac{4}{\hat uw^2}+\frac{2}{\hat uw}
\end{aligned}
$$

**NLO** dim-5: $\mu_\pi^2$

$$
\begin{aligned}
W_1^{(G,1)}=&-\frac{2}{3}w\Big[G_1+\Big(\tfrac{C_F}{4}\Big(1+8w-5\frac{w^2\ln w}{1-w}\Big)-\tfrac{C_A}{4}(1+2w)\Big)\delta'(\hat u) \qquad \text{(A.9)}\\
&+C_F(5+\tfrac{2}{w^2}-\tfrac{2}{w})\Big(\Big[\frac{\ln\hat u}{\hat u}\Big]_+ -2\ln w\Big[\frac{1}{\hat u}\Big]_+ +L_w\delta(\hat u)\Big)\Big]\\
&-\frac{2}{3}\Big(\tfrac{C_A}{4}(8-5w)+C_F\Big(\frac{4}{w}-3+\frac{5w}{4}\Big)\Big)\Big[\frac{1}{\hat u}\Big]_+\\
&-\frac{1}{3}\Big(C_A\tfrac{5w^3-34w^2+51w-20}{2(w-1)^2}+C_F\tfrac{10w^5-21w^4+7w^3-10w^2+28w-16}{(w-1)^2w}\Big)\ln w\,\delta(\hat u)\\
&-\frac{1}{3}\Big(C_A\tfrac{2w^4+2w^3-3w^2+5w-4}{2(1-w)w}+C_F\tfrac{35w^3-25w^2-10w-8}{4(1-w)}\Big)\delta(\hat u)+\mathcal{R}_1^{(G)}\theta(\hat u)
\end{aligned}
$$

$$
\begin{aligned}
W_2^{(G,1)}=&-\frac{8}{3}\Big[G_2+\Big(C_F\Big(\frac{1}{w}-\frac{11}{4}+2w-(1-\frac{5w}{4})\ln w\Big)+\tfrac{C_A}{4}(3-2w)\Big)\delta'(\hat u)\\
&+2C_F\tfrac{w^2-w-1}{w^3}\Big(\Big[\frac{\ln\hat u}{\hat u}\Big]_+ -2\ln w\Big[\frac{1}{\hat u}\Big]_+ +L_w\delta(\hat u)\Big)\\
&+\Big(C_A(\tfrac{8}{w}-\tfrac{9}{2w^2}-\tfrac{2}{w^3}-\tfrac{9}{4})+C_F(\tfrac{7}{w^2}-\tfrac{6}{w^3}-\tfrac{5}{2})\Big)\Big[\frac{1}{\hat u}\Big]_+ \qquad \text{(A.10)}\\
&+\Big(C_A\tfrac{9w^3-56w^2+40w+16}{4w^3}+C_F\tfrac{5w^4-6w^3+3w^2-12w+12}{w^3}\Big)\ln w\,\delta(\hat u)\\
&-\Big(C_A\tfrac{2w^3+4w^2-23w+16}{4w^2}+C_F\tfrac{35w^4-98w^3+134w^2-120w+32}{8w^3}\Big)\delta(\hat u)\Big]+\mathcal{R}_2^{(G)}\theta(\hat u)
\end{aligned}
$$

$$
\begin{aligned}
W_3^{(G,1)}=&-\frac{4}{3}\Big[G_3+\Big(C_F\Big(\frac{1}{4}+\frac{5w}{2}-\frac{5w^2\ln w}{4(1-w)}\Big)-\tfrac{C_A}{4}(1+w)\Big)\delta'(\hat u)\Big]\\
&-\frac{2}{3}C_F\tfrac{5w^2+4w+4}{w^2}\Big(\Big[\frac{\ln\hat u}{\hat u}\Big]_+ -2\ln w\Big[\frac{1}{\hat u}\Big]_+ +L_w\delta(\hat u)\Big)\\
&-\frac{2}{3}\Big(C_A(\tfrac{2}{w^2}+\tfrac{5}{w}-\tfrac{7}{2})+C_F(\tfrac{4}{w^2}-\tfrac{5}{4})\Big)\Big[\frac{1}{\hat u}\Big]_+ \qquad \text{(A.11)}\\
&-\frac{1}{3}\Big(C_A\tfrac{7w^4-40w^3+49w^2-6w-8}{(w-1)^2w^2}+C_F\tfrac{(20w^5-37w^4-w^3+6w^2+24w-16)}{(w-1)^2w^2}\Big)\ln w\,\delta(\hat u)\\
&-\frac{2}{3}\Big(C_A\tfrac{w^2-w+1}{1-w}+C_F\tfrac{35w^4-85w^3+66w^2-8w-16}{4(1-w)w^2}\Big)\delta(\hat u)+\mathcal{R}_3^{(G)}\theta(\hat u)
\end{aligned}
$$

$$
\begin{aligned}
G_i=&\Big(1+\frac{5}{2}w-4\delta_{i2}\Big)\Big[C_F\Big(\frac{3-8\ln w}{4}\Big[\frac{1}{\hat u^2}\Big]_+ +\Big[\frac{\ln\hat u}{\hat u^2}\Big]_+ -L_w\delta'(\hat u)\Big)+\tfrac{C_A}{2}\ln\frac{\mu}{m_b}\delta'(\hat u)\Big]\\
&+C_A\Big[\frac{1+w}{2}\Big[\Big[\frac{1}{\hat u^2}\Big]_+ +\ln w\,\delta'(\hat u)\Big]-\delta_{i2}\Big(\frac{1+2w}{2w}\Big[\frac{1}{\hat u^2}\Big]_+ +\frac{\ln w}{w}\delta'(\hat u)\Big)\Big]-\tfrac{3C_A}{4}\frac{w_i^{(G,0)}}{w_i^{(0)}}\ln\frac{\mu}{m_b}\delta(\hat u)\\
&+C_A\Big(\frac{1+4w}{2w^2}-\frac{1+2w}{w^3}\delta_{i2}\Big)\Big[\Big[\frac{\ln\hat u}{\hat u}\Big]_+ -2\ln w\Big[\frac{1}{\hat u}\Big]_+ +L_w\delta(\hat u)\Big] \qquad \text{(A.12)}
\end{aligned}
$$

where

$$
w_1^{(G,0)}=-\frac{2}{3}(4-5w),\qquad w_2^{(G,0)}=0,\qquad w_3^{(G,0)}=\frac{10}{3}, \qquad \text{(A.13)}
$$

$$
\begin{aligned}
\mathcal{R}_1^{(G)}=&\frac{C_A}{3}\Big[\tfrac{1}{2}+\tfrac{\hat u+13w-16}{\tilde\lambda}+\tfrac{4w+1}{\hat uw}\ln\tfrac{\hat u}{w^2}+\Big(\tfrac{4w+1-6\hat u}{\hat u}+2\tfrac{3\hat u(\hat u-3+w)+4w}{\tilde\lambda}\Big)\mathcal{I}_1\Big] \qquad \text{(A.14)}\\
&+\frac{C_F}{3}\Big[\tfrac{15\hat u-5\hat uw-5\hat u^2-11w+20}{\tilde\lambda}-\tfrac{4w}{\hat u\tilde\lambda}-\tfrac{10w}{\hat u}+\tfrac{8}{\hat uw}+\tfrac{11\hat u+24}{4\hat u}+\Big(\tfrac{5w^2}{\hat u^2}+\tfrac{2(5\hat u+1)w}{\hat u^2}+\tfrac{4-4w}{\hat uw}\Big)\ln\frac{\hat u}{w^2}\\
&+\Big(\tfrac{8-3\hat u^2-13\hat uw+10\hat u-12w}{\tilde\lambda}+\tfrac{5w^3}{\hat u^2}+\tfrac{(15\hat u+2)w^2}{\hat u^2}+\tfrac{3(5\hat u-8)w}{2\hat u}+\tfrac{5\hat u}{2}-2\Big)\mathcal{I}_1\Big]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}_2^{(G)}=&4C_A\Big[\tfrac{16-13\hat u^2-25\hat uw+51\hat u-29w}{\tilde\lambda^2}+\tfrac{22-15\hat uw-9\hat u^2+112\hat u-32w}{6\tilde\lambda\hat u}+\tfrac{w}{3\tilde\lambda\hat u^2}+\tfrac{16\hat u-1}{3\hat u^2w}-\tfrac{3}{\hat uw^2}-\tfrac{4}{3\hat uw^3}\\
&+\tfrac{4w^2-3w-2}{3w^3\hat u}\ln\frac{\hat u}{w^2}+\Big(\tfrac{14\hat u^2-26\hat uw+58\hat u-3w-2}{3\tilde\lambda\hat u}-\tfrac{2\left(3\hat u^2w+3\hat u^3-5\hat u^2+20\hat uw-25\hat u\right)}{\tilde\lambda^2}-\tfrac{8w}{\tilde\lambda^2}+\tfrac{4}{3\hat u}\Big)\mathcal{I}_1\Big]\\
&+4C_F\Big[\tfrac{5\hat u^2w+42\hat uw+5\hat u^3-4\hat u^2-55\hat u+39w-36}{\tilde\lambda^2}+\tfrac{4w}{\tilde\lambda^2\hat u}+\tfrac{53\hat uw-20\hat u^2-155\hat u+44w-52}{6\tilde\lambda\hat u}+\tfrac{14}{3\hat uw^2}-\tfrac{4}{\hat uw^3}-\tfrac{10}{3\hat u}\\
&+\tfrac{4\hat u\left(w^2-w-1\right)+(5w-6)w^3}{3\hat u^2w^3}\ln\frac{\hat u}{w^2}+\Big(\tfrac{23\hat u^2w+13\hat u^3-37\hat u^2+47\hat uw-58\hat u+20w-8}{\tilde\lambda^2} \qquad \text{(A.15)}\\
&+\tfrac{25\hat u^2w+15\hat u^3-114\hat u^2+76\hat uw-150\hat u+16w-8}{6\tilde\lambda\hat u}+\tfrac{5w^2}{3\hat u^2}+\tfrac{(5\hat u-6)w}{3\hat u^2}-\tfrac{5\hat u+8}{2\hat u}\Big)\mathcal{I}_1\Big]
\end{aligned}
$$

