華南師範大學 SOUTH CHINA NORMAL UNIVERSITY

LPC Lattice Parton Collaboration

The Baryon LCDA from Lattice QCD

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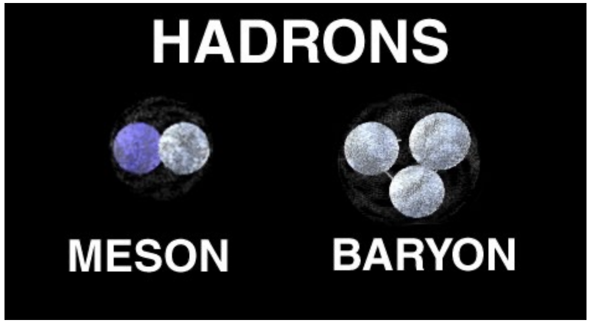
第二十一届全国重味物理和CP破坏研讨会

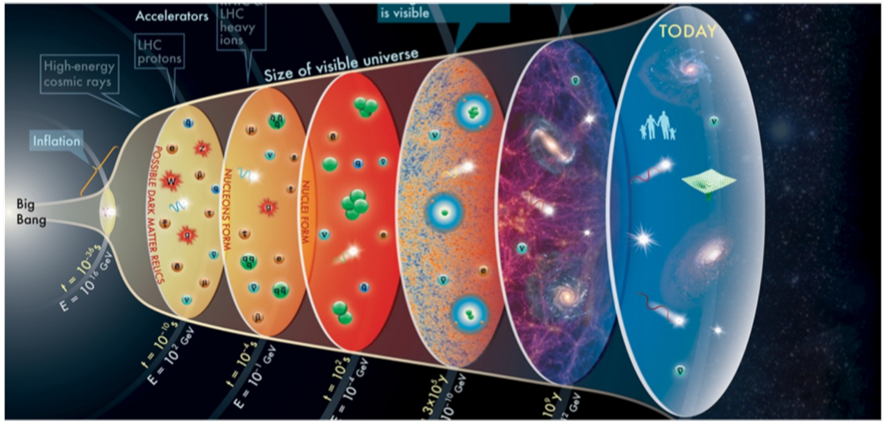
CONTENTs

Motivation

- The visible matter of the Universe is mainly made of baryons.

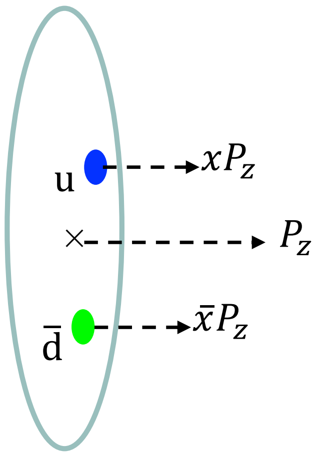
- Baryons play an important role in the evolution of the Universe, such as baryogenesis and big-bang nucleosythesis.



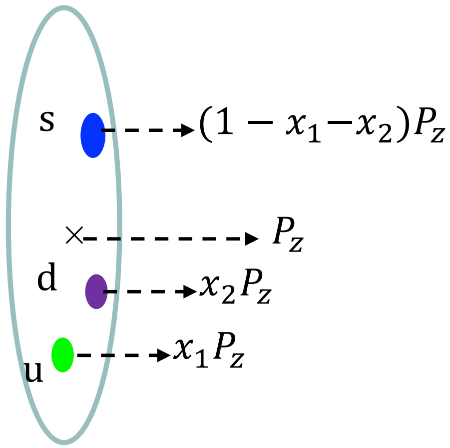


Motivation

Light meson π/K…



Baryon Λ, proton…



- **CKM matrix**
- **CP violation**
- **New physics ···**

- Sakharov conditions for Baryogenesis:
  1) Baryon number violation
  2) C and CP violation
  3) Out of thermal equilibrium

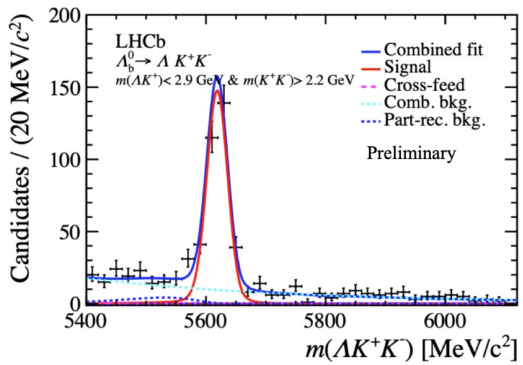
- CPV well established in K, B and D mesons
- But CPV never established in any baryon

Motivation

CP violation in Baryon

15:10-15:30	LHCb 上的重子 CP 破坏研究 戴鑫琛（清华大学）	
15:50-16:10	CPV of Baryon Decays with $N\pi$ Rescatterings 汪建鹏（兰州大学）	
16:40-17:00	Observable CPV in charmed baryons decays with SU(3) symmetry analysis 邢志鹏（南京师范大学）	
8:50-9:10	Recent results on baryons and charmed baryons from Belle and Belle II 李素娴（复旦大学）	
9:50-10:10	CPV of Λ_b decays in PQCD 韩佳杰（兰州大学）	
17:00-17:15	质子电磁形状因子的微扰 QCD 研究 余纪新（兰州大学）	Tensor analysis for topological diagrams of charmed baryon decays 王迪（湖南师范大学）

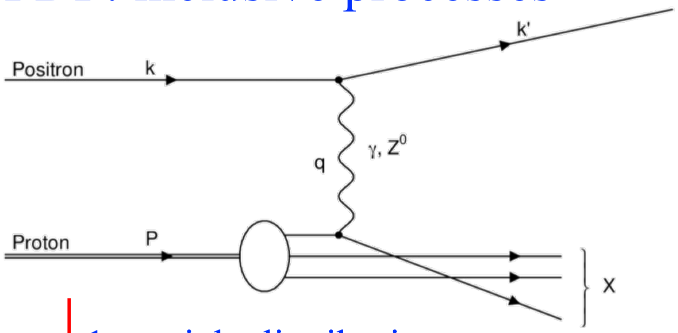
……



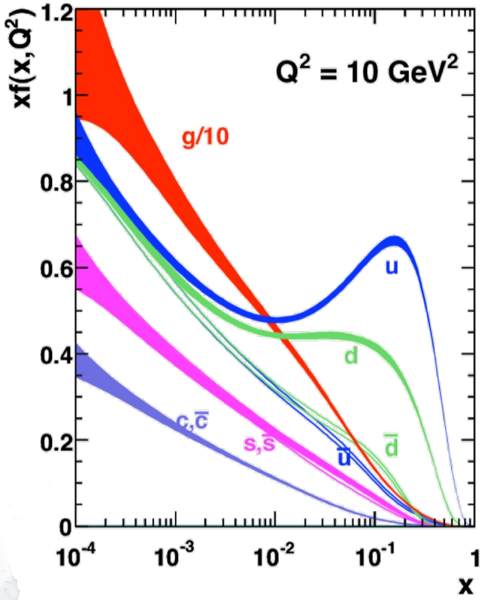
Channel	$m(h^+h'^-)$	$m(\Lambda h^+)$	$\mathcal{A}^{CP}$	
$\Lambda_b^0 \to \Lambda K^+K^-$	$< 1.10\ \mathrm{GeV}/c^2$	/	$0.150 \pm 0.062 \pm 0.021$	2.1σ
$\Lambda_b^0 \to \Lambda K^+K^-$	$> 2.20\ \mathrm{GeV}/c^2$	$< 2.90\ \mathrm{GeV}/c^2$	$0.165 \pm 0.050 \pm 0.017$	3.2σ
$\Lambda_b^0 \to \Lambda K^+K^-$	$> 2.20\ \mathrm{GeV}/c^2$	$> 2.90\ \mathrm{GeV}/c^2$	$0.011 \pm 0.053 \pm 0.017$	

Motivation

PDF: inclusive processes

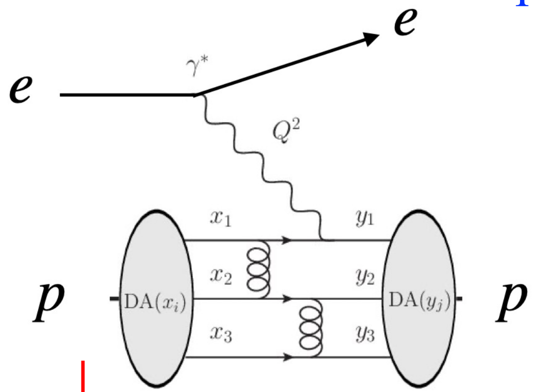


1-particle distributions

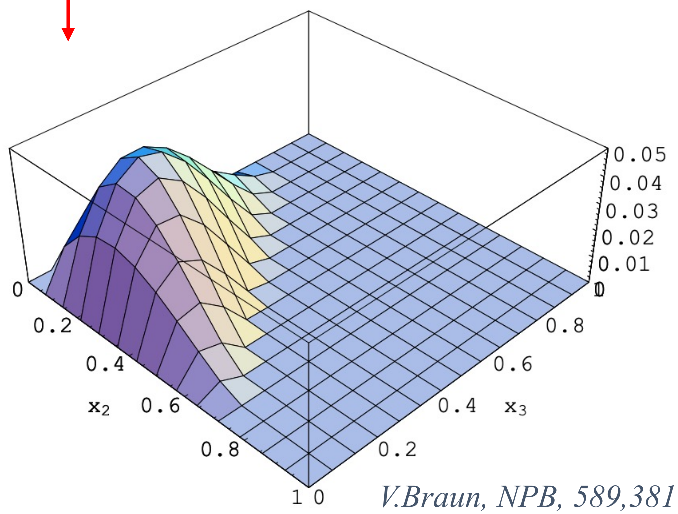


Complementary

LCDA: hard exclusive processes



3-particle distributions



V.Braun, NPB, 589,381(2000)

