$D \to K\pi\ell\nu$ as a probe of low-energy QCD

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Charm at threshold workshop
Beijing, Oct 22 2011

Semileptonic D decays

Semileptonic decays

- Great probes of low-energy QCD
- $D \to \pi\ell\nu$, $D \to K\ell\nu$: extraction of $|V_{cd}|$, $|V_{cs}|$ together with lattice
- But other modes also interesting...

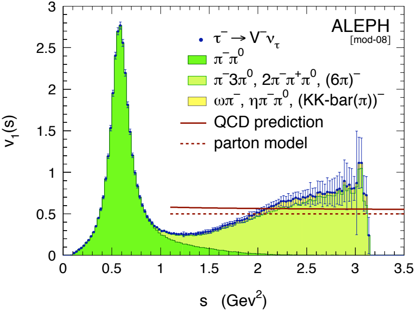
$D \to K\pi\ell\nu$

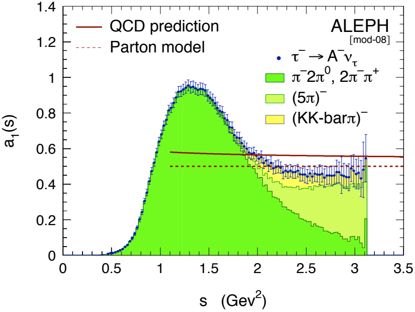
- Dominated by $D \to K^*(892)\ell\nu$, with several form factors to be compared with lattice/hadronic models
- Final-state interaction probes $K\pi$ scattering in S, P and D waves...
- ...related to corresponding resonances ($\kappa(800), K^*(892)\ldots$)
- More generally, valuable information on low-energy QCD dynamics

Low-energy QCD and light quarks

QCD at low energies

- Spectrum degeneracies: isospin multiplets, octets and decuplets...
- Significant differences between opposite parity channels





- Mass gap: pseudoscalar mesons (π, K, η) lighter than others

$SU(3)$ flavour symmetry + spontaneous breakdown of a symmetry

QCD and chiral symmetry

Project over left- and right-handed chiralities $q_{L,R} = \frac{1 \mp \gamma_5}{2} q$

$$\mathcal{L}_q = \sum_q \bar{q}_L i\partial q_L + \bar{q}_R i\partial q_R - m_q(\bar{q}_L q_R + \bar{q}_R q_L)$$

In chiral limit $m_{u,d\ldots} \to 0$, $\mathcal{L}_q$ inv. under $SU(N_f)$ rotations

$$\Psi = \begin{pmatrix} u \\ d \\ \vdots \end{pmatrix} \qquad \begin{array}{ccc} \Psi_L & \to & V_L \Psi_L \\ \Psi_R & \to & V_R \Psi_R \end{array}$$

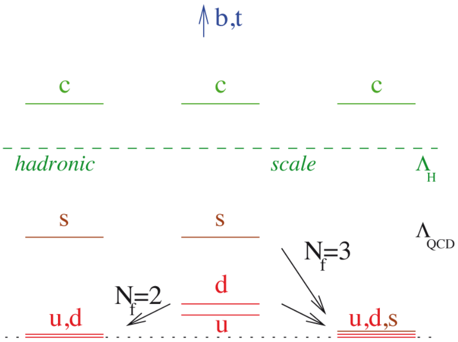
Spontaneous breaking: $SU_L(N_f) \times SU_R(N_f) \to SU_V(N_f)$

Goldstone Theorem

$N_f^2 - 1$ massless bosons whose interactions vanish as $E \to 0$

$\Rightarrow$ Light pseudoscalar mesons (for $m_q \neq 0$)

Two chiral limits of interest



Two a-priori different patterns of chiral symmetry breaking

$m_u, m_d \to 0$

$N_f = 3 \quad : \quad m_s \to 0$
$N_f = 2 \quad : \quad m_s$ physical

Two low-energy effective theories of QCD

$N_f = 2$: π only d.o.f. (very low E, few param. & processes)
$N_f = 3$: π, K, η d.o.f. (higher E, more param. & processes)

