

# Ion Optics of Fragment Separators

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1. **Basic Ion Optics Description**  
(transfer map, lenses, phase space ellipse, beam envelopes)
2. **Ion Optics of Separators**
3. **Separation with Degrader -> Fragment separator**  

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**break**
4. **Higher Order Optics, Large Apertures, Fringe-Fields**
5. **Modern Fragment Separator Systems**
6. **FAIR Facility Status**

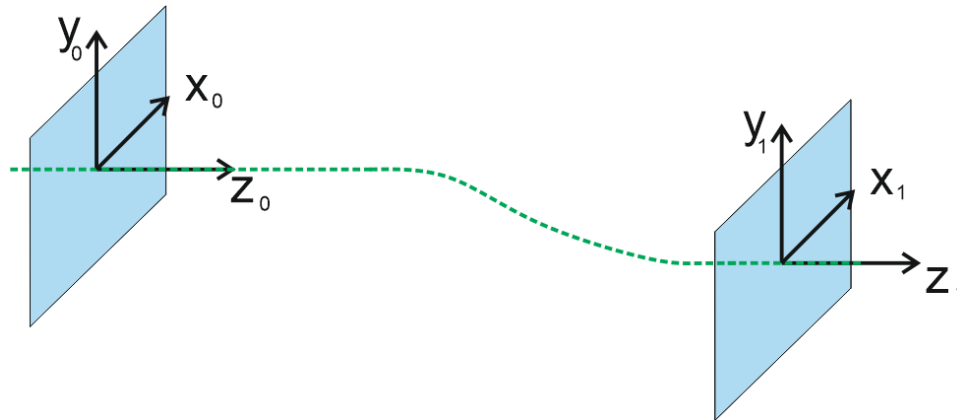


# 1. Basic Ion Optics Description

# Ion Optical Coordinates



We look at beamline, use coordinates relative to the nominal **optical axis**.



Transverse motion:

$$x' = dx / dz$$

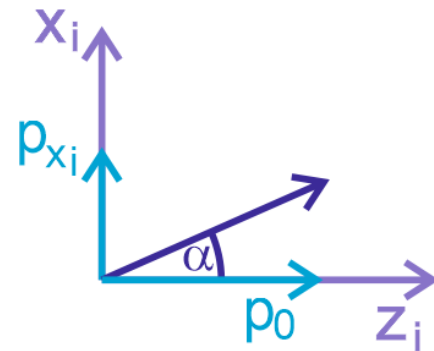
$$y' = dy / dz$$

$$a = p_x / p_0$$

$$b = p_y / p_0$$

Often defined as derivative in path with coordinates of single ions.

With common constant  $p_0$  we can use a normal Hamiltonian.



for same forward momentum  $\mathbf{x}' = \mathbf{a}$ ,  
for small angles  $\mathbf{x}' = \mathbf{a} = \tan(\alpha) \sim \alpha$

# Magnetic Rigidity ( $B\rho$ )

$$B\rho := p / q$$

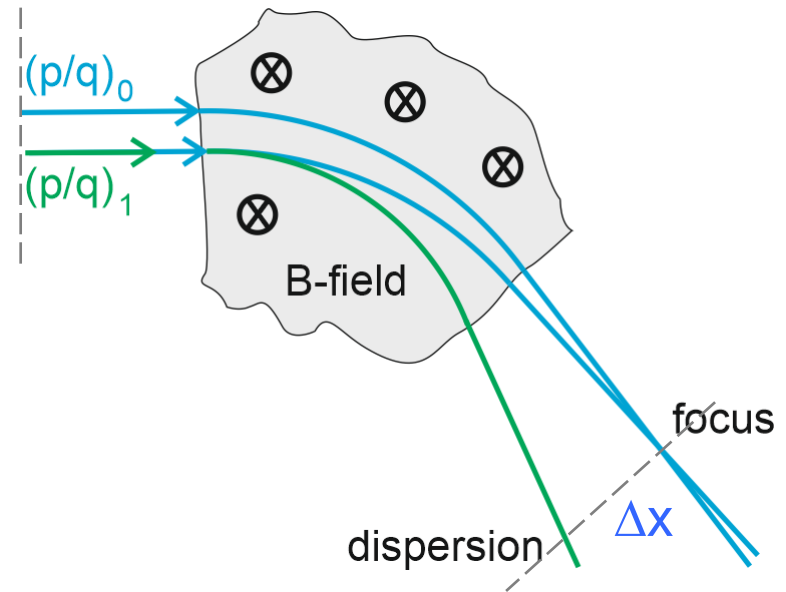
$p$  = momentum

$q$  = charge

In a homogenous field with flux density  $B$  perpendicular to the direction of motion, ions of magnetic rigidity  $B\rho$  are bent on a radius  $\rho$ .

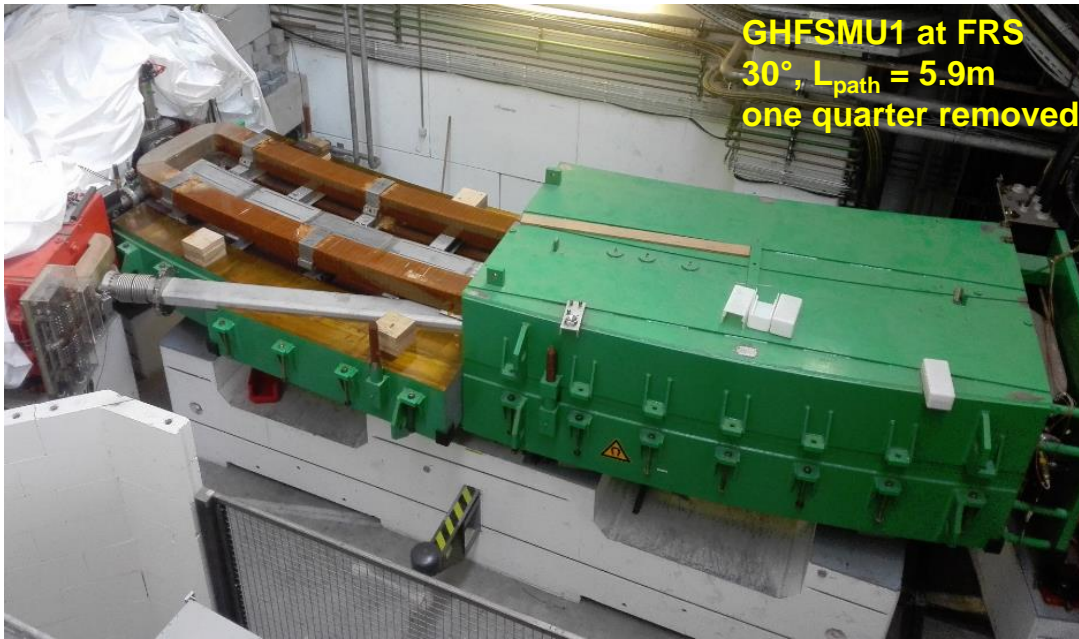
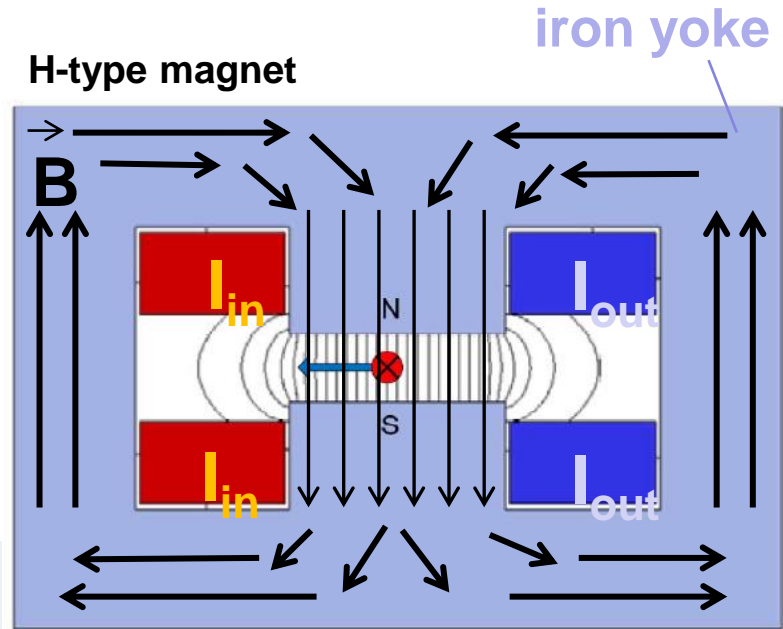
Ions with relative deviation in  $\delta := \Delta B\rho / B\rho_0$  from given reference arrive at a shifted position on the detector. Shift  $\Delta x$  per  $\delta$  is called dispersion coefficient ( $x|\delta$ ).

In magnets not mass, charge or velocity are important only  $B\rho$ . Similar definition for in electrostatic fields  $E\rho := mv^2 / q$



# Optical Elements – Dipole Magnets

Dipole magnets to deflect the beam.  
We want a homogeneous magnetic field  
to bend the beam on a constant radius  
--> sector magnets of H shape.



# Transfer Matrix Description

Transfer function on vector of coordinates

In practise use Taylor expansion of this function,  $(x,a) = \frac{\partial x_f}{\partial a_i}$

1<sup>st</sup> order transfer matrix T :

$$\begin{pmatrix} X \\ a \\ Y \\ b \\ \delta \end{pmatrix}_f = \begin{pmatrix} (X, X) & (X, a) & \boxed{= 0} & (X, \delta) \\ (a, X) & (a, a) & & (a, \delta) \\ \boxed{= 0} & & (Y, Y) & (Y, b) & \boxed{= 0} \\ & & (b, Y) & (b, b) & \\ \boxed{= 0} & & & & \boxed{= 1} \end{pmatrix} \begin{pmatrix} X \\ a \\ Y \\ b \\ \delta \end{pmatrix}_i$$

Det ( T ) = 1  
Liouville's theorem

with **bending only in one plane**  
**only forces in x or y direction**  
**momentum conservation**

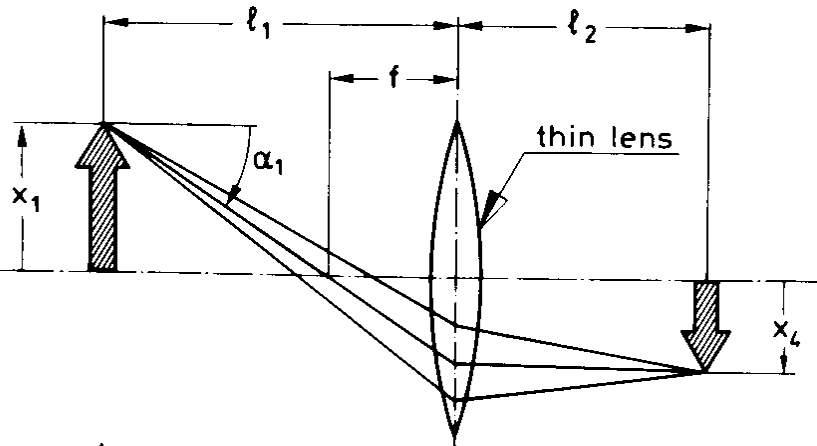
Full system

$$T_{\text{tot}} = T_n * \dots * T_3 * T_2 * T_1$$

# Transfer Matrix Example

## focusing with one thin lens

$$\begin{pmatrix} x(z) \\ a(z) \end{pmatrix} = \begin{pmatrix} (x|x) & (x|a) \\ (a|x) & (a|a) \end{pmatrix} \begin{pmatrix} x_1 \\ a_1 \end{pmatrix}$$



Transfer matrices for:

free space with length  $l_1$   
 $x \rightarrow x + l_1 \tan(\alpha)$

$$T_{21} = \begin{pmatrix} 1 & l_1 \\ 0 & 1 \end{pmatrix}$$

thin lens with  
 focal length  $f$

$$T_{32} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

free space with length  $l_2$

$$T_{43} = \begin{pmatrix} 1 & l_2 \\ 0 & 1 \end{pmatrix}$$

combined system:

$$\begin{pmatrix} x_4 \\ \tan \alpha_4 \end{pmatrix} = \begin{pmatrix} 1 & l_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & l_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ \tan \alpha_1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - l_2/f & =0 \\ -1/f & 1 - (l_1/f) \end{pmatrix} \begin{pmatrix} x_1 \\ \tan \alpha_1 \end{pmatrix}$$

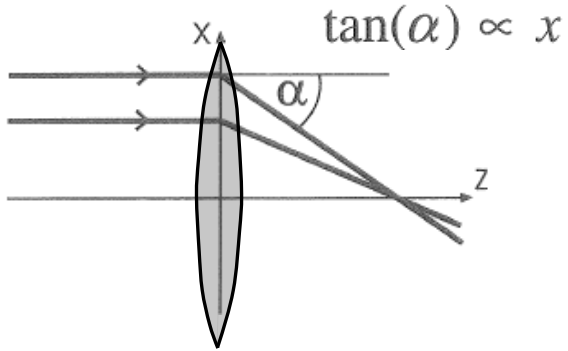
image (point to point)  
 when  $(x|a) = 0$

magnification

$(x|x) = 1 - l_2/f \rightarrow -1$  (for  $f = l_2/2$ )

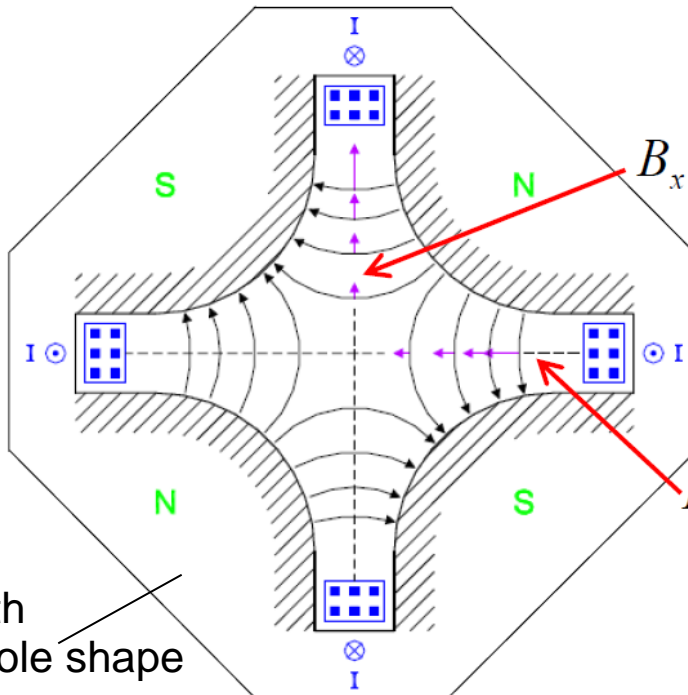
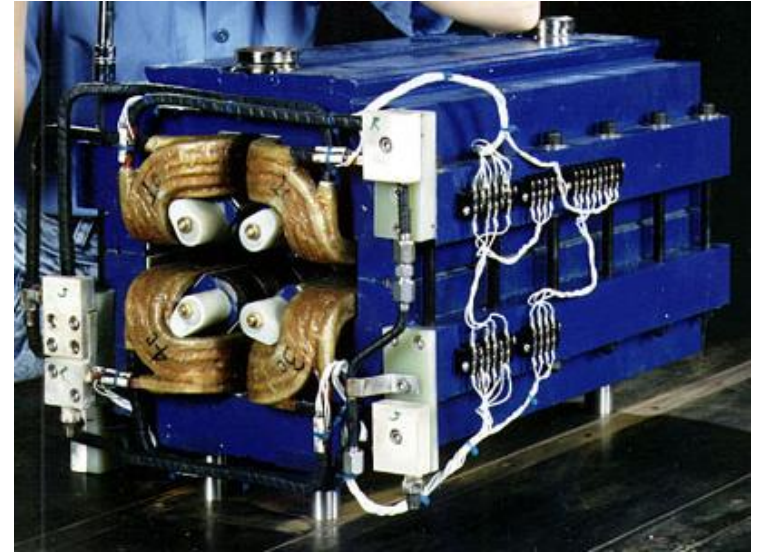
# Focusing Elements

## perfect linear lens



requires force  $F_x$  proportional to  $x$

## realisation as quadrupole magnet



$$B_x = g \cdot y \Rightarrow F_y = g \cdot y$$

defocusing

$$B_y = -g \cdot x \Rightarrow F_x = -g \cdot x$$

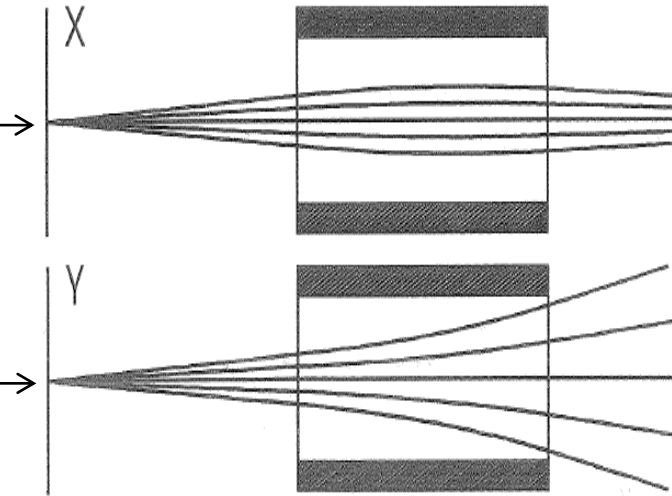
focusing !

iron yoke with hyperbolic pole shape

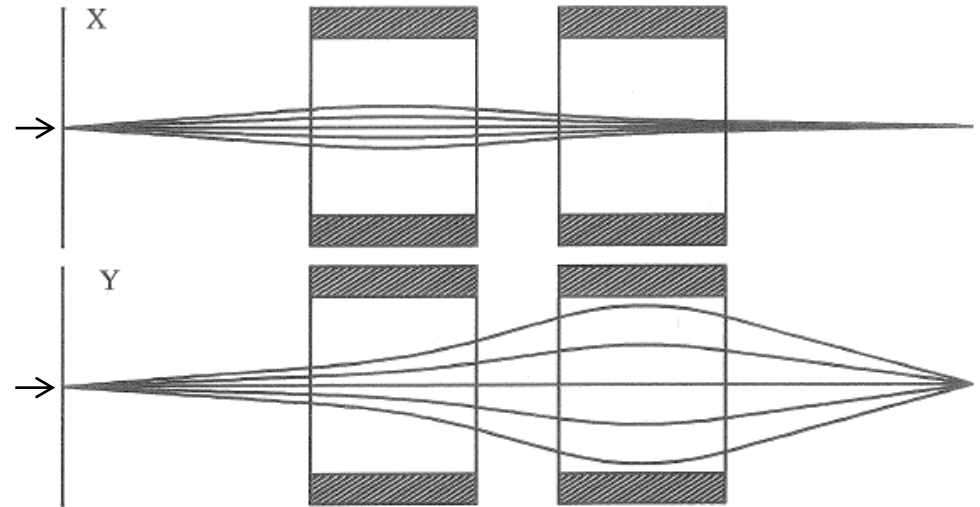


# Focusing Elements

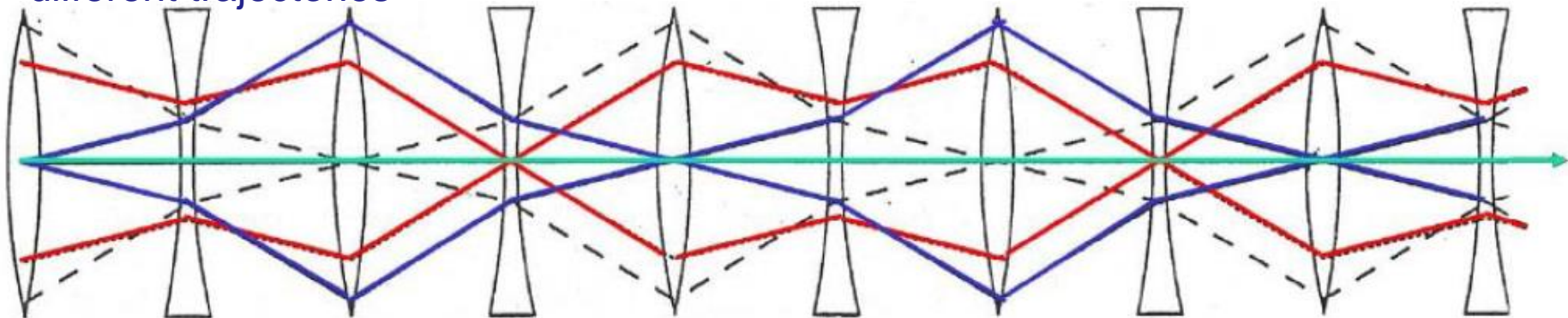
one quadrupole does not solve the problem



many quadrupole magnets combined can focus in x and y

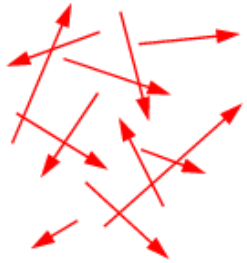


quadrupole channel with different trajectories

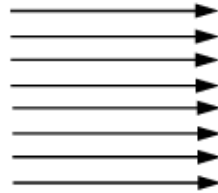


# How to Describe a Beam

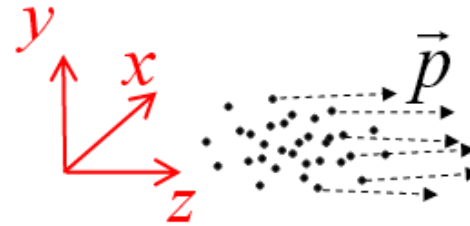
## Beam emittance



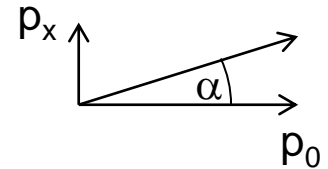
plasma



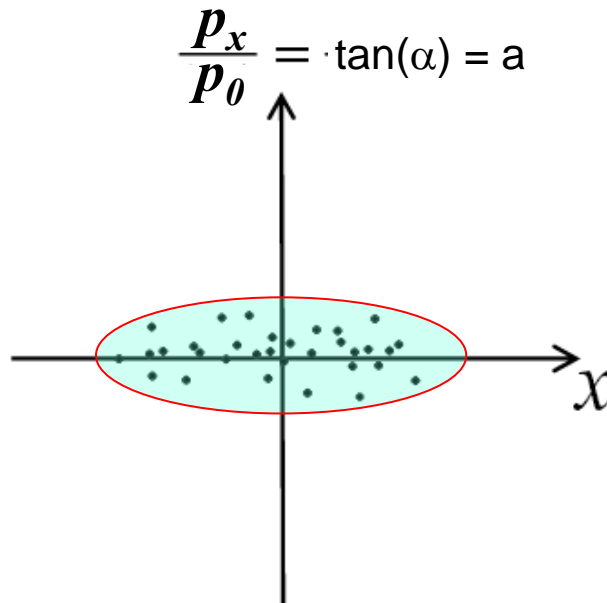
beam



real beam



## Phase space diagram



Area is often described well by an ellipse.

emittance  $\epsilon = \text{ellipse area} / \pi$

same with  $y$ ,  $b$  and in longitudinal ( $z$ ) direction

$\delta_p = \Delta p_z / p_0$  vs.  $z$ , or  
 $\delta_E = \Delta E / E_0$  vs. time  $t$ .

# Definition of Phase-Space Ellipse

Described by beam parameters\*  $\alpha, \beta, \gamma$  and emittance  $\varepsilon$ .  
Under linear transformation ellipse stays ellipse.

$$\gamma x^2 + 2\alpha xa + \beta a^2 = \varepsilon$$

Any linear transformation again leads to an ellipse.

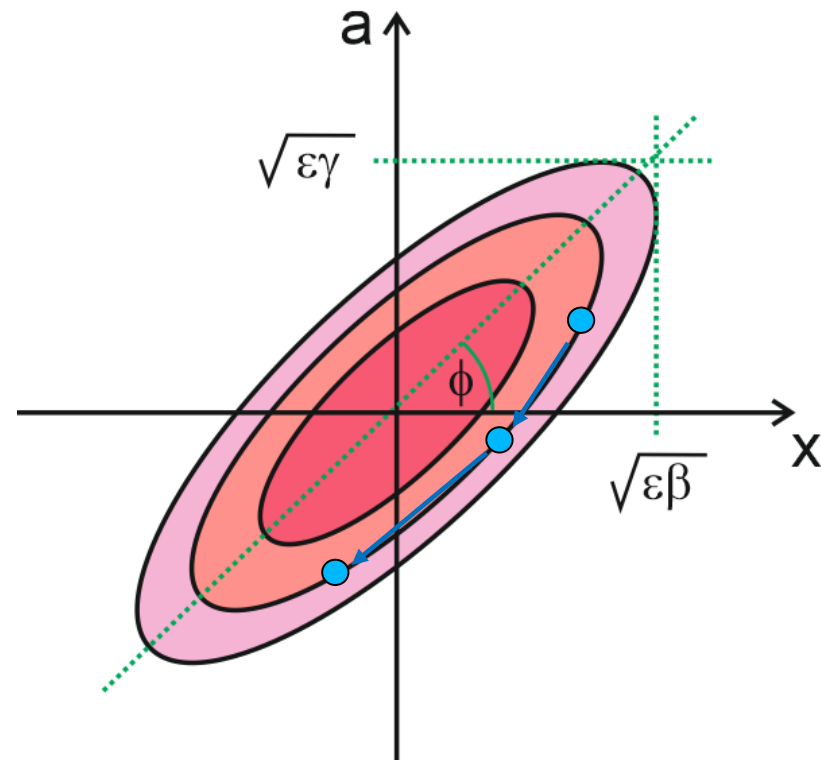
$$\begin{bmatrix} x \\ a \end{bmatrix}_f = \begin{bmatrix} (x,x) & (x,a) \\ (a,x) & (a,a) \end{bmatrix} \cdot \begin{bmatrix} x \\ a \end{bmatrix}_i$$

Emittance is constant (Liouville in const. electro-magn. fields), valid also for subellipses.

⇒ ions travel on ellipse, back and fourth.

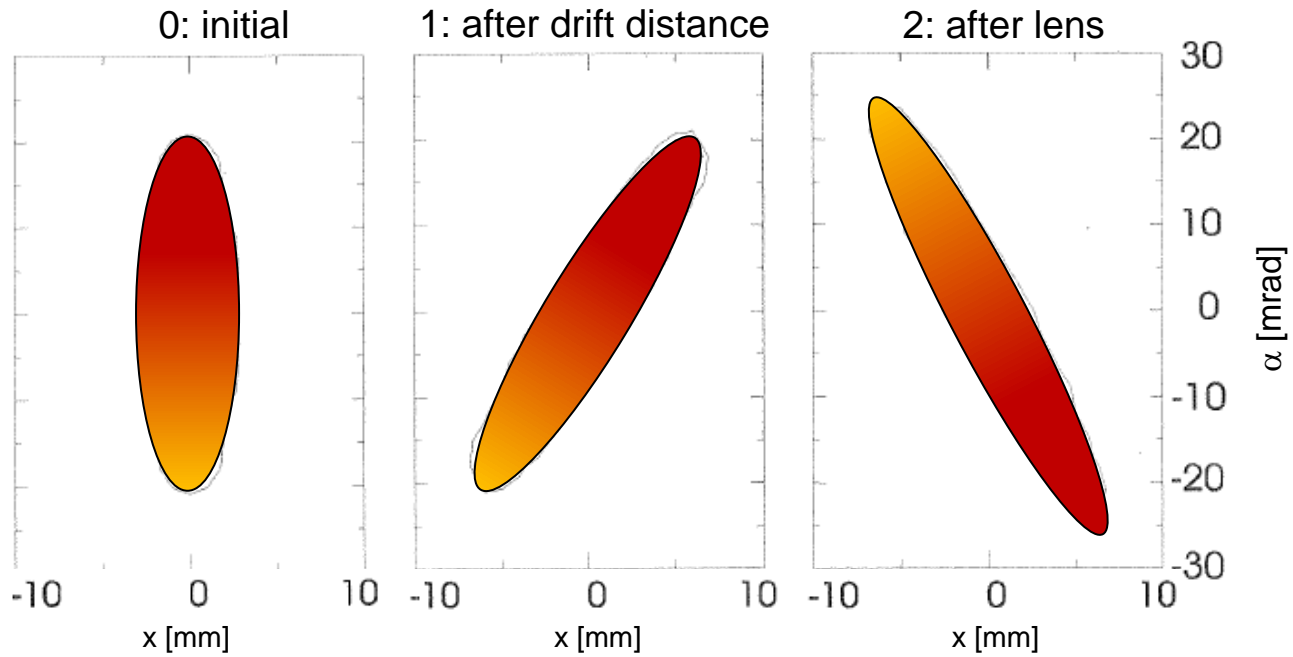
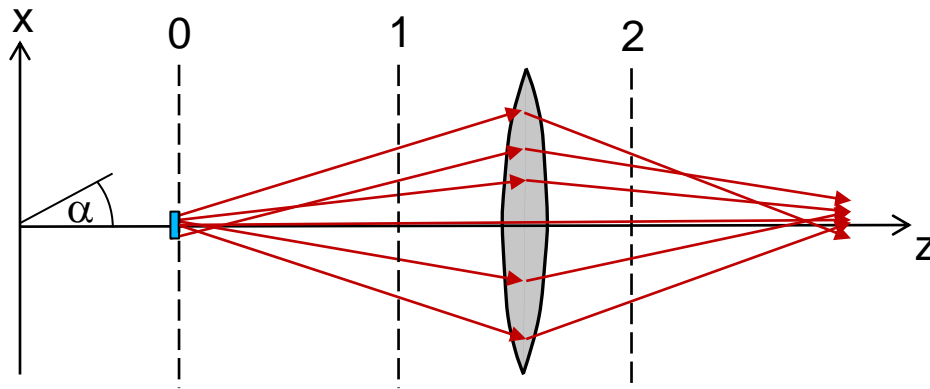
In systems with independent planes use separate ellipses for x and y

Beam diameter defined by parameter  $\beta$ , scales with emittance  $\varepsilon$ , i.e.  $\beta\gamma - \alpha^2 = 1$



\* first used by Courant & Snyder 1953, sometimes also called Twiss parameters.