**NLO** dim-5: $\mu_G^2$

$$
\begin{aligned}
\mathcal{R}_3^{(G)}=&\frac{4C_A}{3}\Big[\tfrac{15\hat u-3\hat u^2-3\hat uw-5w-2}{2\tilde\lambda\hat u}+\tfrac{1}{\hat uw^2}+\tfrac{5}{2\hat uw}+\tfrac{1+4w}{2w^2\hat u}\ln\tfrac{\hat u}{w^2}+\tfrac{w-5\hat u-2w\hat u+4w^2}{2\tilde\lambda\hat u}\mathcal{I}_1\Big] \qquad \text{(A.16)}\\
&+\frac{4C_F}{3}\Big[\tfrac{2\hat u^2+7\hat uw-9\hat u+3w}{\tilde\lambda\hat u}+\tfrac{2}{\hat uw^2}-\tfrac{5}{\hat u}+\Big(\tfrac{5w}{2\hat u^2}+\tfrac{5\hat u+2}{2\hat u^2}+\tfrac{2}{\hat uw^2}+\tfrac{2}{\hat uw}\Big)\ln\tfrac{\hat u}{w^2}\\
&+\Big(\tfrac{5\hat u^2+5\hat uw-16\hat u+12w-12}{2\tilde\lambda}+\tfrac{5w^2}{2\hat u^2}+\tfrac{(5\hat u+1)w}{\hat u^2}-\tfrac{5\hat u+8}{4\hat u}\Big)\mathcal{I}_1\Big]
\end{aligned}
$$

[Capdevila et al. '22]

# Theoretical Framework

## Analytical Result

$$\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}y} = \{2(3-2y)y^2\theta(1-y) - \frac{2\lambda_1}{m_b^2}\left[-\frac{5}{3}y^3\theta(1-y) + \frac{1}{6}\delta(1-y)\theta(1^+-y) + \frac{1}{6}\delta'(1-y)\theta(1^+-y)\right] - \frac{2\lambda_2}{m_b^2}\left[-y^2(6+5y)\theta(1-y) + \frac{11}{2}\delta(1-y)\theta(1^+-y)\right]\}$$

**LO: dim-3 and dim-5** $y = \frac{2E_e}{m_c}$

$$\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}y} \supset \frac{1}{360}\Big(5y(-82+y(153-86y+8\pi^2(-3+2y))) - 24y^3(-50+y(5+y(-28+9y)))\,\mathrm{ArcCoth}[1+2\sqrt{y}] + 12y^3(125-2(-5+y)y(-1+9y))\,\mathrm{ArcTanh}[1-2\sqrt{y}] - 816\,\mathrm{Log}[1-\sqrt{y}] - 600\,\mathrm{Log}[1+\sqrt{y}] - 108\,\mathrm{Log}[(1+\sqrt{y})^2] + 406\,\mathrm{Log}[1-y] + 4\big(6y^4(1+y)(5+9y)\,\mathrm{ArcTanh}[1-2y] + 6y^3(25+9y^2)\,\mathrm{Log}[1+\sqrt{y}] + 90y\,\mathrm{Log}[1-y] - 105y^2\,\mathrm{Log}[1-y] - 260y^3\,\mathrm{Log}[1-y] - 180y^5\,\mathrm{Log}[1-y] + 150\,\mathrm{Log}[1-y]^2 - 360y\,\mathrm{Log}[1-y]^2 + 135y^2\,\mathrm{Log}[1-y]^2 + 30y^3\,\mathrm{Log}[1-y]^2 - 75y^3\,\mathrm{Log}[y] + 153y^5\,\mathrm{Log}[y] - 30y^3\,\mathrm{Log}[y^5]\big) + \ldots$$

**... NLO: dim-3**

$$\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}y} \supset \frac{1}{810}\Big(480\pi^2 + 695y + \frac{7615y^2}{2} - 165\pi^2y^2 - \frac{545y^3}{2} + 540\pi^2y^3 - 6030y^4 + 795\pi^2y^4 + 1800y^5 - 1230\pi^2y^5 + 300\pi^2y^6 + 1440\,i\,\pi\,\mathrm{Log}[2] + 2880y\,\mathrm{Log}[2] - 7740y^2\,\mathrm{Log}[2] + \ldots$$

**... NLO: dim-5**

## Some tips

- Up to finite power, the obtained differential decay rate is **NOT** the experimental spectrum
- Observables require integration over final states

$$\Gamma = \int \frac{d\Gamma}{dy}dy$$

$$\langle E_\ell \rangle = \frac{1}{\Gamma}\int \frac{d\Gamma}{dy}E_\ell\, dy$$

$$\langle E_\ell^2 \rangle = \frac{1}{\Gamma}\int \frac{d\Gamma}{dy}E_\ell^2\, dy$$

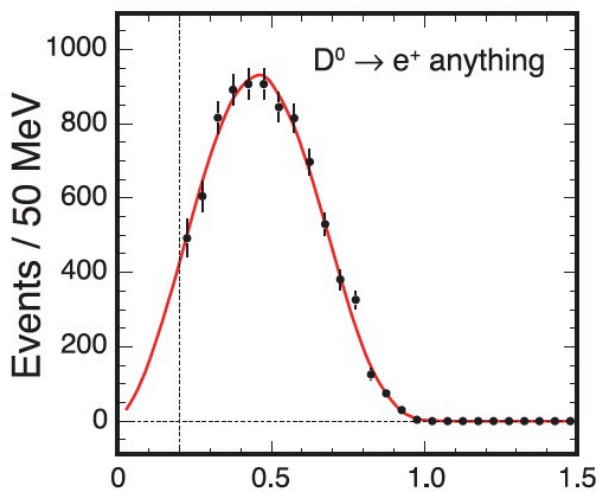
...

# 3. Experimental status

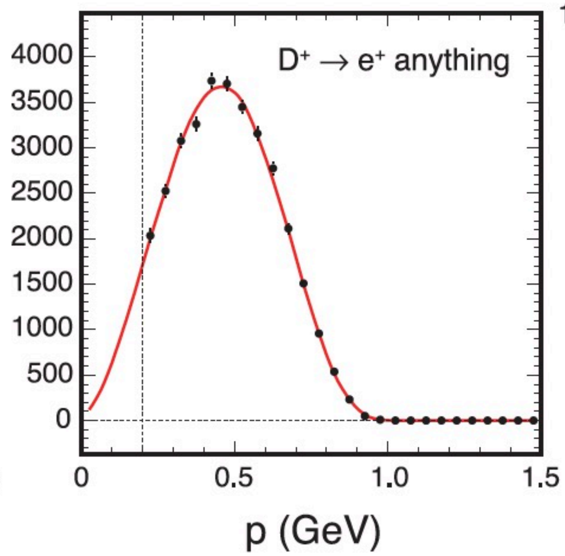
# Experimental status

## CLEO measurements

$D^0 \to e^+X$




$D^+ \to e^+X$




$D_s^+ \to e^+X$

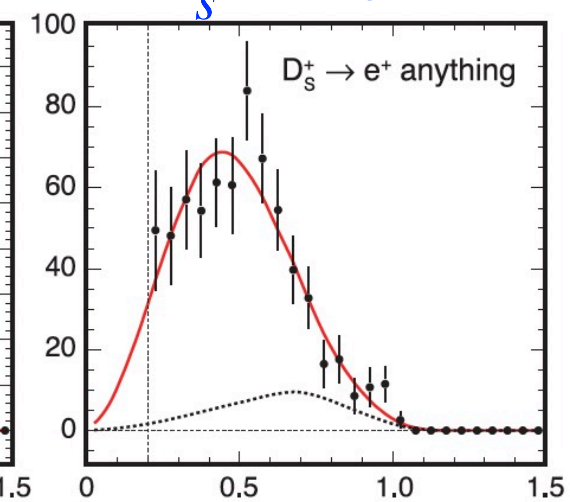



Fig 7

$$B\left(D^0 \to Xe^+\nu_e\right) = (6.46 \pm 0.09 \pm 0.11)\,\%$$

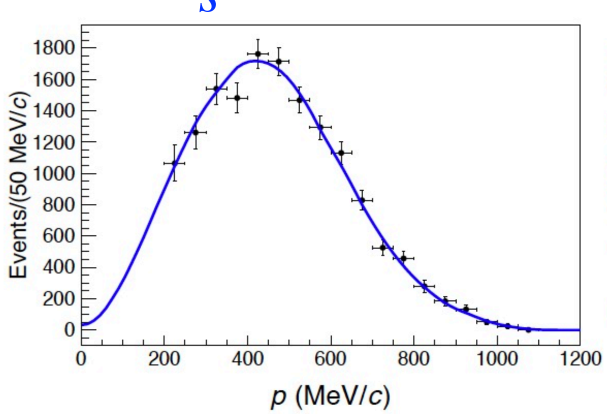
$$B\left(D^0 \to Xe^+\nu_e\right) = (6.46 \pm 0.09 \pm 0.11)\,\%$$

$$B\left(D_s^+ \to Xe^+\nu_e\right) = (6.52 \pm 0.39 \pm 0.15)\,\%$$

[CLEO, '09]

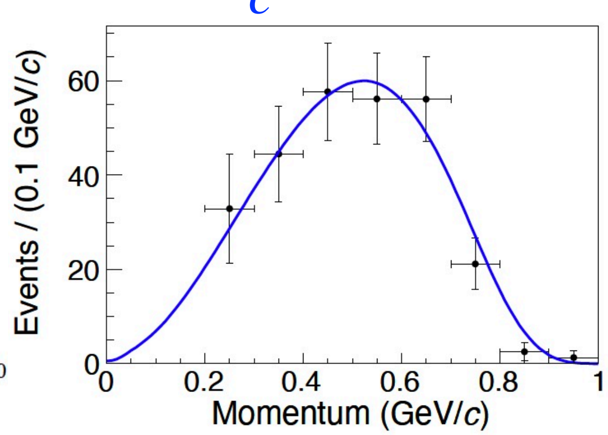
## BESIII measurements

$D_s^+ \to e^+X$




[BESIII, '21]

