



**$B^+ \rightarrow K^+ \nu \bar{\nu}$  excess @ Belle II,**  
**(Dark) SMEFT and NP flavour structure**

**Xing-Bo Yuan (袁兴博)**

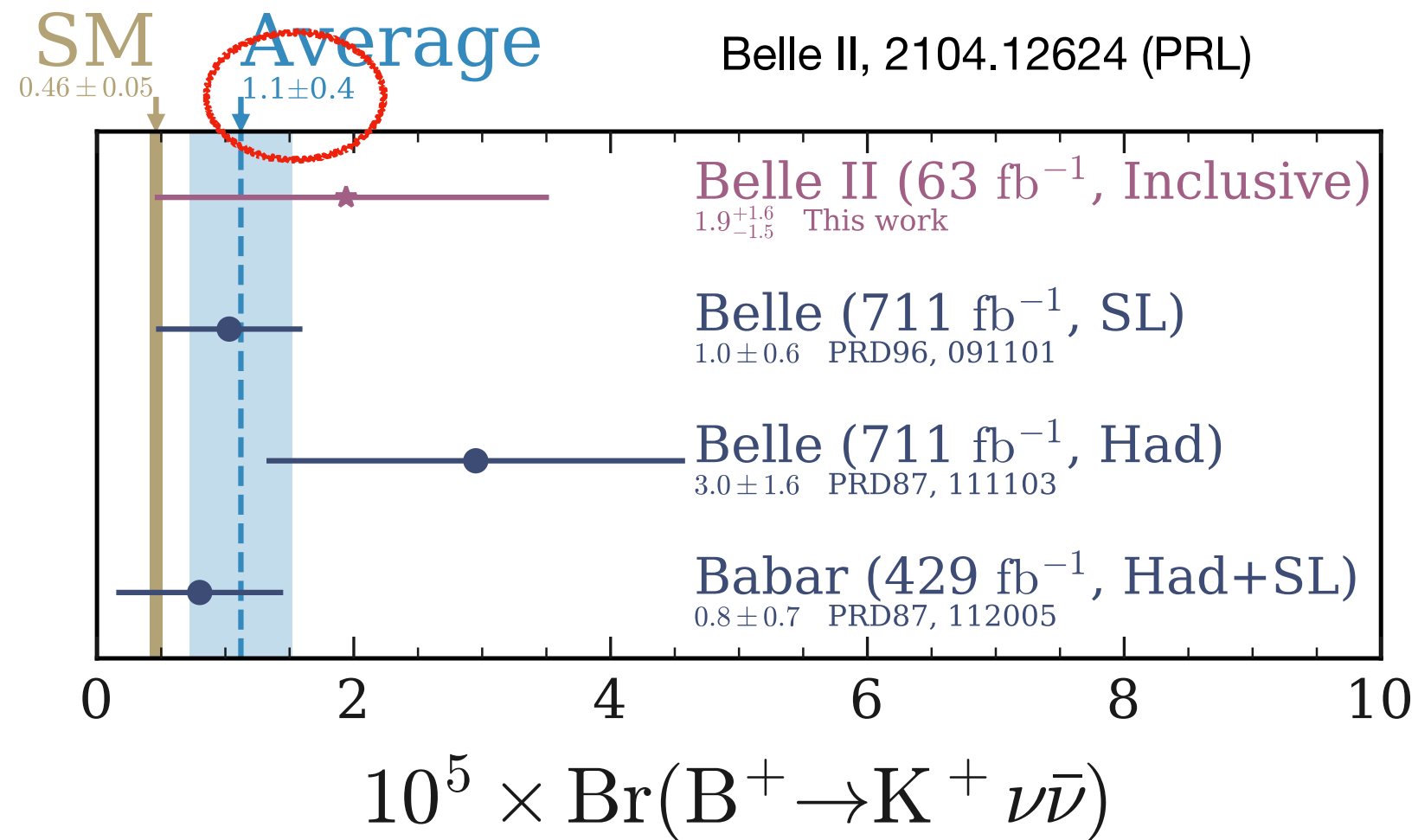
**Central China Normal University (华中师范大学)**

侯镖锋, 李新强, 沈萌, 杨亚东, 袁兴博, arXiv: 2402.19208 [JHEP]

高孟超, 李新强, 杨亚东, 袁兴博, 张欣, work in progress

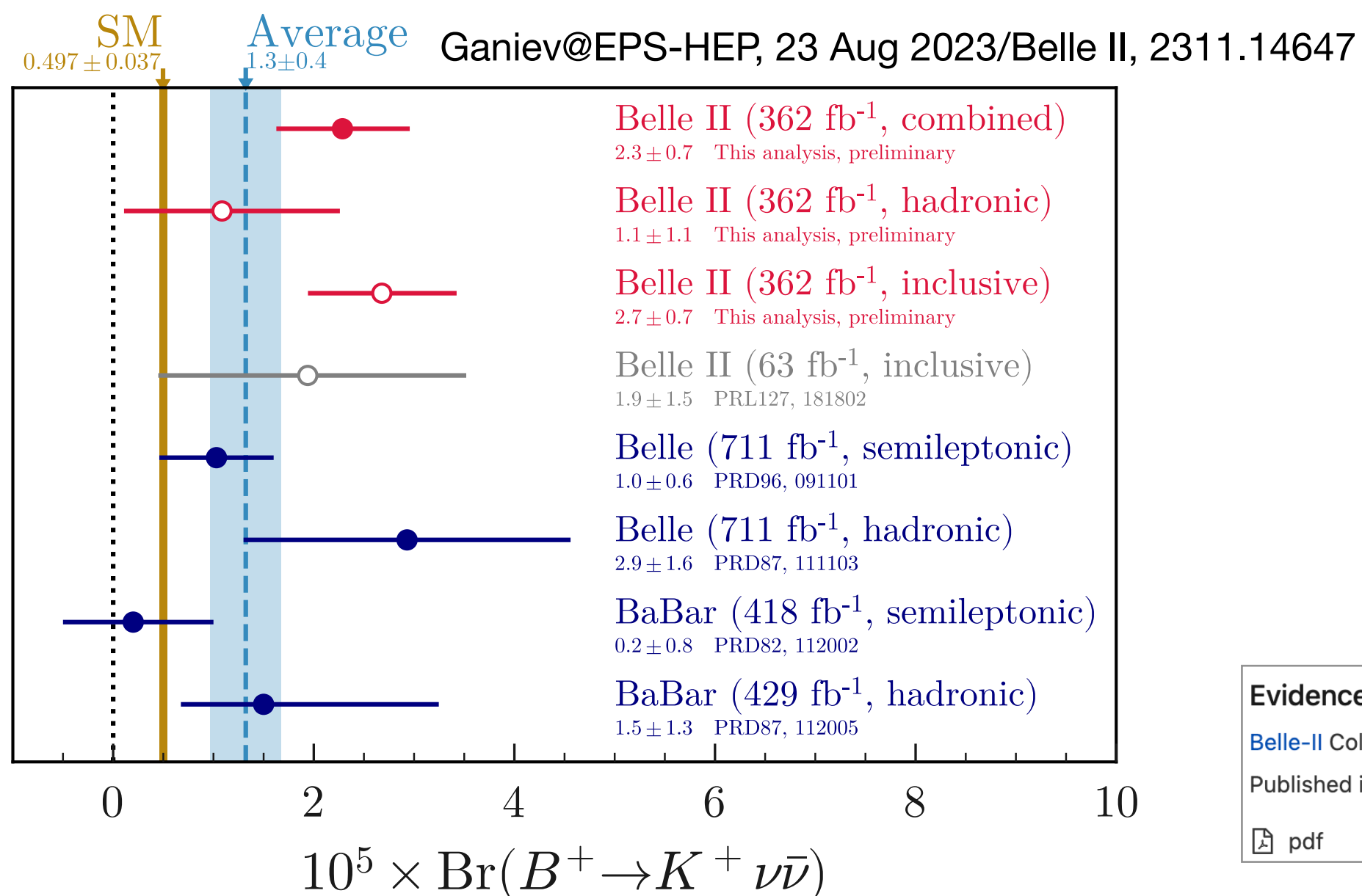
# $b \rightarrow s\nu\bar{\nu}$ : exp & theory

► 2021 Apr



► 2023 Aug

see also 贾森's talk



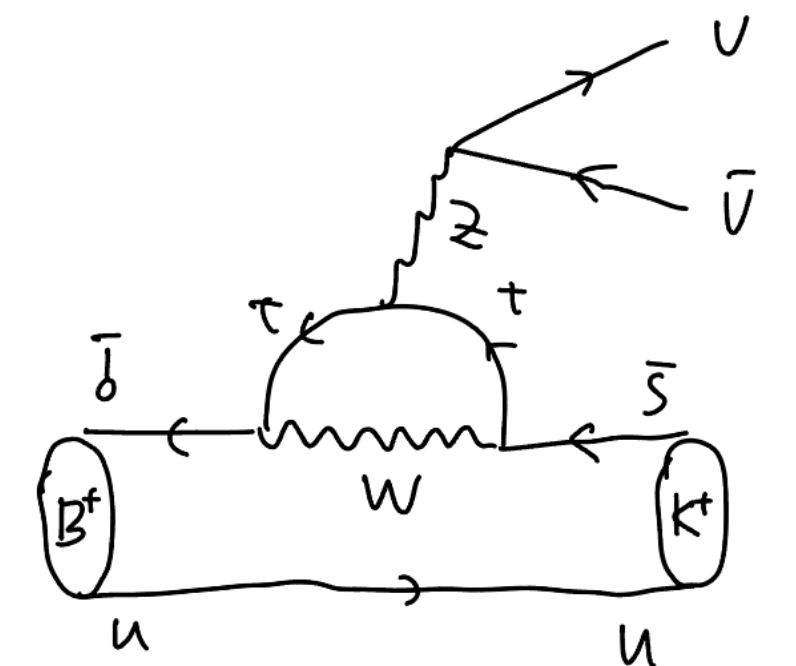
► Exp vs SM [10<sup>-6</sup>]

$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{SM}} = 4.16 \pm 0.57$$

$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{exp}} = 23 \pm 7$$

$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{exp}} \gtrsim 10 \text{ (} 2\sigma \text{ lower bound)}$$

2.7σ difference  
NP/SM ≳ 2



► Theoretical prediction

form factor: 高婧, 李东浩, 沈月龙, WIP (see also 李东浩's talk)

Factorization

$$\mathcal{A} \propto C_L \cdot \langle K | \bar{s} \gamma^\mu b | \bar{B} \rangle \cdot \bar{\nu} \gamma_\mu \nu$$

Wilson coef    quark current    neutrino current

theoretically, simple and clean  
one of the cleanest channels in  
flavour physics

$$\mathcal{O}_L = (\bar{s} \gamma_\mu P_L b) (\bar{\nu} \gamma^\mu P_L \nu) \text{ in the SM}$$

$$\mathcal{O}_L = (\bar{s} P_L b) (\bar{\nu} P_L \nu) \times$$

$$\mathcal{O}_R = (\bar{s} \gamma_\mu P_R b) (\bar{\nu} \gamma^\mu P_L \nu) \text{ possible in BSM}$$

$$\mathcal{O}_R = (\bar{s} P_R b) (\bar{\nu} P_R \nu) \times$$

$$\mathcal{O}_T = (\bar{s} \sigma_{\mu\nu} b) (\bar{\nu} \sigma^{\mu\nu} \nu) \times$$

operator structure highly  
constrained by LH neutrino

$$\mathcal{O}_{T5} = (\bar{s} \sigma_{\mu\nu} \gamma_5 b) (\bar{\nu} \sigma^{\mu\nu} \nu) \times$$

Evidence for  $B^+ \rightarrow K^+ \nu \bar{\nu}$  decays #1

Belle-II Collaboration · I. Adachi et al. (Nov 24, 2023)

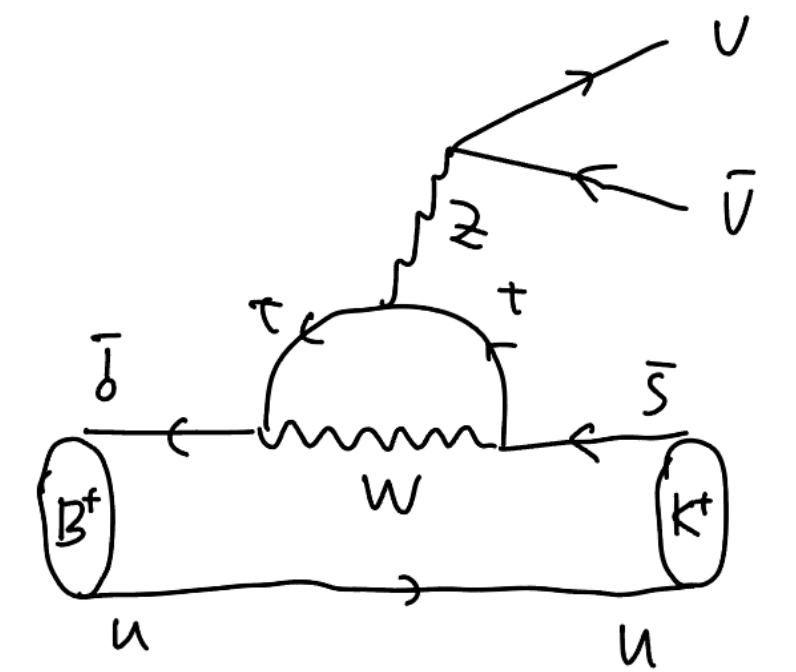
Published in: *Phys.Rev.D* 109 (2024) 11, 112006 · e-Print: 2311.14647 [hep-ex]

pdf    DOI    cite    claim

reference search

83 citations

# $b \rightarrow s\nu\bar{\nu}$ : exp & theory



	Observable	SM	Exp	Unit
$b \rightarrow s$	$\mathcal{B}(B^+ \rightarrow K^+ \nu\bar{\nu})$	$4.16 \pm 0.57$	$23 \pm 5_{-4}^{+5}$	$10^{-6}$
	$\mathcal{B}(B^0 \rightarrow K^0 \nu\bar{\nu})$	$3.85 \pm 0.52$	$< 26$	$10^{-6}$
	$\mathcal{B}(B^+ \rightarrow K^{*+} \nu\bar{\nu})$	$9.70 \pm 0.94$	$< 61$	$10^{-6}$
	$\mathcal{B}(B^0 \rightarrow K^{*0} \nu\bar{\nu})$	$9.00 \pm 0.87$	$< 18$	$10^{-6}$
	$\mathcal{B}(B_s \rightarrow \phi \nu\bar{\nu})$	$9.93 \pm 0.72$	$< 5400$	$10^{-6}$
	$\mathcal{B}(B_s \rightarrow \nu\bar{\nu})$	$\approx 0$	$< 5.9$	$10^{-4}$
$b \rightarrow d$	$\mathcal{B}(B^+ \rightarrow \pi^+ \nu\bar{\nu})$	$1.40 \pm 0.18$	$< 140$	$10^{-7}$
	$\mathcal{B}(B^0 \rightarrow \pi^0 \nu\bar{\nu})$	$6.52 \pm 0.85$	$< 900$	$10^{-8}$
	$\mathcal{B}(B^+ \rightarrow \rho^+ \nu\bar{\nu})$	$4.06 \pm 0.79$	$< 300$	$10^{-7}$
	$\mathcal{B}(B^0 \rightarrow \rho^0 \nu\bar{\nu})$	$1.89 \pm 0.36$	$< 400$	$10^{-7}$
	$\mathcal{B}(B^0 \rightarrow \nu\bar{\nu})$	$\approx 0$	$< 1.4$	$10^{-4}$
$s \rightarrow d$	$\mathcal{B}(K^+ \rightarrow \pi^+ \nu\bar{\nu})$	$8.42 \pm 0.61$	$10.6_{-3.4}^{+4.0} \pm 0.9$	$10^{-11}$
	$\mathcal{B}(K_L \rightarrow \pi^0 \nu\bar{\nu})$	$3.41 \pm 0.45$	$< 300$	$10^{-11}$

## ► Exp vs SM [10<sup>-6</sup>]

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$$\mathcal{B}(B^+ \rightarrow K^+ \nu\bar{\nu})_{\text{exp}} \gtrsim 10 \text{ (} 2\sigma \text{ lower bound)}$$

**2.7 $\sigma$  difference**  
**NP/SM  $\gtrsim 2$**

## ► Theoretical prediction form factor: 高婧, 李东浩, 沈月龙, WIP (see also 李东浩's talk)

### Factorization

$$\mathcal{A} \propto C_L \cdot \langle K | \bar{s} \gamma^\mu b | \bar{B} \rangle \cdot \bar{\nu} \gamma_\mu \nu$$

Wilson coef    quark current    neutrino current

**theoretically, simple and clean**  
**one of the cleanest channels in flavour physics**

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operator structure highly constrained by LH neutrino

$$\mathcal{O}_{T5} = (\bar{s} \sigma_{\mu\nu} \gamma_5 b)(\bar{\nu} \sigma^{\mu\nu} \nu) \times$$

**Why such a large NP effect has not shown up in other  $b \rightarrow s$  decays ?**  
**in  $b \rightarrow d, s \rightarrow d$  decays ?**



# $b \rightarrow s\nu\bar{\nu}$ : exp & theory

	Observable	SM	Exp	Unit
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Why such a large NP effect has not shown up in other  $b \rightarrow s$  decays ?  
in  $b \rightarrow d, s \rightarrow d$  decays ? **NP flavour structure**

**SMEFT**

$$\mathcal{Q}_{Hq}^{(1)} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{q}_p \gamma^\mu q_r),$$

$$\mathcal{Q}_{Hq}^{(3)} = (H^\dagger i \overleftrightarrow{D}_\mu^I H) (\bar{q}_p \tau^I \gamma^\mu q_r),$$

$$\mathcal{Q}_{Hd} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{d}_p \gamma^\mu d_r),$$

$$\mathcal{Q}_{ld} = (\bar{l}_p \gamma^\mu l_r) (\bar{d}_s \gamma_\mu d_t),$$

$$\mathcal{Q}_{lq}^{(1)} = (\bar{l}_p \gamma^\mu l_r) (\bar{q}_s \gamma_\mu q_t),$$

$$\mathcal{Q}_{lq}^{(3)} = (\bar{l}_p \gamma^\mu \tau^I l_r) (\bar{q}_s \tau^I \gamma_\mu q_t),$$

$\mu_{EW}$

**LEFT**

$$\mathcal{O}_L^{\nu_i \nu_j} = (\bar{s} \gamma_\mu P_L b) (\bar{\nu}_i \gamma^\mu P_L \nu_j)$$

$$\mathcal{O}_R^{\nu_i \nu_j} = (\bar{s} \gamma_\mu P_R b) (\bar{\nu}_i \gamma^\mu P_L \nu_j)$$

$\mu_b$

operator structure highly constrained by Left-handed neutrino



# Minimal Flavour Violation

- ▶ Flavour symmetry without Yukawa

$$G_{\text{QF}} = SU(3)_q \otimes SU(3)_u \otimes SU(3)_d$$

- ▶ Flavour symmetry breaking only from SM Yukawa

$$-\mathcal{L}_Y = \bar{q} Y_d H d + \bar{q} Y_u \tilde{H} u + \text{h.c.}$$

- ▶ Flavour symmetry recovering: Yukawa coupling  $\implies$  spurion field

$$Y_u \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}) \quad Y_d \sim (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}})$$

D'Ambrosio, Giudice, Isidori, Strumia, 2009

- ▶ EFT with MFV: operators, constructed from SM and Yukawa spurion fields, are invariant under CP and  $G_{\text{QF}}$

$$\mathcal{C}^{\text{MFV}} = \begin{cases} f(A, B) & \text{for } \bar{q}\gamma^\mu \mathcal{C}q, \\ f(A, B)Y_d & \text{for } \bar{q}\mathcal{C}d, \bar{q}\sigma^{\mu\nu}\mathcal{C}d, \\ \epsilon_0\mathbb{1} + Y_d^\dagger g(A, B)Y_d & \text{for } \bar{d}\gamma^\mu \mathcal{C}d, \end{cases} \quad \begin{aligned} f(A, B) &= \epsilon_0\mathbb{1} + \epsilon_1 A + \epsilon_2 B + \epsilon_3 A^2 + \epsilon_4 B^2 + \epsilon_5 AB + \dots \\ A &= Y_u Y_u^\dagger \\ B &= Y_d Y_d^\dagger \end{aligned}$$

# Minimal Flavour Violation

- ▶ Spurion function

$$f(A, B) = \epsilon_0 \mathbb{1} + \epsilon_1 A + \epsilon_2 B + \epsilon_3 A^2 + \epsilon_4 B^2 + \epsilon_5 AB + \dots$$

- ▶ Cayley-Hamilton identity for  $3 \times 3$  invertible matrix  $X$

$$X^3 = \text{Det}X \cdot \mathbb{1} + \frac{1}{2}[\text{Tr}X^2 - (\text{Tr}X)^2] \cdot X + \text{Tr}X \cdot X^2$$

- ▶ Spurion function after resummation

$$f(A, B) = \epsilon_0 \mathbb{1} + \epsilon_1 A + \epsilon_3 A^2 + \epsilon_5 AB + \epsilon_7 ABA + \epsilon_{10} AB^2 + \epsilon_{12} A^2 B^2 + \epsilon_{14} B^2 AB + \epsilon_{15} AB^2 A^2 \\ + \epsilon_2 B + \epsilon_4 B^2 + \epsilon_6 BA + \epsilon_9 BAB + \epsilon_8 BA^2 + \epsilon_{13} B^2 A^2 + \epsilon_{11} ABA^2 + \epsilon_{16} B^2 A^2 B.$$

- ▶ assumption #1: neglect tiny imaginary parts of  $\epsilon_i$
- ▶ assumption #2: neglect spurion B (suppressed by  $\mathcal{O}(\lambda_d^2)$ )

$$f(A, B) \approx \epsilon_0 \mathbb{1} + \epsilon_1 A + \epsilon_2 A^2$$

Colangelo, Nikolidakis, Smith, 2009  
Mercolli, Smith, 2009

# Minimal Flavour Violation

- ▶ MFV coupling      FCNC controlled by CKM

$$C^{\text{MFV}} = \begin{cases} \epsilon_0 1 + \epsilon_1 \Delta_q & \text{for } \bar{d}_L \gamma^\mu C d_L \\ \epsilon_0 \hat{\lambda}_d + \epsilon_1 \Delta_q \hat{\lambda}_d & \text{for } \bar{d}_L C d_R, \bar{d}_L \sigma^{\mu\nu} C d_R \\ \epsilon_0 1 & \text{for } \bar{d}_R \gamma^\mu C d_R \end{cases} \quad \Delta_q = V^\dagger \hat{\lambda}_u^2 V$$

**No Right-handed down-type FCNC !**

- ▶ Numerics

$$\Delta_q = \begin{pmatrix} 0.8 & -3.3 - 1.5i & 79.3 + 35.4i \\ -3.3 + 1.5i & 16.6 & -397.5 + 8.1i \\ 79.3 - 35.4i & -397.5 - 8.1i & 9839.0 \end{pmatrix} \times 10^{-4}$$

$$\Delta_q \hat{\lambda}_d = \begin{pmatrix} 0.0021 & -0.18 - 0.08i & 191.3 + 85.4i \\ -0.009 + 0.004i & 0.88 & -958.7 + 19.6i \\ 0.21 - 0.10i & -21.1 - 0.4i & 23728.1 \end{pmatrix} \times 10^{-6}$$



# $b \rightarrow s\nu\bar{\nu}$ : SMEFT with MFV

## ► Prediction

$$\frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})}{\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})} = \frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{SM}}}{\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{SM}}} = 0.46 \pm 0.07$$

$$\frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})}{\mathcal{B}(B^+ \rightarrow \pi^+ \nu \bar{\nu})} = \frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{SM}}}{\mathcal{B}(B^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}}} = 29.7 \pm 5.6$$

## ► prediction

$$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{SM}} = (9.00 \pm 0.87) \times 10^{-6}$$

$$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{MFV}} = (50_{-16}^{+17}) \times 10^{-6}$$

$$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{exp}} < 18 \times 10^{-6}$$

$$\mathcal{B}(B^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = (1.40 \pm 0.18) \times 10^{-7}$$

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SMEFT

$$Q_{Hq}^{(1)} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{q}_p \gamma^\mu q_r),$$

$$Q_{Hq}^{(3)} = (H^\dagger i \overleftrightarrow{D}_\mu^I H) (\bar{q}_p \tau^I \gamma^\mu q_r),$$

$$Q_{Hd} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{d}_p \gamma^\mu d_r),$$

induce  $\bar{s}bZ$  interaction,  
Thus, universally affect  
 $b \rightarrow se^+e^-, \mu^+\mu^-, \tau^+\tau^-$

$$Q_{ld} = (\bar{l}_p \gamma^\mu l_r) (\bar{d}_s \gamma_\mu d_t),$$

forbidden by MFV

$$Q_{lq}^{(1)} = (\bar{l}_p \gamma^\mu l_r) (\bar{q}_s \gamma_\mu q_t),$$

$$Q_{lq}^{(3)} = (\bar{l}_p \gamma^\mu \tau^I l_r) (\bar{q}_s \tau^I \gamma_\mu q_t),$$

$\mu_{\text{EW}}$

LEFT

$$O_L^{\nu_i \nu_j} = (\bar{s} \gamma_\mu P_L b) (\bar{\nu}_i \gamma^\mu P_L \nu_j)$$

$$O_R^{\nu_i \nu_j} = (\bar{s} \gamma_\mu P_R b) (\bar{\nu}_i \gamma^\mu P_L \nu_j)$$

**one LEFT operator !**  
**just the SM operator**

$\mu_b$

# $b \rightarrow s\nu\bar{\nu}$ : SMEFT with MFV

## ► Prediction

$$\frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})}{\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})} = \frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{SM}}}{\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{SM}}} = 0.46 \pm 0.07$$
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## ► prediction

$$\left. \begin{aligned} \mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{SM}} &= (9.00 \pm 0.87) \times 10^{-6} \\ \mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{MFV}} &= (50^{+17}_{-16}) \times 10^{-6} \\ \mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{exp}} &< 18 \times 10^{-6} \end{aligned} \right\} \text{Inconsistent} \longrightarrow$$

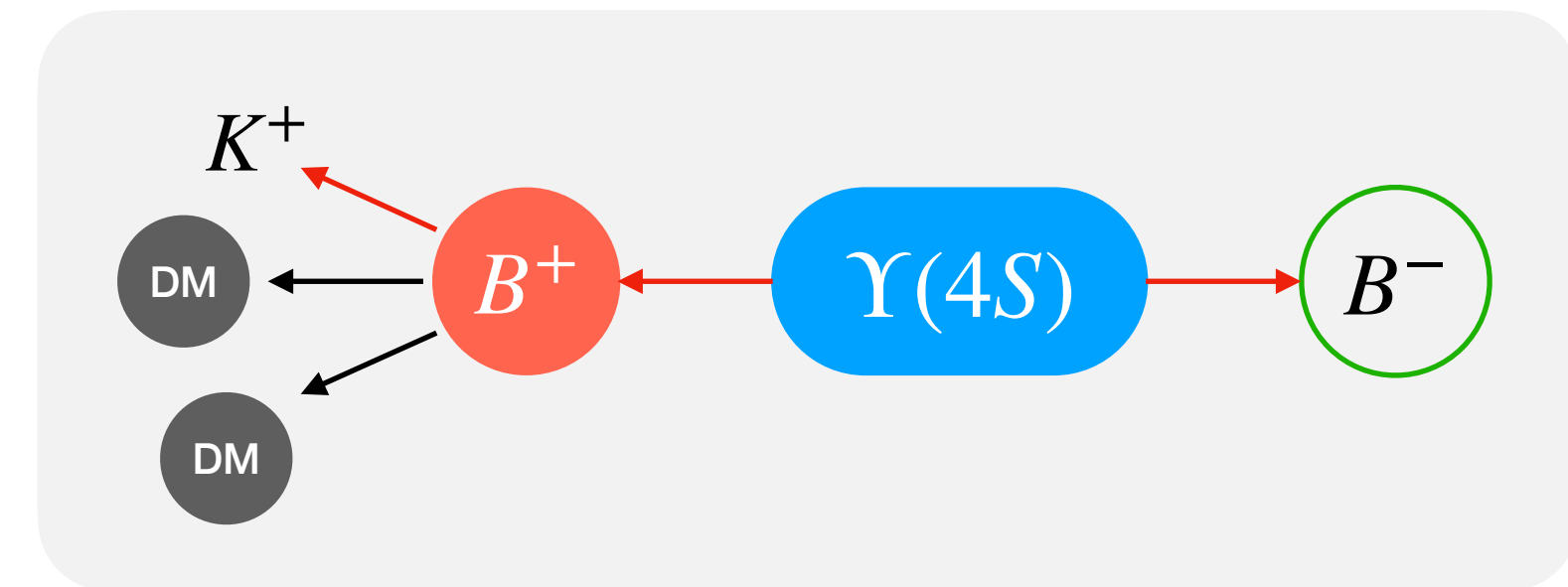
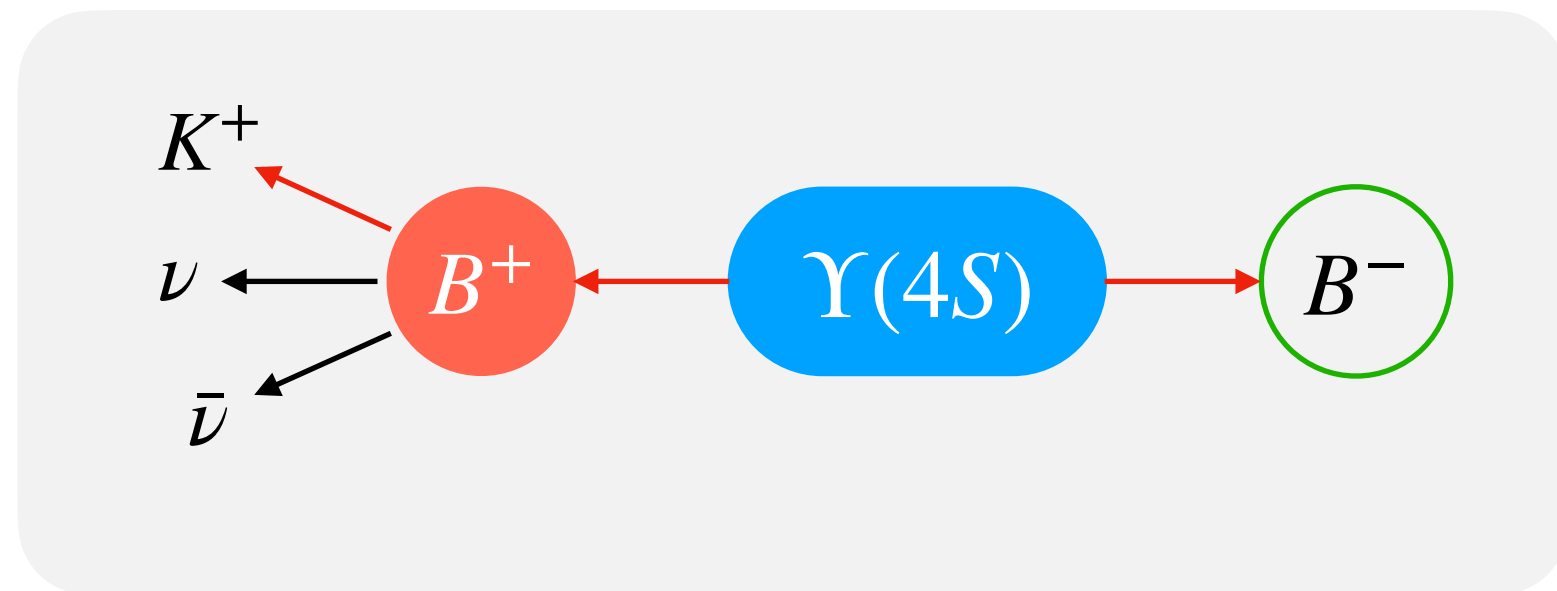
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$$\mathcal{B}(B^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{exp}} < 140 \times 10^{-7}$$

Belle II excess (if confirmed in the future) implies:

- impossible to explain in SMEFT with MFV
- NP flavour structure is highly non-trivial
- **NP structure in quark sector is beyond MFV**
- **flavour violation is beyond Yukawa coupling**

This conclusion only assumes the quark MFV.  
No lepton flavour structure is assumed.