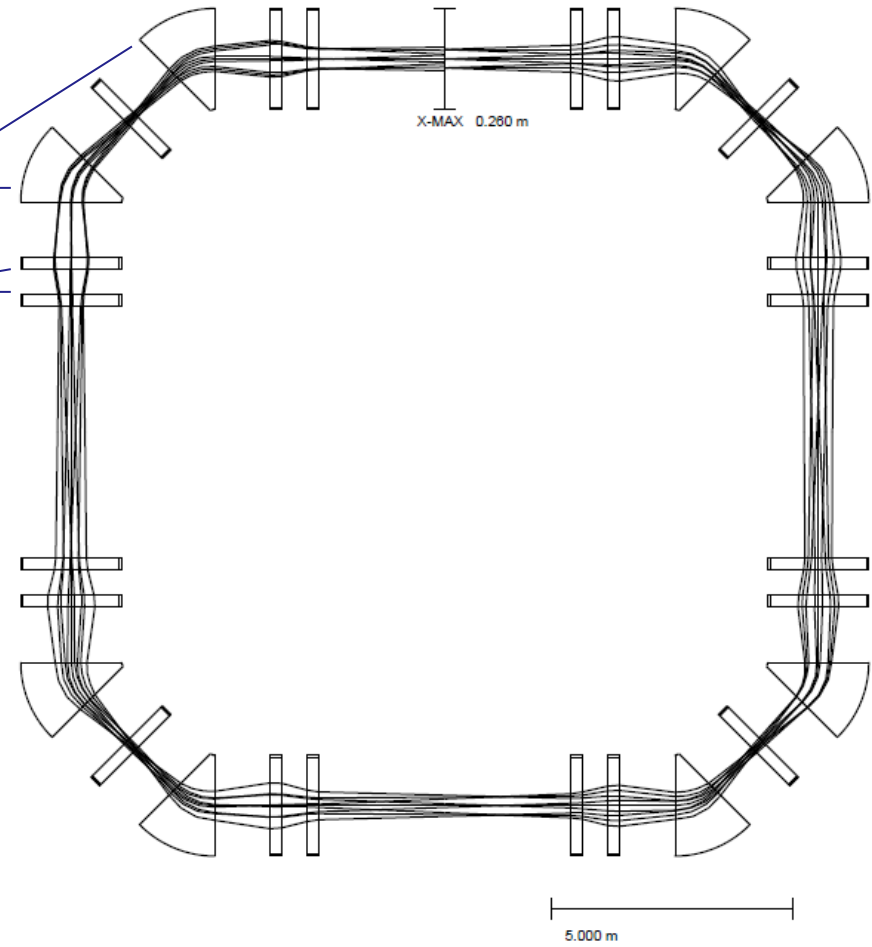
# Dynamics of Phase-Space Ellipse



# Old TSR storage ring in Heidelberg

Dipoles

Quadrupoles

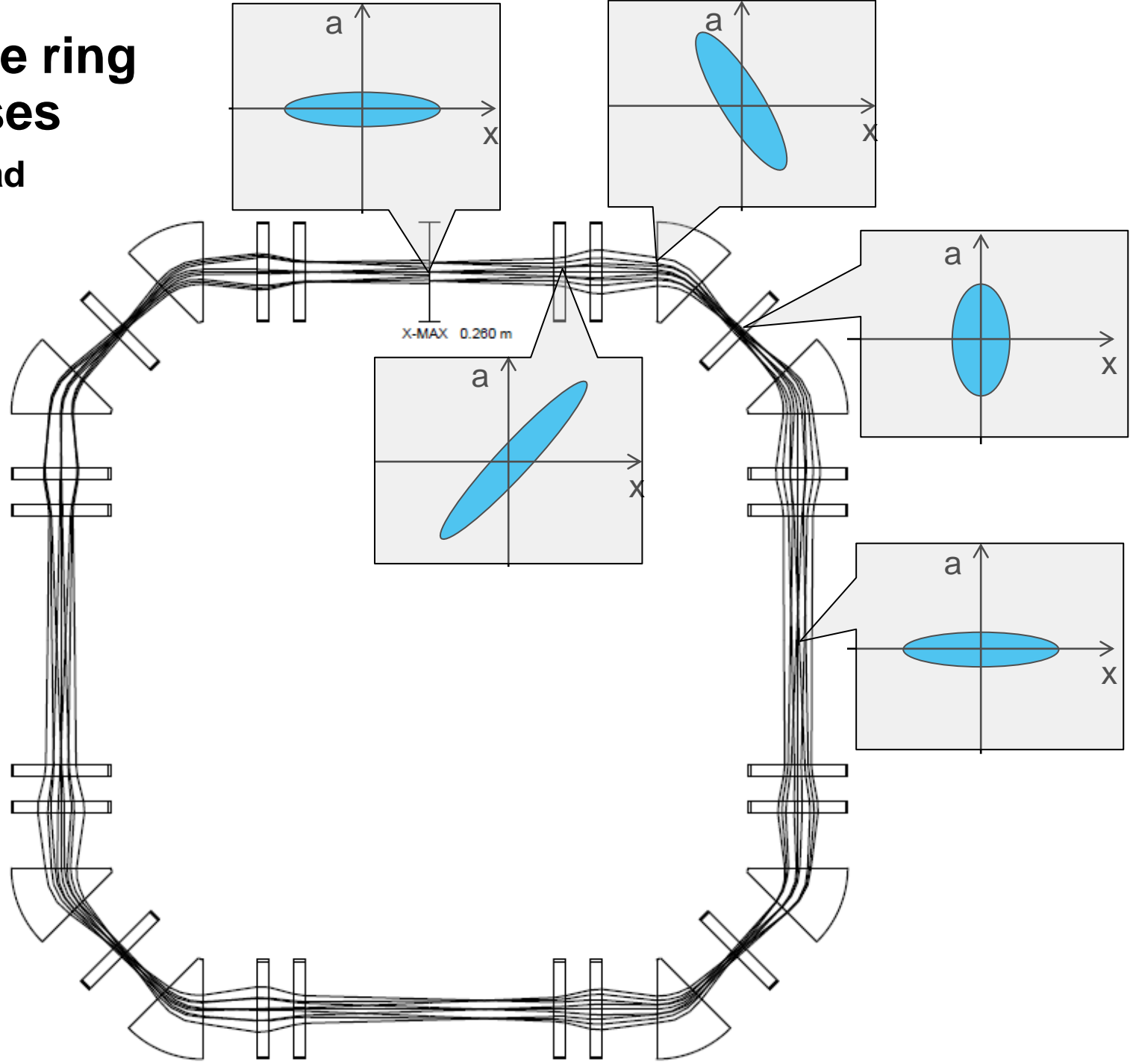


5,000 m

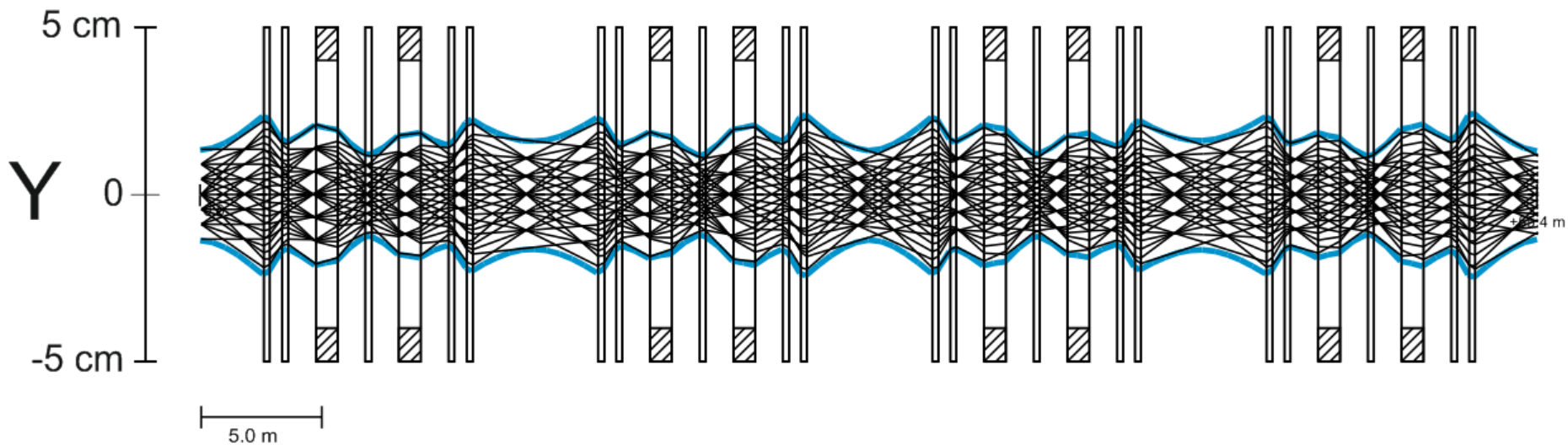
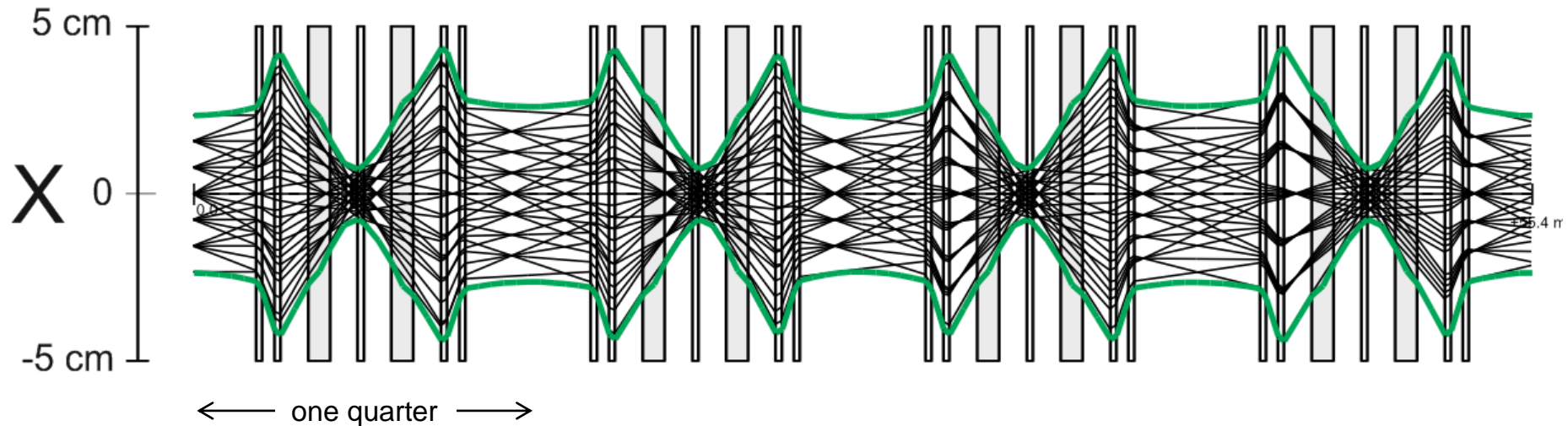


# TSR storage ring beam ellipses

$\epsilon = 100 \text{ mm mrad}$



# Beam envelopes in TSR storage ring



Periodic structure, will envelope stay within fixed limits or diverge?

Elliptic or hyperbolic solution of 2<sup>nd</sup> order ODE.

Stable when for single cell:  $\text{Trace}(\mathbf{T}) = |(x,x) + (a,a)| \leq 2$

The header features a collage of scientific and technical illustrations. On the left, there are blue and white diagrams of particle paths or structures. In the center, there are orange and yellow abstract shapes and lines. On the right, there is a detailed illustration of a particle accelerator or detector component, showing various tubes and a central structure.

## **2. Ion Optics of Separators**



# Resolution of Separator

Resolving power

$$R = \frac{(x, \delta)}{\Delta x_f} = \frac{(x, \delta)}{(x, x) \Delta x_i}$$

High R needed to separate not only by mass number but also by binding energy,  $R > 10\,000$ .

Use dipole magnet most effectively.

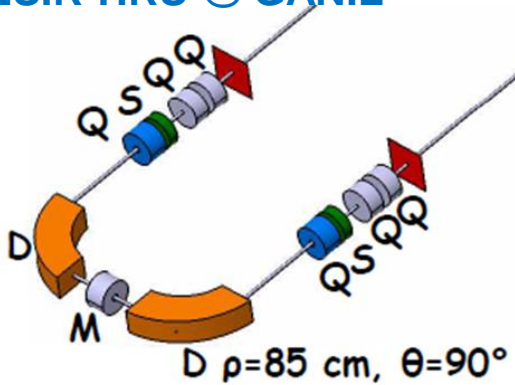
In an almost parallel beam small changes of deflection angle are most sensitive.

→ stretch phase space, make beam very wide.

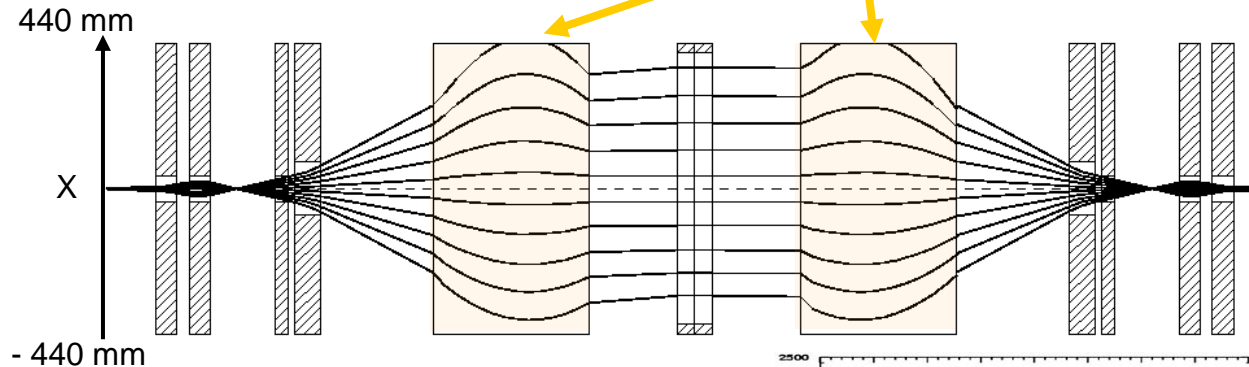
$$R = \frac{1}{x_0 a_0} \int \frac{\vec{B}(s)}{B\rho} d\vec{f}$$

maximize area in dipoles

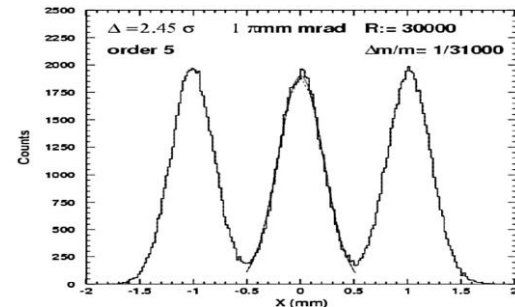
## DESIR-HRS @ GANIL



T. Kurtukian-Nieto et al., NIM B (2013) 317



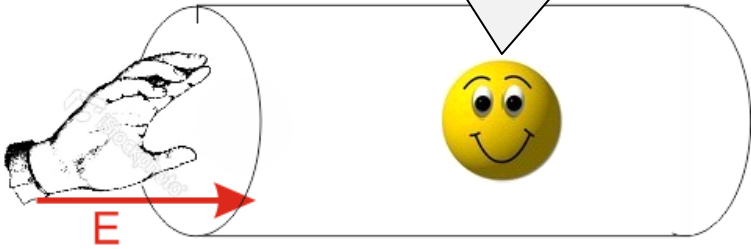
trajectories  
 $x_0 a_0 = 0.5\text{mm} \times 7\text{ mrad}$



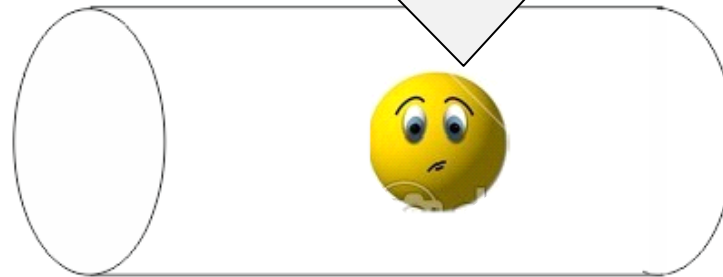
Small emittance of course helps (beam cooling).

# Beam Interaction

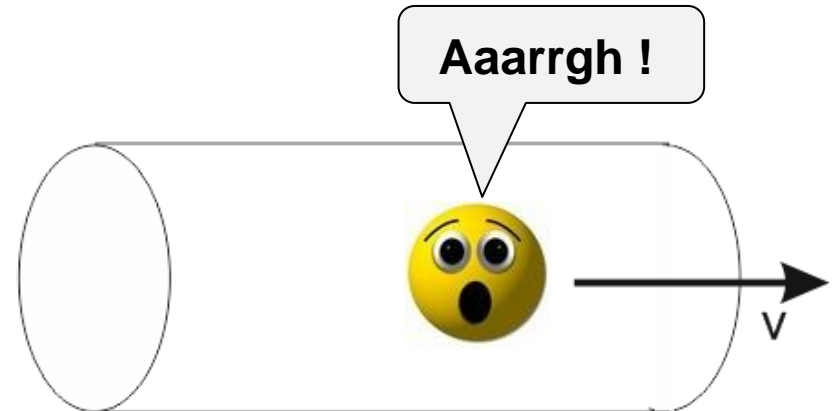
Yeah, I am an accelerated ion.



By the way, what do you do with accelerated ions?

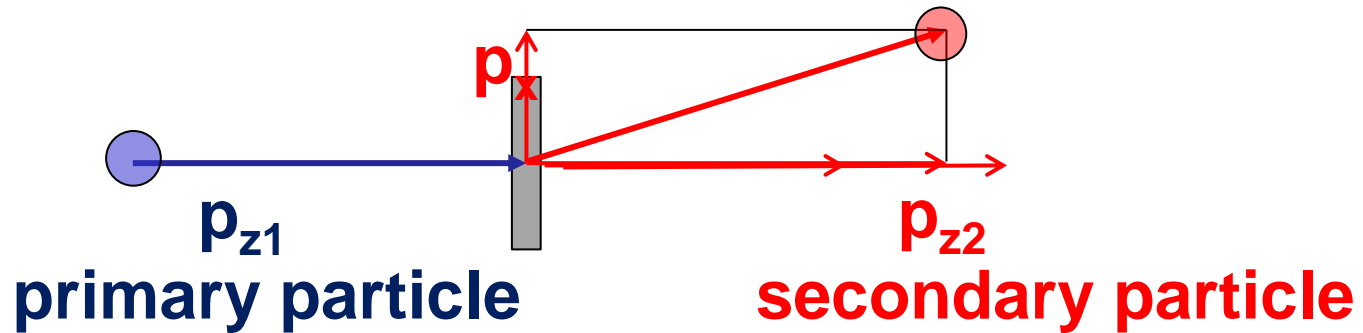


Aaarrgh !

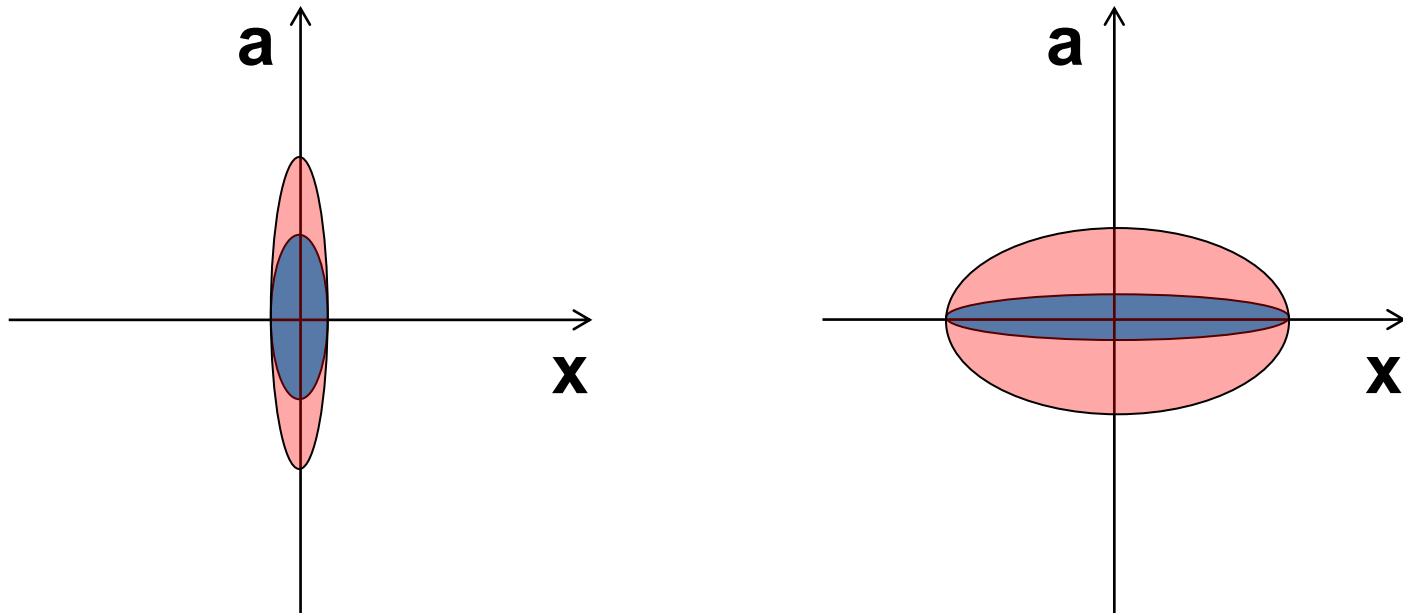


# Emittance Growth with Targets

Momentum transfer by reaction in target increases transverse momentum spread, but in a thin target  $\Delta x$  does not change much.



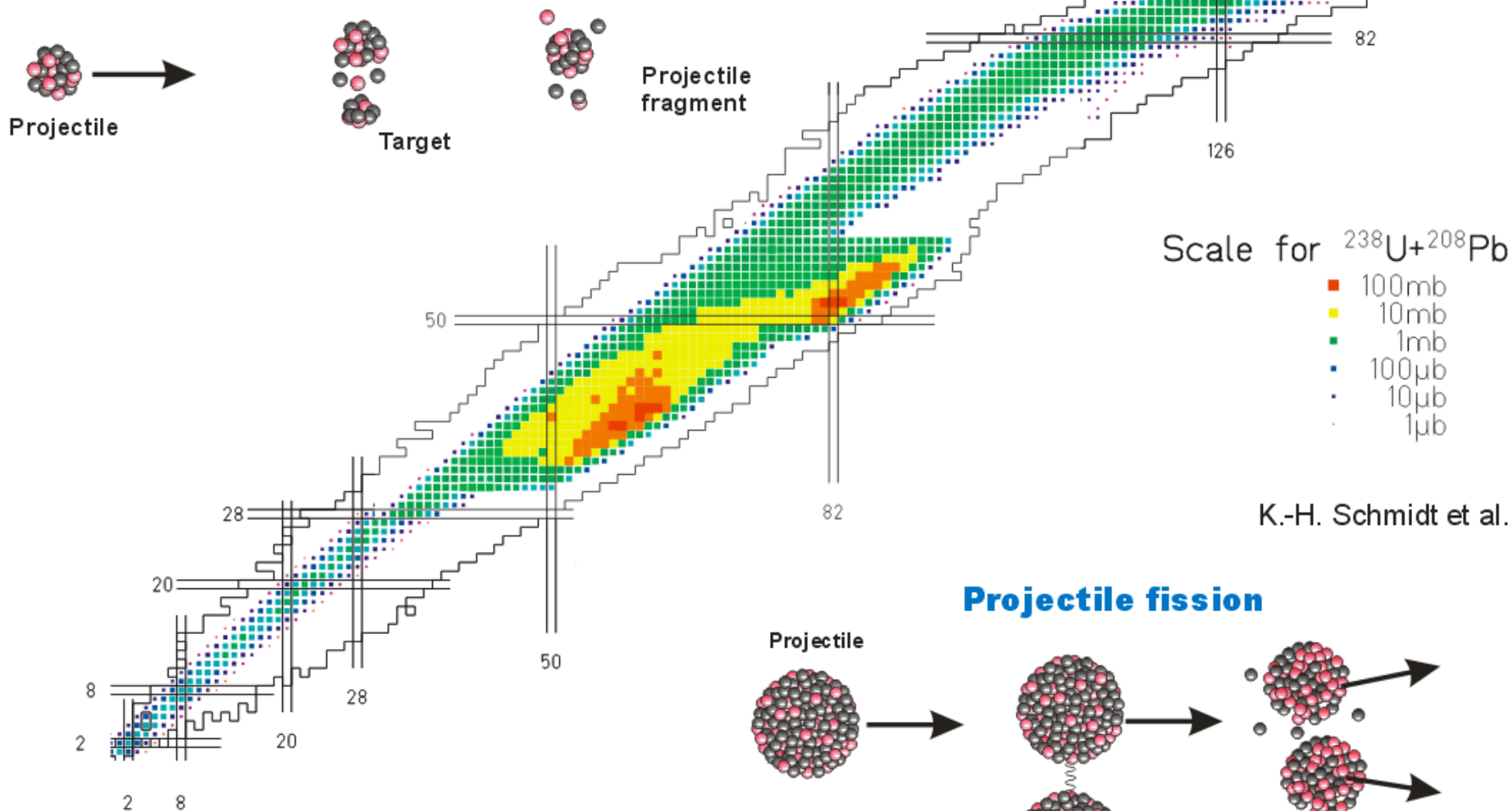
Make small beam spot to avoid large emittance for secondary beam



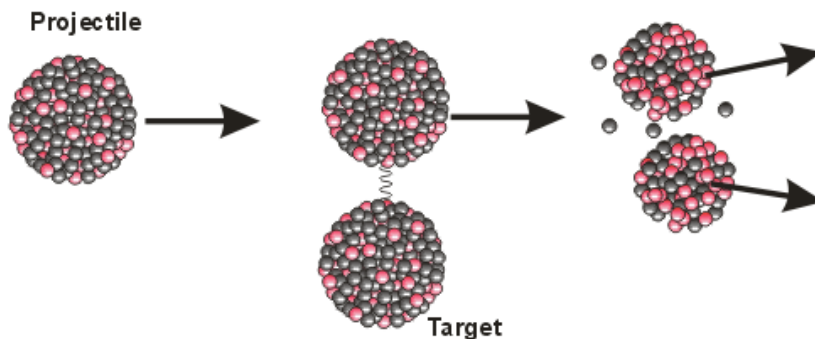
# Fragmentation and Fission as source of rare isotopes



## Nuclear projectile fragmentation



## Projectile fission



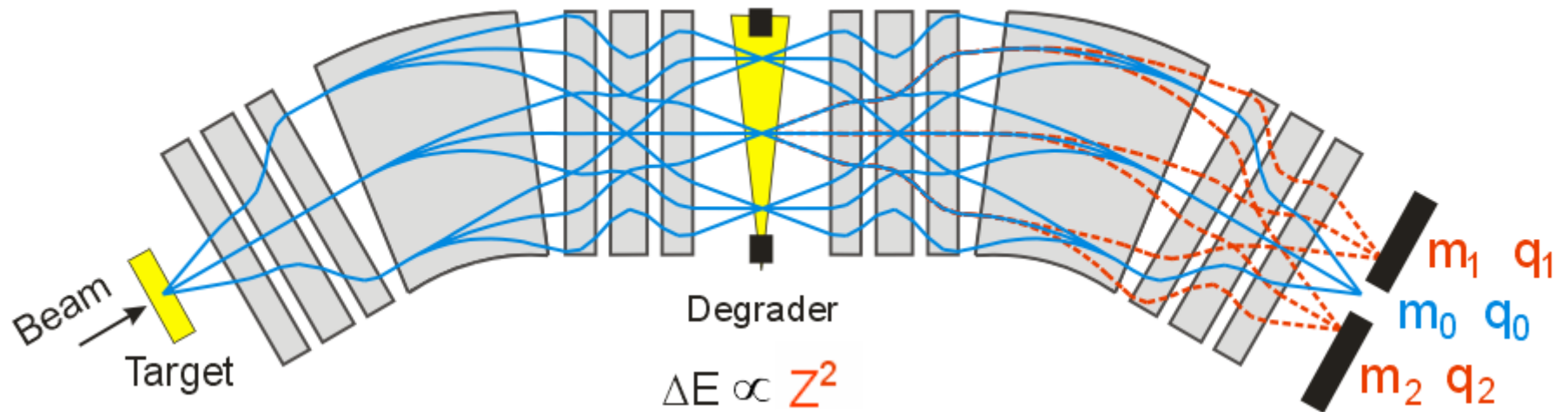


### **3. Separation with Degrader**

**select ions by mass and charge separately  
→ fragment separator**

# Principle: $B\rho$ - $\Delta E$ - $B\rho$ Separation

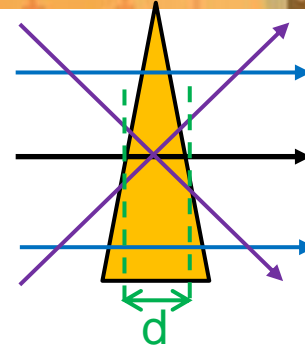
Achromatic in **velocity**, but dispersive in **mass** and **charge**



shaped to keep overall system achromatic

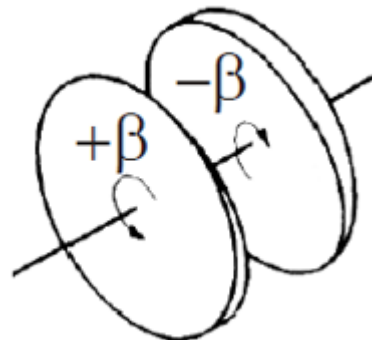
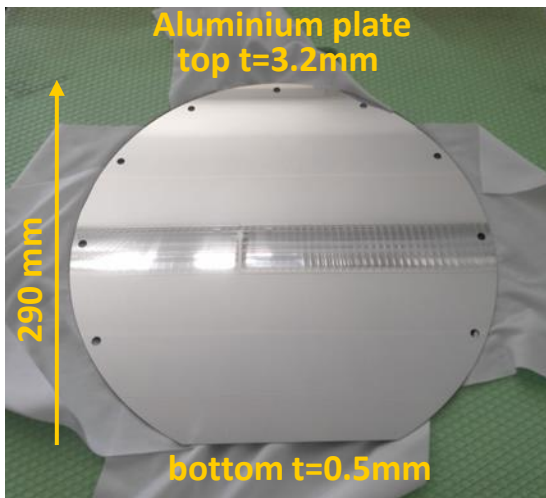
# Degrader Wedge

→ wedge shape ( $\delta|x$ )  
for symmetric wedge ( $\delta|a|=0$ )

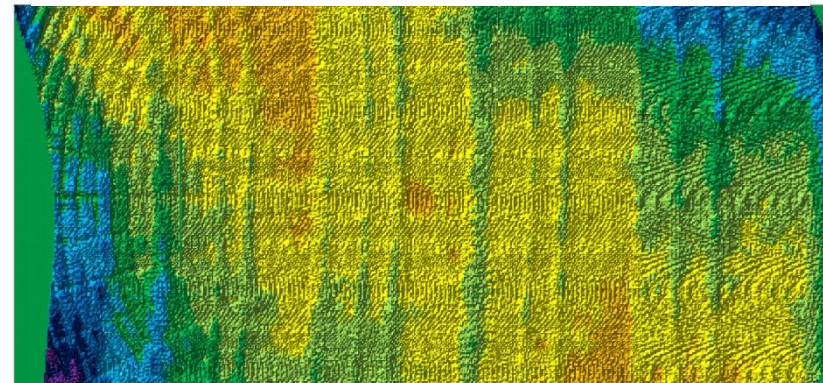


Physics limitation by energy-loss straggling in stochastic process of many atomic collisions:  $\sigma_E^2 = \underbrace{d\Omega^2/dz}_{\text{from atomic physics}} d \rightarrow \delta_{\text{str}} \sim 2e-3 \dots 2e-4$   
ultimate limitation of technique

Real wedges are thin metal plates. Combine many plates for easy adjustment. Shape and surface flatness must be very precise, on the level of wanted resolution.



