第九届XYZ研讨会

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Spectrum of the heavy hadronic molecules:
from the known to the unknown

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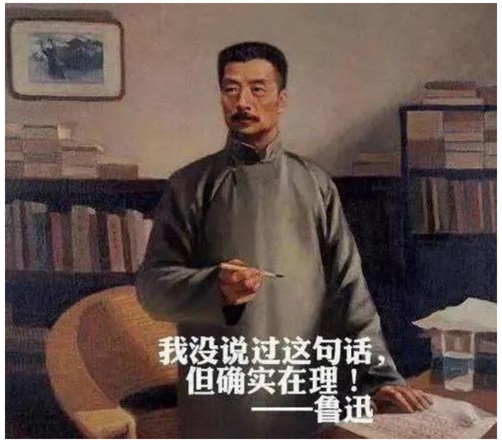
Outline

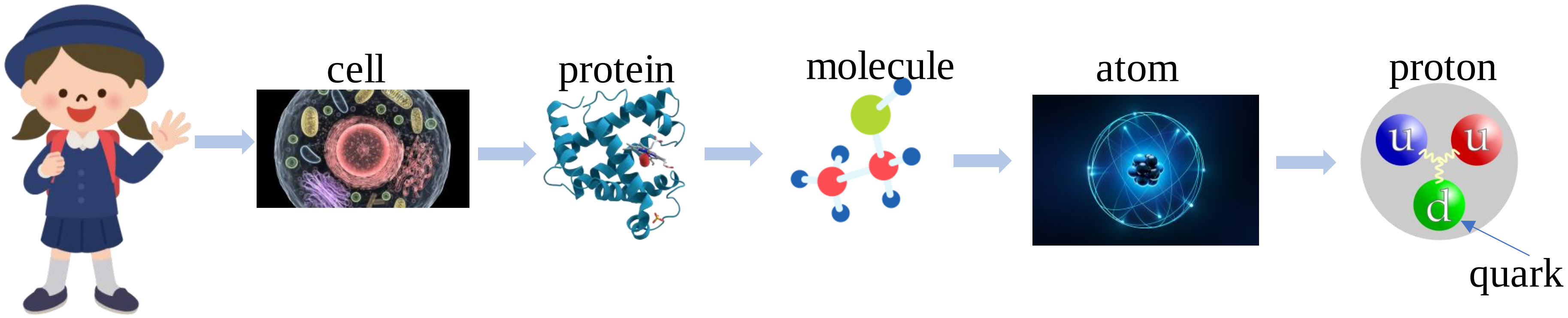
- **Prelude and background**
- **An introduction to our framework**
- **Spectrum of the heavy hadronic molecules**
- **Summary and outlook**

Naive considerations

森严的“等级制度”是大自然与生俱来的属性。

（层级结构）

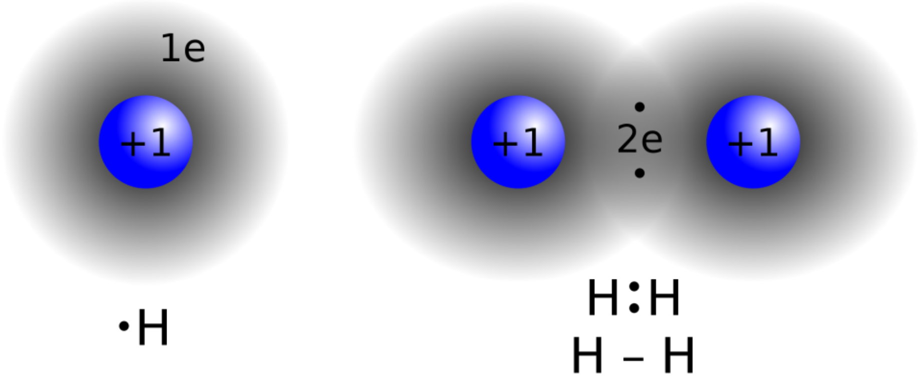




Naive considerations

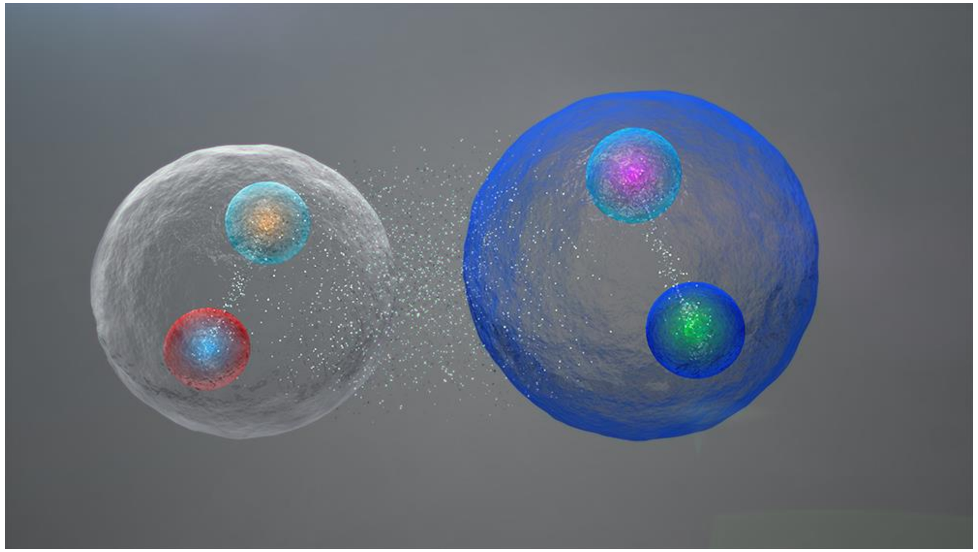
电中性原子之间通过剩余电磁相互作用可以形成分子

QED：

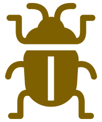
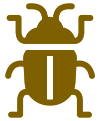


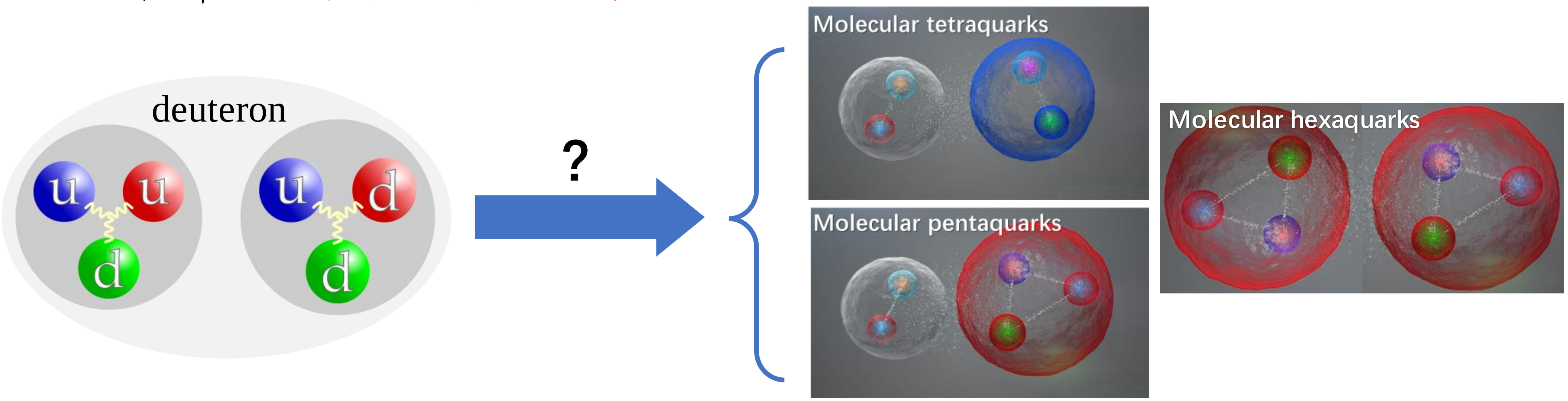
色中性强子之间通过剩余强相互作用可以形成强子分子?