Motivation

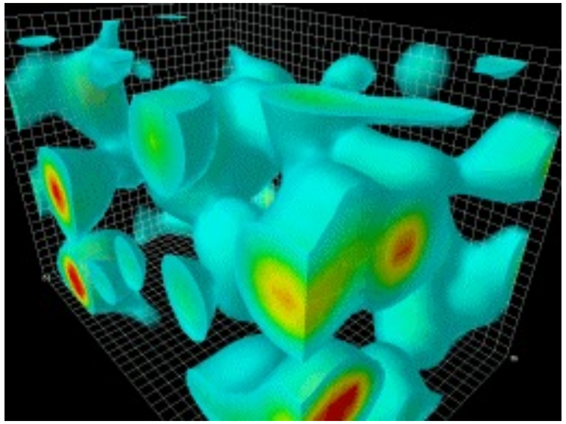
➢ Light meson LCDAs have been extensively pursued: (1970s - now)

- **Asymptotic LCDAs**
  Chernyak, Zhitnitsky, 1977; Lepage, Brodsky, 1979;
  Efremov, Radyushkin, 1980
- **Dyson-Schwinger Equation**
  Chang, Cloet, Cobos-Martinez, Roberts, Schmidt, 2013;
  Gao, Chang, Liu, Roberts, Schmidt, 2014;
  Roberts, Richards, Chang, 2021
- **Sum rules**
  Chernyak, Zhitnitsky, 1982; Braun, Filyanov, 1989;
  Ball, Braun, 1998; Khodjamirian, Mannel, Melcher, 2004;
  Lu, Wang, Hao, 2006; Ball,, Lenz, 2007;
- **Inverse Method**
  Li, 2022
- **Models**
  Arriola, Broniowski, 2002, 2006;
  Zhong, Zhu, Fu, Wu, Huang, 2021;
- **Global Fits**
  Stefanis, 2020; Cheng, Khodjamirian, Rusov, 2020;
  Hua, Li, Lu, Wang, Xing, 2021
- **Lattice with current-current correlation**
  Bali, Braun, Gläßle, Göckeler, Gruber, 2017, 2018;
- **Lattice with OPE**
  Martinelli, Sachrajda, 1987; Braun, Bruns, et al., 2016;
  RQCD collaboration, 2019, 2020
- **Lattice with LaMET**
  Zhang, Chen, Ji, Jin, Lin, 2017; LP3 Collaboration, 2019;
  Zhang, Honkala, Lin, Chen, 2020; Lin, Chen, Fan, Zhang^2, 2021;
  LPC Collaboration, 2021, 2022
- **Quantum Computing**
  QuNu Collaboration, 2023, 2024

Motivation

➢ Light Baryon LCDAs: (1980s - now)

- Asymptotic LCDAs
  Chernyak, Zhitnitsky, 1983
- Sum rules
  King, Sachrajda, 1987; Chernyak, Ogloblin, Zhitnitsky 1989;
  Stefanis, Bergmann, 1993; Braun, Fries, Stein 2000;
  Braun, Lenz, Wittmann 2006;
- Model parametrization
  Bell, Feldmann, Wang, Matthew 2013
- Lattice with OPE
  QCDSF collaboration, 2008, 2009;
  RQCD collaboration, 2016, 2019
- What's next ··· ?



- Lattice is powerful
- Moments is not enough

Motivation

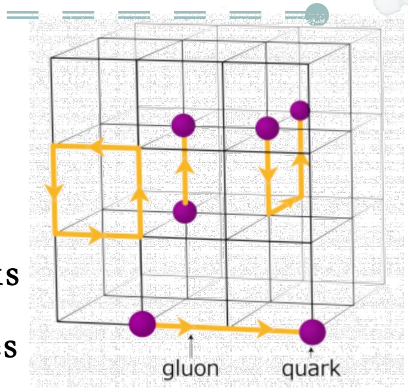
LQCD is formulated as a Feynman path integral on a discrete 4D Euclidean grid.

Numerical simulations based on a QCD Lagrangian with discrete from:

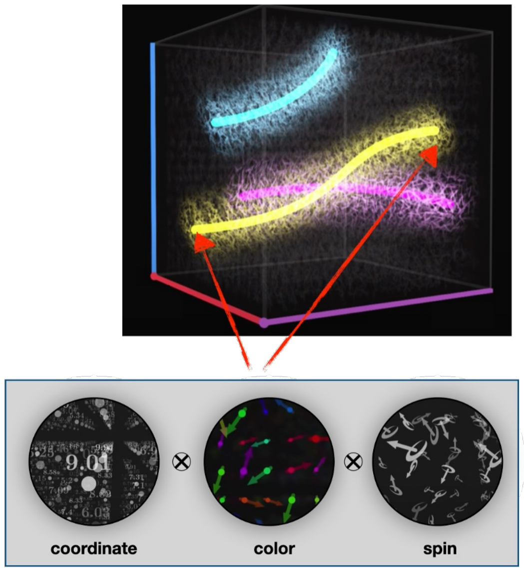
$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi - \frac{1}{4}G^{a}_{\mu\nu}G^{a,\mu\nu}$$

$$S_E^{\text{latt}} = -\sum_{\square}\frac{6}{g^2}\,\mathrm{Re}\,\mathrm{tr}_N\left(U_{\square,\mu\nu}\right) - \sum_q \bar{q}\left(D_\mu^{\text{lat}}\gamma_\mu + am_q\right)q$$

- Gluon fields on links
- Quark fields on sites



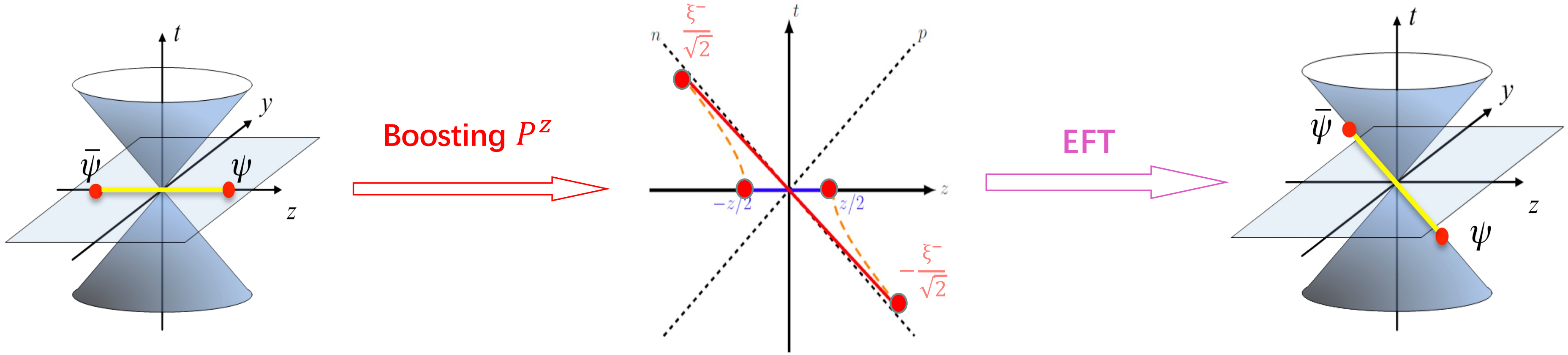
- lattice spacing $a \rightarrow$ UV regulator;
- box length $L \rightarrow$ IR regulator;
- Chiral extrapolation ($M_\pi \rightarrow 135\text{MeV}$);
- Numerical sampling with highly dimension $n_s^3 \times n_t \times N_{\text{color}} \times N_{\text{spin}}$



- Building blocks: ensembles of gauge configurations; quark propagators
- Hadron & interactions put in as external probes: N-point correlation function

Motivation

Large Momentum Effective Theory (LaMET)



- Instead of taking $P^z \to \infty$ calcuation, one can perform an expansion for large but finite P^z:
- For meson LCDA:

Quasi-DA, LCDA

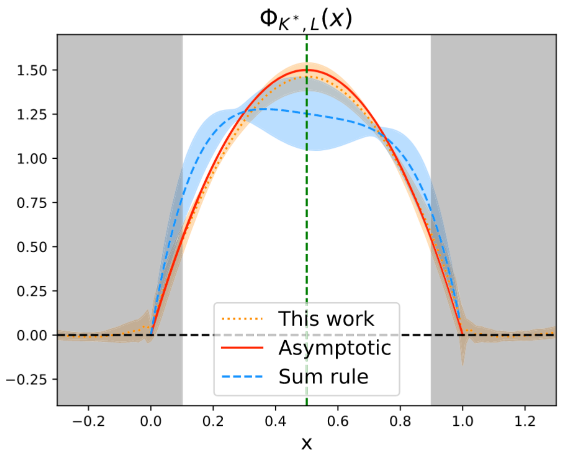
$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C(x, y, P^z, \mu) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2, M^2}{(P^z)^2}\right)$$

X.Ji PRL110 262002 (2013)

Matching kernel, High power correction

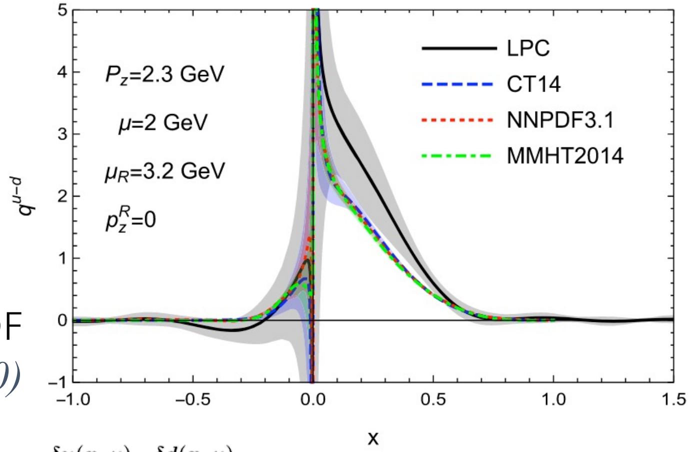
Motivation

Large Momentum Effective Theory (LaMET)



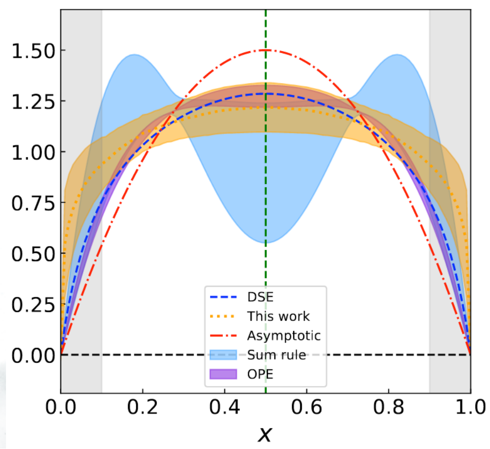
K^*, ϕ LCDA

(LPC) PRL127 062002 (2021)



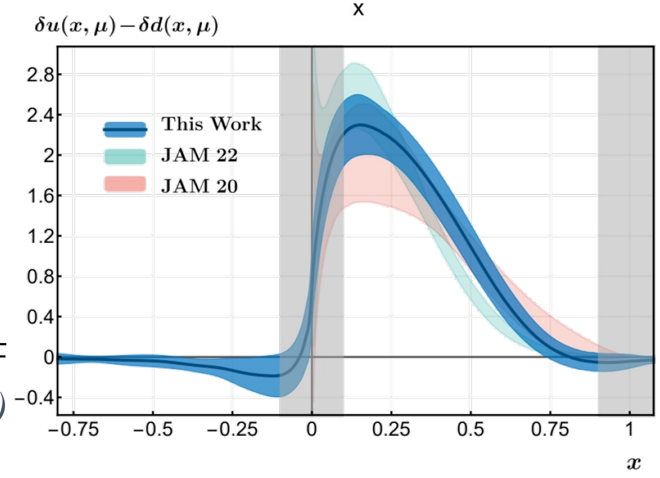
Unpolarized PDF

(LPC) PRD101 034020 (2020)



π, K LCDA

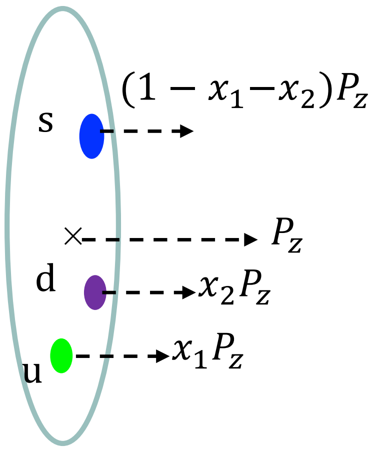
(LPC) PRL129 132001 (2022)



Transversity PDF

(LPC) PRL 131 261901 (2022)

Quasi Distribution



- Definition of light cone baryon LCDA:

$$\int \frac{d\xi_1^-}{2\pi}\frac{d\xi_2^-}{2\pi} e^{ix_1p^+\xi_1^-} e^{ix_2p^+\xi_2^-}\epsilon^{ijk}\left\langle 0\left|W^{ii'}(\infty,\xi_1^-)\psi_\alpha^{i'}(\xi_1^-)\Gamma_{\alpha\beta}W^{jj'}(\infty,\xi_2^-)\,\psi_\beta^{j'}(\xi_2^-)\,\psi_\gamma^{j}(\infty,0)\right|M(P)\right\rangle$$
$$= if_M(p_1\cdot n)(p_2\cdot n)\phi_M(x_1,x_2).$$

- Twists for baryon LCDA:

C.Han JHEP 07019 (2024)
V.L.C & I.R.Z NPB 24652(1984)
G.R.Farrar et.al. NPB 311585(1989)

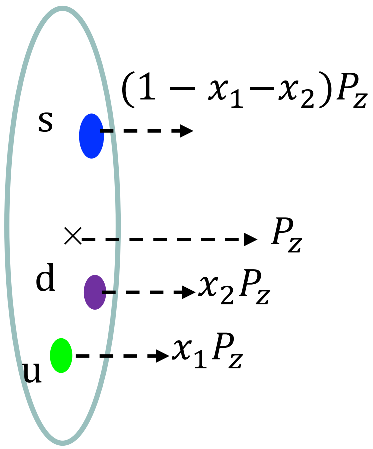
- Octet baryon (3 terms for leading twist):

$$\langle 0\,|f_\alpha(z_1n)\,g_\beta(z_2n)\,h_\gamma(z_3n)|\,B(P_B,\lambda)\rangle$$
$$=\frac{1}{4}f_V\left[(\not P_B C)_{\alpha\beta}(\gamma_5 u_B)_\gamma V^B(z_in\cdot P_B)+(\not P_B\gamma_5 C)_{\alpha\beta}(u_B)_\gamma A^B(z_in\cdot P_B)\right]$$
$$+\frac{1}{4}f_T\,(i\sigma_{\mu\nu}P_B^\nu C)_{\alpha\beta}(\gamma^\mu\gamma_5 u_B)_\gamma T^B(z_in\cdot P_B)\,,$$

- Decuplet baryon (4 terms for leading twist):

$$\langle 0|f_\alpha(z_1n)g_\beta(z_2n)h_\gamma(z_3n)|B(P_B,\lambda)\rangle$$
$$=\frac{1}{4}\lambda_V\left[(\gamma_\mu C)_{\alpha\beta}\Delta_\gamma^\mu V^B(z_in\cdot P_B)+(\gamma_\mu\gamma_5 C)_{\alpha\beta}(\gamma_5\Delta^\mu)_\gamma A^B(z_in\cdot P_B)\right]$$
$$-\frac{1}{8}\lambda_T(i\sigma_{\mu\nu}C)_{\alpha\beta}(\gamma^\mu\Delta^\nu)_\gamma T^B(z_in\cdot P_B)-\frac{1}{4}\lambda_\varphi\left[(i\sigma_{\mu\nu}C)_{\alpha\beta}\left(P_B^\mu\Delta^\nu-\frac{1}{2}M_B\gamma^\mu\Delta^\nu\right)_\gamma\varphi^B(z_in\cdot P_B)\right],$$

Quasi Distribution



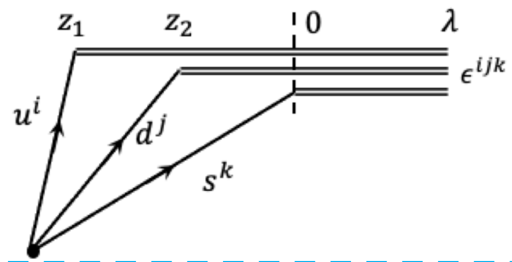
- Corresponding quasi-DA on Euclidean lattice:

$$f_M(p_1,p_2)\tilde{\Phi}^0(x_1,x_2) = \int \frac{p_{z1}dz_1}{2\pi}\frac{p_{z2}dz_2}{2\pi} e^{-i(x_1p_{z1}z_1+x_2p_{z2}z_2)}\langle 0|\hat{O}\Big(z_1,z_2,\tilde{\Gamma}\Big)|P^z\rangle$$

$$\hat{O}_\gamma(\vec{x},t;z_1,z_2) = \epsilon^{ijk}W^{ii'}(\infty,\vec{x}+z_1n_z)u^{i'}_\alpha(\vec{x}+z_1n_z,t) \times \tilde{\Gamma}_{\alpha\beta}W^{jj'}(\infty,\vec{x}+z_2n_z,t)d^{j'}_\beta(\vec{x}+z_2n_z) \times W^{kk'}(\infty,\vec{x})s^{k'}_\gamma(\vec{x},t)$$

$$\epsilon^{ijk}U^{ii\prime}U^{jj\prime}U^{kk\prime} = det(U)\epsilon^{i\prime j\prime k\prime}$$

$$det(U) = 1$$



$$\hat{O}_\gamma(\vec{x},t;z_1,z_2) = \epsilon^{ijk}W^{ii'}(\vec{x},\vec{x}+z_1n_z)u^{i'}_\alpha(\vec{x}+z_1n_z,t) \times \tilde{\Gamma}_{\alpha\beta}W^{jj'}(\vec{x},\vec{x}+z_2n_z)d^{j'}_\beta(\vec{x}+z_2n_z,t) \times s^k_\gamma(\vec{x},t)$$