deviation from wedge shape  
area: 280 mm x 134 mm



# Transfer Matrix Description

ion-optical  
coordinates

$$x, a = p_x/p_0, y, b = p_y/p_0, \delta = \Delta B\rho/B\rho_0$$

$$B\rho = c_0\beta\gamma \quad m/q = v \quad m/q$$

$$\delta_m = \Delta m/m, \delta_q = \Delta q/q, \delta_v = \Delta v/v_0$$

Separator  
stage, D

$$\begin{pmatrix} x \\ a \\ \delta_m \\ \delta_q \\ \delta_v \end{pmatrix}_f = \begin{pmatrix} (x|x) & \cancel{(x|a)} & (x|\delta) & -(x|\delta) & (x|\delta) \\ (a|x) & (a|a) & (a|\delta) & -(a|\delta) & (a|\delta) \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ a \\ \delta_m \\ \delta_q \\ \delta_v \end{pmatrix}_i$$

Degrader  
wedge W

$$\begin{pmatrix} x \\ a \\ \delta_m \\ \delta_q \\ \delta_v \end{pmatrix}_f = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ (\delta_v|x) & (\delta_v|a) & (\delta_v|\delta_m) & (\delta_v|\delta_q) & (\delta_v|\delta_v) \end{pmatrix} \begin{pmatrix} x \\ a \\ \delta_m \\ \delta_q \\ \delta_v \end{pmatrix}_i$$

Full system

$$= \mathbf{D}_2 \cdot \mathbf{W} \cdot \mathbf{D}_1$$

require  $(x|\delta_v)_{\text{tot}} = 0, (a|\delta_v)_{\text{tot}} = 0$



# Transfer Matrix Description

H. Geissel et al, NIM B247 (2006) 368.

## Momentum resolving power of one half

$$R = \frac{(x|\delta)}{\Delta x_f} \sim \mathbf{1000 \dots 2000}$$

## Energy-loss straggling

$$\delta_{v, \text{str}} \sim \mathbf{1e-3 \dots 3e-3}$$

## Resolution in mass, charge

$$R_m = \frac{(x|\delta_m)}{\Delta x_f}$$

$$R_q = \frac{(x|\delta_q)}{\Delta x_f}$$

## for achromatic separator $(x|\delta_v)_{\text{tot}} = 0$

$$R_m = \frac{[(\delta_v|\delta_m) + 1 - (\delta_v|\delta_v)] / (\delta_v|\delta_v)}{\sqrt{1/R^2 + (\delta_{v, \text{str}}/(\delta_v|\delta_v))^2}} = R \frac{[(\delta_v|\delta_m) + 1 - (\delta_v|\delta_v)]}{(\delta_v|\delta_v)}$$

$\uparrow$  for  $\delta_{v, \text{str}} \rightarrow 0$

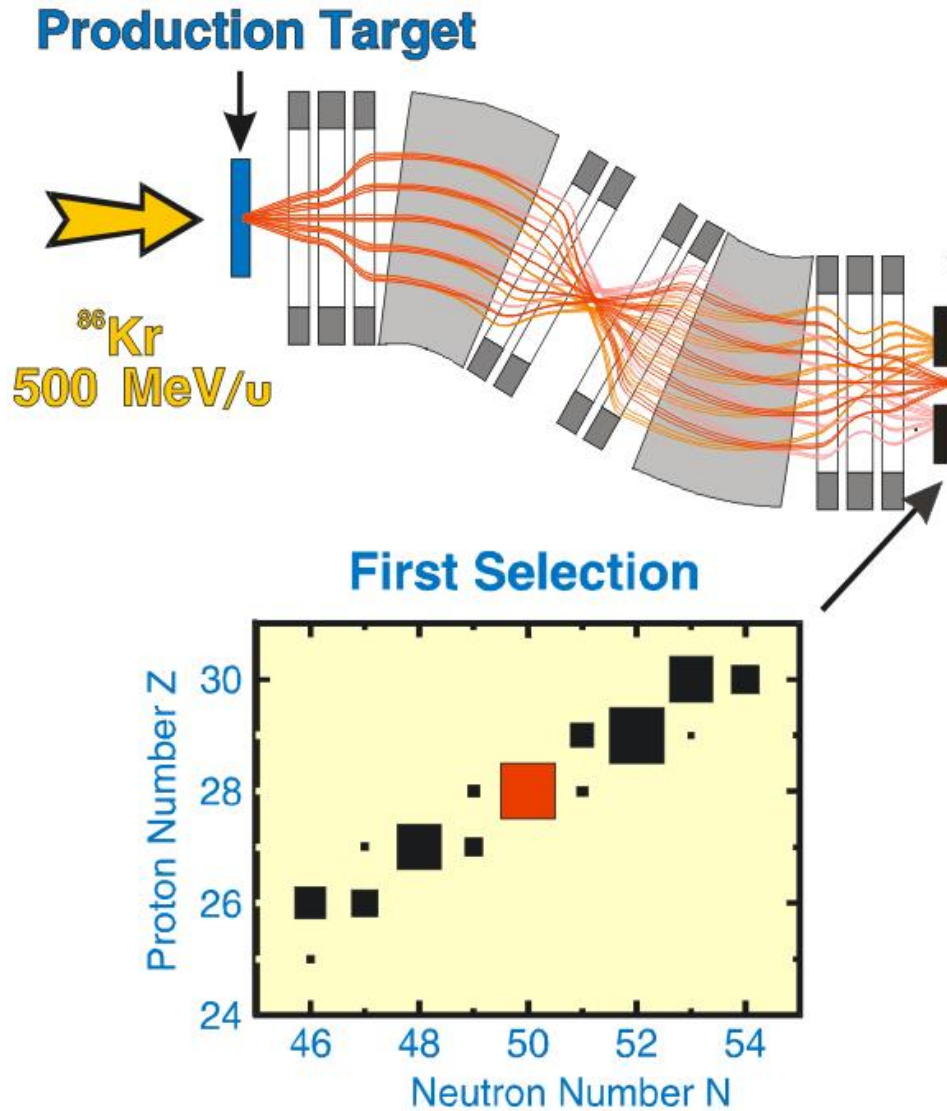
$$R_q = \frac{[(\delta_v|\delta_q) - 1 + (\delta_v|\delta_v)] / (\delta_v|\delta_v)}{\sqrt{1/R^2 + (\delta_{v, \text{str}}/(\delta_v|\delta_v))^2}} = R \frac{[(\delta_v|\delta_q) - 1 + (\delta_v|\delta_v)]}{(\delta_v|\delta_v)} \sim \mathbf{0 \dots 200}$$

$< 1$

$> 1$

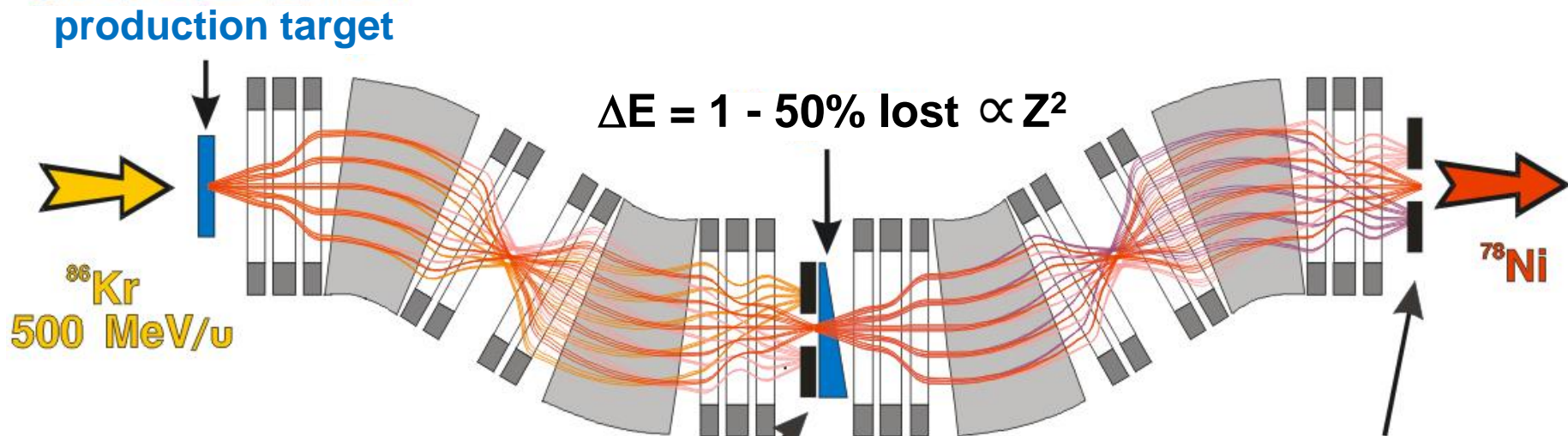
$> 1$

# $B\rho$ - $\Delta E$ - $B\rho$ Separation Method in FRS

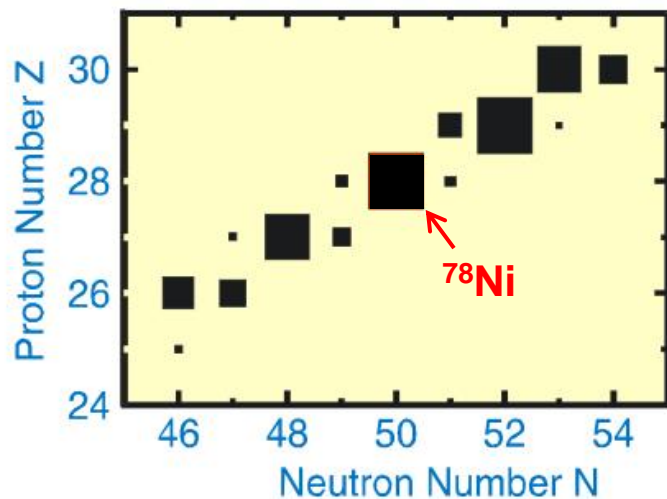


# B $\rho$ - $\Delta E$ -B $\rho$ Separation Method

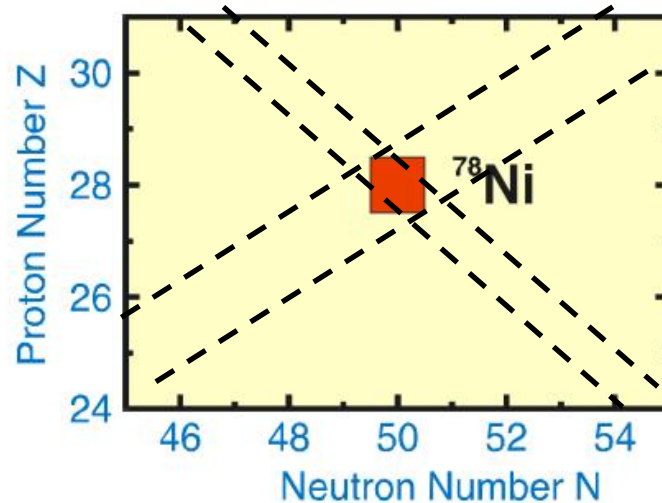
scheme of FRS @ GSI, L=72m



### First Selection

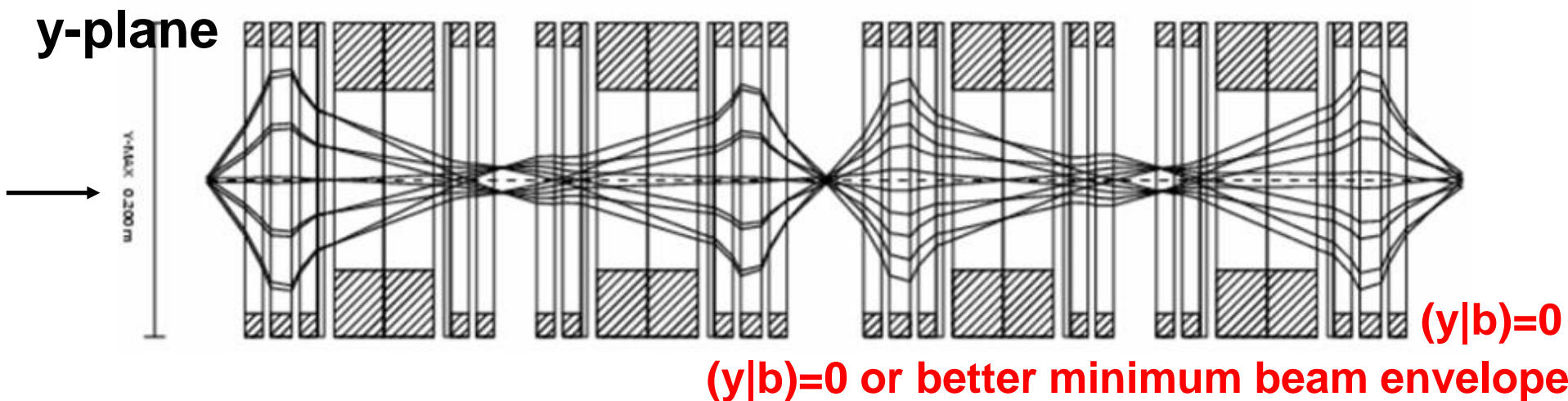
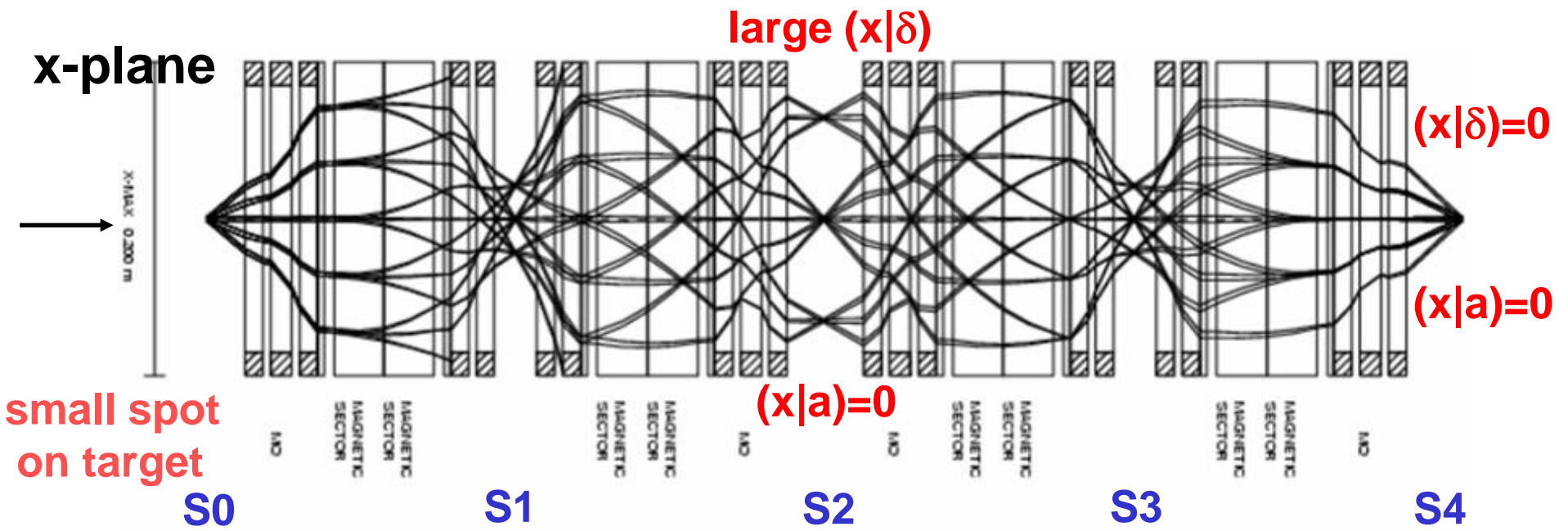


### First and Second Selection



# Optics Mode FRS

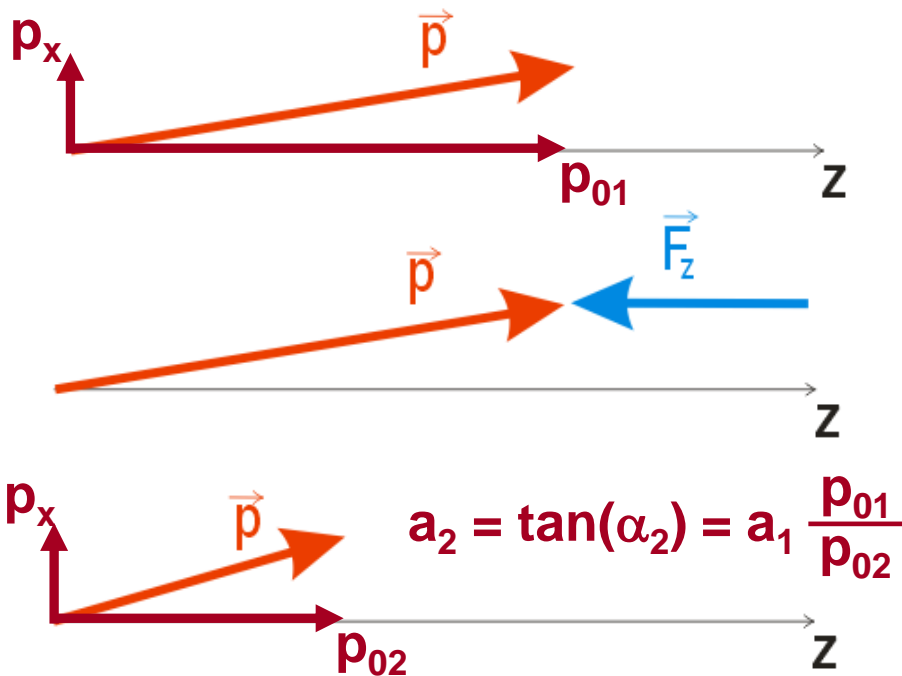
Ions starting with different angles and momenta  
 $\delta = \pm 1\%$ ,  $a = \pm 5, \pm 10$  mrad,  $b = \pm 7.5, \pm 15$  mrad



# Slowing Down in Matter

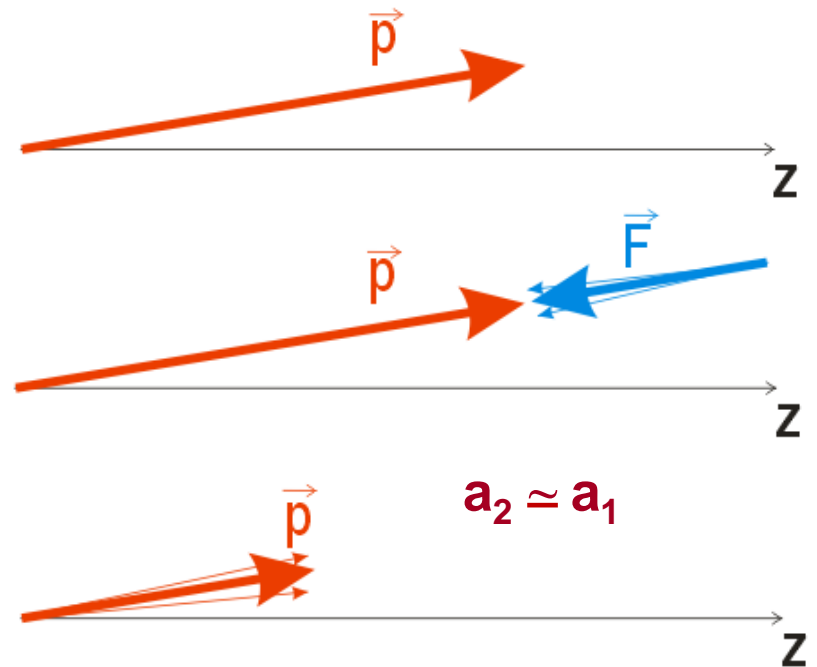
## Accelerator

longitudinal momentum change



## in Matter

Average Force ( $dE/ds$ ) opposite to direction of motion.



Large emittance growth in accelerator, less in matter.

Large changes require a new reference  $p_{02} = p_{01} \frac{\beta_1 \gamma_1}{\beta_2 \gamma_2}$

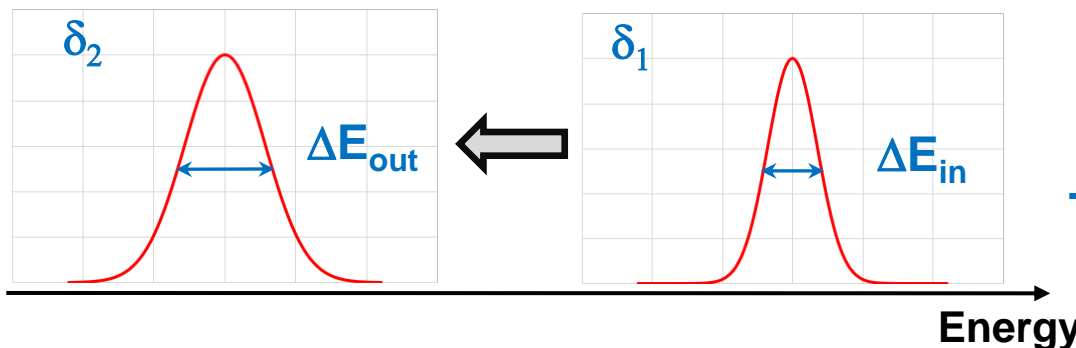
$\delta_1 = \Delta p_{z1}/p_{01} \rightarrow \delta_2 = \Delta p_{z2}/p_{02}$ , to preserve a symmetric separator  $\delta_1 = \delta_2$

# Energy-Loss of Heavy Ions

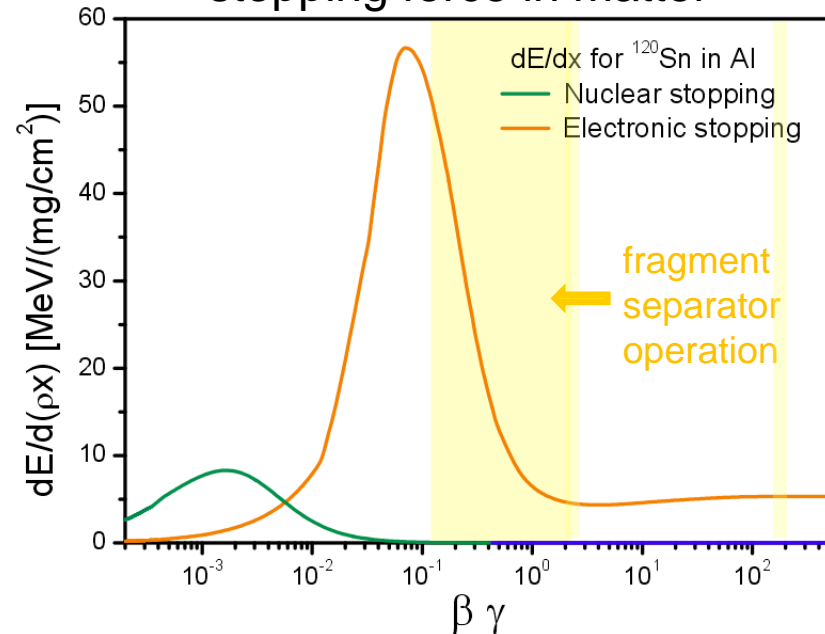
No closed system (non-Liouvillean),  $(\delta|\delta)$  does not just depend on beam momenta  $p_{01}, p_{02}$ .

Slowing down depends on the atomic processes described by average stopping force.

different energy loss within peak  
=> stretching of distribution



stopping force in matter



$$\frac{\Delta E_{in}}{\Delta E_{out}} = \frac{(dE/d\rho x)_{in}^2}{(dE/d\rho x)_{out}^2}$$

→ additional contribution to  $(\delta|\delta)$

$$\delta_E = \delta_p (1 + 1/\gamma)$$

# Effect of degrader at different velocity

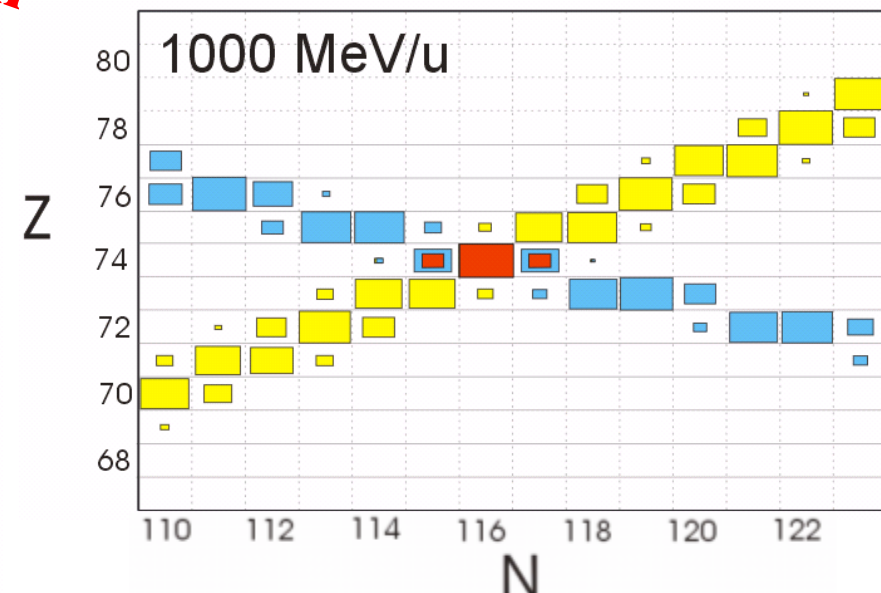
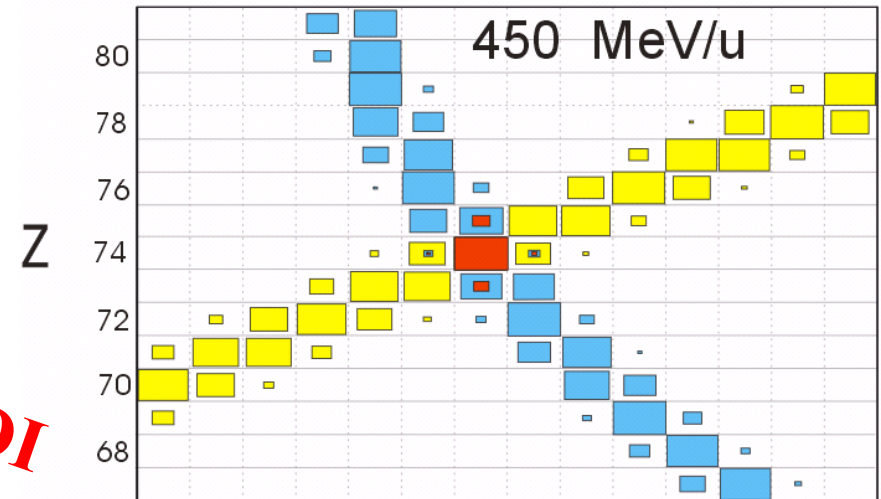
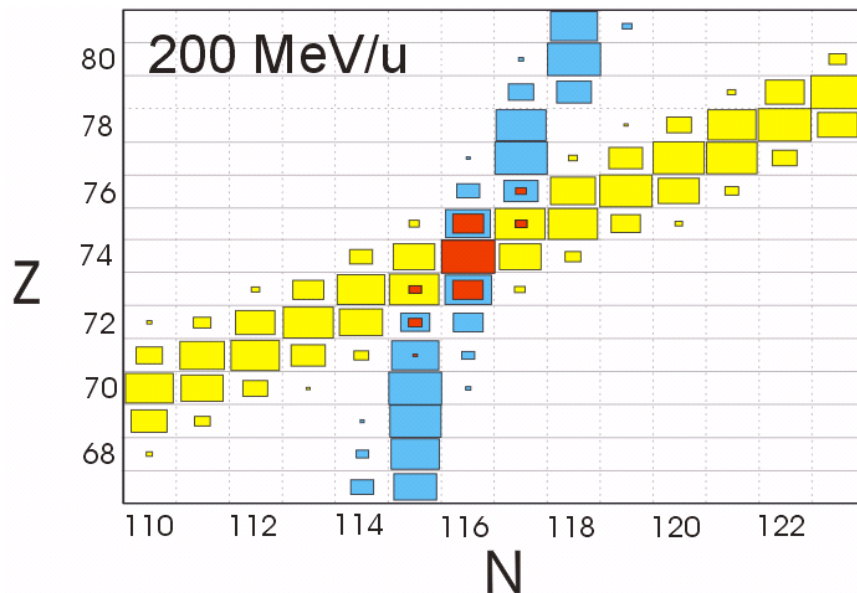
fragment  $^{190}\text{W}$  in achromatic FRS  
with Al degrader  $d/r=0.5$   
transmission

1st slits at  $\pm 2\sigma_x$

2nd slits at  $\pm 2\sigma_x$

both slits at  $\pm 2\sigma_x$

**MOCADI**  
simulation



# Phase Space Consideration

Determinant of transfer matrix is a measure of phase space volume.  
 For overall achromatic system imaging in each stage,  $(x|a)=0$ .

transverse, 2-dim

$$\text{Det}_t = (x|x)_{\text{tot}} (a|a)_{\text{tot}}$$

$$= - (\delta_v|\delta_v) \frac{(x|\delta)_2}{(x|x)_2 (x|\delta)_1} \neq 1$$

Non-Liouvillean

longitudinal, 2-dim

$$\text{Det}_l = (\delta_v|\delta_v) \underbrace{(\Delta z|\Delta z)}_{=1 \text{ no time dependence}} = - \frac{(x|x)_2 (x|\delta)_1}{(x|\delta)_2}$$

combined, 4-dim.


$$\text{Det}_{\text{comb}} = \text{Det}_t \text{Det}_l = (\delta_v|\delta_v) \rightarrow 1$$

quasi Liouvillean

for  $dE/dz (v) = \text{const.}$   
 Neglecting straggling

Wedge shaped degrader couples transverse and longitudinal motion.  
 One increased by factor, the other reduced by same factor.





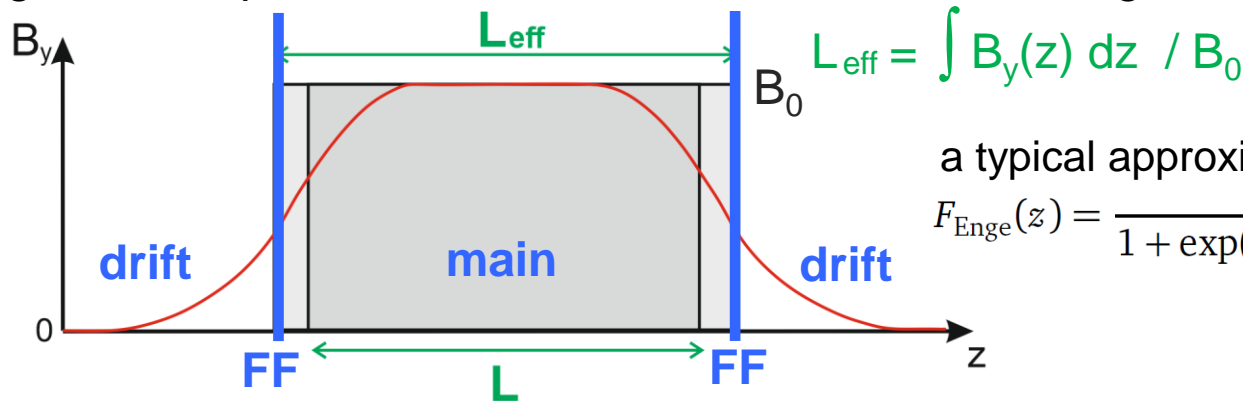
## **4. Large Apertures, Higher Order Optics Fringe Fields**

# Large Aperture Optical Elements

Emittances of secondary beams are large, e.g. after RIB production. Magnets with large aperture compared to their lengths are required.

→ Fringe fields cannot be neglected.

