Nuclear Fission and Multinucleon Transfer Reactions

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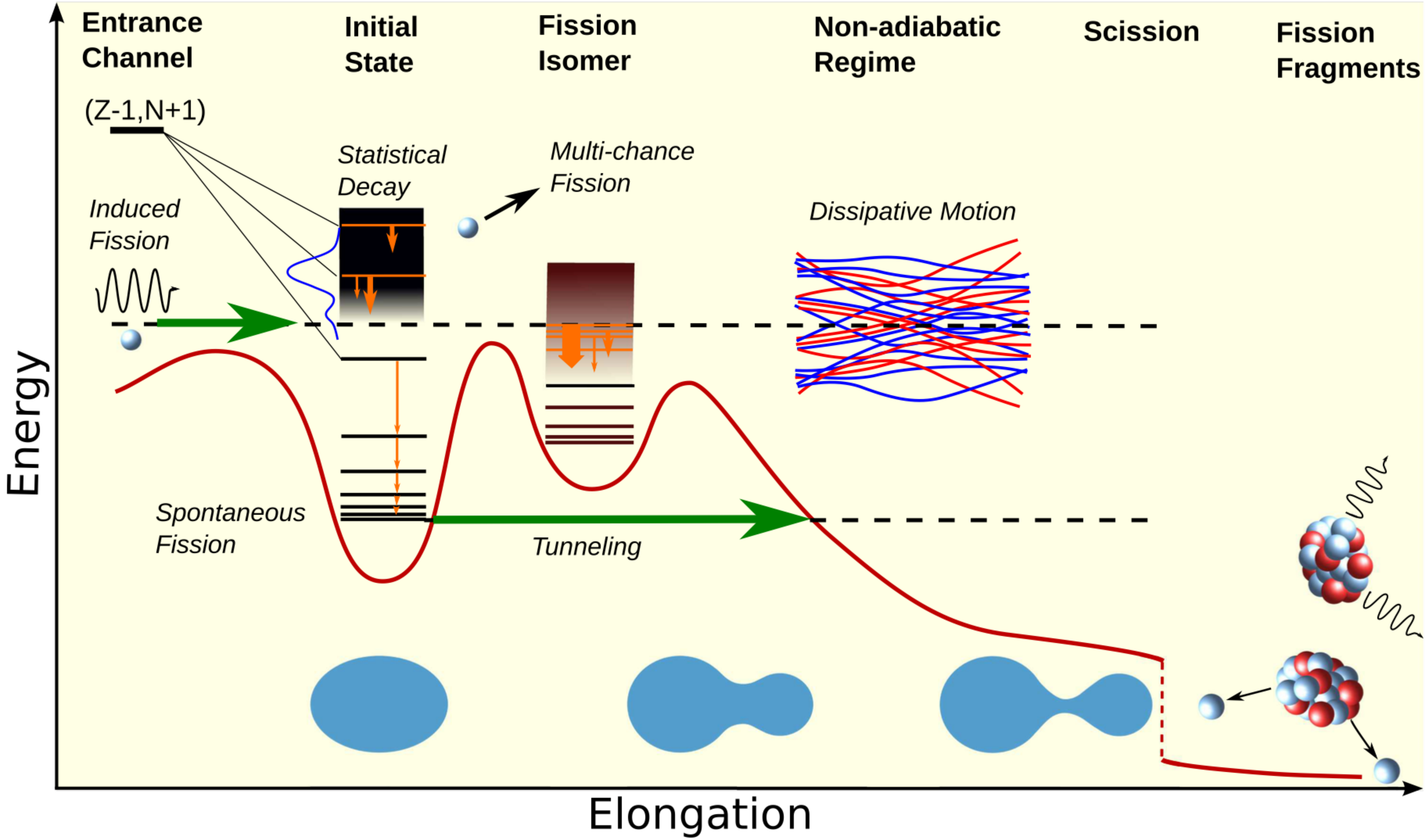
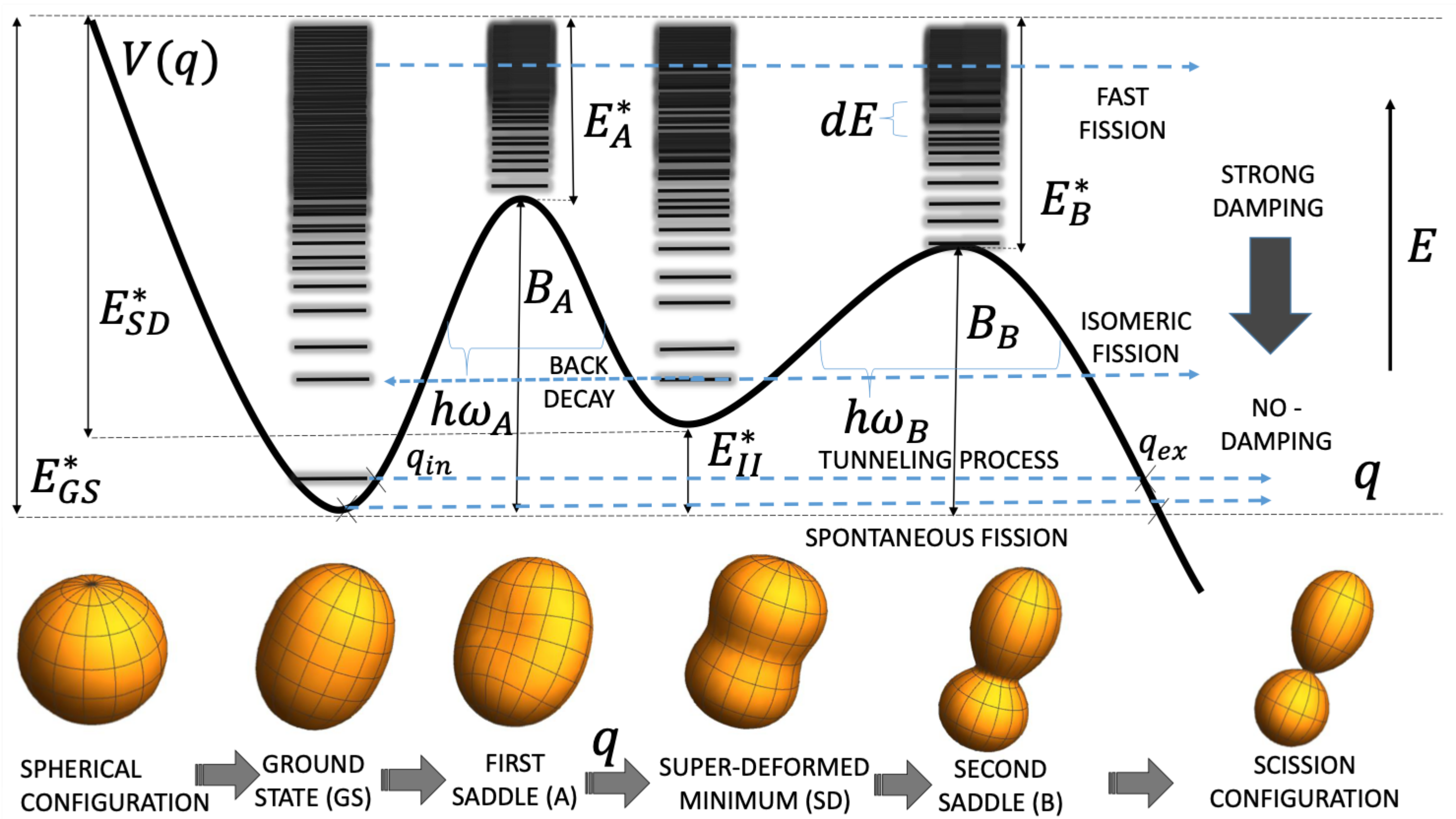
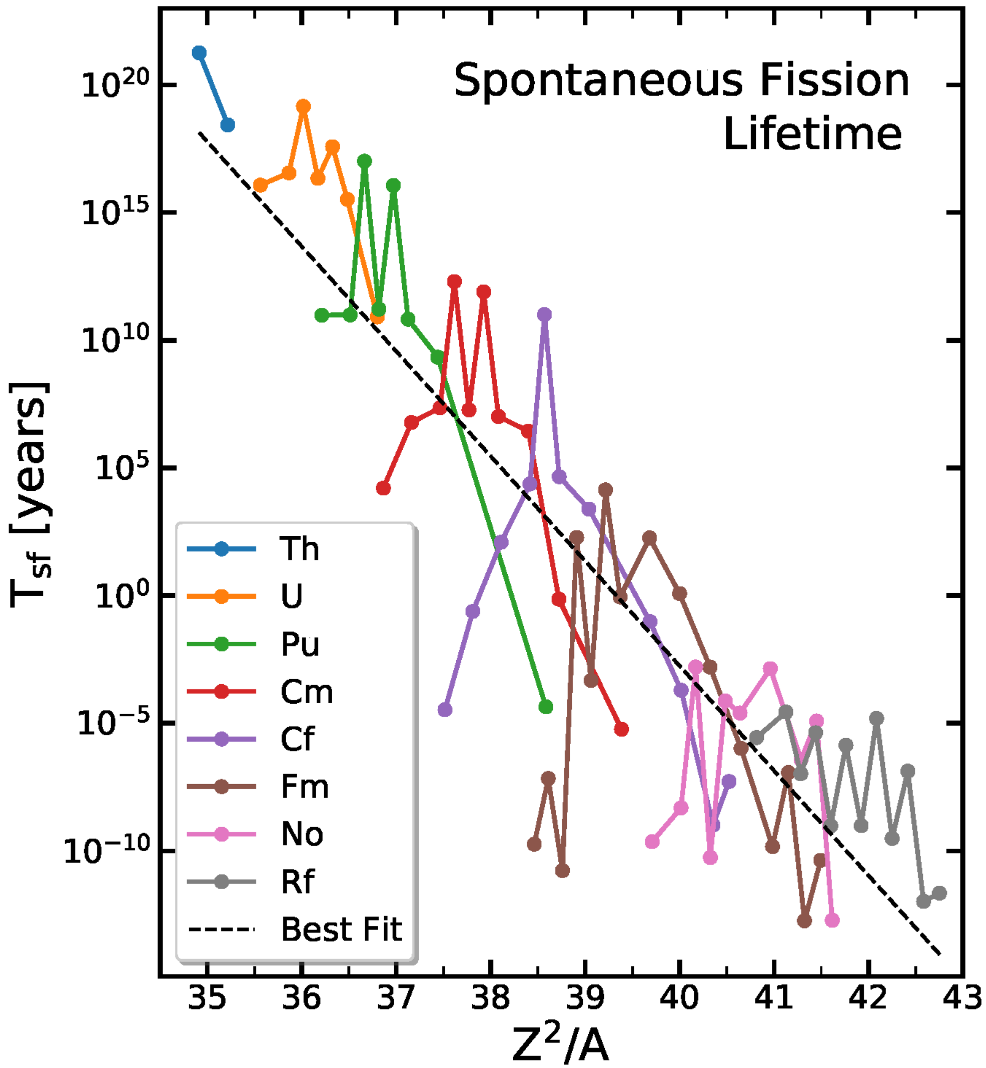