Lattice Setup

- Nonlocal 2pt related to baryon quasi DA:

$$C_2\left(z_1, z_2; t, \vec{P}\right) = \int d^3x e^{-i\vec{P}\vec{x}} \langle 0|\hat{O}_\gamma(\vec{x}, t; z_1, z_2) \times \hat{\bar{O}}_{\gamma'}(0, 0; 0, 0) T^{\gamma\gamma'}|0\rangle,$$

$$\hat{O}_\gamma(\vec{x}, t; z_1, z_2) = \epsilon^{ijk} W^{ii'}(\vec{x}, \vec{x} + z_1 n_z) u_\alpha^{i'}(\vec{x} + z_1 n_z, t) \times \tilde{\Gamma}_{\alpha\beta} W^{jj'}(\vec{x}, \vec{x} + z_2 n_z) d_\beta^{j'}(\vec{x} + z_2 n_z, t) \times s_\gamma^k(\vec{x}, t)$$

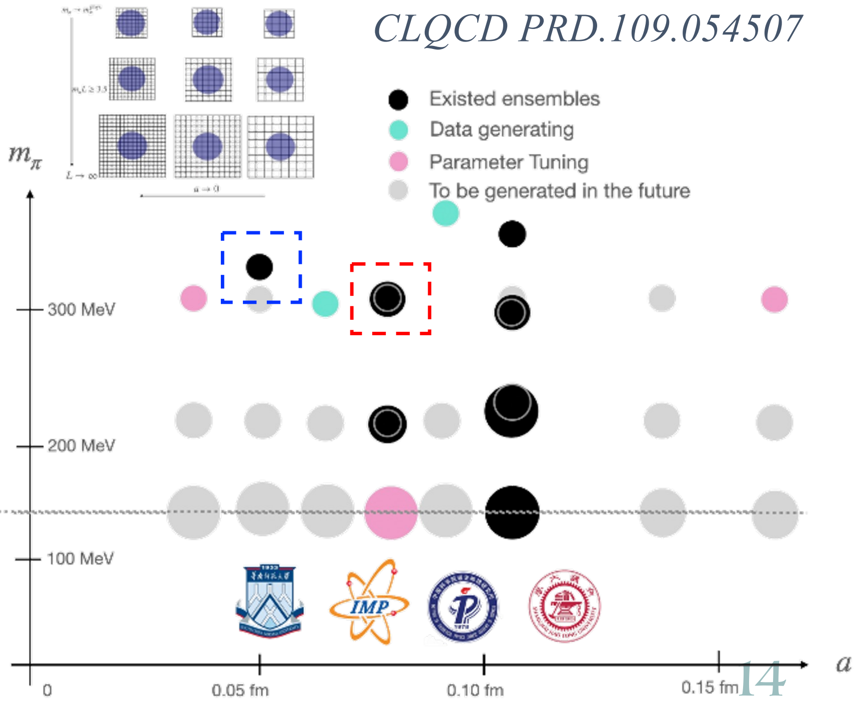
$$\tilde{\Gamma} = C\gamma_5\gamma^t/\gamma^z$$
$$T = (1 + \gamma^4)/2$$

- Lattice setup:

CLQCD Ensembles:

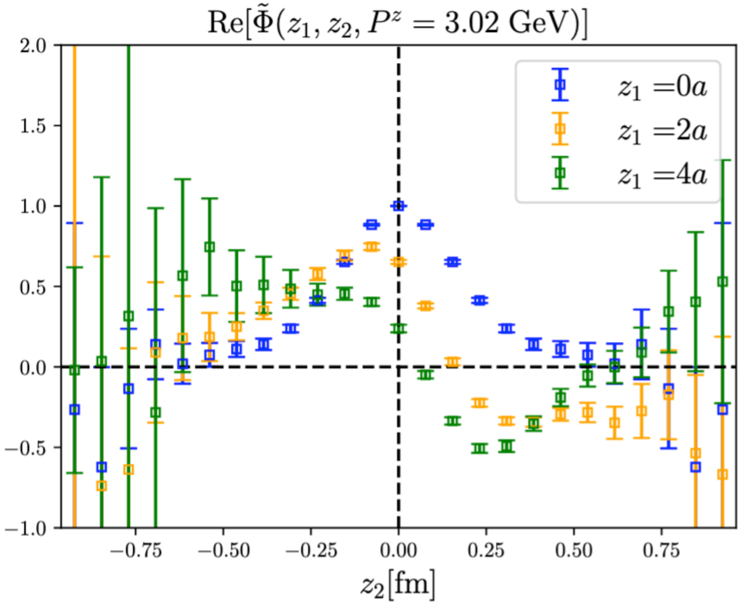
Ensemble	Volume	Lattice spacing	π mass	η_s mass	conf
F32P30	$32^3 \times 96$	0.077fm	290MeV	640MeV	777(*32)
H48P32	$48^3 \times 144$	0.055fm	300MeV	650MeV	except
Momentum	2.01, 2.51, 3.02 GeV				

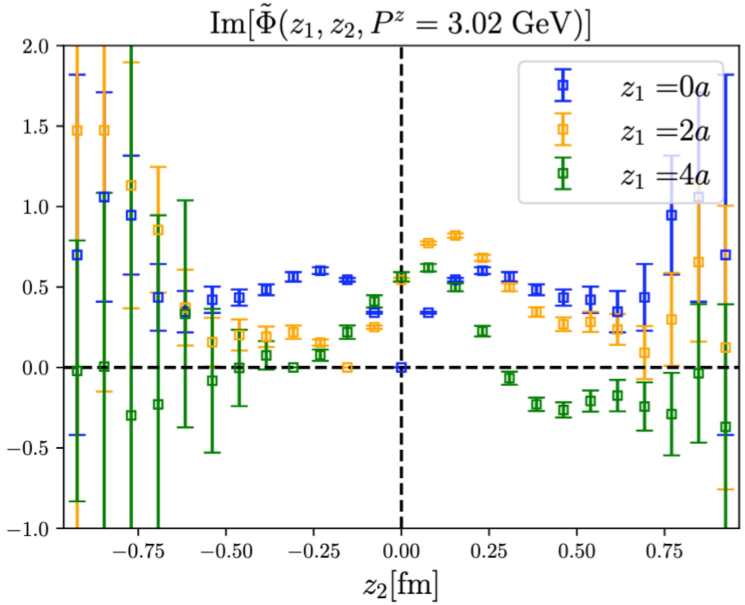
CLQCD PRD.109.054507



Quasi Distribution

- Renormalized quasi-DA $\tilde{\psi}(z_1, z_2, P^z)$ with fixed z_1 (Renormalized by ratio scheme)





