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Heavy quark mass dependence of heavy meson LCDA in QCD

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Preliminary results

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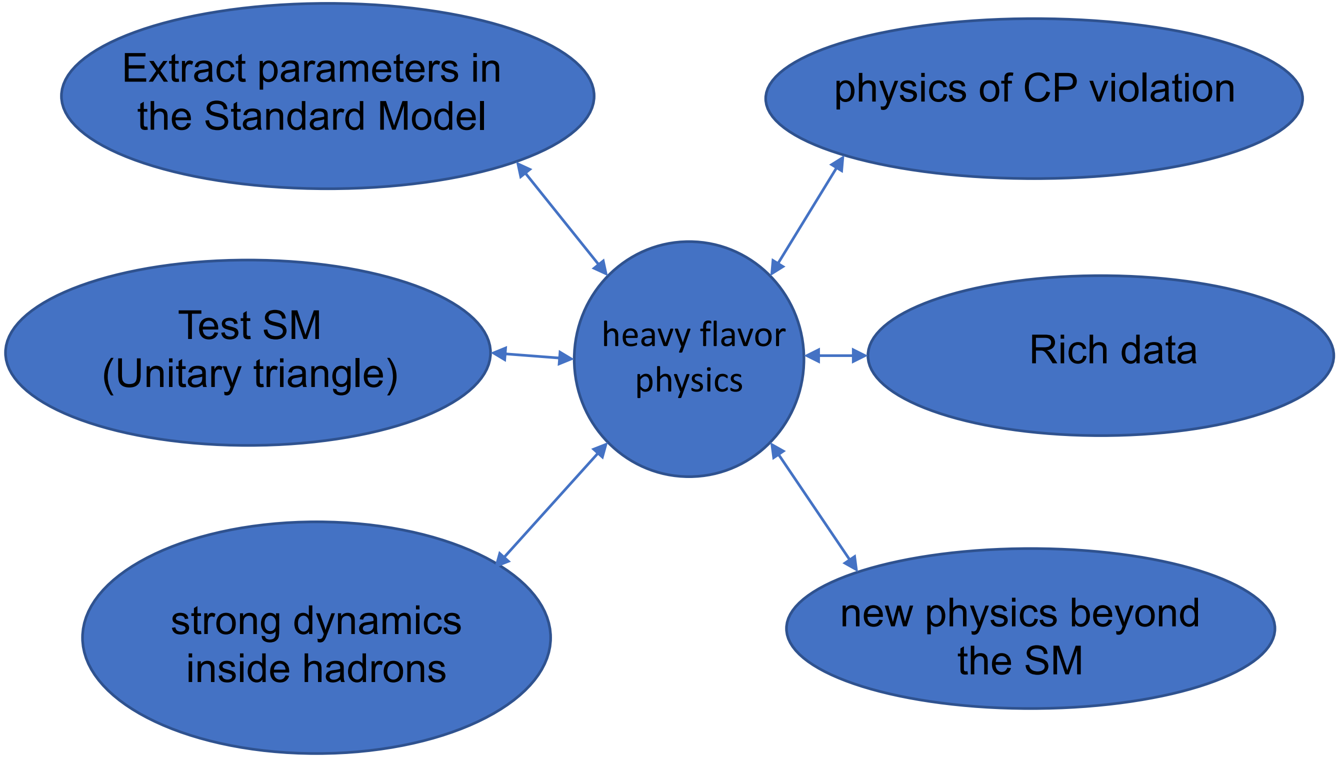
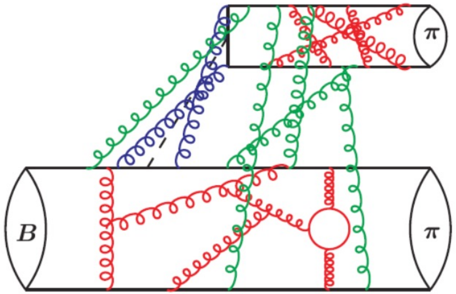
The beauty of heavy flavor physics