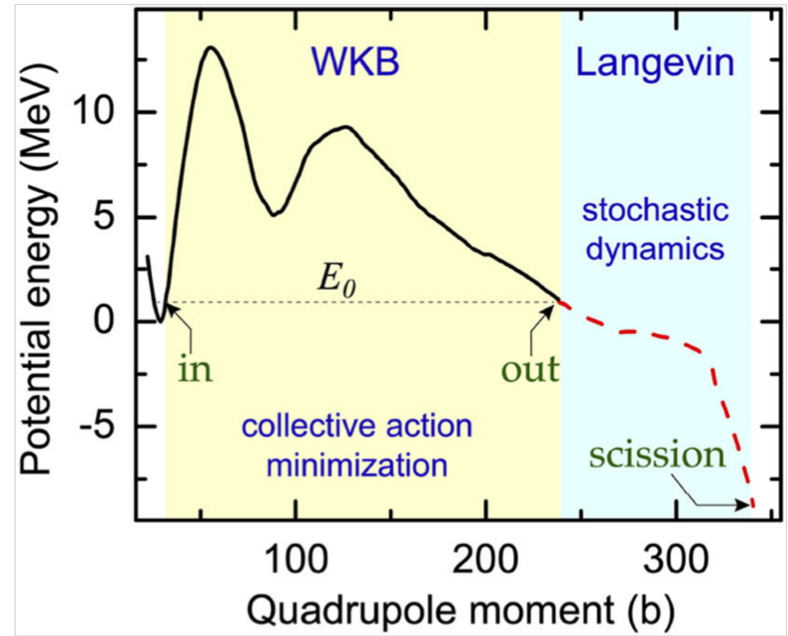
Fission Shapes, Barriers and Isomers



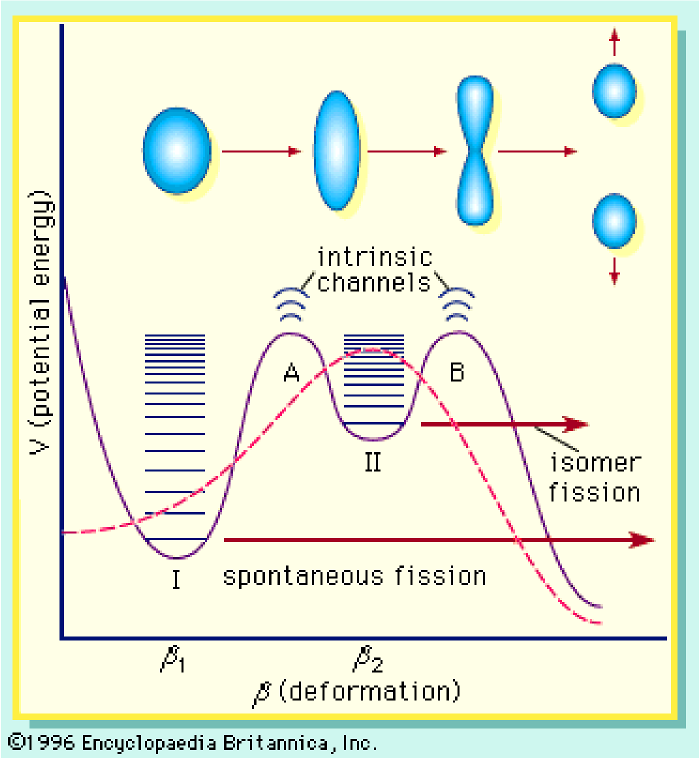
Kowal, M., Skalski, J. (2023).