Often a division into drifts, fringe fields (FF) and main field is possible. Main field with effective length (same integral along axis) and short fringe field maps describe the deviation to a real soft edge.



a typical approximation:

$$F_{\text{Enge}}(z) = \frac{1}{1 + \exp(a_1 + a_2 \cdot (z/D) + \dots + a_6 \cdot (z/D)^5)}$$

Sharp drops violate Maxwell's equations. A field curvature along axis requires similar opposite curvature in transverse direction. Fringe fields will always contain transverse non-linear components. For a given multipole symmetry the distribution along the axis defines the transverse distribution.

Field derived from potential (outside and const. in time):

$$\vec{E} = -\vec{\nabla} \Phi, \quad \Delta \Phi = 0$$

$$\vec{B} = \vec{\nabla} \times \vec{A}, \quad \vec{\nabla} \cdot \vec{A} = 0$$

# Quadrupole with Fringe Fields

Main field has analytical solution.

$$Q = \begin{bmatrix} (x,x) & (x,a) \\ (a,x) & (a,a) \end{bmatrix} = \begin{bmatrix} \cos(k_0 L) & \sin(k_0 L) / k_0 \\ -k \sin(k_0 L) & \cos(k_0 L) \end{bmatrix}$$

$$\text{with } k_0^2 = \frac{B_0}{r_0 B \rho}, \quad L \rightarrow L_{\text{eff}}$$

Use approximate solutions for fringe fields.

For example by step-wise integration (Picard iteration).

$$FF = \begin{bmatrix} (x,x) & (x,a) \\ (a,x) & (a,a) \end{bmatrix} \sim \begin{bmatrix} 1 \pm k_0 I_1 & -2k_0 I_2 \\ 0 & 1 \mp k_0 I_1 \end{bmatrix}$$

$I_1, I_2, \dots$  = fringe field integrals

$$I_1 = \frac{1}{k_0} \iint k(z) d^2z - z_b^2/2$$

$$I_2 = \frac{1}{k_0} \iint k^2(z) d^2z - z_b^3/2$$

Matsuda, Wollnik, NIM 102 (1972) 117.

Integrals depend only on shape, different weighting with  $k(z)$ , can later be scaled with  $r_0$

Some are even independent of the detailed shape, e.g.  $(x,xxx) \sim k_0/12$ ,  $(x,xyy) \sim k_0/4$

→ Any fringe field is better than none, typical ones often are good enough.

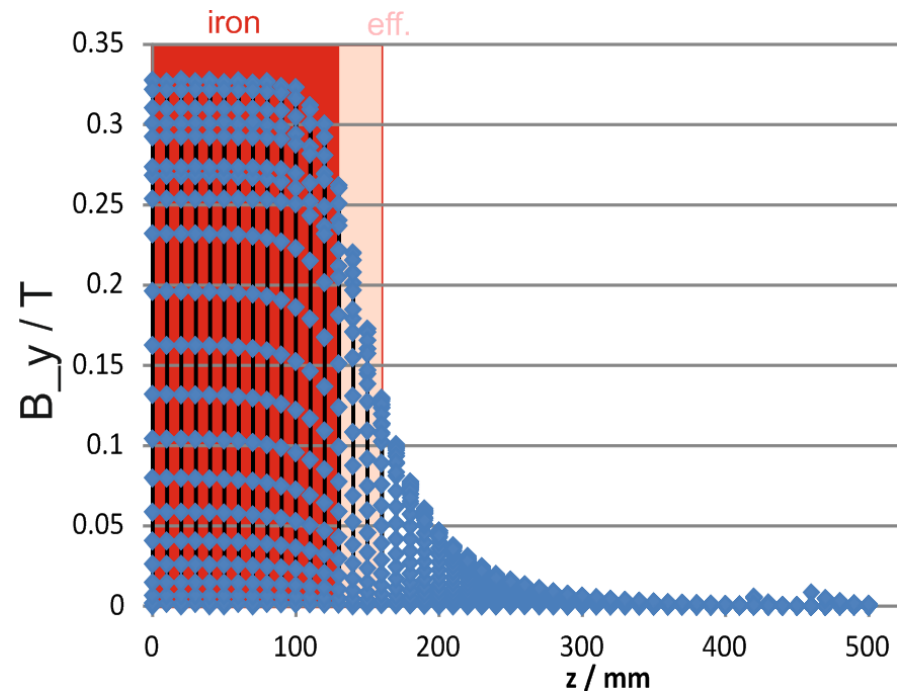
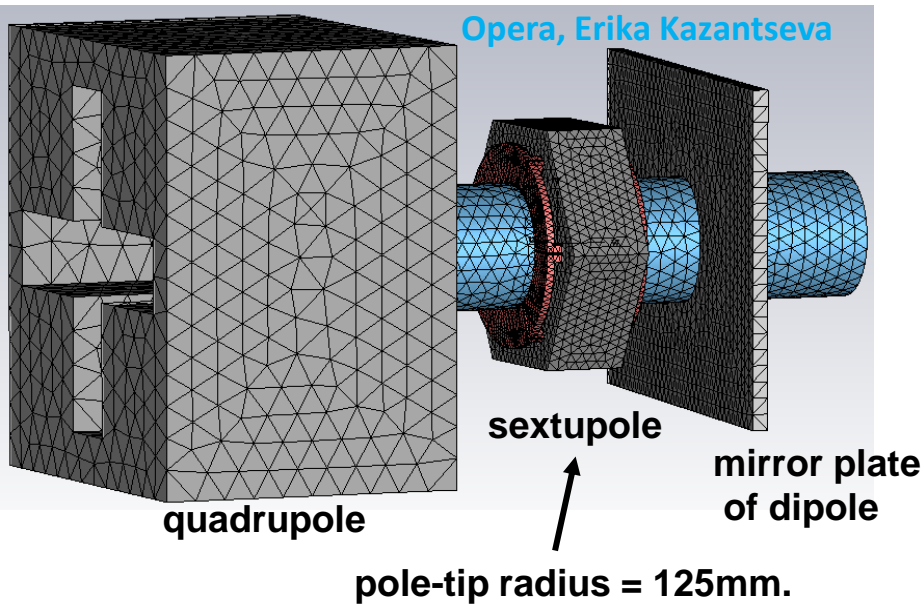
For very hard cases, e.g. no main field can be defined, use

numerical integration directly of algebraic expressions of matrix elements

(differential algebra). Is relatively fast and avoids problem of

differences of differences on higher order optics coefficients. → COSY INFINITY

# Effective Length of FRS Sextupole



$L_{\text{iron\_geometric}} = 260 \text{ mm}$   
 $L_{\text{eff}} \text{ (standalone)} = 330 \text{ mm (measured)}$   
 $L_{\text{eff}} \text{ (combined)} = 319 \text{ mm (calculated)}$

# Higher Order Aberrations

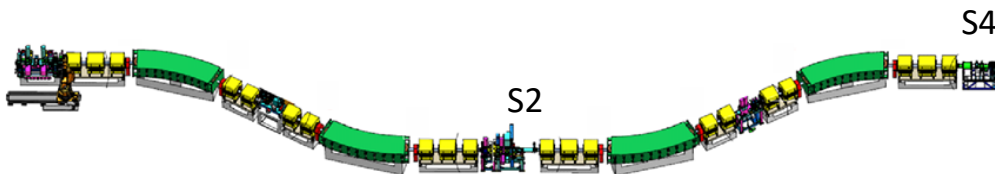
#3

Described by higher order coefficients of transfer map (Taylor expansion)

$$x_2 = (x,x) x + (x,a) a + (x,\delta) \delta + \underbrace{(x,xx) x^2 + (x,xa) xa + (x,aa) a^2 + (x,bb)b^2}_{\text{geometric}} + ..$$

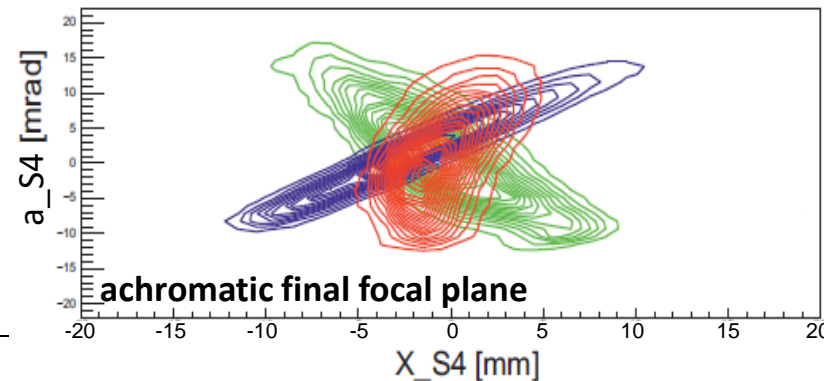
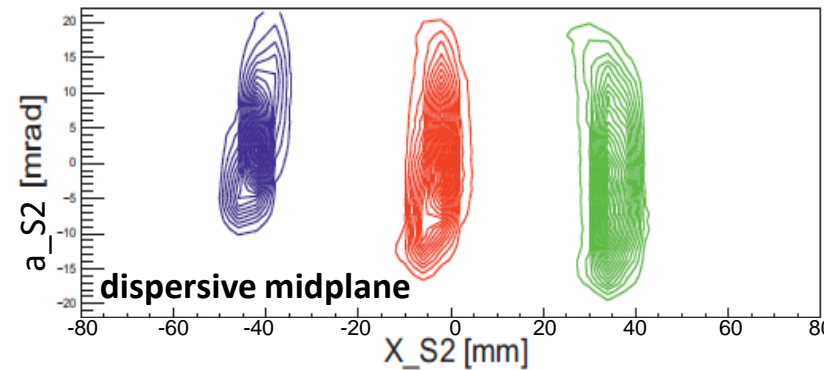
$$+ \underbrace{(x,a\delta) a\delta + (x,x\delta) x\delta}_{\text{chromatic}} + .. + \underbrace{(x,\delta\delta) \delta^2}_{\text{non-lin. dispersion}}$$

$$+ (x,xxx) x^3 + .. + (x,a\delta\delta) a\delta^2 + .. + (x,\delta\delta\delta) \delta^3 + ...$$



high B<sub>p</sub> is focused less -> natural chromaticity

Chromaticity at FRS:  
Wide angle test beam with shifted B<sub>p</sub>,  $\delta = -0.6\%$ ,  $0\%$ ,  $+0.6\%$



# Higher Order Corrections

#3

schematic sextupole

Compensate by non-linear fields at the right positions.  
Use magnets with defined multipole order.

$$B_y(x, y) + iB_x(x, y) = B_0 \sum_{n=1}^{\infty} (b_n + ia_n) \left( \frac{x + iy}{r_0} \right)^{n-1}$$

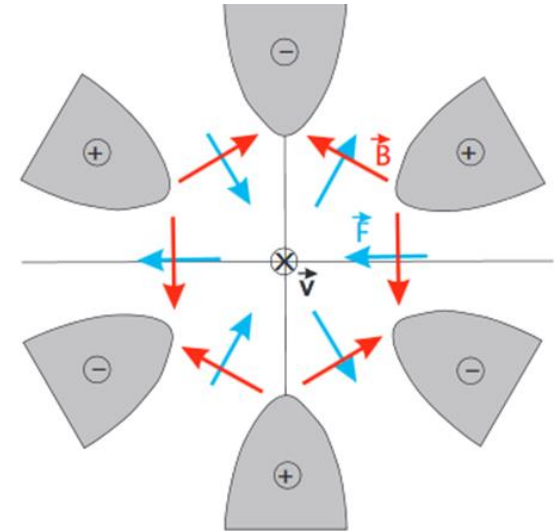
$n=1$ , dipole, 2 poles,  $B_y(x) = B_0 = \text{const.}$

$n=2$ , quadrupole, 4 poles,  $B_y(x) = B_0 b_2 (x/r_0)$

$n=3$ , sextupole, 6 poles,  $B_y(x) = B_0 b_3 (x/r_0)^2$

$n=4$ , octupole, 8 poles,  $B_y(x) = B_0 b_4 (x/r_0)^3$

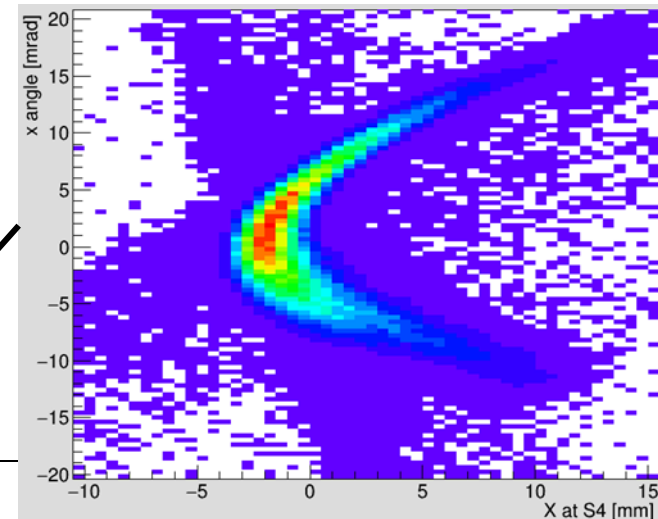
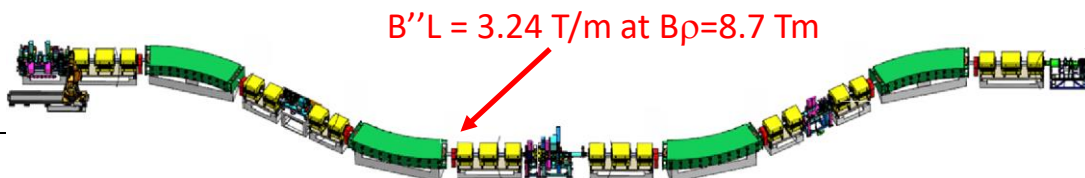
$n$  multipole,  $2n$  poles,  $B_y(x) = B_0 b_n (x/r_0)^{n-1}$



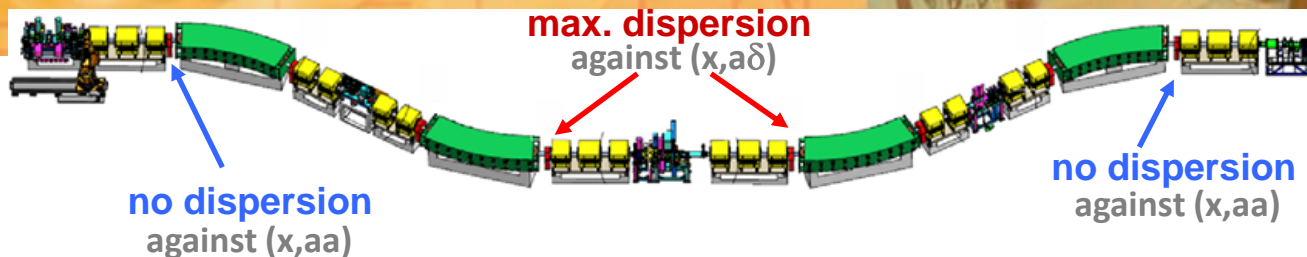
$(a,xx) > 0$  for  $q > 0$  ions

Terms of different orders are linear independent,  
For  $n^{\text{th}}$  order multipole matrix with order  $n < 0$  is like drift,  
 $\Rightarrow$  An  $n^{\text{th}}$  order multipole magnet can only change  
the overall transfer map of order  $\geq n$ .

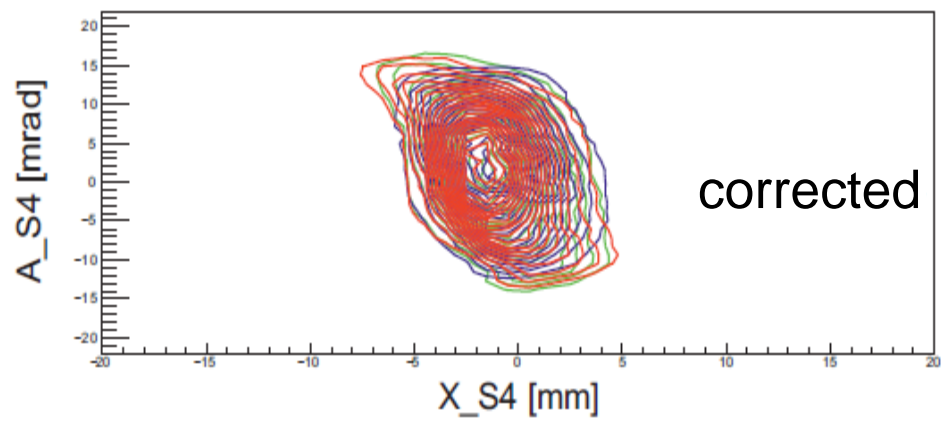
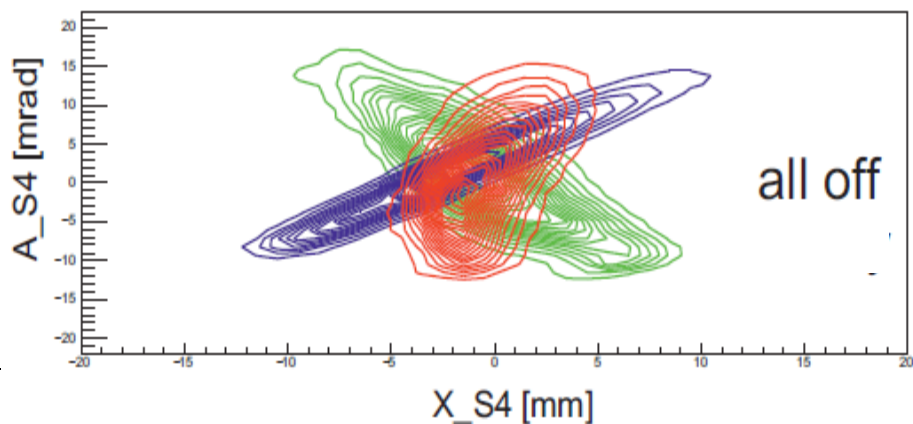
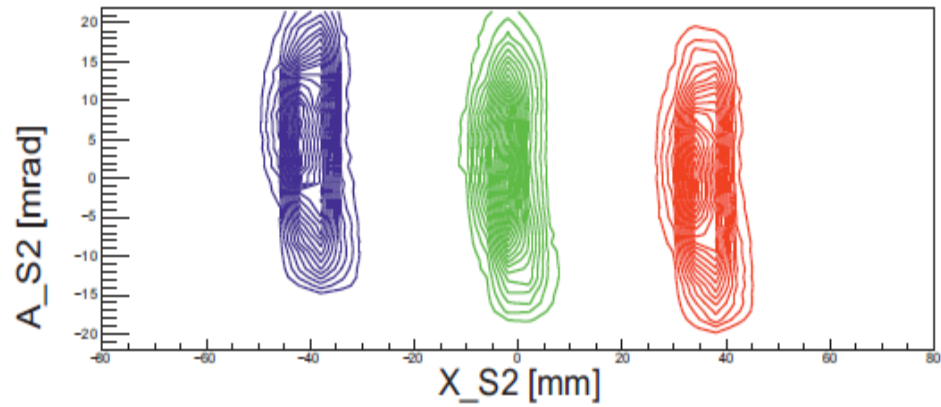
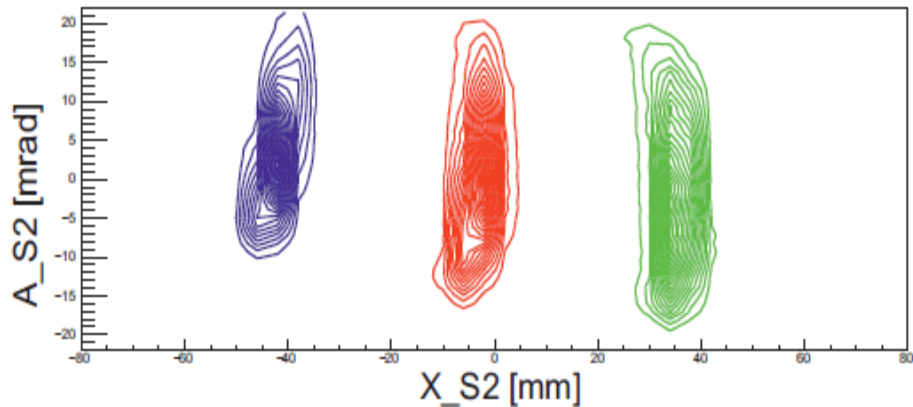
Effect of one excited sextupole in FRS positioned  
where the beam is wide.  $(a,xx)$  of the sextupole  
leads to a parabolic deformation of phase space.



# FRS 2<sup>nd</sup> Order Correction

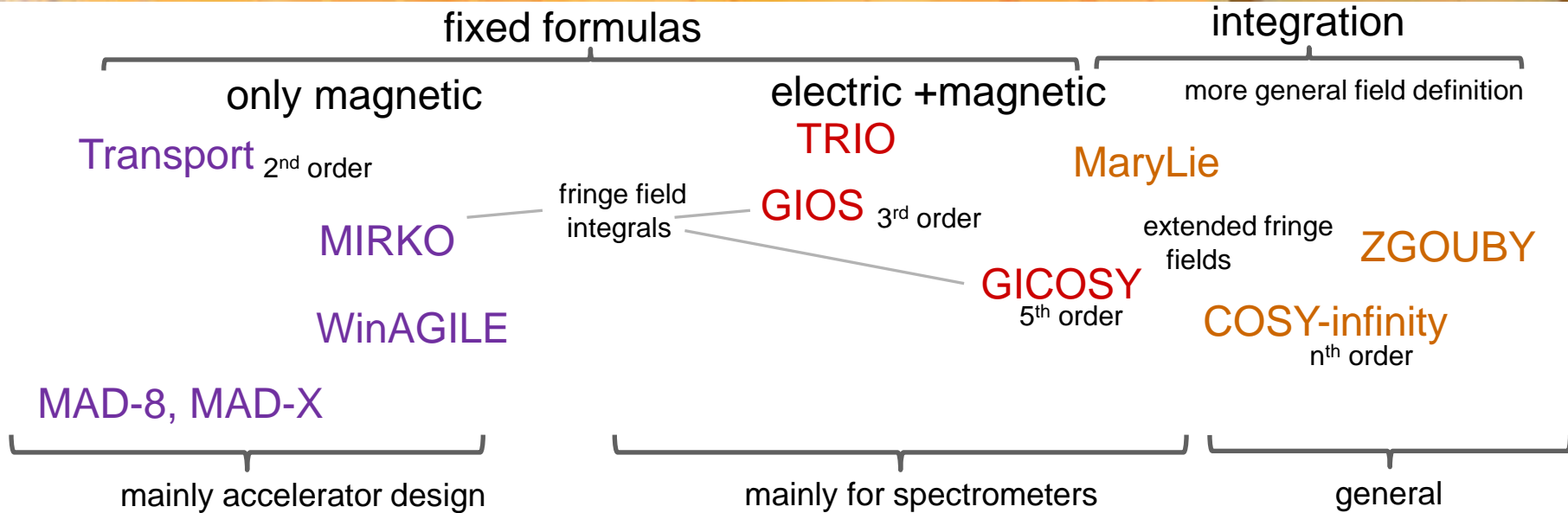


Ta-6045 mg/cm<sup>2</sup> target  
in 600 MeV/u <sup>12</sup>C beam,  
FRS scaled by  $\pm 0.6\%$



# Ion Optics Codes

only the most wide spread



Many add-ons exist:  
e.g. for tracing ions  
in Monte-Carlo mode

PTC, Trace-3D  
Turtle, Raytrace

MOCADI (includes matter in beamline)  
LISE++ (convolution method incl. matter)

Field computation codes  
can trace single ions:

Simion (arbitrary electrode geometry)  
Opera, ANSYS, COMSOL, ...

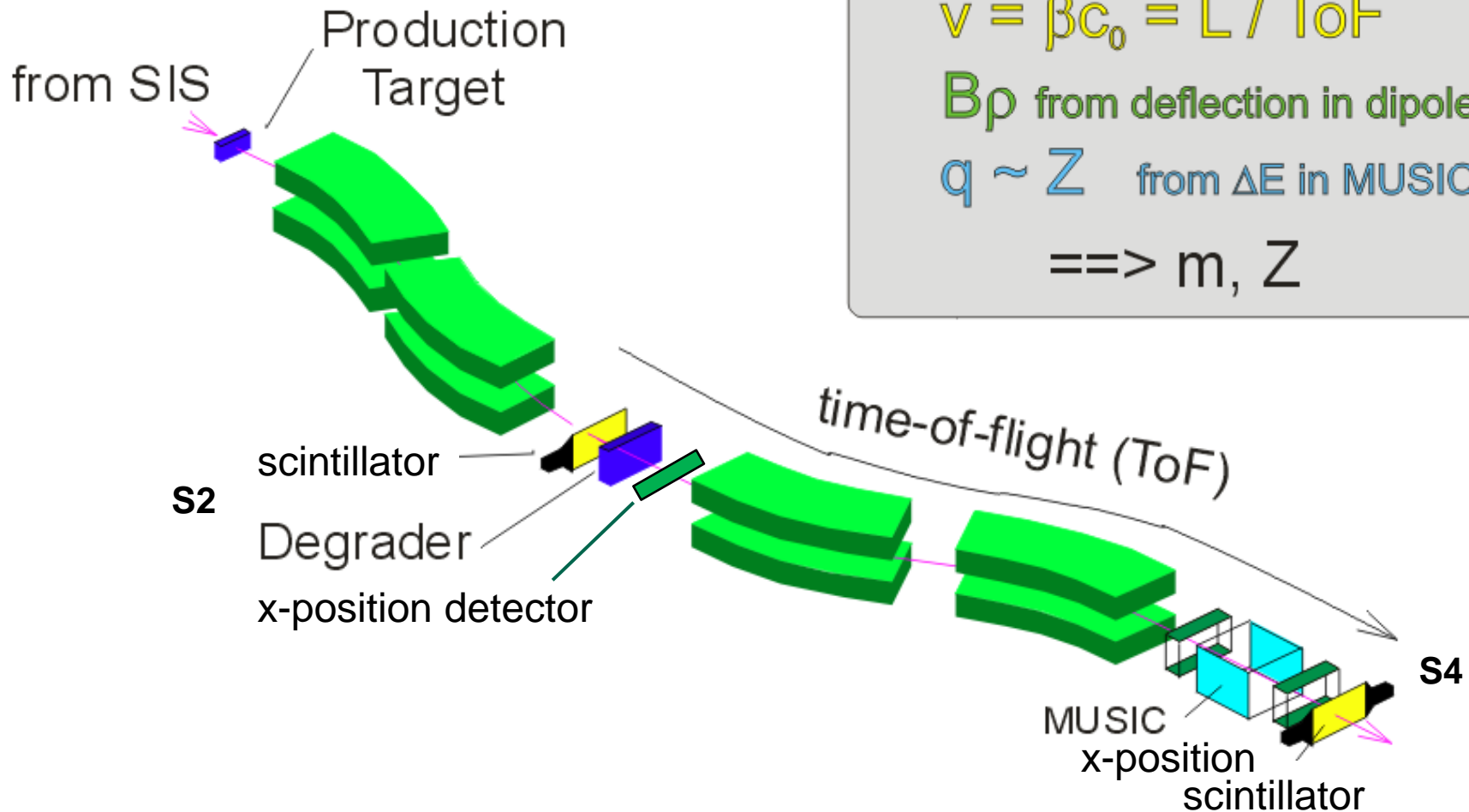
BDSIM, GEANT4beamline and FLUKA focus on interaction





## **4.5. Fragment Identification in Flight**

# Identification In-Flight



$$B\rho = \beta\gamma c_0 m / q$$

$$v = \beta c_0 = L / \text{ToF}$$

$B\rho$  from deflection in dipoles

$q \sim Z$  from  $\Delta E$  in MUSIC

$\implies m, Z$

# Identification In-Flight

$$B\rho = m/q \beta\gamma c_0$$

- $B\rho$  from magnet setting and position detectors at focal planes

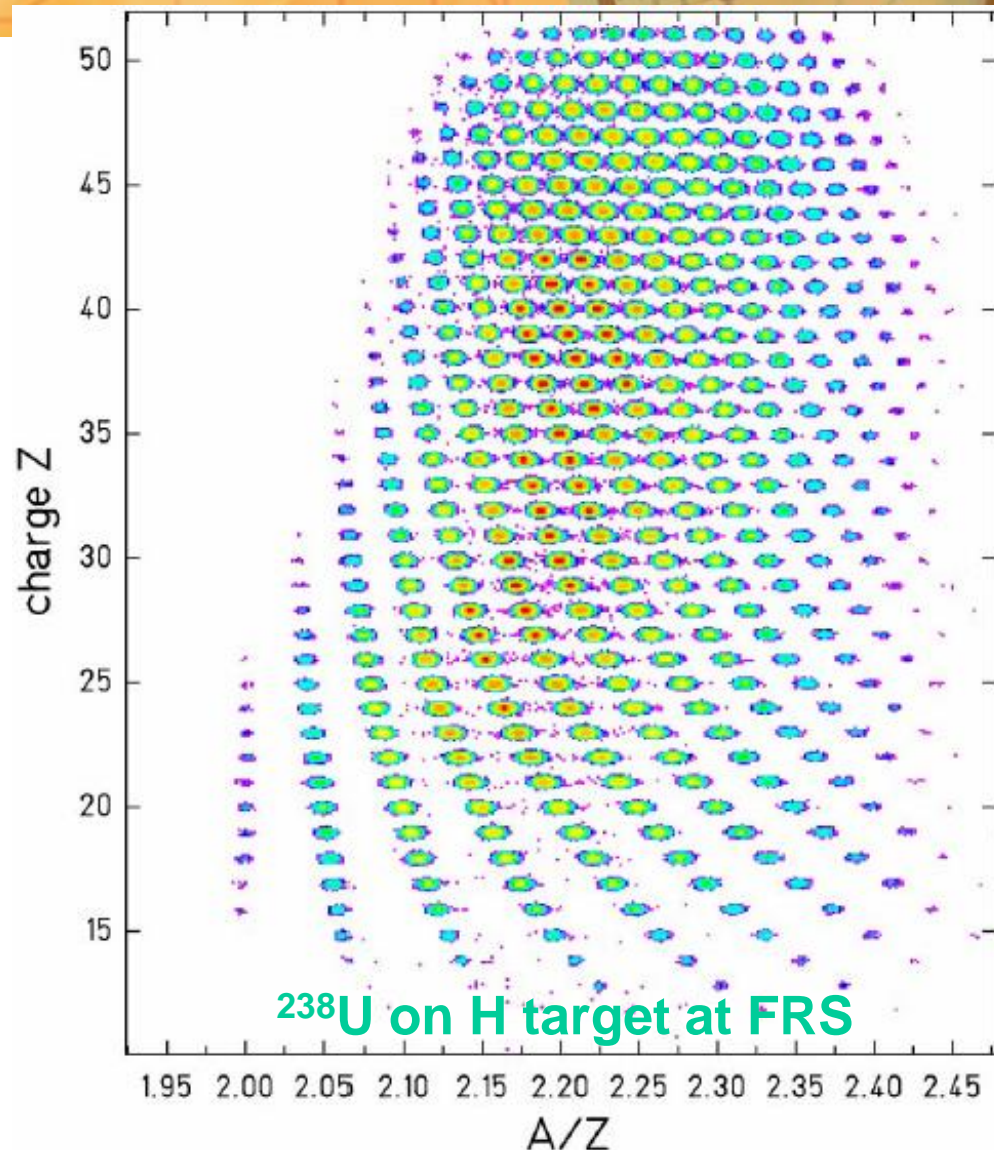
$$B\rho = B\rho_0 \left[ 1 + \frac{x_{S4} - (x|x) x_{S2}}{(x|\delta)} \right]$$

- velocity from ToF between two scintillators at S2 and S4.

$$\beta = L(\alpha, \delta) / \text{ToF} / c_0$$

- MUSIC: average all anodes

$$Z = Z_0 * \sqrt{\frac{\Delta E_{\text{ADC}}}{\text{Polynomial}(\beta)}}$$



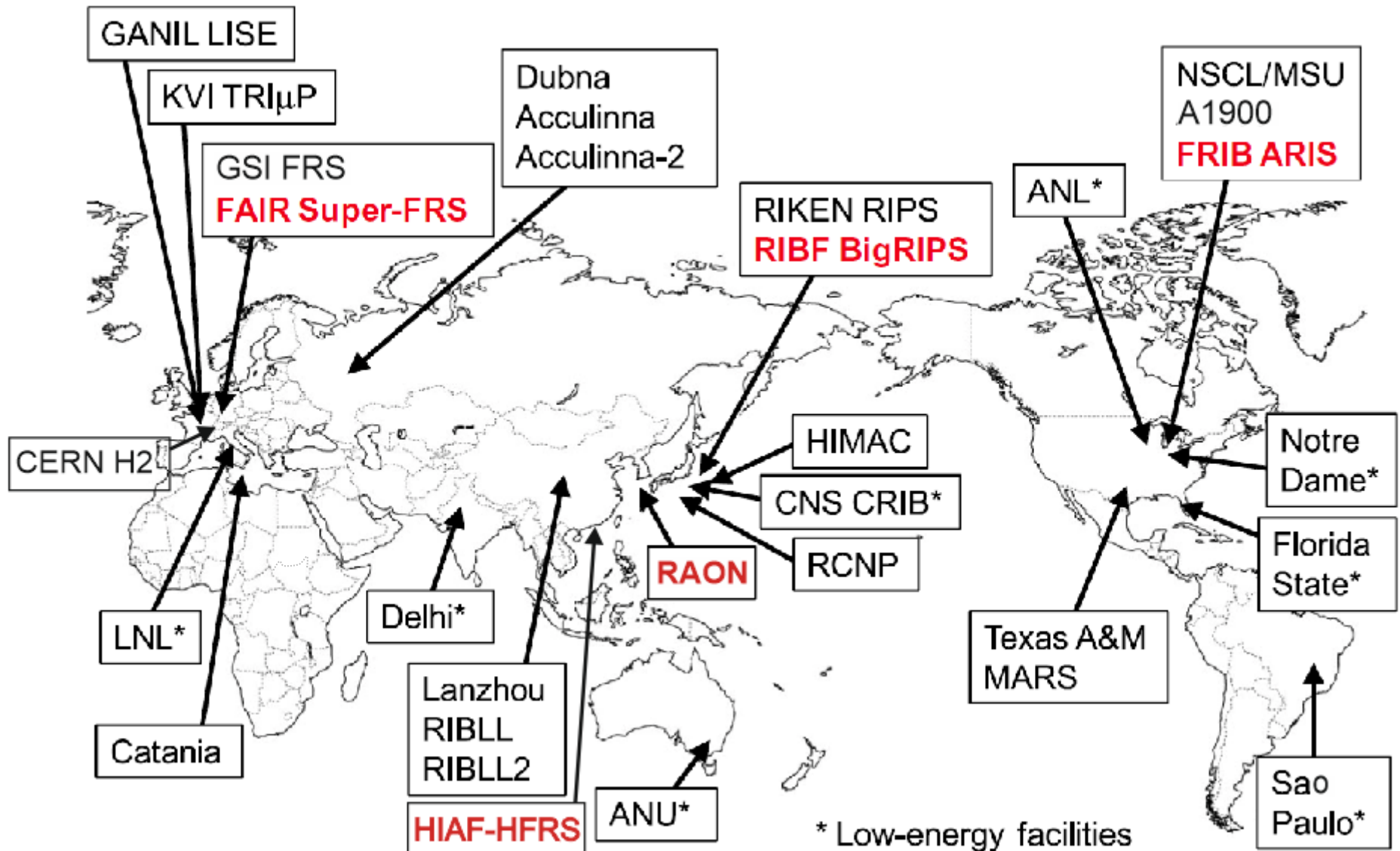


## 5. Modern Fragment Separator Systems

Built for much higher beam intensities.