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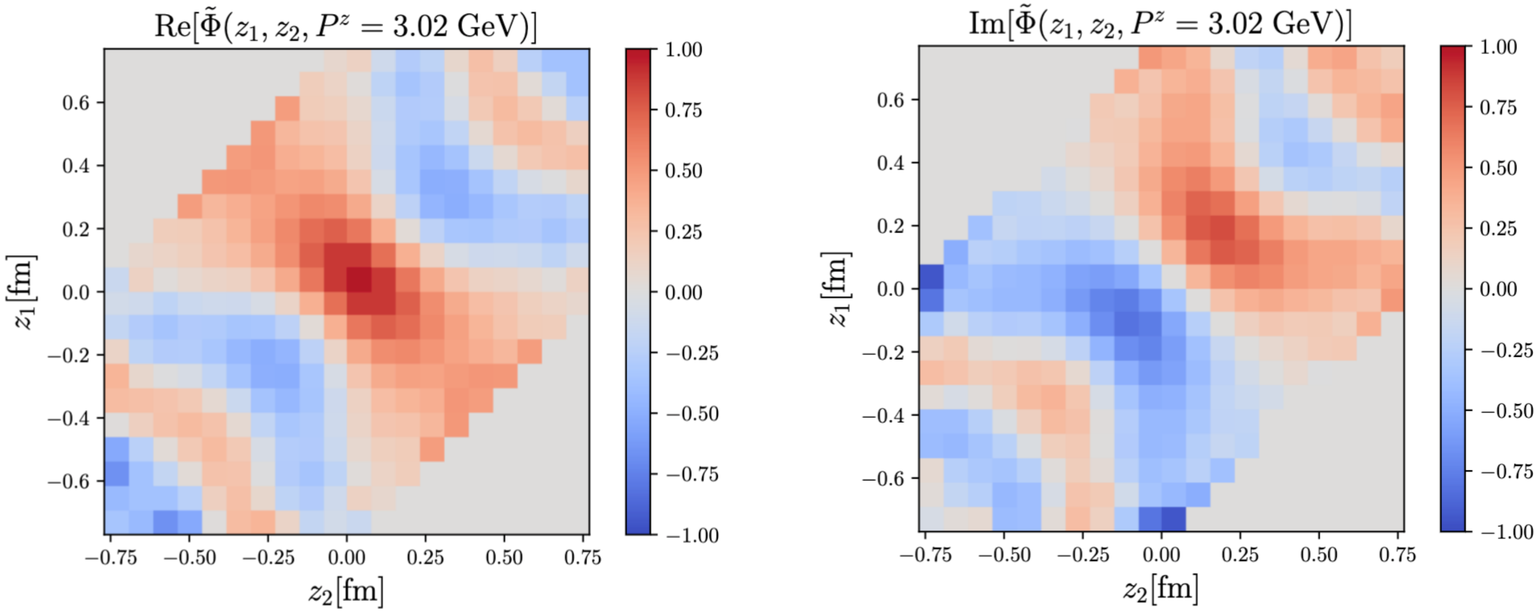
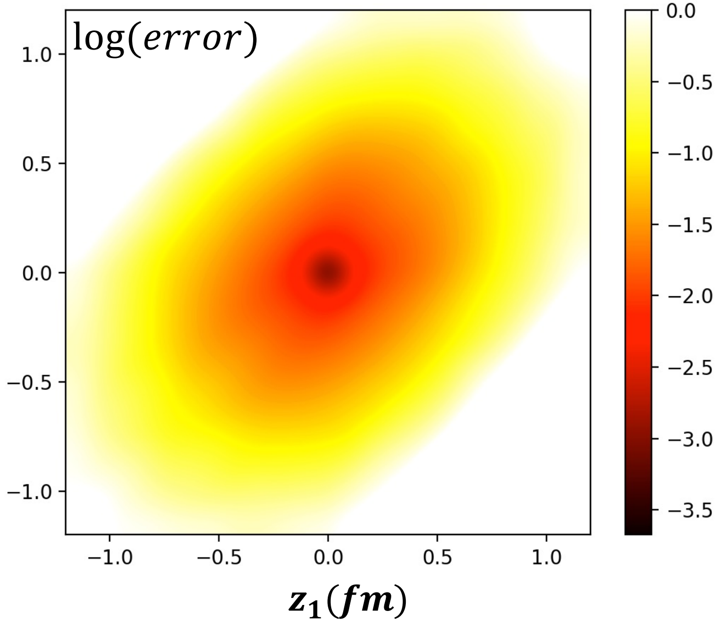
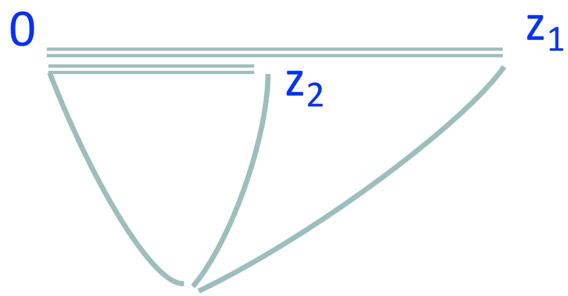


Quasi Distribution

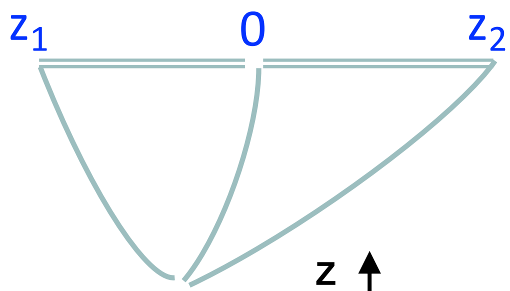


$\tilde{\psi}(z_1, z_2, P^z)$

- z_1+, z_2+
- z_1-, z_2-

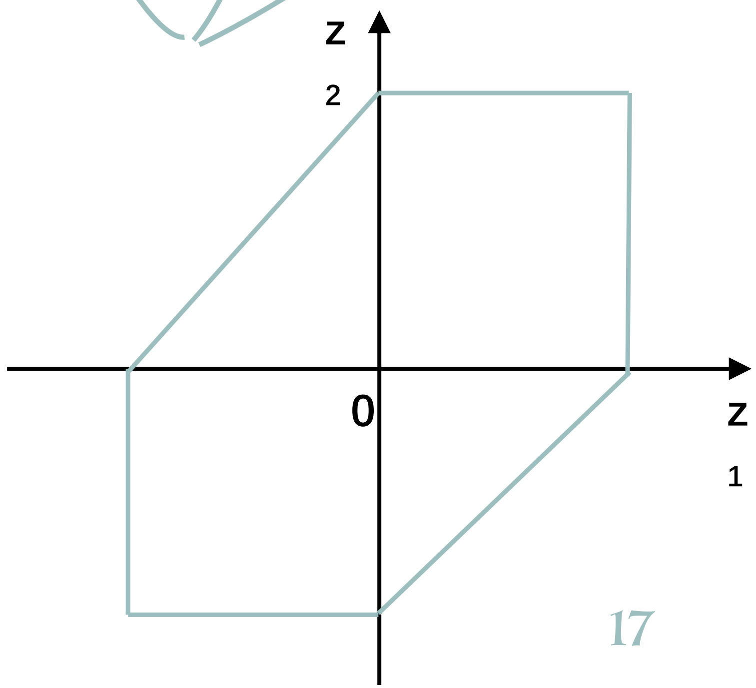


- z_1+, z_2-
- z_1-, z_2+

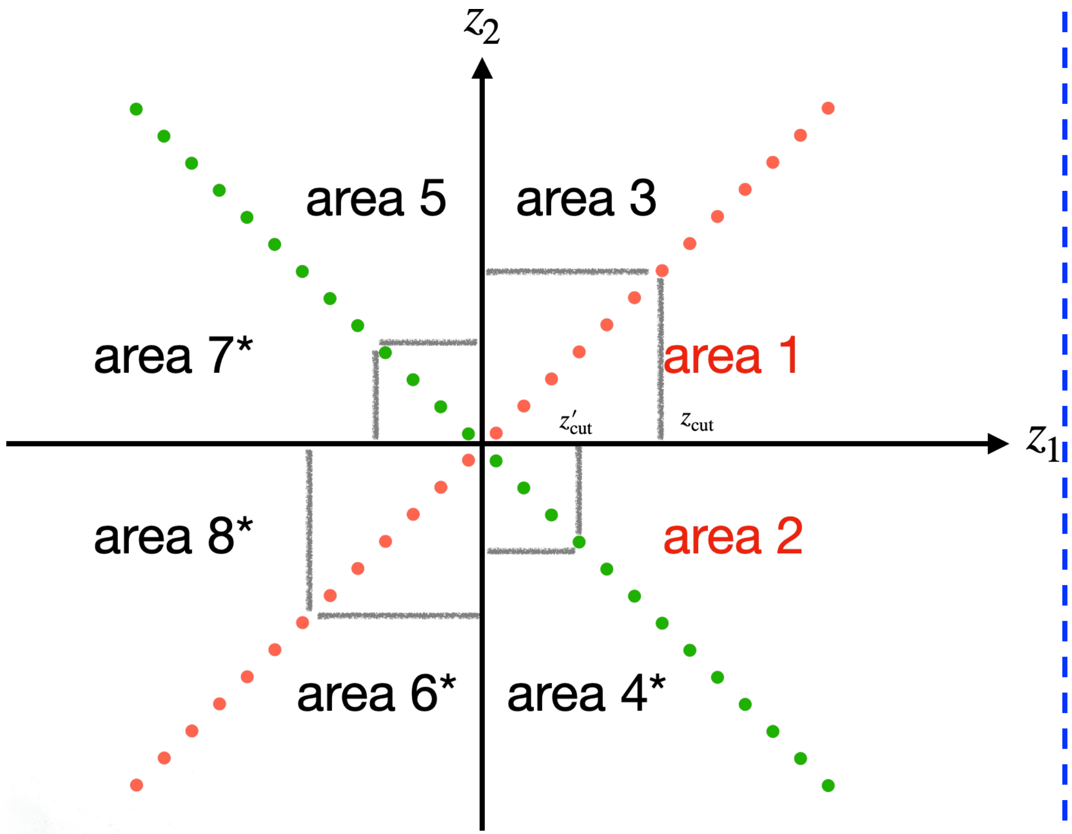


$$\hat{O}_\gamma(\vec{x}, t; z_1, z_2) = \epsilon^{ijk} W^{ii'}(\vec{x}, \vec{x} + z_1 n_z) u_\alpha^{i'}(\vec{x} + z_1 n_z, t) \times \tilde{\Gamma}_{\alpha\beta} W^{jj'}(\vec{x}, \vec{x} + z_2 n_z) d_\beta^{j'}(\vec{x} + z_2 n_z, t) \times s_\gamma^k(\vec{x}, t)$$