Spontaneous Fission

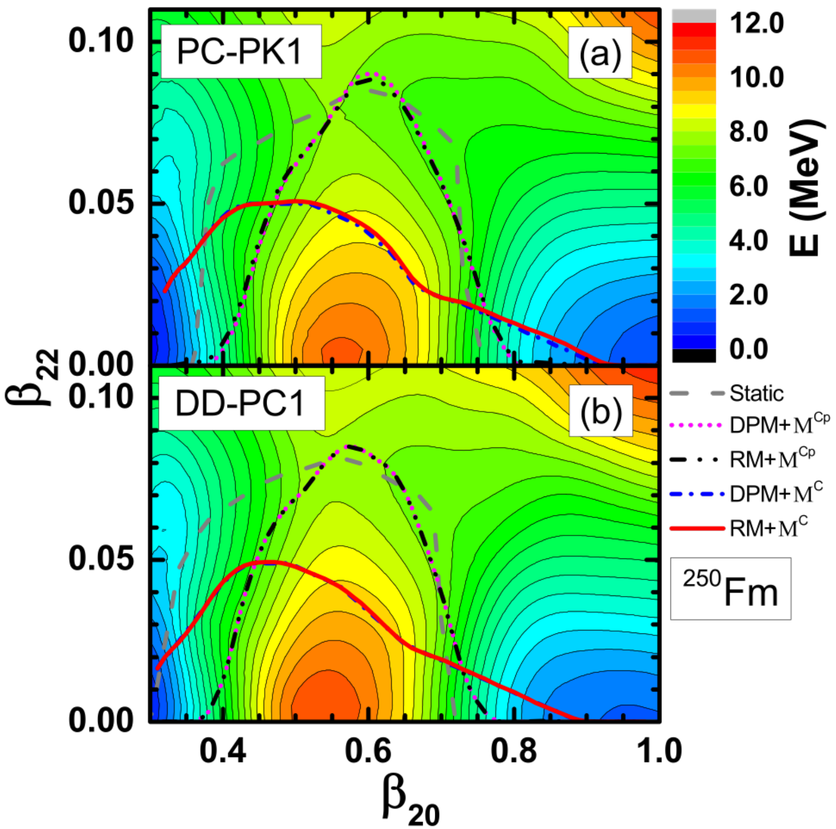




Sadhukhan J (2020) Microscopic Theory for Spontaneous Fission. Front. Phys. 8:567171.



… the fission path *L* is embedded in a multi-dimensional collective space. The actual path is determined by minimizing the action integral.



$$S(L) = \int_{s_{\text{in}}}^{s_{\text{out}}} \frac{1}{\hbar}\sqrt{2\mathcal{M}_{\text{eff}}(s)[V_{\text{eff}}(s) - E_0]}\, ds$$

ground-state energy

The effective inertia and collective potential calculated in a SCMF approach based on EDFs.

… penetration probability:

$$P = \frac{1}{1 + \exp[2S(L)]}$$

$$T_{1/2} = \ln 2/(nP)$$

colective coordinates - functions of the path's length.

... collective inertia

$$\mathcal{M}_{\text{eff}}(s) = \sum_{ij} \mathcal{M}_{ij} \frac{dq_i}{ds} \frac{dq_j}{ds}$$

Dynamical coupling between shape and pairing degrees of freedom

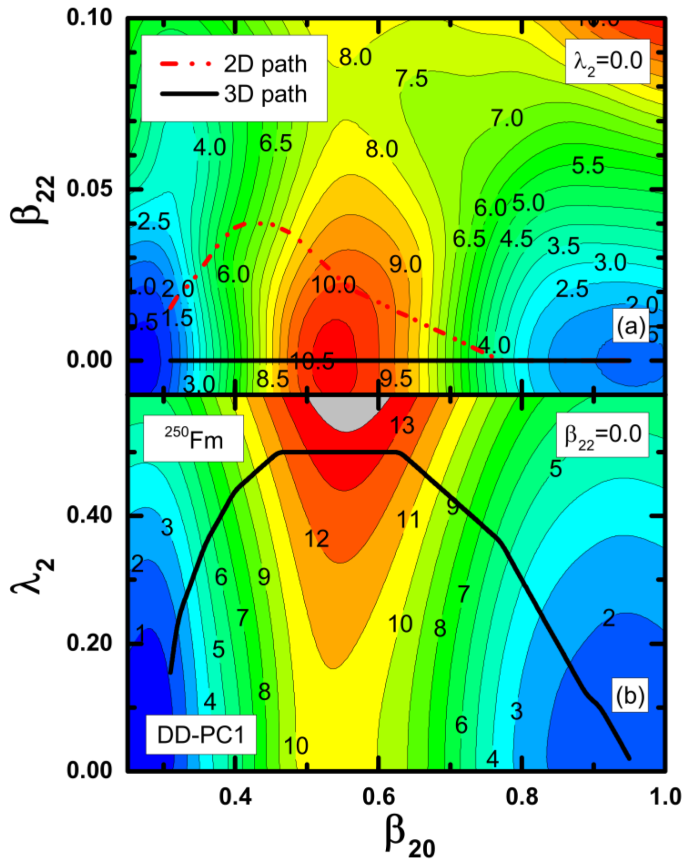
The effective inertia and collective potential depend on the strength of pairing correlations:

$$\mathcal{M} \sim \Delta^{-2}$$

$$V \sim (\Delta - \Delta_0)^2$$

self-consistent stationary gap

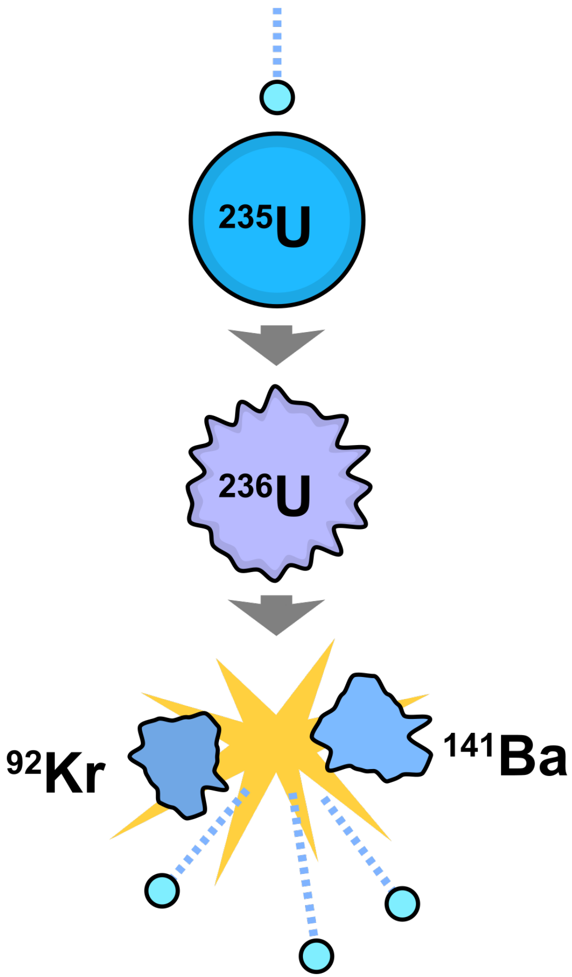
...when the gap parameter is treated as a dynamical variable, an enhancement of pairing reduces the effective inertia and thus minimizes the action integral along the fission path.