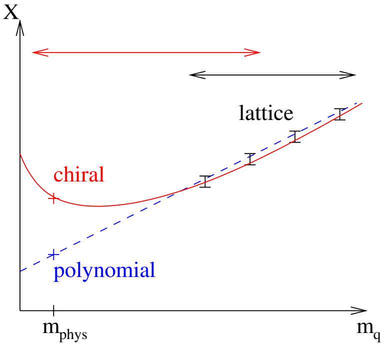
Chiral Perturbation Theory (χPT): expansion in powers of m_q and p

χPT and lattice

χPT : structure of π or π, K, η interactions but not values of couplings

Symbiotic relation with lattice

- Light masses for χPT, but unknown constants
- Heavier masses for lattice (especially for $m_{u,d}$), but extrapolation



Impact of *chiral logarithms* on extrapolation

$$M_\pi^2 \log \frac{M_\pi^2}{\mu^2}$$

Good understanding of pattern(s) of chiral symmetry breaking needed for accurate determination of form factors (e.g., $K \to \pi\ell\nu$ for $|V_{us}|$)

$\Longrightarrow$Useful to determine chiral symmetry breaking from $\pi\pi$ and πK scatterings

$\pi\pi$ scattering

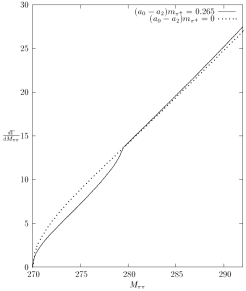
$\pi\pi$ experimental information

$\pi\pi$ (re)scattering projected onto partial waves of definite isospin t^I_ℓ
described via partial-wave phase shifts δ^I_ℓ and inelasticity η^I_ℓ

$$t^I_\ell(s) = \frac{1}{2i\sigma(s)}\left\{\eta^I_\ell(s)\, e^{2i\delta^I_\ell(s)} - 1\right\}. \qquad \sigma(s) = \sqrt{1 - \frac{4M_\pi^2}{s}}.$$

Parametrisation in terms of scattering lengths a^ℓ_I

$$\mathrm{Re}\, t^I_\ell(s) = q^{2\ell}\,\{a^I_\ell + q^2\, b^I_\ell + q^4\, c^I_\ell + \ldots\} \qquad s = 4(M_\pi^2 + q^2)$$

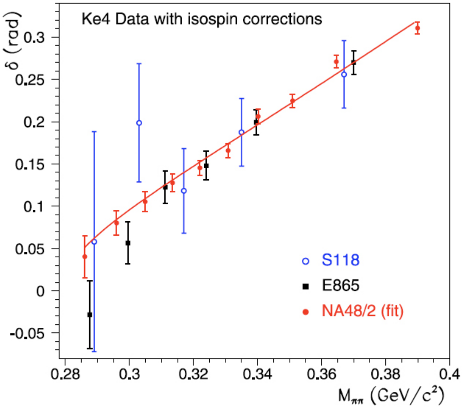


Constraints from

- Pion production on nucleon
- $K^+ \to \pi^+\pi^0\pi^0$ cusp related to $\pi^+\pi^- \to \pi^0\pi^0$ (re)scattering
  N.Cabibbo, G.Isidori; G.Colangelo et al.; NA48/2
- Pionium lifetime *DIRAC*
- $K \to \pi\pi\ell\nu$ decays *NA48/2*

$K \to \pi\pi\ell\nu$: *S*-*P* interference

$K_{\ell 4}$ dominated by *S* and *P* waves, with interference: $\delta_0^0 - \delta_1^1$ for $4M_\pi^2 \leq s \leq M_K^2$
very clean probe of $\pi\pi$ scattering thanks to Watson theorem

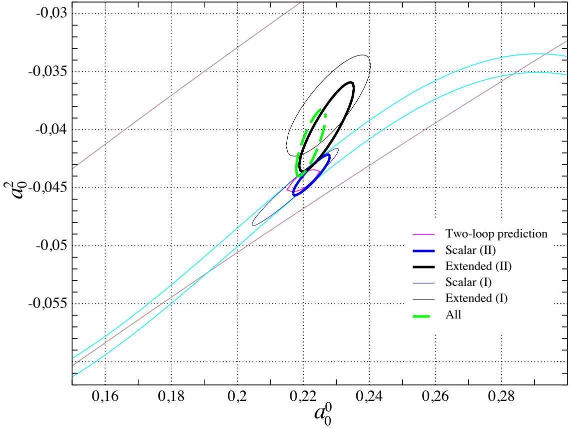


- For a long time, only "high-statistics" Geneva-Saclay exp.
- 2000 : E865 (Brookhaven)
- 2007-2010 : NA48/2 dominating the sample ($K^\pm \to \pi^+\pi^-\ell^\pm\nu$)
- At such accuracy, isospin breaking matters: experiment includes real & virtual photons (Coulomb corrections, Photos), but not $M_{\pi^+} \neq M_{\pi^0}$
- Corrections to Watson theorem before extracting information on $N_f = 2$ chiral symmetry breaking