$\Lambda_c^+ \to e^+X$




$\Lambda_c^+ \to e^+X$

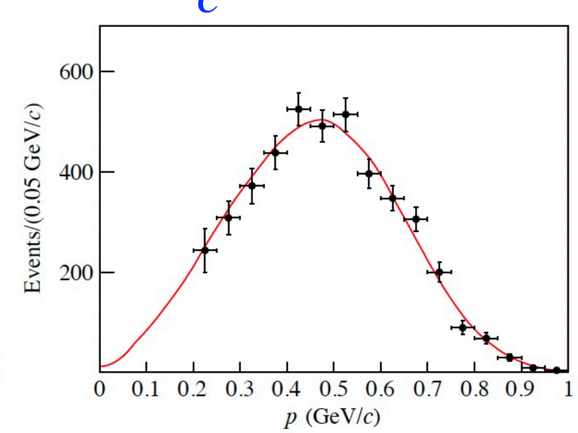



Fig 8

$$B\left(D_s^+ \to Xe^+\nu_e\right) = (6.30 \pm 0.13 \pm 0.10)\,\%$$

[BESIII, '21]

$$B\left(\Lambda_c^+ \to Xe^+\nu_e\right) = (3.95 \pm 0.34 \pm 0.09)\,\%$$

[BESIII (567 pb$^{-1}$), '18]

$$B\left(\Lambda_c^+ \to Xe^+\nu_e\right) = \left(4.06 \pm 0.10_{\text{stat.}} \pm 0.09_{\text{syst}}\right)\%$$

[BESIII (4.5 fb$^{-1}$), '23]

# Experimental status

## Electronic energy momentum

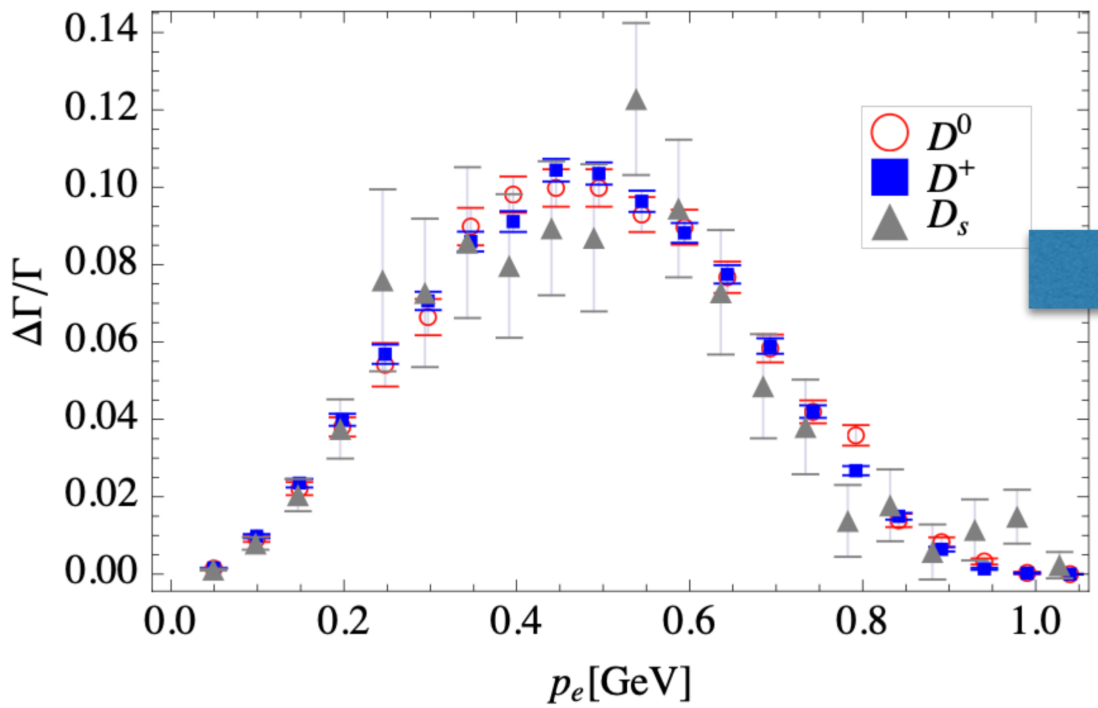



Fig 9

The laboratory frame of the D meson

$$\langle E_e\rangle^{D^0}_{lab} = 0.465(3)\mathrm{GeV}\,,$$
$$\langle E_e\rangle^{D^+}_{lab} = 0.459(1)\mathrm{GeV}\,,$$
$$\langle E_e\rangle^{D_s}_{lab} = 0.466(12)\mathrm{GeV}\,,$$

$$\langle E_e^2\rangle^{D^0}_{lab} = 0.248(2)\mathrm{GeV}^2\,,$$
$$\langle E_e^2\rangle^{D^+}_{lab} = 0.242(1)\mathrm{GeV}^2\,,$$
$$\langle E_e^2\rangle^{D_s}_{lab} = 0.254(13)\mathrm{GeV}^2\,.$$

D mesons rest frame

$$\langle E_\ell\rangle^{D^0}_{exp} = 0.459(3)\mathrm{GeV}\,, \qquad \langle E_\ell^2\rangle^{D^0}_{exp} = 0.240(2)\mathrm{GeV}^2\,,$$
$$\langle E_\ell\rangle^{D^+}_{exp} = 0.455(1)\mathrm{GeV}\,, \qquad \langle E_\ell^2\rangle^{D^+}_{exp} = 0.236(1)\mathrm{GeV}^2\,,$$
$$\langle E_\ell\rangle^{D_s}_{exp} = 0.456(11)\mathrm{GeV}\,, \qquad \langle E_\ell^2\rangle^{D_s}_{exp} = 0.239(12)\mathrm{GeV}^2\,,$$

[Gambino,Kamenik, ’10]

# 4. Phenomenological Analysis (preliminary)

https://github.com/ChunHuangPhy/CompactOject

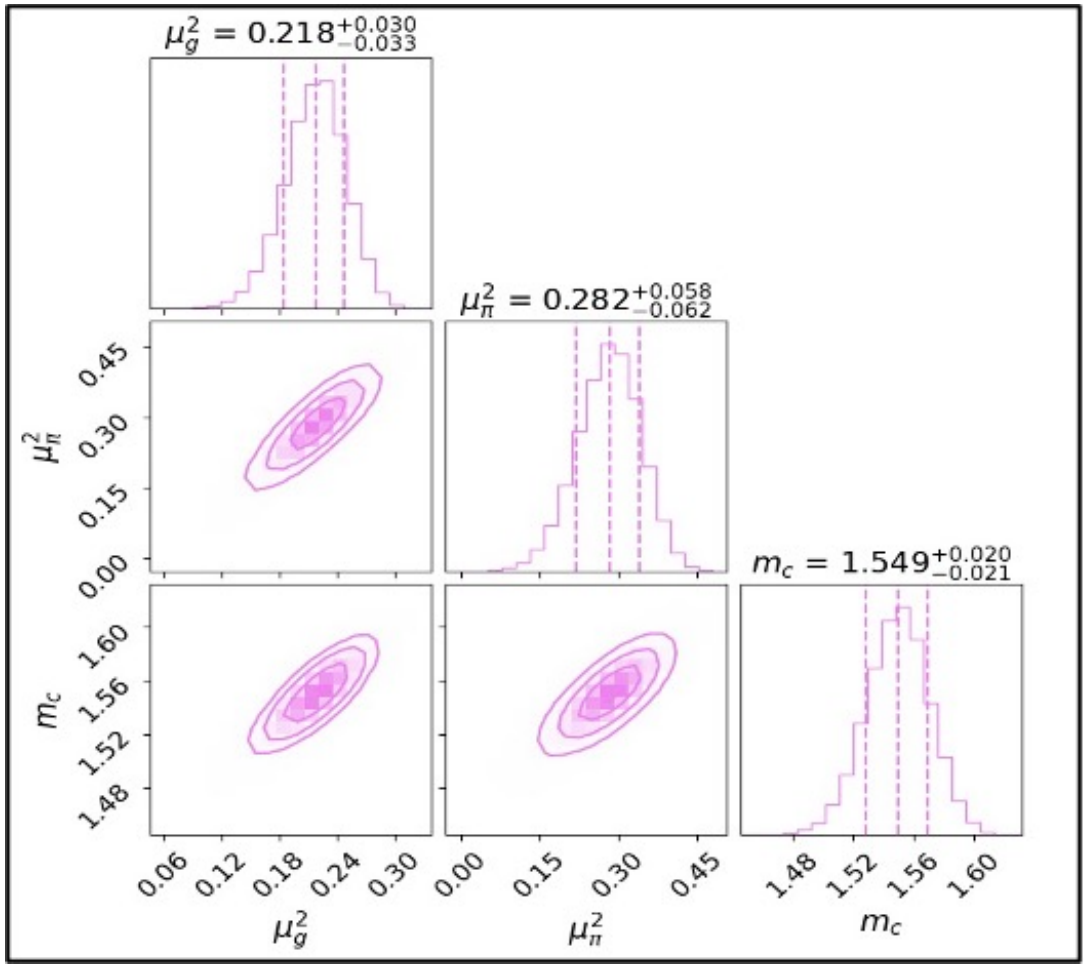

shaokk18@lzu.edu.cn

# Phenomenological Analysis (preliminary)

**CompactObject**

[Huang et al, '2024]

⭐ **For the first time,** we systematically extract the **Model-independent** fundamental parameters of HQE from experimental data on semi-leptonic inclusive decays of D mesons.