QCD：

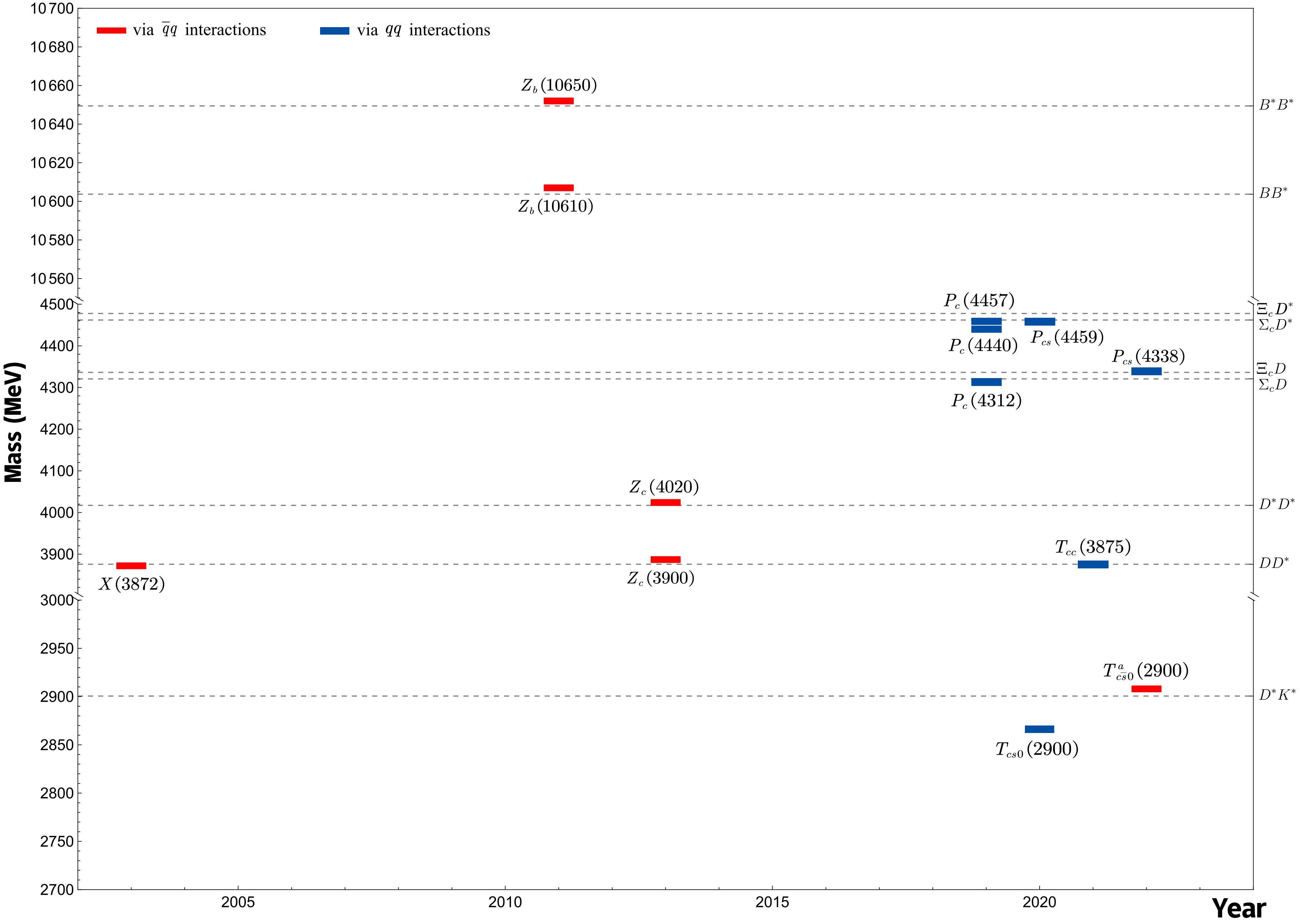


Naive considerations

- 当在房间里发现一只的时候，说明很可能在未注意到的地方也存在很多。
- 当在强子谱中发现一个分子态的时候，说明很可能在未注意到的体系中也存在很多分子态。



Spectrum of the near-threshold exotic hadrons



Two salient features:

1. Near-threshold

2. Strong isospin tropism

Are these states independent or correlated entities?

Recent reviews:

H.-X. Chen et al, Phys. Rept. 639, 1 (2016)

F.-K. Guo et al, Rev. Mod. Phys. 90, 015004 (2018)

Y.-R. Liu et al, Prog. Part. Nucl. Phys. 107, 237 (2019)

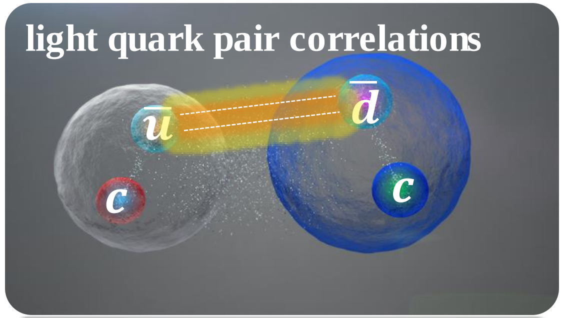
N. Brambilla et al, Phys. Rept. 873, 1 (2020)

H.-X. Chen et al, Rept. Prog. Phys. 86, 026201 (2023)

L. Meng et al, Phys. Rept. 1019, 1 (2023)

Implications of the spectrum

Are these states independent or correlated entities?



2. Strong isospin tropism

1. Near-threshold → Highly plausible molecule candidates → Their existence is reliant on the light quark pair correlations

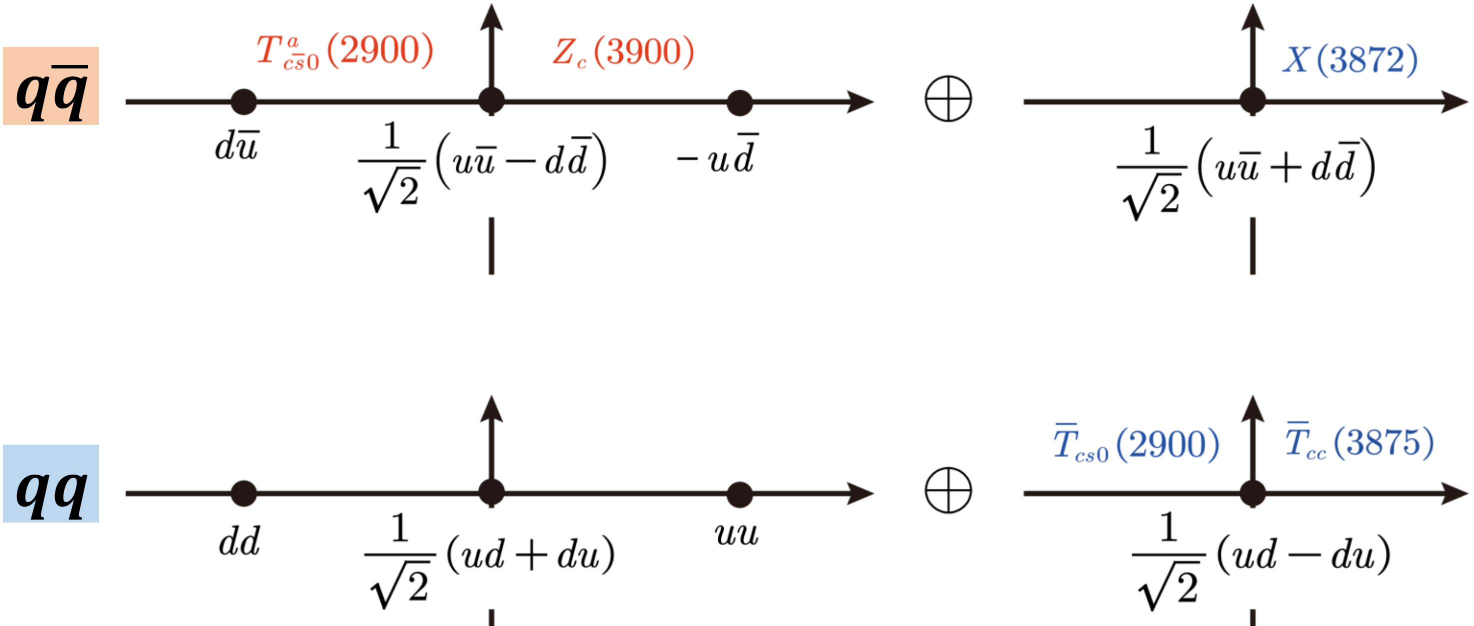
↓

These exotic states do not exist as independent entities

3. Flavor symmetry

Occam's Razor

Light flavor wave functions



B. Wang et al., PhysRevD.109.034027

Are there underlying connections between

- $T^{a}_{c\bar{s}0}(2900)$ and $Z_c(3900)$, $X(3872)$
- $T_{cs0}(2900)$ and $T_{cc}(3875)$