Multiscale Problem in heavy flavor physics



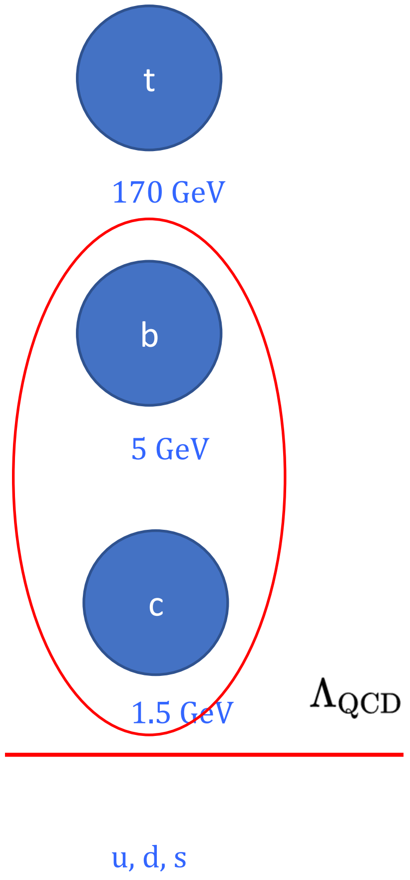
- ✓ m_W: 90GeV
- ✓ m_b: 5GeV
- ✓ $E_\pi \sim \sqrt{\Lambda * m_b}$: 2GeV
- ✓ Λ: 0.5GeV

- Separate scales
- Effective theories and factorization theorems

HQET, NRQCD, pNRQCD, etc.

- Short distance effects: calculated with perturbation theory
- Long distance effects: universal, number reduced with approximate symmetries.

Heavy quark meson



- Heavy quark: $m_Q \gg \Lambda_{\mathrm{QCD}}$
- There exist hard scales.

Asymptotic freedom, perturbation theory can be used

- Heavy-light meson

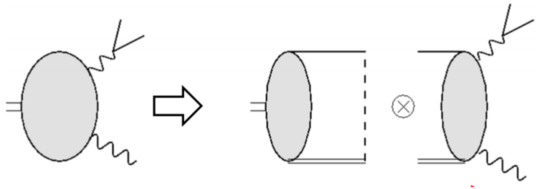
To resolve heavy quark quantum number hard probe is needed. $\sim 1/m_Q$

Gluon exchange between heavy and light quark is $\sim 1/\Lambda_{\mathrm{QCD}} \gg 1/m_Q$

Light degree of freedom is blind to heavy quark flavor, spin, …

Factorization theorems in B exclusive decay

- B-meson exclusive decay provides important information for understanding the CP violation
- LCDA plays dominate role in factorization theorem for B-meson exclusive decay/QCD (SCET) sum rules.
- Example: B meson radiative decay $B \to \gamma l \nu$



See the talks by X.Q. Li and D.S. Yang

$$Amp \propto \int_0^\infty \frac{d\omega}{\omega} T(\omega, m_b, \mu; \alpha_s) \phi_+^B(\omega, \mu)$$

B-meson LCDA

- The light cone HQET matrix element Grozin, Neubert, 1997

$$\langle 0|\bar{q}_\beta(z)\,[z,0]\,h_{v\alpha}(0)|\bar{B}(v)\rangle = -\frac{i\tilde{f}_B m_B}{4}\left[\frac{1+\not{v}}{2}\left\{2\,\tilde{\phi}_B^+(t,\mu)+\frac{\tilde{\phi}_B^-(t,\mu)-\tilde{\phi}_B^+(t,\mu)}{t}\,\not{z}\right\}\gamma_5\right]_{\alpha\beta}$$

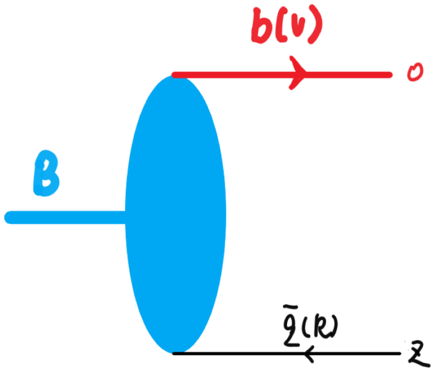
$t \equiv z \cdot v$

- h_v: heavy quark field in HQET

v: velocity of B meson

$\tilde{f}_B$ decay constant of B meson

[z,0]: gauge link along light cone direction



- LCDA describes the light quark momentum distribution in B meson

B meson LCDA on the lattice

Positive moments do not exist. No local OPE

Braun, Ivanov, Korchemsky, 2003

Inverse (logarithmic) moments are not defined with local operators. Need full LCDA