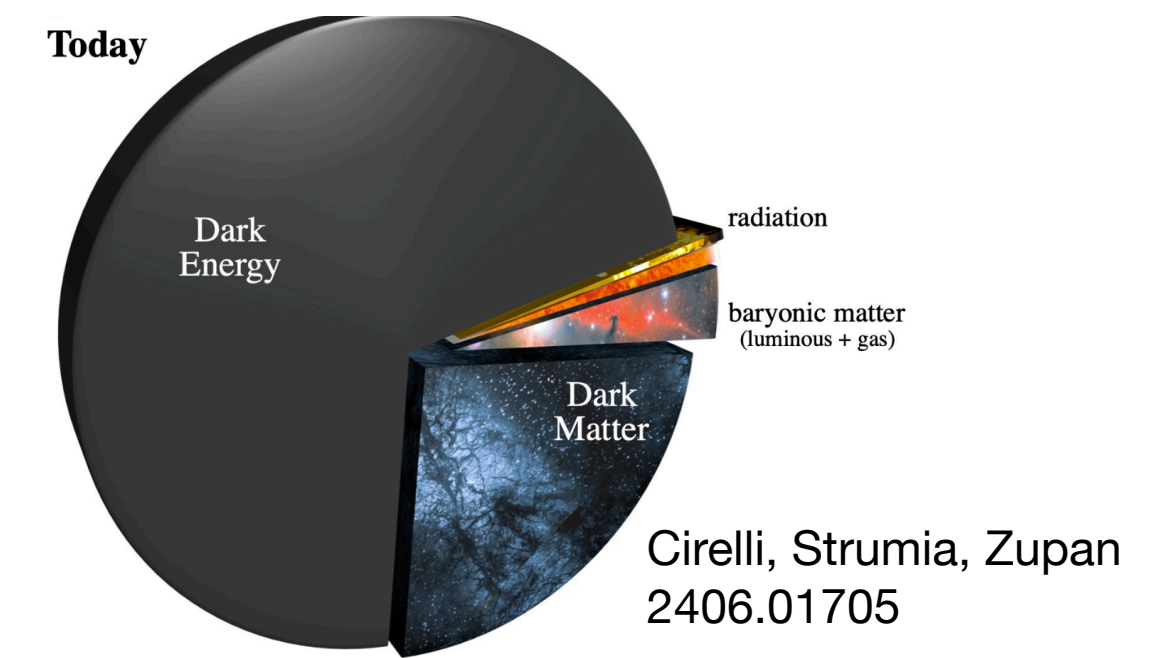
$b \rightarrow s\nu\bar{\nu}$ : exp picture





# Light DM: a bottom-up view

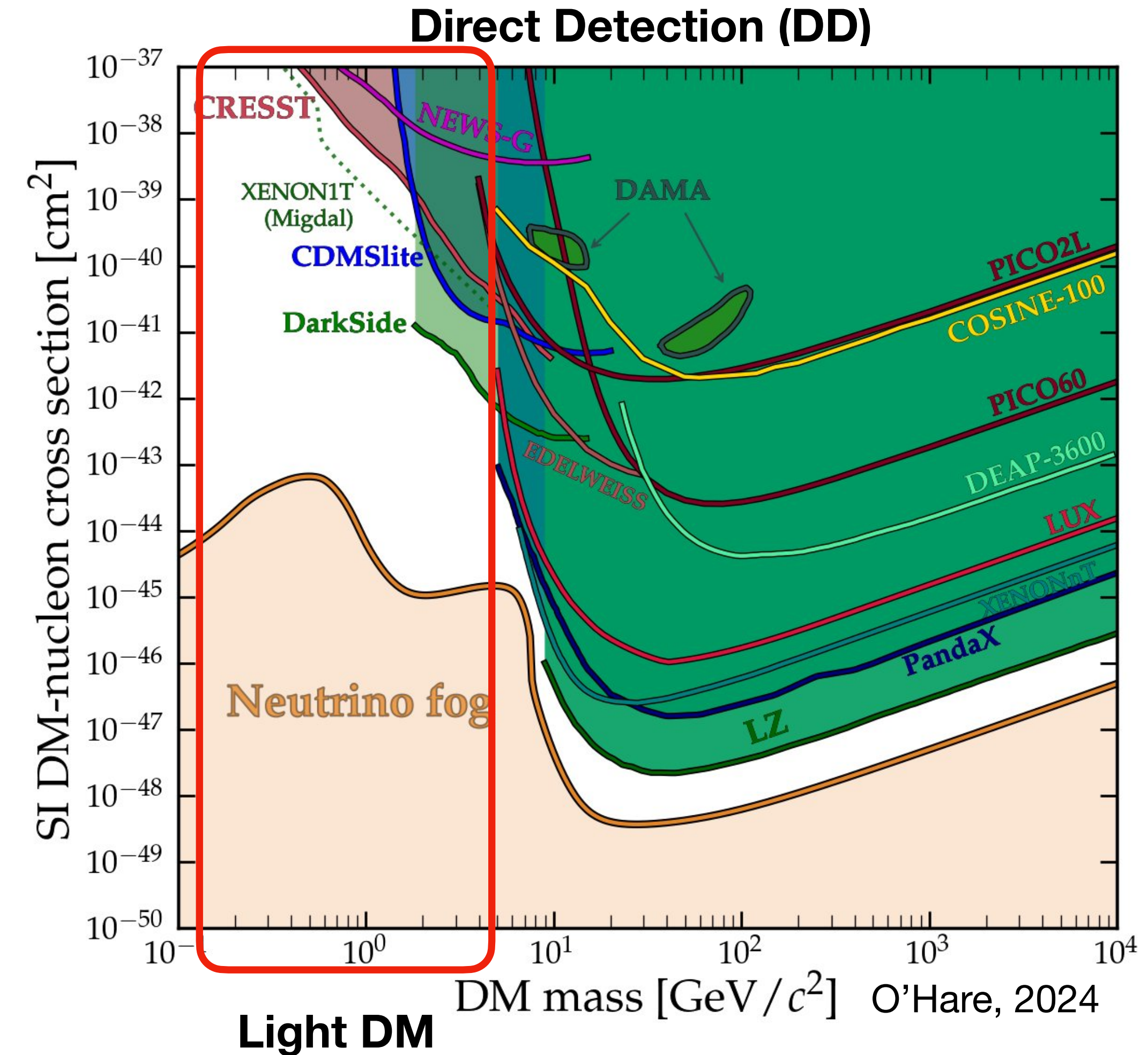
DM is electrically neutral !  $\implies$  DM only have FC or FCNC couplings to quarks !



	$d$	$s$	$b$
$d$	DD	NA62/KOTO	Belle II
$s$		DD	Belle II
$b$			Belle II/LHC
	$u$	$c$	$t$
$u$	DD	BES/STCF	LHC
$c$		BES/STCF	LHC
$t$			LHC

example:  
 $B^+ \rightarrow K^+ + \text{DM} + \text{DM}$   
 $K^+ \rightarrow \pi^+ + \text{DM} + \text{DM}$   
 $D^0 \rightarrow \pi^0 + \text{DM} + \text{DM}$

   means related to the DM relic density



# Effective Field Theory approach to combine the various experimental searches

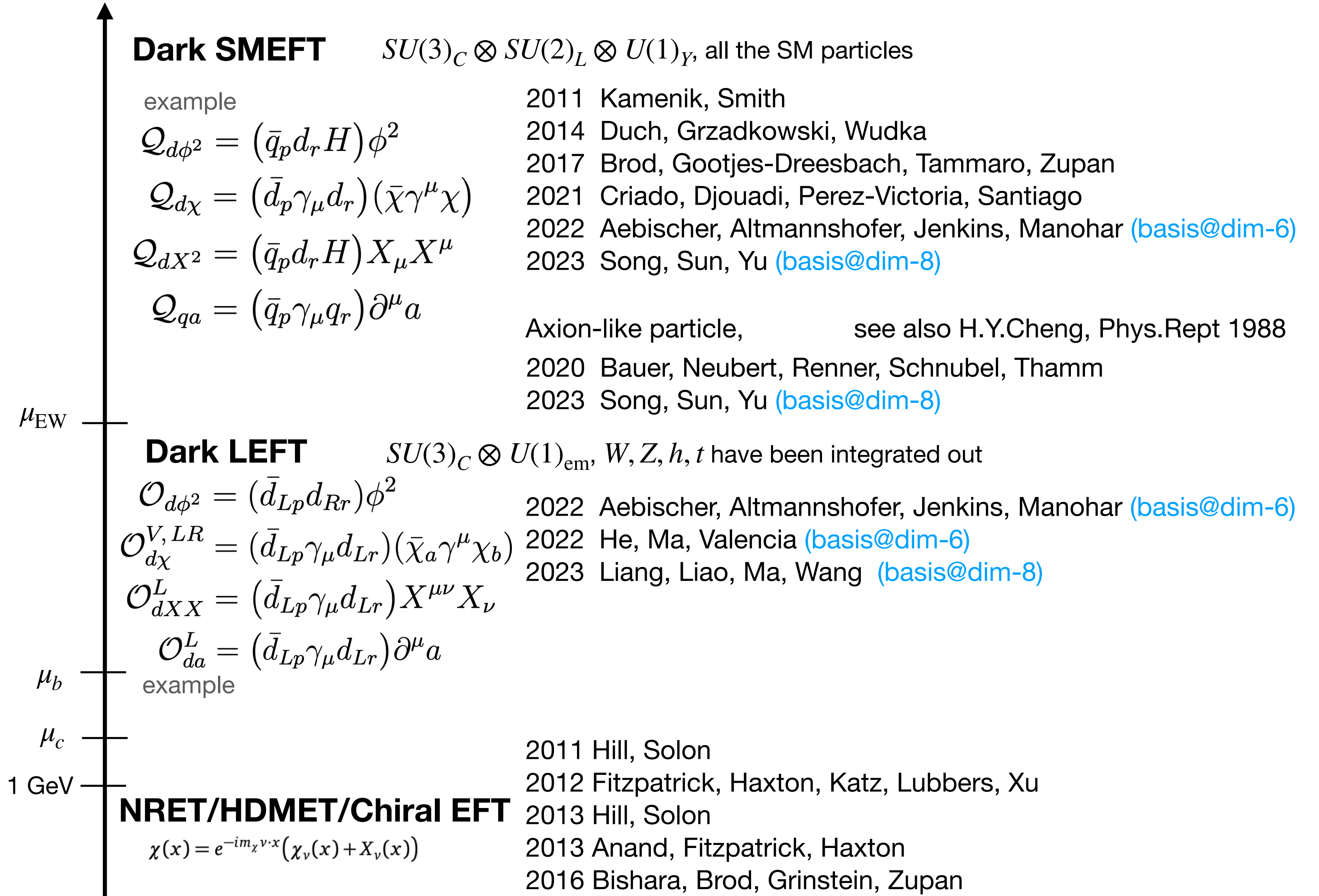
In EFT, DM is a just singlet under the SM gauge group.

connect to UV model

direct detection

relic density

hadron decay





# $H_1 \rightarrow H_2 + \mathbf{DM}$ theoretical calculation and experimental searches

## ▶ $d_i \rightarrow d_j + \phi + \phi$

2011 Kamenik, Smith

2004 Bird, Jackson, Kowalewski, Pospelov

2019 G.Li, J.Y. Su, Tandean

$$\Lambda \rightarrow n + \phi\phi, \Sigma^+ \rightarrow p + \phi\phi, \Xi^0 \rightarrow \Lambda + \phi\phi,$$

$$\Xi^- \rightarrow \Sigma^- \phi\phi, \Omega^- \rightarrow \Sigma^- + \phi\phi$$

2020 X.G. He, X.D. Ma, Tandean, Valencia

2020 C.Q.Geng, Tandean,  $K \rightarrow \pi\pi + \phi\phi$

2021 G. Li, T. Wang, J.B. Zhang, G.L. Wang

2022 Kling, S. Li, H. Song, S. Su, W. Su

## ▶ $d_i \rightarrow d_j + \chi + \chi$

2011 Kamenik, Smith

2019 J.Y. Su, Tandean

2020 G. Li, T. Wang, Y. Jiang, J.B. Zhang, G.L. Wang

2021 Felkl, S. L. Li, Schmidt

## ▶ $d_i \rightarrow d_j + X + X$

2011 Kamenik, Smith

2021 G. Li, T. Wang, J.B. Zhang, G.L. Wang

2022 X.G. He, X.D. Ma, Valencia

## ▶ $d_i \rightarrow d_j + a$

2020 Camalich, Pospelov, Vuong, Ziegler, Zupan,

2021 Bauer, Neubert, Renner, Schnubel, Thamm

2022 Guerrero and S. Rigolin

▶ theoretically clean:  $A \propto C \cdot \langle H_1 | O | H_2 \rangle$  (tiny LD contribution)

▶ no GIM suppression

▶ possibly two-body decay

} **enhancement**

Observable
$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$
$\mathcal{B}(B^0 \rightarrow K^0 \nu \bar{\nu})$
$\mathcal{B}(B^+ \rightarrow K^{*+} \nu \bar{\nu})$
$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})$
$\mathcal{B}(B_s \rightarrow \phi \nu \bar{\nu})$
$\mathcal{B}(B^+ \rightarrow \pi^+ \nu \bar{\nu})$
$\mathcal{B}(B^0 \rightarrow \pi^0 \nu \bar{\nu})$
$\mathcal{B}(B^+ \rightarrow \rho^+ \nu \bar{\nu})$
$\mathcal{B}(B^0 \rightarrow \rho^0 \nu \bar{\nu})$
$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$
$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$
$\mathcal{B}(B_s \rightarrow \nu \bar{\nu})$
$\mathcal{B}(B^0 \rightarrow \nu \bar{\nu})$
$\nu \rightarrow \mathbf{DM}$

## ▶ $c \rightarrow u + \mathbf{DM}$

2022 C.Q.Geng, G.Li

2023 G.Li, Tandean

$$D^0 \rightarrow s\bar{s}'$$

$$D^0 \rightarrow \gamma s\bar{s}'$$

$$D^0 \rightarrow \pi^0 s\bar{s}'$$

$$D^+ \rightarrow \pi^+ s\bar{s}'$$

$$D_s^+ \rightarrow K^+ s\bar{s}'$$

$$D^0 \rightarrow \rho^0 s\bar{s}'$$

$$D^+ \rightarrow \rho^+ s\bar{s}'$$

$$D_s^+ \rightarrow K^{*+} s\bar{s}'$$

$$\Lambda_c^+ \rightarrow p s\bar{s}'$$

$$\Xi_c^+ \rightarrow \Sigma^+ s\bar{s}'$$

$$\Xi_c^0 \rightarrow \Sigma^0 s\bar{s}'$$

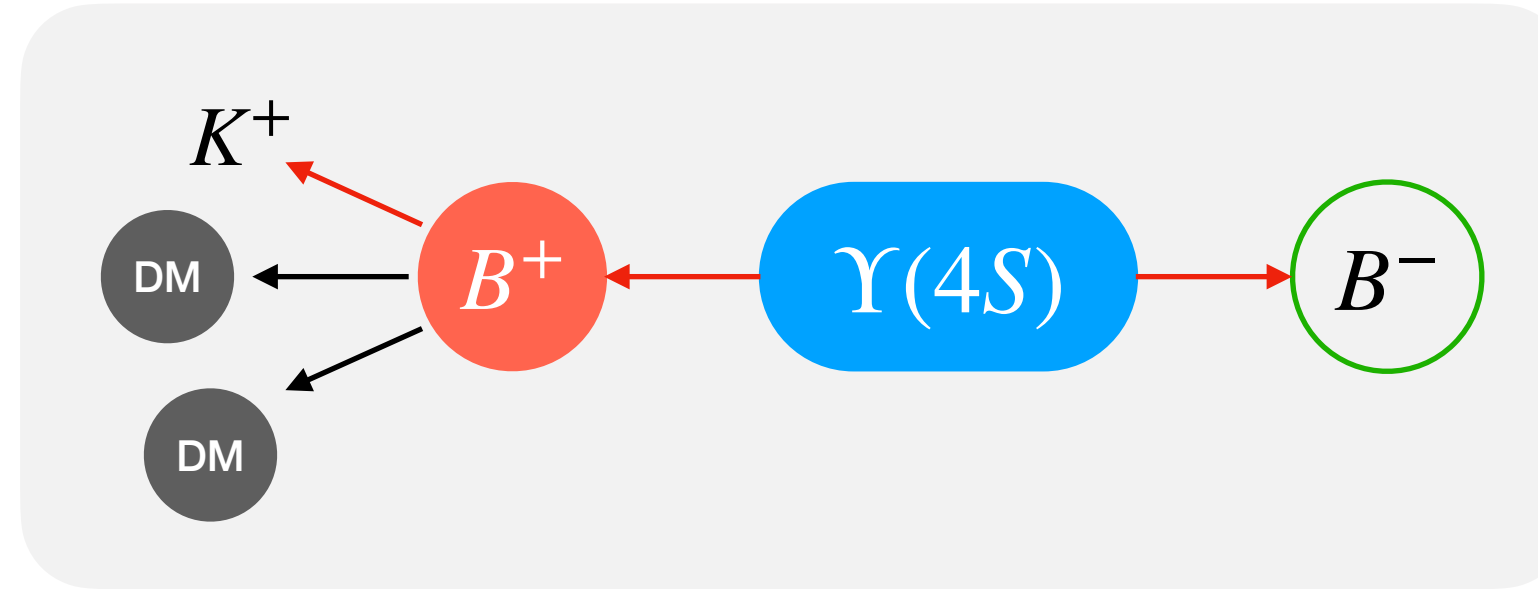
$$\Xi_c^0 \rightarrow \Lambda s\bar{s}'$$

**HadronToNP**: a package to calculate decay of hadron to new particles  
 B.F. Hou, X.Q. Li, H.Yan, Y.D. Yang, **XY** *to be finished*



# $b \rightarrow s\nu\bar{\nu}$ : DSMEFT

Can DSMEFT operators explain the Belle II excess, while satisfy other  $b \rightarrow s$  bounds ?



Observable	SM	Exp	Unit
$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$	$4.16 \pm 0.57$	$23 \pm 5_{-4}^{+5}$	$10^{-6}$
$\mathcal{B}(B^0 \rightarrow K^0 \nu \bar{\nu})$	$3.85 \pm 0.52$	$< 26$	$10^{-6}$
$\mathcal{B}(B^+ \rightarrow K^{*+} \nu \bar{\nu})$	$9.70 \pm 0.94$	$< 61$	$10^{-6}$
$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})$	$9.00 \pm 0.87$	$< 18$	$10^{-6}$
$\mathcal{B}(B_s \rightarrow \phi \nu \bar{\nu})$	$9.93 \pm 0.72$	$< 5400$	$10^{-6}$
$\mathcal{B}(B_s \rightarrow \nu \bar{\nu})$	$\approx 0$	$< 5.9$	$10^{-4}$
$\mathcal{B}(B^+ \rightarrow \pi^+ \nu \bar{\nu})$	$1.40 \pm 0.18$	$< 140$	$10^{-7}$
$\mathcal{B}(B^0 \rightarrow \pi^0 \nu \bar{\nu})$	$6.52 \pm 0.85$	$< 900$	$10^{-8}$
$\mathcal{B}(B^+ \rightarrow \rho^+ \nu \bar{\nu})$	$4.06 \pm 0.79$	$< 300$	$10^{-7}$
$\mathcal{B}(B^0 \rightarrow \rho^0 \nu \bar{\nu})$	$1.89 \pm 0.36$	$< 400$	$10^{-7}$
$\mathcal{B}(B^0 \rightarrow \nu \bar{\nu})$	$\approx 0$	$< 1.4$	$10^{-4}$
$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$	$8.42 \pm 0.61$	$10.6_{-3.4}^{+4.0} \pm 0.9$	$10^{-11}$
$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$	$3.41 \pm 0.45$	$< 300$	$10^{-11}$



## Dark SMEFT

$$\mathcal{Q}_{d\phi} = (\bar{q}_p d_r H) \phi + \text{h.c.}, \quad \mathcal{Q}_{d\phi^2} = (\bar{q}_p d_r H) \phi^2 + \text{h.c.},$$

$$\mathcal{Q}_{\phi q} = (\bar{q}_p \gamma_\mu q_r) (i\phi_1 \overleftrightarrow{\partial}^\mu \phi_2), \quad \mathcal{Q}_{\phi d} = (\bar{d}_p \gamma_\mu d_r) (i\phi_1 \overleftrightarrow{\partial}^\mu \phi_2),$$

$$\mathcal{Q}_{q\chi} = (\bar{q}_p \gamma_\mu q_r) (\bar{\chi} \gamma^\mu \chi), \quad \mathcal{Q}_{d\chi} = (\bar{d}_p \gamma_\mu d_r) (\bar{\chi} \gamma^\mu \chi),$$

$$\mathcal{Q}_{dHX} = (\bar{q}_p \sigma_{\mu\nu} d_r) H X^{\mu\nu} \quad \mathcal{Q}_{dX^2} = (\bar{q}_p d_r H) X_\mu X^\mu$$

$$\mathcal{Q}_{qa} = (\bar{q}_p \gamma_\mu q_r) \partial^\mu a \quad \mathcal{Q}_{da} = (\bar{d}_p \gamma_\mu d_r) \partial^\mu a$$

scalar: 4

fermion: 2

vector: 1+13

ALP: 2

## Dark LEFT

$$\mathcal{O}_{d\phi} = (\bar{d}_{Lp} d_{Rr}) \phi + \text{h.c.}, \quad \mathcal{O}_{\phi d}^L = (\bar{d}_{Lp} \gamma_\mu d_{Lr}) (i\phi_1 \overleftrightarrow{\partial}^\mu \phi_2),$$

$$\mathcal{O}_{d\chi}^{V,LR} = (\bar{d}_{Lp} \gamma_\mu d_{Lr}) (\bar{\chi}_a \gamma^\mu \chi_b), \quad \mathcal{O}_{d\chi}^{V,RR} = (\bar{d}_{Rp} \gamma_\mu d_{Rr}) (\bar{\chi}_a \gamma^\mu \chi_b),$$

$$\mathcal{O}_{dX}^T = (\bar{d}_{Lp} \sigma_{\mu\nu} d_{Rr}) X_a^{\mu\nu} \quad \mathcal{O}_{dXX}^L = (\bar{d}_{Lp} \gamma_\mu d_{Lr}) X^{\mu\nu} X_\nu$$

$$\mathcal{O}_{da}^L = (\bar{d}_{Lp} \gamma_\mu d_{Lr}) \partial^\mu a, \quad \mathcal{O}_{da}^R = (\bar{d}_{Rp} \gamma_\mu d_{Rr}) \partial^\mu a.$$