- The Wilson line length(non-local separation) is smaller with z1z2 in same direction than in opposite
- Thus the good signal region is rhombic



Quasi Distribution



- Two symmetries for Quasi-DA:
  - iso-spin symmetry for “u, d” quarks:

    $$\tilde{\phi}(z_1, z_2) = \tilde{\phi}(z_2, z_1)$$

  - The constrain by real $\tilde{\phi}(x_1, x_2)$:

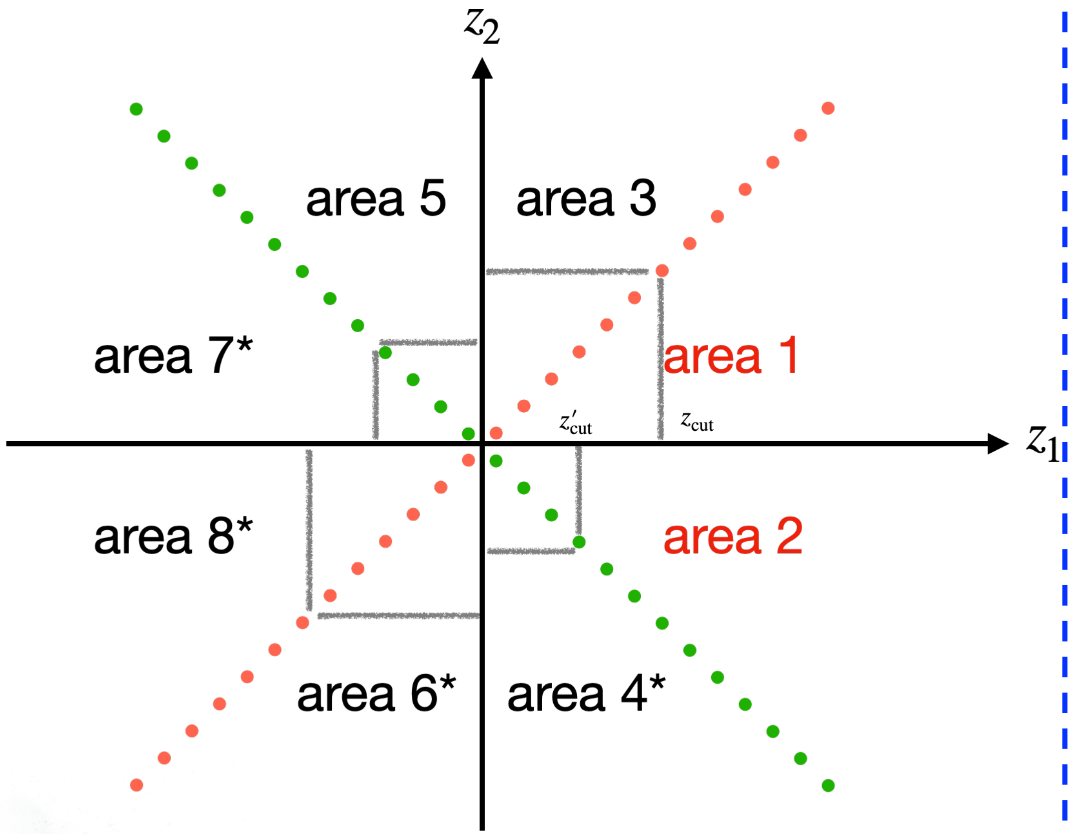
    $$\tilde{\phi}(z_1, z_2) = \tilde{\phi}^*(-z_1, -z_2)$$

- Thus for these areas: $1 = 3 = 6^* = 8^*$, $2 = 4^* = 5 = 7^*$

  only area 1,2 are independent

- $\tilde{\phi}(z_1 = -z_2)_{Im} = 0$

Quasi Distribution



➢ Renormalization scheme:

- Perturbative 0 momentum quasi-DA:

$$\hat{M}_p(z_1, z_2, 0, 0, \mu) = 1 + \frac{\alpha_s C_F}{2\pi}$$

$$\left[\frac{1}{8}\ln\left(\frac{z_1^2\mu^2 e^{2\gamma_E}}{4}\right) + \frac{1}{8}\ln\left(\frac{z_2^2\mu^2 e^{2\gamma_E}}{4}\right) + \frac{1}{4}\ln\left(\frac{(z_1 - z_2)^2\mu^2 e^{2\gamma_E}}{4}\right) + 4\right]$$

C. Han et.al. JHEP12044(2023)

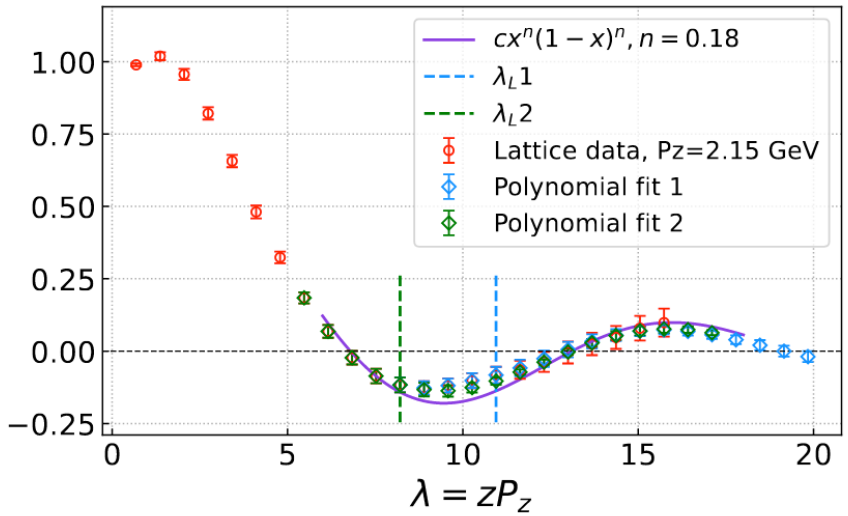
1 pole at $z_1 = z_2$

- Hybrid scheme need a match with the perturbative quasi-DA

  At least $a < 0.06fm$ to apply hybrid scheme

- For simplification: Ratio scheme $\frac{\tilde{\psi}(z_1, z_2, P^z, a)}{\tilde{\psi}(z_1, z_2, P^z = 0, a)}$

Quasi Distribution



(LPC)PRL129 132001(2022)

- **Extrapolation & Fourier transformation(1D):**
  - For meson LCDA :
    - Asymptotic in momentum space:

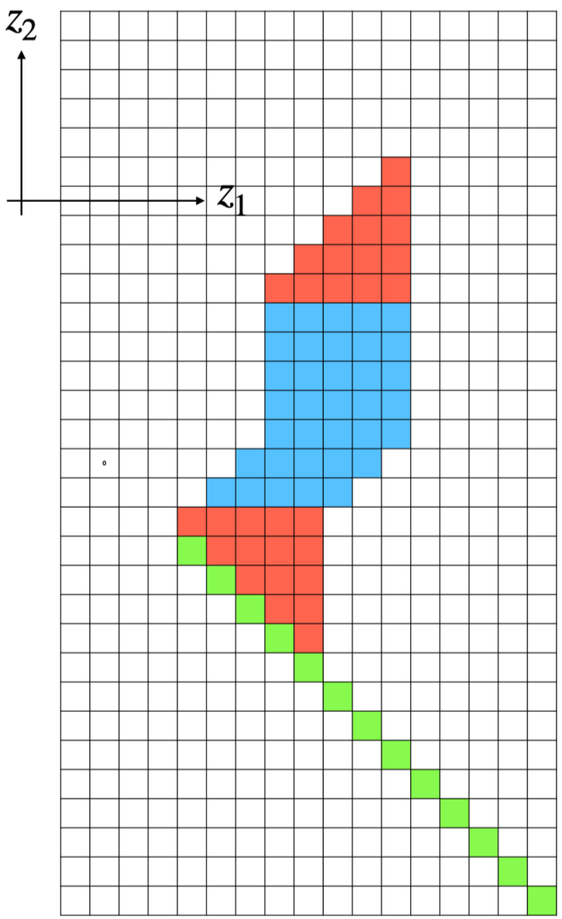
$$F(x) = cx^{d_1}(1-x)^{d_2}$$

Analytic FT then simplify

- Extrapolation form in coordinate space

$$H_m^{\mathrm{R}}(z, P_z) = \left[\frac{c_1}{(i\lambda)^a} + e^{-i\lambda}\frac{c_2}{(-i\lambda)^b}\right]e^{-\lambda/\lambda_0},$$

Quasi Distribution



➢ Extrapolation & Fourier transformation(2D):