G. Colangelo, J. Gasser and A. Rusetski; V. Bernard, SDG and M. Knecht

Results before and after NA48/2

- Dispersive constraints (Roy equations): amplitudes in terms of (a_0^0, a_0^2)
- (a_0^0, a_0^2) extracted from $\delta_0^0 - \delta_1^1$ measured in $K_{\ell 4}$
  - adding theoretical information on pion scalar radius (Scalar)
  - or experimental info on $I = 2, \ell = 0$ channel (Extended)
  - or including info on $I = 2, \ell = 0$ channel and $K \to 3\pi$ cusp (All)

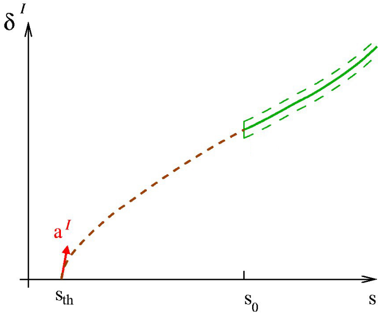


- Sharpening constraints once NA48/2 included (thin → thick ellipses)
- Good agreement with NNLO prediction from χPT with pions only
- $N_f = 2$ quark condensate $\langle 0|\bar{u}u|0\rangle$ dominating $\pi\pi$ interactions

What about πK scatt. ?

πK scattering

πK theoretical description



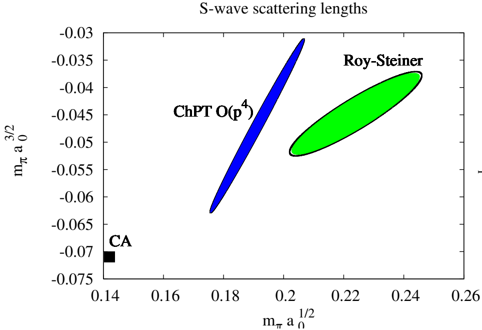
(Dispersive) Roy-Steiner eqs. yield low-energy phase shifts for πK scattering from higher-energy data in terms of two subtraction constants

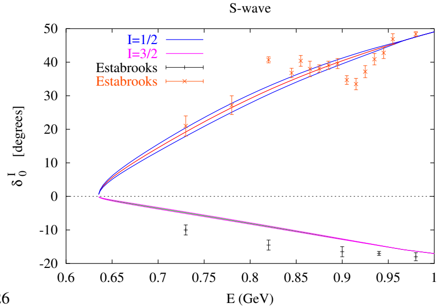
$$(a_0^{1/2}, a_0^{3/2})$$
$$\mathrm{Im}\ t \text{ for } s_0 \leq s \leq s_2$$
$$\Downarrow$$
$$\delta_{\ell=0,1}^{I=1/2,3/2} \text{ for } s_{\mathrm{th}} \leq s \leq s_0$$

P. Büttiker, SDG and B. Moussallam

- High-E data [$s \geq (0.97\ \mathrm{GeV})^2$] already constrain low-$E$ πK scattering
- But more efficient to combine dispersive contraints with both high- and low-E data to constrain structure of $N_f = 3$ chiral symmetry breaking
- …limiting impact of high-energy data or theoretical assumptions