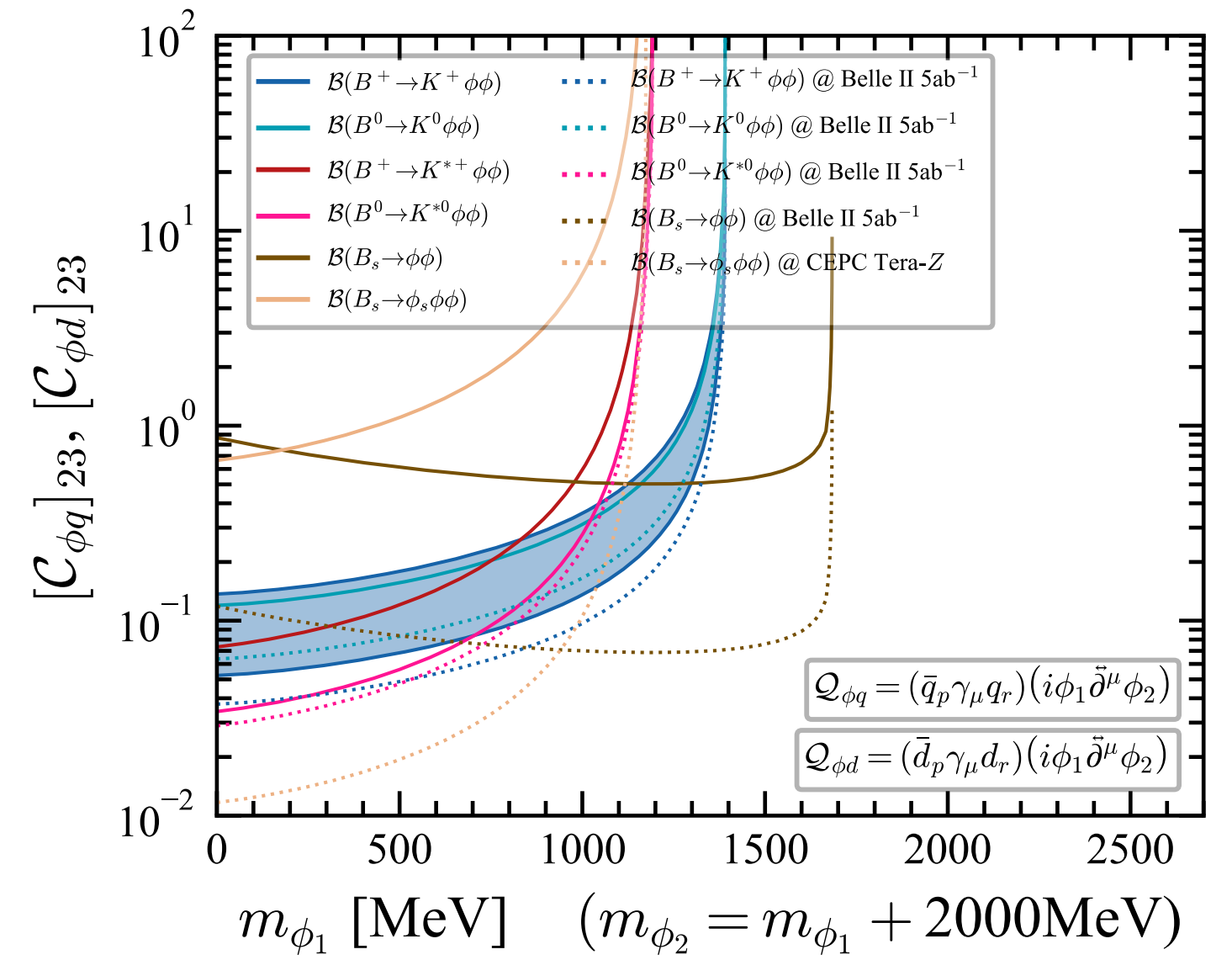
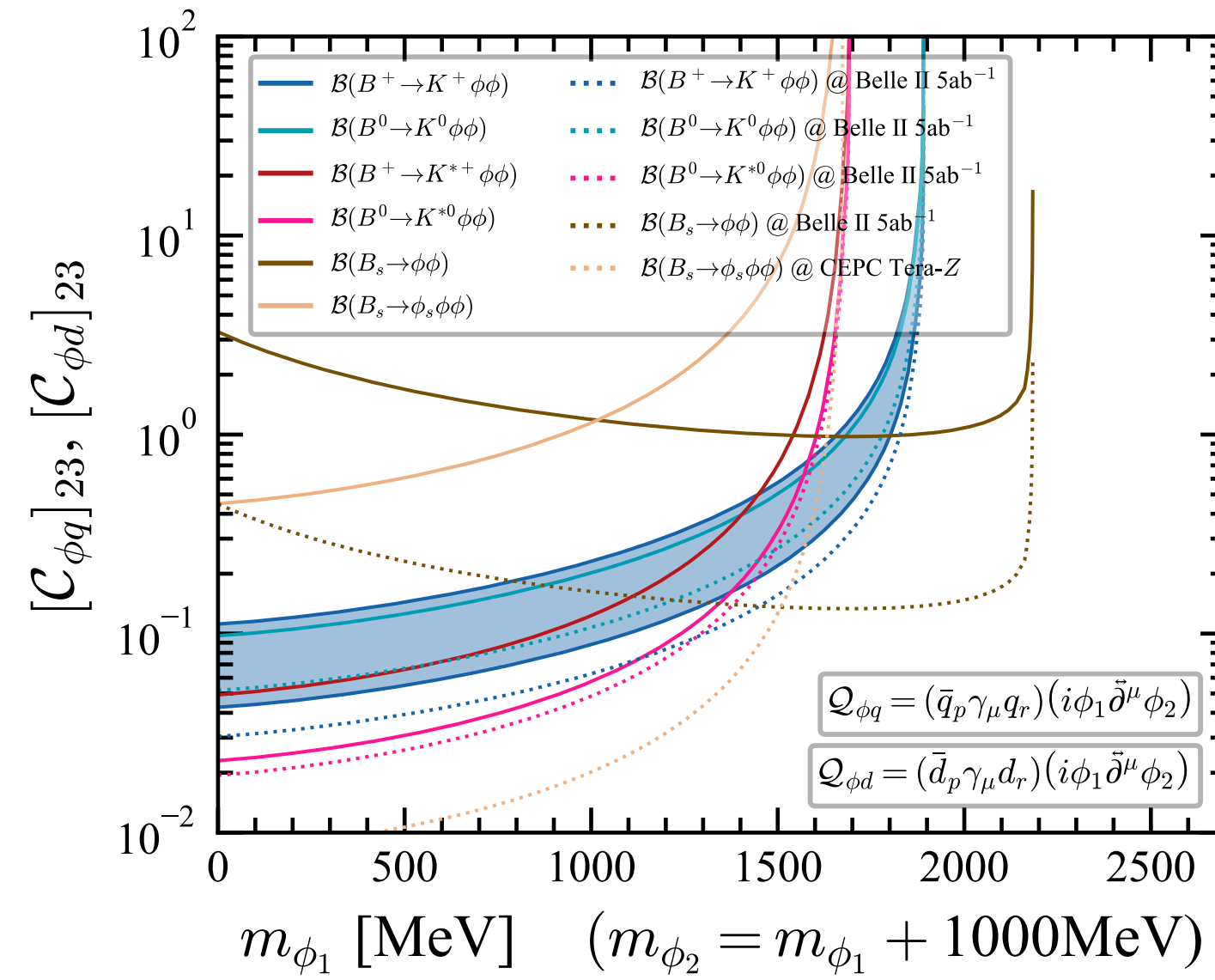
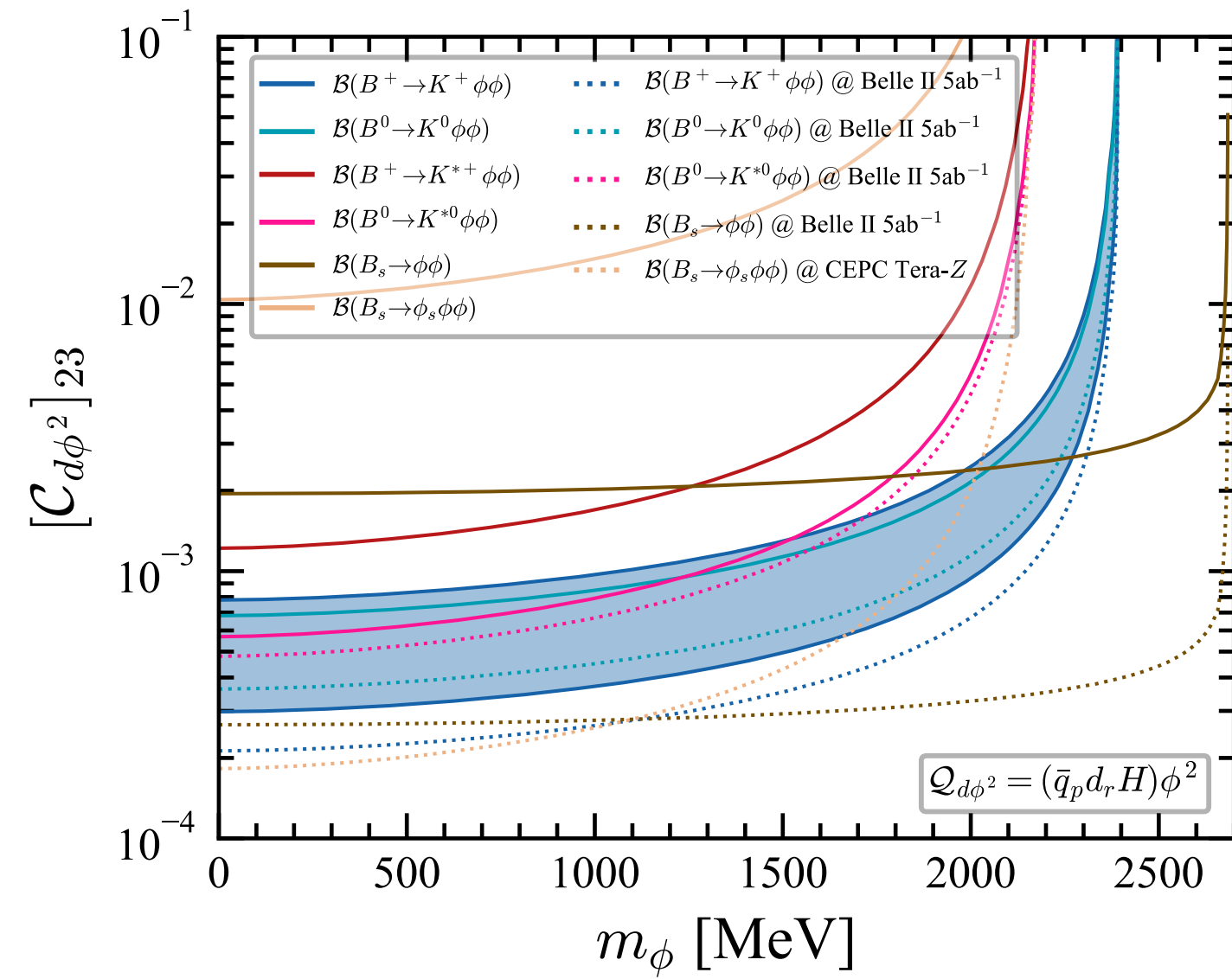
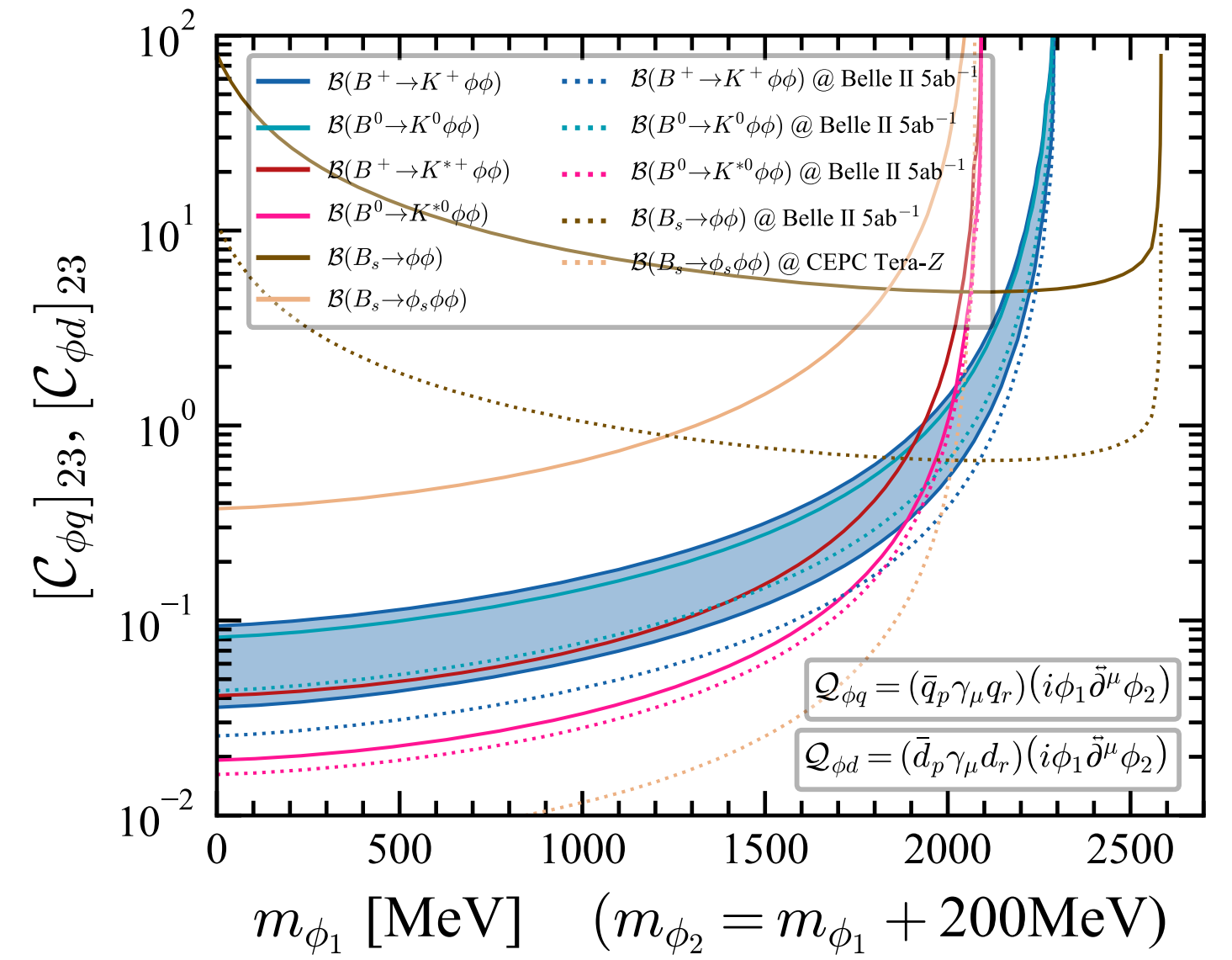
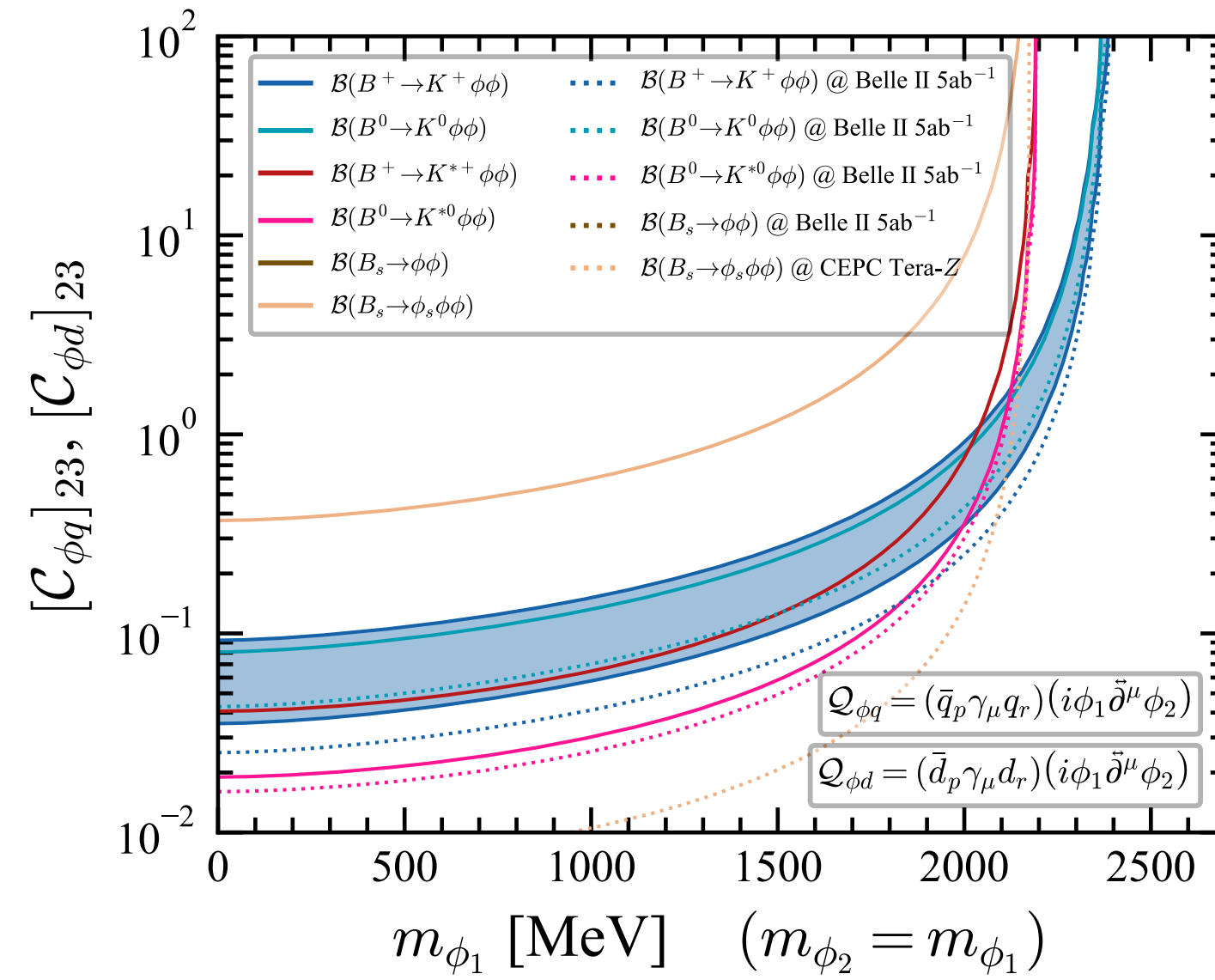
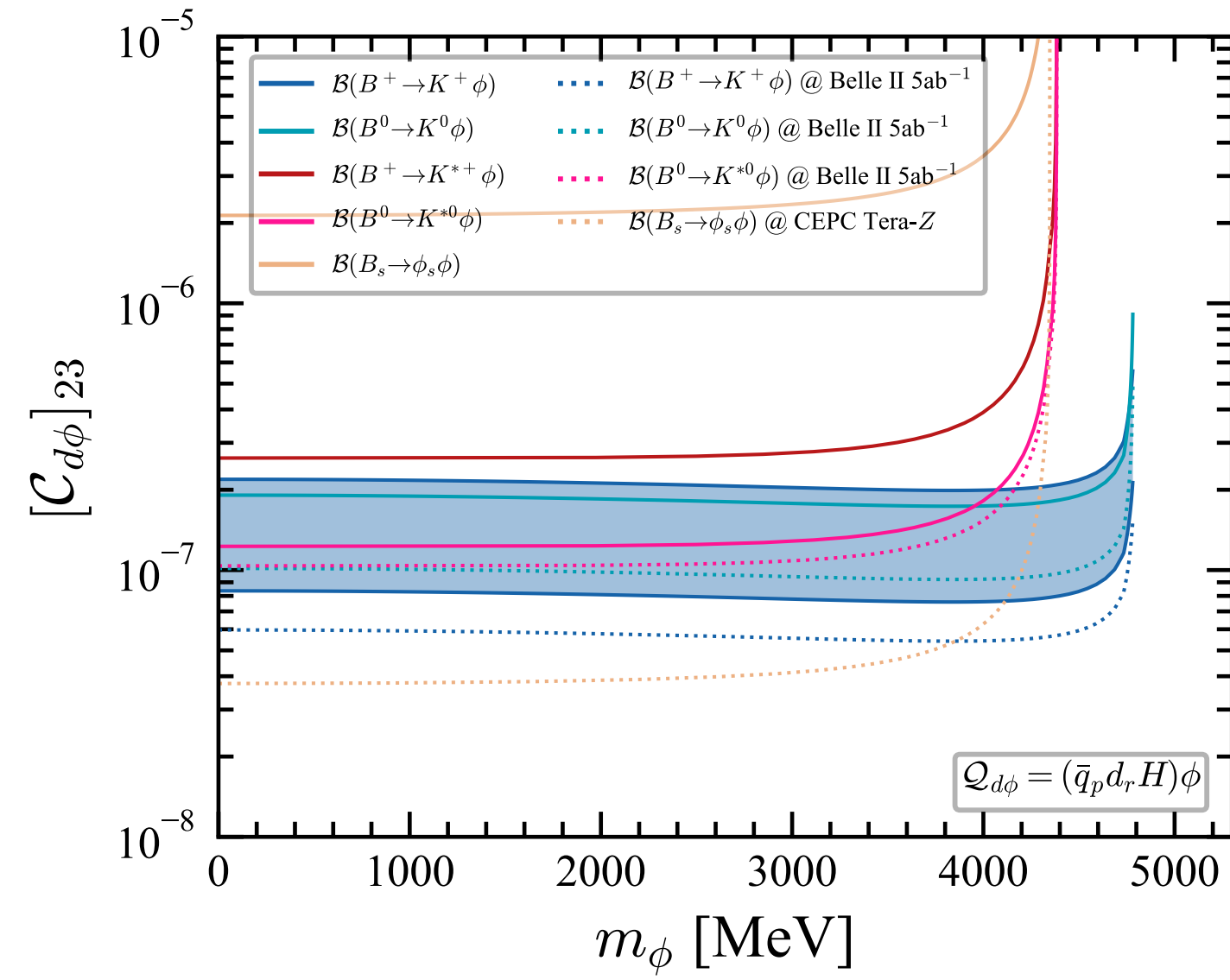
scalar: 4

