Semileptonic Processes in AdS / QCD Soft Wall Models

Alfredo Vega

The 21st National Symposium on Heavy Flavor Physics and CP Violation

Hengyang, China

October 27, 2024

Outline

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General Ideas about AdS / QCD Models

⋆ Basic ideas about AdS / QCD models.

- Gauge / Gravity can be used to study hadron properties
- There are two ways to extend AdS/CFT through to QCD:
  - Top-Down Approach.
    Starting from a string theory, in low energy limits, and compactifiying extra dimensions, try to get a QCD-like theory at the border.
  - Bottom-Up Approach.
    Hadron properties are used to build a model in a gravity frame with extra dimensions.
- Exists a dictionary that relates quantities at both sides of the holographic correspondence.
- In Bottom-Up approach we consider

$$S = \int d^{d+1}x \sqrt{g}\, e^{-\Phi(z)} \left(\mathcal{L}_{Part} + \mathcal{L}_{Int} \right)$$

$$S = \int d^{d+1}x\sqrt{g}e^{-\Phi(z)}\mathcal{L}_{Part} + \int d^{d+1}x\sqrt{g}e^{-\Phi(z)}\mathcal{L}_{Int}$$

Example: Scalar hadrons case [1]

$$\mathcal{L} = g^{MN}\,\partial_M\psi\left(x,z\right)\,\partial_N\psi\left(x,z\right) + m_5^2\psi^2\left(x,z\right),$$

with $M, N = 0, 1, 2, 3, z$.

- z correspond to the holographic coordinate.
- $\phi\left(z\right)$ is a dilaton field introduced to discretize spectrum (you can use a hard cut-off also).
- m_5 is the mass in bulk. It is related to the dimension of operators that create hadrons. For scalars $m_5^2 R^2 = \Delta\left(\Delta - 4\right)$

Δ_0	(nQ)(mG)
3	(2Q)
4	(2G)

[1] e.g., see A. V and I. Schmidt Phys.Rev.D 78 (2008) 017703

$$S = \int d^{d+1}x\sqrt{g}e^{-\Phi(z)}\mathcal{L}_{Part} + \int d^{d+1}x\sqrt{g}e^{-\Phi(z)}\mathcal{L}_{Int}$$

Example: Scalar hadrons case

$$\mathcal{L} = g^{MN}\,\partial_M\Psi\,(x,z)\;\partial_N\Psi\,(x,z) + m_5^2\Psi^2\,(x,z)\,,$$

A usual choice for hadrons in vacuum at zero temperature is

$$d^2s = e^{2A(z)}\,(z)\,\eta_{MN}dx^M dx^N,$$

where $e^{2A(z)}$ is a warp factor and $\eta_{MN} = diag(-1,1,1,1,1)$.
From this action we obtain an equation of motions in 5 dimensions for scalars, and we use the transformation $\Psi\,(X,z) = e^{-iPX}f\,(z)$, where P and X correspond to momentum and position in a 4D boundary.

$$-f''\,(z) + B'(z)f'\,(z) + e^{2A(z)}m_5^2 f\,(z) = M^2 f\,(z)\,,$$

where $B(z) = \phi(z) - 3A(z)$ and $P^2 = M^2$, i.e., M is the mass of the hadron studied in these kinds of models.

Considering the transformation

$$f(z) = e^{\frac{1}{2}B(z)}\psi(z),$$

we obtain a Schrödinger like equation

$$\left(-\partial_z^2 + V(z)\right)\psi(z) = M^2\psi(z).$$

In terms of AdS metric ($A(z) = Ln(1/z)$), $\phi(z)$ and m_5, potential is

$$V(z) = \frac{15}{4z^2} + \frac{m_5^2 R^2}{z^2} - \frac{3}{2z}\phi'(z) + \frac{1}{4}\phi'^2(z) - \frac{1}{2}\phi''(z).$$

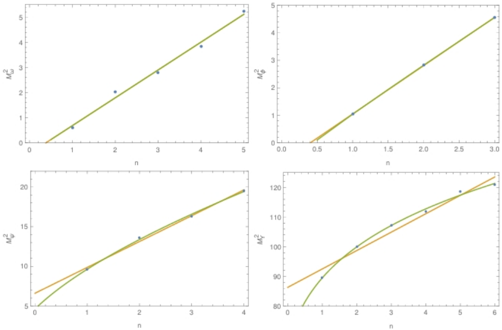
Examples:

- $\phi(z) = cte$ with a hard cutoff at z_0. (Hard Wall model)
- $\phi(z) = cz^2$ (Traditional Soft Wall Model)

$$V(z) = \frac{15}{4z^2} + \frac{m_5^2 R^2}{z^2} + c^2 z^2 + 2c.$$

Warning: Vectors [2] appear on this slide but the same happens with scalars.