$$\mu_\pi^2(D) = (0.48 \pm 0.20)\mathrm{GeV}^2$$

$$\mu_G^2(D) = (0.34 \pm 0.10)\mathrm{GeV}^2$$

[Lenz et al, '22]

$$\mu_\pi^2(D) = (0.282^{+0.058}_{-0.062})\mathrm{GeV}^2$$

$$\mu_G^2(D) = (0.218^{+0.030}_{-0.033})\mathrm{GeV}^2$$

$$m_c = (1.549^{+0.020}_{-0.021})\mathrm{GeV}$$

**[Our results]**

☑ **Higher** precision

☑ **Stronger** constraints

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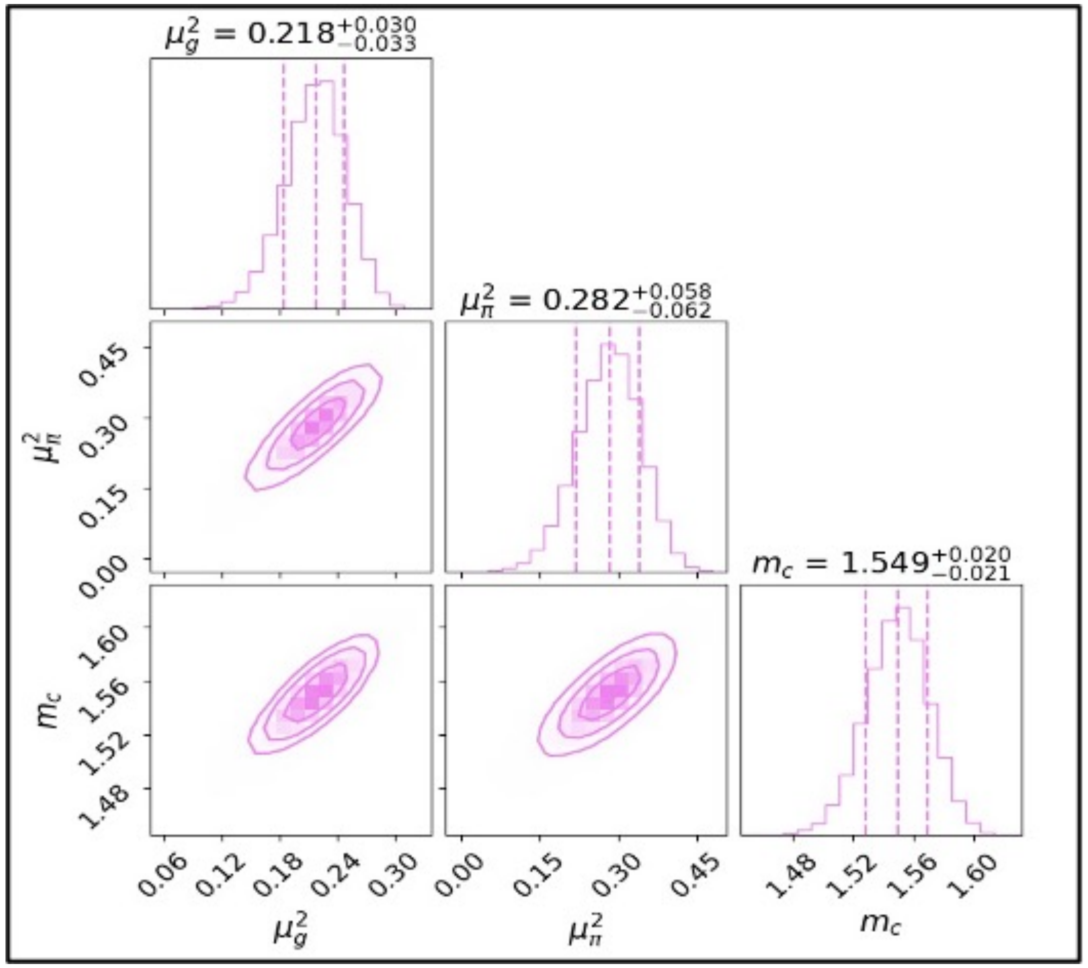


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# Phenomenological Analysis (preliminary)

**CompactObject**

[Huang et al, '2024]

Prior Distribution for Free Parameters

$$m_c \in [1,2]\mathrm{GeV}$$

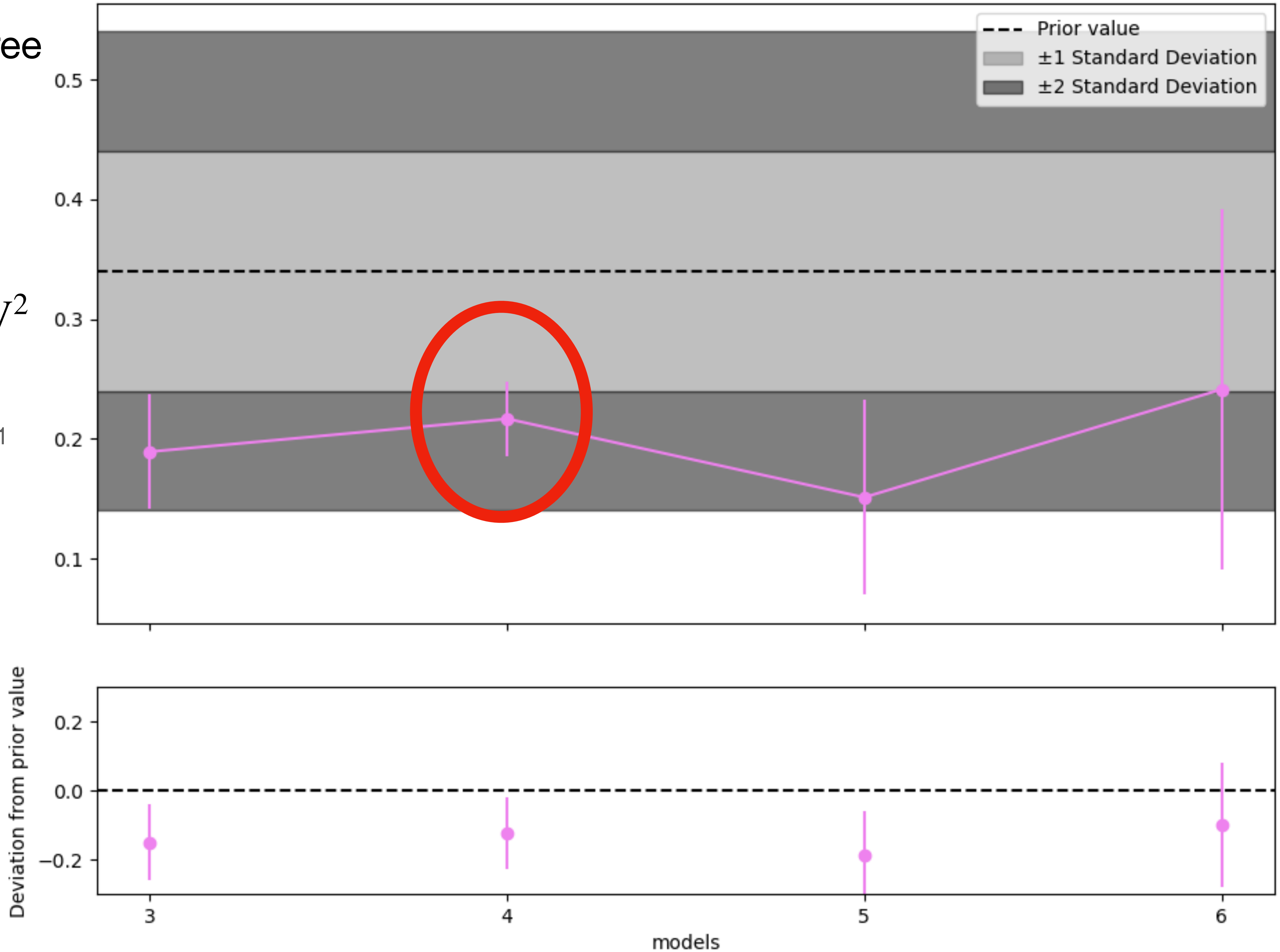
$$\mu_\pi^2 \in [-0.28, 1.08]\mathrm{GeV}^2$$

$$\mu_G^2 \in [0.04, 0.64]\mathrm{GeV}^2$$

JHEP08(2022)241

$$\alpha_s\left(\bar{m}_c = 1.273\mathrm{GeV}\right) = 0.378387$$

$$\alpha_s\left(m_z = 91.1880\mathrm{GeV}\right) = 0.1179$$



# Phenomenological Analysis (preliminary)

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[Huang et al, '2024]