fermion: 5

vector: 1+10

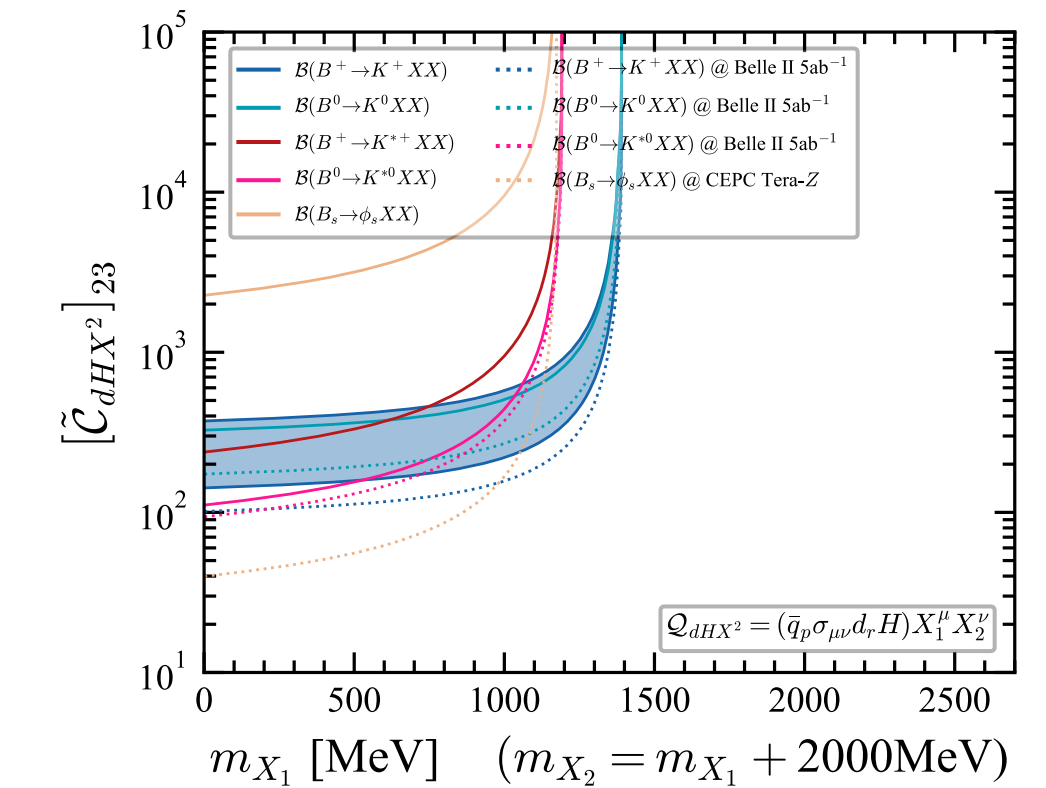
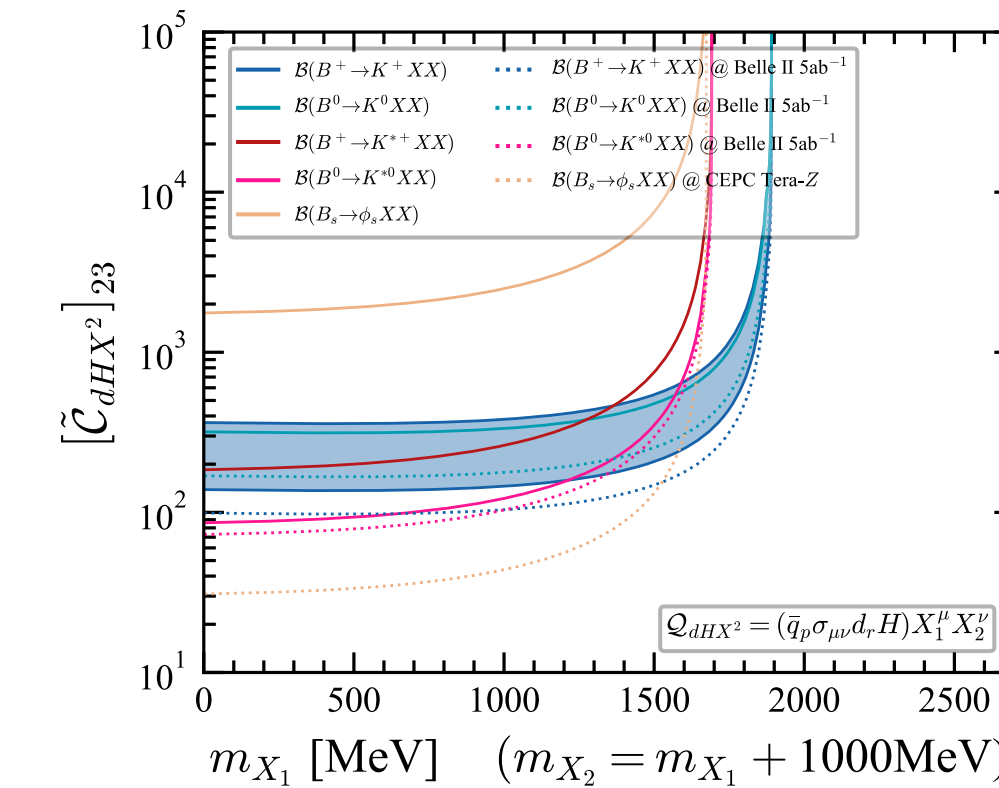
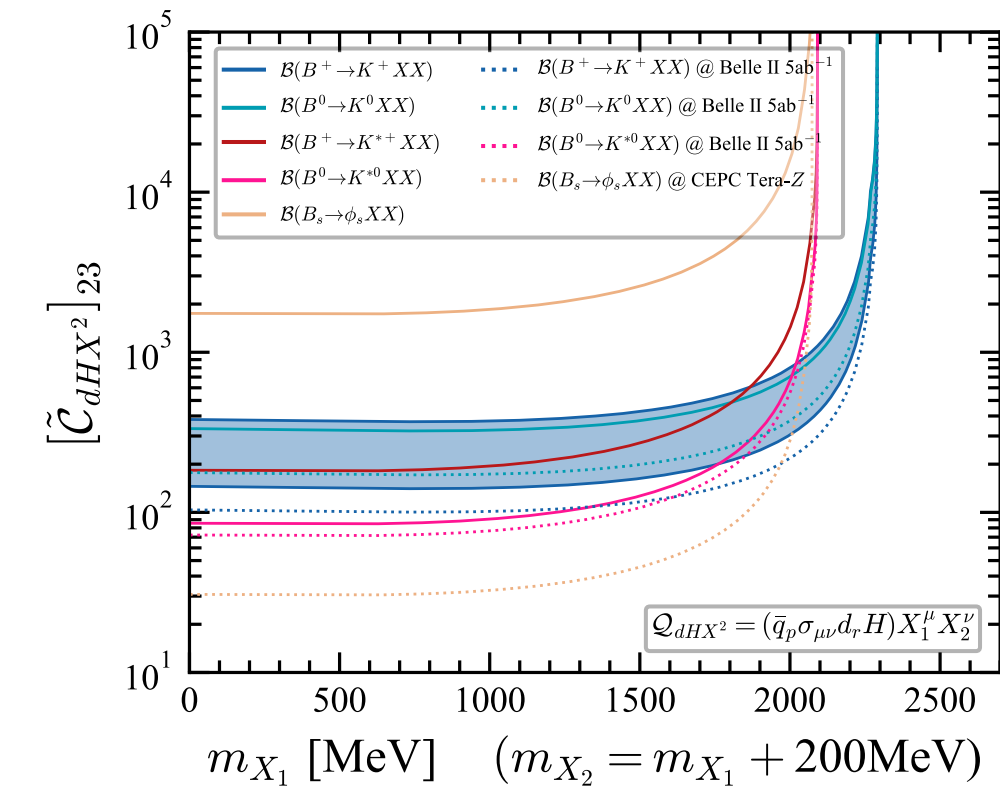
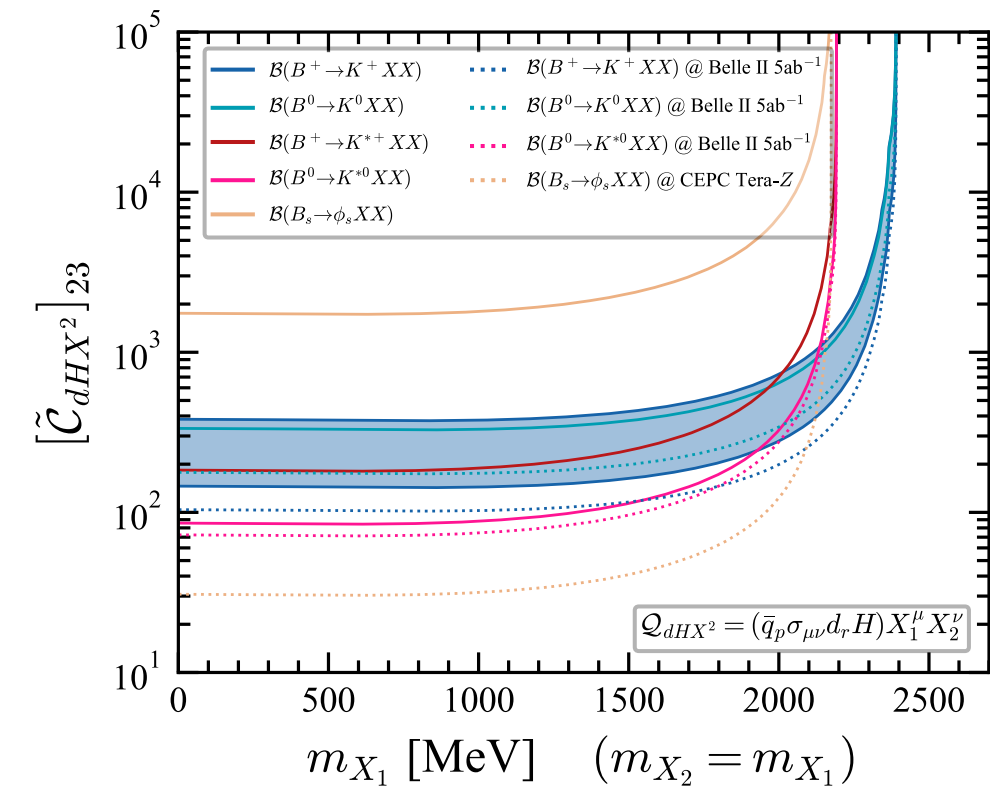
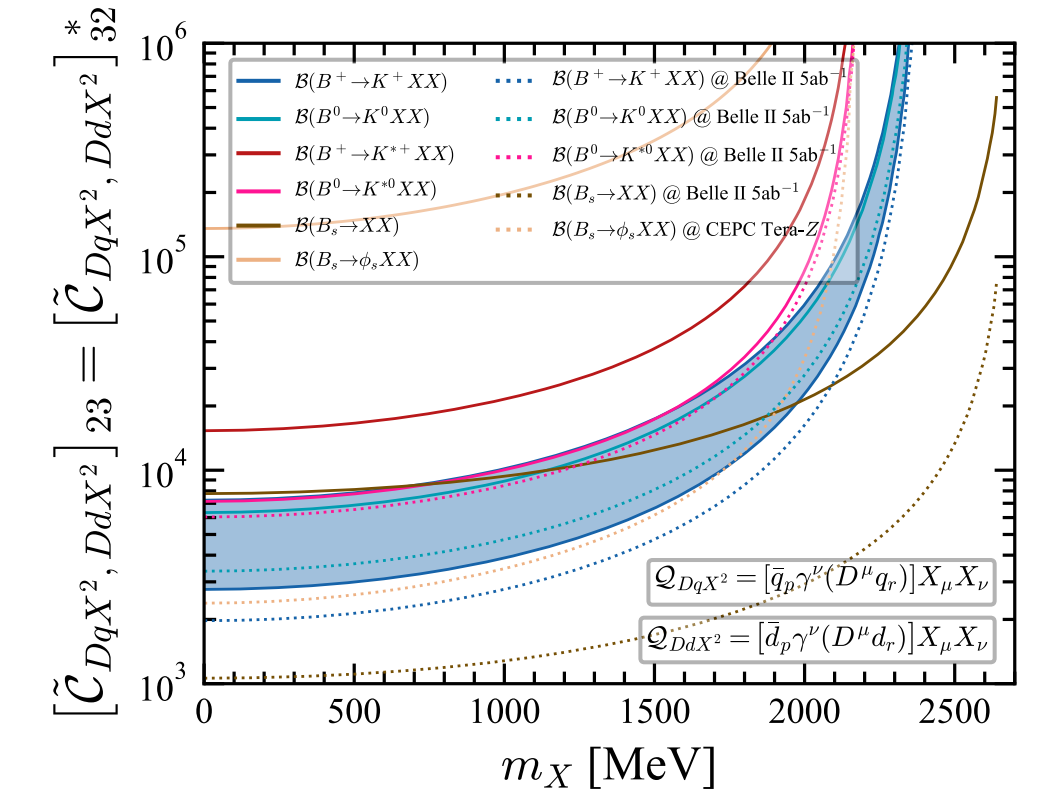
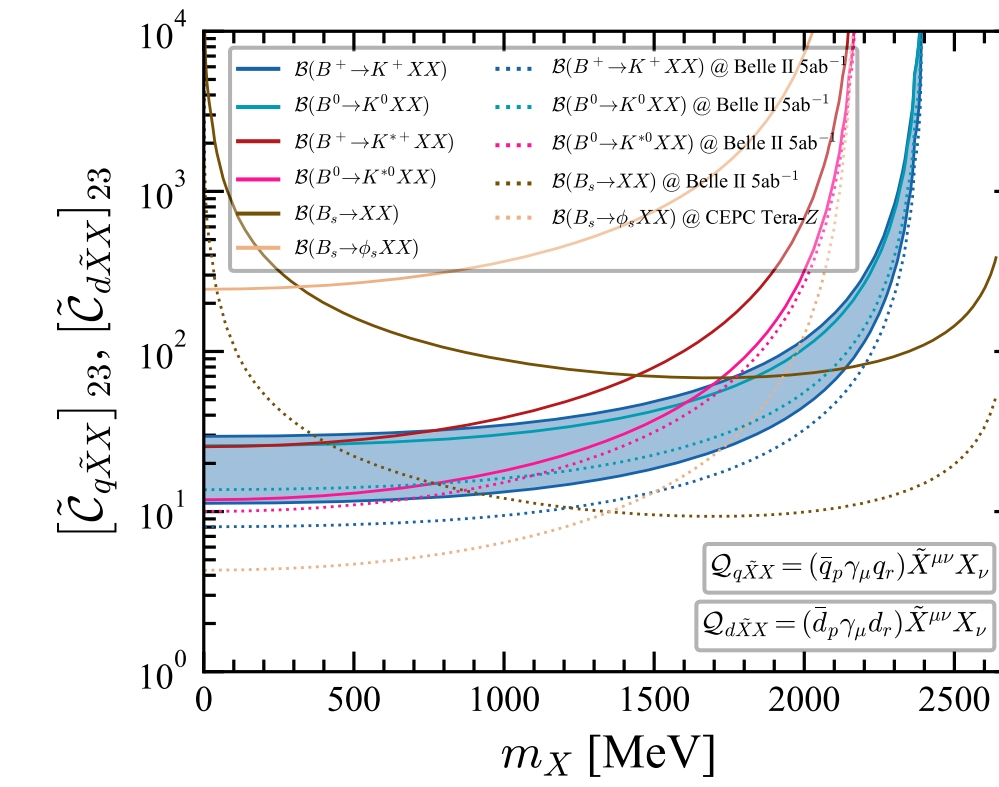
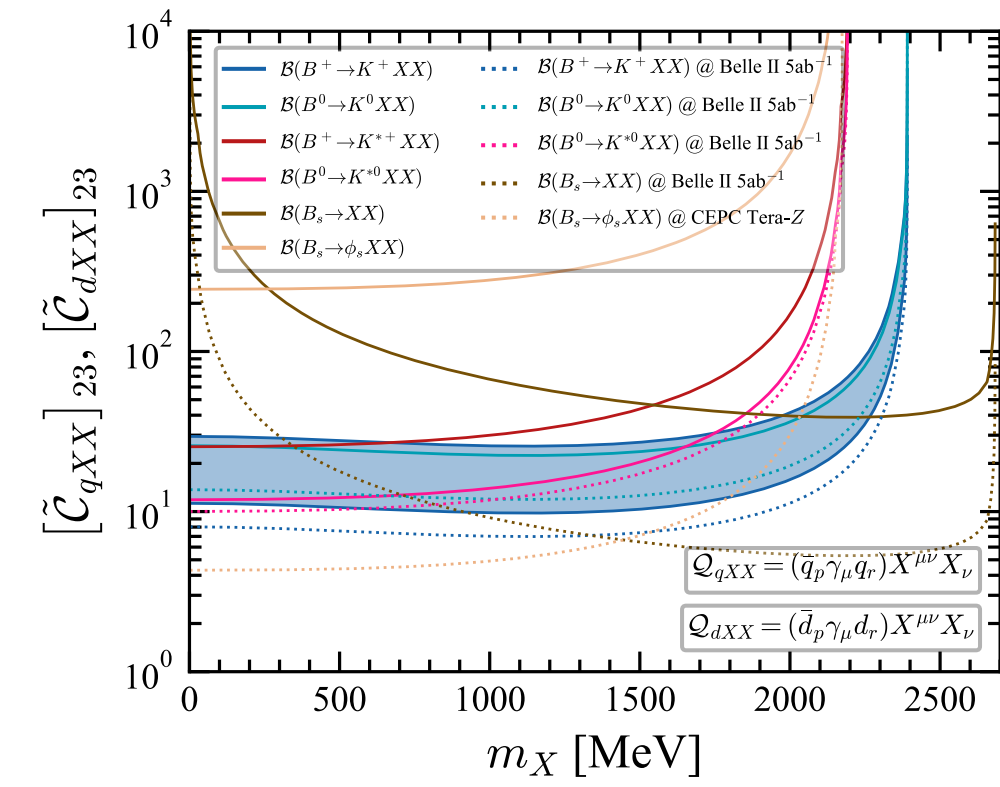
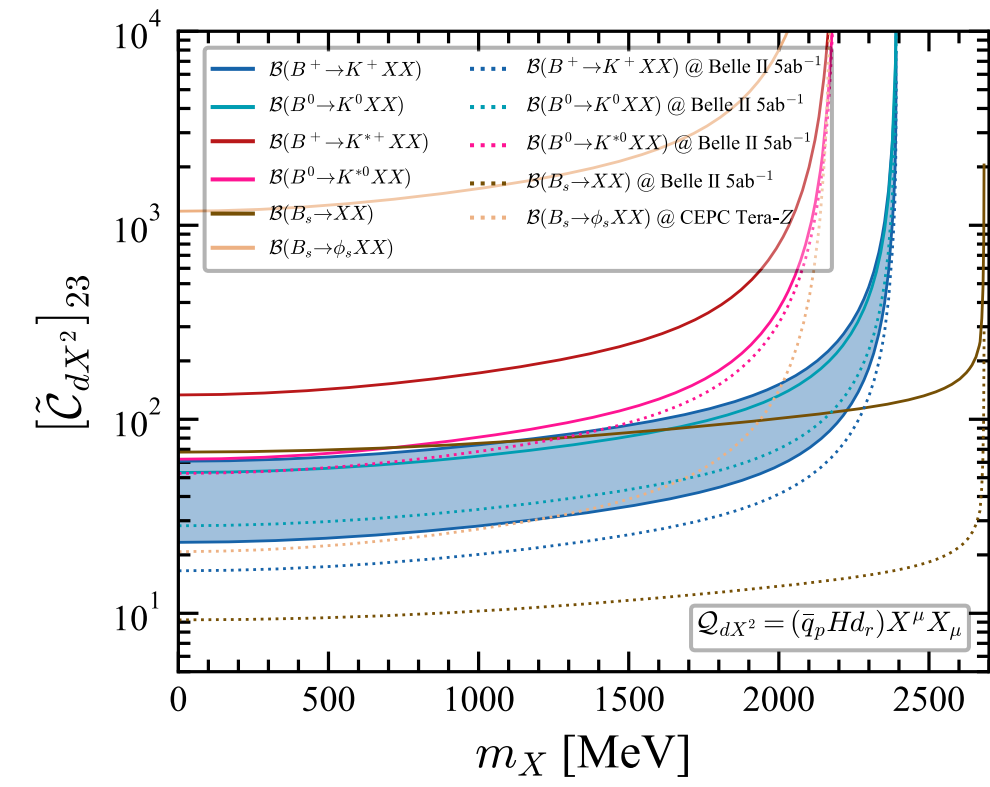
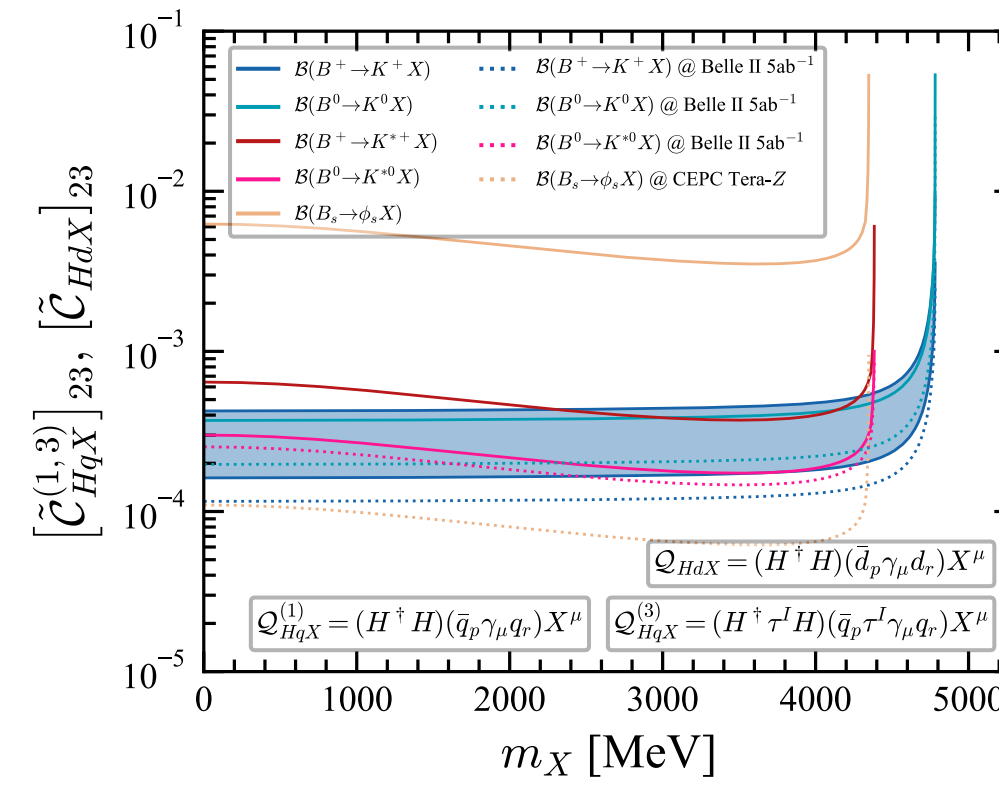
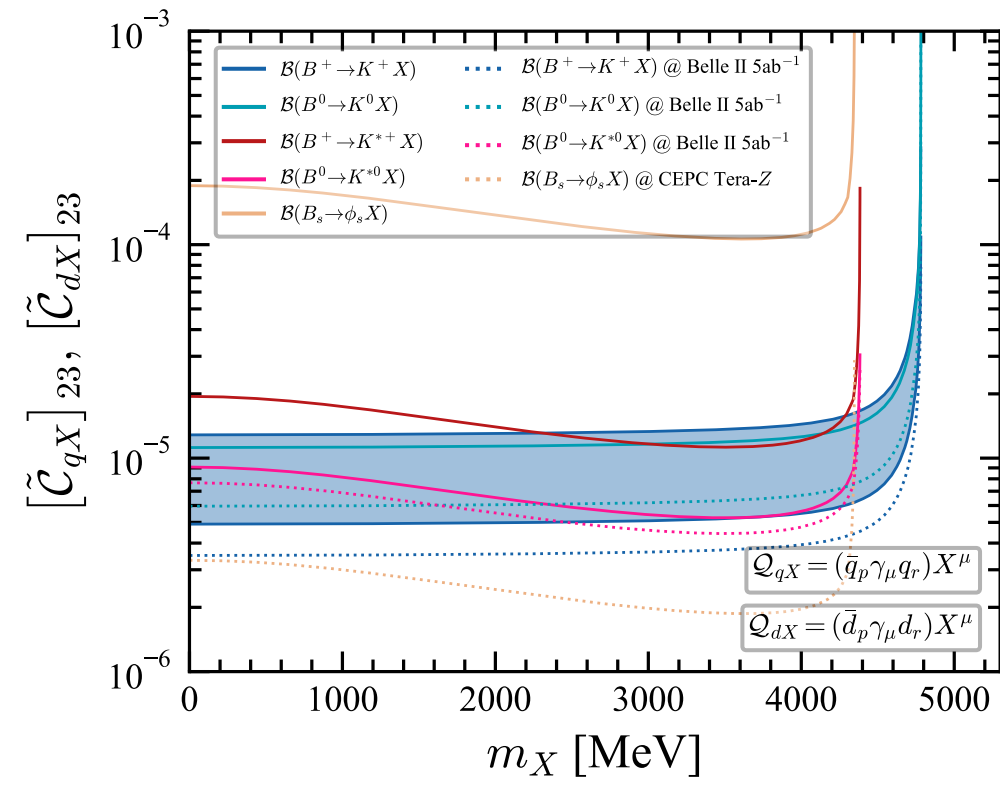
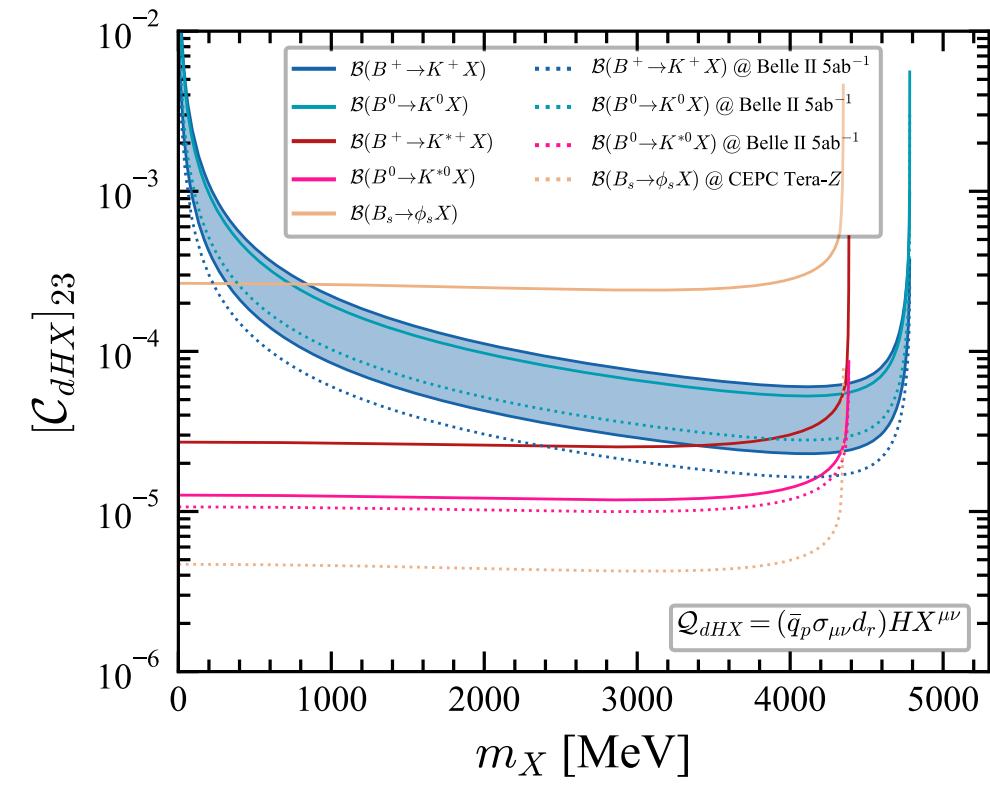
ALP: 2

# Dark SMEFT: Scalar

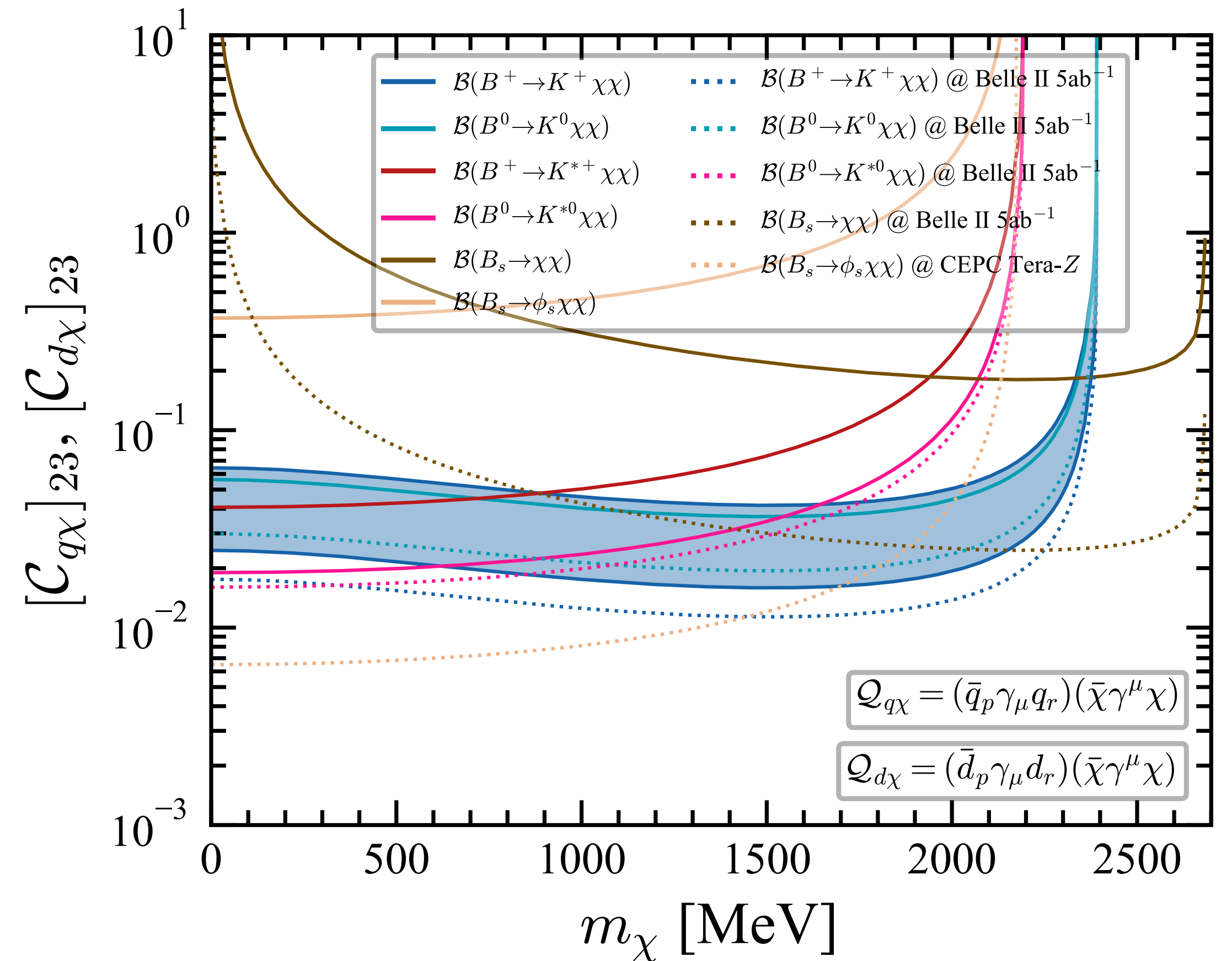
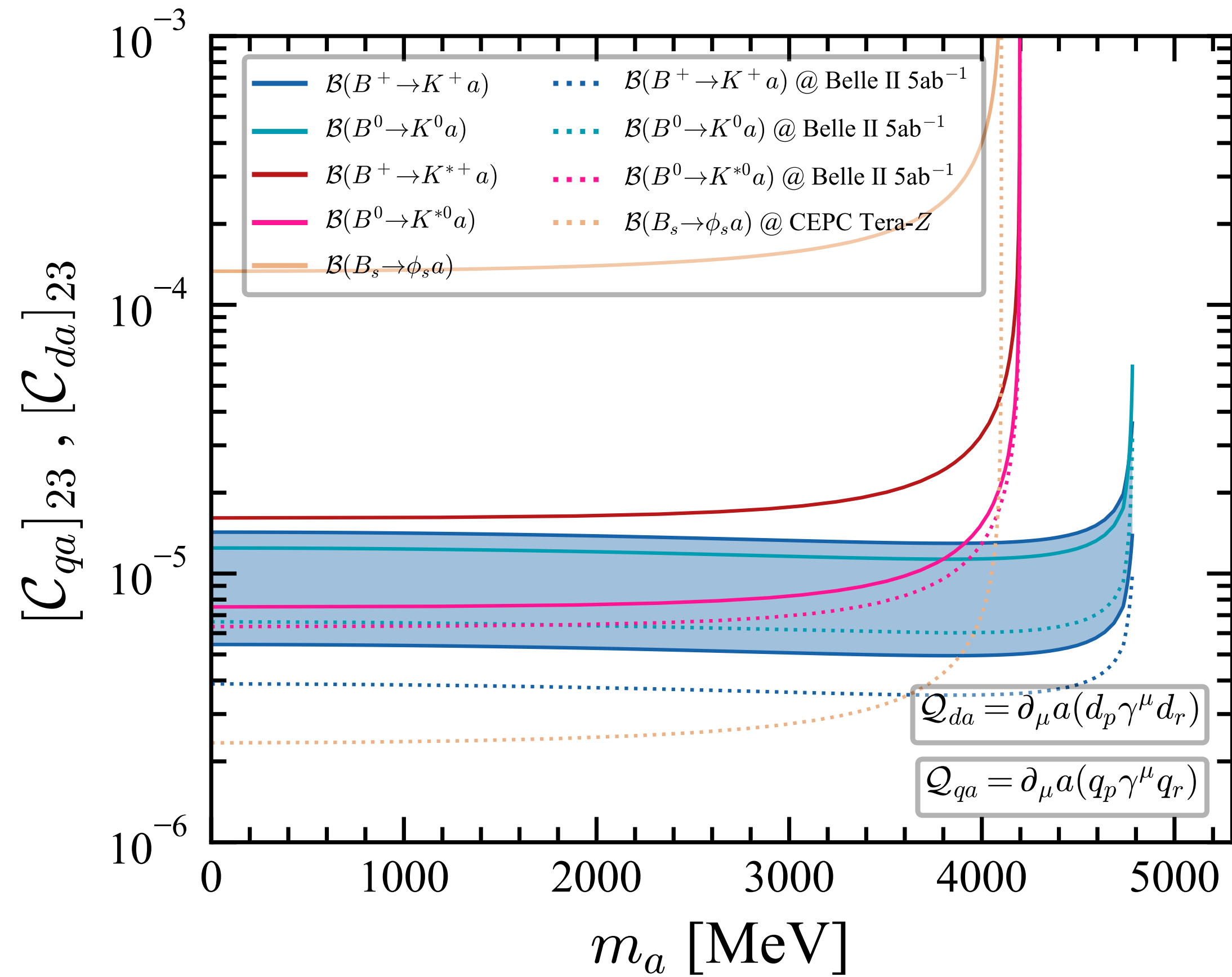




# Dark SMEFT: Vector



# Dark SMEFT: Fermion, ALP



All the operators survive from the constraints of the various FCNC decays.

In the future, all the parameter space to explain the Belle II anomaly can be covered by combining the Belle II (e.g.,  $B^0 \rightarrow K^0 + \text{inv}$ ) and CEPC (e.g.,  $B_s \rightarrow \phi + \text{inv}$  and  $B_s \rightarrow \text{inv}$ ) measurements.



# Dark SMEFT with MFV

- ▶ MFV coupling  $b \rightarrow s, b \rightarrow d, s \rightarrow d$  are connected with each other.

$$c_i^{\text{MFV}} = \begin{cases} \epsilon_0^i \hat{\lambda}_d + \epsilon_1^i \Delta_q \hat{\lambda}_d & \text{for } Q_i = Q_{d\phi}, Q_{d\phi^2}, Q_{dHX}, Q_{dHX^2}, Q_{dX^2}, \\ \epsilon_0^i \mathbb{1} + \epsilon_1^i \Delta_q & \text{for } Q_i = Q_{\phi q}, Q_{q\chi}, Q_{qXX}, Q_{q\tilde{X}X}, Q_{DqX^2}, Q_{qX}, Q_{HqX}^{(1,3)}, Q_{qa}, \\ \epsilon_0^i \mathbb{1} & \text{for } Q_i = Q_{\phi d}, Q_{d\chi}, Q_{dXX}, Q_{d\tilde{X}X}, Q_{DdX^2}, Q_{dX}, Q_{HdX}, Q_{da}, \end{cases}$$

**8 operators are eliminated**

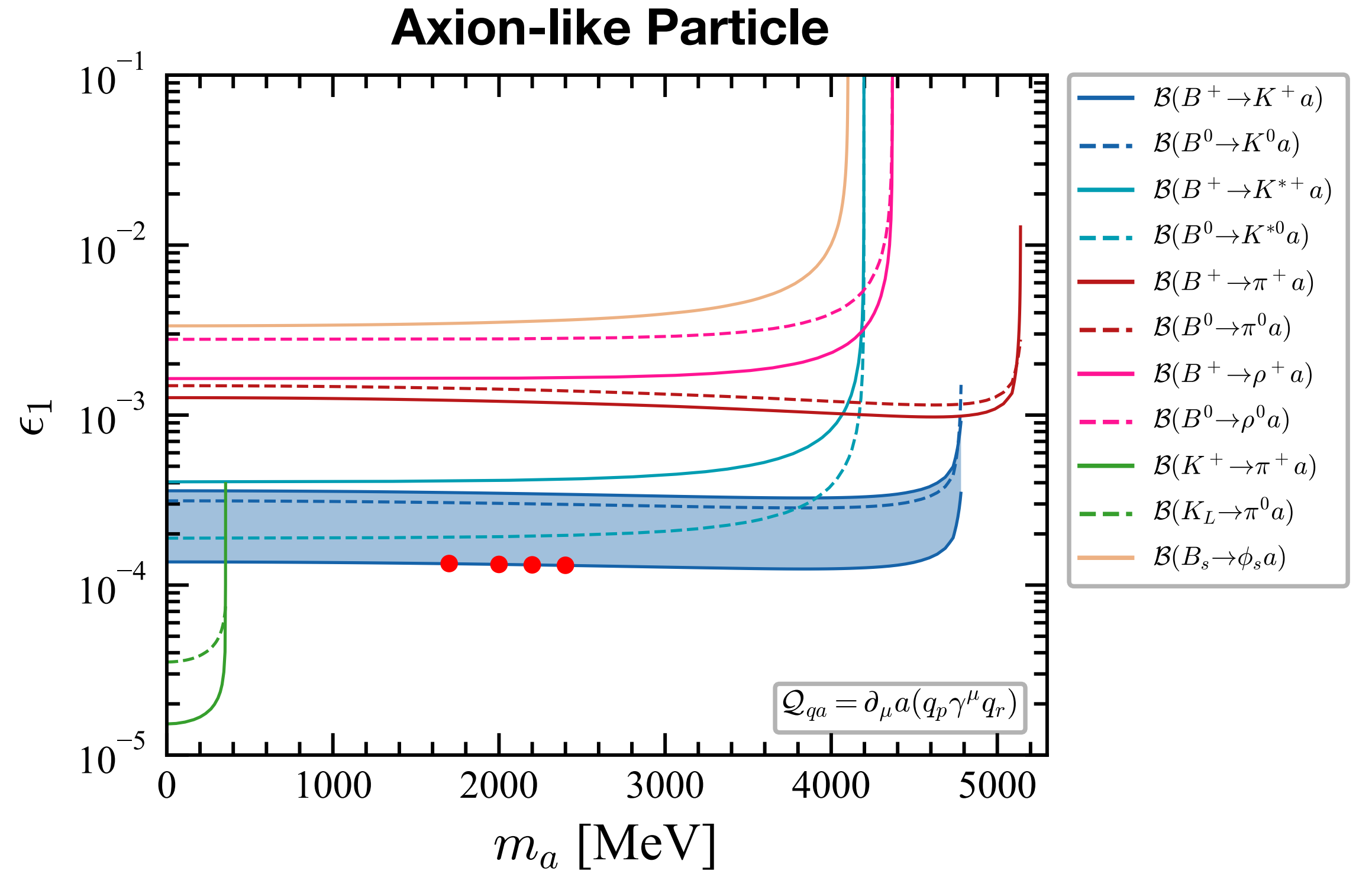
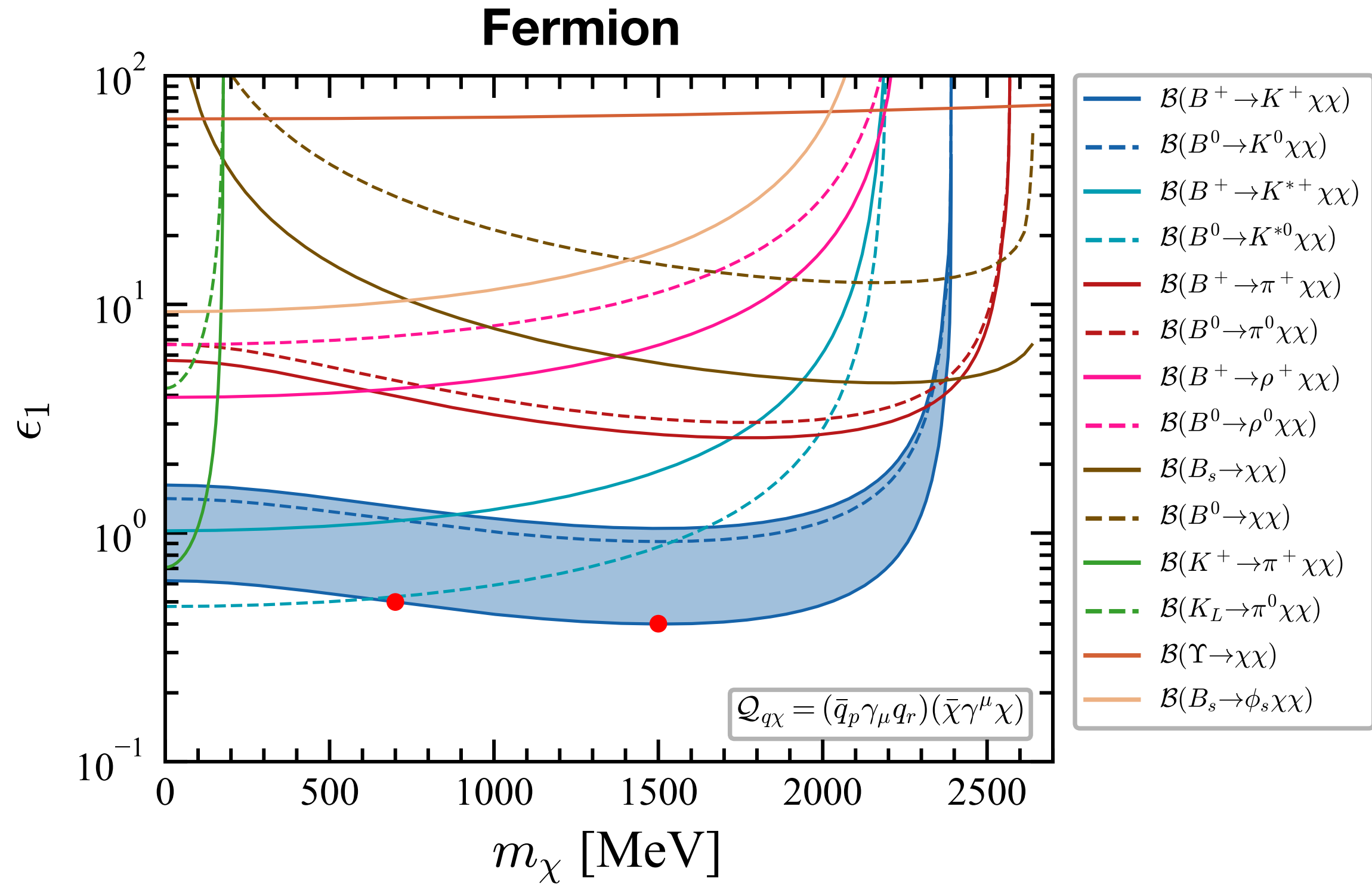
- ▶ Numerics

$$\Delta_q = \begin{pmatrix} 0.8 & -3.3 - 1.5i & 79.3 + 35.4i \\ -3.3 + 1.5i & 16.6 & -397.5 + 8.1i \\ 79.3 - 35.4i & -397.5 - 8.1i & 9839.0 \end{pmatrix} \times 10^{-4}$$

$$\Delta_q \hat{\lambda}_d = \begin{pmatrix} 0.0021 & -0.18 - 0.08i & 191.3 + 85.4i \\ -0.009 + 0.004i & 0.88 & -958.7 + 19.6i \\ 0.21 - 0.10i & -21.1 - 0.4i & 23728.1 \end{pmatrix} \times 10^{-6}$$