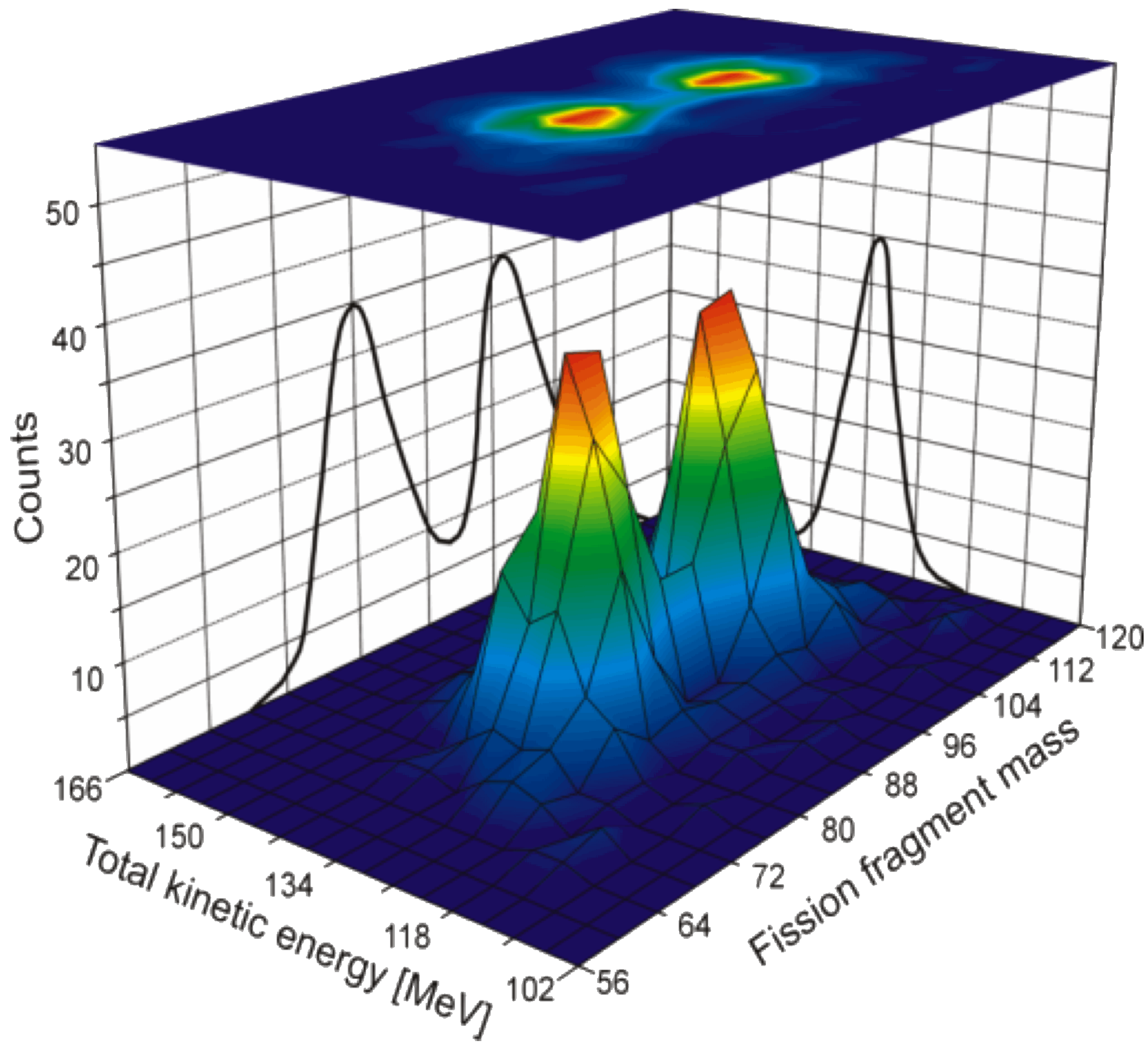
Action integrals and SF half-lives

Nucleus	Path	$S(L)$	$\log_{10}(T_{1/2}/\text{yr})$
^{264}Fm	2D	19.58	– 11.03
	3D	14.15	– 15.75
^{250}Fm	2D	32.09	– 0.16
	3D	22.33	– 8.64

Induced Fission



Fragment Mass Distributions



Japan Atomic Energy Agency

Time-dependent density functional theory (TDDFT) for induced fission

$$i\frac{\partial}{\partial t}\psi_k(\boldsymbol{r},t) = \left[\hat{h}(\boldsymbol{r},t) - \varepsilon_k(t)\right]\psi_k(\boldsymbol{r},t),$$

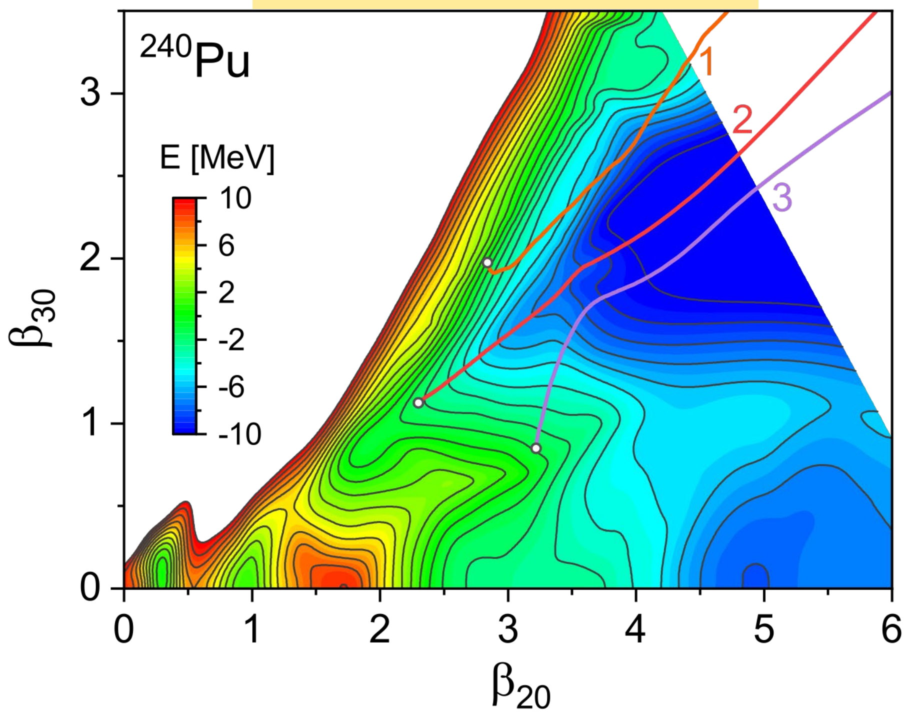
$$i\frac{d}{dt}n_k(t) = n_k(t)\Delta_k^*(t) - n_k^*(t)\Delta_k(t),$$

$$i\frac{d}{dt}\kappa_k(t) = [\varepsilon_k(t) + \varepsilon_{\bar{k}}(t)]\kappa_k(t) + \Delta_k(t)[2n_k(t) - 1].$$

