QCD LCDA of Heavy Mesons from bHQET

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Deng, Wang, **YBW** and Zeng: 2409.00632

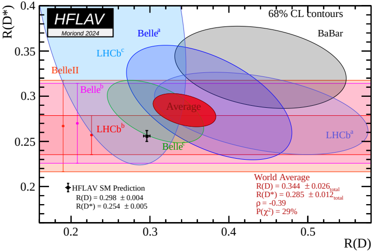
Outline

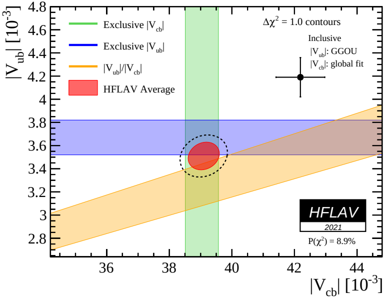
- LCDA of heavy mesons
- Introduction to bHQET
- Factorization of QCD LCDA
- Numeric applications

B Physics

New physics beyond the SM

- Direct search: new particles
- Indirect search: flavour physics
  CPV, $R(D^{(*)})$, $|V_{ub}|$, $|V_{cb}|$, $\cdots$





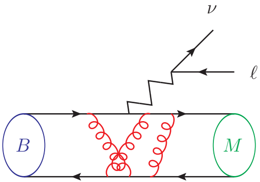
- BaBar, Belle
- LHC, Belle-II
- HL-LHC

HQET LCDA of heavy meson

$$\langle H_v|\bar{h}_v(0)\not{n}_+\gamma^5[0,tn_+]q_s(tn_+)|0\rangle = -i\,\widetilde{f}_H(\mu)\,n_+\cdot v\int_0^\infty d\omega\,e^{i\omega tn_+\cdot v}\varphi_+(\omega)$$

Most important long-distance function in exclusive B decays

- Non-leptonic decays: $B\to\pi\pi$, $B\to\pi K\cdots$
- Semi-leptonic decays: $B\to D^{(*)}\ell\nu$, $B\to K^{(*)}\ell\nu\,\cdots$
- Radiative decay: $B\to\gamma\ell\nu$, $B\to\gamma\gamma\,\cdots$



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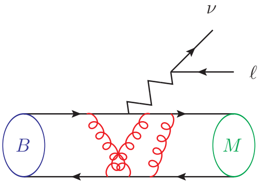

⋇ Inverse moments: [Braun, Ivanov and Korchemsky, 03'], [Belle, 18'], [Han, et al. 24']

⋇ RG evolution properties: [Lange and Neubert, 03'], [Braun, Ji and Manashov. 19']

⋇ asymptotic behavior, generalized LCDA, EOM ...

Non-perturbative LCDA of hadrons

Light-cone distribution amplitudes (LCDA): non-perturbative physics

- Calculate with LQCD [LPC collaboration, 22']
- Extract from the experiments: clean process $B \to \gamma \ell \nu$

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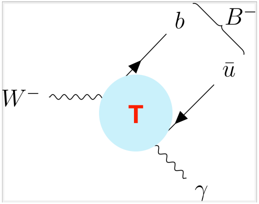
Rare decay $W^- \to B^- \gamma$ [Grossman, König and Neubert, 15']

$$A(W^- \to B^- \gamma) = \int_0^1 du\, T(u)\, \phi(u)$$



LaMET [Ji, 13']: quasi DA $\to$ QCD LCDA $\to$ HQET LCDA [Han, et al. 24']

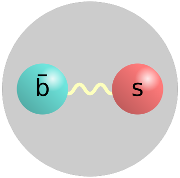
QCD LCDA to HQET LCDA

LCDA of heavy meson

The heavy meson QCD LCDA [Braun and Filyanov, 89']

$$\langle H(p_H)|\bar{Q}(0)\not{n}_+\gamma^5[0,tn_+]q(tn_+)|0\rangle = -if_H n_+\cdot p_H\int_0^1 du\, e^{iutn_+\cdot p_H}\,\phi(u)$$