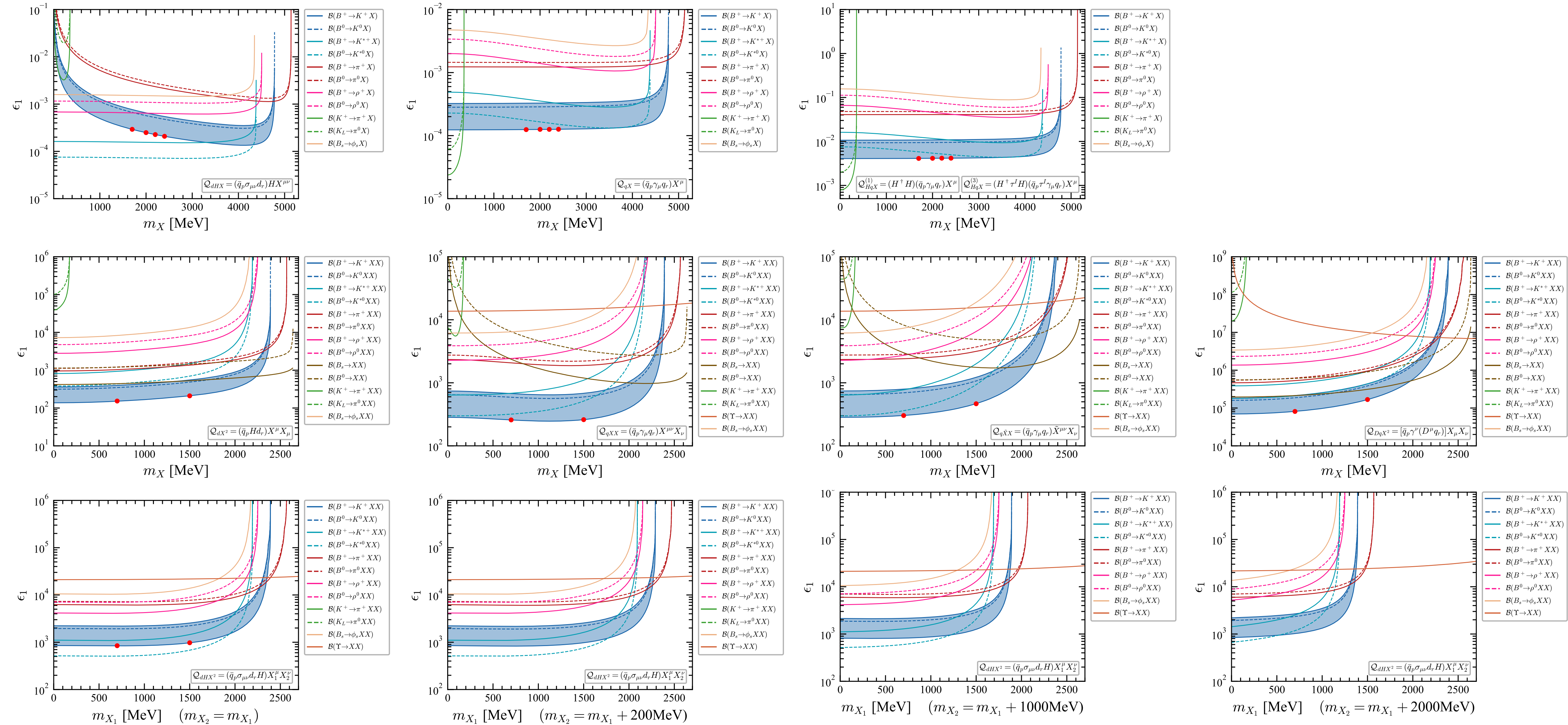
# Dark SMEFT with MFV: Fermion, ALP



**all the operators survive**



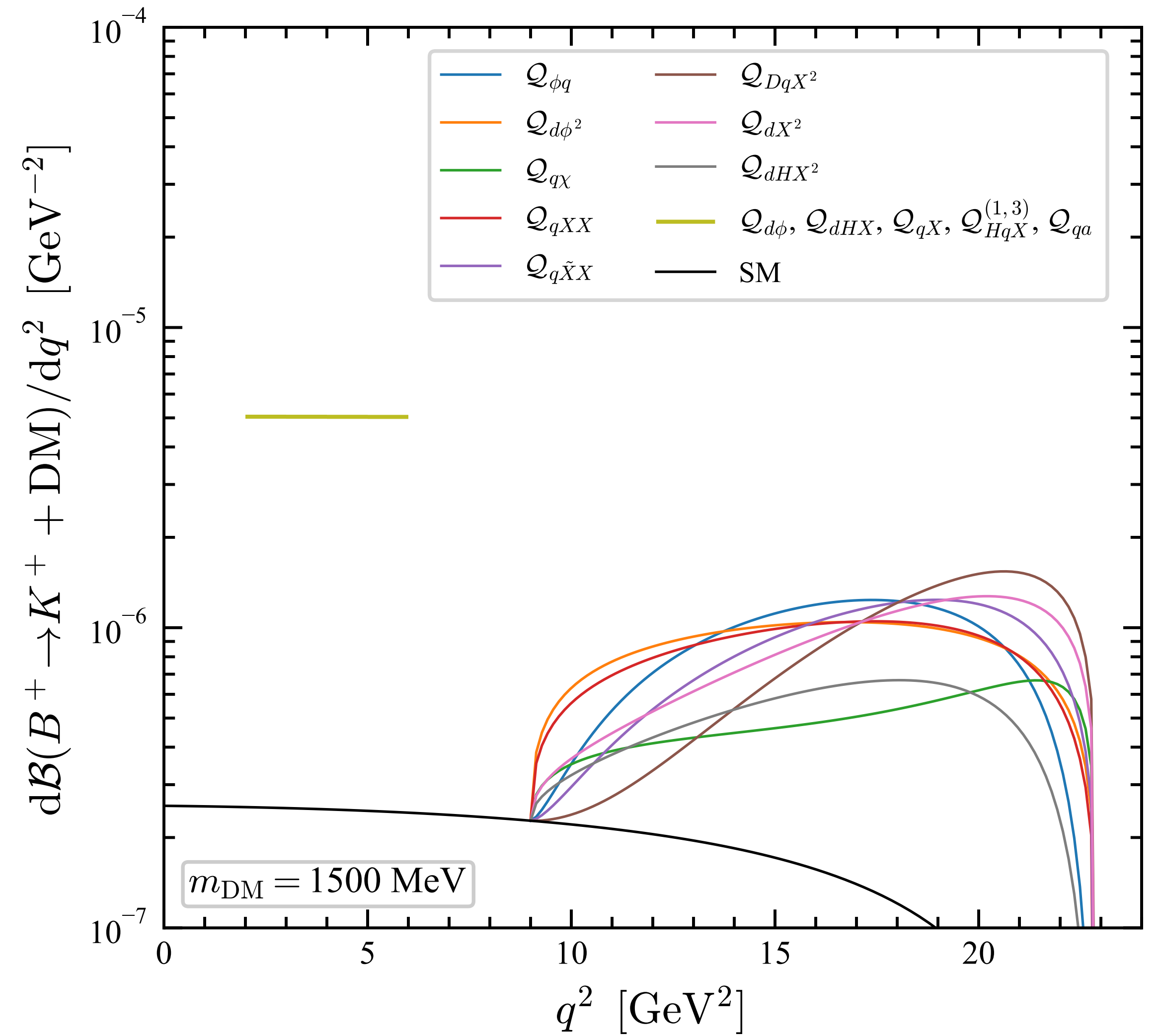
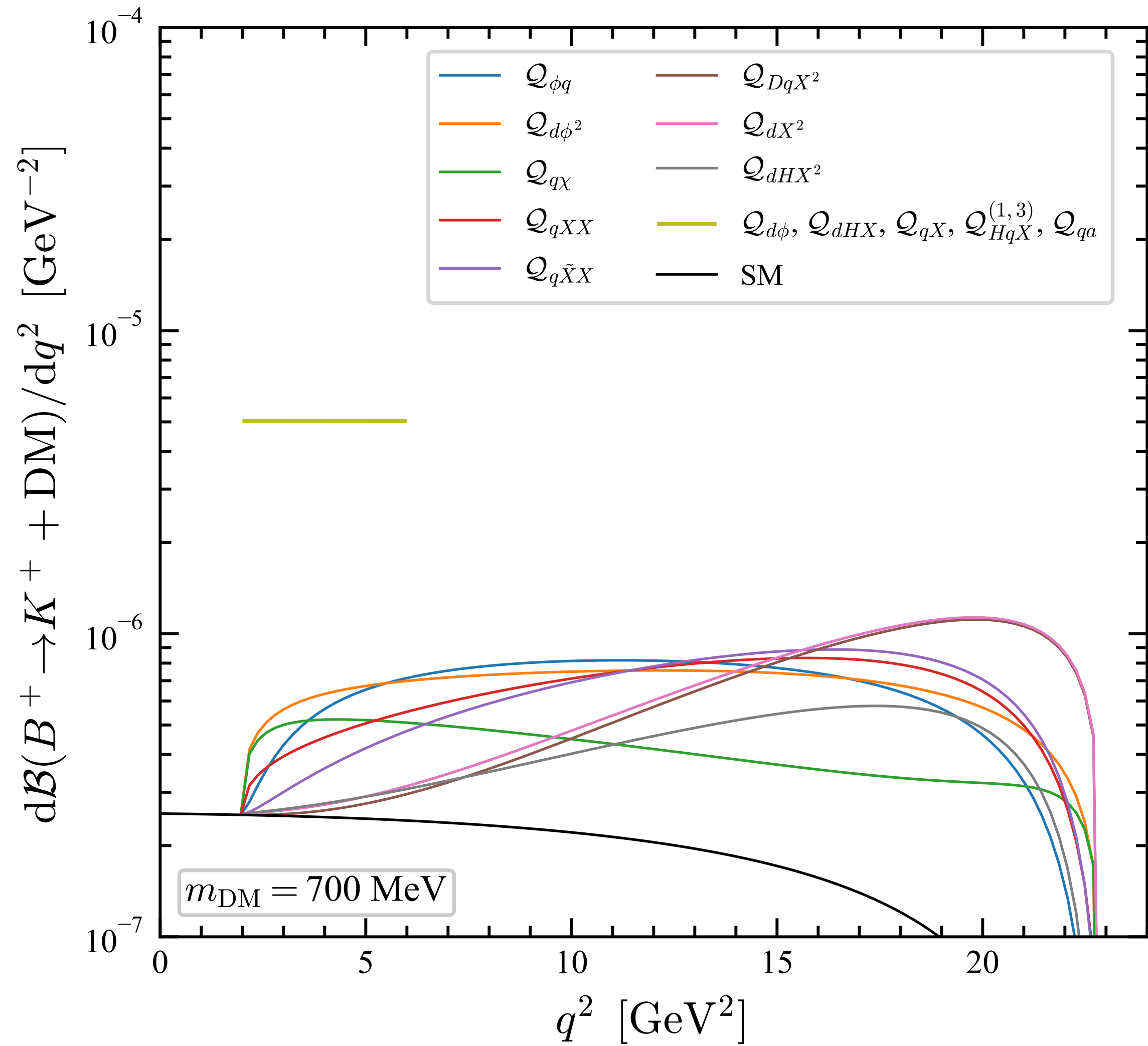
# Dark SMEFT with MFV: Vector



all the operators survive, some ones highly constrained



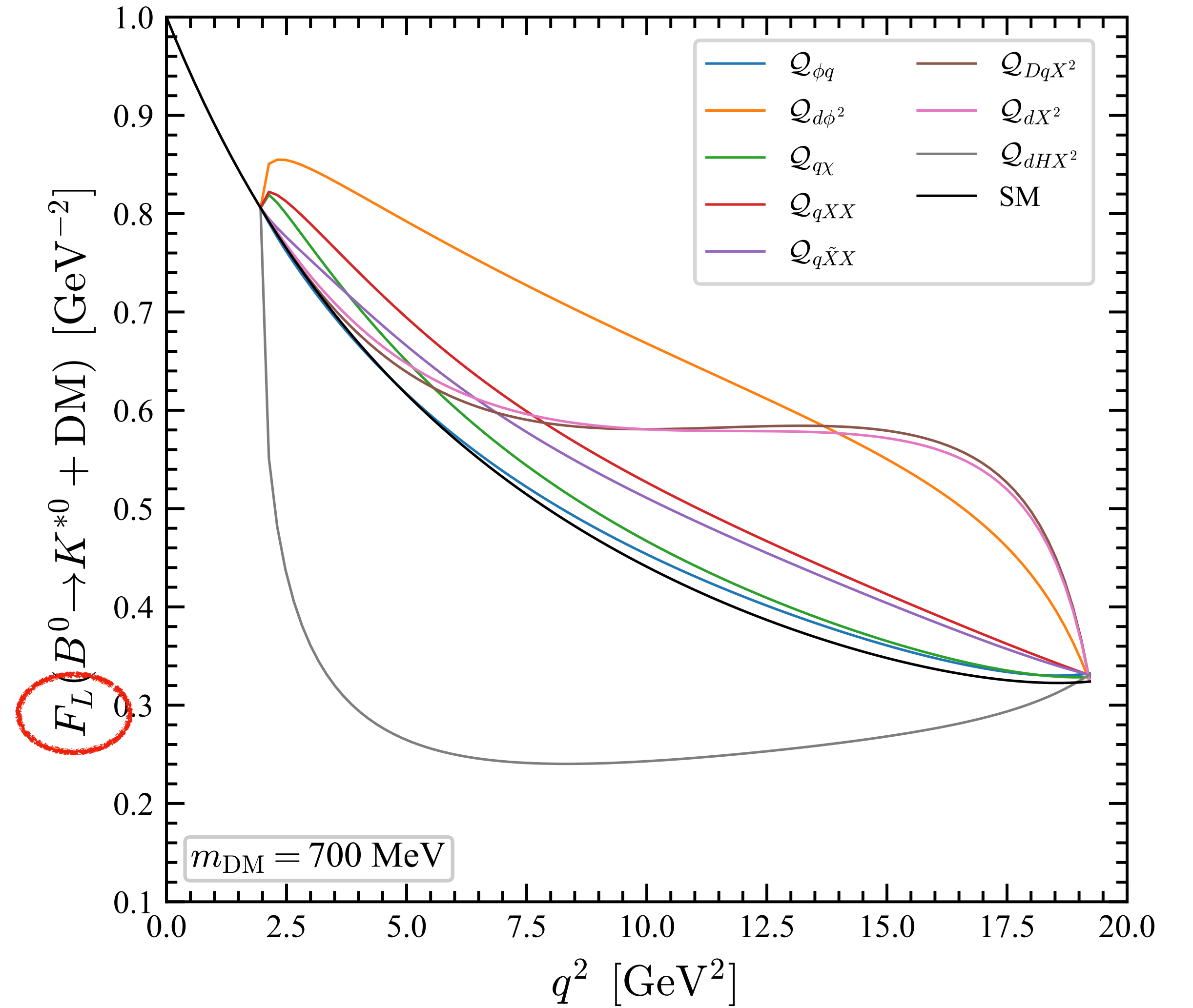
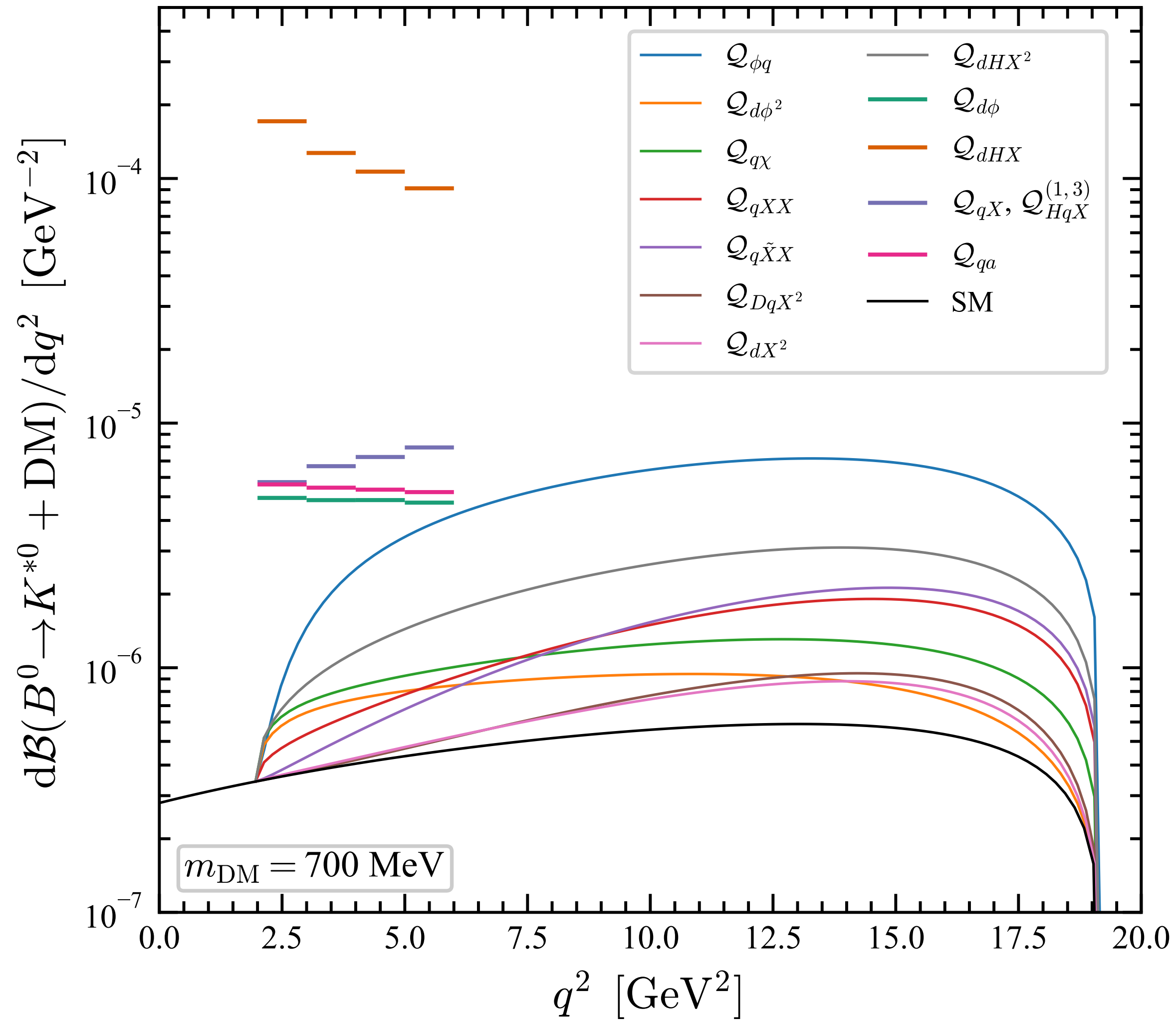
# Dark SMEFT: $dB/dq^2$



Difficult to distinguish the DSMEFT operators by considering only the  $B^+ \rightarrow K^+ \nu \bar{\nu}$  decay. However,

# Dark SMEFT: $dB/dq^2, F_L$

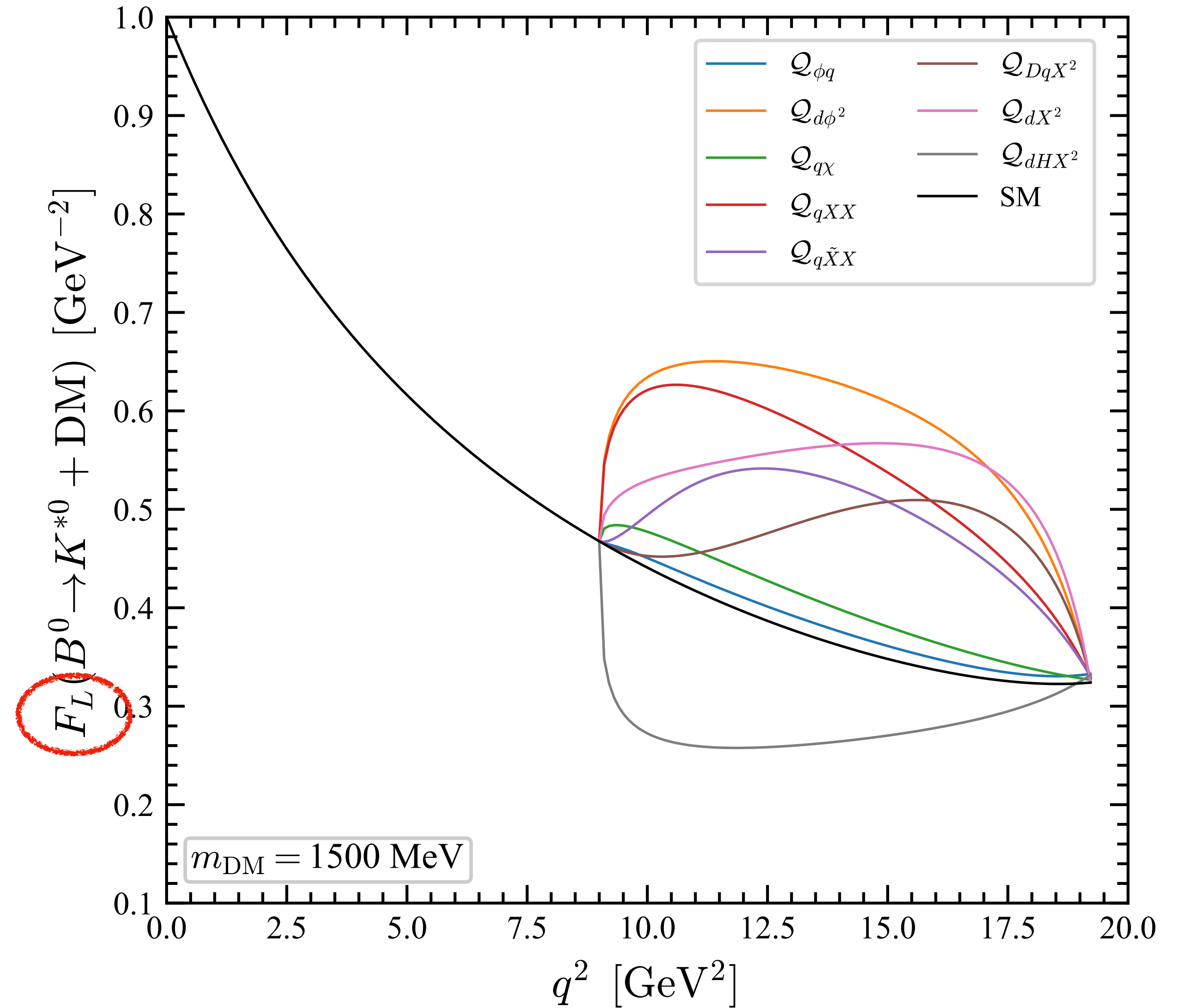
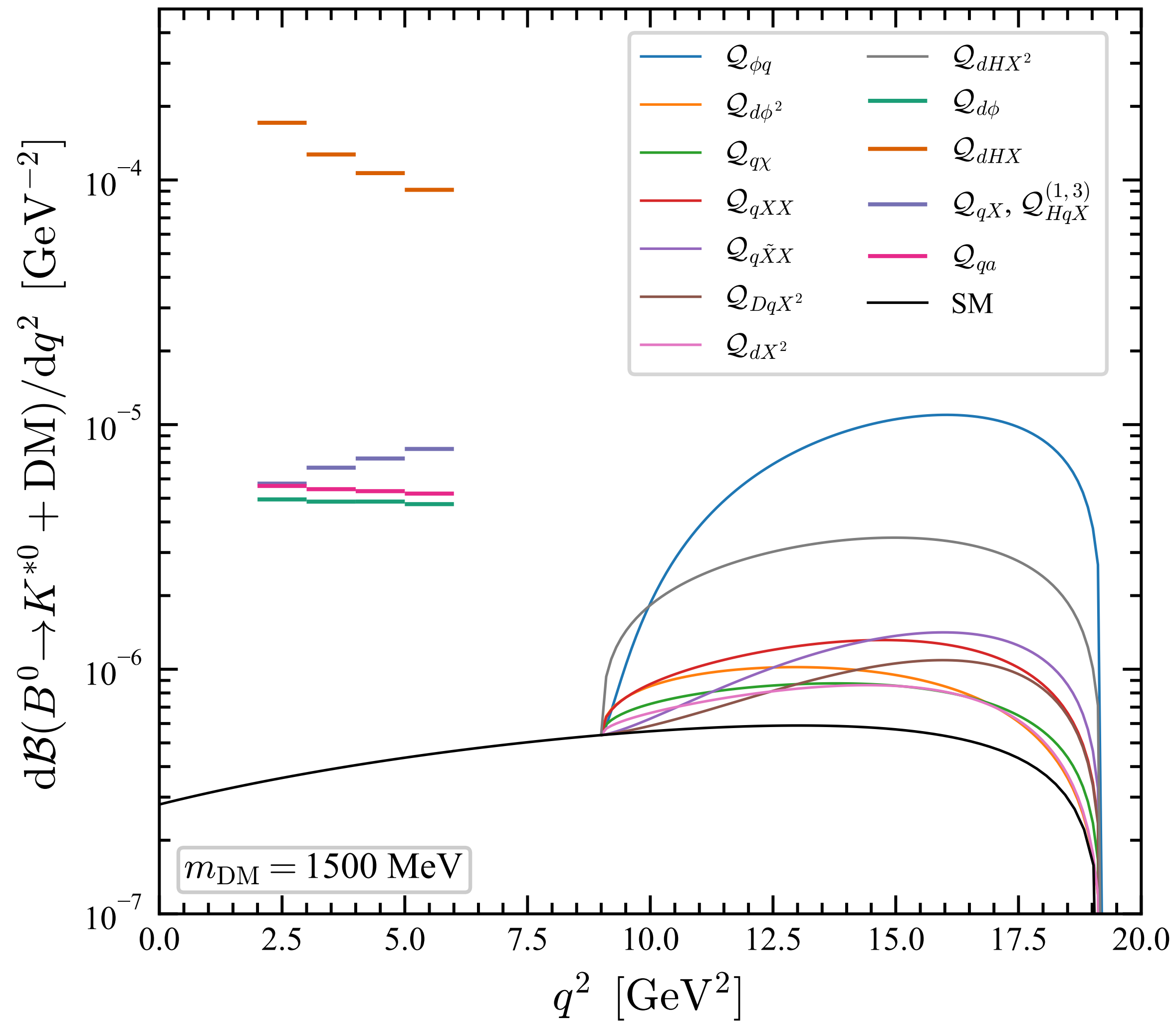
$m_{\text{DM}} = 700 \text{ MeV}$



All the operators are distinguishable from each other by combing these observables, except  $\mathcal{Q}_{qXX}$  and  $\mathcal{Q}_{q\tilde{X}X}$   
 $\mathcal{Q}_{dX^2}$  and  $\mathcal{Q}_{DqX^2}$

# Dark SMEFT: $dB/dq^2, F_L$

$m_{\text{DM}} = 1500 \text{ MeV}$

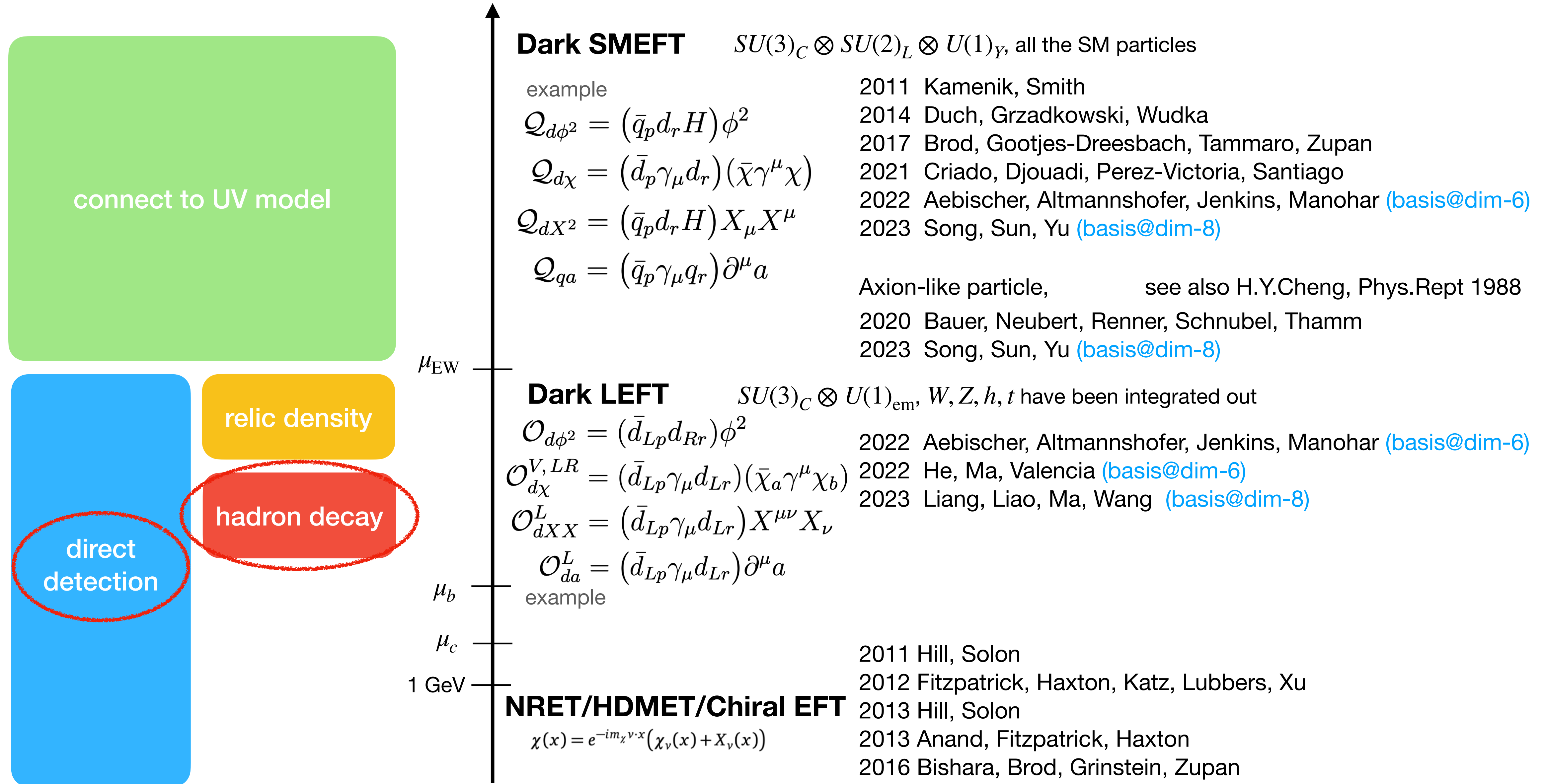


All the operators are distinguishable from each other by combining these observables, except

~~$Q_{qXX}$  and  $Q_{qX\tilde{X}}$~~   
 ~~$Q_{dX^2}$  and  $Q_{DqX^2}$~~

# Effective Field Theory approach to combine the various experimental searches

In EFT, DM is a just singlet under the SM gauge group.