An introduction to our framework

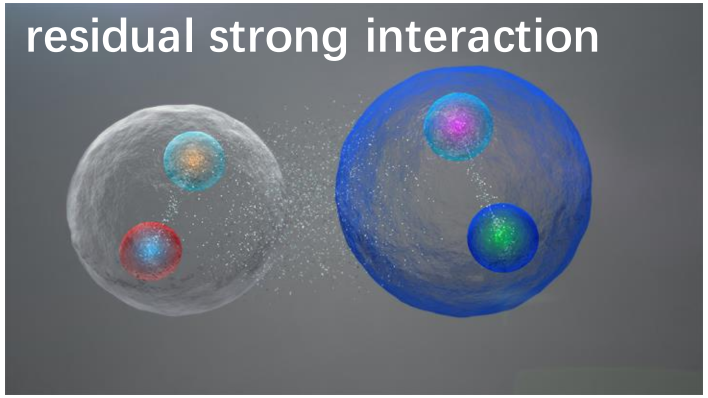
States	Nearest thresholds	Quark contents	V_{type}
$X(3872)$	$D\bar{D}^*$	$[c\bar{q}][\bar{c}q]$	$V_{\bar{q}q}$
$Z_c(3900)$	$D\bar{D}^*$	$[c\bar{q}][\bar{c}q]$	
$T^a_{c\bar{s}0}(2900)$	D^*K^*	$[c\bar{q}][\bar{s}q]$	
P_c	$\Sigma_c\bar{D}^{(*)}$	$[cqq][\bar{c}q]$	V_{qq}
$\bar{T}_{cc}(3875)$	$\bar{D}\bar{D}^*$	$[\bar{c}q][\bar{c}q]$	
$\bar{T}_{cs0}(2900)$	$\bar{D}^*K^*$	$[\bar{c}q][\bar{s}q]$	

B. Wang et al., PhysRevD.109.034027

Strange quark: light or *heavy*?

$m_s < \Lambda_{\text{QCD}}$,
in contrast to the $m_{u,d} \ll \Lambda_{\text{QCD}}$
SU(3) symmetry is not good!

$m_s^{\text{QM}} \sim 500$ MeV
For shallow bound hadronic molecules:
$\gamma_b^{\text{ty}} = \sqrt{2\mu E_b} \lesssim 100$ MeV$\ll m_s^{\text{QM}}$
such as for the $DD^*/D\bar{D}^*$ systems:
$E_b \lesssim 10$ MeV

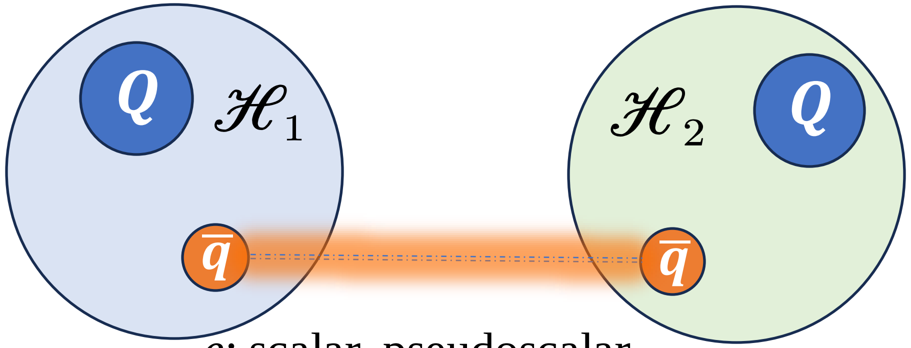


- ✓ The near-threshold interactions are too weak to excite the strange quarks inside the *heavy* hadrons.
- ✓ The strange quark will behave like an inert source (*heavy* quark) from the view of the residual strong interactions at the near-threshold energy scale.

An introduction to our framework

Constructing the effective potentials from a quark-level perspective

$$V_{\rm eff} \sim \sum_e \frac{\{1, \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2, (\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}), \dots\}}{\boldsymbol{q}^2 + M_e^2}$$

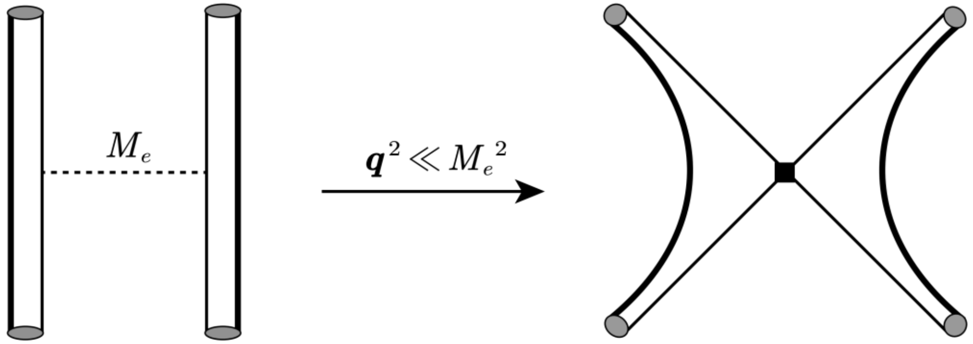


e: scalar, pseudoscalar, vector, axial-vector, tensor,…

For the near-threshold interactions: $\boldsymbol{q}^2 \ll M_e^2$

$$\frac{1}{\boldsymbol{q}^2 + M_e^2} = \frac{1}{M_e^2}\left(1 - \frac{\boldsymbol{q}^2}{M_e^2} + \dots\right)$$

From "H" to a "butterfly"



Keep the leading order ($\boldsymbol{q}^0$) of the expansion

B. Wang et al., PhysRevD.109.034027

An introduction to our framework

- ✓ Each exchanged meson fields contain both the isospin triplet and singlet:

$$\mathscr{S} = \mathscr{S}_i \tau_i + \frac{1}{\sqrt{2}} \mathscr{S}_1$$

- ✓ The net contributions from the exchanged particles will give rise to the non-relativistic effective potentials for the light qq and $q\bar{q}$ with the following forms, respectively,

$$V_{qq} = \left(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \frac{1}{2} \tau_{0,1} \cdot \tau_{0,2} \right) (c_s + c_a \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2),$$

$$V_{\bar{q}q} = \left(-\boldsymbol{\tau}_1^* \cdot \boldsymbol{\tau}_2 + \frac{1}{2} \tau_{0,1} \cdot \tau_{0,2} \right) (\tilde{c}_s + \tilde{c}_a \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2),$$

Pauli matrix in isospin space

Pauli matrix in spin space

B. Wang et al., PhysRevD.109.034027

An introduction to our framework

✓ Translating the quark-level potential to the hadron-level:

$$V^{I,J}_{\mathscr{H}_1\mathscr{H}_2} = \left\langle [\mathscr{H}_1\mathscr{H}_2]^I_J \left| V_{qq} \right| [\mathscr{H}_1\mathscr{H}_2]^I_J \right\rangle$$

spin-flavor wave function of $\mathscr{H}_1\mathscr{H}_2$ di-hadron systems

✓ Equivalent to using a hadron-level contact Lagrangians:

$$\mathcal{L}_{\bar{D}^{(*)}K^*} = C_1(\bar{\tilde{\mathcal{H}}}\tilde{\mathcal{H}})(K^{*\mu\dagger}K^*_\mu) + C_2(\bar{\tilde{\mathcal{H}}}\tau_i\tilde{\mathcal{H}})(K^{*\mu\dagger}\tau_i K^*_\mu)$$
$$+iC_3(\bar{\tilde{\mathcal{H}}}\sigma^{\mu\nu}\tilde{\mathcal{H}})(K^{*\dagger}_\mu K^*_\nu)+iC_4(\bar{\tilde{\mathcal{H}}}\sigma^{\mu\nu}\tau_i\tilde{\mathcal{H}})(K^{*\dagger}_\mu\tau_i K^*_\nu)$$

B. Wang et al., PhysRevD.109.034027

An introduction to our framework

✓ To search for the bound/virtual state poles,

$$t = v + vGt, \longrightarrow t^{-1} = v^{-1} - G$$

$$G(E+i\epsilon) = \int_0^\Lambda \frac{k^2 dk}{(2\pi)^3} \frac{2\mu}{p^2 - k^2 + i\epsilon} = \frac{2\mu}{(2\pi)^3}\left[p \tanh^{-1}\left(\frac{p}{\Lambda}\right) - \Lambda - \frac{i\pi}{2}p\right]$$