Selectivity up to of 1 out of  $10^{20}$   
at efficiency  $\sim 50\%$

# Production of RIBs In-Flight



# Super-FRS



Initial acceptance:

$$\Delta a = \pm 40 \text{ mrad}$$

$$\Delta b = \pm 20 \text{ mrad}$$

$$\Delta p/p = \pm 2.5\%$$

$$x_0 = 0.5 - 4 \text{ mm}$$

after thick shaped degraders

$$\Delta a, \Delta b \sim \pm 20 \text{ mrad}, \Delta p/p = \pm 2.5\%$$

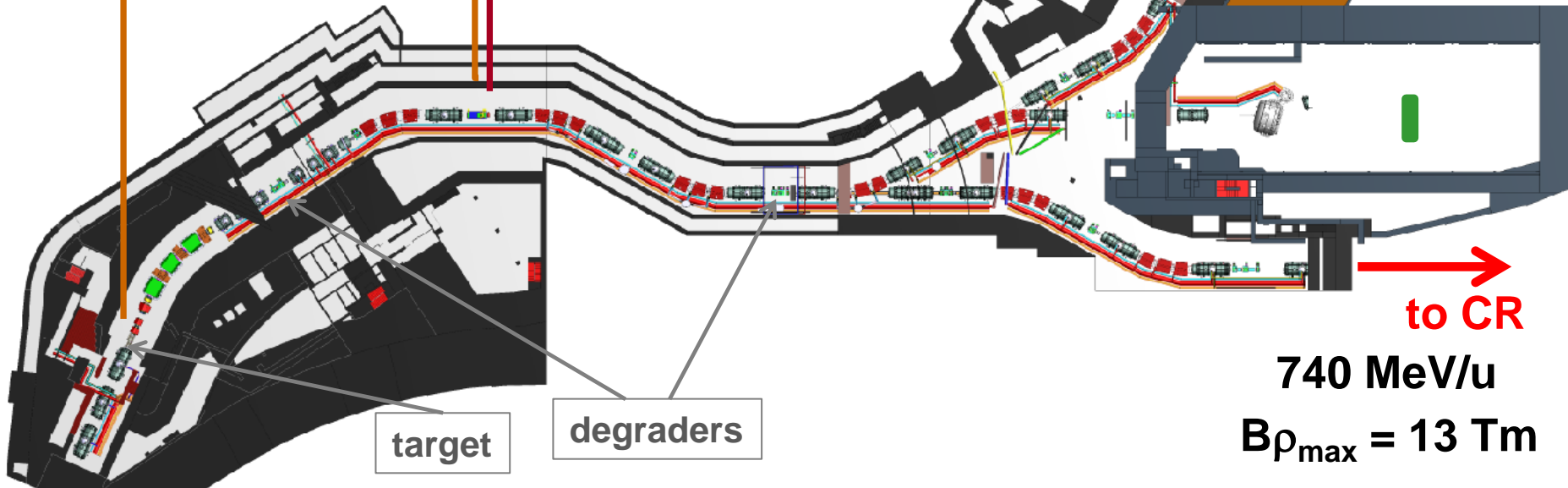
$$\text{but } \varepsilon_{x,y} \sim 100\text{-}300 \text{ mm mrad}$$

$$B\rho_{\text{max}} = 7 \text{ Tm}$$
$$\sim 300 \text{ MeV/u}$$

main sep

$$B\rho_{\text{max}} = 20 \text{ Tm}$$

presep



to CR

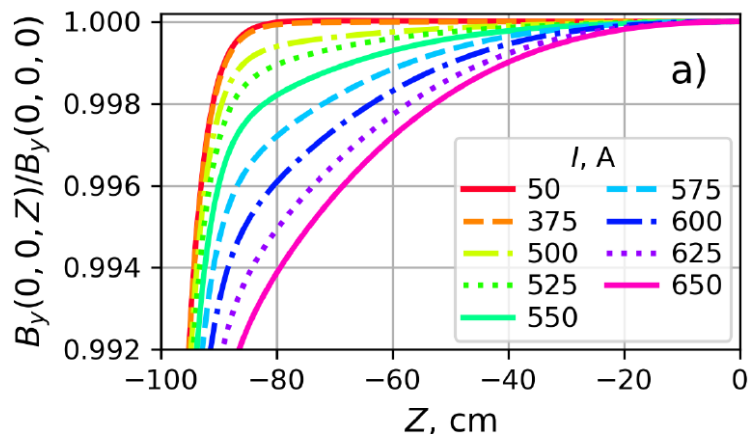
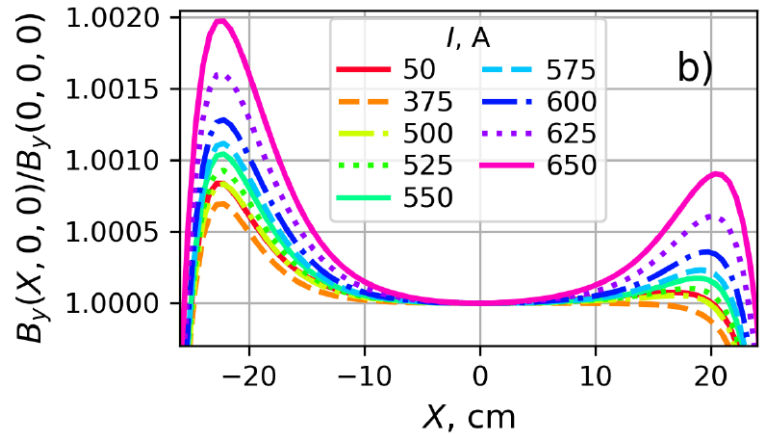
$$740 \text{ MeV/u}$$

$$B\rho_{\text{max}} = 13 \text{ Tm}$$

$$^{238}\text{U}^{28+} \quad 0.4 - 2.7 \text{ GeV/u}$$

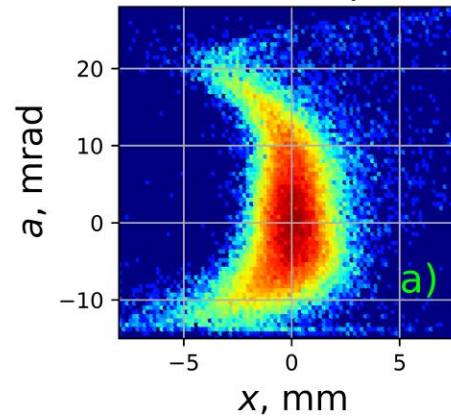
# Super-FRS in Higher Order

field shapes in nc-dipoles  
in reality full 3D distribution

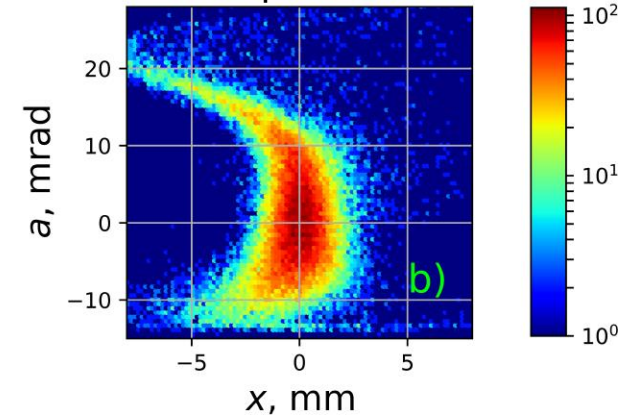


Simulation in COSY infinity 13<sup>th</sup> order with two different descriptions of dipole field. Corrections up to octupoles (3<sup>rd</sup> order).

3<sup>rd</sup> o. least square fit



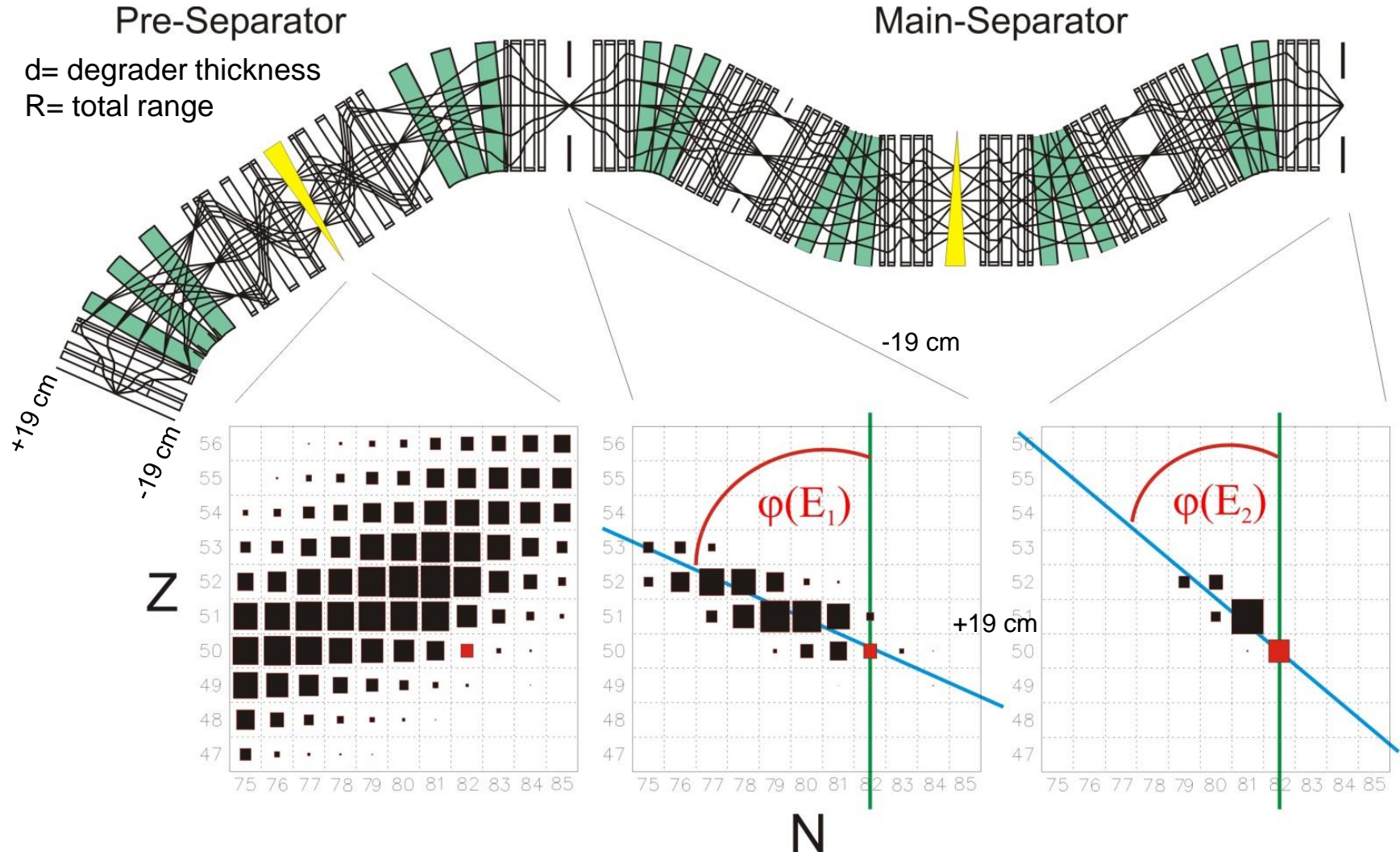
3<sup>rd</sup> o. part of full fit



Different effective field lengths and shapes depending on magnetic field strength, due to iron saturation. No exact prediction possible, but find a scheme for correction.

# Optics of Super-FRS at FAIR

1.1 A GeV  $^{238}\text{U}$  on 4 g/cm<sup>2</sup> C target, two Al degraders d/R=0.3, d/R=0.7



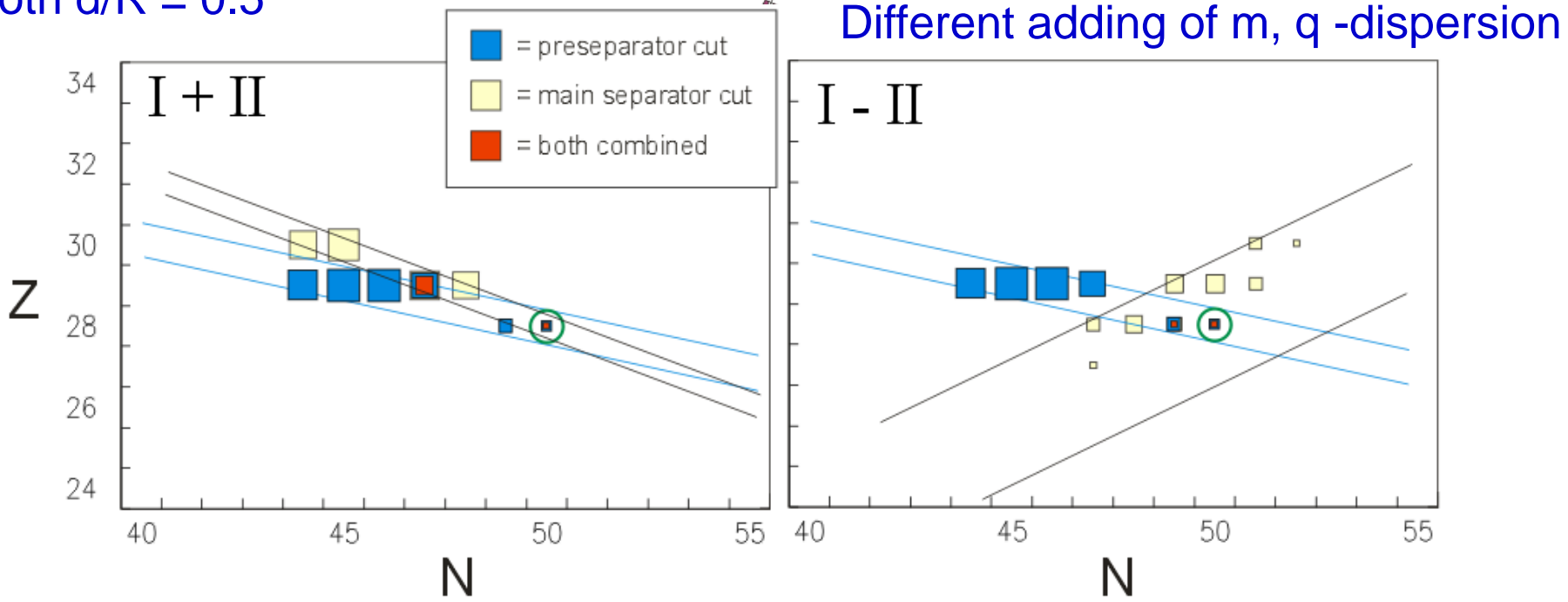
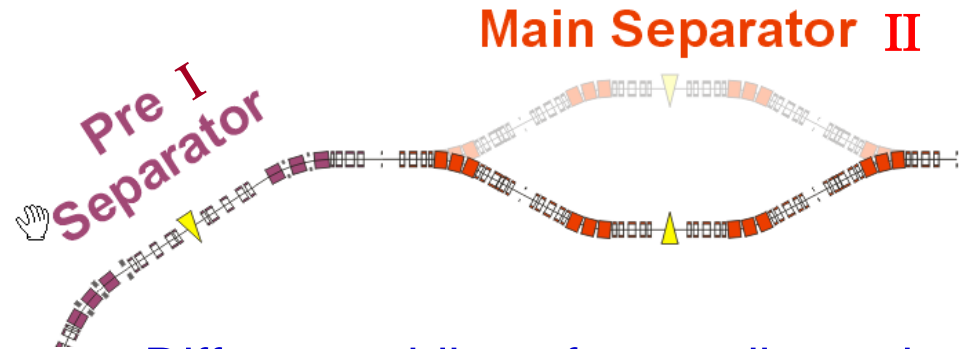
1.1 A GeV  $^{238}\text{U}$  on 4 g/cm<sup>2</sup> C target, two Al degraders d/R=0.3, d/R=0.7  
 For fission fragments separation is difficult, other beams more pure.



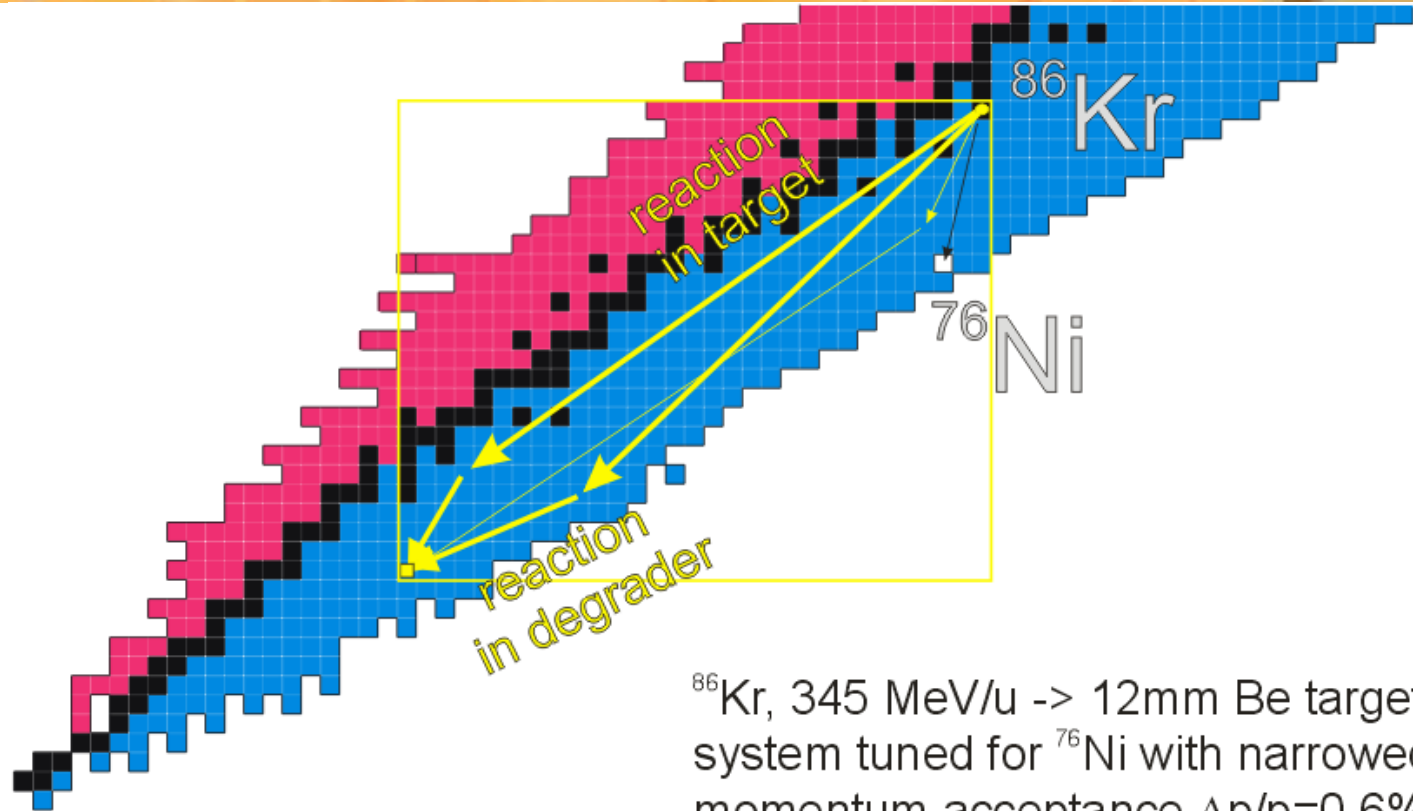
# Coupling of many Stages

Wide momentum spread after reaction, simple  $B\rho$  cut not selective enough.  
What helps? Two achromatic degraders at different energy.

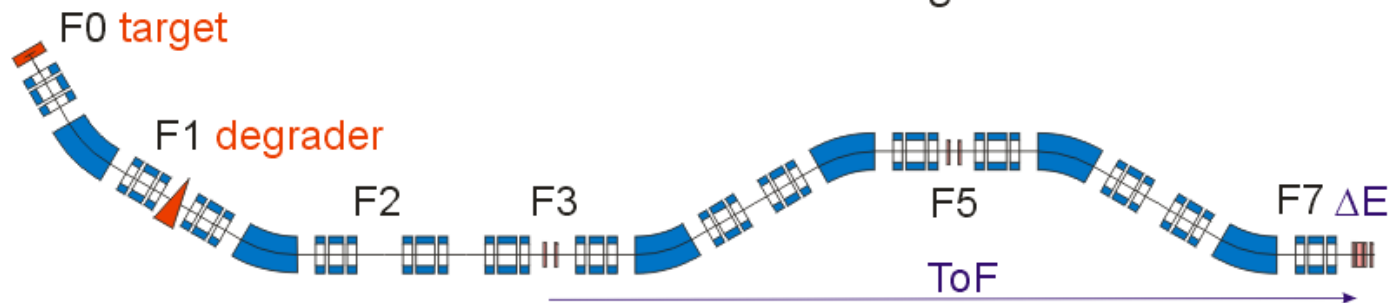
Example  $^{78}\text{Ni}$  with degrader matter distributed to two degraders in pre (I) + main(II) separator, both  $d/R = 0.5$



# Fragments from Degradator BigRIPS Commissioning at RIKEN

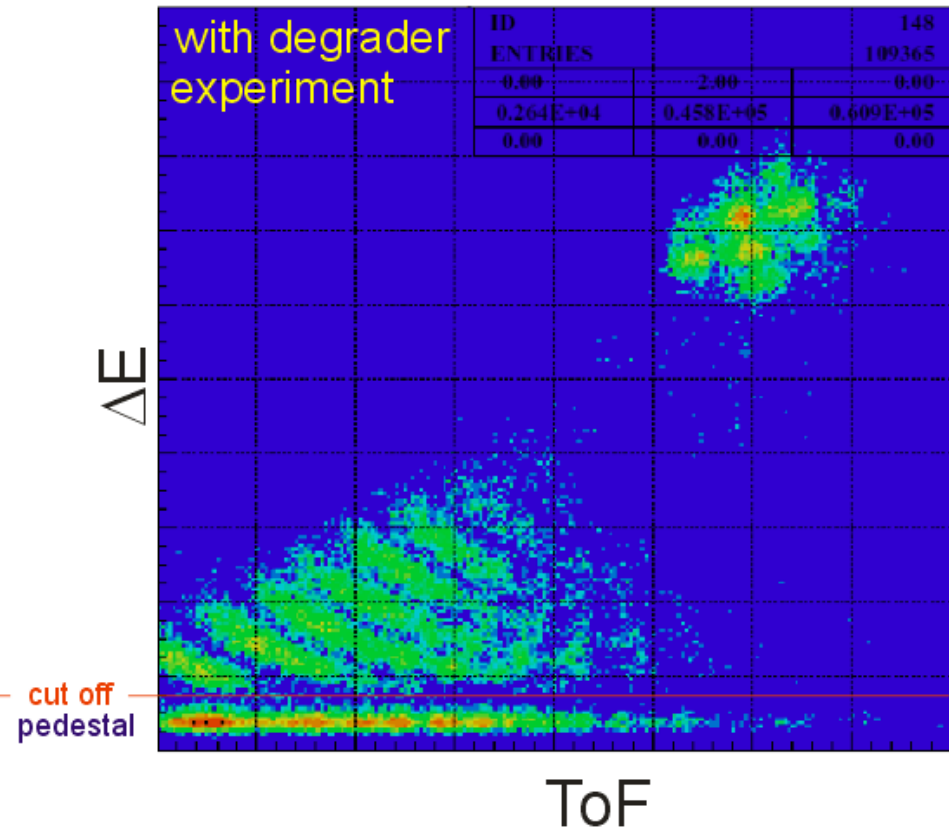
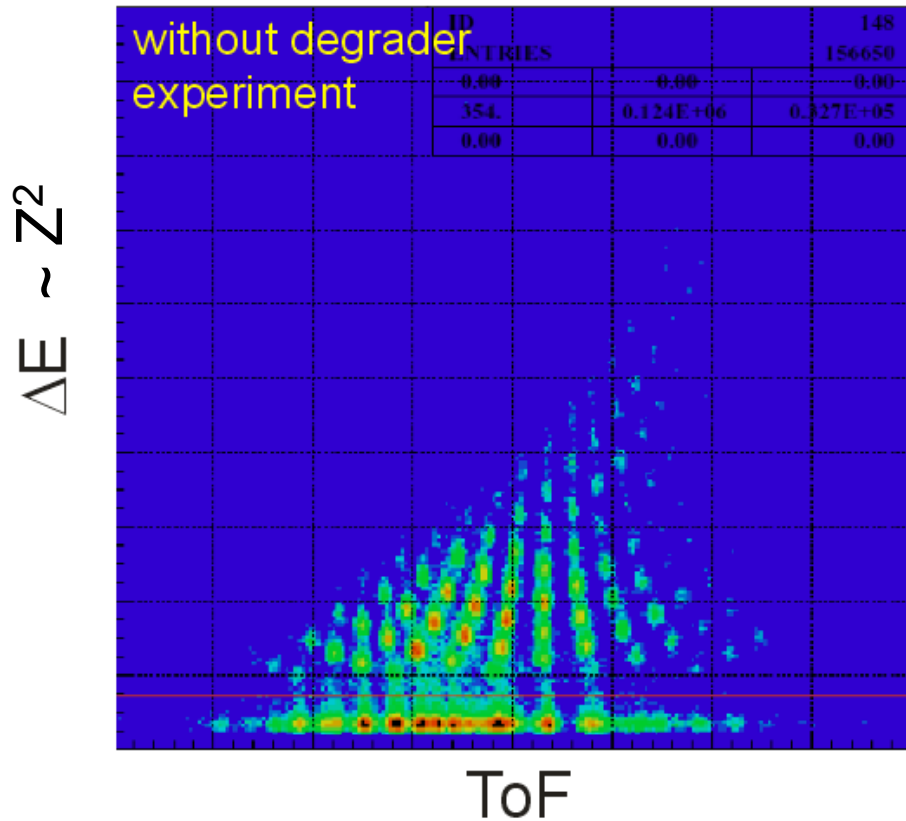


$^{86}\text{Kr}$ , 345 MeV/u  $\rightarrow$  12mm Be target system tuned for  $^{76}\text{Ni}$  with narrowed momentum acceptance  $\Delta p/p=0.6\%$ , a **6mm Al** degrader at F1 was used.