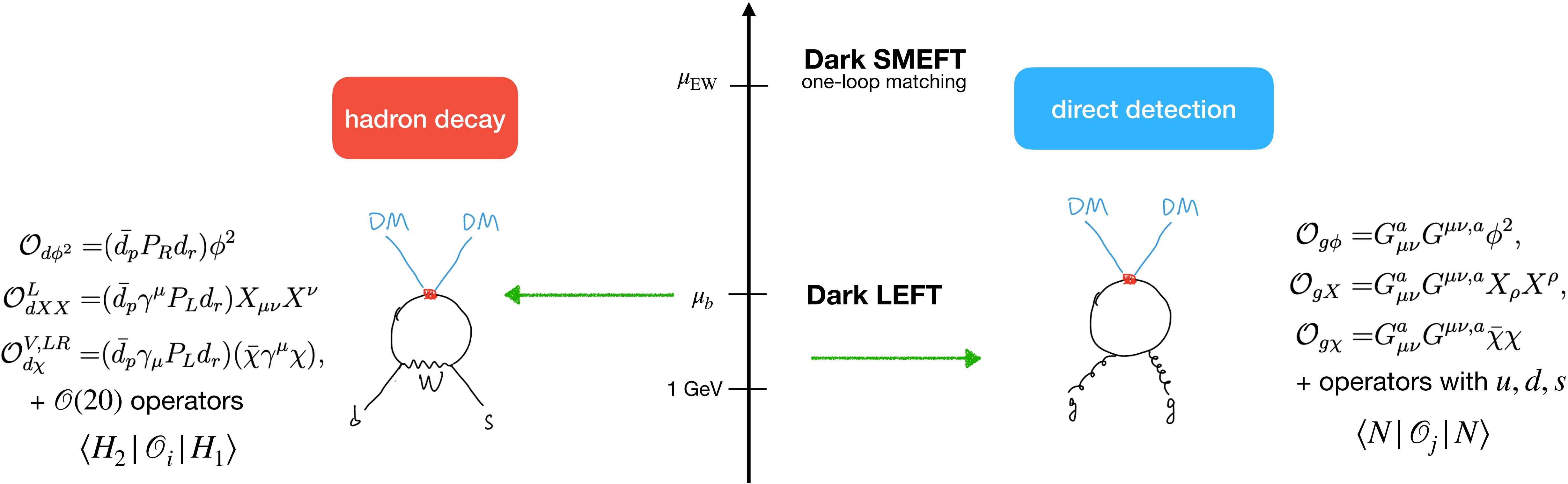


# Top-flavored DM

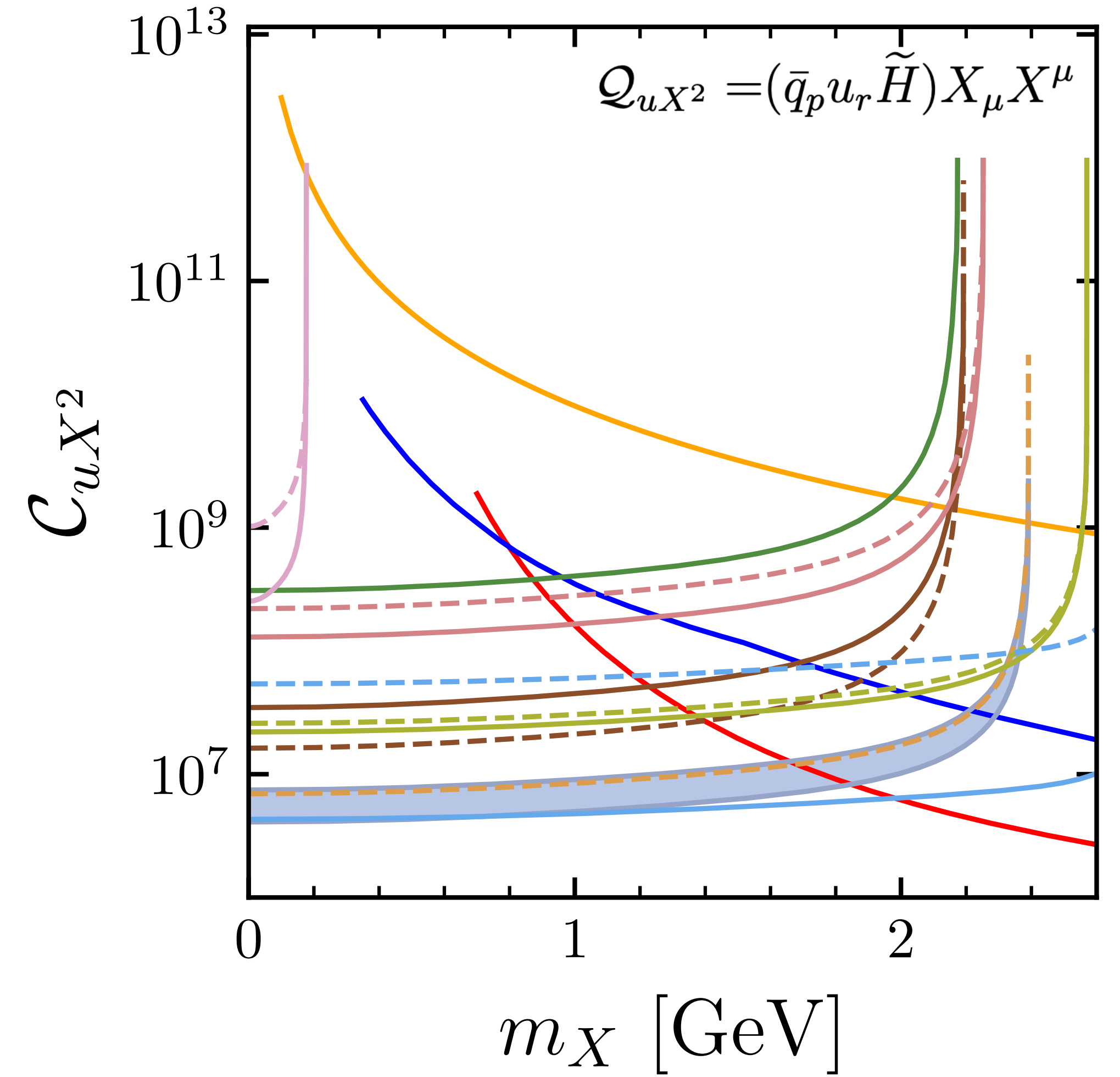
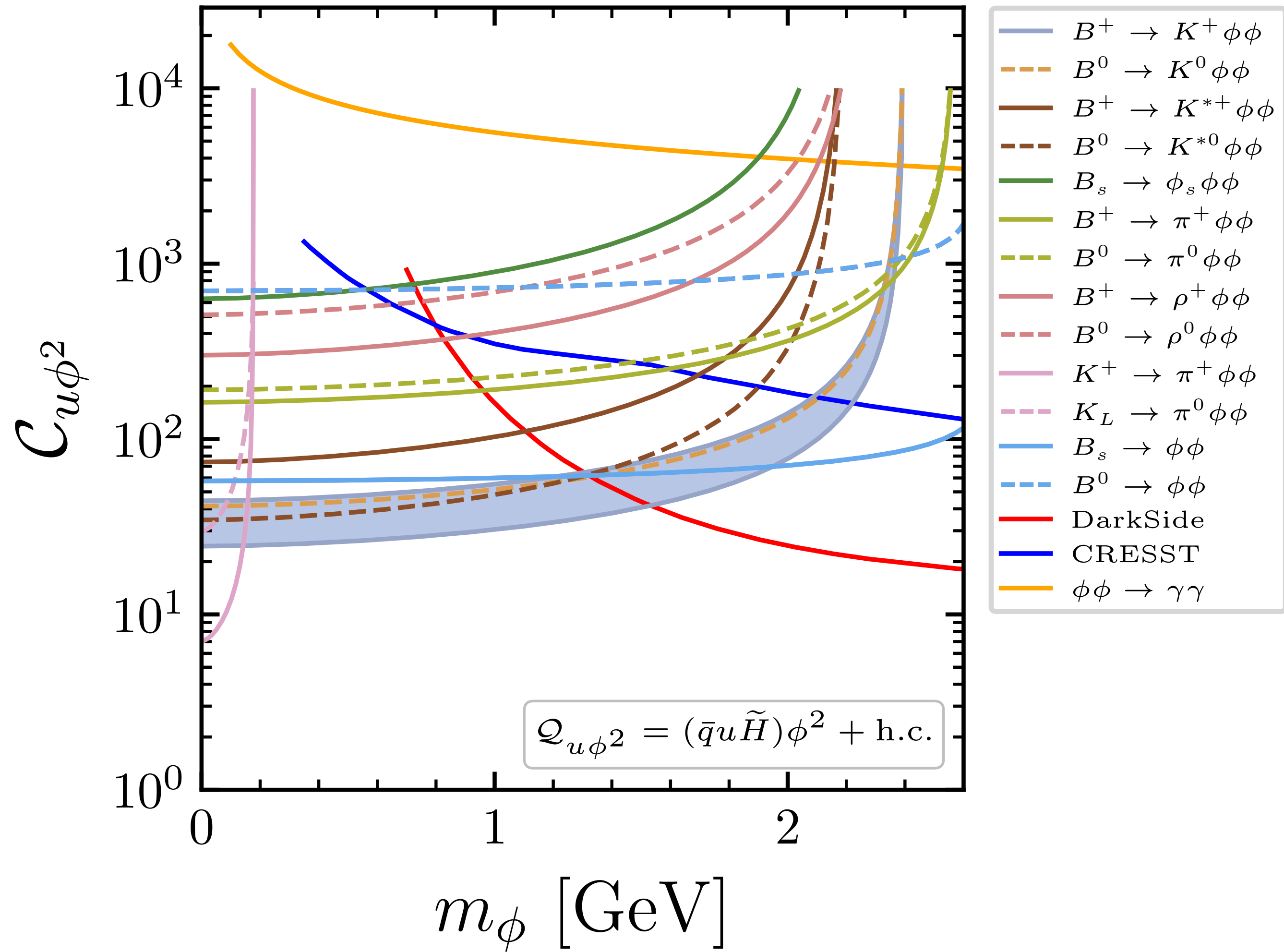
► Dark SMEFT with 3rd generation @  $\mu_{EW}$

$$\begin{aligned}
 \mathcal{Q}_{u\chi} &= (\bar{u}_p \gamma_\mu u_r)(\bar{\chi} \gamma^\mu \chi), \implies (\bar{t}_R \gamma_\mu t_R)(\bar{\chi} \gamma^\mu \chi) \\
 \mathcal{Q}_{q\chi} &= (\bar{q}_p \gamma_\mu q_r)(\bar{\chi} \gamma^\mu \chi), \\
 \mathcal{Q}_{u\chi^2} &= (\bar{q}_p u_r \tilde{H})(\bar{\chi} \chi), \quad \mathcal{C} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathcal{C}_{33}
 \end{aligned}$$

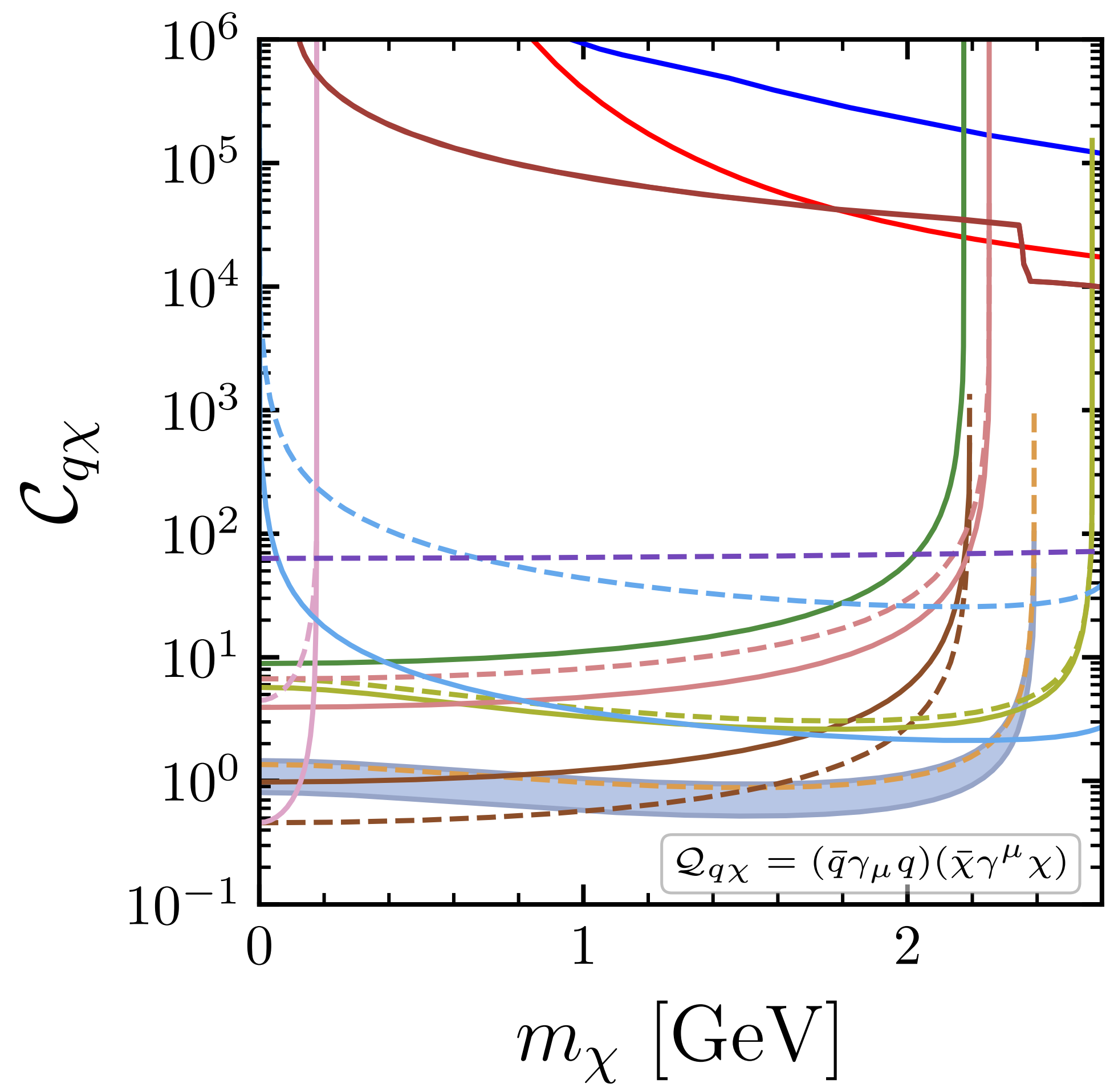
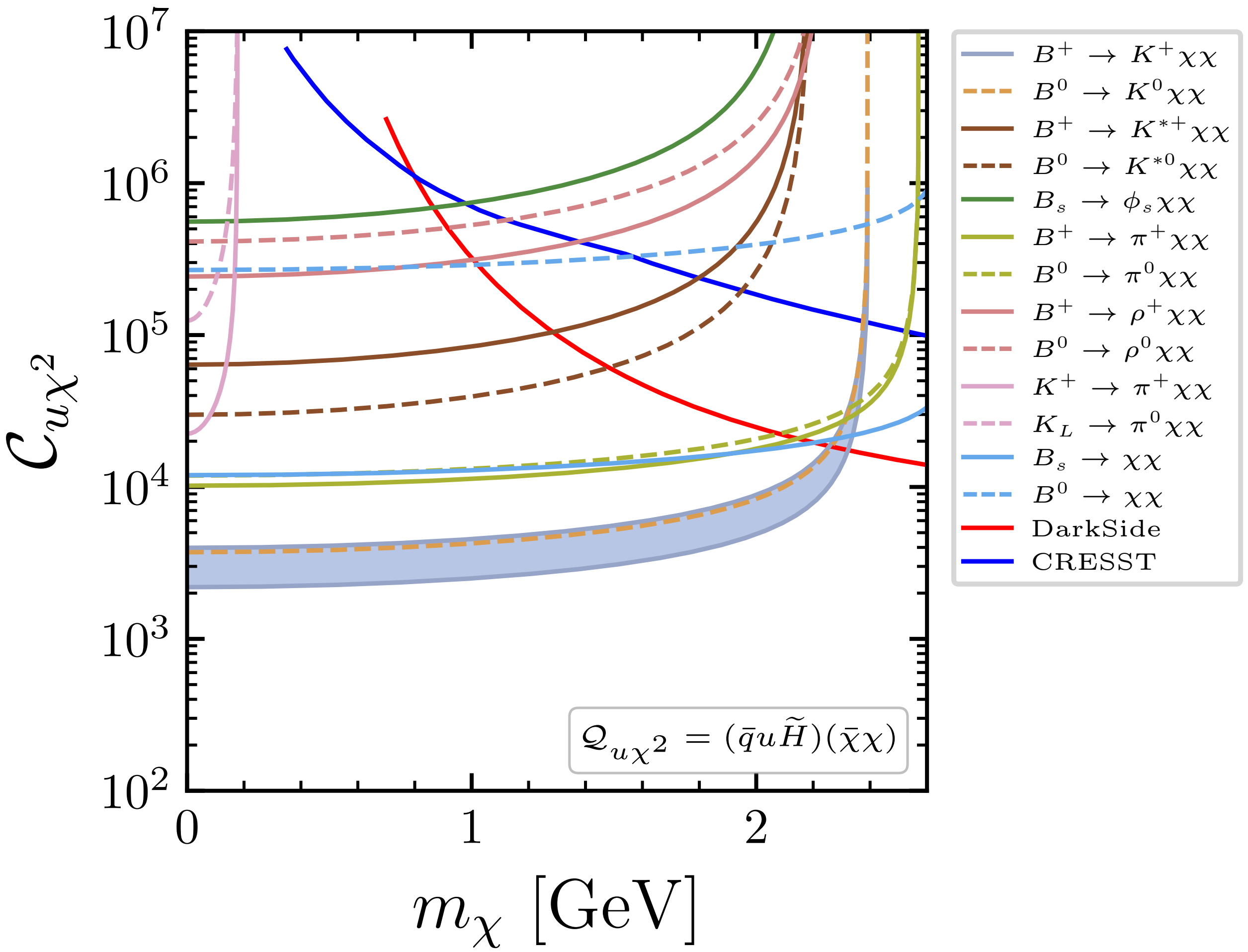
2013 Tongyan Lin, Kolb, Lian-Tao Wang  
 2015 Kilic, Klimek, Jiang-Hao Yu  
 2015 Haisch, Re  
 2015 Boucheneb, Cacciapaglia, Deandrea, Fuks  
 2017 Blanke, Kast  
 2021 Blanke, Pani, Polesello, Rovelli  
 2021 Haisch, Polesello, Schulte  
 2021 Hermanna, Worek  
 ... ..  
 2019 ATLAS [JHEP05(2019)142]



# Hadron decay vs direct detection



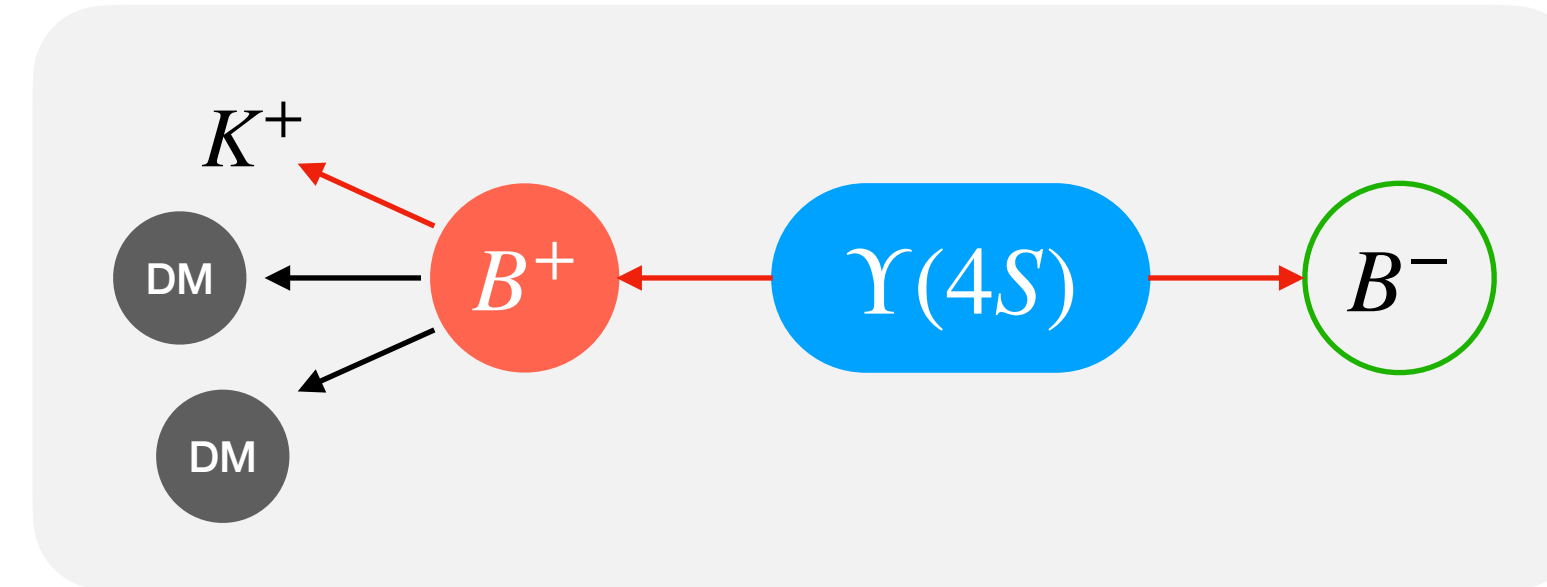
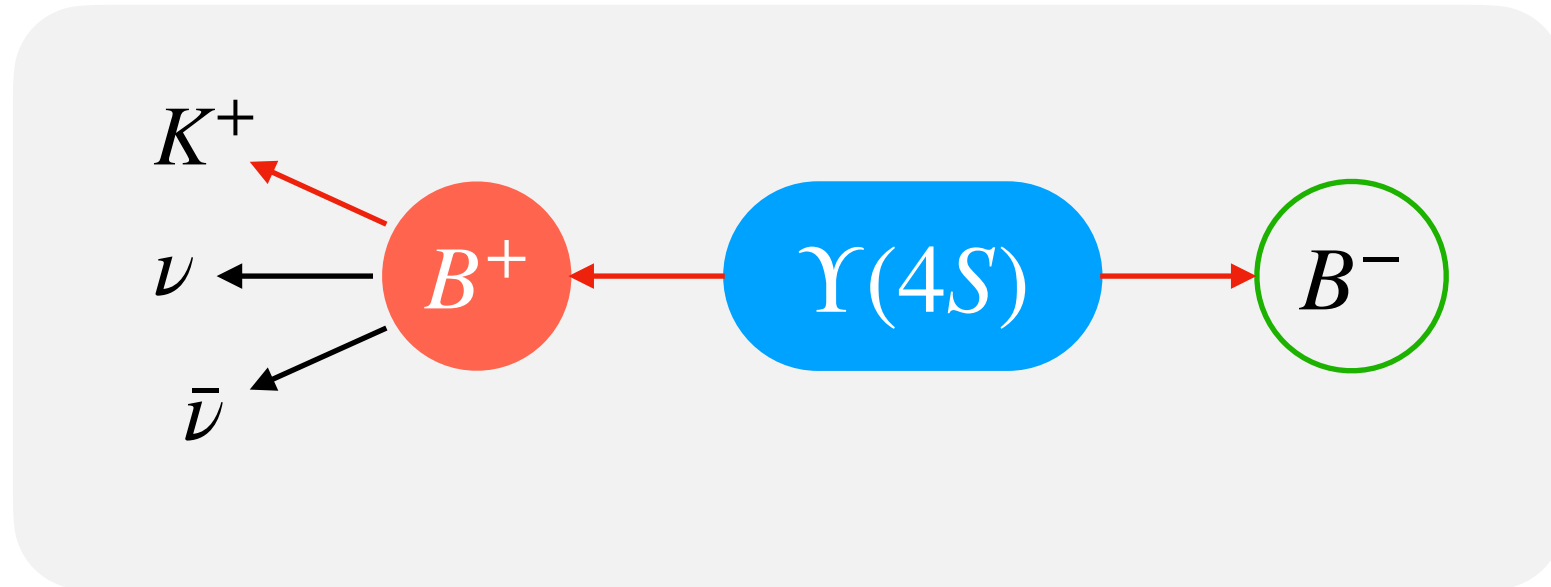
# Hadron decay vs direct detection



# Conclusion

HadronToNP: a package to calculate decay of hadron to new particles

$B \rightarrow K + \text{DM}$ ,  $B \rightarrow \rho + \text{DM}$ ,  $\Lambda_b \rightarrow \Lambda + \text{DM}$ ,  $\Upsilon \rightarrow \text{DM}$ , ... *to be finished*  
 $D \rightarrow \pi + \text{DM}$ ,  $D \rightarrow \rho + \text{DM}$ ,  $\Xi_c \rightarrow \Xi + \text{DM}$ ,  $J/\psi \rightarrow \text{DM}$ , ...