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JHEP08(2022)241

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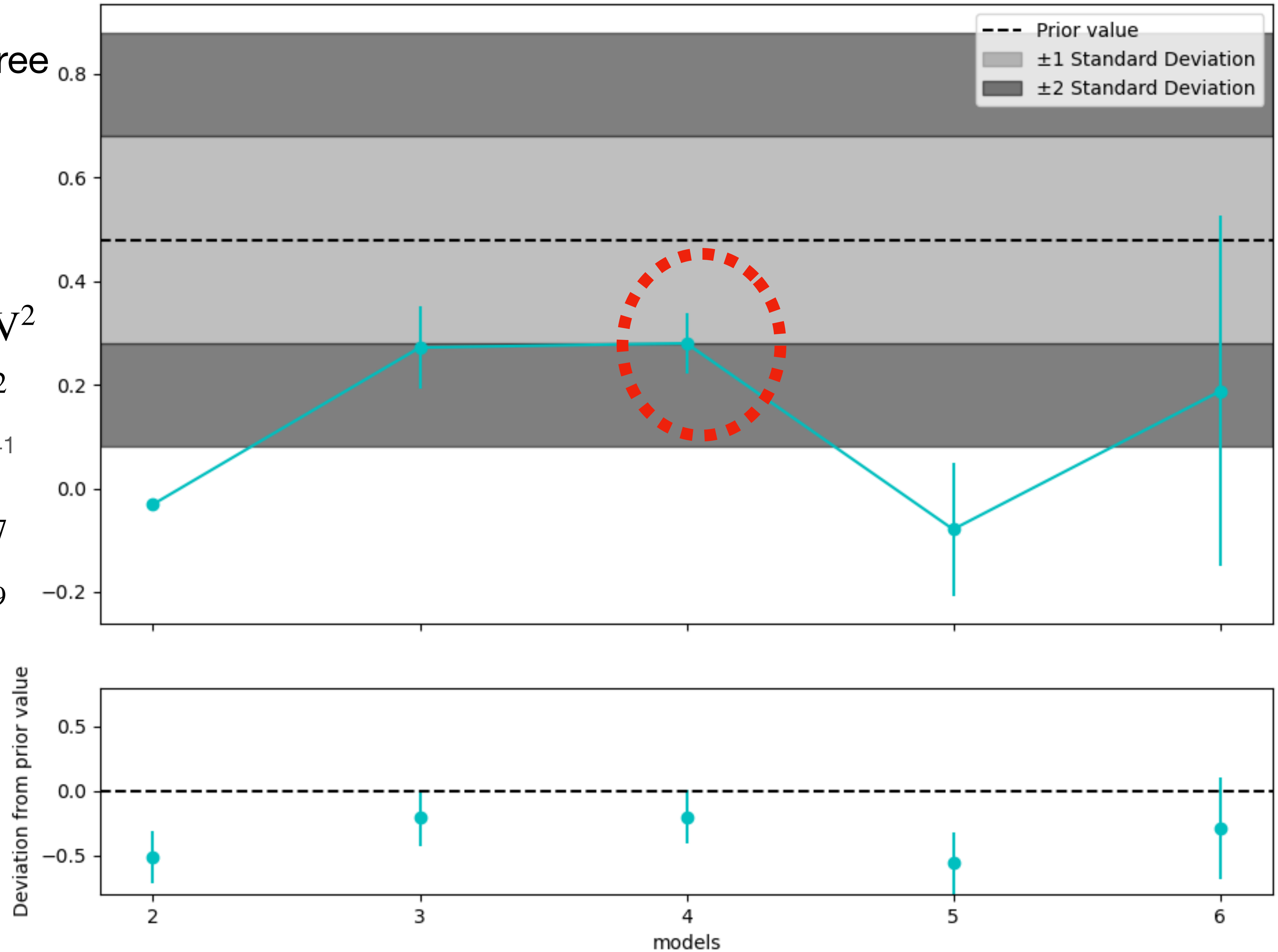


# What did we do?

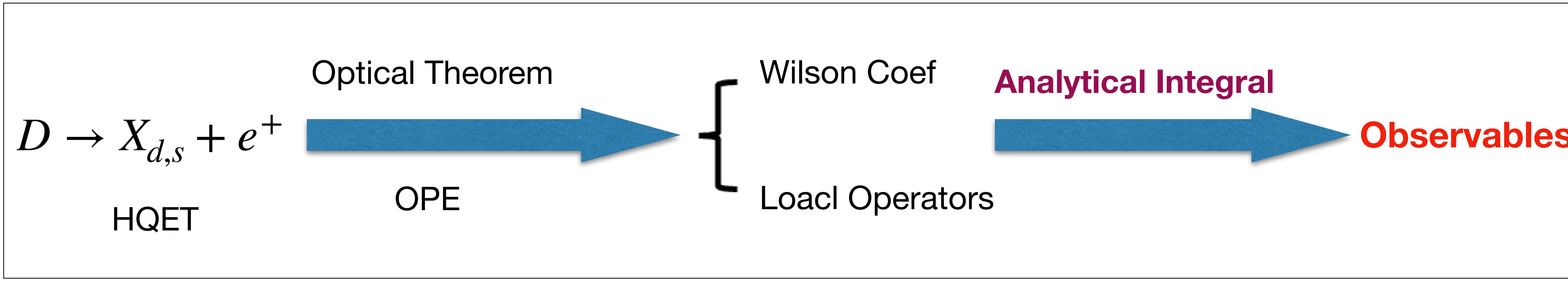
**Theory:**

**Experiment:**

**Phenomenological Analysis:**

# What did we do?

## Theory:




## Experiment:

## Phenomenological Analysis:

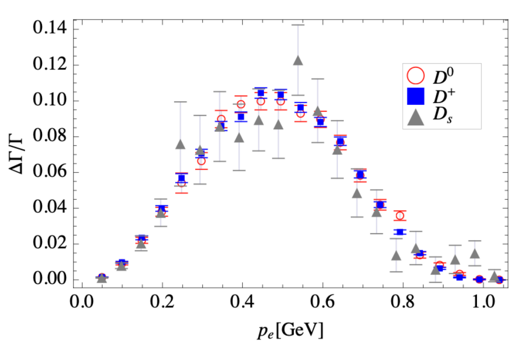
# What did we do?

## Theory:

$D \to X_{d,s} + e^+$ — Optical Theorem / OPE → { Wilson Coef, Loacl Operators } — **Analytical Integral** → **Observables**

HQET

## Experiment:




→ **Observables**

## Phenomenological Analysis:

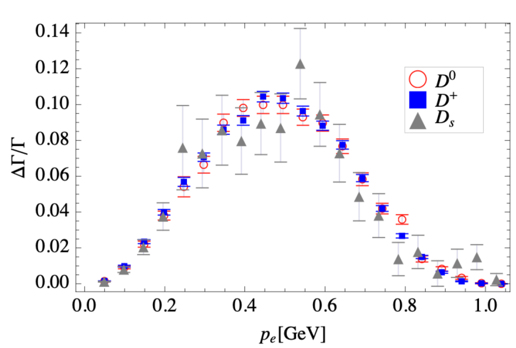
# What did we do?

## Theory:

$D \to X_{d,s} + e^+$ — Optical Theorem / OPE → $\left\{ \begin{array}{l} \text{Wilson Coef} \\ \text{Loacl Operators} \end{array} \right.$ — **Analytical Integral** → **Observables**

HQET

## Experiment:




→ **Observables**

## Phenomenological Analysis:

**Observables**

**Observables**

**Bayesian analysis** →

$\mu_\pi^2(D)$

$\mu_G^2(D)$

$m_c$

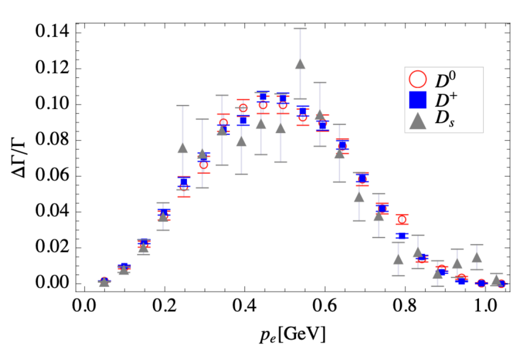
# What did we do?

## Theory:

$D \to X_{d,s} + e^{+}$ — Optical Theorem / OPE → { Wilson Coef, Loacl Operators } — **Analytical Integral** → **Observables**

HQET

## Experiment:




→ **Observables**

## Phenomenological Analysis:

**Observables**

**Observables**

— **Bayesian analysis** →

$\mu_\pi^2(D)$

$\mu_G^2(D)$

$m_c$

# Wishlist

- Precision measurements of leptonic energy spectrum **in the rest frame** of charmed hadrons

- $q^2$ spectrum, good for **higher-dimensional operators**

- Separate $X_d$, $X_s$, to give **first measurements** of $V_{cd}$, $V_{cs}$

- Rare decays: $D \to X_u \ell \ell$

**Thank you!**

**Ying-ao Tang（唐迎澳）, Ji-xin Yu（余纪新）， Yong-Zheng（郑勇）， Bo-nan zhang（张博楠）， Wen-jie Song（宋雯捷）， Yin-fa Shen（沈胤发）et, al**

# Appendix

### Summary: observable

$\Gamma = \dfrac{\dfrac{G_F^2 m_c^5 |V_{CKM}| \ldots}{192\pi^3}}{\text{Cons}}$

$\langle E_e \rangle = \dfrac{G_F^2 m_c^5 |V_{CKM}| \ldots}{192\pi^3 \Gamma}$

$\langle E_e^2 \rangle = \dfrac{G_F^2 m_c^5 |V_{CKM} \ldots}{192\pi^3 \Gamma}$

M1=M1
M2=M1+M2
M3=M1+M2+M3
M4=M1+M2+M3+M4

Cons= $2.28252 \times 10^{-14}\ mc^5$

## Prior Distribution for Free Parameters

$m_c \in$ [1, 2] GeV

$\mu_\pi^2 \in$ [-0.28, 1.08] $GeV^2$

$\mu_G^2 \in$ [0.04, 0.64] $GeV^2$

A: Daniel King, Alexander Lenz at al. “Revisiting inclusive decay widths of charmed mesons”, JHEP08(2022)241.

Fixed $\alpha_s(\bar{m}_c = 1.273GeV)$ = 0.378387

Prof Qin’s code：running from $\alpha_s(m_z = 91.1880GeV) = 0.1179$ at four loop level.