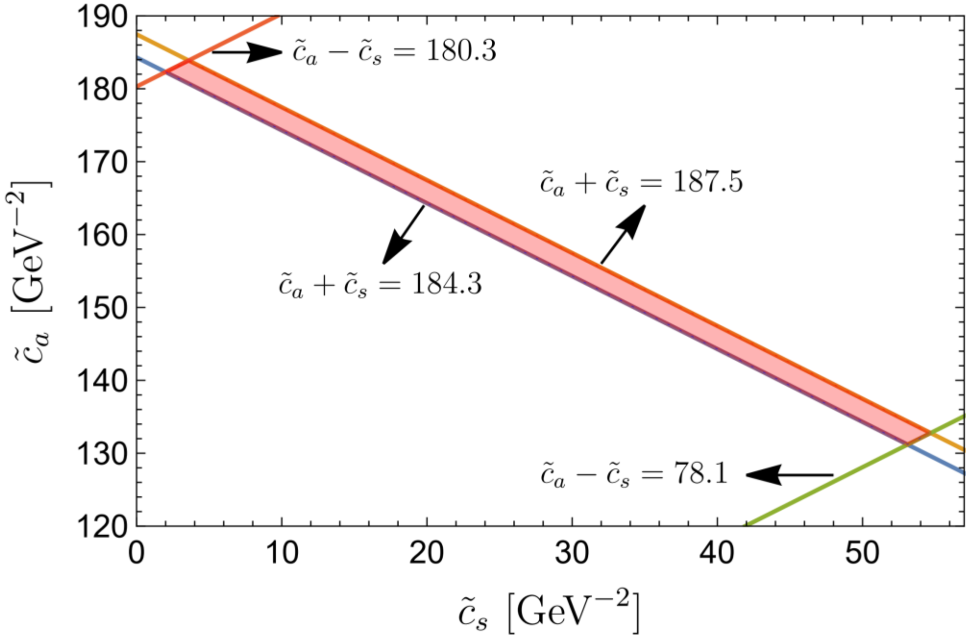
Bound state pole in Sheet-I (physical) : $G(E+i\epsilon)$

Virtual state pole in Sheet-II (unphysical) : $G(E+i\epsilon) + i\frac{\mu}{4\pi^2}p$

P_c and T_{cc} as inputs

$$c_s = 146.4 \pm 10.8\ \text{GeV}^{-2},$$
$$c_a = -7.3 \pm 10.5\ \text{GeV}^{-2}.$$

$X(3872)$ and $Z_c(3900)$ as inputs



Spectrum of the molecular tetraquarks: understanding the $T_{cs0}(2900)$

$D^{(*)}D^{(*)}$ systems

Systems	$I(J^P)$	V_{sys}	E_B/E_V	States
DD	$1(0^+)$	$\frac{3}{2}c_s$	...	...
DD^*	$0(1^+)$	$-\frac{5}{2}c_s+\frac{5}{2}c_a$	$[3874.7\pm0.04]_B$	[input] $T_{cc}(3875)$
	$1(1^+)$	$\frac{3}{2}c_s+\frac{3}{2}c_a$	...	...
D^*D^*	$0(1^+)$	$-\frac{5}{2}c_s+\frac{5}{2}c_a$	$[4015.6^{+0.8}_{-2.3}]_B$	$T_{cc}(4015)$
	$1(0^+)$	$\frac{3}{2}c_s-3c_a$	...	...
	$1(2^+)$	$\frac{3}{2}c_s+\frac{3}{2}c_a$	...	...

- E_B: Bound state E_V: Virtual state

$\bar{D}^{(*)}K^*$ systems

Systems	$I(J^P)$	V_{sys}	E_B/E_V	States
$\bar{D}K^*$	$0(1^+)$	$-\frac{5}{2}c_s$	$[2752.5^{+2.6}_{-3.6}]_V$	$T^f_{cs1}(2760)$
	$1(1^+)$	$\frac{3}{2}c_s$	...	...
$\bar{D}^*K^*$	$0(0^+)$	$-\frac{5}{2}c_s+5c_a$	$[2897.8^{+2.6}_{-9.1}]_V$	$T^f_{cs0}(2900)$
	$0(1^+)$	$-\frac{5}{2}c_s+\frac{5}{2}c_a$	$[2896.6^{+3.0}_{-6.3}]_V$	$T^f_{cs1}(2900)$
	$0(2^+)$	$-\frac{5}{2}c_s-\frac{5}{2}c_a$	$[2892.8^{+5.0}_{-9.8}]_V$	$T^f_{cs2}(2900)$
	$1(0^+)$	$\frac{3}{2}c_s-3c_a$	...	...
	$1(1^+)$	$\frac{3}{2}c_s-\frac{3}{2}c_a$	...	...
	$1(2^+)$	$\frac{3}{2}c_s+\frac{3}{2}c_a$	...	...

B. Wang et al., PhysRevD.109.034027

Highlights of our predictions

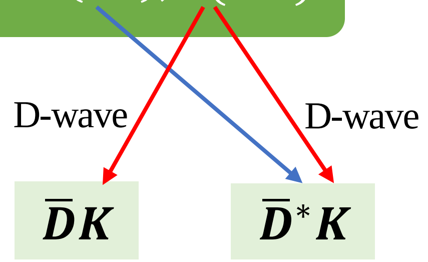
$D^*D^*: 0(1^+)$

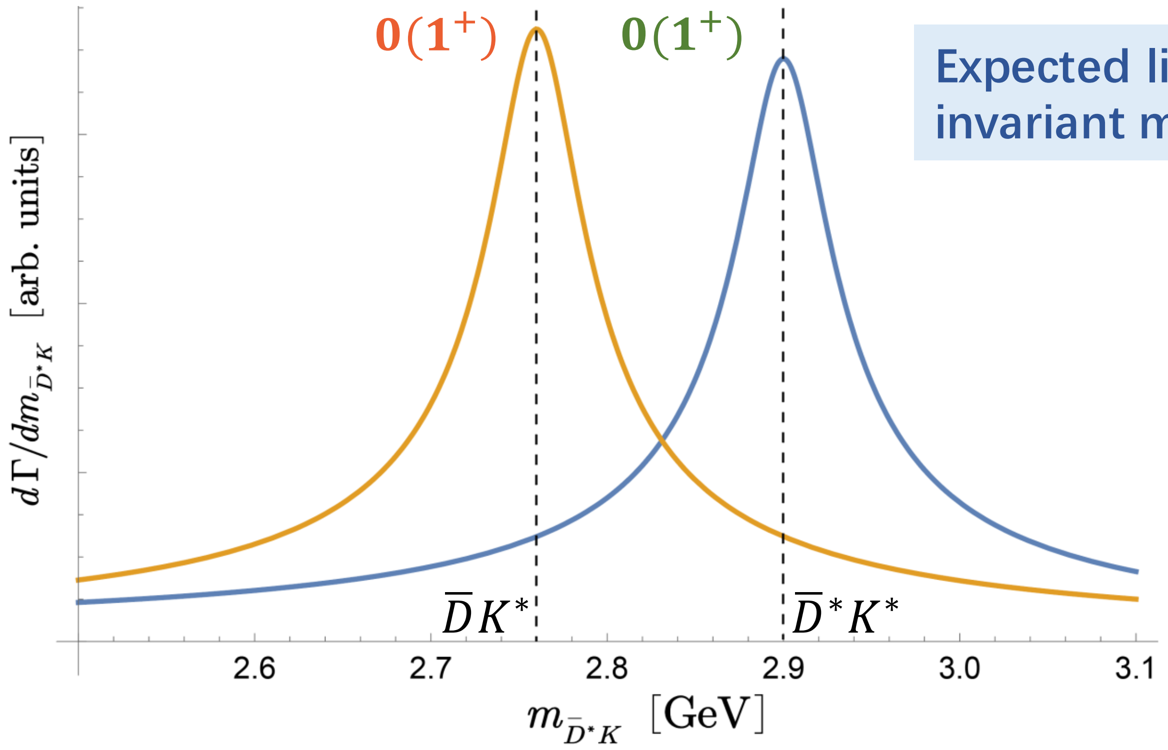
also expected in many works, such as Chen et al,. Eur. Phys. J. C 82, 581 (2022); Li et al., Phys. Rev. D 88, 114008 (2013); Liu et al., Phys. Rev. D 99, 094018 (2019); Ding et al., Eur. Phys. J. C 80, 1179 (2020); Dong et al., Commun. Theor. Phys. 73, 125201 (2021).

$\bar{D}K^*: 0(1^+)$ → $\bar{D}^*K$

It is expected that there should be a peak at the $\bar{D}K^*$ (2.76 GeV) and $\bar{D}^*K^*$ (2.9 GeV) thresholds in the $\bar{D}^*K$ channel, respectively.