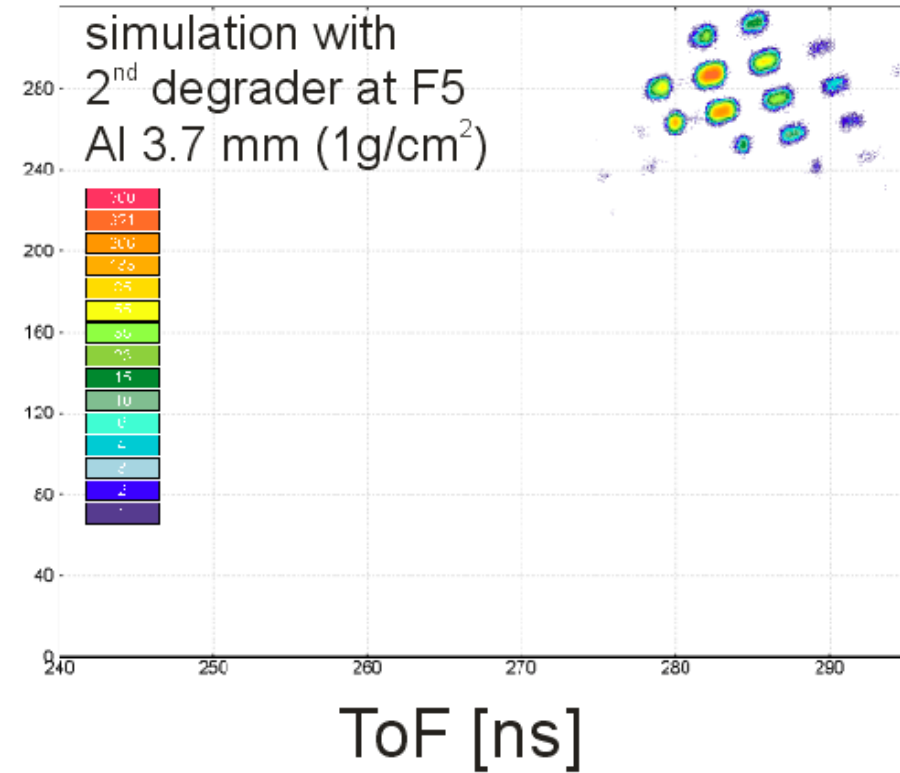
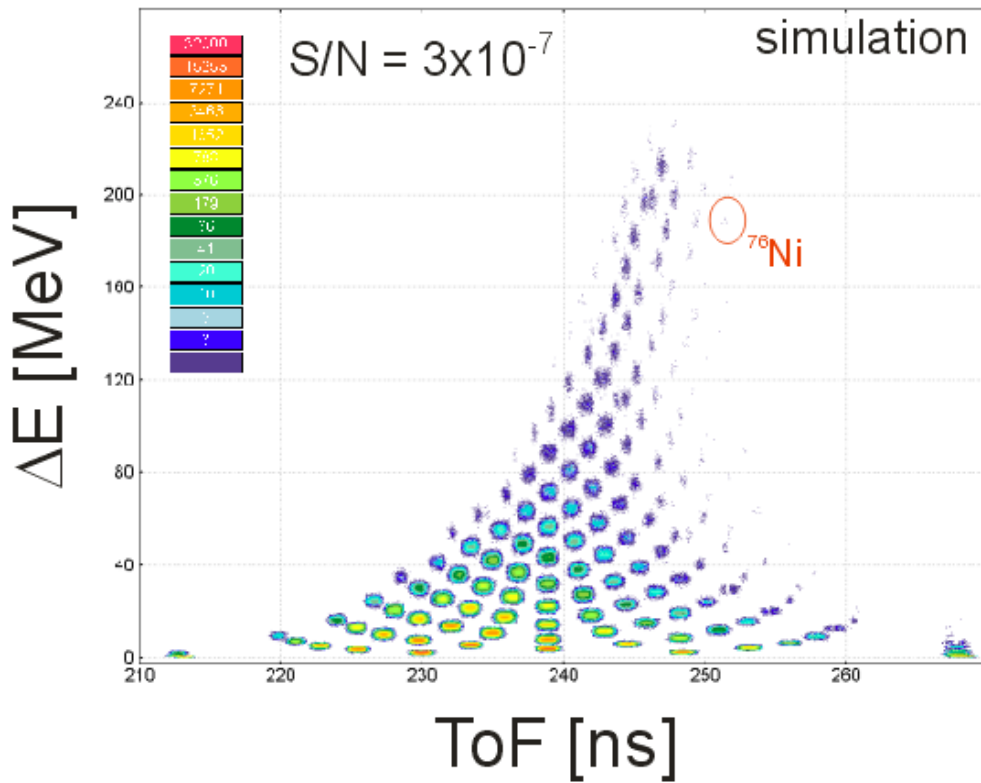
# BigRIPS Commissioning in 2006

## Setting $^{86}\text{Kr} \rightarrow ^{76}\text{Ni}$



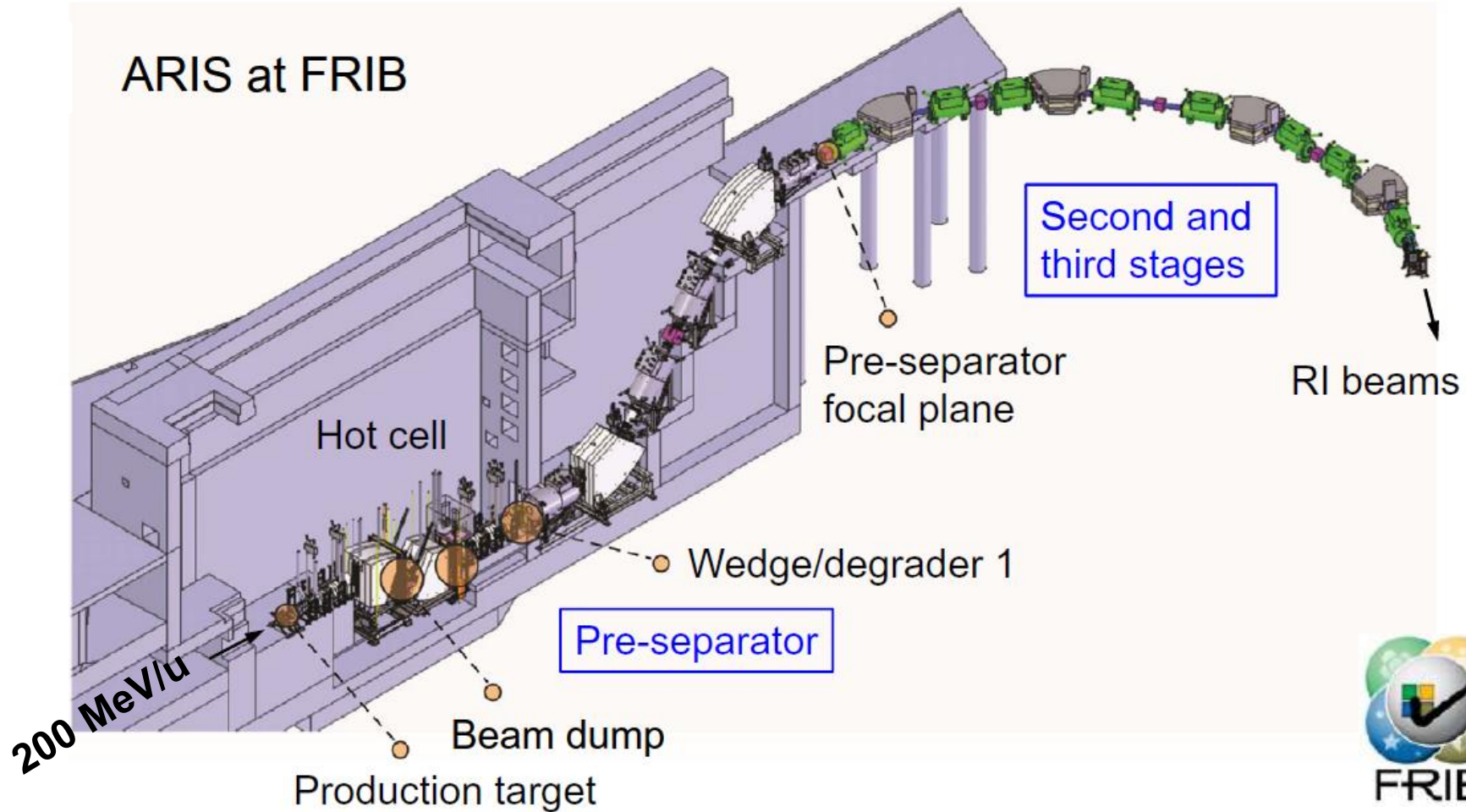
Without degrader  $^{76}\text{Ni}$  region not visible,  
with degrader still lots of lighter fragments.

# BigRIPS $^{86}\text{Kr} \rightarrow ^{76}\text{Ni}$ Simulation with LISE++



Ratio: total rate /  $^{76}\text{Ni}$  rate (=S/N).

# FRIB Separator

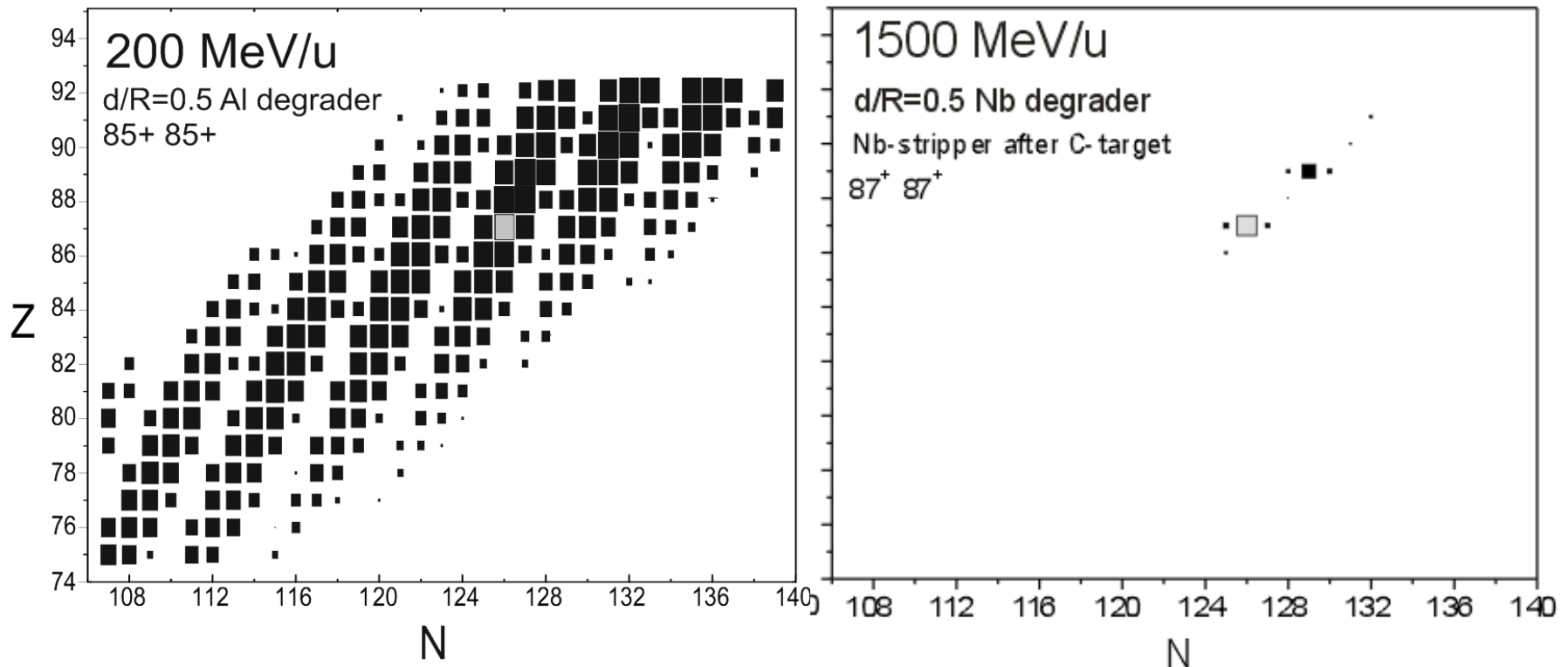


- Asymmetric pre-separator to compress  $B_p$  spread by factor 3
- Pre and main separator in different planes

# Q-State Effect on Separator

## Separation of $^{213}\text{Fr}$

optimum combination of q-states and stripper materials at different energy, FRS with achromatic S2 degrader in LISE<sup>++</sup>



**Change in q in degrader fools the  $B\rho\text{-}\Delta E\text{-}B\rho$  separation**

# Separator Comparison

	stages	[Tm] Brho	in LAB			normalized to $B\rho = 18 \text{ Tm}$ for $m/q = 2.56$		
			[mrad] a	[mrad] b	[%] dp/p	[mrad] a	[mrad] b	[%] dp/p
FRS	1	18	11	17	1.2	11.0	17.0	1.2
A1900-MSU	1	6	30	50	2.9	10.0	16.7	1.7
BigRIPS	2	9	55	40	3.0	27.5	20.0	2.2
Super-FRS	2	20	40	20	2.5	44.4	22.2	
ARIS-FRIB	3	7	40	40	4.5	15.6	15.6	2.9
HFRS	2	12.75-25 limited by BRing	30	25	2.0	21-42	18-35	

Forward focusing (Lorentz transformation)

transverse:  $a = p_x / p_0$

longitudinal:  $\Delta p_z / p = \gamma p_z / p_0$

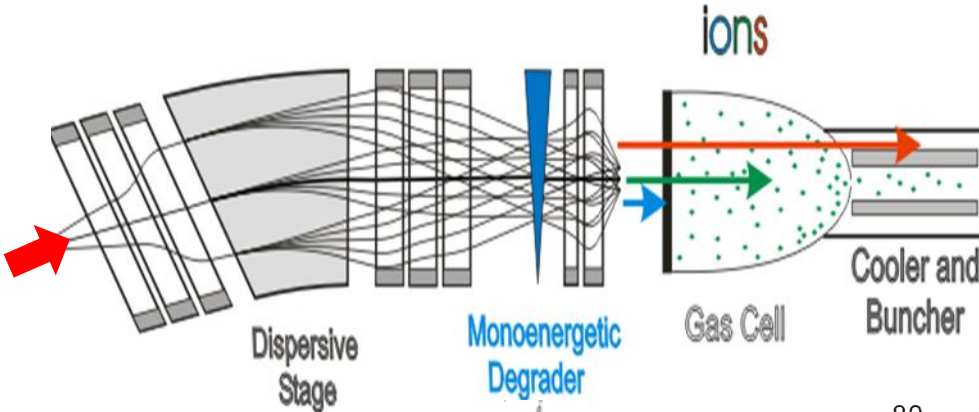
Resolution limit by energy-loss straggling in half range thickness degrader,  $R \sim 800$ .

One can also use large area dipoles for high-resolution physics experiments, spectrometer for secondary beam.

	[mm] $x_0$	$R = \frac{(x \delta)}{2(x x)x_0}$	[m <sup>2</sup> ] dipole area*
FRS	1.2	3385	2.36
A1900-MSU	1.0	1480	0.68
BigRIPS	0.5	2850	1.28
Super-FRS	1.0	2900	4.08
ARIS-FRIB	0.5	1300	0.96
HFRS	1.0	1100	

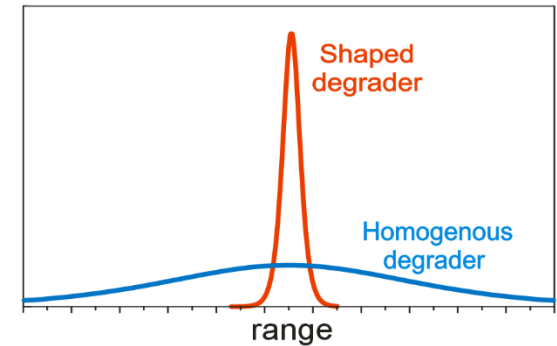
\* for 2 dipole stages

# Range Bunching Monoenergetic Degradator



$^{56}\text{Ni}$  at 360 MeV/u  
 $\Delta p/p = \pm 0.45\%$

$A\alpha$



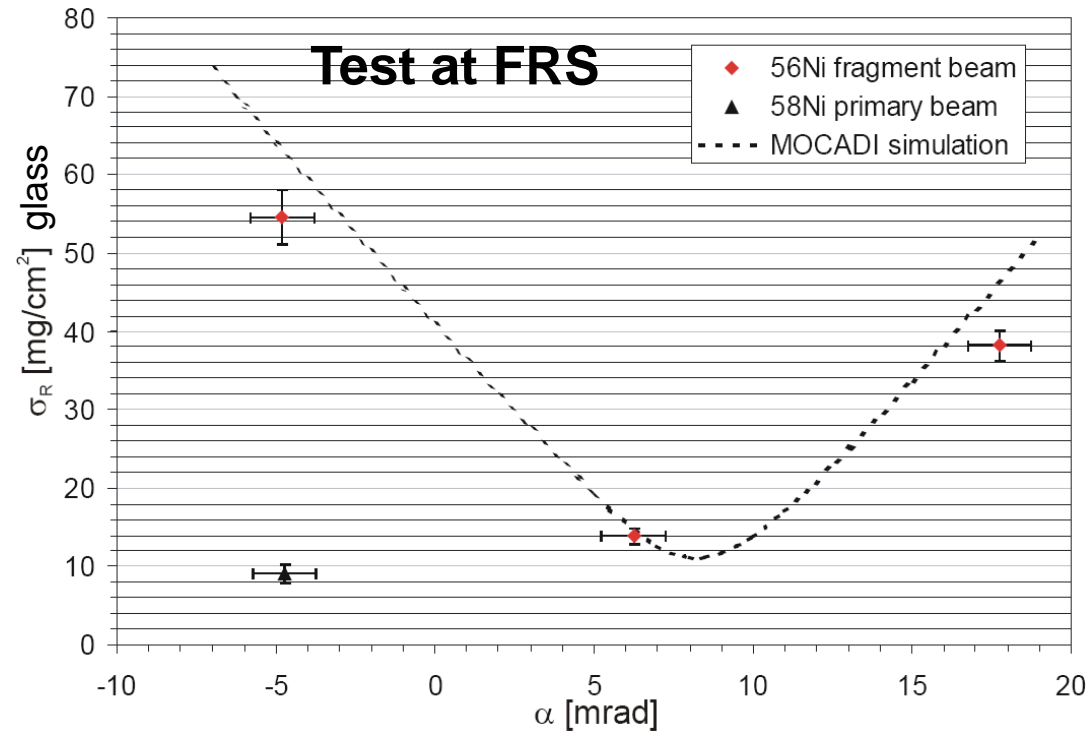
Limit by energy-loss straggling

$$\sigma_R = 9.2 \text{ mg/cm}^2 \text{ Al}$$

a typical sheet of paper is 8 mg/cm<sup>2</sup>

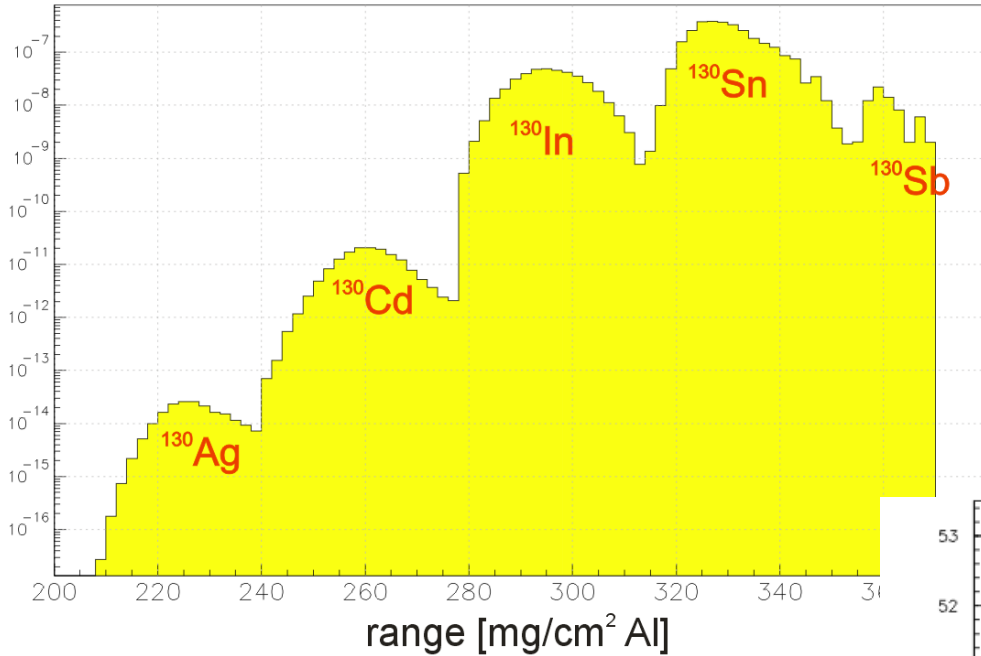
momentum compression

$$\sigma_p/p < 5 \times 10^{-4}$$





# Separation in Range

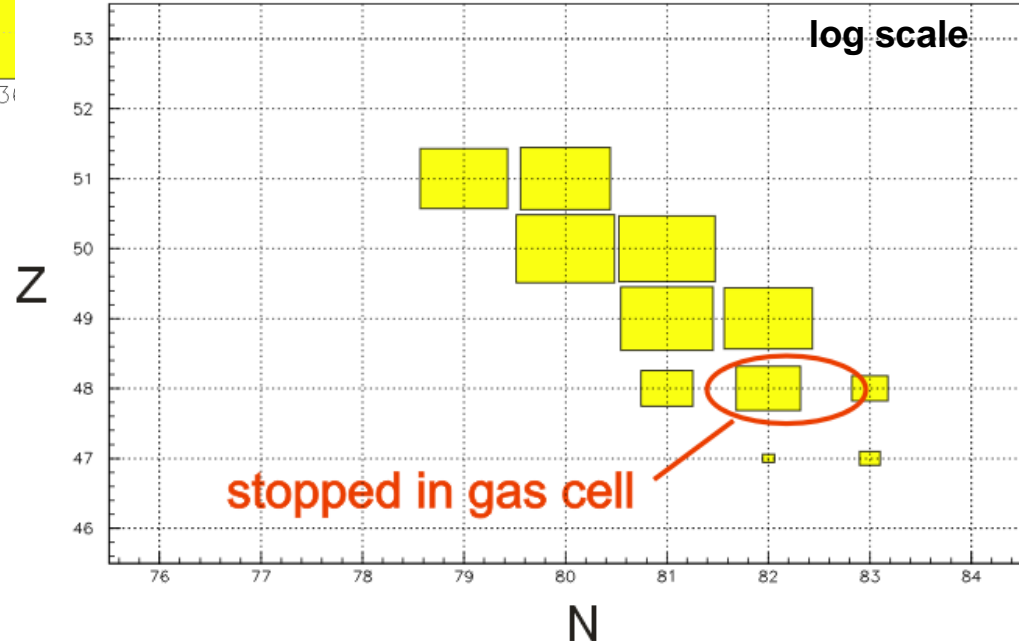


Separation of isobars in range for stopping in gas cell.

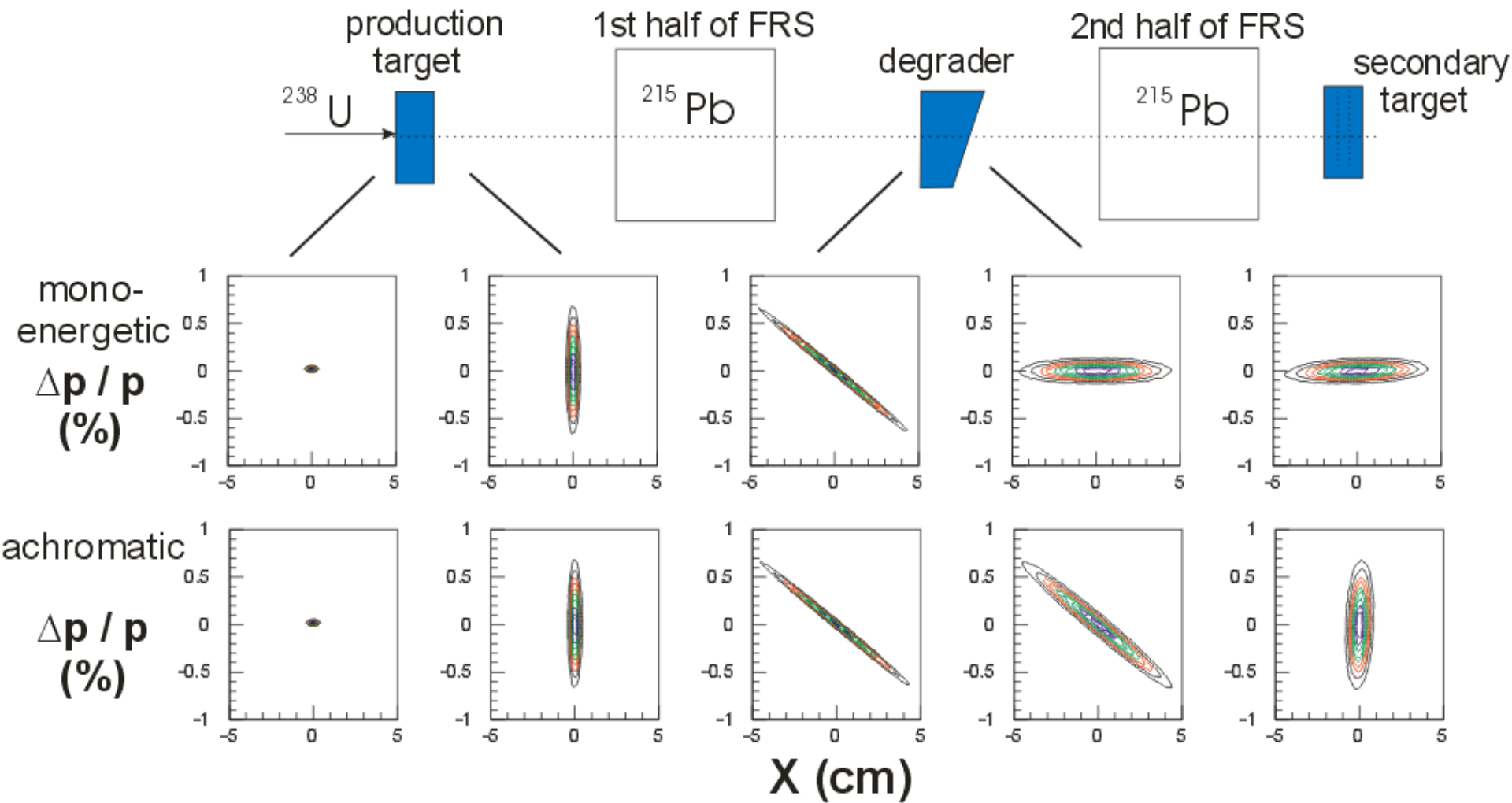
$$\sigma_R (^{130}\text{Cd}) = 7.5 \text{ mg/cm}^2$$

Separation in range also not hindered by transverse emittance increase.

MOCADI simulation with extra stage behind Super-FRS.



# Influence of Degradator Shape



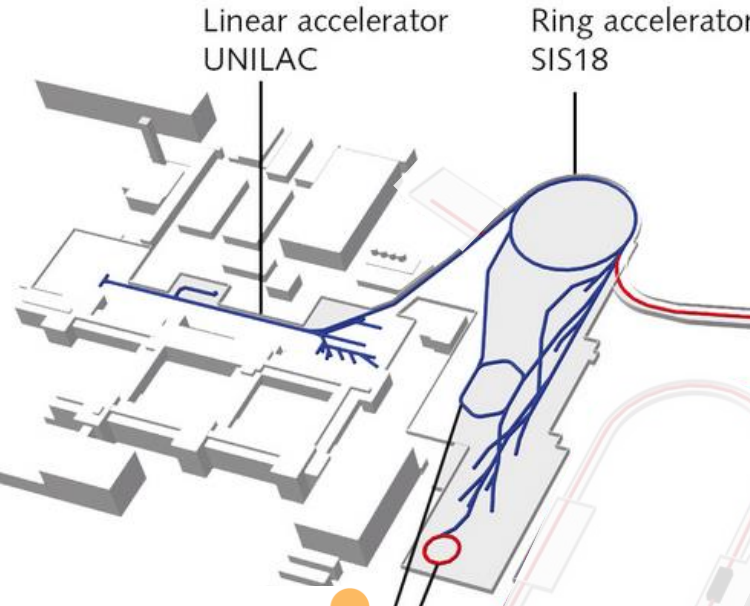


## **6. FAIR Status**

# FAIR - Facility for Antiproton and Ion Research



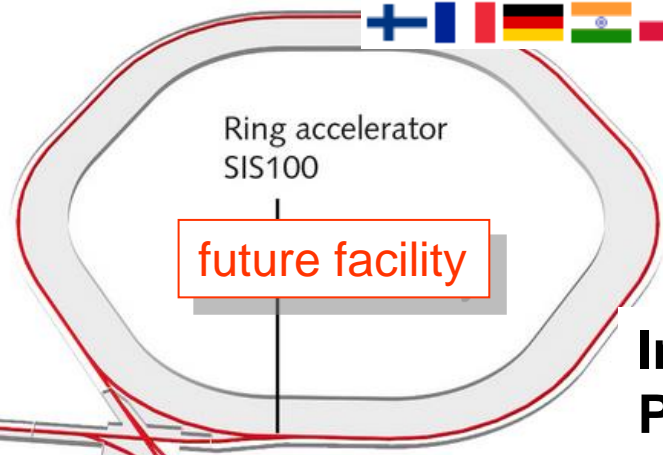
GSI today



100 metres

**GSI**

Experimental and storage ring



**FAIR**  
FAIR GmbH

International Partners 8+2,  
Germany >50%

beams with energies of  
11 GeV/u  $^{238}\text{U}$ , 27 GeV p  
high intensity up to  
 $5 \times 10^{11}$  U Ions/pulse  
28.5 kJ per pulse

Experiments on:  
nuclear structure, astro physics  
hadron physics, compressed  
nuclear matter, plasma physics  
atomic physics, material science,  
bio physics