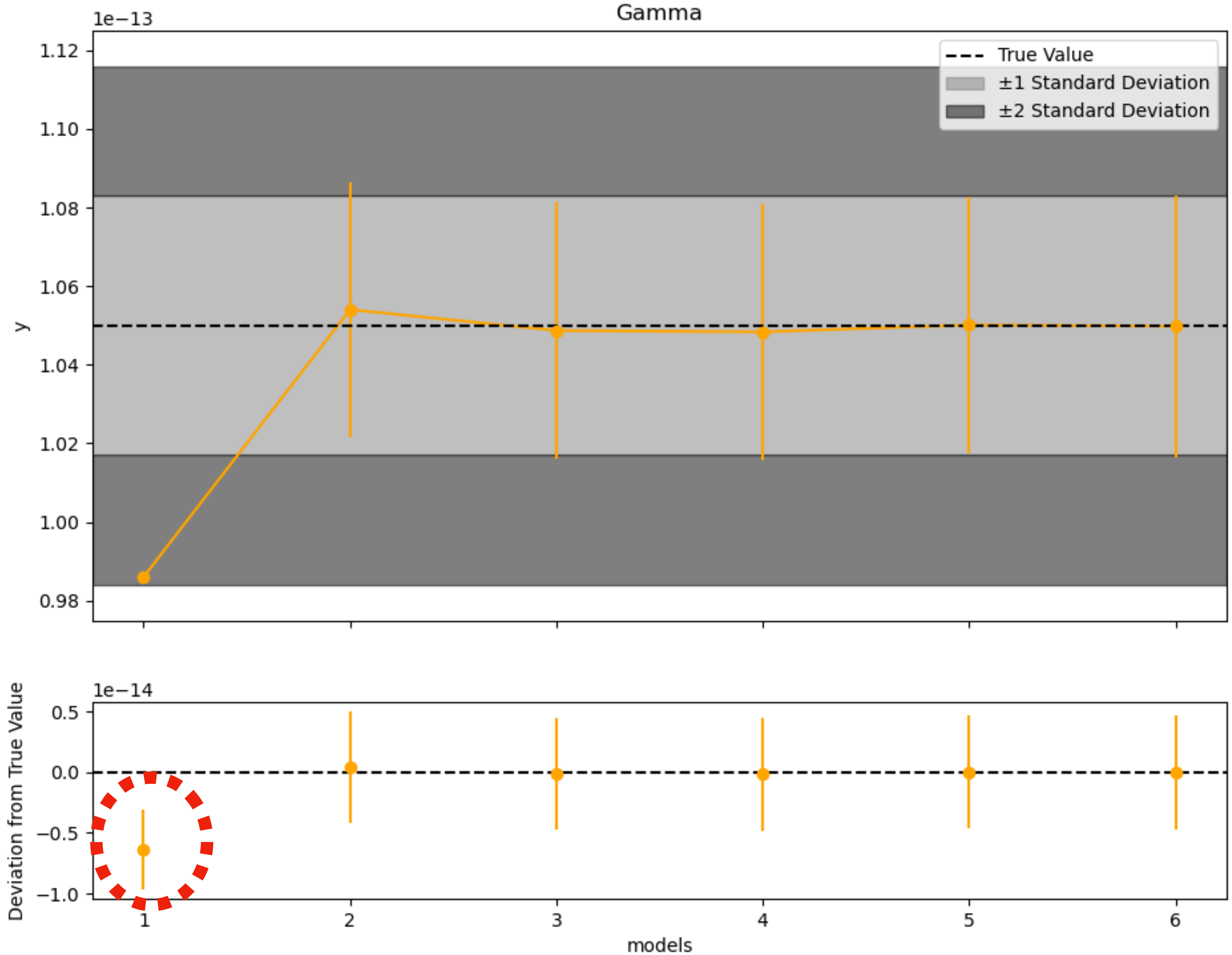
## Note!

| | |
|---|---|
| M1=M1 | Tree Level dim-3 operator |
| M2=M1+M2 | Tree Level : dim-3 operator+ $\mu_\pi^2$ |
| M3=M1+M2+M3 | Tree Level : dim-3 operator+ $\mu_\pi^2$+ $\mu_G^2$ |
| M4=M1+M2+M3+M4 | Tree Level : dim-3 operator+ $\mu_\pi^2$ + $\mu_G^2$ +dim-3 （NLO) |

# Phenomenological Analysis (preliminary)

**CompactObject**

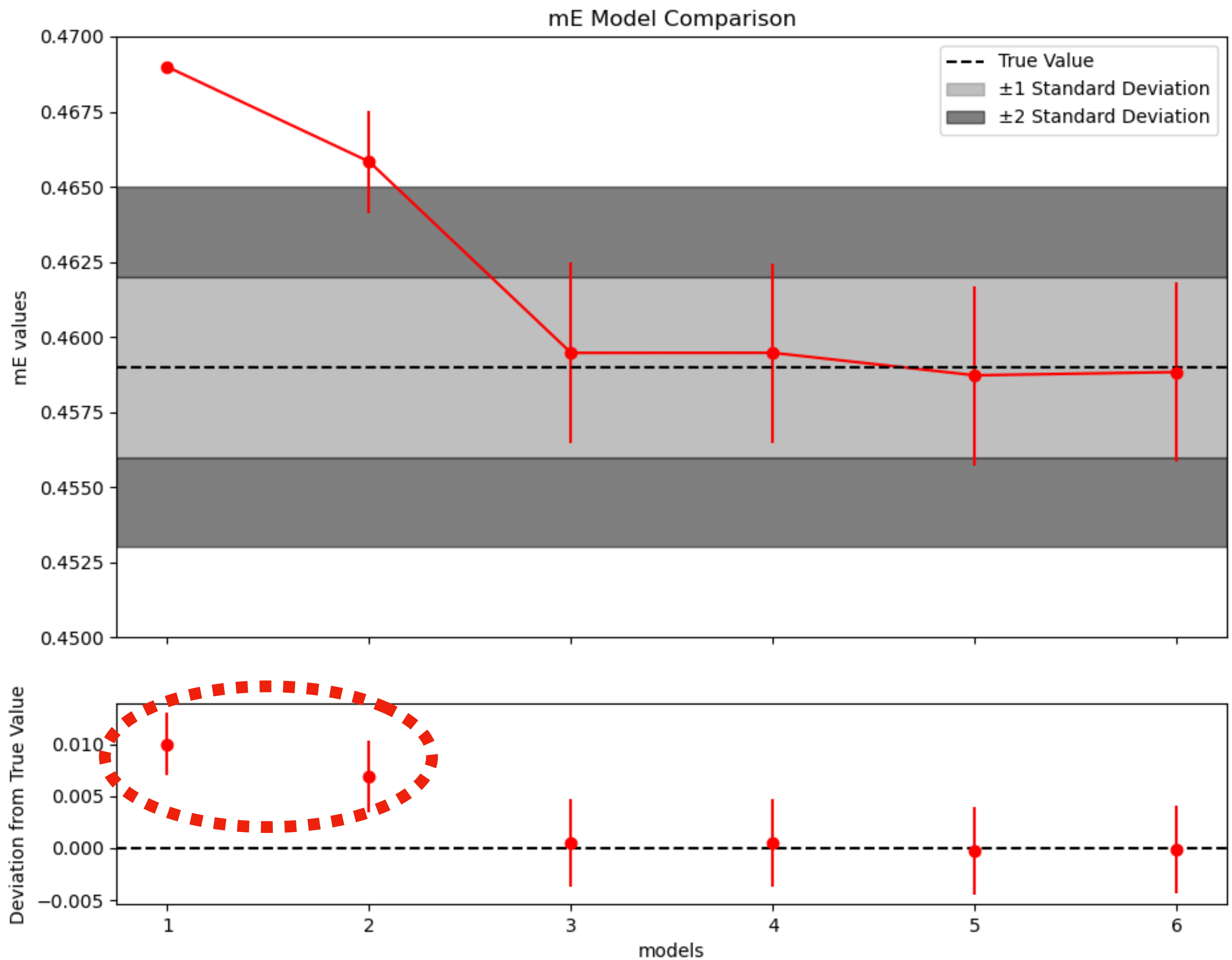
[Huang et al, '2024]



# Phenomenological Analysis (preliminary)

**CompactObject**

[Huang et al, '2024]