$\bar{D}^*K^*: 0(1^+), 0(2^+)$





Expected line-shape of the $\bar{D}^*K$ invariant mass distribution

Spectrum of the molecular tetraquarks: understanding the $T^{a}_{c\bar{s}0}(2900)$

Systems	$I(J^P)$	V_{sys}	E_B/E_V	States
$D\bar{D}$	$0^+(0^{++})$	$-\frac{5}{2}\tilde{c}_s$	$[\sim 3696.5]_V$	$X(3700)^{\#}$
	$1^-(0^{++})$	$\frac{3}{2}\tilde{c}_s$	…	…
$D\bar{D}^*$	$0^+(1^{++})$	$-\frac{5}{2}\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	$[3871.2,3871.6]_B$	[input] $X(3872)$
	$0^-(1^{+-})$	$\frac{5}{2}\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	…	…
	$1^+(1^{+-})$	$-\frac{3}{2}\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	$[3825.8,3874.8]_V$	[input] $Z_c(3900)$
	$1^-(1^{++})$	$\frac{3}{2}\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	…	…
$D^*\bar{D}^*$	$0^+(0^{++})$	$5\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	…	…
	$0^-(1^{+-})$	$\frac{5}{2}\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	…	…
	$0^+(2^{++})$	$-\frac{5}{2}\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	$[4011.8,4012.2]_B$	$X_2(4012)$
	$1^-(0^{++})$	$-3\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	$[4007.2,4016.7]_B$	$Z_c(4010)$
	$1^+(1^{+-})$	$-\frac{3}{2}\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	$[3973.6,4014.5]_V$	$Z_c(4020)$
	$1^-(2^{++})$	$\frac{3}{2}\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	…	…

Systems	$I(J^P)$	V_{sys}	E_B/E_V	States
DK^*	$0(1^+)$	$-\frac{5}{2}\tilde{c}_s$	$[\sim 2586.0]_V$	$T^{f}_{c\bar{s}0}(2760)^{\#}$
	$1(1^+)$	$\frac{3}{2}\tilde{c}_s$	…	…
D^*K^*	$0(0^+)$	$5\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	…	…
	$0(1^+)$	$\frac{5}{2}\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	…	…
	$0(2^+)$	$-\frac{5}{2}\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	$[2900.2,2900.3]_V$	$T^{f}_{c\bar{s}2}(2900)$
	$1(0^+)$	$-3\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	$[2887.6,2900.5]_V$	$T^{a}_{c\bar{s}0}(2900)$
	$1(1^+)$	$-\frac{3}{2}\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	$[2676.2,2876.3]_V$	$T^{a}_{c\bar{s}1}(2900)$
	$1(2^+)$	$\frac{3}{2}\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	…	…

Highlights of our predictions

$V[D\bar{D}, 0^+(0^{++})] = V[DK^*, 0(1^+)]$

$D\bar{D}: 0^+(0^{++})$ virtual state/nonexistent

investigated in Phys. Rev. D 86, 056004 (2012); Phys. Rev. D 87, 076006 (2013) ; Phys. Rev. D 74, 014013 (2006) ; Phys. Rev. D 76, 074016 (2007); JHEP 06, 035 (2021).

1. **Limited decay channels with tiny decay widths**
2. **Difficult to be detected in experiments**

$DK^*: 0(1^+)$ virtual state/nonexistent

can be used to infer the existence of the $0^+(0^{++})$ $D\bar{D}$

D^*K

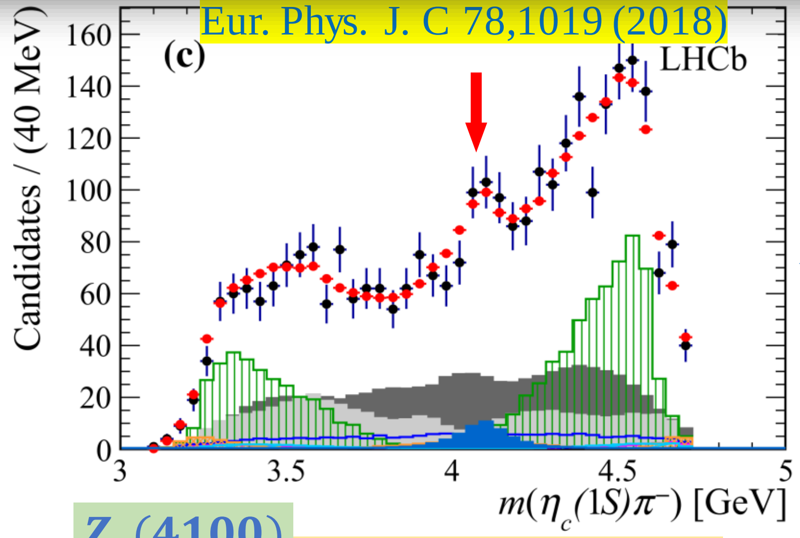
Highlights of our predictions

$D^*\bar{D}^*: 0^+(2^{++}), 1^-(0^{++})$

- $0^+(2^{++})$: tensor partner of X(3872); see the investigations Phys. Rev. D 86, 056004 (2012) ; Phys.Rev. D 87, 076006 (2013) ; Chin. Phys. C 36, 194 (2012) ; Phys. Rev. D 88, 054007 (2013) ; Eur. Phys. J. C 75, 547 (2015) ; Phys. Lett. B 763, 20 (2016) .
- $1^-(0^{++})$: $\eta_c\pi, D\bar{D}, J/\psi\rho, \chi_{c1}\pi$ (P wave)

Evidence for an $\eta_c(1S)\pi^-$ resonance in $B^0 \to \eta_c(1S)K^+\pi^-$ decays

Eur. Phys. J. C 78,1019 (2018)



$Z_c(4100)$

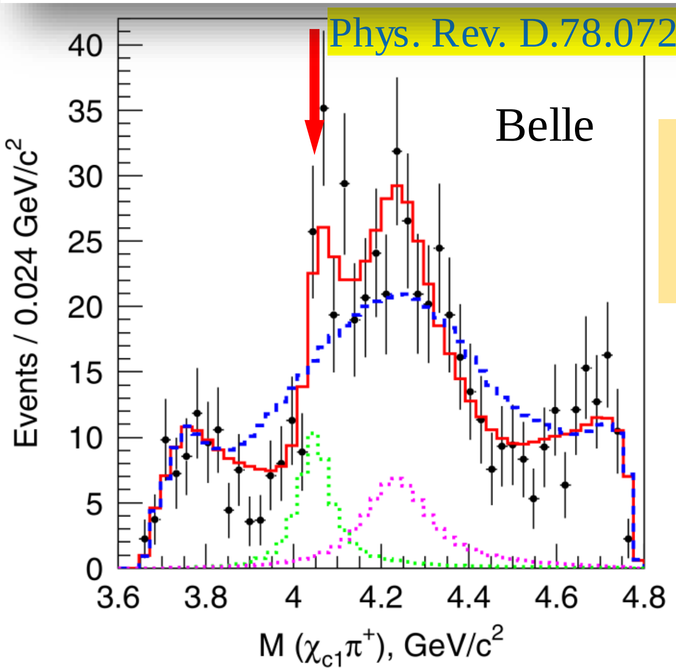
$m = 4096 \pm 20^{+18}_{-22}$ MeV

$\Gamma = 152 \pm 58^{+60}_{-35}$ MeV

Same state?

Observation of two resonancelike structures in the $\pi^+\chi_{c1}$ mass distribution in exclusive $\bar{B}^0 \to K^-\pi^+\chi_{c1}$ decays

Phys. Rev. D.78.072004 (2008)