# FAIR civil construction



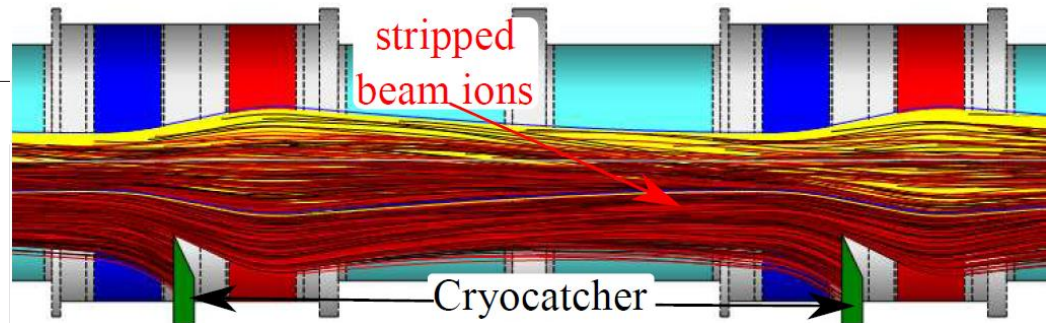
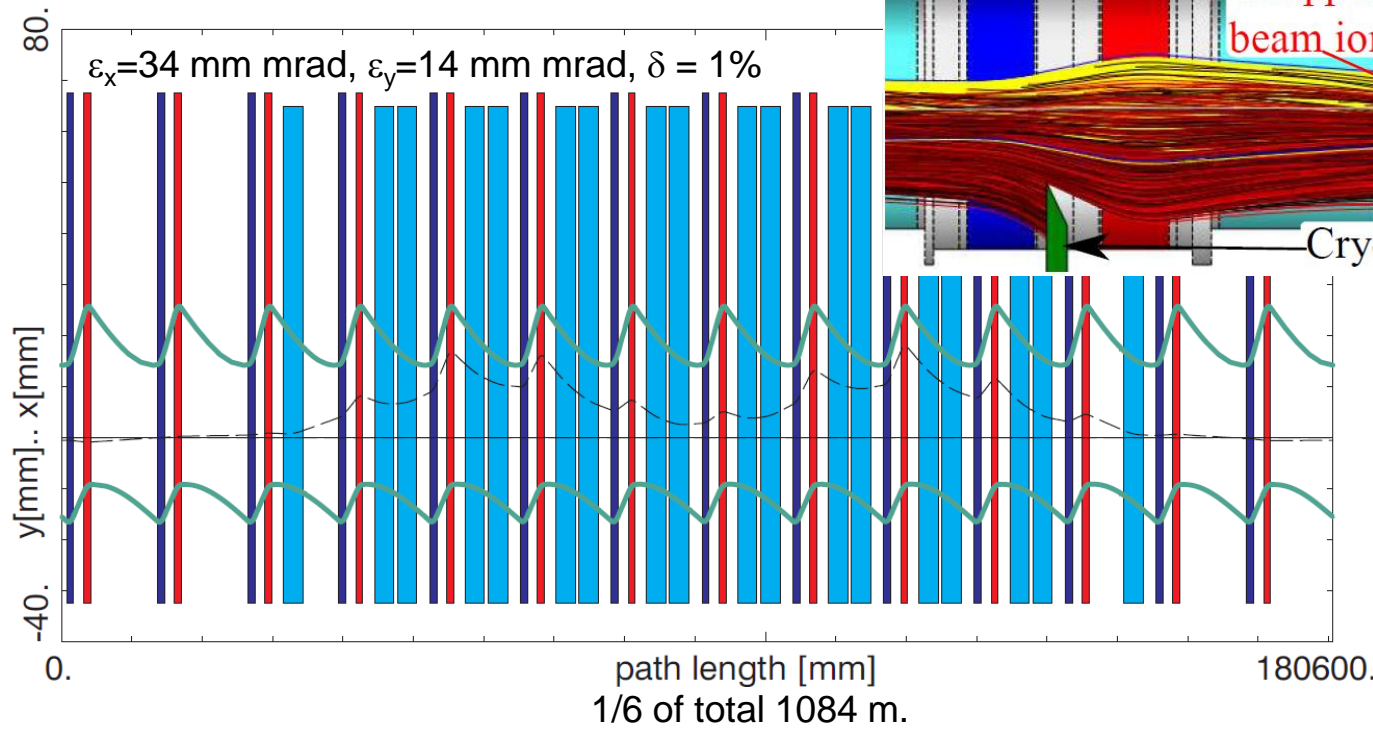
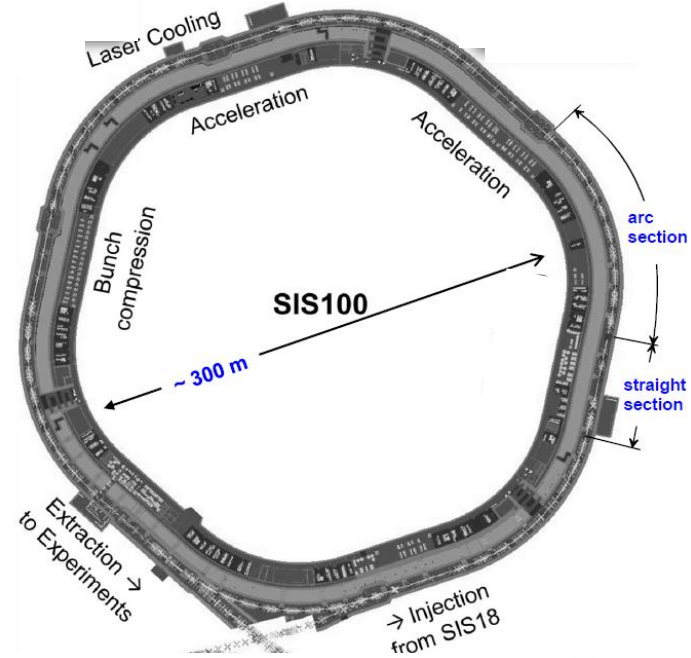
April 2024

# SIS-100

Ion synchrotron for p - U

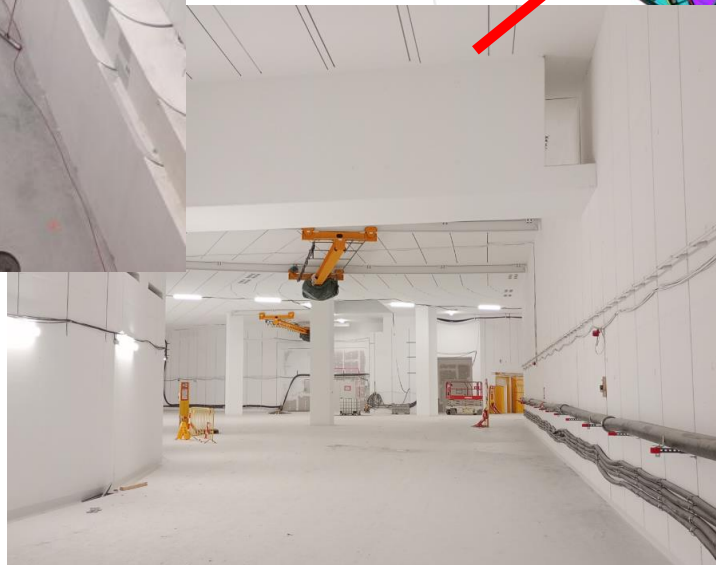
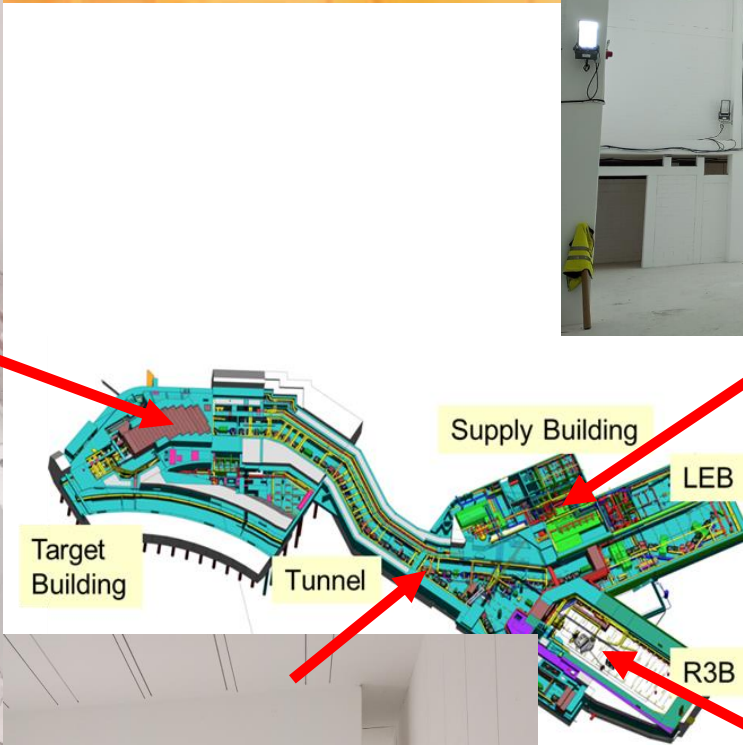
$$B\rho_{\max} = 100 \text{ Tm}$$

Accelerate  $U^{28+}$  to avoid space charge and one stripper. UHV in danger by beam losses. Cryogenic catchers for ions changing charge. FODO cells but asymmetric.



a little bit of separator

# Building Construction Super-FRS



# FAIR Magnets

external storage hall, beamline magnets



SIS100 sc magnets



big rad. hard dipole 95 t

sc multiplets  
for SuperFRS



BE42 storage at GSI

in SIS100 tunnel



Pre-assembly place at GSI







**The END**



## 7. Simulation

# Input for Simulation

nuclear physics:

reaction kinematics  
production cross sections  
nuclear absorption

**codes:**

**MOCADI**  
**LISE++**

ion optics:

ion optics of the separator

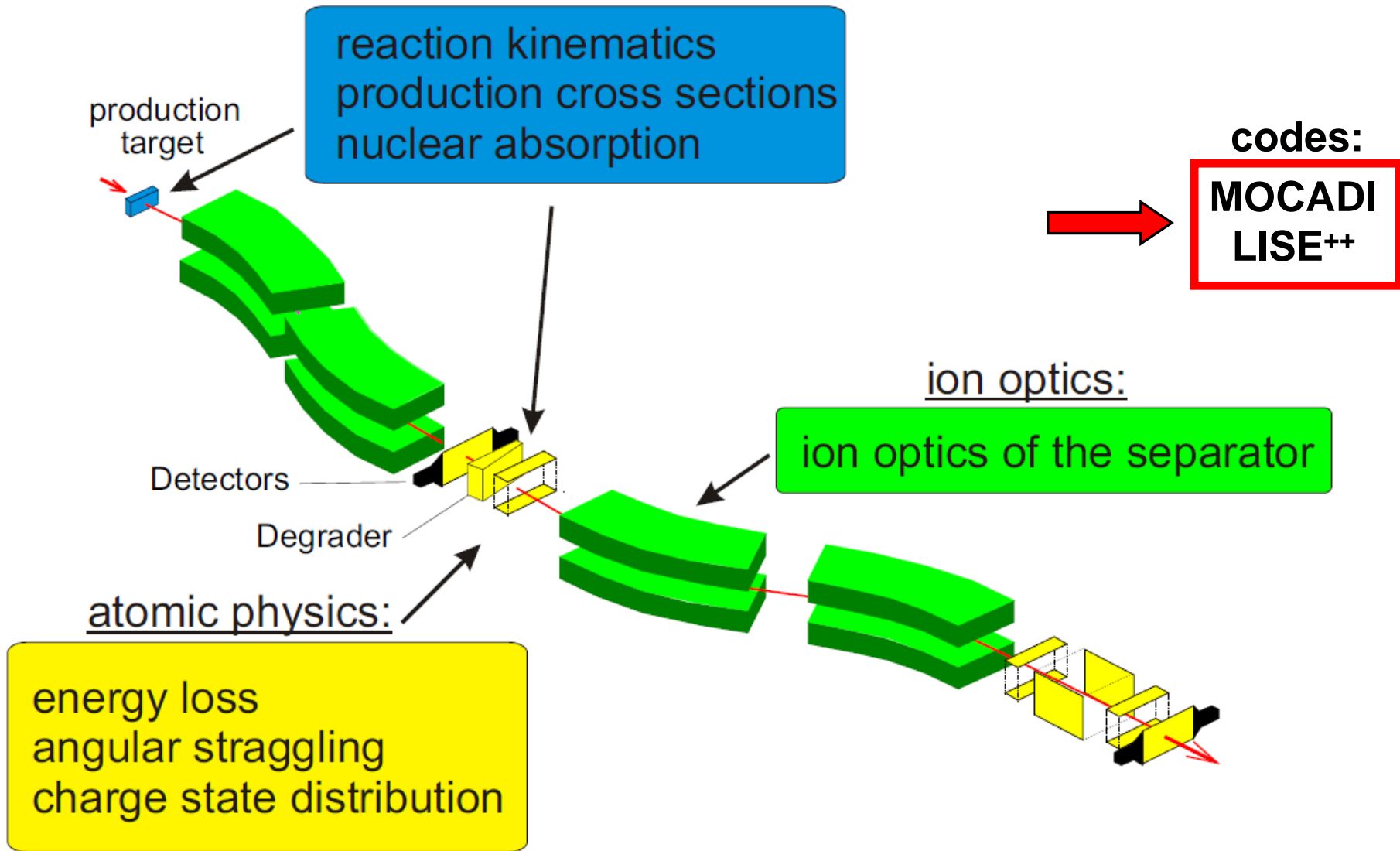
atomic physics:

energy loss  
angular straggling  
charge state distribution

production target

Detectors

Degrader



# Monte Carlo

## MOCADI / LISE<sup>++</sup> MC

**MOCADI/LISE<sup>++</sup> MC are optimized for beamlines with matter:**  
ion always fly in forward direction, no multiplicity,  
physics routines adjusted for beamline needs,  
GEANT would be orders of magnitude slower and  
more complicated to setup.

### **Speed:**

Do not evaluate physics for each ion,  
use parametrizations (Goldhaber, Morrissey, EPAX)  
and precalculated results (ATIMA spline tables for energy loss),  
optics parametrized by transfer matrices, no magnetic fields.

### **Biassing:**

$10^{10}$  ions/s like in reality are impossible for MC,  
Do not create fragments with probability like in reality.  
Calculate a certain statistics for one nuclide and biassing  
is done by the very different production cross sections.

# Example MOCADI

**$B\rho$  distribution after target**  
 $^{68}\text{Ni}$  fragments from  $^{86}\text{Kr}$

PAW: nt/plot 1.brho(1)  
 Root: T->Draw("brho[0]")

## list of variables in output

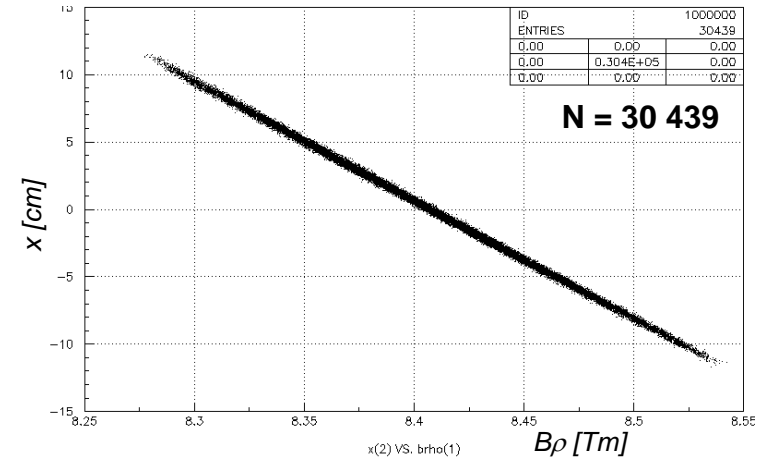
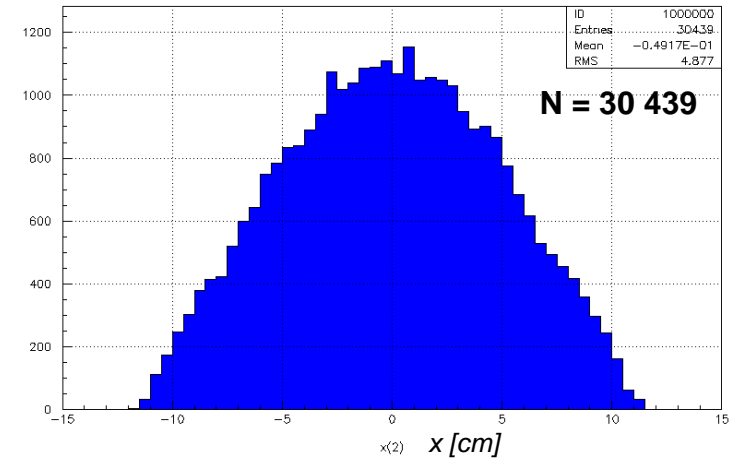
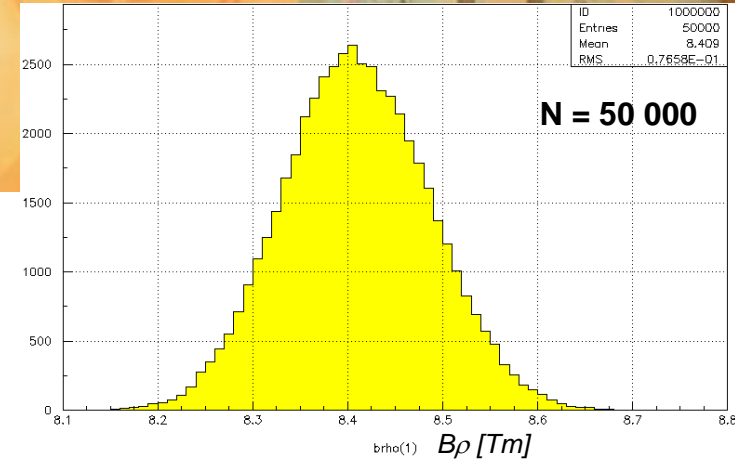
N  
 X(N)  
 A(N)  
 Y(N)  
 B(N)  
 ENERGY(N)  
 TIME(N)  
 MASS(N)  
 Z(N)  
 ELNUM(N)  
 TOF(N)  
 DE(N)  
 BRHO(N)  
 WEIGHT(N)  
 RANGE  
 tpos

**x position at dispersive focal plane S2**

nt/plot 1.x(2)  
 T->Draw("x[1]")

**Correlation**

nt/plot 1.x(2)%brho(1)  
 T->Draw("x[1]:brho[0]")

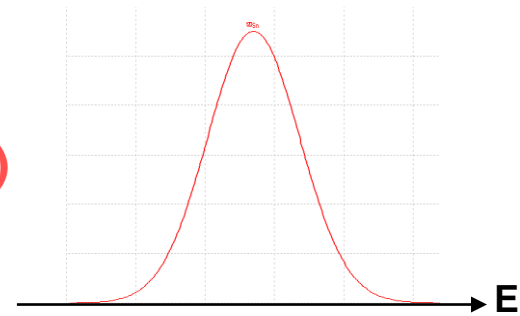


# Convolution Technique

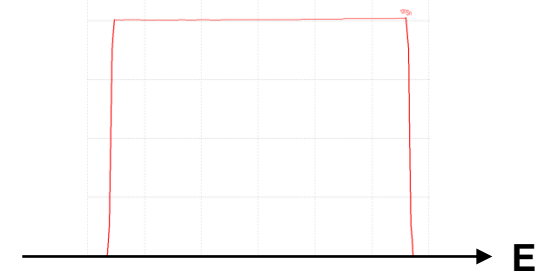
Each target, each piece of matter in beamline, each collimator reshapes the distributions in position, angle, energy of an ion species

Example:

Energy distribution after thin target,  
Goldhaber distribution, Gaussian shape,  $f(E)$



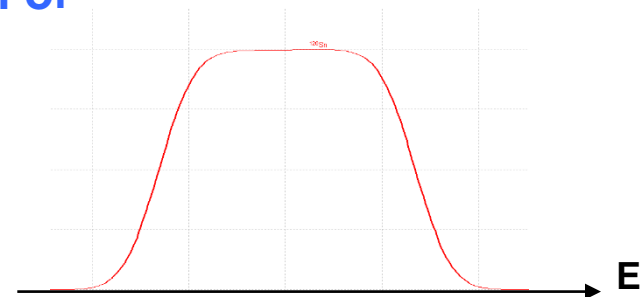
Energy difference by slowing down in thick target,  
difference in  $dE/dx$  for projectile and fragment  
depending on  $Z_p$  vs.  $Z_f$  and reaction position,  $g(E)$



Combined effect given by the convolution of  
the two distributions

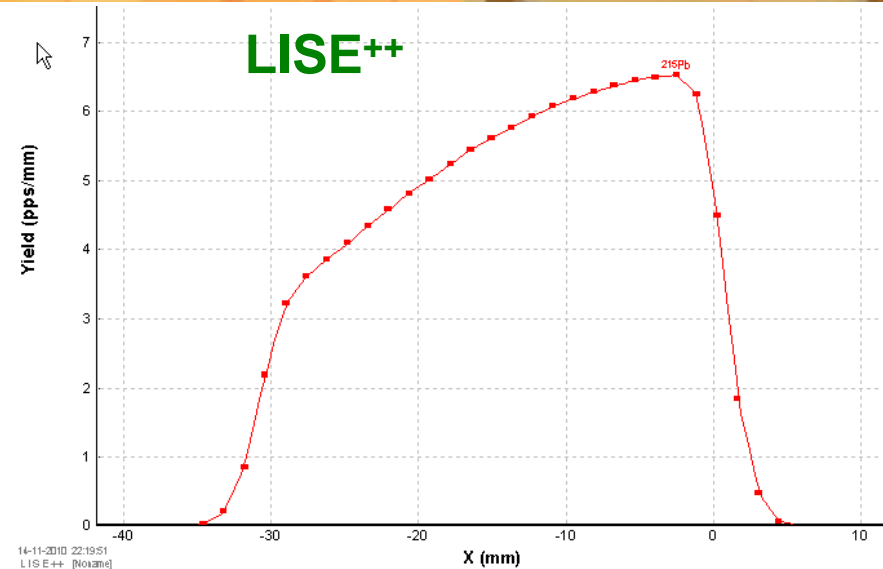
$$h(E) = (g * f)(E)$$

$$= \int_{-\infty}^{\infty} f(E-E') g(E') dE'$$



# Convolution Technique (2)

programs:  
LISE++ or LIESCHEN



**x-distribution described by 32 points**

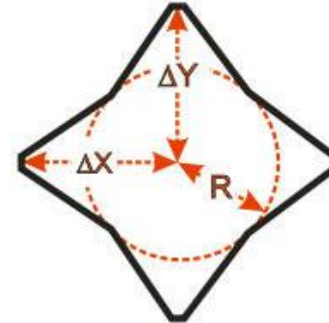
- + Fast as only a low number of points has to be calculated. Calculation of all fragments possible while watching.
- + Parametrization on log scale, even small tails are still visible
- Becomes very difficult with many cross correlations.
- Usually limited to linear transformations (only 1<sup>st</sup> order optics).

# Simplifications for Convolution

Example in FRS:

star shaped vacuum chamber is difficult to describe in convolution technique.

Only use independent cuts in x or y distribution.



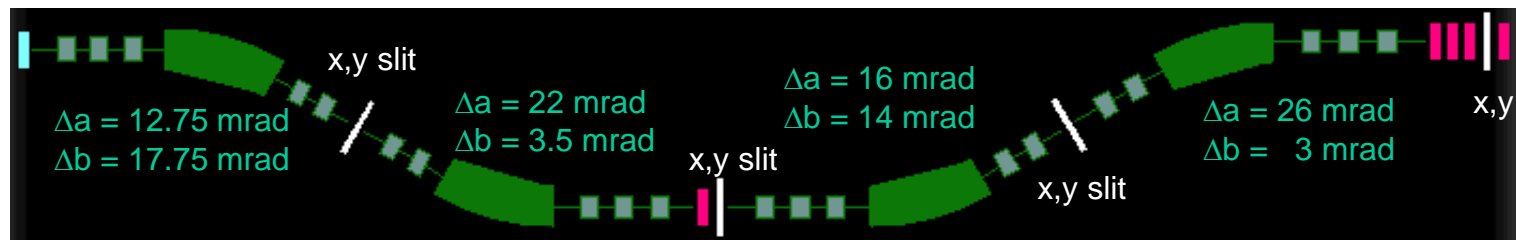
Details cannot be taken into account, so we do not try.

This also means more simplifications are allowed.

Replace single aperture cuts by effective cut for whole section.

One cut (x, y) after each separator stage,  
one angular acceptance (a, b) for each stage (TA-S1, S1-S2, S2-S3, S3-S4).

FRS-TA2-S4  
\_2014.lcn



Values used in LISE are adjusted to values of a MOCADI simulation.  
In normal FRS operation agreement of transmission within 20%



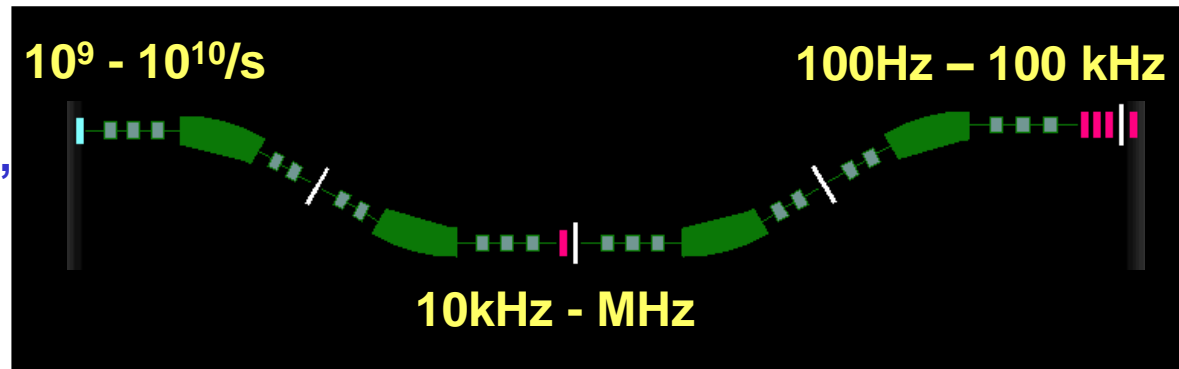
# Goals

**We can separate to single nuclide species  
(exception very high Z and A), but we do not have to !**

**The goal is to reduce the count rate enough so that detectors will work.  
Separation will always cause additional losses, slits or nuclear reactions in  
degraders, angular scattering, ...**

**Typical limits:**

**DAQ (kHz), MUSIC (10 kHz),  
Sci(<MHz), TPC (10-100 kHz),  
Si implantation (100 Hz)**

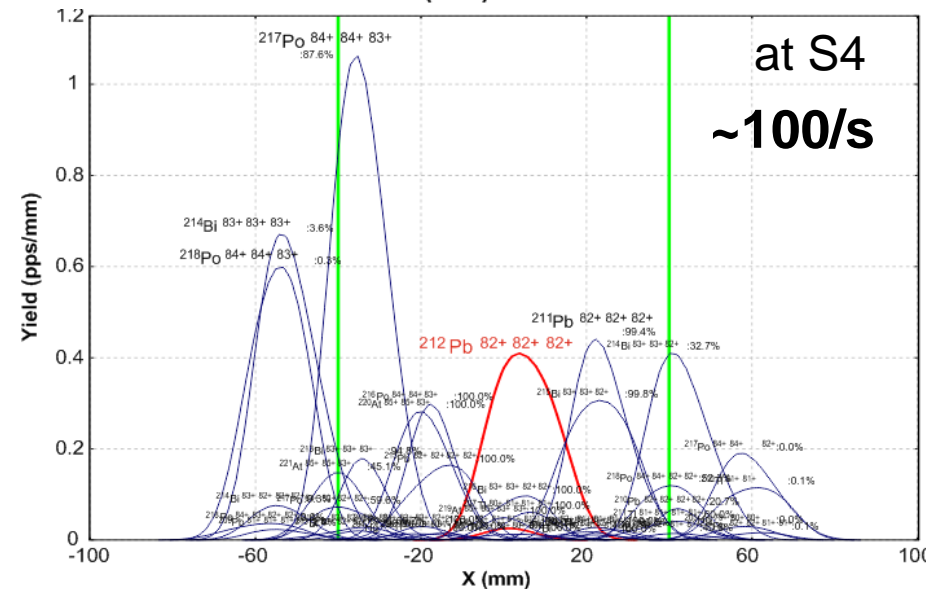
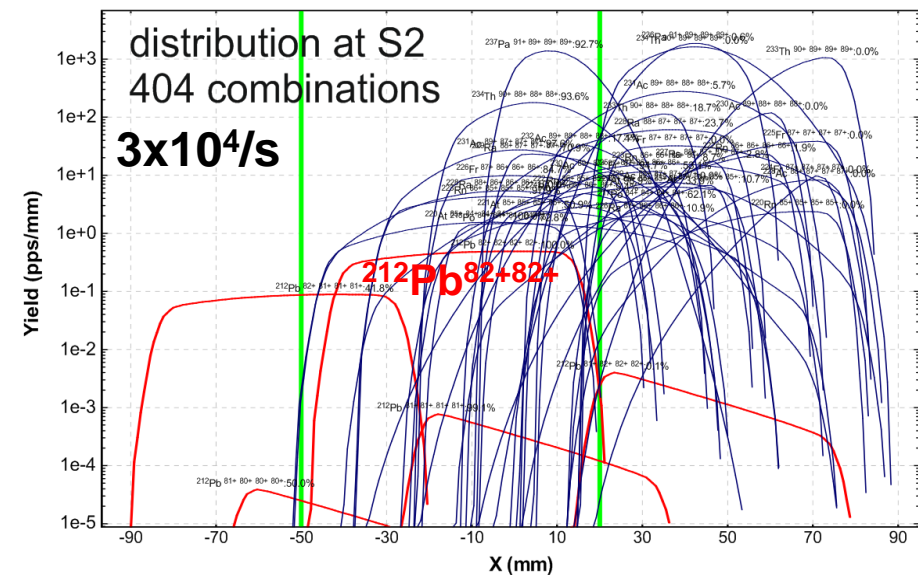
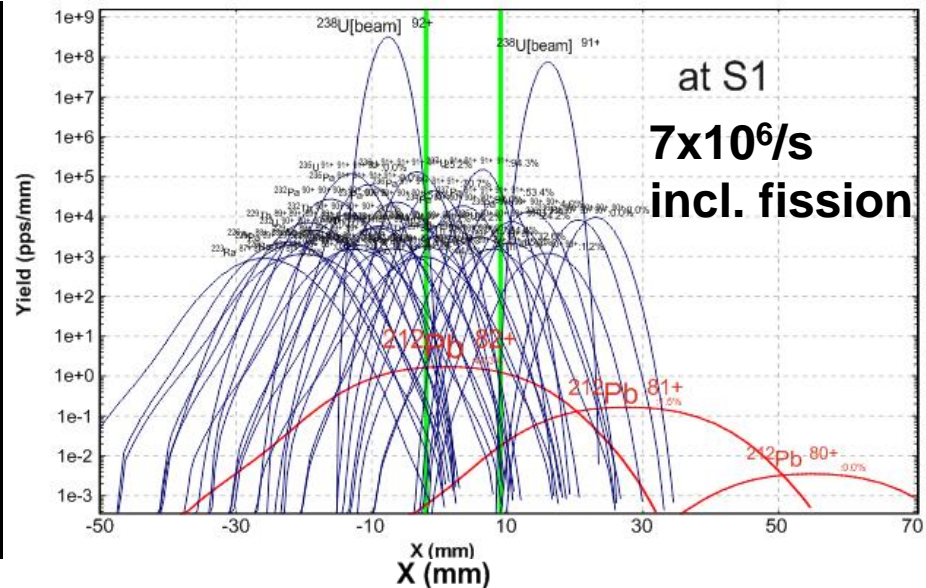
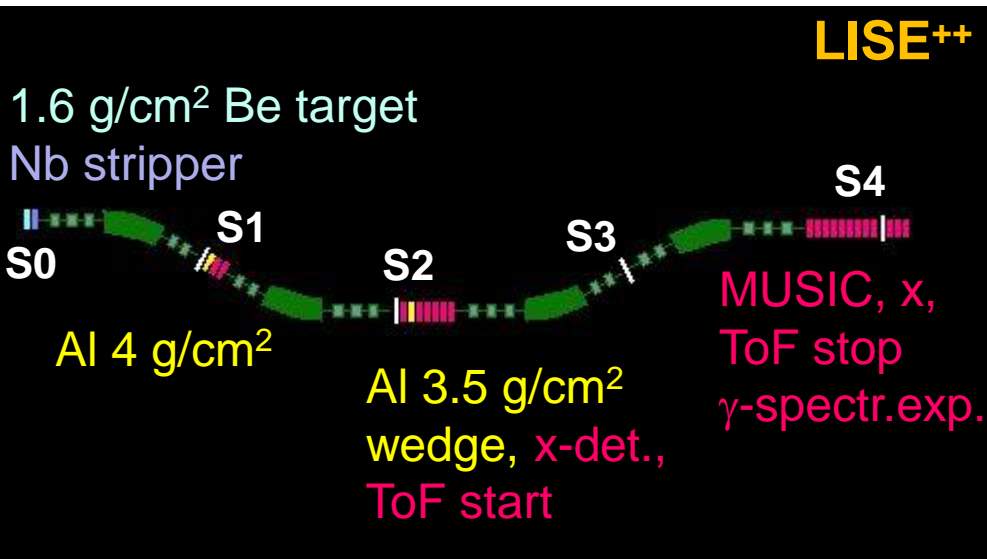