	Linear Regge Trajectory: $M^2 = a(n+b)$			Non Linear Regge Trajectory ($M^2 = a(n+b)^\nu$)			
Meson	**a**	**b**	R^2	**a**	**b**	ν	R^2
ω	1.1074	−0.3781	0.9978	1.1078	−0.3784	0.9998	0.9978
ϕ	1.7595	−0.4048	0.9999	1.8545	−0.4524	0.9617	1.000
ψ	3.2607	2.0259	0.9997	7.6516	0.4460	0.6249	0.9999
Υ	6.2015	13.9182	0.9996	85.3116	0.2849	0.1917	0.9999



[2] M. A. Martín and A. V, Phys.Rev.D 102 (2020) 4, 046007

Example: Gauge boson fields [3]

For a five dimensional gauge boson field, we have:

$$S = \int d^5x \sqrt{-g}\ \frac{1}{4} V_{mn} V^{mn} \,,$$

where the electromagnetic tensor V^{mn} is written as usual as $V_{mn} = \partial_m V_n - \partial_n V_m$.

$$V(x,z)_\mu = \epsilon_\mu e^{-iQ\cdot x} J(Q^2, z) \text{ and } V_z = 0$$

with

$$\mathrm{J}(\mathrm{q}^2, z) = \Gamma\left(1 - \frac{q^2}{4\kappa^2}\right) U\left(-\frac{q^2}{4\kappa^2}, 0, z^2\kappa^2\right).$$

For quadratic dilaton case.

[3] e.g., see T. Gutsche, V. Lyubovitskij, A. V and I. Schmidt.

Electromagnetic Form Factors in AdS / QCD models

$$S = \int d^{d+1}x\sqrt{g}e^{-\Phi(z)}\mathcal{L}_{Part} + \int d^{d+1}x\sqrt{g}e^{-\Phi(z)}\mathcal{L}_{Int}$$

The matrix element for the spin-S current and the spin-J hadrons is given by

$$\langle b|J_{\mu_1\mu_2\ldots\mu_s}|a\rangle = (\text{charge})(\text{kinematic factor})F_{ab}(Q^2),$$

where $F_{ab}(Q^2)$ is the form factor.

Example: For scalar mesons, the hadronic matrix element for the electromagnetic current

$$\langle M(P')|J_\mu^{EM}|M(P)\rangle = (P+P')_\mu f_+(q^2) + (P-P')_\mu f_-(q^2).$$

By current conservation $f_-(q^2) = 0$

$$\langle M(P')|J_\mu^{EM}|M(P)\rangle = (P+P')_\mu F_\pi(q^2).$$

Considering an interaction term in AdS/QCD side as [4]

$$S_{int} = ig_5 \int d^4x dz \sqrt{g} e^{-\phi(z)} V^l(x,z) \Psi_{p'}(x,z) \overleftrightarrow{\partial}_l \Psi_p(x,z).$$

where g_5 is an effective coupling constant and $A \overleftrightarrow{\partial} B = A(\partial B) + (\partial A)B$.

$\Psi_P(x,z) \sim e^{P\cdot x}\Psi_P(z)$, where x and P is position and momentum on the Minkowski part and $V_\mu(x,z) \sim \epsilon_\mu e^{q\cdot x} J(q^2,z)$, $A_z(z) = 0$.
Then form factors related to scalar hadrons in AdS is

$$F(Q^2) = \int_0^\infty dz\, e^{3A(z)-\phi(z)} \Psi(z) J(Q^2,z) \Psi(z),$$

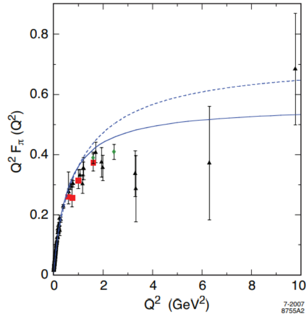
where $\Psi(z)$ and $J(Q^2,z)$ are the AdS modes dual to scalar hadrons and photons. These modes are the solutions of the bulk EOM in the Sturm Liouville form associated with each bulk field.