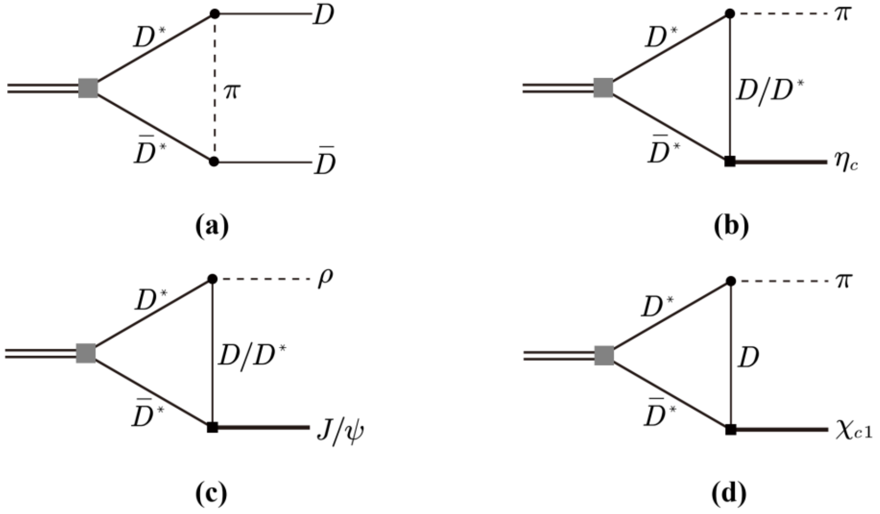


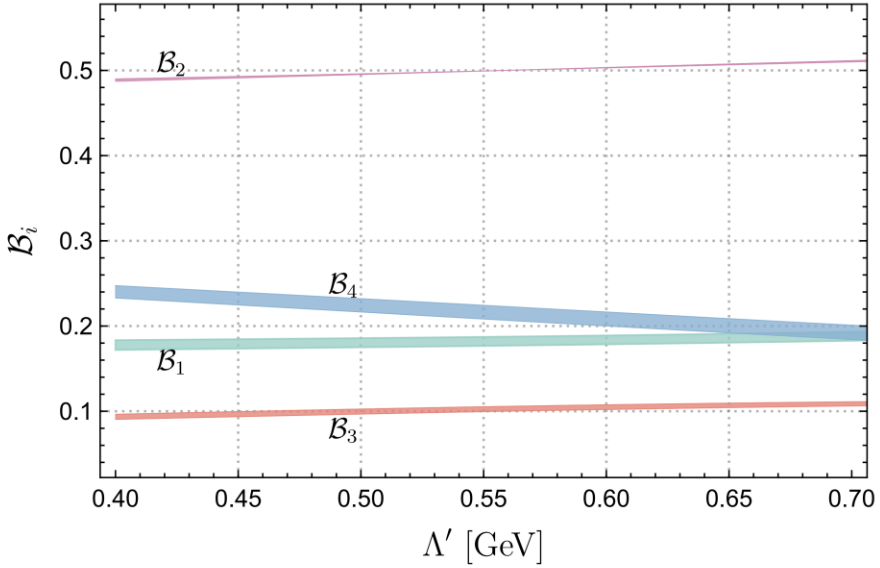
$Z_1(4050)^+$

$m = 4051 \pm 14^{+20}_{-41}$ MeV

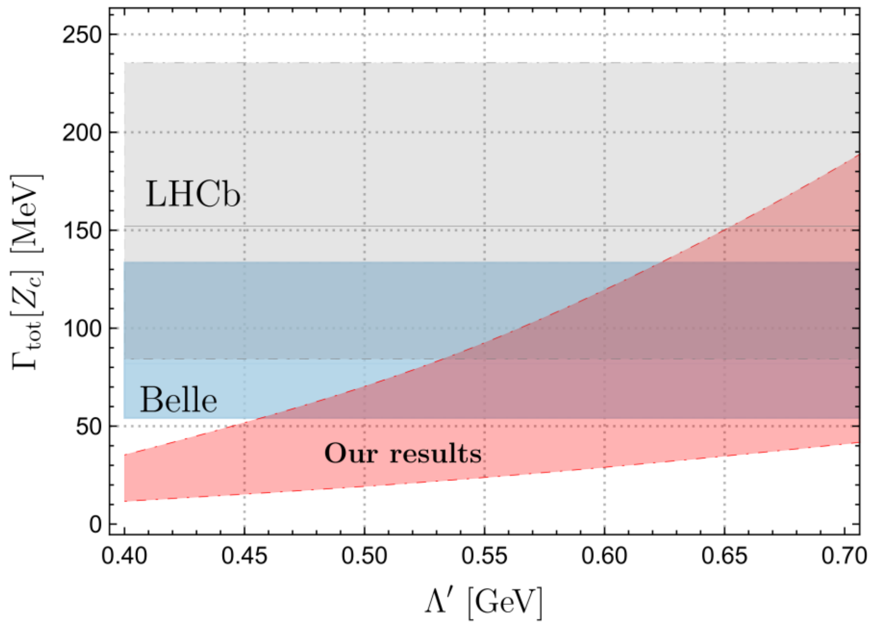
$\Gamma = 82^{+21+47}_{-17-22}$ MeV

Strong decays of the isovector-scalar $D^*\bar{D}^*$ molecule





in preparation   arXiv: 2406.xxxxx



$$\mathcal{B}_1 = \frac{\Gamma[Z_c \to D\bar{D}]}{\Gamma_{\rm tot}}, \qquad \mathcal{B}_2 = \frac{\Gamma[Z_c \to \eta_c \pi]}{\Gamma_{\rm tot}},$$

$$\mathcal{B}_3 = \frac{\Gamma[Z_c \to J/\psi \rho]}{\Gamma_{\rm tot}}, \qquad \mathcal{B}_4 = \frac{\Gamma[Z_c \to \chi_{c1} \pi]}{\Gamma_{\rm tot}},$$

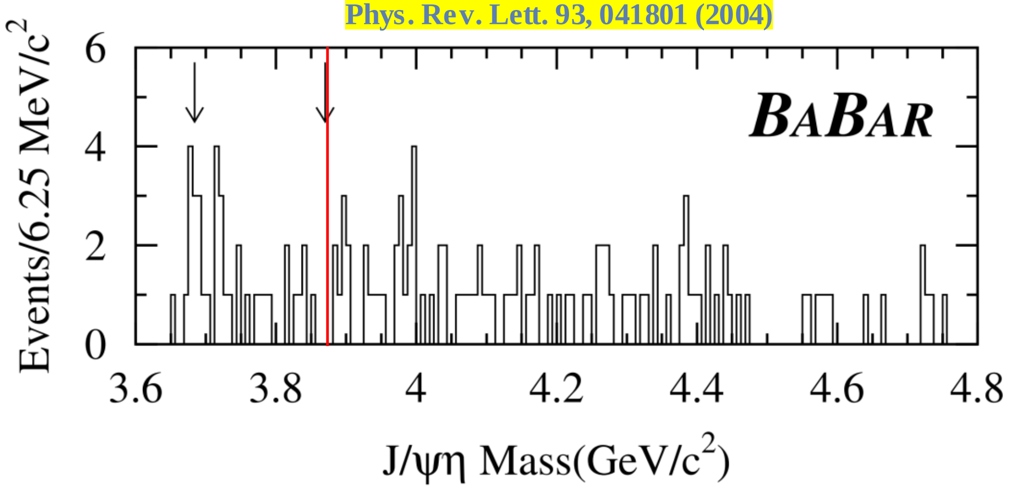
Highlights of our predictions

Does the $\tilde{X}$ (3872), the C-parity odd partner of $X(3872)$ exist?

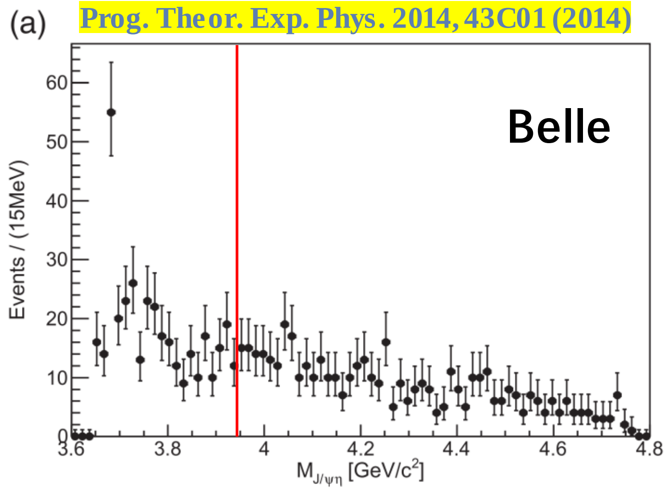
In our framework, $V[D\bar{D}^*, 0^-(1^{+-})] > 0$ ⟶ $\tilde{X}$ (3872) **does not exist!**

$\tilde{X}(3872)[0^-(1^{+-})] \to J/\psi[0^-(1^{--})] + \eta[0^+(0^{-+})]$

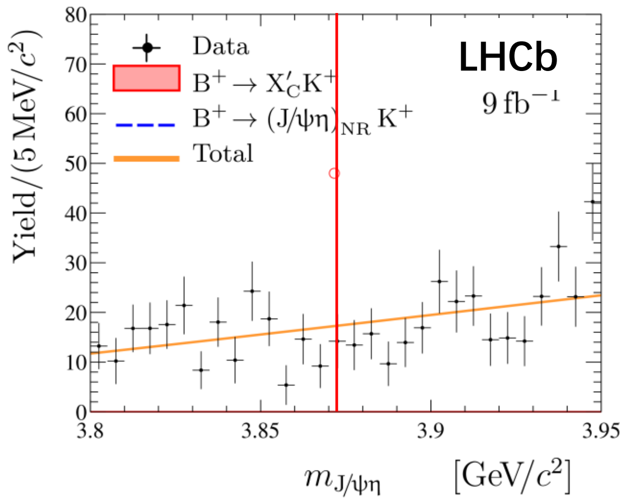
Phys. Rev. Lett. 93, 041801 (2004)



Prog. Theor. Exp. Phys. 2014, 43C01 (2014)