- Asymptotic in momentum space:

$$F(x_1, x_2, d_1, d_2) = Cx_1^{d_1} x_2^{d_1} (1 - x_1 - x_2)^{d_2}$$

↓ FT

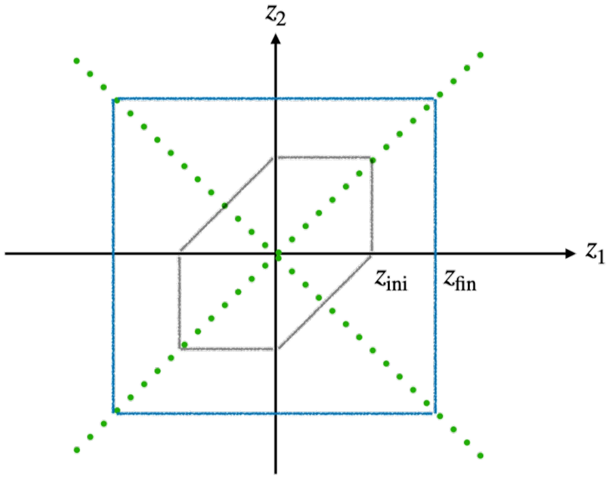
$$\tilde{\phi}(z_1, z_2; d_1, d_2) = \int_0^1 dx_1 \int_0^1 dx_2\, e^{-ix_1 z_1 P^z} e^{-ix_2 z_2 P^z} C x_1^{d_1} x_2^{d_1} (1 - x_1 - x_2)^{d_2}$$

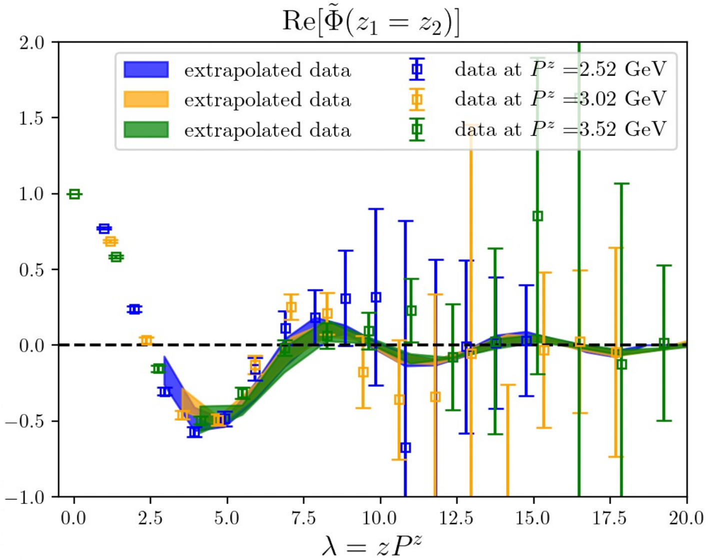
- ▫ Numerically FT as the fit from
  [computing source cost]
- ▫ Analytica FT and simplified as the fit form
  [complicated form for baryon]

Area2$[z_1 \gg 0, z_2 \gg 0]$:

$$\frac{\Psi(\lambda_1, \lambda_2)}{e^{-\frac{|\lambda_1|}{\lambda_0} - \frac{|\lambda_2|}{\lambda_0} - \frac{|\lambda_1 - \lambda_2|}{\lambda_0}}} = c_1 \left[\frac{1}{(\lambda_1 - \lambda_2)^{d_1} \lambda_1^{d_1}} + \frac{1}{(\lambda_1 - \lambda_2)^{d_1} (-\lambda_2)^{d1}} \right] + c_2 \frac{\cos\left[\frac{1}{2}(d_1\pi + d_2\pi - 2\lambda_1)\right]}{(\lambda_1 - \lambda_2)^{d_1} \lambda_1^{d_2}} + c_2 \frac{\cos\left[\frac{1}{2}(d_1\pi + d_2\pi + 2\lambda_2)\right]}{(\lambda_1 - \lambda_2)^{d_1} (-\lambda_2)^{d_2}}$$
$$- i \left(c_2 \frac{\sin\left[\frac{1}{2}(d_1\pi + d_2\pi - 2\lambda_1)\right]}{(\lambda_1 - \lambda_2)^{d_1} \lambda_1^{d_2}} - c_2 \frac{\sin\left[\frac{1}{2}(d_1\pi + d_2\pi + 2\lambda_2)\right]}{(\lambda_1 - \lambda_2)^{d_1} (-\lambda_2)^{d_2}} \right)$$

Quasi Distribution





➢ Extrapolation & Fourier transformation(2D):

- Asymptotic in momentum space:

$$F(x_1, x_2, d_1, d_2) = C x_1^{d_1} x_2^{d_1} (1 - x_1 - x_2)^{d_2}$$

↓ FT

$$\tilde{\phi}(z_1, z_2; d_1, d_2) = \int_0^1 dx_1 \int_0^1 dx_2\, e^{-ix_1 z_1 P^z} e^{-ix_2 z_2 P^z} C x_1^{d_1} x_2^{d_1} (1 - x_1 - x_2)^{d_2}$$

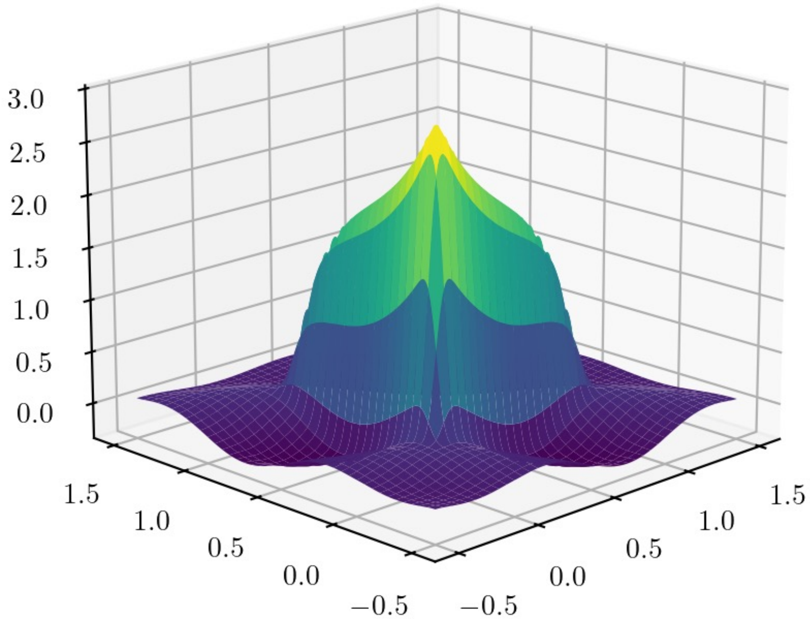
  - Numerically FT as the fit from [computing source cost]
  - Analytica FT and simplified as the fit form [complicated form for baryon]

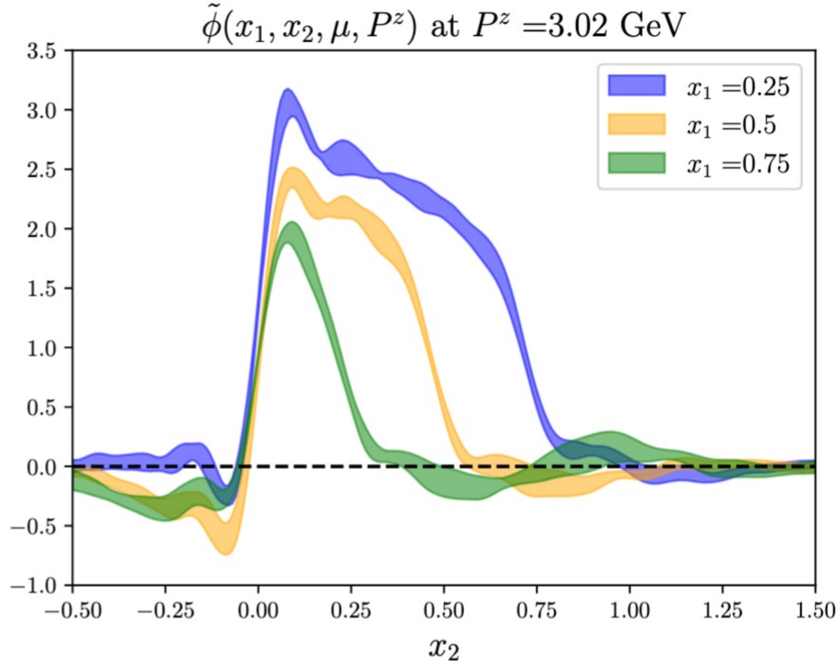
Area2$[z_1 \gg 0, z_2 \gg 0]$:

$$\frac{\Psi(\lambda_1,\lambda_2)}{e^{-\frac{|\lambda_1|}{\lambda_0}-\frac{|\lambda_2|}{\lambda_0}-\frac{|\lambda_1-\lambda_2|}{\lambda_0}}} = c_1\left[\frac{1}{(\lambda_1-\lambda_2)^{d_1}\lambda_1^{d_1}}+\frac{1}{(\lambda_1-\lambda_2)^{d_1}(-\lambda_2)^{d1}}\right] + c_2\frac{\cos\left[\frac{1}{2}(d_1\pi+d_2\pi-2\lambda_1)\right]}{(\lambda_1-\lambda_2)^{d_1}\lambda_1^{d_2}} + c_2\frac{\cos\left[\frac{1}{2}(d_1\pi+d_2\pi+2\lambda_2)\right]}{(\lambda_1-\lambda_2)^{d_1}(-\lambda_2)^{d_2}}$$
$$-i\left(c_2\frac{\sin\left[\frac{1}{2}(d_1\pi+d_2\pi-2\lambda_1)\right]}{(\lambda_1-\lambda_2)^{d_1}\lambda_1^{d_2}} - c_2\frac{\sin\left[\frac{1}{2}(d_1\pi+d_2\pi+2\lambda_2)\right]}{(\lambda_1-\lambda_2)^{d_1}(-\lambda_2)^{d_2}}\right)$$