[4] e.g, see S. Brodsky and G. de Teramond, Phys.Rev.D 77 (2008) 056007

The latter form factor can be transformed into the following expression

$$F(Q^2) = \int_0^\infty dz\, \psi(z) J(Q^2, z) \psi(z)\,,$$

where $\psi(z)$ are solutions of the EOM transformed in Schrödinger-like form, while $J(Q^2, z)$ remains as the same solution used before.



Semileptonic Processes in AdS / QCD models

Considering the decay $M_1(P) \to M_2(P') + l(k_1) + \bar{\nu}_l(k_2)$, the current of interest for us can be decomposed as follows

$$\langle M_2(P')|J_\mu(0)|M_1(P)\rangle = (P+P')_\mu f_+(q^2) + (P-P')_\mu f_-(q^2).$$

where q^2 varies within the range $m_l^2 \leq q^2 \leq (M_1^2 - M_2^2)^2 = q^2_{max}$.

From these form factors it is possible to calculate branching ratios,

$$\frac{\mathcal{B}(H_1 \to H_2 l \bar{\nu}_l)}{\tau(H_1)} = \int_0^{(m_1-m_2)^2} \frac{d\Gamma(H_1 \to H_2 l \bar{\nu}_l)}{dq^2} dq^2.$$

In AdS side these form factors can be written as

$$f_\pm(q^2) = g_\pm \int_0^\infty dz \; J(q^2, z)\psi_{P'}(z)\psi_P(z).$$

$$\langle M_2(P')|J^\mu(0)|M_1(P)\rangle = (P+P')^\mu f_+(q^2) + (P-P')^\mu f_-(q^2).$$

In AdS side these form factors can be written as

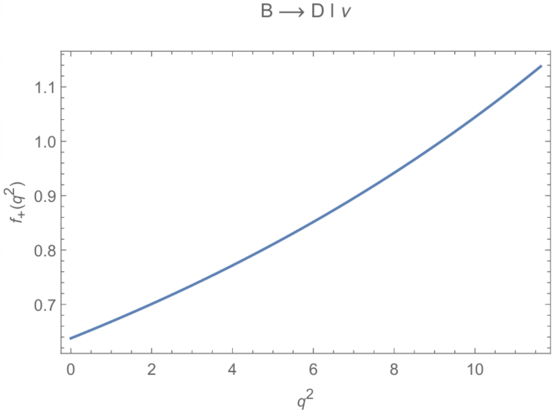
$$f_\pm(q^2) = g_\pm \int_0^\infty dz\ J(q^2,z)\psi_{P'}(z)\psi_P(z).$$

Some processes of this kind are,

- $(V_{ud})\ \pi^- \rightarrow \pi^0 e^- \bar{\nu}_e$
- $(V_{us})\ K^- \rightarrow \pi^0 l^- \bar{\nu}_l$
- $(V_{cd})\ D^- \rightarrow \pi^0 l^- \bar{\nu}_l$
- $(V_{cs})\ D^- \rightarrow K^0 l^- \bar{\nu}_l$
- $(V_{ub})\ B^- \rightarrow \pi^0 l^- \bar{\nu}_l$
- $(V_{cb})\ B^- \rightarrow D^0 l^- \bar{\nu}_l$

An holographic approach to Isgur - Wise function

$$\xi_{AdS}(v \cdot v') = g_{\pm} \frac{2\sqrt{M_1 M_2}}{M_1 \pm M_2} \int_0^{\infty} dz J(q^2, z) \psi_{P'}(z) \psi_P(z).$$



Final Comments

- In the context of holographic dualities it is possible to build phenomenological models to study hadron properties.
- These models in the past have been used extensively to study several properties, as electromagnetic form factors, but a small amount of paper explores the uses of holography in the electroweak sector.
- Applicability of AdS / QCD models to study electroweak processes, as semileptonic decays is possible and by now there is a small amount of work in this direction.


That's all Folks!