# Phenomenological Analysis (preliminary)

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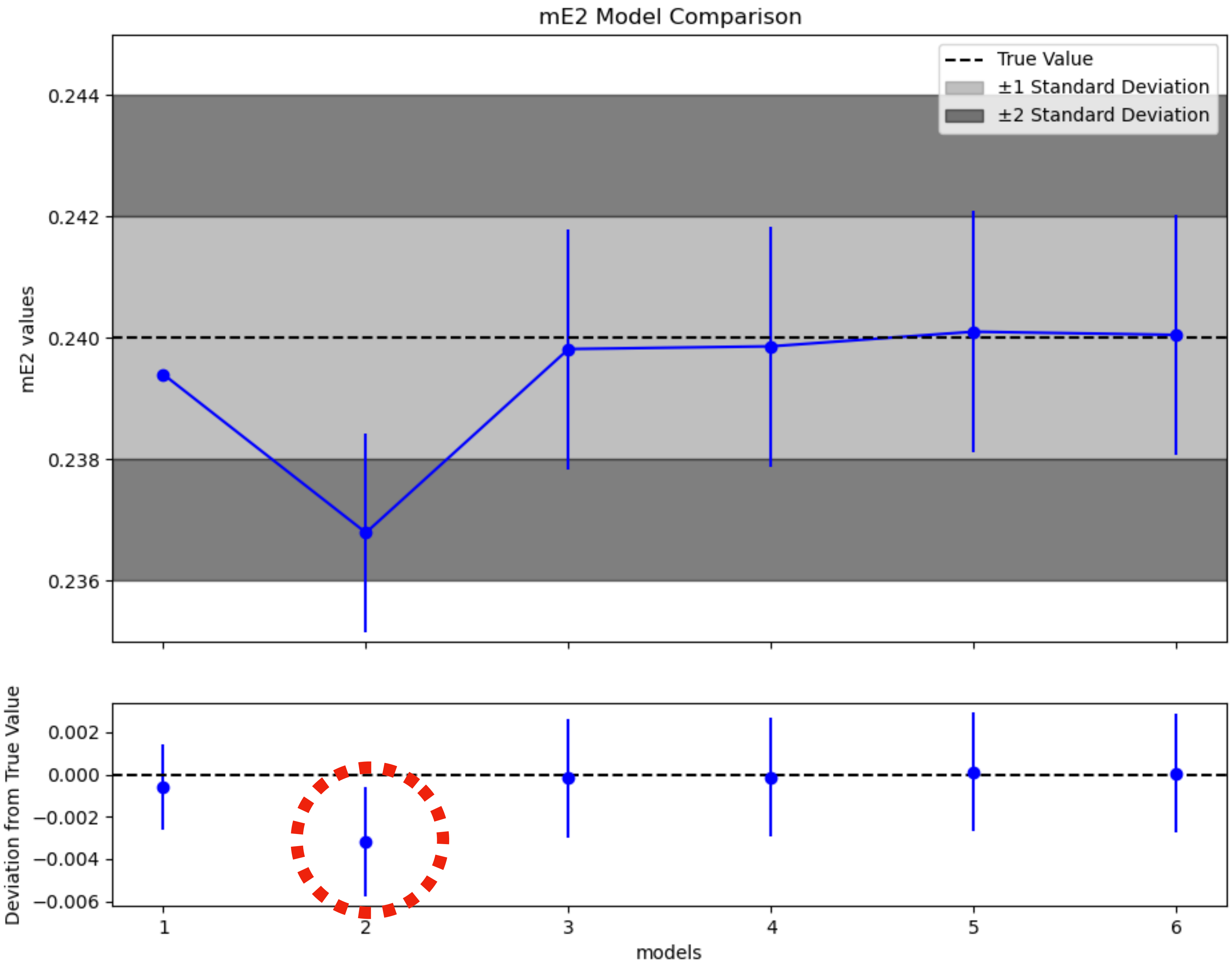
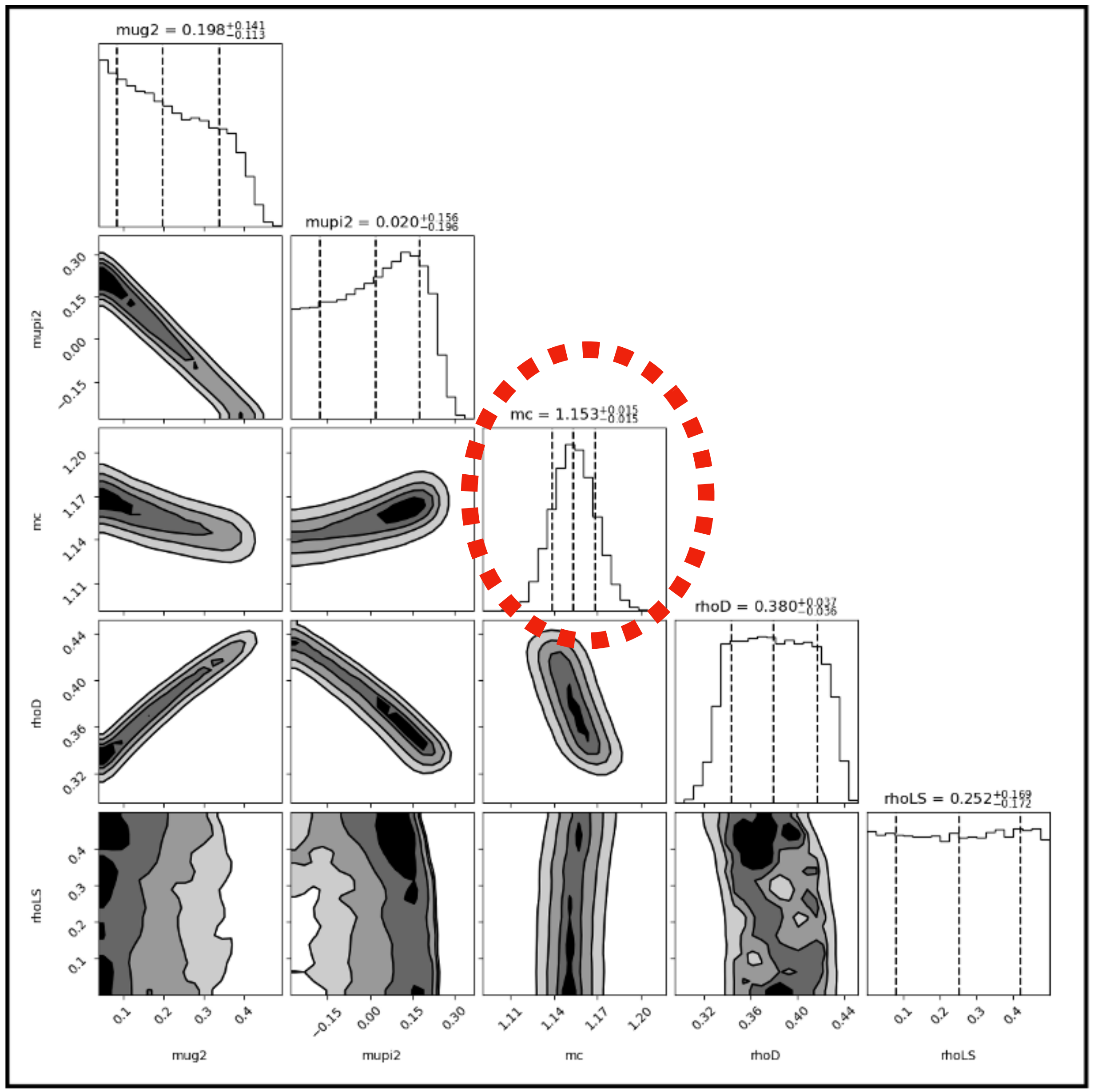
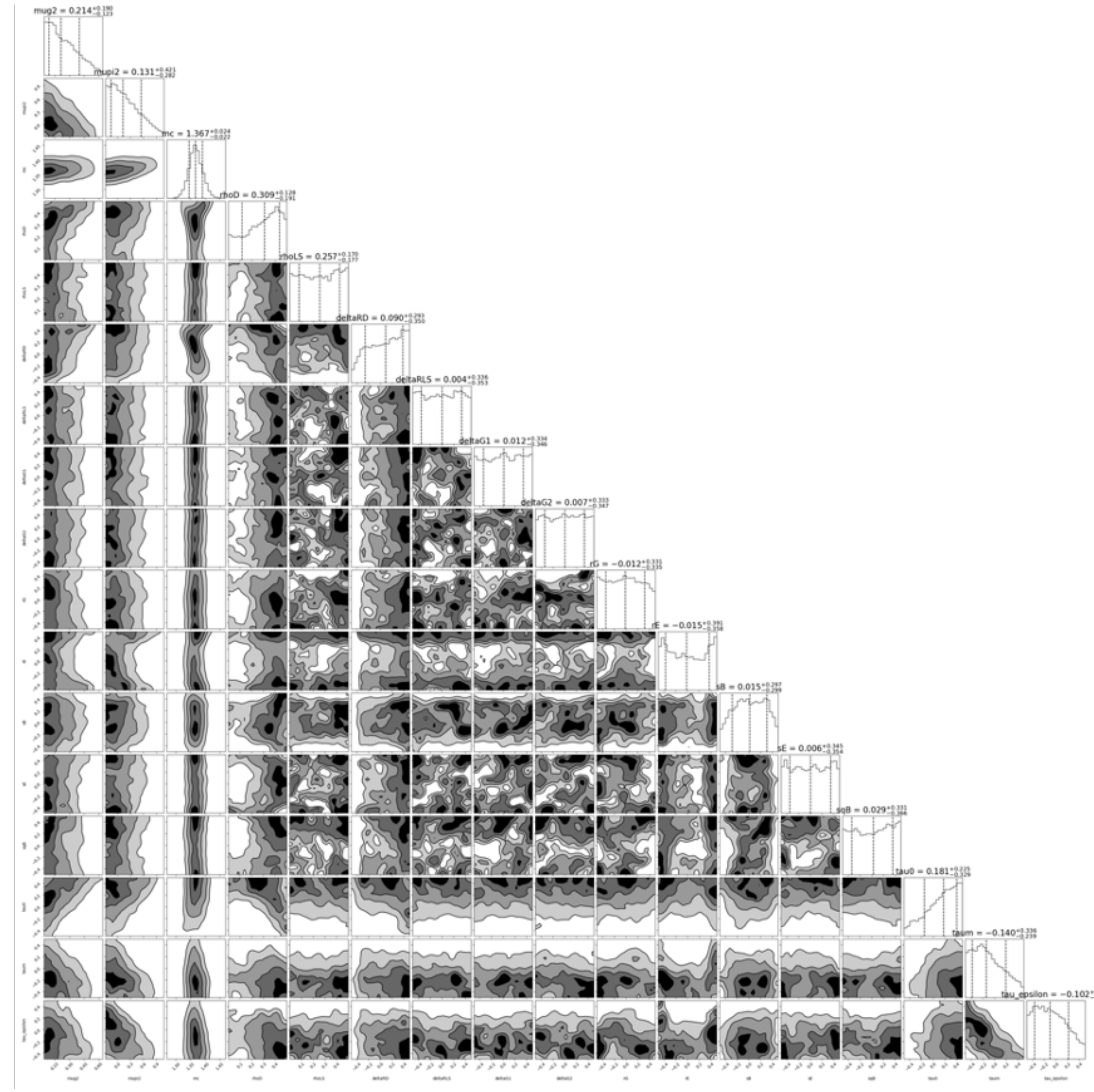
[Huang et al, ’2024]



# Appendix




[M5]



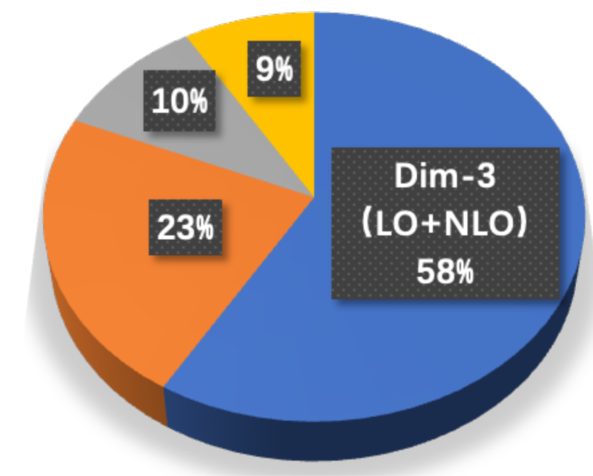
[M6]