SMEFT

Dark SMEFT

$$\frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})}{\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})} = \frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{SM}}}{\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{SM}}} = 0.46 \pm 0.07$$

$$\frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})}{\mathcal{B}(B^+ \rightarrow \pi^+ \nu \bar{\nu})} = \frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{SM}}}{\mathcal{B}(B^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}}} = 29.7 \pm 5.6$$

Belle II excess (if confirmed in the future) implies:

- impossible to explain in SMEFT with MFV
- NP flavour structure is highly non-trivial
- **NP structure in quark sector is beyond MFV**
- **flavour violation is beyond Yukawa coupling**

$\mu_{\text{EW}}$   
LEFT

Dark LEFT

$\mu_b$

A Possible Flavour Path to New Physics

	$d$	$s$	$b$
$d$	DD	NA62/KOTO	Belle II
$s$		DD	Belle II
$b$			Belle II/LHC
	$u$	$c$	$t$
$u$	DD	BES/STCF	LHC
$c$		BES/STCF	LHC
$t$			LHC



# Backup

# $b \rightarrow s\nu\bar{\nu}$ : SMEFT

SMEFT

$$\mathcal{Q}_{Hq}^{(1)} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{q}_p \gamma^\mu q_r),$$

$$\mathcal{Q}_{Hq}^{(3)} = (H^\dagger i \overleftrightarrow{D}_\mu^I H) (\bar{q}_p \tau^I \gamma^\mu q_r),$$

$$\mathcal{Q}_{Hd} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{d}_p \gamma^\mu d_r),$$

$$\mathcal{Q}_{ld} = (\bar{l}_p \gamma^\mu l_r) (\bar{d}_s \gamma_\mu d_t),$$

$$\mathcal{Q}_{lq}^{(1)} = (\bar{l}_p \gamma^\mu l_r) (\bar{q}_s \gamma_\mu q_t),$$

$$\mathcal{Q}_{lq}^{(3)} = (\bar{l}_p \gamma^\mu \tau^I l_r) (\bar{q}_s \tau^I \gamma_\mu q_t),$$

$\mu_{EW}$

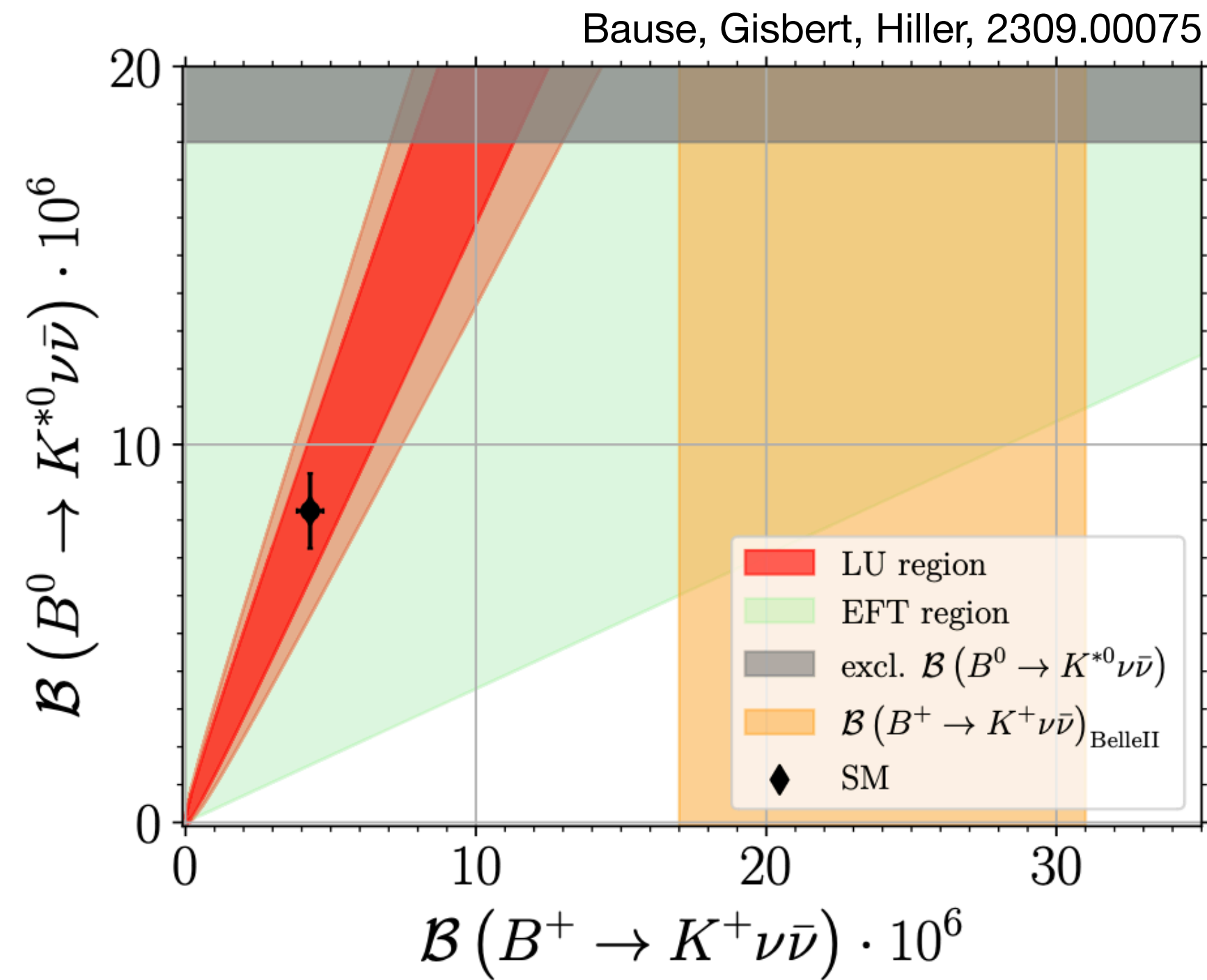
LEFT

$$\mathcal{O}_L^{\nu_i \nu_j} = (\bar{s} \gamma_\mu P_L b) (\bar{\nu}_i \gamma^\mu P_L \nu_j)$$

$$\mathcal{O}_R^{\nu_i \nu_j} = (\bar{s} \gamma_\mu P_R b) (\bar{\nu}_i \gamma^\mu P_L \nu_j)$$

$\mu_b$

operator structure highly  
constrained by Left-handed neutrino



$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu}) = A_+^{BK} x^+,$$

$$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu}) = A_+^{BK^*} x^+ + A_-^{BK^*} x^-,$$

$$x^\pm = \sum_{\nu, \nu'} |C_L^{\nu\nu'} \pm C_R^{\nu\nu'}|^2,$$

Bause, Gisbert, Hiller, 2309.00075

Allwicher, Becirevic, Piazza, Rosauero-Alcaraz, Sumensari, 2309.02246

Chen, Wen, Xu, 2401.11552

# $b \rightarrow s\nu\bar{\nu}$ : SMEFT

<b>SMEFT</b>	$\mathcal{Q}_{Hq}^{(1)} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{q}_p \gamma^\mu q_r),$ $\mathcal{Q}_{Hq}^{(3)} = (H^\dagger i \overleftrightarrow{D}_\mu^I H) (\bar{q}_p \tau^I \gamma^\mu q_r),$ $\mathcal{Q}_{Hd} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{d}_p \gamma^\mu d_r),$ $\mathcal{Q}_{ld} = (\bar{l}_p \gamma^\mu l_r) (\bar{d}_s \gamma_\mu d_t),$ $\mathcal{Q}_{lq}^{(1)} = (\bar{l}_p \gamma^\mu l_r) (\bar{q}_s \gamma_\mu q_t),$ $\mathcal{Q}_{lq}^{(3)} = (\bar{l}_p \gamma^\mu \tau^I l_r) (\bar{q}_s \tau^I \gamma_\mu q_t),$
$\mu_{EW}$	
<b>LEFT</b>	$\mathcal{O}_L^{\nu_i \nu_j} = (\bar{s} \gamma_\mu P_L b) (\bar{\nu}_i \gamma^\mu P_L \nu_j)$ $\mathcal{O}_R^{\nu_i \nu_j} = (\bar{s} \gamma_\mu P_R b) (\bar{\nu}_i \gamma^\mu P_L \nu_j)$
$\mu_b$	operator structure highly constrained by Left-handed neutrino

	Observable	SM	Exp	Unit
$b \rightarrow s$	$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})$	$4.16 \pm 0.57$	$23 \pm 5_{-4}^{+5}$	$10^{-6}$
	$\mathcal{B}(B^0 \rightarrow K^0 \nu \bar{\nu})$	$3.85 \pm 0.52$	$< 26$	$10^{-6}$
	$\mathcal{B}(B^+ \rightarrow K^{*+} \nu \bar{\nu})$	$9.70 \pm 0.94$	$< 61$	$10^{-6}$
	$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})$	$9.00 \pm 0.87$	$< 18$	$10^{-6}$
	$\mathcal{B}(B_s \rightarrow \phi \nu \bar{\nu})$	$9.93 \pm 0.72$	$< 5400$	$10^{-6}$
	$\mathcal{B}(B_s \rightarrow \nu \bar{\nu})$	$\approx 0$	$< 5.9$	$10^{-4}$
$b \rightarrow d$	$\mathcal{B}(B^+ \rightarrow \pi^+ \nu \bar{\nu})$	$1.40 \pm 0.18$	$< 140$	$10^{-7}$
	$\mathcal{B}(B^0 \rightarrow \pi^0 \nu \bar{\nu})$	$6.52 \pm 0.85$	$< 900$	$10^{-8}$
	$\mathcal{B}(B^+ \rightarrow \rho^+ \nu \bar{\nu})$	$4.06 \pm 0.79$	$< 300$	$10^{-7}$
	$\mathcal{B}(B^0 \rightarrow \rho^0 \nu \bar{\nu})$	$1.89 \pm 0.36$	$< 400$	$10^{-7}$
	$\mathcal{B}(B^0 \rightarrow \nu \bar{\nu})$	$\approx 0$	$< 1.4$	$10^{-4}$
	$s \rightarrow d$	$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$	$8.42 \pm 0.61$	$10.6_{-3.4}^{+4.0} \pm 0.9$
$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$		$3.41 \pm 0.45$	$< 300$	$10^{-11}$

Why such a large NP effect has not shown up in other  $b \rightarrow s$  decays ?  
in  $b \rightarrow d, s \rightarrow d$  decays ? **NP flavour structure**



# $b \rightarrow s\nu\bar{\nu}$ and $b \rightarrow s\ell\bar{\ell}$

SMEFT notation:  $l = \begin{pmatrix} \nu \\ e \end{pmatrix}_L, q = \begin{pmatrix} u \\ d \end{pmatrix}_L, d = d_R$

B.F.Hou, X.Q.Li, M.Shen, Y.D.Yang, **XBY**, 2402.19208

## ► Prediction

$$\frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})}{\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})} = \frac{\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu})_{\text{SM}}}{\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{SM}}} = 0.46 \pm 0.07$$

## ► prediction

$$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{SM}} = (9.00 \pm 0.87) \times 10^{-6}$$

$$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{SMEFT}} = (50^{+17}_{-16}) \times 10^{-6} \quad \text{conflict}$$

$$\mathcal{B}(B^0 \rightarrow K^{*0} \nu \bar{\nu})_{\text{exp}} < 18 \times 10^{-6}$$

## ► Only $\mathcal{O}_{lq}^{(3)}$ is relevant with $R_{D^{(*)}}$

## ► $\mathcal{O}_{ld}$ can explain the $B^+ \rightarrow K^+ \nu \bar{\nu}$ data

## ► $\mathcal{O}_{ld}$ also induce $O'_{9,ij}$ and $O'_{10,ij}$

## ► They can't improve the $b \rightarrow s\ell\bar{\ell}$ fit

►  $O'_{9e}$  and  $O'_{10\mu}$  worsen the fit. **weird** (LFV,  $\tau\tau \gg ee, \mu\mu$ )

►  $O'_{9,ij}$  and  $O'_{10,ij}$  with  $i = j = \tau$  has no effect.

►  $O'_{9,ij}$  and  $O'_{10,ij}$  with  $i \neq j$  (i.e. LFV) has no effect.

SMEFT

$$\mathcal{Q}_{Hq}^{(1)} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{q}_p \gamma^\mu q_r),$$

$$\mathcal{Q}_{Hq}^{(3)} = (H^\dagger i \overleftrightarrow{D}_\mu^I H) (\bar{q}_p \tau^I \gamma^\mu q_r),$$

$$\mathcal{Q}_{Hd} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{d}_p \gamma^\mu d_r),$$

$$\mathcal{Q}_{ld} = (\bar{l}_p \gamma^\mu l_r) (\bar{d}_s \gamma_\mu d_t),$$

$$\mathcal{Q}_{lq}^{(1)} = (\bar{l}_p \gamma^\mu l_r) (\bar{q}_s \gamma_\mu q_t),$$

$$\mathcal{Q}_{lq}^{(3)} = (\bar{l}_p \gamma^\mu \tau^I l_r) (\bar{q}_s \tau^I \gamma_\mu q_t),$$

$\mu_{\text{EW}}$

LEFT

$$\mathcal{O}_L^{\nu_i \nu_j} = (\bar{s} \gamma_\mu P_L b) (\bar{\nu}_i \gamma^\mu P_L \nu_j)$$

$$\mathcal{O}_R^{\nu_i \nu_j} = (\bar{s} \gamma_\mu P_R b) (\bar{\nu}_i \gamma^\mu P_L \nu_j)$$

$\mu_b$

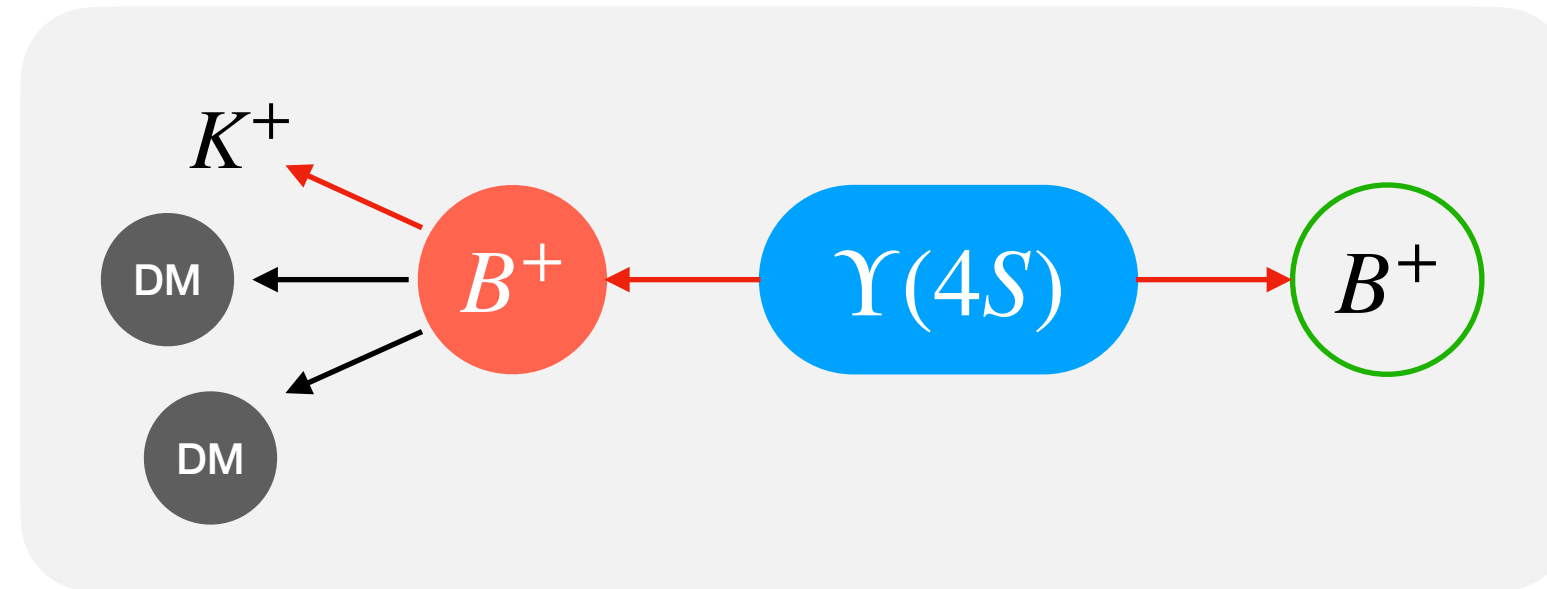
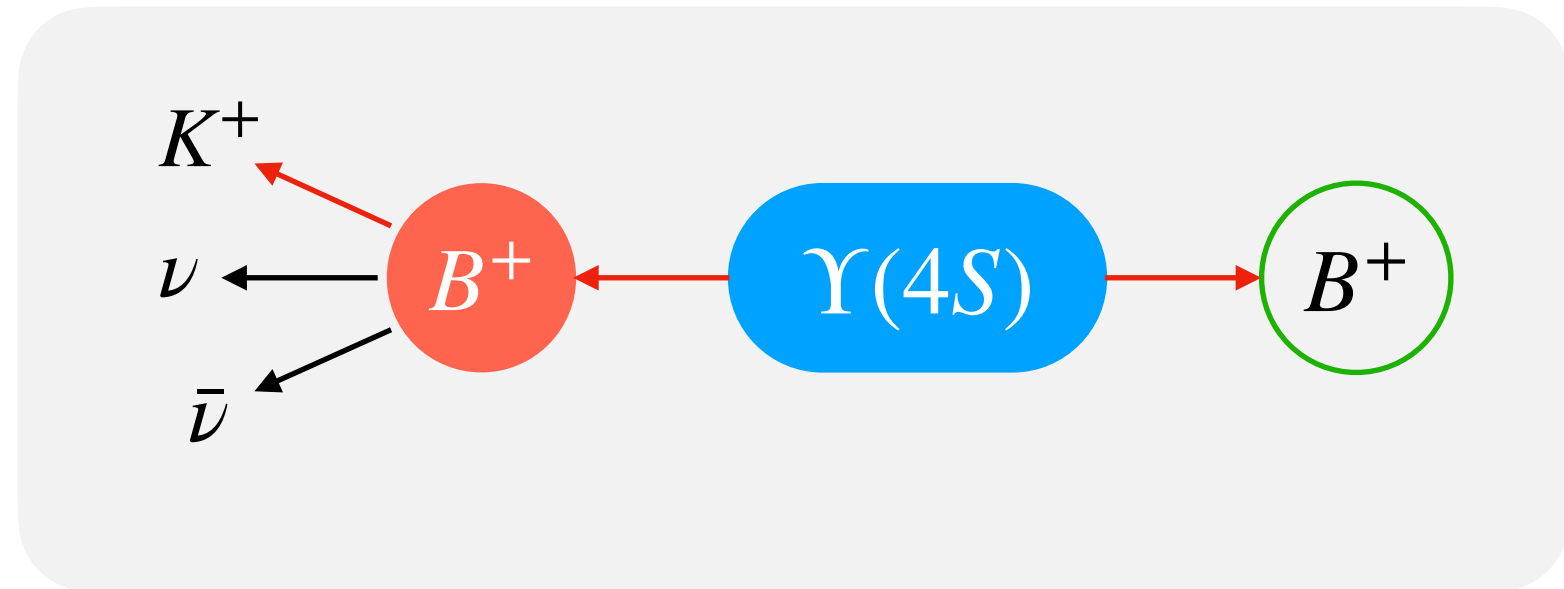
$$O'_{9,ij} = (\bar{b} \gamma^\mu P_{RS}) (\bar{\ell}_i \gamma_\mu \ell_j)$$

$$O'_{10,ij} = (\bar{b} \gamma^\mu P_{RS}) (\bar{\ell}_i \gamma_\mu \gamma_5 \ell_j)$$

induce  $\bar{s}bZ$  interaction,  
Thus, universally affect  
 $b \rightarrow se^+e^-, \mu^+\mu^-, \tau^+\tau^-$

**one LEFT operator!**  
just the SM operator

# $b \rightarrow s\nu\bar{\nu}$ : exp picture



$$Q_{Hq}^{(1)} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{q}_p \gamma^\mu q_r),$$

$$Q_{Hq}^{(3)} = (H^\dagger i \overleftrightarrow{D}_\mu^I H) (\bar{q}_p \tau^I \gamma^\mu q_r),$$

$$Q_{Hd} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{d}_p \gamma^\mu d_r),$$

$$Q_{ld} = (\bar{l}_p \gamma^\mu l_r) (\bar{d}_s \gamma_\mu d_t),$$

$$Q_{lq}^{(1)} = (\bar{l}_p \gamma^\mu l_r) (\bar{q}_s \gamma_\mu q_t),$$

$$Q_{lq}^{(3)} = (\bar{l}_p \gamma^\mu \tau^I l_r) (\bar{q}_s \tau^I \gamma_\mu q_t),$$

$$\mathcal{O}_L^{\nu_i \nu_j} = (\bar{s} \gamma_\mu P_L b) (\bar{\nu}_i \gamma^\mu P_L \nu_j)$$

$$\mathcal{O}_R^{\nu_i \nu_j} = (\bar{s} \gamma_\mu P_R b) (\bar{\nu}_i \gamma^\mu P_L \nu_j)$$

**SMEFT**

**Dark SMEFT**

example

$$Q_{d\phi^2} = (\bar{q}_p d_r H) \phi^2$$

$$Q_{d\chi} = (\bar{d}_p \gamma_\mu d_r) (\bar{\chi} \gamma^\mu \chi)$$

$$Q_{dX^2} = (\bar{q}_p d_r H) X_\mu X^\mu$$

$$Q_{qa} = (\bar{q}_p \gamma_\mu q_r) \partial^\mu a$$

- 2011 Kamenik, Smith
- 2014 Duch, Grzadkowski, Wudka
- 2017 Brod, Gootjes-Dreesbach, Tamaro, Zupan
- 2021 Criado, Djouadi, Perez-Victoria, Santiago
- 2022 Aebischer, Altmannshofer, Jenkins, Manohar ([basis@dim-6](mailto:basis@dim-6))
- 2023 Song, Sun, Yu ([basis@dim-8](mailto:basis@dim-8))

- Axion-like particle, see also H.Y.Cheng, Phys.Rept 1988
- 2020 Bauer, Neubert, Renner, Schnubel, Thamm
  - 2023 Song, Sun, Yu ([basis@dim-8](mailto:basis@dim-8))

$\mu_{EW}$   
**LEFT**

**Dark LEFT**

$$\mathcal{O}_{d\phi^2} = (\bar{d}_{Lp} d_{Rr}) \phi^2$$

$$\mathcal{O}_{d\chi}^{V,LR} = (\bar{d}_{Lp} \gamma_\mu d_{Lr}) (\bar{\chi}_a \gamma^\mu \chi_b)$$

$$\mathcal{O}_{dXX}^L = (\bar{d}_{Lp} \gamma_\mu d_{Lr}) X^{\mu\nu} X_\nu$$

$$\mathcal{O}_{da}^L = (\bar{d}_{Lp} \gamma_\mu d_{Lr}) \partial^\mu a$$

example

- 2022 Aebischer, Altmannshofer, Jenkins, Manohar ([basis@dim-6](mailto:basis@dim-6))
- 2022 He, Ma, Valencia ([basis@dim-6](mailto:basis@dim-6))
- 2023 Liang, Liao, Ma, Wang ([basis@dim-8](mailto:basis@dim-8))

$\mu_b$

# Backup

$$\begin{aligned}
 \mathcal{Q}_{d\phi} &= (\bar{q}_p d_r H) \phi + \text{h.c.}, & \mathcal{Q}_{d\phi^2} &= (\bar{q}_p d_r H) \phi^2 + \text{h.c.}, \\
 \mathcal{Q}_{\phi q} &= (\bar{q}_p \gamma_\mu q_r) (i\phi_1 \overleftrightarrow{\partial}^\mu \phi_2), & \mathcal{Q}_{\phi d} &= (\bar{d}_p \gamma_\mu d_r) (i\phi_1 \overleftrightarrow{\partial}^\mu \phi_2),
 \end{aligned} \tag{4.2}$$

$$\mathcal{Q}_{q\chi} = (\bar{q}_p \gamma_\mu q_r) (\bar{\chi} \gamma^\mu \chi), \quad \mathcal{Q}_{d\chi} = (\bar{d}_p \gamma_\mu d_r) (\bar{\chi} \gamma^\mu \chi), \tag{4.3}$$

$$\mathcal{Q}_{dHX} = (\bar{q}_p \sigma_{\mu\nu} d_r) H X^{\mu\nu} + \text{h.c.}, \tag{4.4}$$

$$\begin{aligned}
 \mathcal{Q}_{dX} &= (\bar{d}_p \gamma_\mu d_r) X^\mu, & \mathcal{Q}_{HdX} &= (H^\dagger H) (\bar{d}_p \gamma^\mu d_r) X_\mu, \\
 \mathcal{Q}_{qX} &= (\bar{q}_p \gamma_\mu q_r) X^\mu, & \mathcal{Q}_{HqX}^{(1)} &= (H^\dagger H) (\bar{q}_p \gamma^\mu q_r) X_\mu, \\
 \mathcal{Q}_{dX^2} &= (\bar{q}_p d_r H) X_\mu X^\mu + \text{h.c.}, & \mathcal{Q}_{HqX}^{(3)} &= (H^\dagger \tau^I H) (\bar{q}_p \tau^I \gamma^\mu q_r) X_\mu, \\
 \mathcal{Q}_{qXX} &= (\bar{q}_p \gamma_\mu q_r) X^{\mu\nu} X_\nu, & \mathcal{Q}_{dXX} &= (\bar{d}_p \gamma_\mu d_r) X^{\mu\nu} X_\nu, \\
 \mathcal{Q}_{q\tilde{X}X} &= (\bar{q}_p \gamma_\mu q_r) \tilde{X}^{\mu\nu} X_\nu, & \mathcal{Q}_{d\tilde{X}X} &= (\bar{d}_p \gamma_\mu d_r) \tilde{X}^{\mu\nu} X_\nu, \\
 \mathcal{Q}_{DqX^2} &= i(\bar{q}_p \gamma^\mu D^\nu q_r) X_\mu X_\nu + \text{h.c.}, & \mathcal{Q}_{DdX^2} &= i(\bar{d}_p \gamma^\mu D^\nu d_r) X_\mu X_\nu + \text{h.c.}, \\
 \mathcal{Q}_{dHX^2} &= (\bar{q}_p \sigma_{\mu\nu} d_r H) X_1^\mu X_2^\nu + \text{h.c.}, & &
 \end{aligned} \tag{4.5}$$

$$c_i = \tilde{c}_i \cdot \begin{cases} (m_X/\Lambda)^2 & \text{for } \mathcal{Q}_i = \mathcal{Q}_{dX^2}, \mathcal{Q}_{DdX^2}, \mathcal{Q}_{DqX^2}, \mathcal{Q}_{dHX^2}, \\ (m_X/\Lambda) & \text{for } \mathcal{Q}_i = \text{others.} \end{cases}$$

$$\mathcal{Q}_{qa} = (\bar{q}_p \gamma_\mu q_r) \partial^\mu a, \quad \mathcal{Q}_{da} = (\bar{d}_p \gamma_\mu d_r) \partial^\mu a, \tag{4.7}$$



# Backup

One can also apply the MFV hypothesis to the lepton sector. However, since the mechanism of neutrino mass generation is still unknown, there are different approaches to formulate the leptonic MFV [73–79]. Here, we consider the realization of leptonic MFV within the so-called minimal field content [73, 74], in which the neutrino masses are generated by the Weinberg operator. In this case, the Yukawa interactions in the lepton sector can be written as

$$-\Delta\mathcal{L} = \bar{e}Y_e H^\dagger l + \frac{1}{2\Lambda_{\text{LN}}}(\bar{l}^c\tau_2 H)Y_\nu(H^T\tau_2 l) + \text{h.c.}, \quad (2.18)$$

where  $l$  denotes the left-handed lepton doublet with the charge conjugated field given by  $l^c = -i\gamma_2 l^*$ , and  $e$  is the right-handed charged lepton singlet.  $\Lambda_{\text{LN}}$  denotes the breaking scale of the lepton number symmetry  $U(1)_{\text{LN}}$ .  $Y_e$  and  $Y_\nu$  stand for the  $3 \times 3$  Yukawa coupling matrices in flavour space. In the absence of these Yukawa couplings, the lepton sector respects the flavour symmetry

$$G_{\text{LF}} = SU(3)_l \otimes SU(3)_e. \quad (2.19)$$

finite polynomial of  $\mathbf{A}_\ell$  and  $\mathbf{B}_\ell$ . After neglecting all the terms involving  $\mathbf{B}_\ell$ , which are suppressed by the small lepton Yukawa couplings  $Y_e$ , we obtain