**Predict the degrader settings to get beam at the wanted energy  
or to implant ions.**

**Provide input of beam sizes, energy spread,  
also for further simulations with more detectors.**

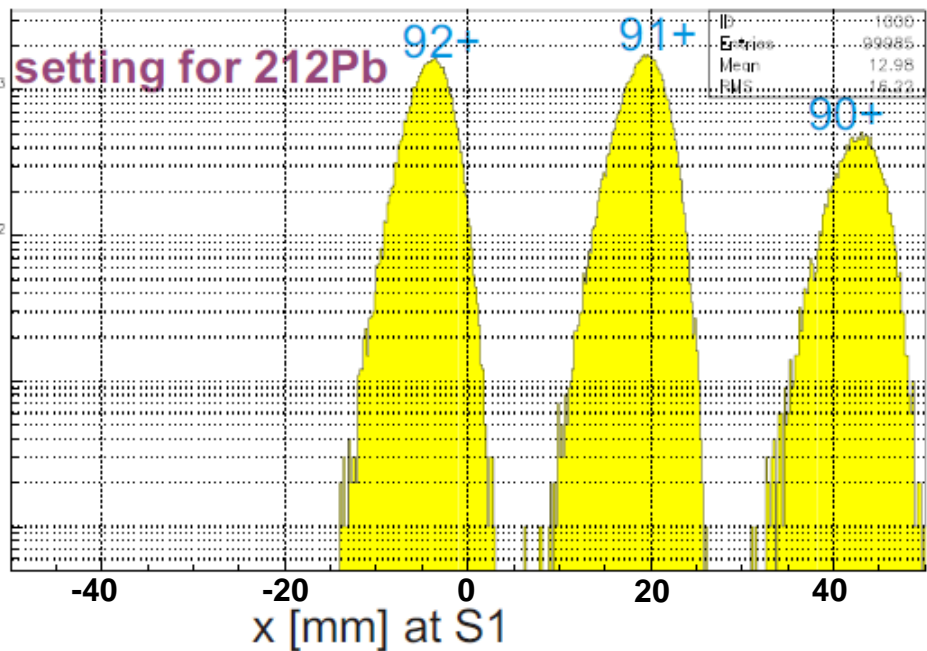
# Separator Setting for $^{212}\text{Pb}$

from older FRS experiment

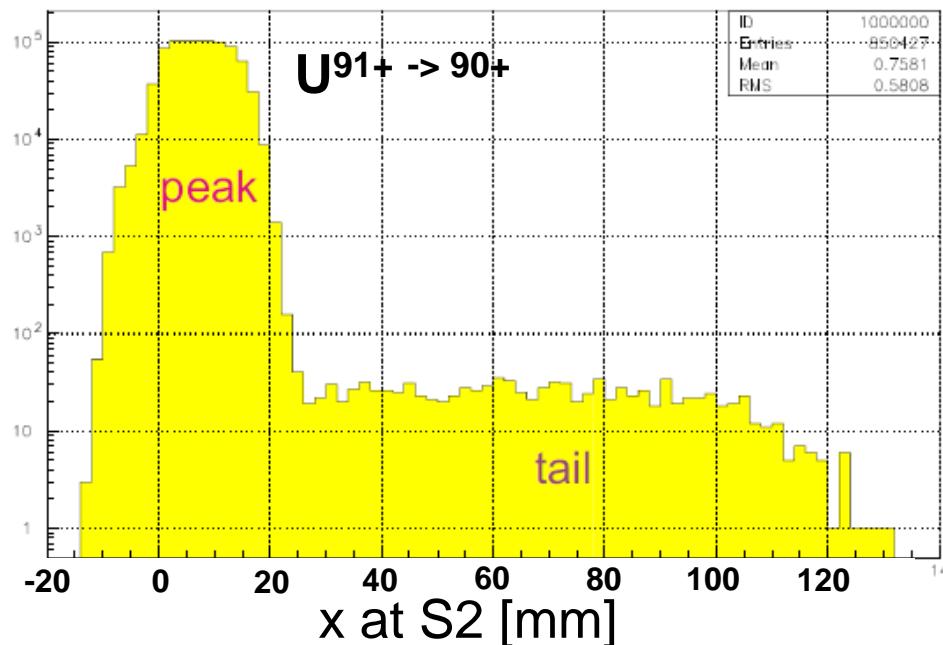
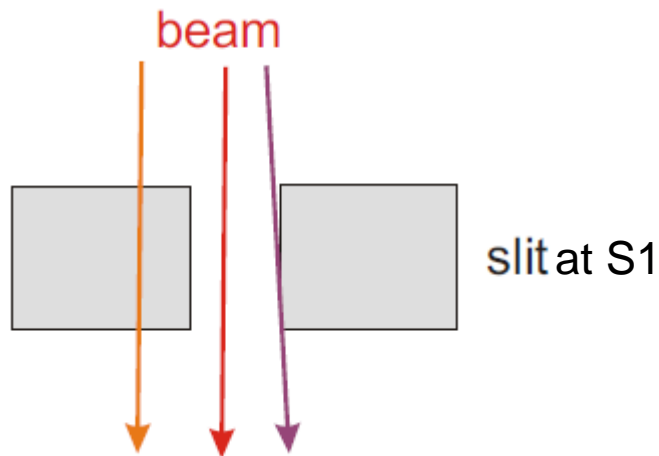


# Details with Monte-Carlo

primary U beam at S1



MOCADI

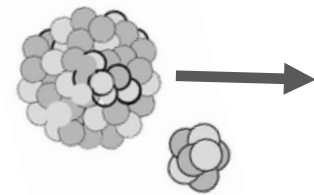




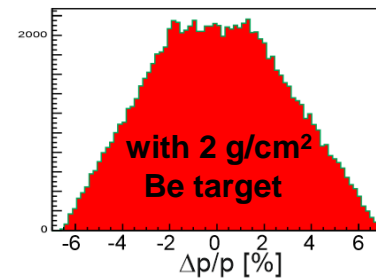
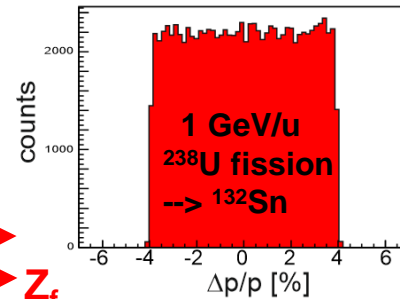
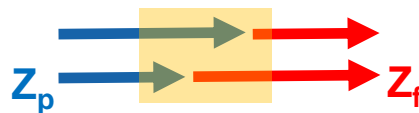
**Really the END**

# Requirements / Boundaries

High nuclear production cross section for fragmentation,  $\sim A_t^{2/3}$   
 Not too high energy loss in target, less electrons (low  $Z_t$ )  
 → best fragmentation yield for low  $Z_t$  materials (Li, Be, C).

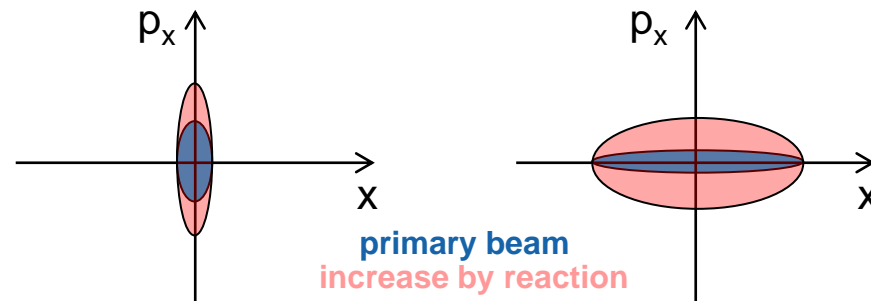


No big difference between projectile and fragment, at best  $Z_p^2/A_p \sim Z_f^2/A_f$   
 Or increased energy spread by location straggling inside thick target.



Thicker target → lower energy → larger emittance → lower transmission  
 Very thick → destruction of wanted fragment, but also possibility of enhancement via intermediate fragments.

Small beam spot on target to keep emittance increase low.  
 → Not too large longitudinal target extension, otherwise not all in focus.



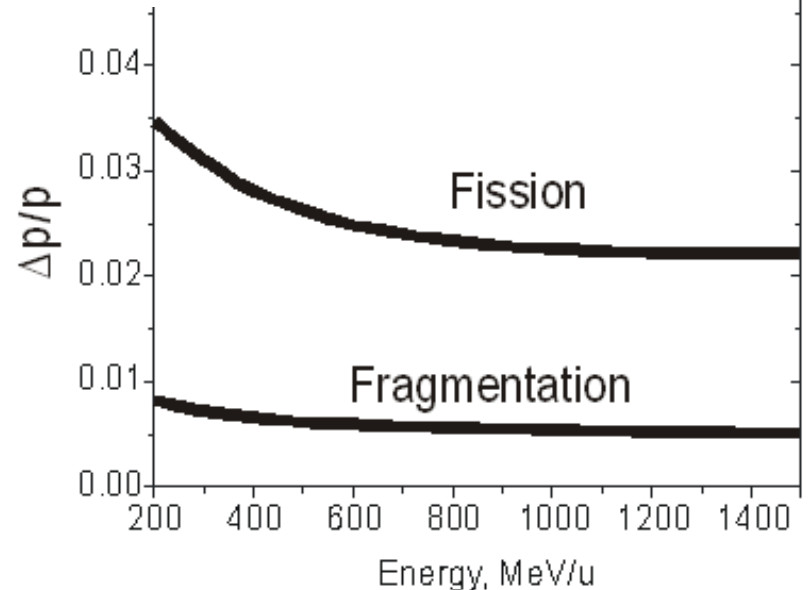
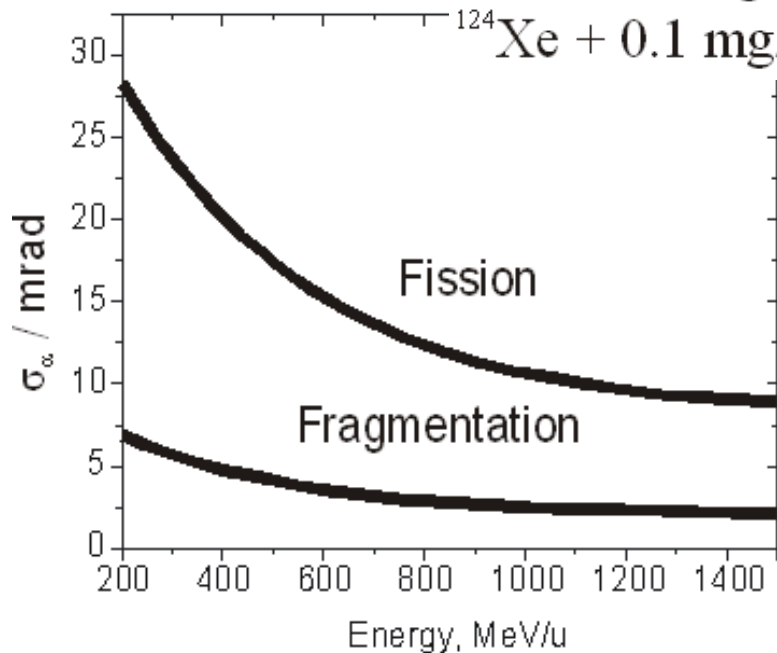
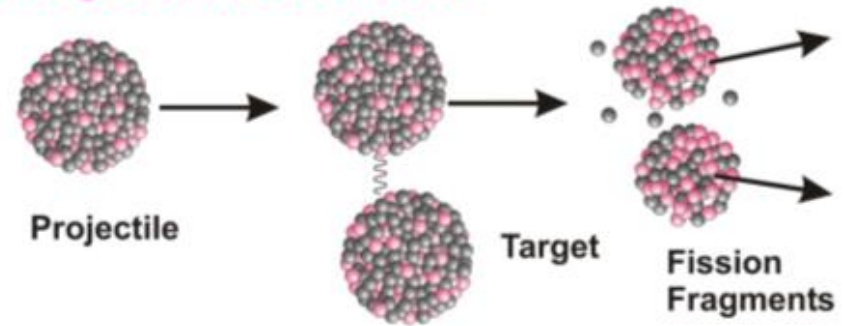
# Production of Rare Isotopes

(Reactions, kinematics in-flight)

## Projectile Fragmentation

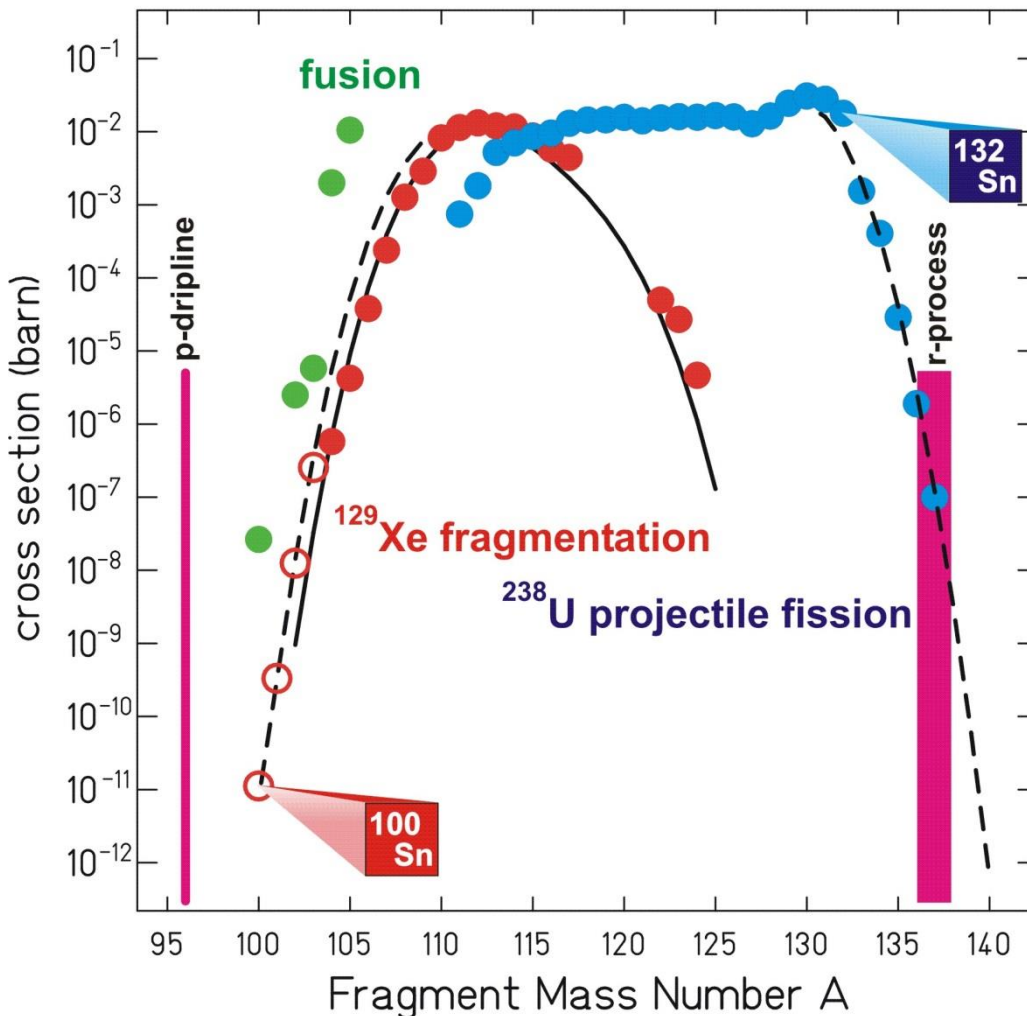


## Projectile Fission



# Production Cross sections

## Sn-Isotopes from Fusion, Fragmentation and Fission



## codes:

### EPAX 3

parametrization of cross sections  
K. Sümmerer, B. Blank

### ABRABLA

J. Benlliure, K.H. Schmidt, et al.

### LISE Abrasion/Ablation

O. Tarasov

+ Experimental data

# Material in Fast Pulsed Beam

## a simple temperature and stress calculation

Instantaneous energy deposition

$$\frac{dQ}{dm} = \frac{dE}{\rho dx} \frac{n}{\Delta x \Delta y}$$

stopping power  
number of ions  
spot size

$$\Delta T = \frac{dQ}{dm} \frac{A}{c_{mol}}$$

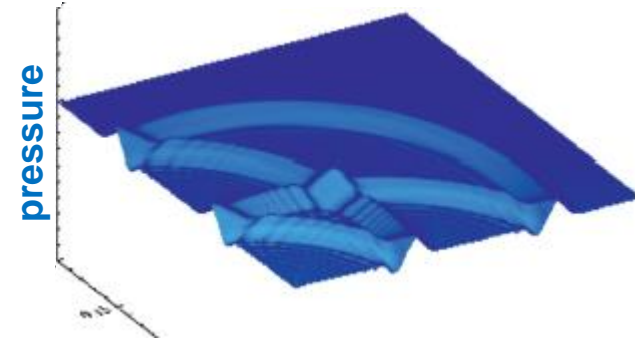
molar mass  
heat capacity

at high T  
 $c_{mol} \sim 25 \frac{J}{mol K}$

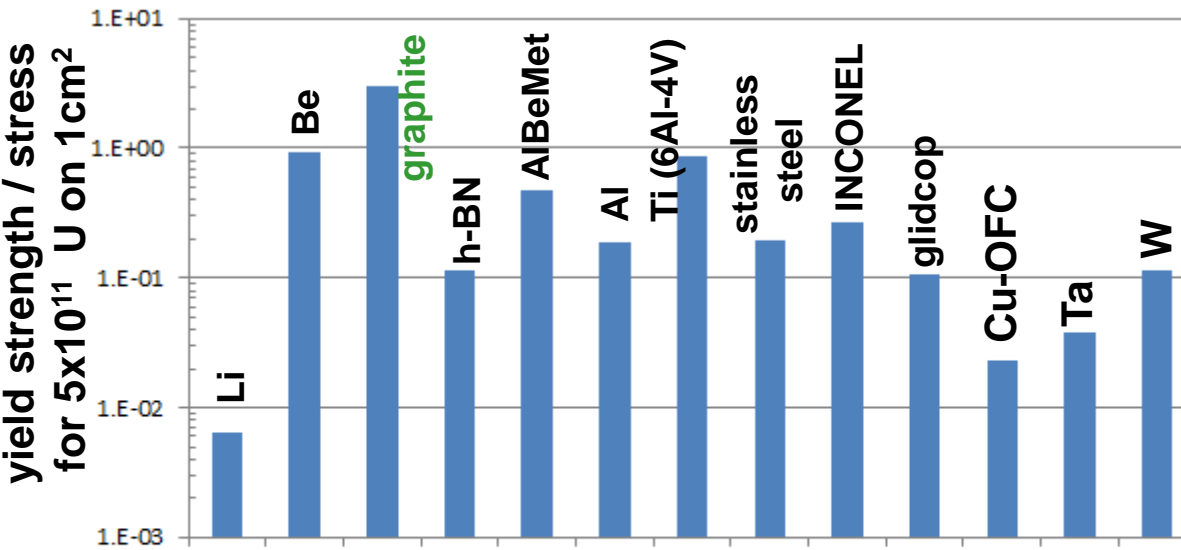
low A → low ΔT

$$P = K \alpha \Delta T$$

bulk modulus  
thermal expansion coeff.



polycrystalline graphite,  
more nuclear x-section  
less slowing-down by  
electrons.

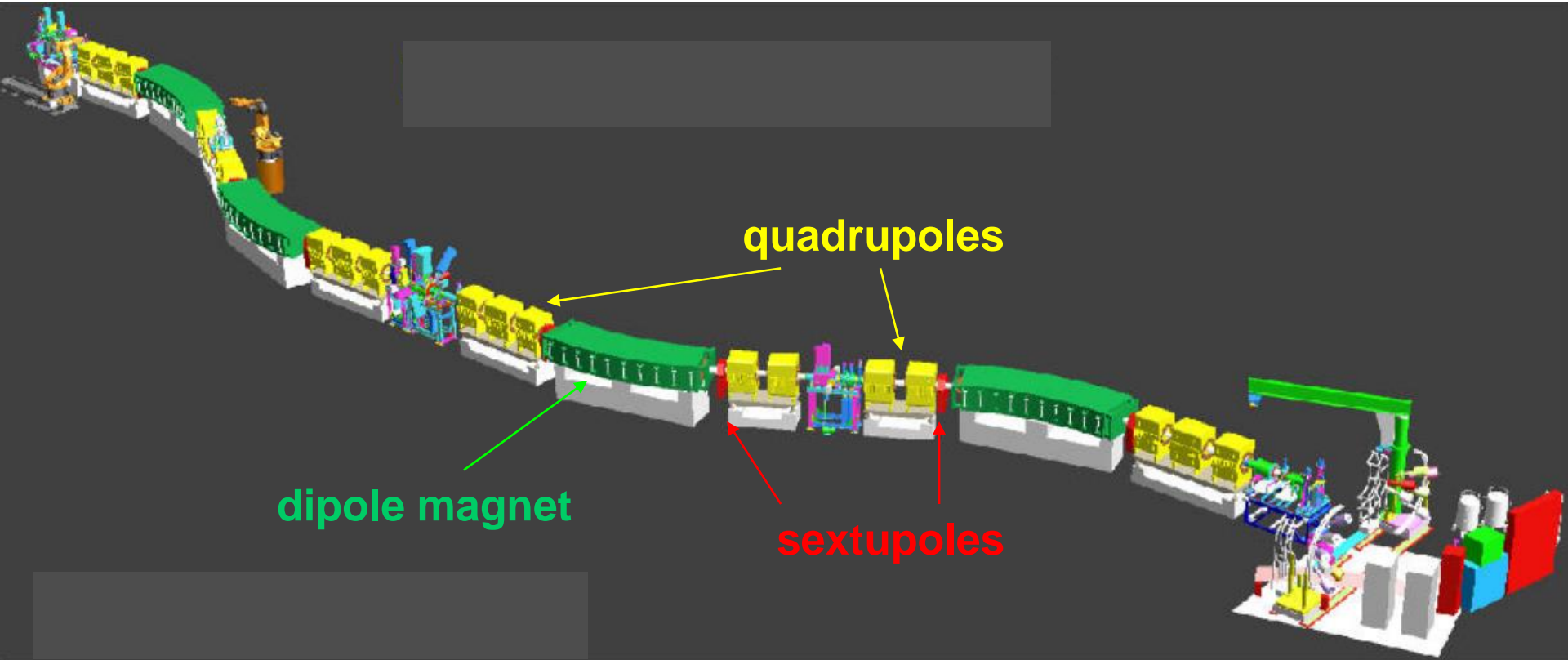


Initial compressive pressure,  
wave propagates to boundary  
→ tensile stress.

plastic deformation  
not exactly elastic,  
cyclic stress, cracks?

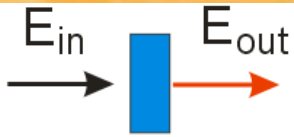


# Optical Elements of FRS



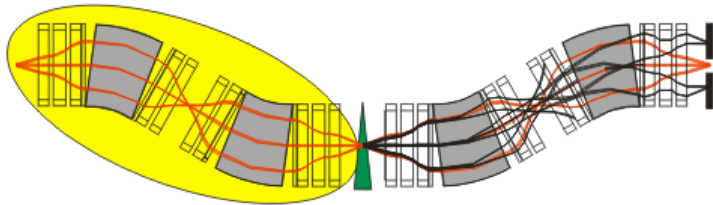
# Steps

Choose target, primary beam, initial energy

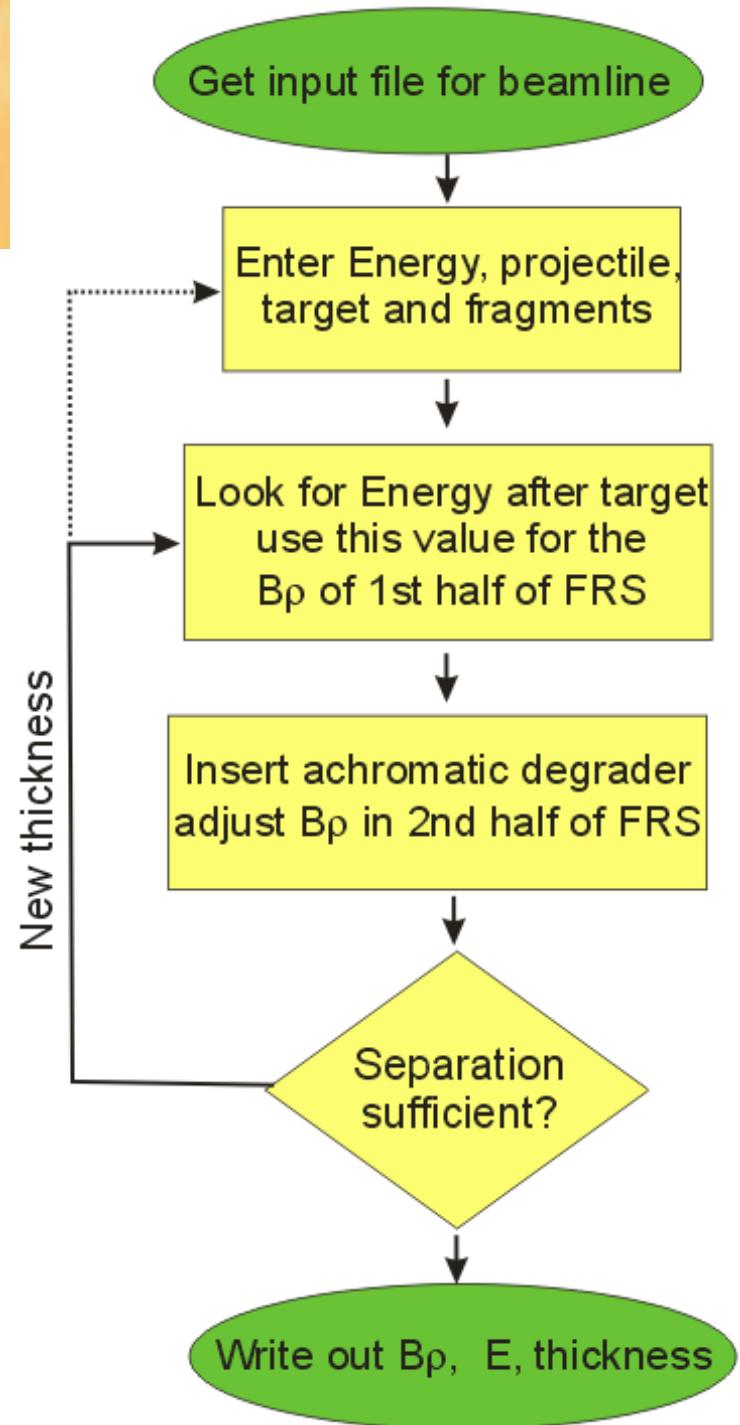
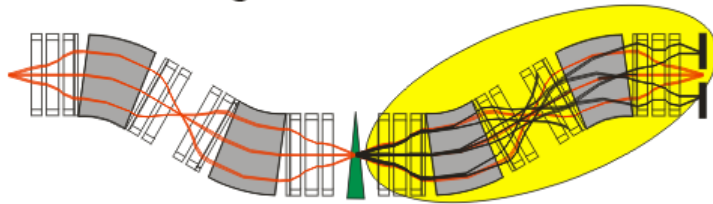


$E_{in}$   $E_{out}$

Scale magnets of 1st half of FRS



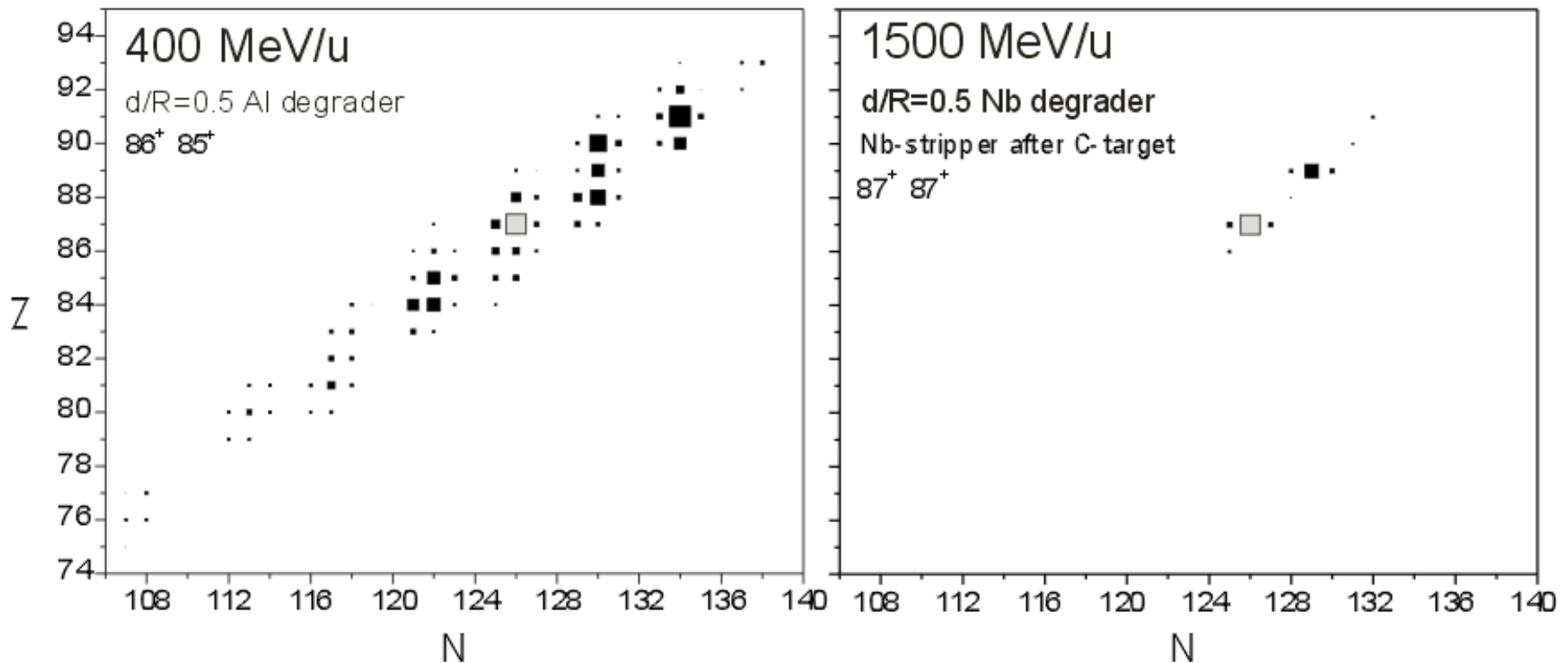
Scale magnets of 2nd half of FRS



# Effect on Fragment Separator

## Separation of $^{213}\text{Fr}$

optimal combination of q-states and stripper materials at different energy, FRS with achromatic S2 degrader in LISE<sup>++</sup>



**change in q at S2 fools the  $B\rho$ - $\Delta E$ - $B\rho$  separation**