- ϕ: $\ln\Lambda_{\mathrm{QCD}}/m_Q$ from m_Q and Λ_{QCD}



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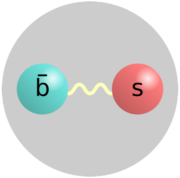


The leading-twist heavy-meson LCDA in (b)HQET [Grozin and Neubert, 96']

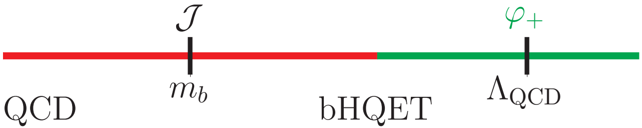
$$\langle H_v|\bar{h}_v(0)\not{n}_+\gamma^5[0,tn_+]q_s(tn_+)|0\rangle = -i\widetilde{f}_H(\mu)\,n_+\cdot v\int_0^\infty d\omega\, e^{i\omega tn_+\cdot v}\varphi_+(\omega)$$

- φ_+: Λ_{QCD}

Factorization of the QCD LCDA

- Momentum space matching of QCD to HQET @ NLO [Ishaq, Jia, Xiong and Yang, 19']

$$\phi(u) = \mathcal{J}(u,\omega) \otimes \varphi_+(\omega)$$



NLO: [Bell and Feldmann, 08']

- Coordinate space matching @NLO [Zhao, 19']

Momentum modes

A given momentum in the light-cone coordinate

$$l^\mu = (n_+ l,\ l_\perp,\ n_- l)$$

In the rest frame of the heavy meson

- heavy quark (hard): $(1,1,1)m_Q$
- light degree of freedom (soft): $(1,1,1)\Lambda_{\mathrm{QCD}}$

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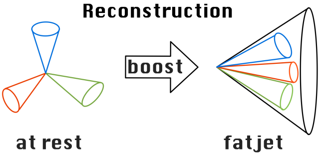

For a boosted heavy meson: $b \sim \frac{m_Q}{Q}$

- heavy quark (hard-collinear): $(\frac{1}{b}, 1, b)m_Q = (Q, m_Q, \frac{m_Q^2}{Q})$
- light degree of freedom (soft-collinear): $(\frac{Q}{m_Q}, 1, \frac{m_Q}{Q})\Lambda_{\mathrm{QCD}}$

Boosted HQET

The heavy quark field in bHQET [Fleming, Hoang, Mantry and Stewart, 07'], [Dai, Kim and Leibovich, 21']

$$h_n(x) \equiv \sqrt{\frac{2}{n_+ v}} e^{i m_Q v \cdot x} \frac{\not{n}_- \not{n}_+}{4} Q(x)$$

The bHQET Lagrangian could be derived from HQET or SCET

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The bHQET Lagrangian could be derived from HQET or SCET

Then one find out the LP operators

$$\hat{\mathcal{O}}_k = \frac{1}{n_+ v} \sqrt{\frac{n_+ v}{2}} \bar{h}_n \not{n}_+ \left(n_+ v \frac{i \not{D}_\perp}{i n_+ D} \right)^k \gamma^5 \xi_{sc}$$

Only operators $\hat{\mathcal{O}}_0$ and $\hat{\mathcal{O}}_1$ will appear at LP [Beneke, Finauri, Vos and **YBW**, 23']

Factorization of the QCD LCDA

The factorization formula

$$\phi(u) = \begin{cases} u \sim \delta: & \mathcal{J}_p(u,\omega) \otimes \varphi_+(\omega)\,, \qquad \mathcal{O}_{\rm QCD} = \mathcal{J}_p \otimes \mathcal{O}_0, \\ u \sim 1: & \mathcal{J}_{\rm tail}(u)\,, \qquad \mathcal{O}_{\rm QCD} = \mathcal{J}_0 \mathcal{O}_0 + \mathcal{J}_1 \mathcal{O}_1\,, \end{cases}$$