πK theoretical constraints

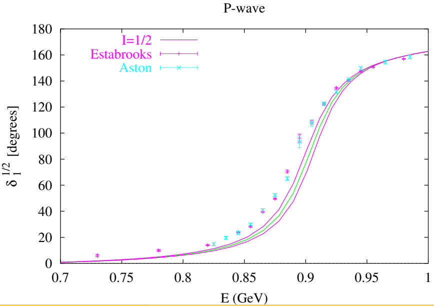




Using only consistency between high and low energies to predict low E

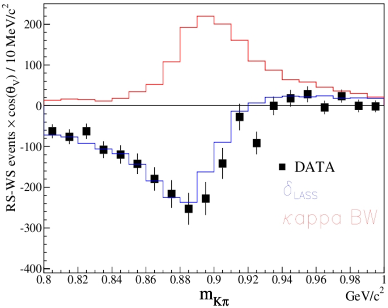
$$M_\pi\, a_0^{1/2} \simeq 0.224 \pm 0.022$$
$$M_\pi\, a_0^{3/2} \simeq (-0.448 \pm 0.077) \cdot 10^{-1}$$

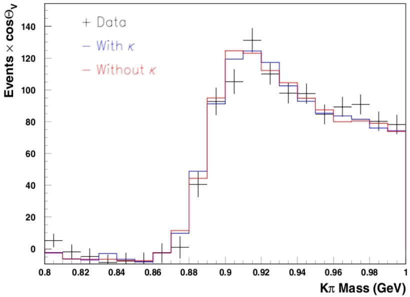
Not very good agreement with χPT
Contrary to $\pi\pi$ scattering



πK experimental information

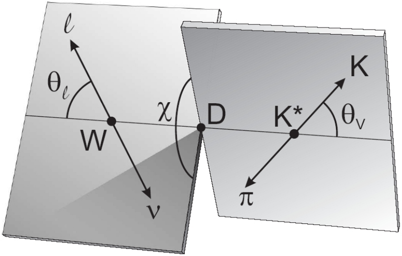
- $Kp \to K\pi N$ interaction, but higher-E [Estabrooks et al, LASS]
- $\tau \to K\pi\nu_\tau$, but almost P-wave only [Babar, Belle]
- $D \to K\pi\pi$, but hadronic rescattering from 3rd meson
- $D \to K\pi\ell\nu$
  - First studied at FOCUS & CLEO
  - Significant S-P-wave interference visible in angular asymmetries
  - Possibility to measure it, exactly like in the case of $K \to \pi\pi\ell\nu$





FOCUS

$D \to K\pi\ell\nu$ kinematics



$D \to K\pi\ell\nu$ described by
5 kinematic parameters
(3 angles and 2 invariant masses)

$$\mathrm{d}^5\Gamma \propto |V_{cs}|^2\, \mathcal{I}(m^2, q^2, \theta_K, \theta_e, \chi)\mathrm{d}m^2\mathrm{d}q^2\mathrm{d}\cos(\theta_K)\mathrm{d}\cos(\theta_e)\mathrm{d}\chi$$

3 form factors from hadronic matrix element in $\mathcal{I}$

- $\mathcal{F}_1$ (S,P...waves), et $\mathcal{F}_{2,3}$ (P...waves) depending on $s_{K\pi}$, $s_{\ell\nu}$ and $\cos\theta_K$
- Partial-wave phases identical to πK scattering (Watson theorem)
- At low energy, dominated by $I = 1/2$ S- and P-waves

Interference terms

$$\mathcal{I} = \mathcal{I}_1 + \mathcal{I}_2 \cos 2\theta_e + \mathcal{I}_3 \sin^2\theta_e \cos 2\chi + \mathcal{I}_4 \sin 2\theta_e \cos\chi + \mathcal{I}_5 \sin\theta_e \cos\chi$$
$$+\mathcal{I}_6 \cos\theta_e + \mathcal{I}_7 \sin\theta_e \sin\chi + \mathcal{I}_8 \sin 2\theta_e \sin\chi + \mathcal{I}_9 \sin^2\theta_e \sin 2\chi$$

$$\mathcal{I}_1 = \frac{1}{4}\left\{|\mathcal{F}_1|^2 + \frac{3}{2}\sin^2\theta_K \left(|\mathcal{F}_2|^2 + |\mathcal{F}_3|^2\right)\right\}$$

$$\mathcal{I}_2 = -\frac{1}{4}\left\{|\mathcal{F}_1|^2 - \frac{1}{2}\sin^2\theta_K \left(|\mathcal{F}_2|^2 + |\mathcal{F}_3|^2\right)\right\}$$

$$\mathcal{I}_3 = -\frac{1}{4}\left\{|\mathcal{F}_2|^2 - |\mathcal{F}_3|^2\right\}\sin^2\theta_K$$

$$\mathcal{I}_4 = \frac{1}{2}\mathrm{Re}\,(\mathcal{F}_1^*\mathcal{F}_2)\sin\theta_K \qquad \mathcal{I}_5 = \mathrm{Re}\,(\mathcal{F}_1^*\mathcal{F}_3)\sin\theta_K$$

$$\mathcal{I}_6 = \mathrm{Re}\,(\mathcal{F}_2^*\mathcal{F}_3)\sin^2\theta_K \qquad \mathcal{I}_7 = \mathrm{Im}\,(\mathcal{F}_1\mathcal{F}_2^*)\sin\theta_K$$