J. High Energy Phys.04, 046 (2022)

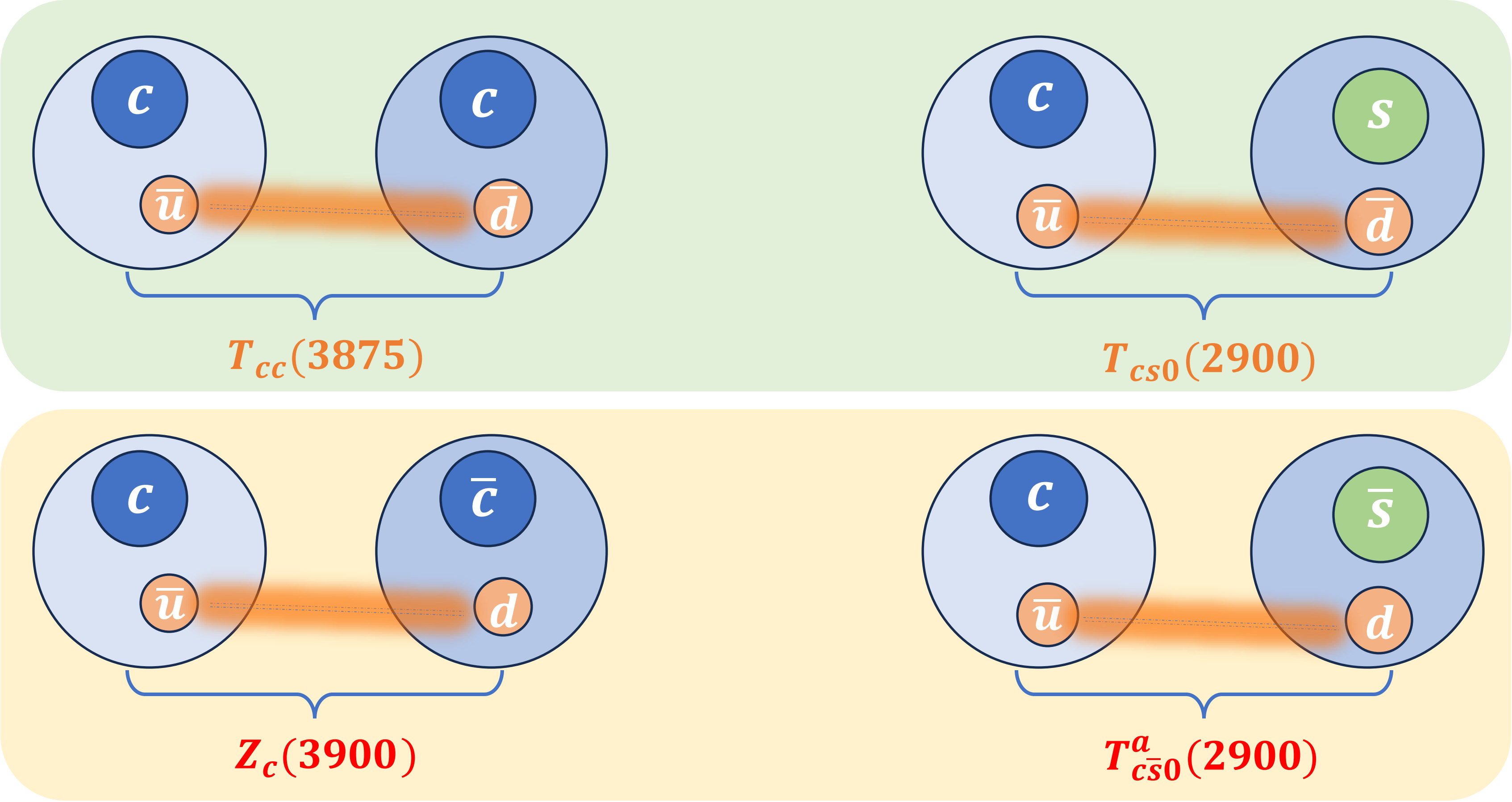


2403.16811

BESIII recently searched for the $e^+e^- \to \eta\tilde{X}$ (3872): "*We do not find any evident signal for the* $\tilde{X}$ (3872)......"

Unraveling the $T_{cs0}(2900)$ and $T^a_{c\bar{s}0}(2900)$

In our calculations, the recently discovered $T_{cs0}(2900)$ and $T^a_{c\bar{s}0}(2900)$ can be confidently identified as the charm-strange counterparts of $T_{cc}(3875)$ and $Z_c(3900)$, respectively.



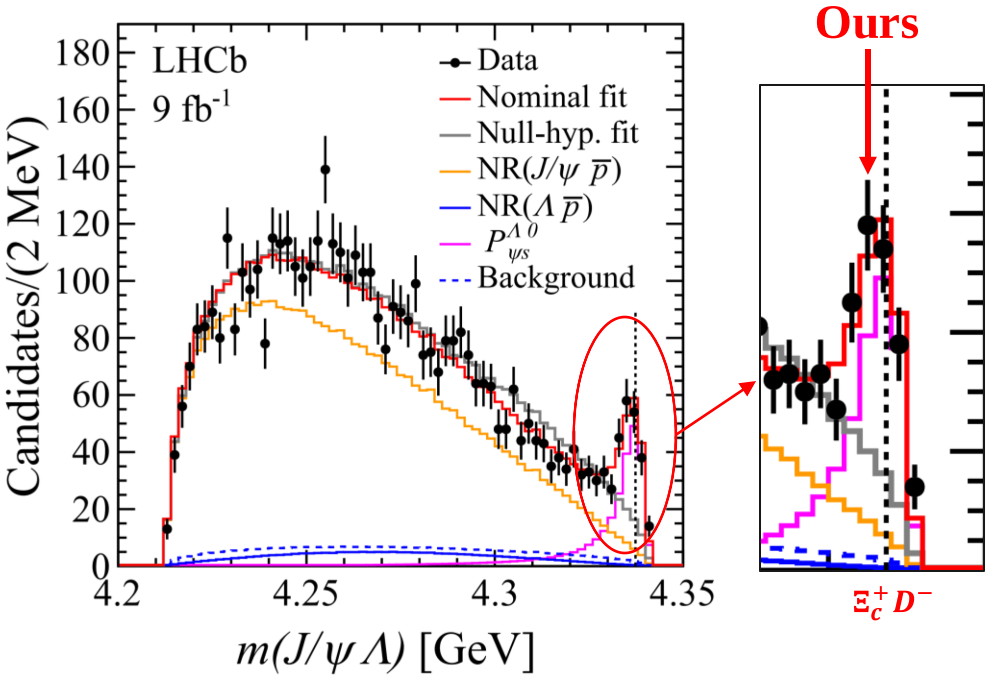
Why the $P_{\psi s}^{\Lambda}(4338)$ is so close to the $\Xi_c\bar{D}$ threshold?

Phys. Rev. D 109.074035 (2024)

Spectrum of the molecular pentaquarks

Bo Wang,[1,2,3] Kan Chen,[4,*] Lu Meng,[5,†] and Shi-Lin Zhu[6,‡]

Type-I: V_{qq}		Type-II: $V_{\bar{q}q}$	
$\Sigma_c^{(*)}\bar{D}^{(*)}$	$[cqq][\bar{c}q]$	$\Sigma_c^{(*)}D^{(*)}$	$[cqq][c\bar{q}]$
$\Xi_c^{(\prime,*)}\bar{D}^{(*)}$	$[csq][\bar{c}q]$	$\Xi_c^{(\prime,*)}D^{(*)}$	$[csq][c\bar{q}]$
$\Sigma_c^{(*)}K^*$	$[cqq][\bar{s}q]$	$\Sigma_c^{(*)}\bar{K}^*$	$[cqq][s\bar{q}]$
$\Xi_c^{(\prime,*)}K^*$	$[csq][\bar{s}q]$	$\Xi_c^{(\prime,*)}\bar{K}^*$	$[csq][s\bar{q}]$
$\Xi_{cc}^{(*)}\bar{D}^{(*)}$	$[ccq][\bar{c}q]$	$\Xi_{cc}^{(*)}D^{(*)}$	$[ccq][c\bar{q}]$
$\Xi_{cc}^{(*)}K^*$	$[ccq][\bar{s}q]$	$\Xi_{cc}^{(*)}\bar{K}^*$	$[ccq][s\bar{q}]$
$\Sigma^{(*)}\bar{D}^{(*)}$	$[sqq][\bar{c}q]$	$\Sigma^{(*)}D^{(*)}$	$[sqq][c\bar{q}]$
$\Xi^{(*)}\bar{D}^{(*)}$	$[ssq][\bar{c}q]$	$\Xi^{(*)}D^{(*)}$	$[ssq][c\bar{q}]$
$\Sigma^{(*)}K^*$	$[sqq][\bar{s}q]$	$\Sigma^{(*)}\bar{K}^*$	$[sqq][s\bar{q}]$
$\Xi^{(*)}K^*$	$[ssq][\bar{s}q]$	$\Xi^{(*)}\bar{K}^*$	$[ssq][s\bar{q}]$