The jet function $\mathcal{J}$: $\mathcal{O}(m_Q)$, HQET LCDA φ_+: $\mathcal{O}(\Lambda_{\rm QCD})$

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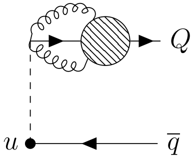
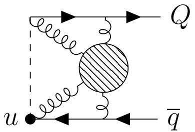


Peak region:

$$\mathcal{J}_p(u,\omega) = \theta(m_Q - \omega)\delta\left(u - \frac{\omega}{m_Q}\right)\left[1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2\right)\right], \qquad L = \ln\frac{\mu^2}{m_Q^2}$$

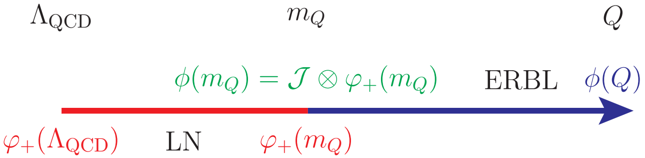


Tail region:

$$\mathcal{J}_{\rm tail}(u) = \frac{\alpha_s C_F}{4\pi}\frac{2\bar{u}}{u}\Big((1+u)[L - 2\ln u] - u + 1\Big)$$

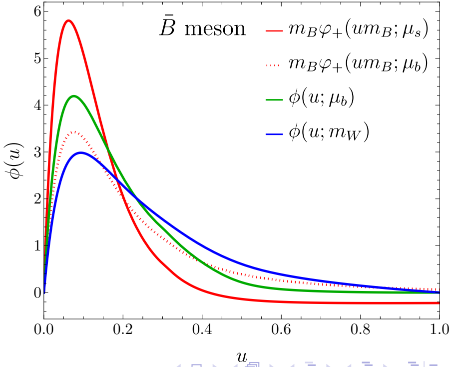
Evolution of the LCDAs: φ_+ and ϕ

Two-step evolutions: $\Lambda_{\rm QCD} \to m_Q \to Q$



$\phi(m_Q) \to \phi(Q)$:
evolve of the Gegenbauer moments

$$\frac{a_n(\mu_h)}{a_n(\mu)} = \left(\frac{\alpha_s(\mu_h)}{\alpha_s(\mu)}\right)^{\frac{\gamma_n}{2\beta_0}}$$



$W \to B\gamma$: branch ratio

The branch ratio

$$\mathrm{Br}(W \to B\gamma) = \frac{\Gamma(W \to B\gamma)}{\Gamma_W} = \frac{\alpha_{\mathrm{em}} m_W f_B^2}{48 v^2 \Gamma_W} |V_{ub}|^2 \left(|F_1^B|^2 + |F_2^B|^2 \right)$$

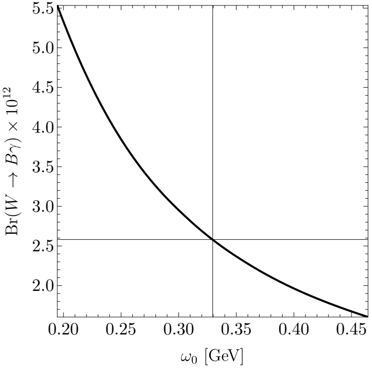
[Grossman, König and Neubert, 15']: QCD LCDA

$$\mathrm{Br} = (1.99 \pm 0.17_{\mathrm{in}} {}^{+0.03}_{-0.06}{}_{\mu_h} {}^{+2.48}_{-0.80}{}_{\lambda_B}) \cdot 10^{-12}$$