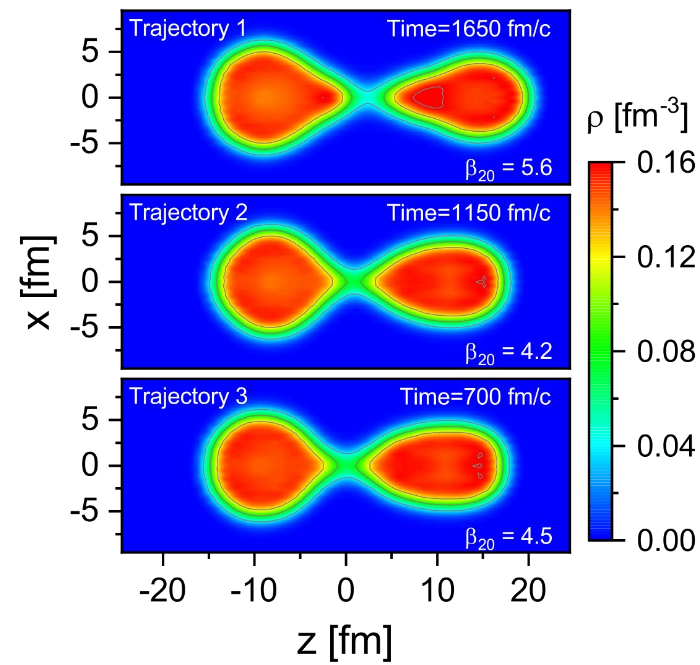
⇒ classical evolution of independent nucleons in mean-field potentials, cannot be applied in classically forbidden regions of the collective space, nor does it take into account quantum fluctuations.

⇒ automatically includes the one-body dissipation mechanism, but can only simulate a single fission event by propagating the nucleons independently.