# Appendix

**In the framework of the HQET,**

**SL：**

$$
\begin{aligned}
\Gamma_{sl}^{D^+} &= \Gamma_0 \Big[ \underbrace{1.02}_{c_3^{\rm LO}} + \underbrace{0.16}_{\Delta c_3^{\rm NLO}} - 0.27\,\frac{\mu_\pi^2(D)}{\rm GeV^2} - 0.84\,\frac{\mu_G^2(D)}{\rm GeV^2} + 2.48\,\frac{\rho_D^3(D)}{\rm GeV^3} + \underbrace{0.00}_{\rm dim-7,VIA} \\
&\quad - 0.28\,\tilde{B}_1^q + 0.28\,\tilde{B}_2^q - 0.09\,\tilde{\epsilon}_1^q + 0.09\,\tilde{\epsilon}_2^q - 5.24\,\tilde{\delta}_1^{sq} + 5.24\,\tilde{\delta}_2^{sq} \Big] \\
&= 1.02\,\Gamma_0 \Big[ 1 + 0.16 - 0.13\,\frac{\mu_\pi^2(D)}{0.48\,\rm GeV^2} - 0.28\,\frac{\mu_G^2(D)}{0.34\,\rm GeV^2} + 0.20\,\frac{\rho_D^3(D)}{0.082\,\rm GeV^3} \\
&\quad - \underbrace{0.00}_{\rm dim-6,7,VIA} - 0.005\,\frac{\delta\tilde{B}_1^q}{0.02} + 0.005\,\frac{\delta\tilde{B}_2^q}{0.02} + 0.004\,\frac{\tilde{\epsilon}_1^q}{-0.04} - 0.004\,\frac{\tilde{\epsilon}_2^q}{-0.04} \\
&\quad - 0.0118\,r_1^{sq} - 0.0088\,r_2^{sq} \Big], \qquad (4.13)
\end{aligned}
$$




Dim-3 operators contributions

Dim-5 operators contributions $\mu_G^2$ $\mu_\pi^2$

Dim-6 operators contributions

…

**SL+NL：**

$$
\begin{aligned}
\Gamma(D^+) &= \Gamma_0 \Big[ \underbrace{6.15}_{c_3^{\rm LO}} + \underbrace{2.95}_{\Delta c_3^{\rm NLO}} - 1.66\,\frac{\mu_\pi^2(D)}{\rm GeV^2} + 0.13\,\frac{\mu_G^2(D)}{\rm GeV^2} + 23.6\,\frac{\rho_D^3(D)}{\rm GeV^3} \\
&\quad - 16.9\,\tilde{B}_1^q + 0.56\,\tilde{B}_2^q + 84.0\,\tilde{\epsilon}_1^q - 1.34\,\tilde{\epsilon}_2^q + \underbrace{6.76}_{\rm dim-7} \\
&\quad - 0.06\,\tilde{\delta}_1^{qq} + 0.06\,\tilde{\delta}_2^{qq} - 16.8\,\tilde{\delta}_3^{qq} + 16.9\,\tilde{\delta}_4^{qq} - 29.3\,\tilde{\delta}_1^{sq} + 28.8\,\tilde{\delta}_2^{sq} + 0.56\,\tilde{\delta}_3^{sq} + 2.36\,\tilde{\delta}_4^{sq} \Big] \\
&= 6.15\,\Gamma_0 \Big[ 1 + 0.48 - 0.13\,\frac{\mu_\pi^2(D)}{0.48\,\rm GeV^2} + 0.01\,\frac{\mu_G^2(D)}{0.34\,\rm GeV^2} + 0.31\,\frac{\rho_D^3(D)}{0.082\,\rm GeV^3} \\
&\quad - \underbrace{2.66}_{\rm dim-6,VIA} - 0.055\,\frac{\delta\tilde{B}_1^q}{0.02} + 0.002\,\frac{\delta\tilde{B}_2^q}{0.02} - 0.546\,\frac{\tilde{\epsilon}_1^q}{-0.04} + 0.009\,\frac{\tilde{\epsilon}_2^q}{-0.04} + \underbrace{1.10}_{\rm dim-7,VIA} \\
&\quad - 0.0000\,r_1^{qq} - 0.0000\,r_2^{qq} + 0.0011\,r_3^{qq} + 0.0008\,r_4^{qq} \\
&\quad - 0.0109\,r_1^{sq} - 0.0080\,r_2^{sq} - 0.0000\,r_3^{sq} + 0.0001\,r_4^{sq} \Big], \qquad (4.6)
\end{aligned}
$$

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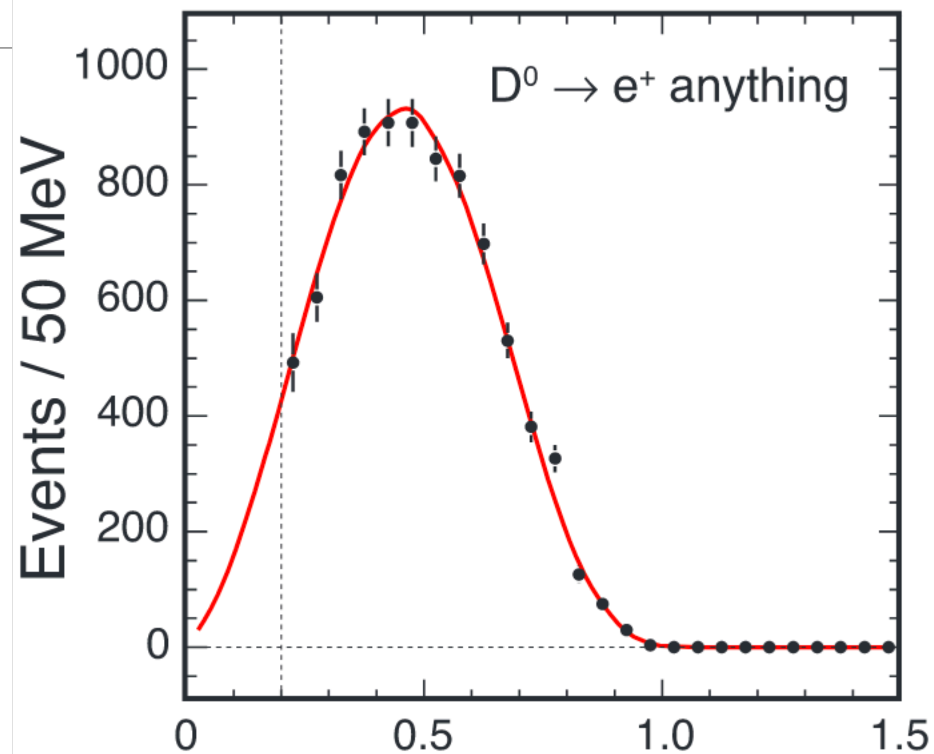
# Discussion about Energy Spectrum

| $p(\mathrm{GeV})$ | $\Delta B(D^0 \to Xe^+\nu_e)(\%)$ |
|---|---|
| 0.200 – 0.250 | $0.347 \pm 0.036$ |
| 0.250 – 0.300 | $0.426 \pm 0.030$ |
| 0.300 – 0.350 | $0.576 \pm 0.031$ |
| 0.350 – 0.400 | $0.629 \pm 0.030$ |
| 0.400 – 0.450 | $0.640 \pm 0.031$ |
| 0.450 – 0.500 | $0.640 \pm 0.031$ |
| 0.500 – 0.550 | $0.596 \pm 0.029$ |
| 0.550 – 0.600 | $0.575 \pm 0.029$ |
| 0.600 – 0.650 | $0.492 \pm 0.026$ |
| 0.650 – 0.700 | $0.374 \pm 0.023$ |
| 0.700 – 0.750 | $0.269 \pm 0.019$ |
| 0.750 – 0.800 | $0.230 \pm 0.017$ |
| 0.800 – 0.850 | $0.089 \pm 0.011$ |
| 0.850 – 0.900 | $0.053 \pm 0.008$ |
| 0.900 – 0.950 | $0.021 \pm 0.005$ |
| 0.950 – 1.000 | $0.002 \pm 0.002$ |
| 1.000 – 1.050 | ... |




$$\mathscr{B}\left(D^0 \to Xe^+\nu_e\right) = (6.46 \pm 0.09 \pm 0.11)\,\%$$

| Channel | $\mathcal{B}(\%)$ | Form factor | Comment |
|---|---|---|---|
| $D^0 \to K^{*-}e^+\nu_e$ | $2.16(17)[1]$ | SPOLE | $r_V = 1.62(8)$ and $r_2 = 0.83(5)[17]$ |
| $D^0 \to K^-e^+\nu_e$ | $3.50(5)[5]$ | BK | $\alpha_{\mathrm{BK}} = 0.30(3)[5]$ |
| $D^0 \to K_1^-e^+\nu_e$ | $0.11(11)$ | ISGW2 | $\mathcal{B}$ from Ref. [10] scaled by Ref. [5] |
| $D^0 \to K_2^{*-}e^+\nu_e$ | $0.11(11)$ | ISGW2 | $\mathcal{B}$ set to same as $D^0 \to K_1^-e^+\nu_e$ |
| $D^0 \to \bar{K}\pi e^+\nu_e$ | $0.12(3)[17,29]$ | PHSP | Nonresonant |
| $D^0 \to \pi^-e^+\nu_e$ | $0.288(9)[5]$ | BK | $\alpha_{\mathrm{BK}} = 0.21(7)[5]$ |
| $D^0 \to \rho^-e^+\nu_e$ | $0.16(2)[2]$ | SPOLE | $r_V = 1.4(3)$ and $r_2 = 0.6(2)[2]$ |

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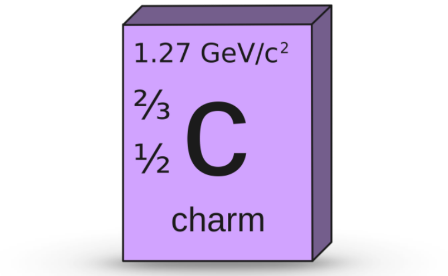


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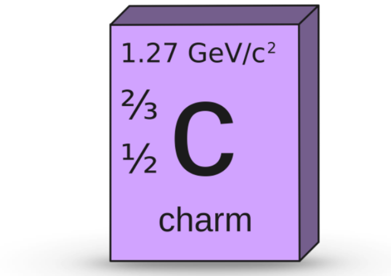


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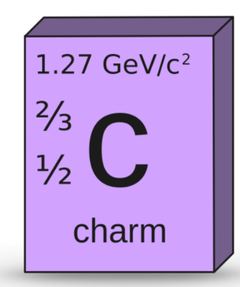


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