Our result: with $\ln \Lambda_{\mathrm{QCD}}/m_Q$ resummation

$$\mathrm{Br} = (2.58 \pm 0.21_{\mathrm{in}} {}^{+0.05}_{-0.08}{}_{\mu_h} {}^{+2.95}_{-0.98}{}_{\lambda_B}) \cdot 10^{-12}$$

30% increase



Summary

※ Introduction to the QCD and HQET LCDAs

※ Match QCD LCDA to the HQET LCDA

- Peak region: $\mathcal{O}_C(u) = \mathcal{J}_p(u,\omega) \otimes \mathcal{O}_h(\omega)$

- Tail region: $\mathcal{O}_C(u) = \mathcal{J}_+(u)\mathcal{O}_+ + \mathcal{J}_-(u)\mathcal{O}_-$

※ $W^- \to B^-\gamma$ decay: 30% enhancement

Thank you!

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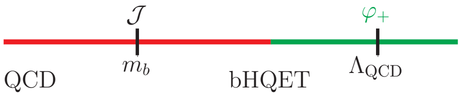

Tree level

$$\mathcal{J}^{(0)}(u,\omega) = \delta\Big(u - \frac{\omega}{\omega + m_Q}\Big)$$

ω and m_Q have different power counting

NLO from [Bell and Feldmann, 08']

- Coordinate space matching @NLO [Zhao, 19']

Inhomogeneous power counting of ϕ

For the matching scale μ ($\delta \sim \Lambda_{\text{QCD}}/m_H$)

- $\mu \gg m_Q$: $\phi(u)$ is symmetric under $u \leftrightarrow 1-u$
- $\mu \lesssim m_Q$: $\phi(u)$ develops a peak at $u \sim \mathcal{O}(\delta)$
  $\phi(u)$ is suppressed at $u \sim \mathcal{O}(1)$
- Normalization condition

$$\phi(u) \sim \begin{cases} \delta^{-1}\,, & \text{for } u \sim \delta \quad (\text{"peak"}) \\ 1\,, & \text{for } u \sim 1 \quad (\text{"tail"}) \end{cases}$$

For consistent power counting: separate $u \sim \mathcal{O}(1)$ and $u \sim \mathcal{O}(\delta)$ region

$W \to B\gamma$ (non-perturbative input)

HQET LCDA with radiative tail [Lee and Neubert, 05']

$$\varphi_+(\omega;\mu_s) = \left(1 + \frac{\alpha_s(\mu_s) C_F}{4\pi}\left[\frac{1}{2} - \frac{\pi^2}{12}\right]\right)\varphi_+^{\text{mod}}(\omega;\mu_s) + \theta(\omega - \sqrt{e}\mu_s)\varphi_+^{\text{asy}}(\omega;\mu_s)$$

Two parameter models

$$\varphi_+^{(\text{I})}(\omega;\mu_s) = \left[1 - \beta + \frac{\beta}{2-\beta}\frac{\omega}{\omega_0}\right]\varphi_+^{\exp}\left(\omega, (1-\beta/2)\omega_0;\mu_s\right), \quad \text{for} \quad 0 \le \beta \le 1,$$

$$\varphi_+^{(\text{II})}(\omega;\mu_s) = \frac{(1+\beta)^\beta}{\Gamma(2+\beta)}\left(\frac{\omega}{\omega_0}\right)^\beta \varphi_+^{\exp}\left(\omega, \frac{\omega_0}{1+\beta};\mu_s\right), \quad \text{for} \quad -\frac{1}{2} < \beta < 1,$$

$$\varphi_+^{(\text{III})}(\omega;\mu_s) = \frac{\sqrt{\pi}}{2\Gamma(3/2+\beta)} U\left(-\beta, \frac{3}{2} - \beta, (1+2\beta)\frac{\omega}{\omega_0}\right) \times \varphi_+^{\exp}\left(\omega, \frac{\omega_0}{1+2\beta};\mu_s\right), \quad \text{for} \quad 0 \le \beta < \frac{1}{2}$$