No light-cone separation on the Euclidean lattice

↓

Lattice quasi/pseudo-distributions in HQET

Ji 2013; Wang, Wang, Xu, SZ 2019; Xu, Zhang, SZ 2022; Xu&Zhang 2022; Hu et al 2023,2024; Radyushkin 2017; SZ&Radyushkin 2020;

Large noise-to-signal ratio in lattice HQET

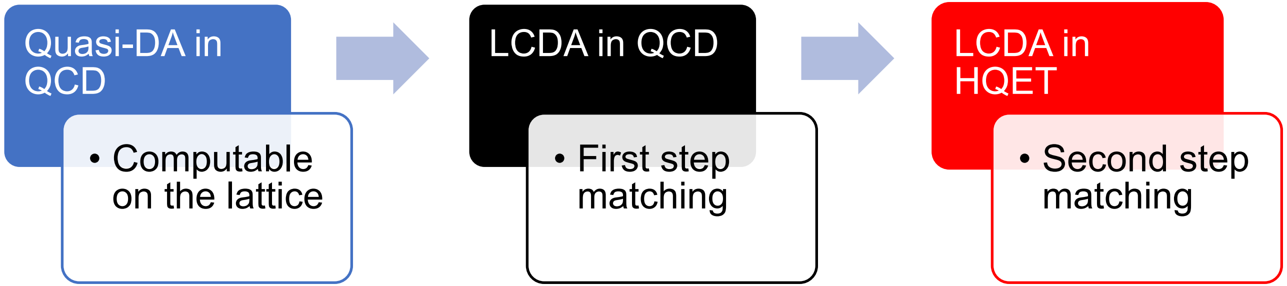
↓

quasi/pseudo/LC distributions in QCD

B-meson QCD DAs

- New method to calculate B-meson (HQET) LCDA on the lattice

$$P^z \gg m_Q \gg \Lambda_{QCD}$$



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See the talks by J. Xu, Q. A. Zhang, J. L. Zhang, J. Zeng

- Resummation of logarithms in $W \to B\,\gamma$

See the talk by Y. B. Wei

B-meson QCD LCDA

- Definition:

$$\phi(u, m_Q; \mu) = -if_H \int \frac{dz^-}{2\pi} e^{iuP^+z^-} \langle 0|\bar{q}(z)[z,0]\not{n}\gamma_5 Q(0)|\bar{H}(P_H)\rangle$$

μ: renormalization scale. m_Q heavy quark mass

$$\mu \geq m_Q \gg \Lambda_{QCD}$$

- Scale evolution: Efremov-Radyushkin-Brodsky-Lepage (ERBL)

Relates LCDAs defined at two different scales

- What about m_Q?

-Given the D meson LCDA, what can be inferred about the B meson QCD LCDA?

-Can one write down an evolution equation for mass dependence?

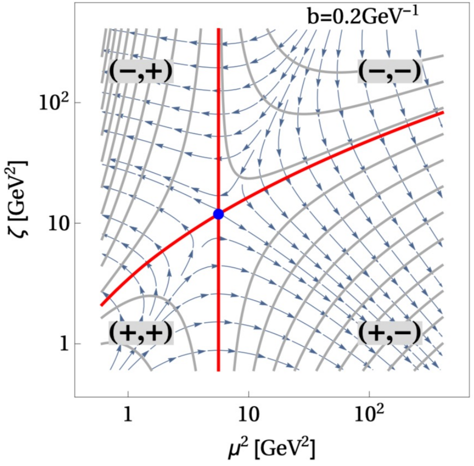
Multiscale evolutions

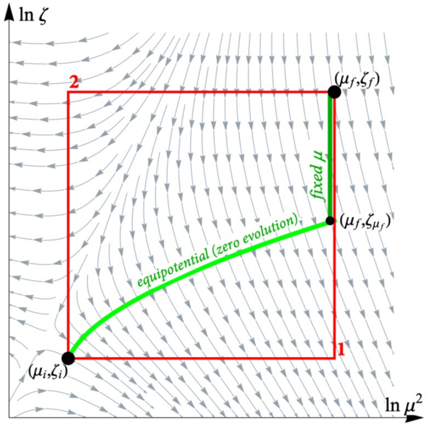
- Multiscale problems are common in QFT

- Example 1: Transverse Momentum Dependent (TMD) distributions

  -Scale evolution: governed by RGE

  -Rapidity evolution: Collins-Soper equation





Scimemi&Vladimirov
1803.11089

Multiscale evolutions

- Example 2: Two scale MS renormalization and evolution

Einhorn&Jones, 1983 Ford&Wiesendanger, 1996

Resum the logarithms of multiscale

Effective potential of Higgs-Yukawa theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{24}\phi^4 + \bar{\psi} i \partial\!\!\!/ \psi + g\bar{\psi}\phi\psi + \Lambda,$$