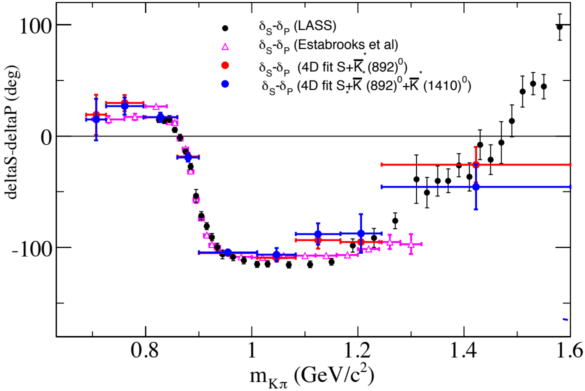
$$\mathcal{I}_8 = \frac{1}{2}\mathrm{Im}\,(\mathcal{F}_1\mathcal{F}_3^*)\sin\theta_K \qquad \mathcal{I}_9 = -\frac{1}{2}\mathrm{Im}\,(\mathcal{F}_2\mathcal{F}_3^*)\sin^2\theta_K$$

$$\int_{-1}^{+1} d\cos\theta_K\, \mathcal{I}_4 \propto |\mathcal{F}_1||\mathcal{F}_2|\cos(\delta_0^{1/2}-\delta_1^{1/2}) \qquad \int_{-1}^{+1} d\cos\theta_K\, \mathcal{I}_7 \propto |\mathcal{F}_1||\mathcal{F}_2|\sin(\delta_0^{1/2}-\delta_1^{1/2})$$

$D \to K\pi\ell\nu$: results on phase shifts

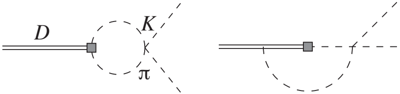
- Babar study with $244 \cdot 10^3$ events for $D^+ \to K^-\pi^+e^+\nu_e$ [347 fb^{-1} of data]
- $Br(D^+ \to K^-\pi^+e^+\nu_e) = (4.00 \pm 0.03 \pm 0.04 \pm 0.09) \cdot 10^{-2}$
- Among limitations: $s_{K\pi}$-dependence of modulus of form factors
  - slow variation of S-wave (similar to form factors in $K \to \pi\pi\ell\nu$)
  - quick variation of P-wave, due to K^*

Babar collaboration



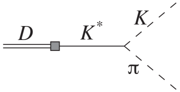
Beyond Breit-Wigner for K^* contribution

- at very low q^2: heavy-meson χPT (expansion in $1/M_D$, $p_{\pi,k}$, $m_{u,d,s}$)



Loops linked to K^* tail and width

- around K^* peak and large-N_c: resonance Lagrangian



Couplings of K^* to $K\pi$ and D

- in elastic regime: Omnès resummation from unitarity

$$f^I_\ell(s,s_l) = Q_{nf}(s,s_l) \exp\left[\frac{(s-s_0)^n}{\pi}\int_{(m_\pi+m_K)^2}^\infty \frac{dz}{(z-s_0)^n}\frac{\delta^I_\ell(z)}{z-s-i\epsilon}\right]$$

where Q_{nf} is (exponential of) subtraction polynomial of order $n-1$

$\Longrightarrow$Put these constraints together to get a realistic model for the $D \to K\pi\ell\nu$ P-wave form factors and study BES potential for $K\pi$ phase shifts