2-D Effective Matching

- Renormalized quasi-DA in momentum space $\tilde{\psi}(x_1, x_2, P^z)$ (Renormalized by ratio scheme)



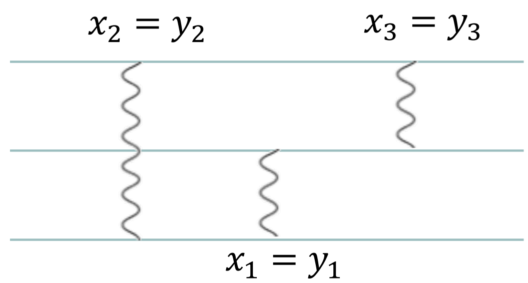


2-D Effective Matching

➢ LaMET factorization for Baryon LCDA:

$$\tilde{\phi}(x_1,x_2)=\int_0^1 dy_1\int_0^{1-y_1} dy_2 C(x_1,x_2,y_1,y_2)\phi(y_1,y_2)+\mathcal{O}\left(\frac{1}{(x_1P^z)^2},\frac{1}{(x_2P^z)^2},\frac{1}{[(1-x_1-x_2)P^z]^2}\right)$$

- Matching kernel:



$$\begin{aligned}C\left(x_1,x_2,y_1,y_2,\mu\right)=&\ \delta\left(x_1-y_1\right)\delta\left(x_2-y_2\right)\\&+\frac{\alpha_s C_F}{2\pi}\left[\left(\frac{1}{4}C_2\left(x_1,x_2,y_1,y_2\right)-\frac{7}{8}\frac{-1}{|x1-y1|}\right)\delta\left(x_2-y_2\right)\right.\\&+\left(\frac{1}{4}C_2\left(x_2,x_1,y_2,y_1\right)-\frac{7}{8}\frac{-1}{|x2-y2|}\right)\delta\left(x_1-y_1\right)\\&\left.+\left(\frac{1}{4}C_3\left(x_1,x_2,y_1,y_2\right)+\frac{1}{4}C_3\left(x_2,x_1,y_2,y_1\right)-\frac{3}{4}\frac{-2}{|x_1-y_1-x_2+y_2|}\right)\delta\left(x_1+x_2-y_1-y_2\right)\right]_{\oplus},\end{aligned}$$

Double plus function

$$\left[g\left(x_1,x_2,y_1,y_2\right)\right]_{\oplus}=g\left(x_1,x_2,y_1,y_2\right)-\delta\left(x_1-y_1\right)\delta\left(x_2-y_2\right)\int dz_1dz_2 g\left(z_1,z_2,y_1,y_2\right)$$

C.Han et.al. JHEP 12 044 (2023), JHEP 07 019 (2024)

2-D Effective Matching

➢ LaMET factorization for Baryon LCDA:

$$\tilde{\phi}(x_1,x_2)=\int_0^1 dy_1\int_0^{1-y_1} dy_2 C(x_1,x_2,y_1,y_2)\phi(y_1,y_2)+\mathcal{O}\left(\frac{1}{(x_1P^z)^2},\frac{1}{(x_2P^z)^2},\frac{1}{[(1-x_1-x_2)P^z]^2}\right)$$

- Inverse matching:

  $C(x_1,x_2,y_1,y_2)$ → 4 Dimensional tensor → Reduce to 2D matrix → inverse

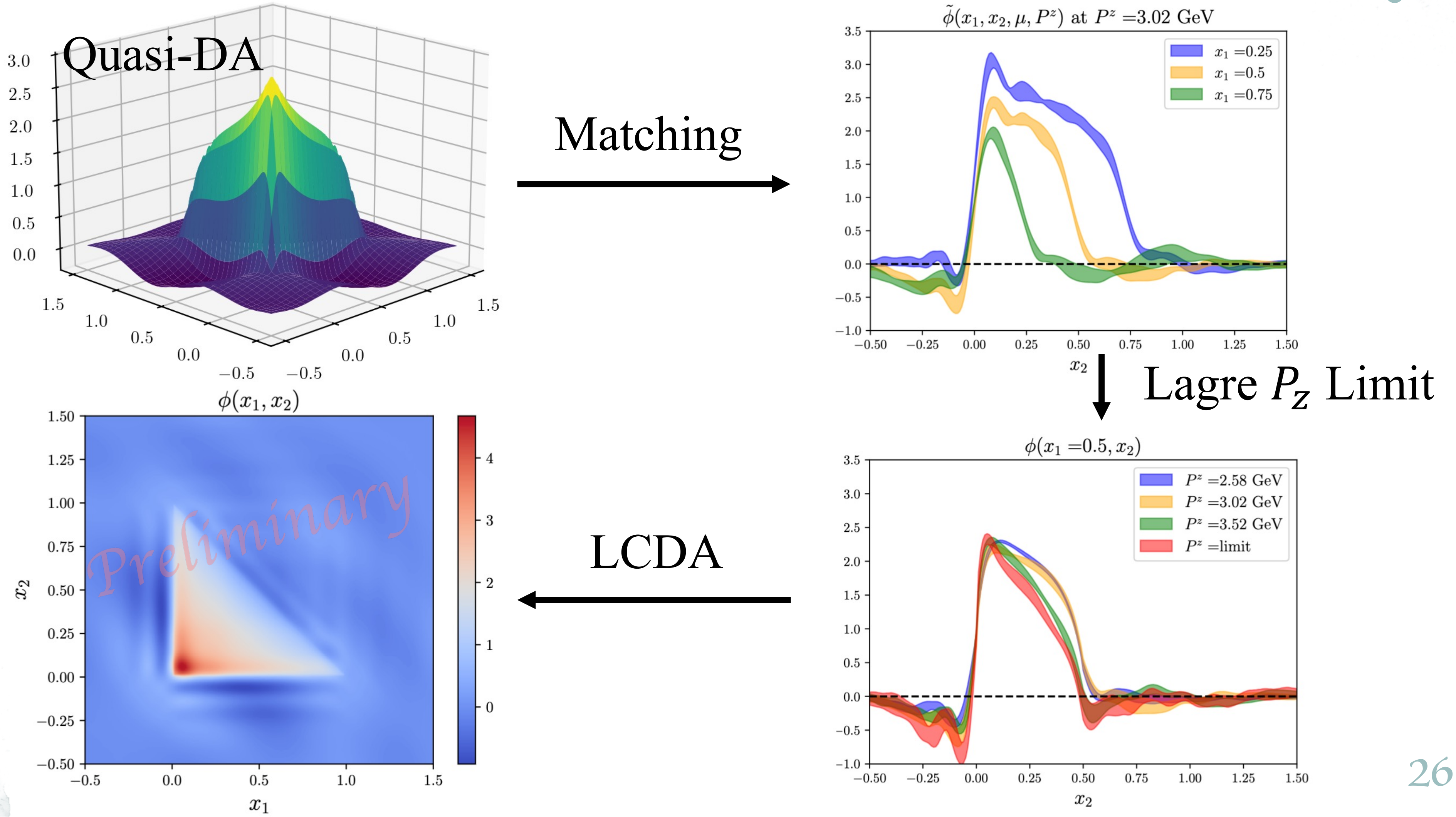
- Iterative matching:

$$\tilde{\phi}(x_1,x_2)=\phi(x_1,x_2)+\frac{\alpha_s C_F}{2\pi}\int_0^1 dy_1\int_0^{1-y_1} dy_2 c^{(1)}(x_1,x_2,y_1,y_2)\phi(y_1,y_2)+\mathcal{O}(\alpha_s^2)$$

The difference between $\tilde{\phi}(x_1,x_2)$ and $\phi(x_1,x_2)$ introduces error only at higher order

$$\phi(x_1,x_2)=\tilde{\phi}(x_1,x_2)-\frac{\alpha_s C_F}{2\pi}\int_0^1 dy_1\int_0^{1-y_1} dy_2 c^{(1)}(x_1,x_2,y_1,y_2)\tilde{\phi}(y_1,y_2)+\mathcal{O}(\alpha_s^2)$$

Numerical results



Summary and outlook

- We made the first attempt to implement the numerical computation of baryon LCDA in the LaMET framework.

- The 3-particle distribution cased 3D structure complexity in several parts:
  - Hybrid renormalization (Match with Perturbative Quasi)
  - Extrapolation and Fourier transformation
  - Matching implementation

- [ ] Calculation with smaller lattice spacing (at least < 0.6 fm)
- [ ] Calculation for all leading twist structure Proton and Lambda LCDA
- [ ] High twist …

Thanks For Your Attention！