$$\begin{aligned} V^{\text{tree}}(\phi) &= \frac{\lambda}{24}\phi^4 + \frac{1}{2}m^2\phi^2 - \Lambda, \\ V^{\text{1-loop}}(\phi) &= \frac{(m^2 + \frac{1}{2}\lambda\phi^2)^2}{4(4\pi)^2}\left[\log\boxed{\frac{m^2 + \frac{1}{2}\lambda\phi^2}{\mu^2}} - \frac{3}{2}\right] - \frac{Ng^4\phi^4}{(4\pi)^2}\left[\log\boxed{\frac{g^2\phi^2}{\mu^2}} - \frac{3}{2}\right]. \end{aligned}$$

Multiscale evolutions

- Example 3: Momentum RGE and quasi distributions   Ji, 2014

Ji, Liu, Liu, Zhang, Zhao 2021

Normal RGE: evolution with renormalization scale

Momentum RGE: $P \gg \Lambda_{QCD}$

In the large-momentum limit, because of asymptotic freedom, the P-dependence is calculable in perturbation theory, and Eq. (27) simplifies. One obtains the momentum or boost RGE (Ji, 2014),

$$\frac{dO(P)}{dP} = \lim_{\Delta P \to 0} [O(P+\Delta P) - O(P)]/\Delta P \qquad (29)$$

$$\xrightarrow{P \gg M} C(\alpha_s(P)) \otimes O(P) + \mathcal{O}(M^2/P^2) \; . \qquad (30)$$

Evolution of QCD LCDA as a two-scale problem

- A mass RGE for QCD LCDA? (just like momentum RGE for quasidistributions)

Unrenormalized HQET LCDA
UV divergent

MS bar: All the UVs are renormalized.

Lange-Neubert (LN) evolution with $\overline{MS}$ scale

Finite quark mass as a cutoff: Not all UVs are regularized.
A RGE associated

Add additional cutoff, e.g., DR
RGE: ERBL

LN → ERBL + mass RGE (?)

A preliminary test: deriving mass RGE with HQET

- General matching formula $\qquad m_Q \gg \Lambda_{QCD}$

$$\phi(u, m_Q; \mu) = \int d\omega C(u, \omega, m_Q; \mu, \mu_F)\varphi_+(\omega, \mu_F),$$

Ishaq, Jia, Xiong, Yang, 2019; SZ, 2019;
Beneke, Finauri, Keri Vos, Wei 2023

- $\mu = \mu_F$: $\quad \phi(u, m_Q; \mu) = \mathcal{J}(m_Q, \mu) m_Q \varphi_+(u m_Q, \mu).$ Multiplicative

Holds when $\Lambda_{QCD} \sim u m_Q \ll m_Q$, "peak region" Beneke, Finauri, Keri Vos, Wei 2023

- Mass RGE (derived from the m_Q independence of φ_+)

$$m_Q \frac{\partial}{\partial m_Q}\phi(u, m_Q; \mu) - u\frac{\partial}{\partial u}\phi(u, m_Q; \mu) - (1 + \gamma(m_Q, \mu))\phi(u, m_Q; \mu) = 0,$$

$$\gamma(m_Q, \mu) \equiv \frac{d \ln \mathcal{J}(m_Q, \mu)}{d \ln m_Q}.$$

Function $\gamma(m_Q, \mu)$

- Matching coefficient

$$\mathcal{J}(m_Q,\mu) = \exp\left[\int_{m_Q}^{\mu} \frac{d\mu'}{\mu'} \frac{\alpha_s(\mu')C_F}{\pi} \left(\ln\frac{\mu'}{m_Q} + 1\right)\right] \times \mathcal{J}(m_Q, m_Q)$$

$$\mathcal{J}(m_Q,m_Q) = 1 + \frac{\alpha_s(m_Q)}{4\pi} C_F \left(4 + \frac{\pi^2}{12}\right) + \mathcal{O}(\alpha_s^2),$$ Beneke, Finauri, Keri Vos, Wei 2023