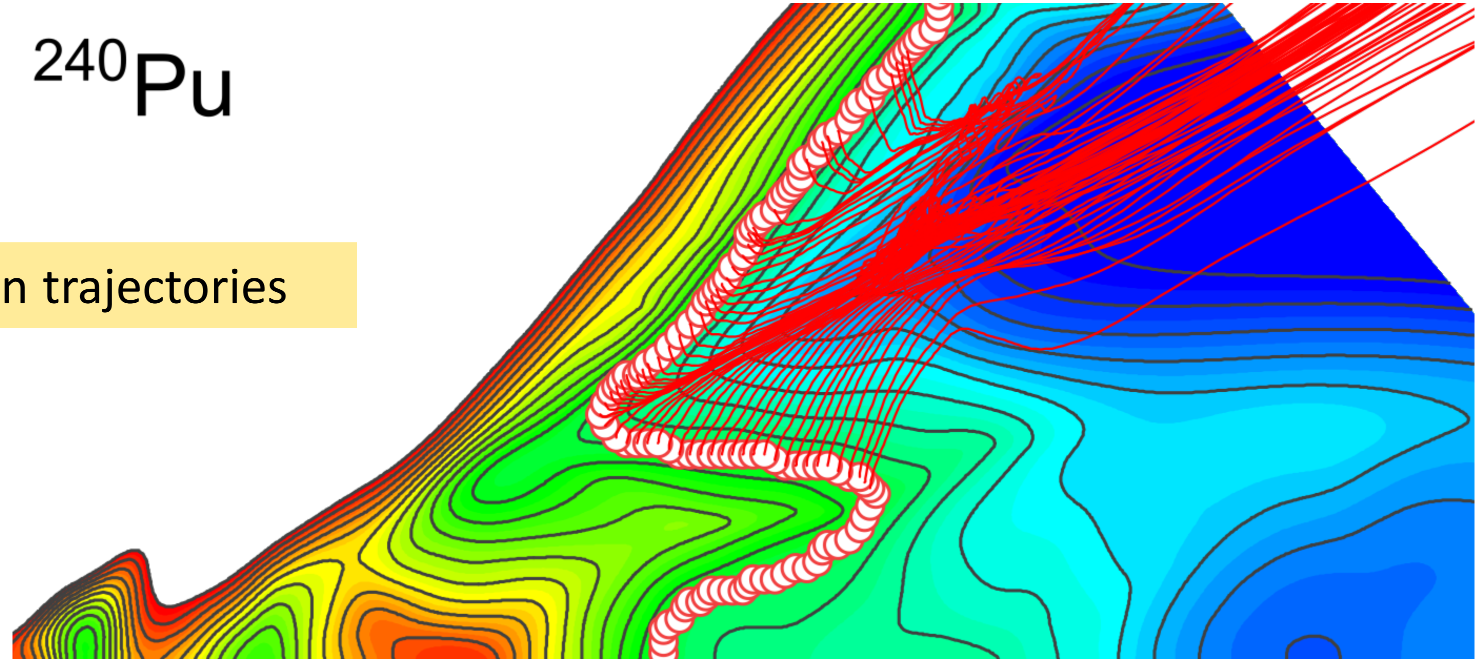
TDDFT fission trajectories



Density profiles at scission

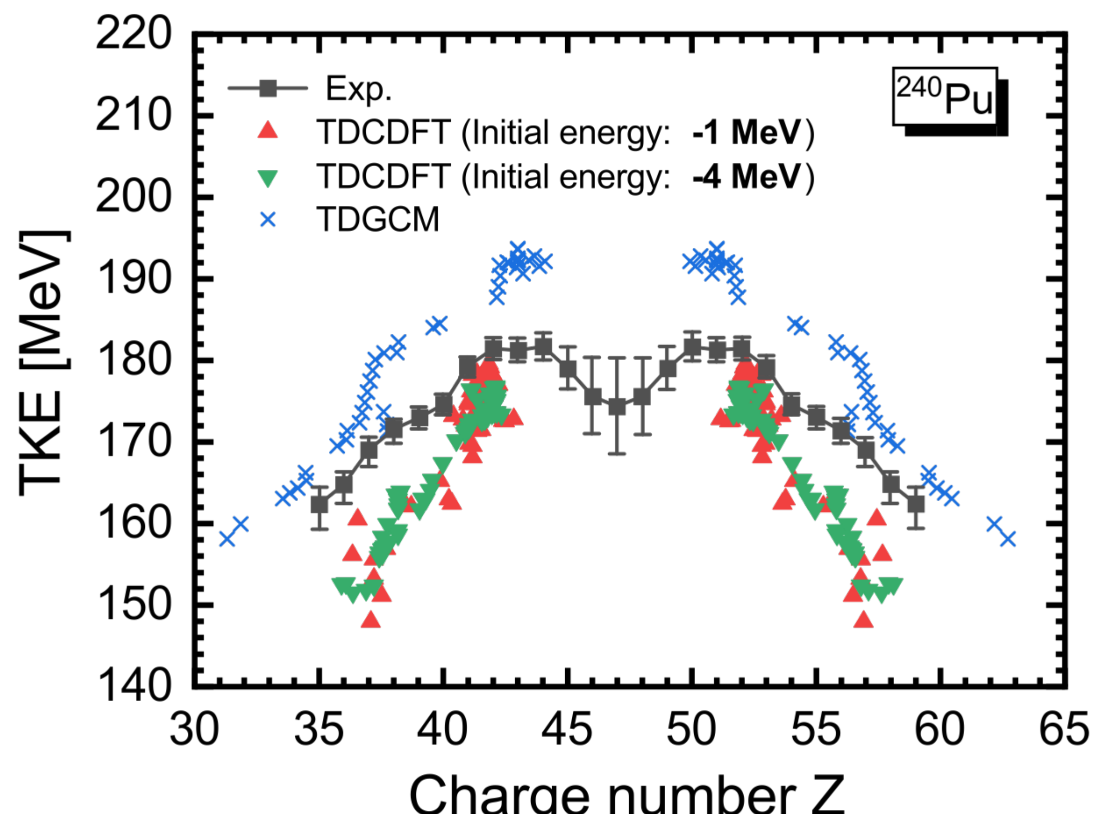


^{240}Pu

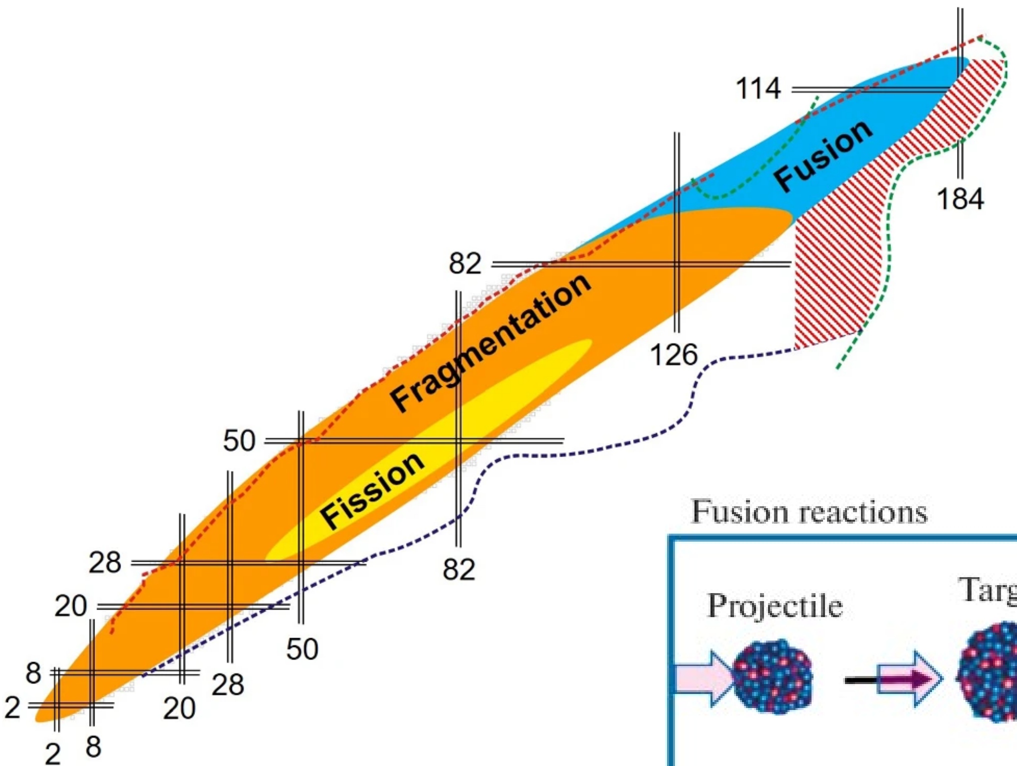


TDDFT fission trajectories

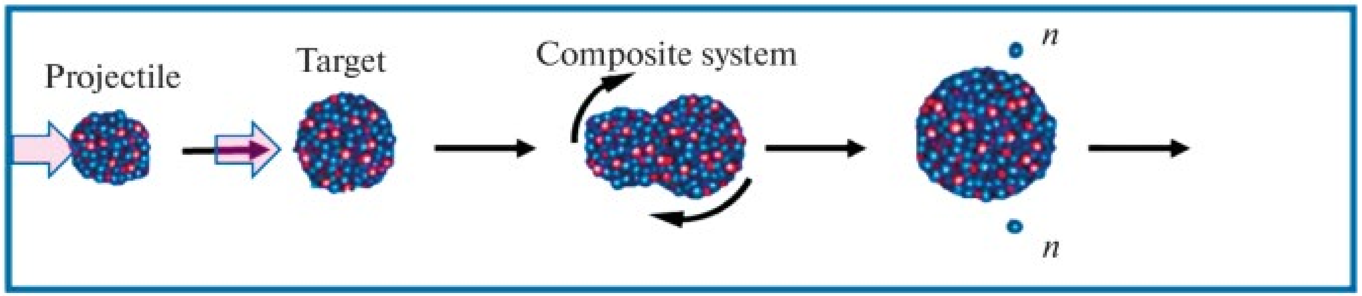
Total kinetic energies (TKEs) of the fragments