J. Charles, SDG, X.W. Kang, H.B. Li, work in progress

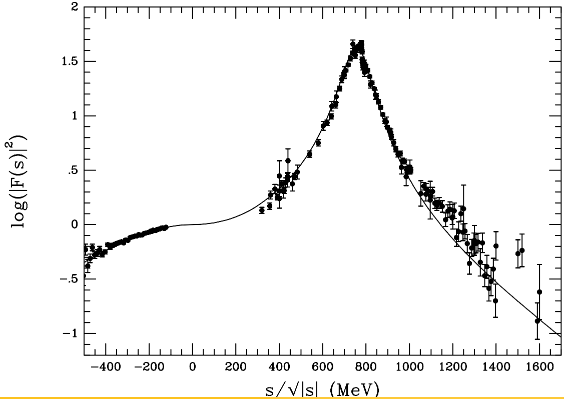
A case where it works well

Pion vector form factor: $\langle \pi^0 \pi^- | \bar{d} \gamma^\mu u | 0 \rangle = \sqrt{2}\, F(s)\, (p_{\pi^-} - p_{\pi^0})^\mu$

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp\left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[\mathrm{Re} A\left(\frac{m_\pi^2}{s}, \frac{m_\pi^2}{M_\rho^2}\right) + \frac{1}{2} \mathrm{Re} A\left(\frac{m_K^2}{s}, \frac{m_K^2}{M_\rho^2}\right) \right] \right\}$$

$$\Gamma_\rho(s) = -\frac{M_\rho\, s}{96\pi^2 f_\pi^2} \, \mathrm{Im} \left[A\left(\frac{m_\pi^2}{s}, \frac{m_\pi^2}{M_\rho^2}\right) + \frac{1}{2} A\left(\frac{m_K^2}{s}, \frac{m_K^2}{M_\rho^2}\right) \right] .$$

with A one-loop scalar integral describing $\pi\pi$ rescattering



F. Guerrero and A. Pich

Outlook

Low-energy dynamics of QCD

- governed by chiral symmetry breaking
- related to dynamics of π, K, η described by Chiral Perturbation Theory
- pattern still to be investigated (e.g., assessment of lattice systematics)

$\pi\pi$ scattering (non-strange sector $N_f = 2 : m_u, m_d \to 0$)

- Several experimental sources, most precise one from $K \to \pi\pi\ell\nu$
- Measurement of S-P wave interference (E865, NA48/2)
- When isospin breaking included, good agreement with χPT expectations

πK scattering (strange sector $N_f = 3 : m_u, m_d, m_s \to 0$)

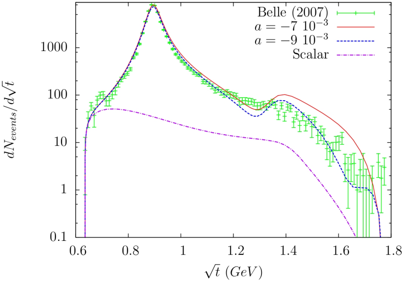
- Theoretical studies (dispersion relations): only mild agreement with χPT
- Need low-energy data, e.g., $D \to K\pi\ell\nu$ (FOCUS, CLEO-c and Babar)
- Beyond Breit-Wigner for P-wave form factors (K^* contribution)

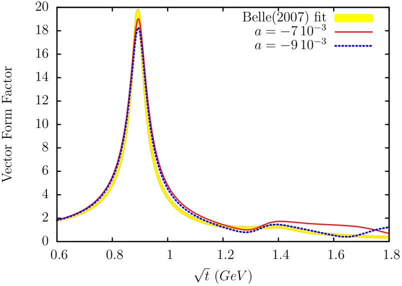
Investigation of BES potential to measure πK phase shifts under way

$\tau \to K\pi\nu_\tau$

- Studied at BELLE
- Dominated essentially by *P*-wave
- Constrains *P*-wave $K\pi$, but not very strongly, due to the importance of additional channels for the correct description of form factors

B. Moussallam





How well χPT is tested ?

- Convergence of fits to NNLO χPT *Bijnens, Jemos*

Relative contribution	LO	NLO	NNLO
M_π^2	1.035	-0.084	0.049
M_K^2	1.106	-0.181	0.075
M_η^2	1.186	-0.224	0.038
F_π	0.705	0.220	0.127
F_K	0.589	0.260	0.216

- Difficulties between $N_f = 3$ χPT and lattice
  - Poor fits of chiral expressions to lattice spectrum (RBC/UKQCD, PACS-CS)
  - Convergence issues with abnormally small NLO compared to NNLO (MILC)