- Evolution function ("anomalous dimension")

$$\gamma(m_Q,\mu) \approx \frac{\alpha_s(m_Q)C_F}{\pi} - \frac{2C_F}{\beta_0} \ln\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}.$$

- The equation can be solved with standard methods

  e.g., method of characteristics

Solve the equation

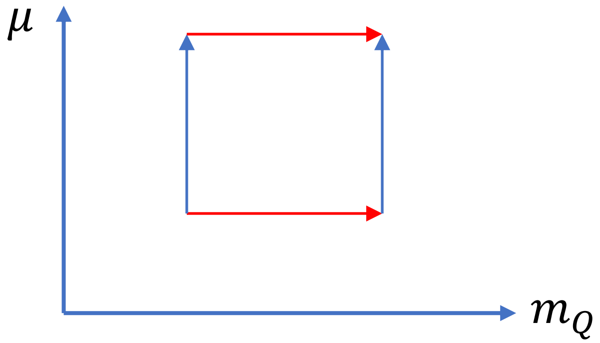
- Given the initial condition

$$\phi(u, m_{Q_0}, \mu) \equiv \phi_0(u)$$

- The solution of the equation

$$\phi(u, m_Q; \mu) = \exp\left[-\int_{m_{Q_0}}^{m_Q} \frac{dm'}{m'}\gamma(m', \mu)\right] \times \frac{m_Q}{m_{Q_0}}\phi_0\left(u\frac{m_Q}{m_{Q_0}}\right).$$

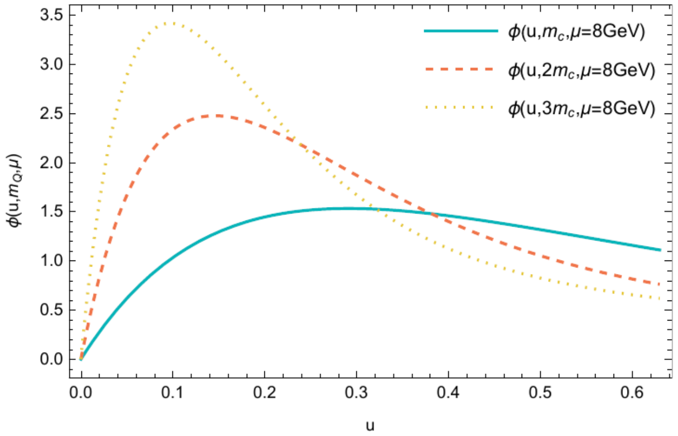
- Related QCD LCDAs of heavy mesons with heavy quark mass m_{Q_0} and m_Q, with μ fixed



Applications

- Effect of the m_Q evolution equation

  -Higher peak
  -Narrower shape
  -Smaller u



- Relations between QCD LCDAs

$$\frac{\phi_B(u,\mu)}{\phi_D\left(u\frac{m_b}{m_c},\mu\right)} \approx \frac{m_b}{m_c}\exp\left[\frac{2C_F}{\beta_0}\ln\frac{\alpha_s(m_b)}{\alpha_s(m_c)} - \frac{4\pi C_F}{\beta_0^2}\left(\frac{1}{\alpha_s(m_c)}\ln\frac{\alpha_s(\mu)}{\alpha_s(m_c)e} - \frac{1}{\alpha_s(m_b)}\ln\frac{\alpha_s(\mu)}{\alpha_s(m_b)e}\right)\right]$$

As a comparison: $$\frac{f_B\sqrt{m_B}}{f_D\sqrt{m_D}} = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}$$ Manohar&Wise, Heavy Quark Physics

- Can be checked with lattice QCD calculations, e.g., lattice simulations+LaMET

Summary

- The heavy meson QCD LCDA plays an important role in lattice QCD calculation of HQET LCDAs and heavy meson exclusive productions

- The evolution of heavy meson QCD LCDA can be treated as a two-scale problem

- A differential equation for heavy quark dependence is derived, aiming at relating QCD LCDAs of different heavy mesons

- Could be checked and applied to lattice simulations

Thank you!