$$V_{\Xi_c\bar{D}}^{0,1/2} = -\frac{5}{2}c_s$$

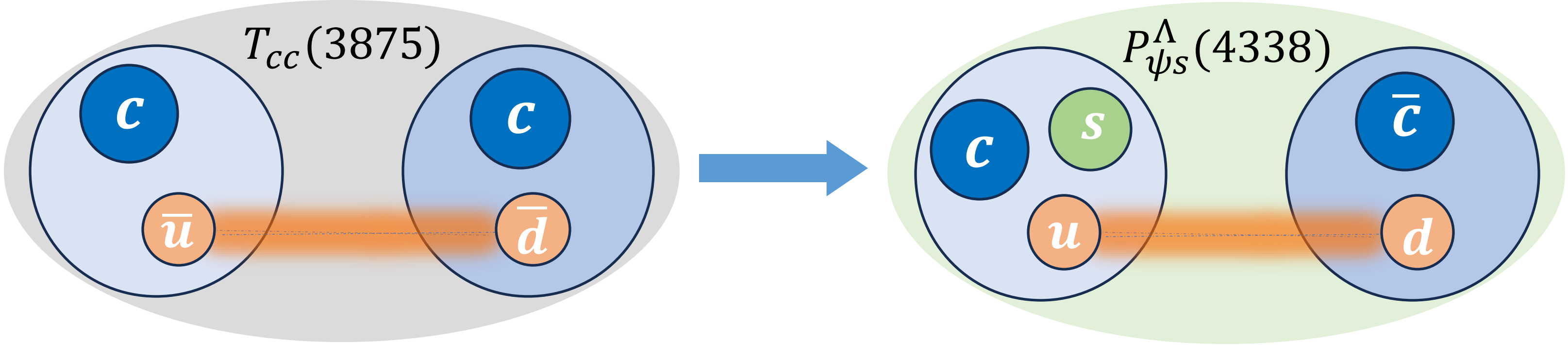
$$V_{DD^*}^{0,1} = -\frac{5}{2}c_s + \frac{5}{2}c_a$$

$$|c_s| \gg |c_a|$$

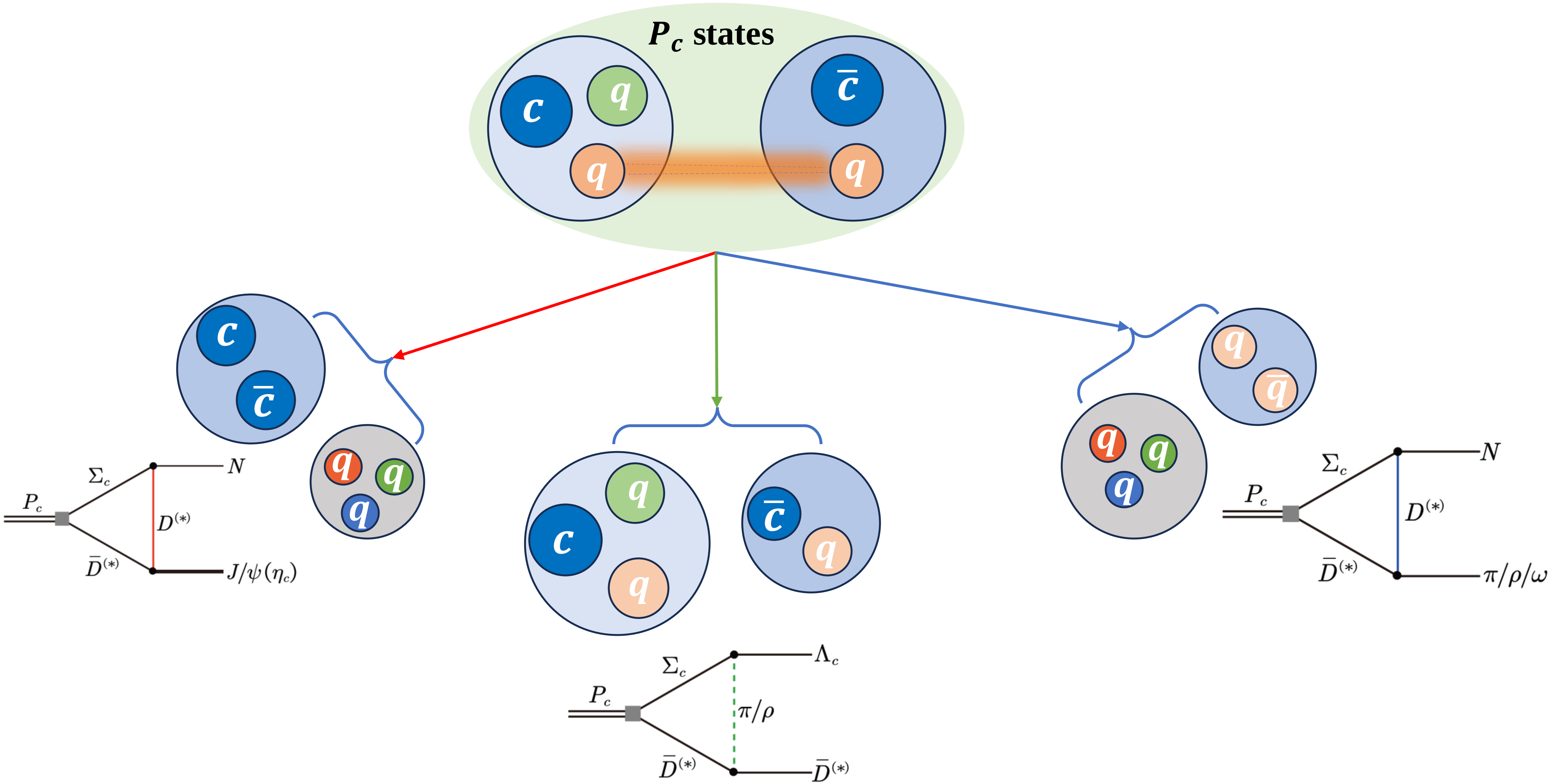
$$V_{\Xi_c\bar{D}}^{0,1/2} \approx V_{DD^*}^{0,1}$$

Our calculation gives:

$$m[P_{\psi s}^{\Lambda}(4338)] = 4334.8_{-1.1}^{+0.8}\ \text{MeV}$$



Decay properties of the hidden-charm pentaquarks



Decay properties of the hidden-charm pentaquarks

State	Decay channels	G.-J. Wang PRD 102, 036012	Y.-J. Xu PRD 102, 034028	Y.-H. Lin PRD 100, 056005	Y. Dong EPJC 80, 341	C.-J. Xiao PRD 100, 014022	Q. Li JHEP 06, 189	Z.-Y. Lin PRD.108.114014
$P_c(4312)$ $\Gamma_{\mathrm{expt.}} = 9.8 \pm 2.7^{+3.7}_{-4.5}$ MeV	$J/\psi p$	0.32	1.67	$10^{-3} - 0.1$	0.033	$3 - 8$	0.17	$\cdots$
	$\eta_c p$	0.98	5.54	$10^{-2} - 0.4$	0.066	$\cdots$	0.085	$\cdots$
	$\Lambda_c \bar{D}^*$	$\cdots$	$\cdots$	10.7	6.16	$\cdots$	8.8	~ 9.8
	$\Lambda_c \bar{D}$	$\cdots$	$\cdots$	0.3	0	$\cdots$	0.026	~ 0

Spectrum of the molecular hexaquarks

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We investigate the mass spectra of molecular-type hexaquark states in the dibaryon systems. These systems are composed of the charmed baryons $[\Sigma_c^{(*)}, \Xi_c^{(\prime,*)}]$, doubly charmed baryons $[\Xi_{cc}^{(*)}]$, and hyperons $[\Sigma^{(*)}, \Xi^{(*)}]$. We consider all possible combinations of particle-particle and particle-antiparticle pairs, including the S-wave spin multiplets in each combination. We establish the underlying connections among the molecular tetraquarks, pentaquarks, and hexaquarks with the effective quark-level interactions. We find that the existence of molecular states in DD^*, $D\bar{D}^*$, and $\Sigma_c\bar{D}^{(*)}$ systems leads to the emergence of a large number of deuteron-like hexaquarks in the heavy flavor sectors. Currently, there have been several experimental candidates for molecular tetraquarks and pentaquarks. The experimental search for near-threshold hexaquarks will further advance the establishment of the underlying dynamical picture of hadronic molecules and deepen our understanding of the role of spin-flavor symmetry in near-threshold residual strong interactions.

Summary and outlook

- The hadronic molecules do not exist as independent entities.
- Their formation crucially depends on the correlations between light quark pairs within separate hadrons.
- We obtained the mass spectra of molecular tetraquarks, pentaquarks and hexaquarks.
- The experimental search for the predicted states in various decay channels continues to be of great importance in advancing our understanding of the underlying physics.

Thank you