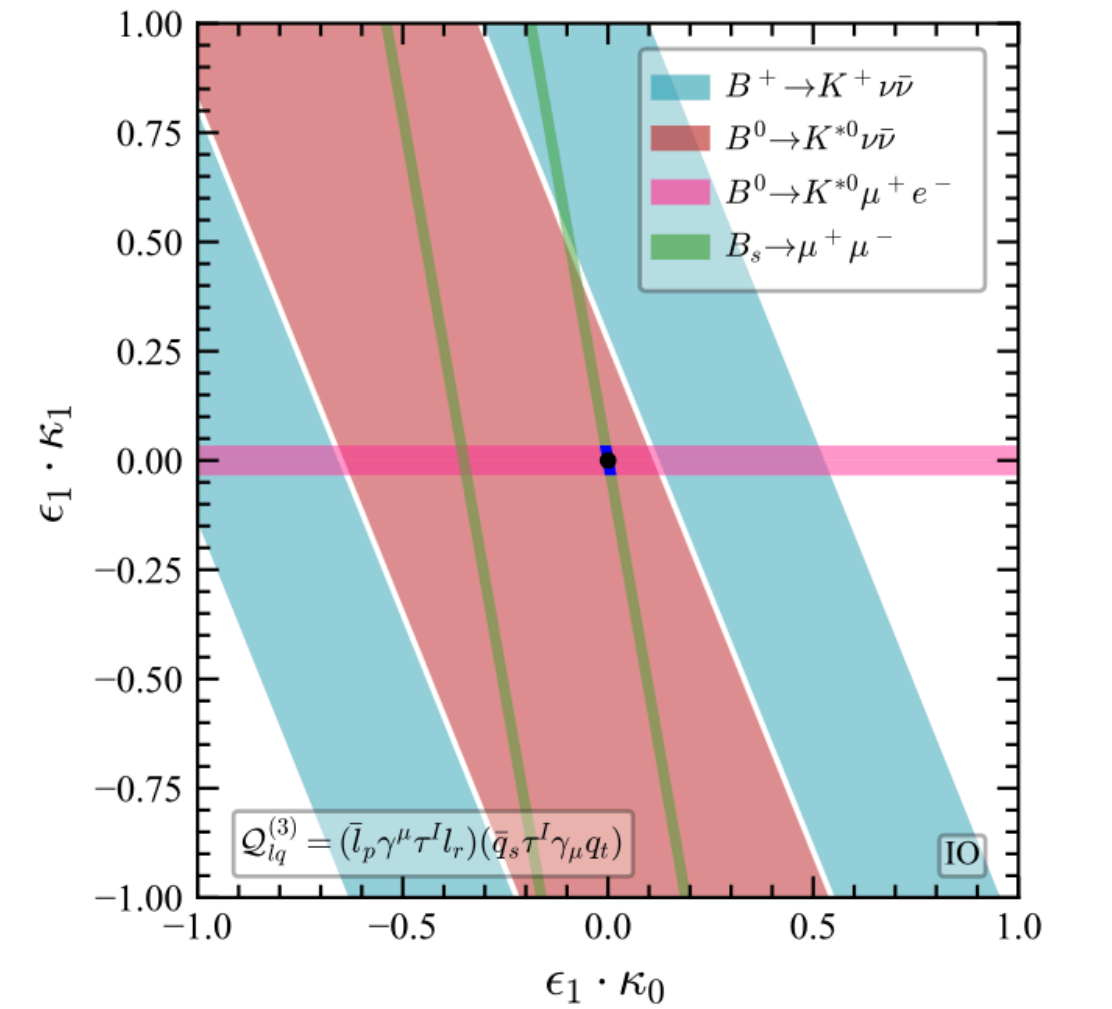
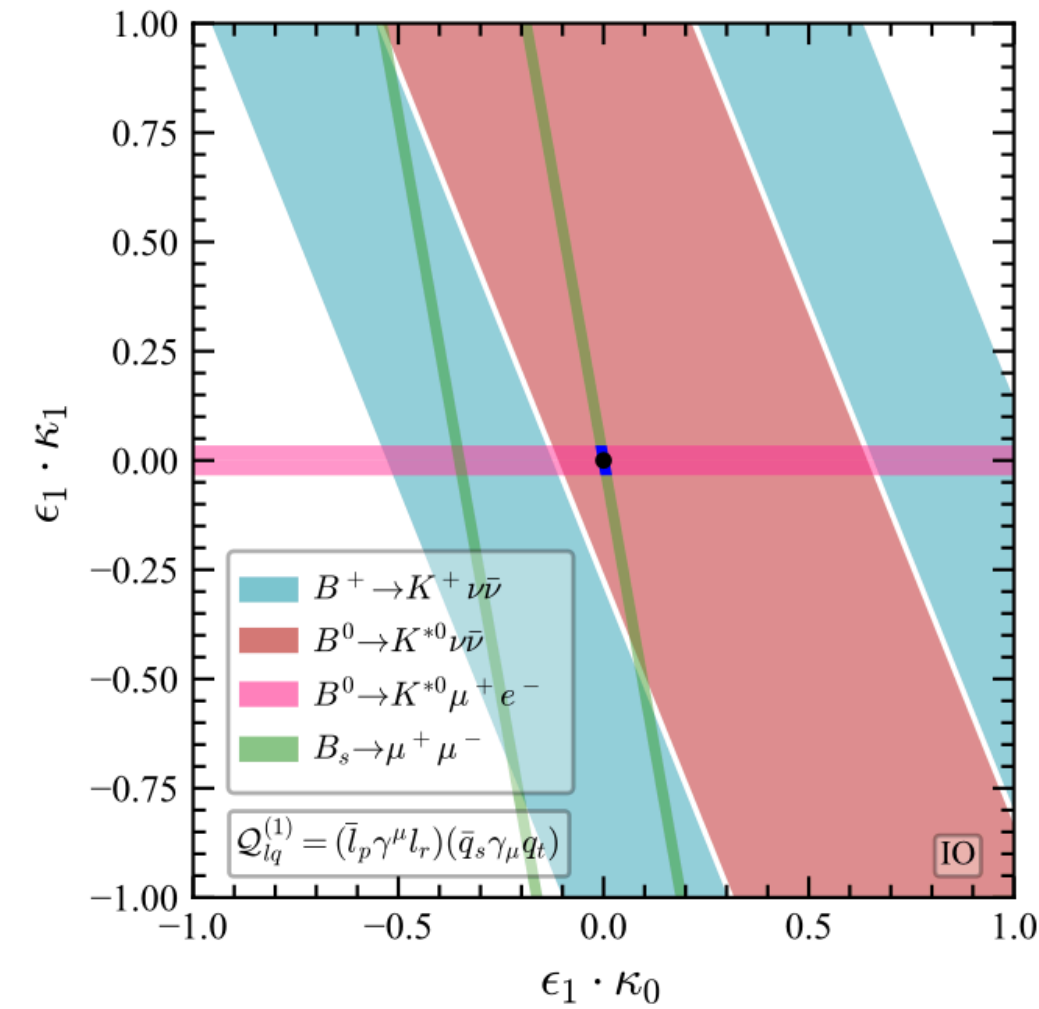
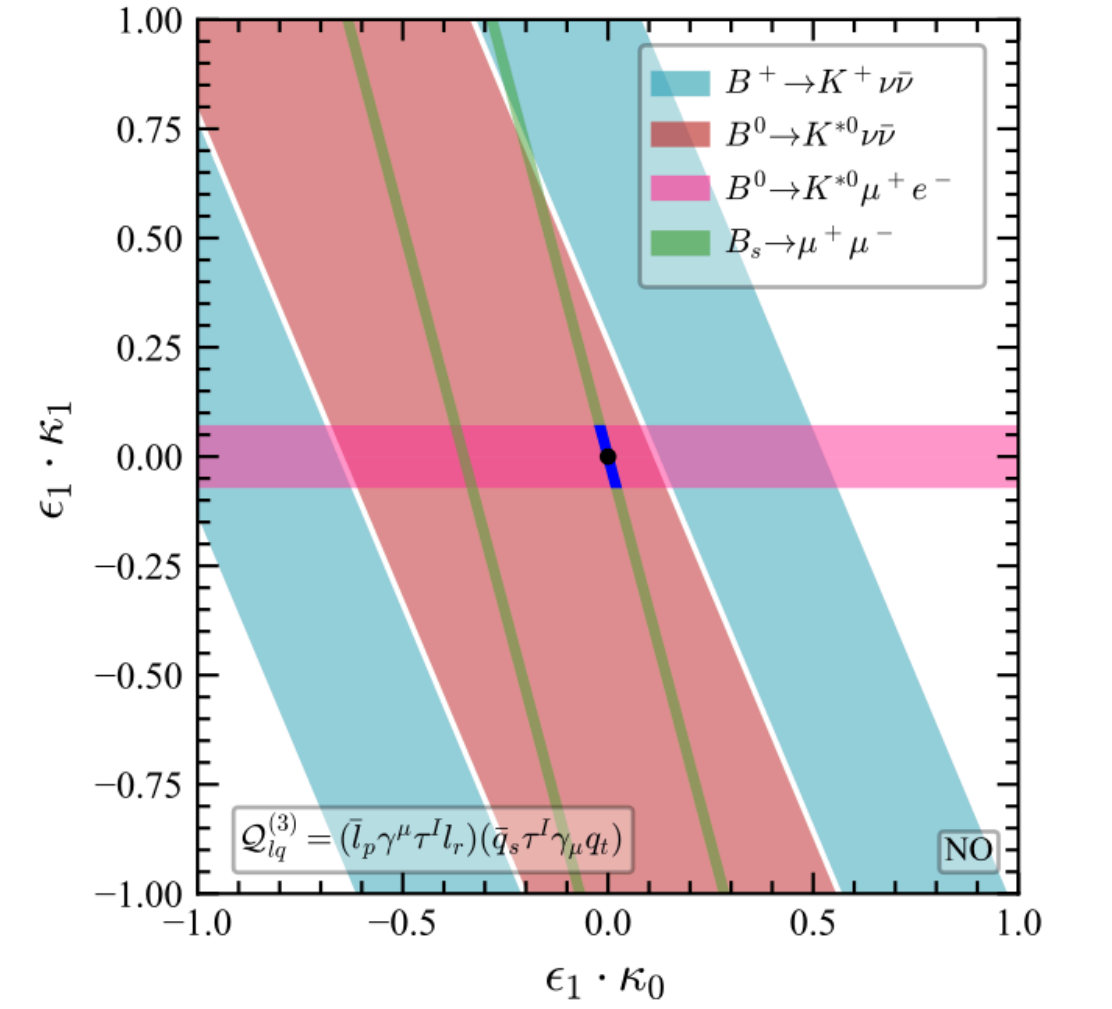
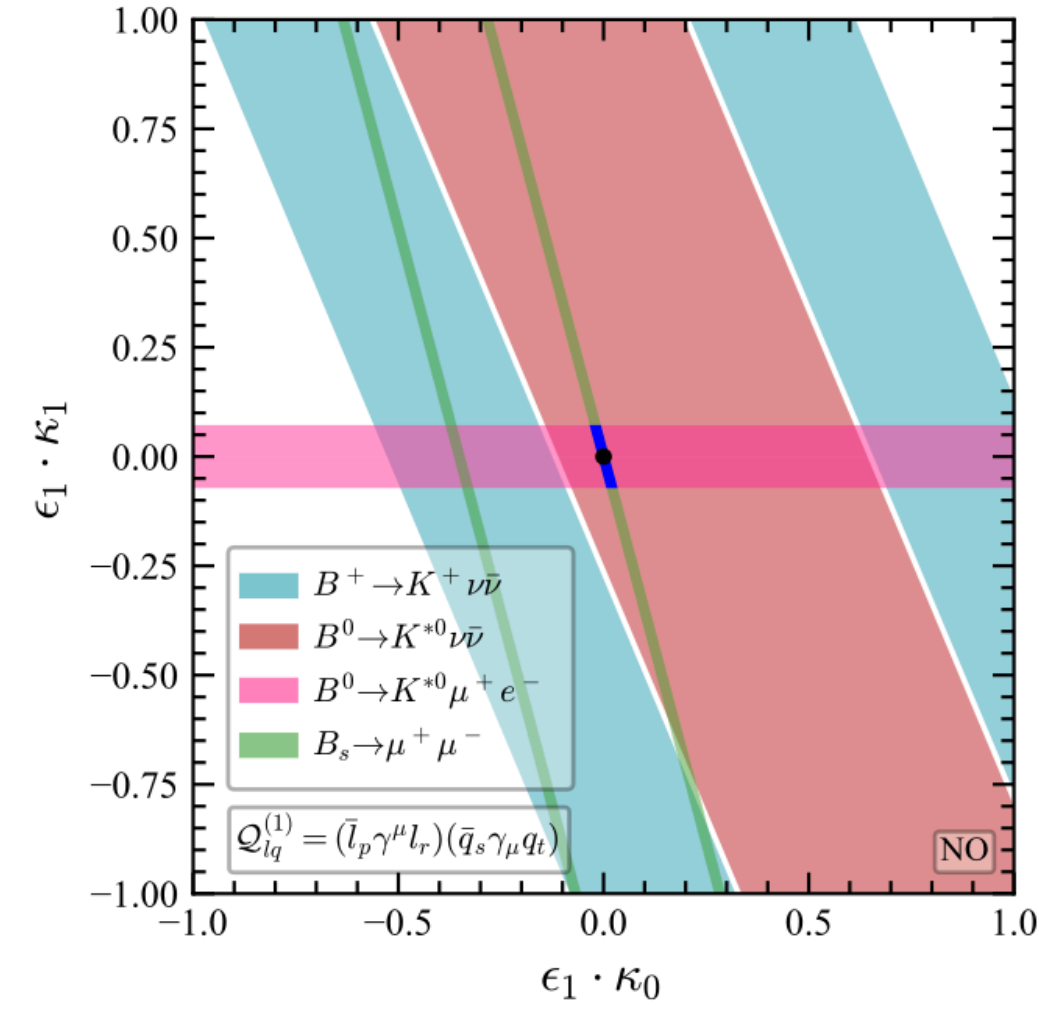
$$\mathcal{C}_{\text{MFV}} \approx \kappa_0 + \kappa_1 \mathbf{A}_\ell + \kappa_2 \mathbf{A}_\ell^2, \quad (2.21)$$

where the coefficients  $\kappa_{0,1,2}$  are free real parameters. In the numerical analysis, we keep only the leading lepton flavour violation term  $\mathbf{A}_\ell$  for simplicity, i.e.,  $\kappa_2 = 0$ . Turning to the lepton mass eigenbasis, the current  $\bar{l}\gamma^\mu C l$  gives in the MFV hypothesis the following interactions:

$$\bar{e}_L \gamma^\mu (\kappa_0 \mathbf{1} + \kappa_0 \Delta_\ell) e_L + \bar{\nu}_L \gamma^\mu (\kappa_0 \mathbf{1} + \kappa_0 \hat{\lambda}_\nu^2) \nu_L, \quad (2.22)$$

where the basic LFV coupling  $\Delta_\ell$  can be obtained from  $\mathbf{A}_\ell$  and takes the form

$$\Delta_\ell = U \hat{\lambda}_\nu^2 U^\dagger, \quad (2.23)$$

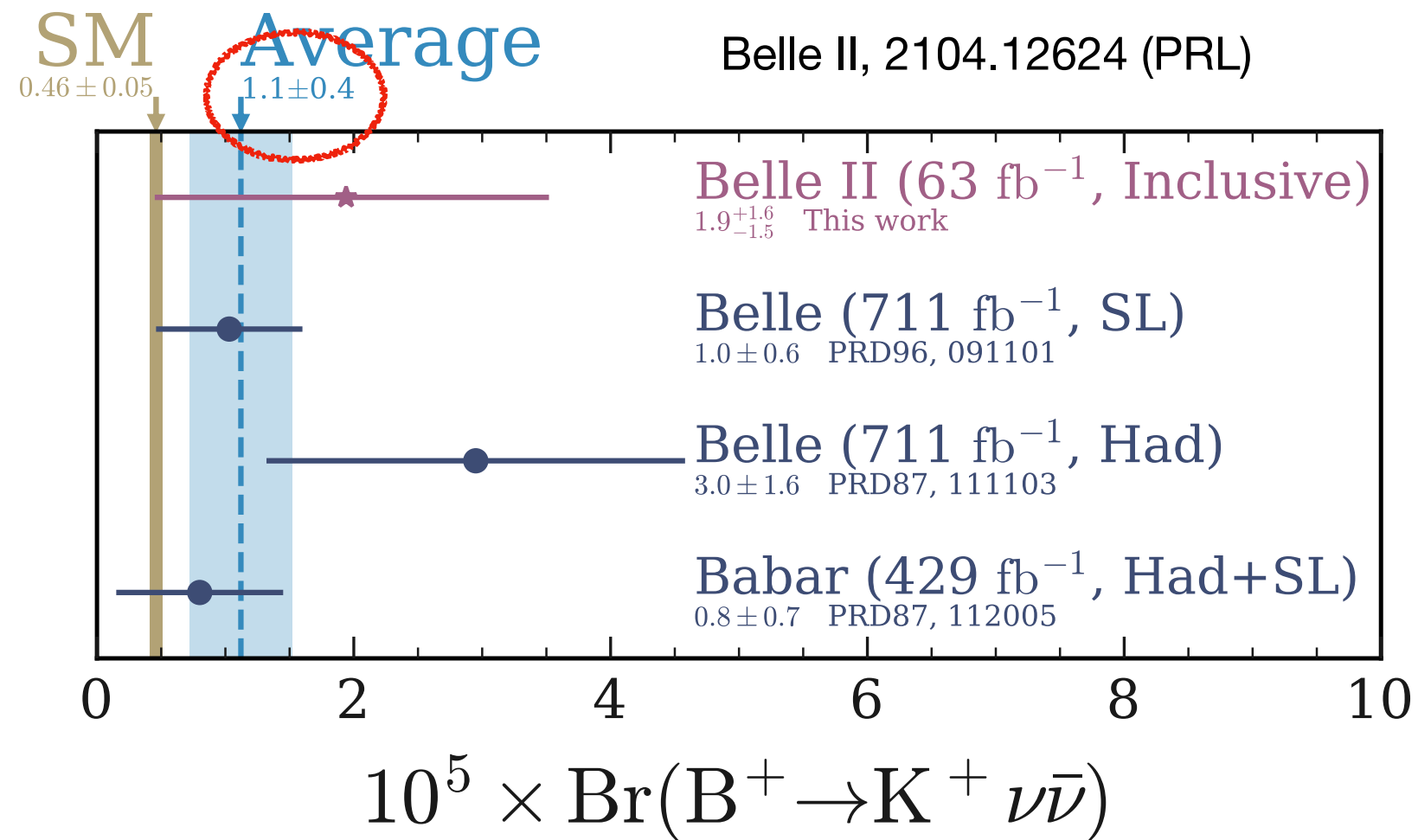


$$\Delta_\ell^{\text{NO}} = \begin{pmatrix} -0.19 - 0.01i & -0.25 - 0.02i & 0.31 - 0.04i \\ 0.12 + 0.01i & 0.28 - 0.00i & 0.29 + 0.04i \\ -0.37 - 0.01i & 0.21 - 0.05i & -0.03 + 0.01i \end{pmatrix}, \quad \Delta_\ell^{\text{IO}} = \begin{pmatrix} 0.21 + 0.09i & -0.34 + 0.05i & 0.03 + 0.11i \\ 0.31 + 0.12i & 0.19 + 0.00i & -0.15 - 0.14i \\ 0.12 - 0.02i & 0.04 - 0.19i & 0.34 - 0.10i \end{pmatrix}$$



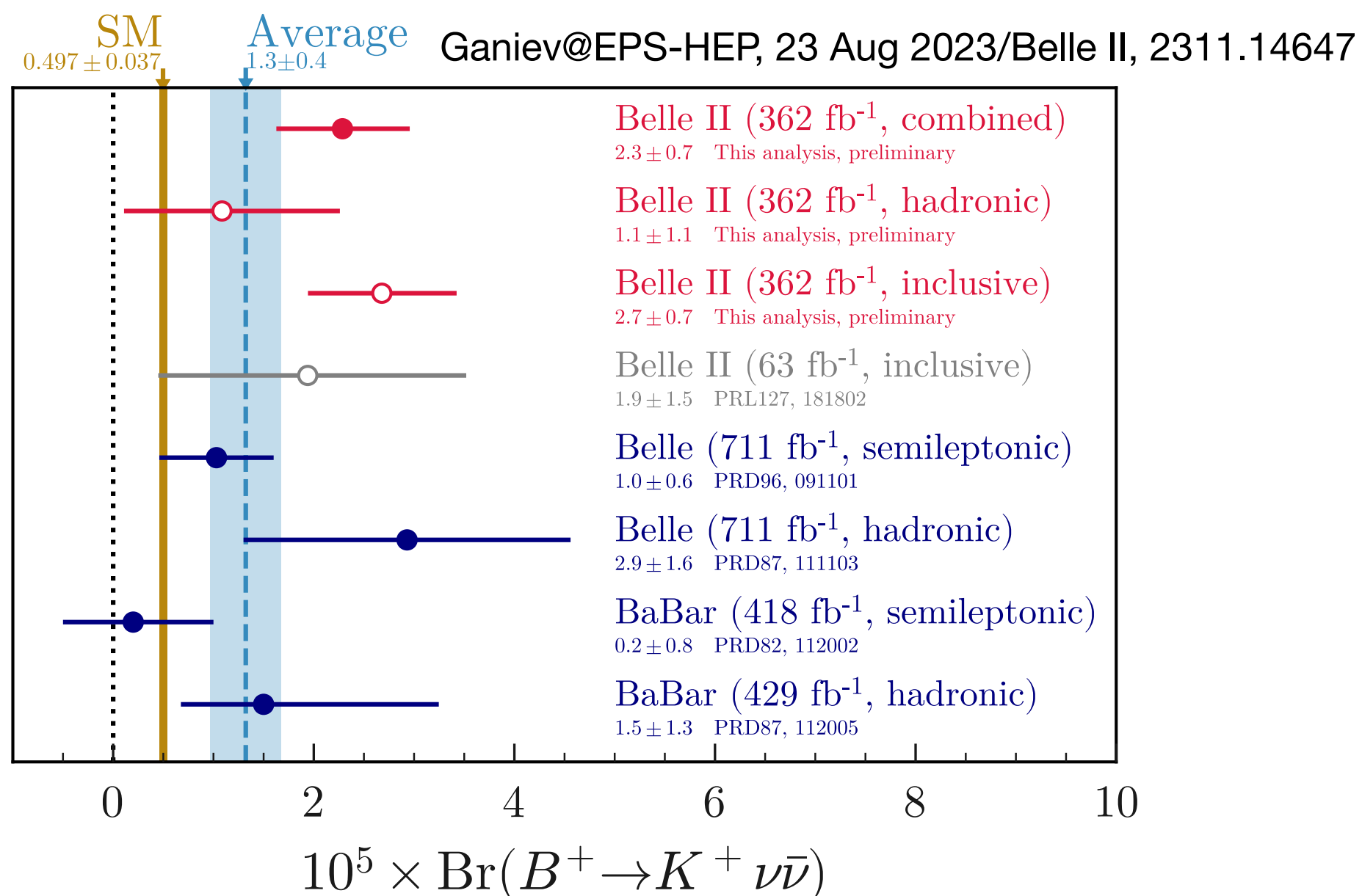
# $b \rightarrow s\nu\bar{\nu}$ : exp & theory

## ▶ 2021 Apr



## ▶ 2023 Aug

see also 贾森's talk



**Impact of  $B \rightarrow K\nu\bar{\nu}$  measurements on beyond the Standard Model theories** #69  
 Thomas E. Browder (Hawaii U.), Nilendra G. Deshpande (Oregon U.), Rusa Mandal (Siegen U.), Rahul Sinha (IMSc, Chennai and Bhubaneswar, Inst. Phys.) (Jul 2, 2021)  
 Published in: *Phys.Rev.D* 104 (2021) 5, 053007 · e-Print: 2107.01080 [hep-ph]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [34 citations](#)

**A tale of invisibility: constraints on new physics in  $b \rightarrow s\nu\bar{\nu}$**  #65  
 Tobias Felki (New South Wales U.), Sze Lok Li (New South Wales U.), Michael A. Schmidt (New South Wales U.) (Nov 8, 2021)  
 Published in: *JHEP* 12 (2021) 118 · e-Print: 2111.04327 [hep-ph]  
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30+ theory papers !

**Phenomenological study of a gauged  $L_\mu - L_\tau$  model with a scalar leptoquark** #42  
 Chuan-Hung Chen (Taiwan, Natl. Cheng Kung U. and NCTS, Taipei), Cheng-Wei Chiang (Taiwan, Natl. Taiwan U. and NCTS, Taipei), Chun-Wei Su (Taiwan, Natl. Taiwan U.) (May 16, 2023)  
 Published in: *Phys.Rev.D* 109 (2024) 5, 5 · e-Print: 2305.09256 [hep-ph]  
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**Higgs portal interpretation of the Belle II  $B^+ \rightarrow K^+ \nu\bar{\nu}$  measurement** #29  
 David McKeen (TRIUMF), John N. Ng (TRIUMF), Douglas Tuckler (TRIUMF and Simon Fraser U.) (Dec 1, 2023)  
 Published in: *Phys.Rev.D* 109 (2024) 7, 075006 · e-Print: 2312.00982 [hep-ph]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [10 citations](#)

**Light new physics in  $B \rightarrow K^{(*)} \nu\bar{\nu}$ ?** #30  
 Wolfgang Altmannshofer (UC, Santa Cruz, Inst. Part. Phys.), Andreas Crivellin (Zurich U.), Huw Haigh (Vienna, OAW), Gianluca Inguglia (Vienna, OAW), Jorge Martin Camalich (IAC, La Laguna) (Nov 24, 2023)  
 Published in: *Phys.Rev.D* 109 (2024) 7, 075008 · e-Print: 2311.14629 [hep-ph]  
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**$B \rightarrow K\nu\bar{\nu}$ , MiniBooNE and muon  $g - 2$  anomalies from a dark sector** #31  
 Alakabha Datta (Mississippi U. and SLAC and UC, Santa Cruz), Danny Marfatia (Hawaii U.), Lopamudra Mukherjee (Nankai U.) (Oct 23, 2023)  
 Published in: *Phys.Rev.D* 109 (2024) 3, L031701 · e-Print: 2310.15136 [hep-ph]  
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**$B \rightarrow K^* M_X$  vs  $B \rightarrow K M_X$  as a probe of a scalar-mediator dark matter scenario** #33  
 Alexander Berezhnoy (SINP, Moscow), Dmitri Melikhov (SINP, Moscow and Dubna, JINR and Vienna U.) (Sep 29, 2023)  
 Published in: *EPL* 145 (2024) 1, 14001 · e-Print: 2309.17191 [hep-ph]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [10 citations](#)

**Flavor anomalies in leptoquark model with gauged  $U(1)_{\nu_\mu - L_\tau}$**  #34  
 Chuan-Hung Chen (Taiwan, Natl. Cheng Kung U. and Unlisted, TW), Cheng-Wei Chiang (Taiwan, Natl. Taiwan U. and Unlisted, TW) (Sep 22, 2023)  
 Published in: *Phys.Rev.D* 109 (2024) 7, 075004 · e-Print: 2309.12904 [hep-ph]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [9 citations](#)

**Revisiting models that enhance  $B^+ \rightarrow K^+ \nu\bar{\nu}$  in light of the new Belle II measurement** #35  
 Belle-II Collaboration · Xiao-Gang He (Tsung-Dao Lee Inst., Shanghai and Taiwan, Natl. Taiwan U.) et al. (Sep 22, 2023)  
 Published in: *Phys.Rev.D* 109 (2024) 7, 075019 · e-Print: 2309.12741 [hep-ph]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [16 citations](#)

**A new look at  $\bar{b} \rightarrow s$  observables in 331 models** #18  
 Francesco Loporco (Jan 22, 2024)  
 e-Print: 2401.11999 [hep-ph]  
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**Correlating  $B \rightarrow K^{(*)} \nu\bar{\nu}$  and flavor anomalies in SMEFT** #19  
 Feng-Zhi Chen, Qiaoyi Wen, Fanrong Xu (Jan 21, 2024)  
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**Recent  $B^+ \rightarrow K^+ \nu\bar{\nu}$  Excess and Muon  $g - 2$  Illuminating Light Dark Sector with Higgs Portal** #20  
 Shu-Yu Ho, Jongkuk Kim, Pyungwon Ko (Jan 18, 2024)  
 e-Print: 2401.10112 [hep-ph]  
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**Explaining the  $B^+ \rightarrow K^+ \nu\bar{\nu}$  excess via a massless dark photon** #16  
 E. Gabrielli, L. Marzola, K. Mürsepp, M. Raidal (Feb 8, 2024)  
 e-Print: 2402.05901 [hep-ph]  
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**Decoding the  $B \rightarrow K \nu\bar{\nu}$  excess at Belle II: kinematics, operators, and masses** #27  
 Kåre Fridell, Mitrajyoti Ghosh, Takemichi Okui, Kohsaku Tobioka (Dec 19, 2023)  
 e-Print: 2312.12507 [hep-ph]  
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**Understanding the first measurement of  $B(B \rightarrow K \nu\bar{\nu})$**  #38  
 Lukas Allwicher (Zurich U.), Damir Becirevic (IJCLab, Orsay), Gioacchino Piazza (IJCLab, Orsay), Salvador Rosairo-Alcaraz (IJCLab, Orsay), Olcyr Sumensari (IJCLab, Orsay) (Sep 5, 2023)  
 Published in: *Phys.Lett.B* 848 (2024) 138411 · e-Print: 2309.02246 [hep-ph]  
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**Implications of an enhanced  $B \rightarrow K \nu\bar{\nu}$  branching ratio** #39  
 Rigo Bause (Tech. U., Dortmund (main)), Hector Gisbert (INFN, Padua and Padua U.), Gudrun Hiller (Tech. U., Dortmund (main) and Sussex U.) (Aug 31, 2023)  
 Published in: *Phys.Rev.D* 109 (2024) 1, 015006 · e-Print: 2309.00075 [hep-ph]  
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**B meson anomalies and large  $B^+ \rightarrow K^+ \nu\bar{\nu}$  in non-universal  $U(1)'$  models** #40  
 Peter Athron (Nanjing Normal U.), R. Martinez (Colombia, U. Natl.), Cristian Sierra (Nanjing Normal U.) (Aug 25, 2023)  
 Published in: *JHEP* 02 (2024) 121 · e-Print: 2308.13426 [hep-ph]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [22 citations](#)

**SMEFT predictions for semileptonic processes** #4  
 Siddhartha Karmakar, Amol Dighe, Rick S. Gupta (Apr 15, 2024)  
 e-Print: 2404.10061 [hep-ph]  
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**Implications of  $B \rightarrow K \nu\bar{\nu}$  under Rank-One Flavor Violation hypothesis** #5  
 David Marzocca, Marco Nardecchia, Alfredo Stanzione, Claudio Toni (Apr 9, 2024)  
 e-Print: 2404.06533 [hep-ph]  
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**The quark flavor-violating ALPs in light of B mesons and hadron colliders** #20  
 Tong Li (Nankai U.), Zhuoni Qian (Hangzhou Normal U.), Michael A. Schmidt (Sydney U. and New South Wales U.), Man Yuan (Nankai U.) (Feb 21, 2024)  
 Published in: *JHEP* 05 (2024) 232 · e-Print: 2402.14232 [hep-ph]  
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**Scalar dark matter explanation of the excess in the Belle II  $B^+ \rightarrow K^+ + \text{invisible}$  measurement** #9  
 Xiao-Gang He, Xiao-Dong Ma, Michael A. Schmidt, German Valencia, Raymond R. Volkas (Mar 19, 2024)  
 e-Print: 2403.12485 [hep-ph]  
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**Status and prospects of rare decays at Belle-II** #10  
 Elisa Manoni (Mar 12, 2024)  
 Published in: *PoS WFAI2023* (2024) 024 · Contribution to: *WFAI 2023*, 024  
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**Rare  $B$  and  $K$  decays in a scotogenic model** #11  
 Chuan-Hung Chen, Cheng-Wei Chiang (Mar 5, 2024)  
 e-Print: 2403.02897 [hep-ph]  
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