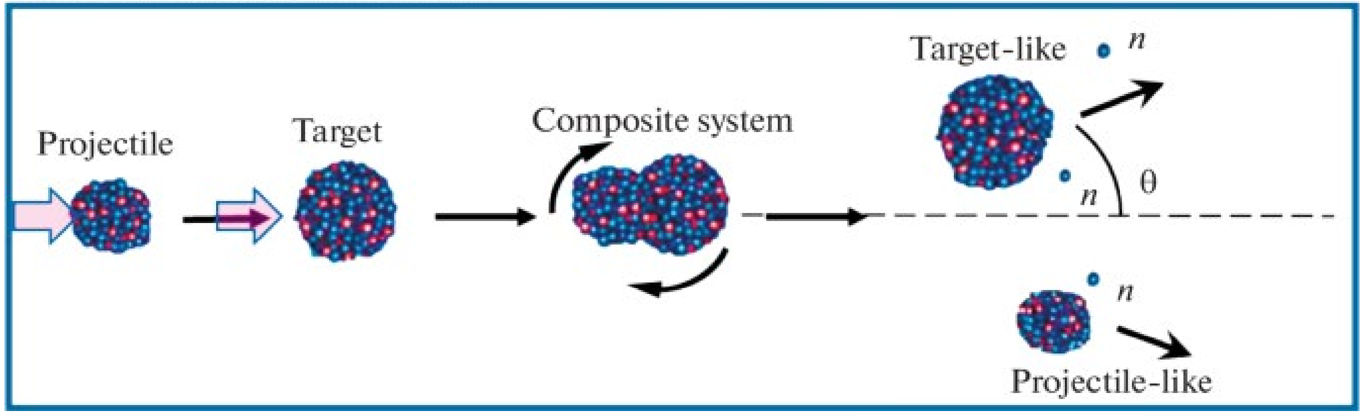
Nucleon Transfer Reactions



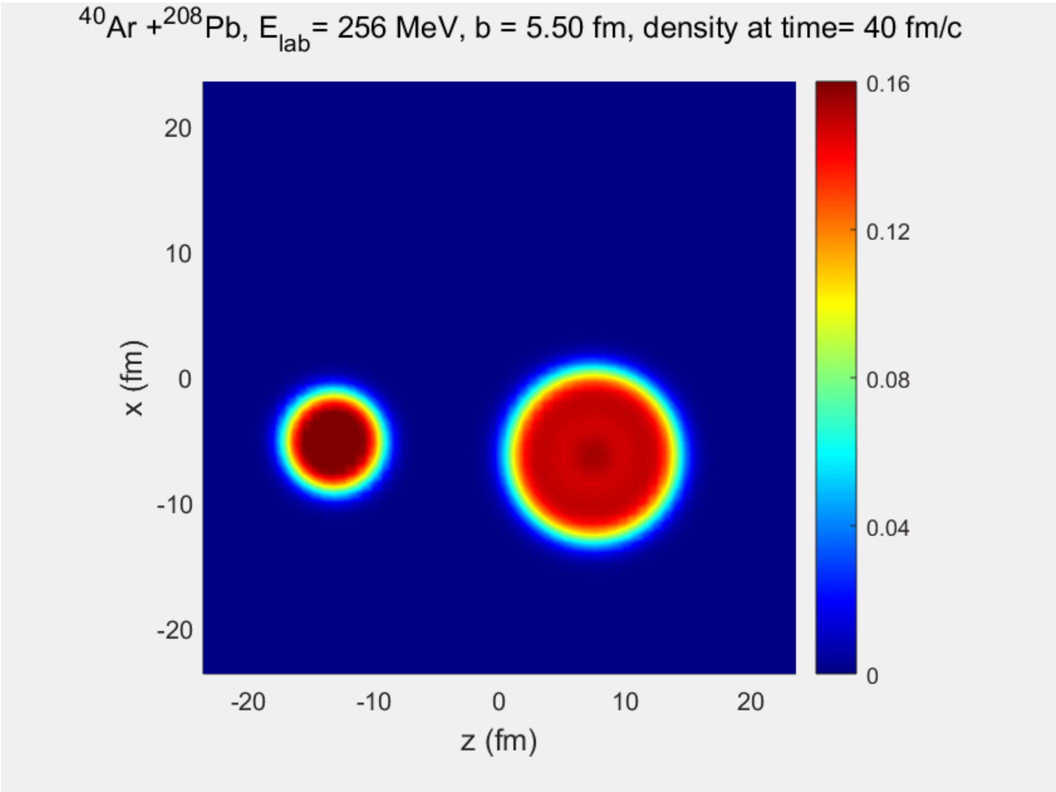
Fusion reactions

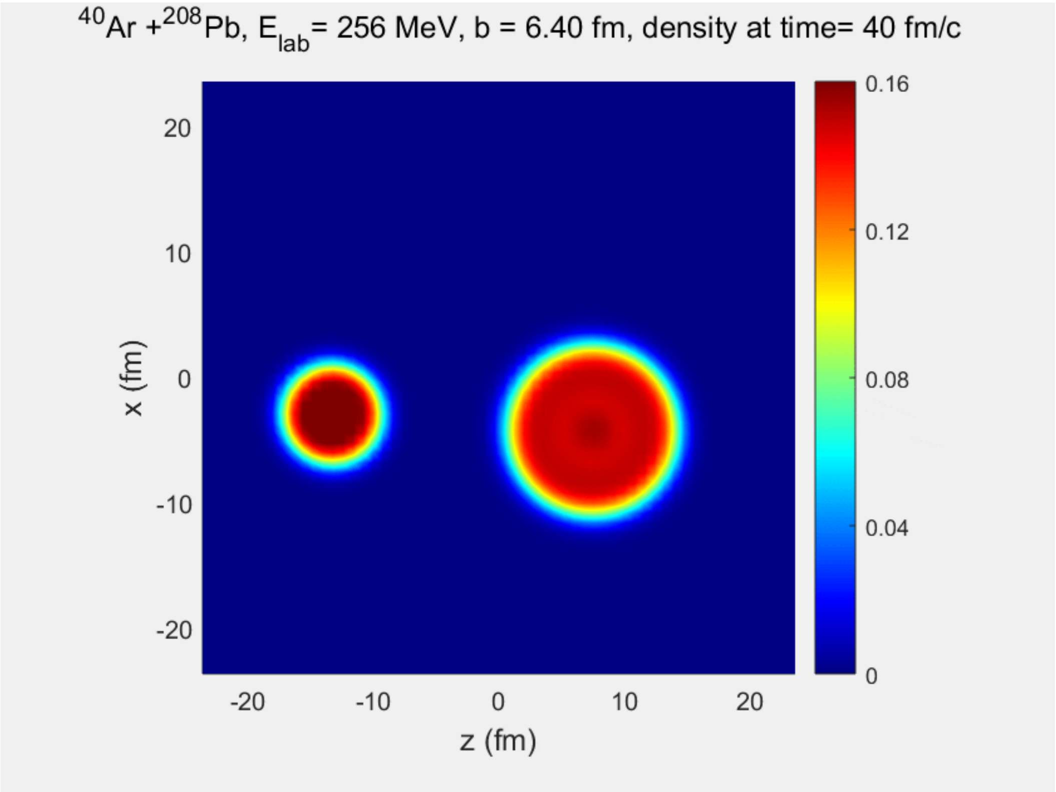


Multi-nucleon transfer reactions



Multi-nucleon transfer reactions





The total wave function $\psi(\boldsymbol{r},t)$ is a single Slater determinant composed of single-particle wave functions:

$$\Psi(\boldsymbol{r},t) = \frac{1}{\sqrt{A!}} \det\{\psi_k(\boldsymbol{r},t)\},$$

The space is divided into the region V, which contains the fragment we are interested in, and the complementary region. The particle number projection operator for neutrons ($q = n$) or protons ($q = p$) in V:

$$\hat{P}_m^{(q)} = \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{i(m - \hat{N}_V^{(q)})\theta}.$$

Where the particle number operator in the region V, is defined as:

$$\hat{N}_V^{(q)} = \int_V d\boldsymbol{r} \sum_{i=1}^{N^{(q)}} \delta(\boldsymbol{r} - \boldsymbol{r}_i) = \sum_{i=1}^{N^{(q)}} \Theta_V(\boldsymbol{r}_i), \qquad \Theta_V(\boldsymbol{r}) = \begin{cases} 1 & \text{if} \quad \boldsymbol{r} \in V, \\ 0 & \text{if} \quad \boldsymbol{r} \notin V. \end{cases}$$

By applying the particle number projection operator to the total wave function $\psi(\boldsymbol{r})$, the specific component with N neutrons and Z protons can be extracted:

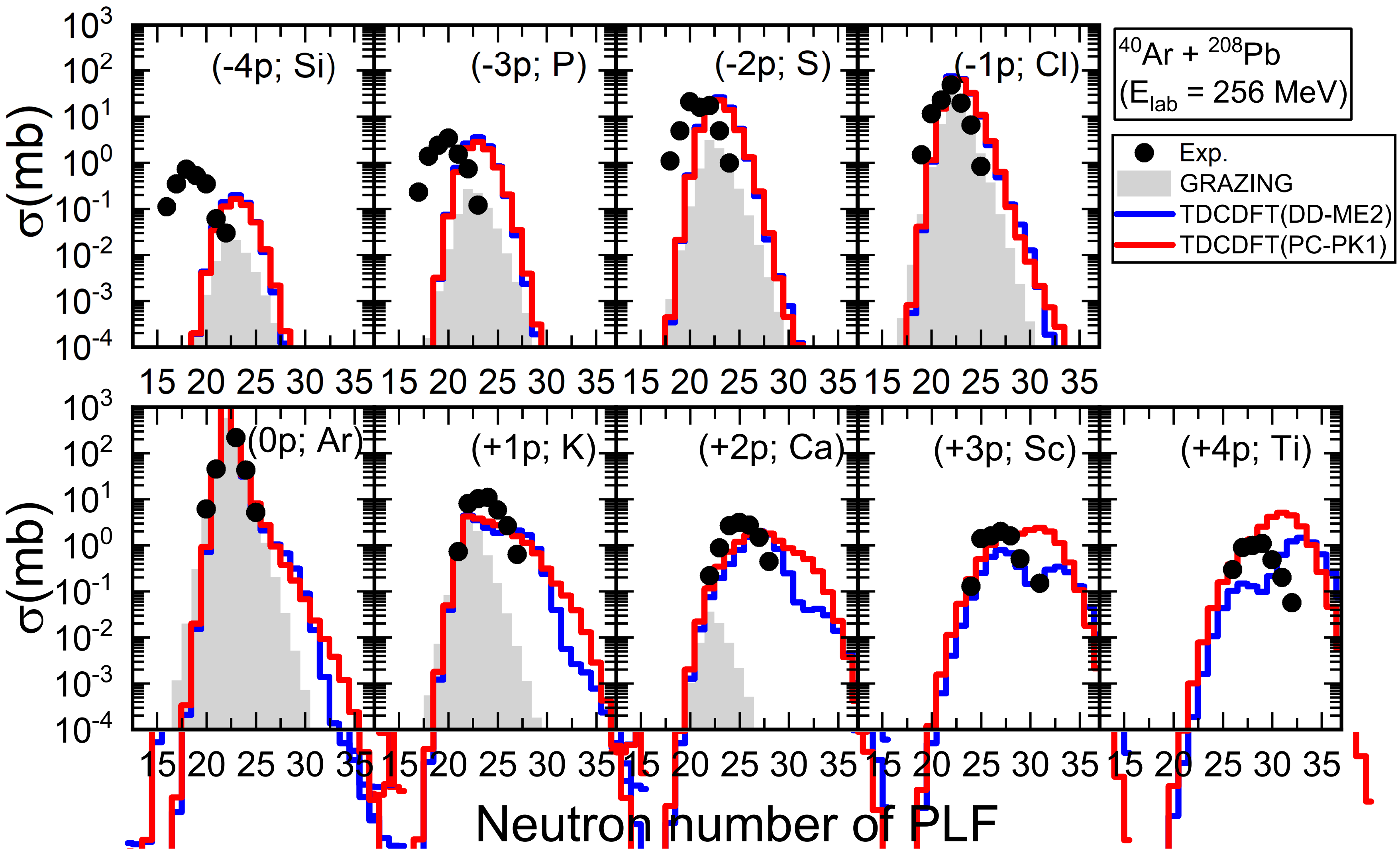
$$|\Psi_{N,Z}\rangle = \hat{P}_N^{(n)} \hat{P}_Z^{(p)} |\Psi\rangle.$$

⇒ the probability of the occurrence of a reaction product composed of N neutrons and Z protons

$$P_{N,Z} = \langle \Psi_{N,Z} | \Psi_{N,Z} \rangle = P_N^{(n)} P_Z^{(p)}.$$

Given specific values for the incident energy *E* and impact parameter *b*, the probability to observe a reaction product with *N* neutrons and *Z* protons in *V* can be determined, denoted as $P_{N,Z}(E, b)$. The cross section for each channel is computed by integrating over the interval of impact parameters,

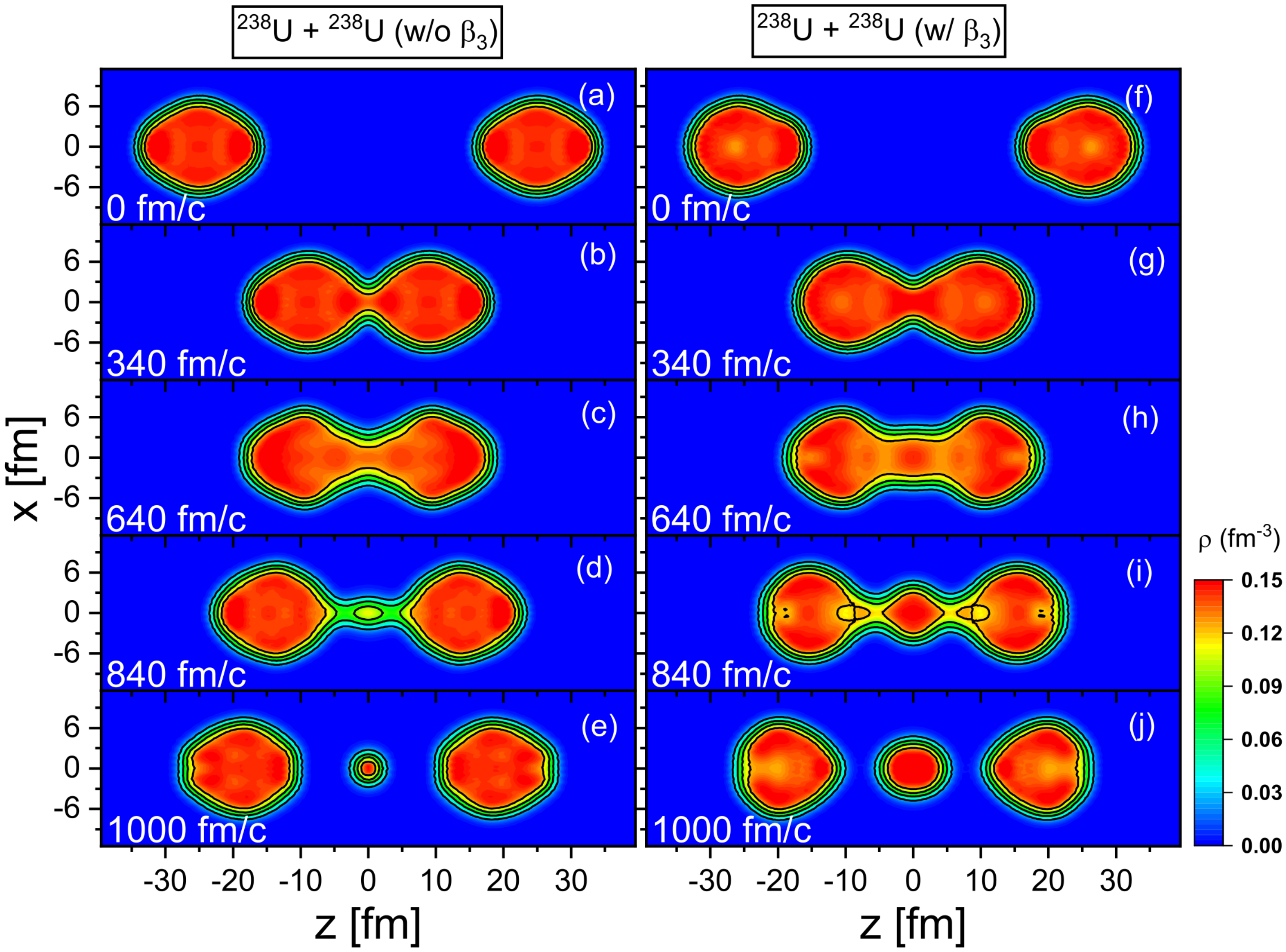
$$\sigma_{N,Z}(E) = 2\pi \int_{b_{\min}}^{b_{\max}} b P_{N,Z}(E, b) db,$$

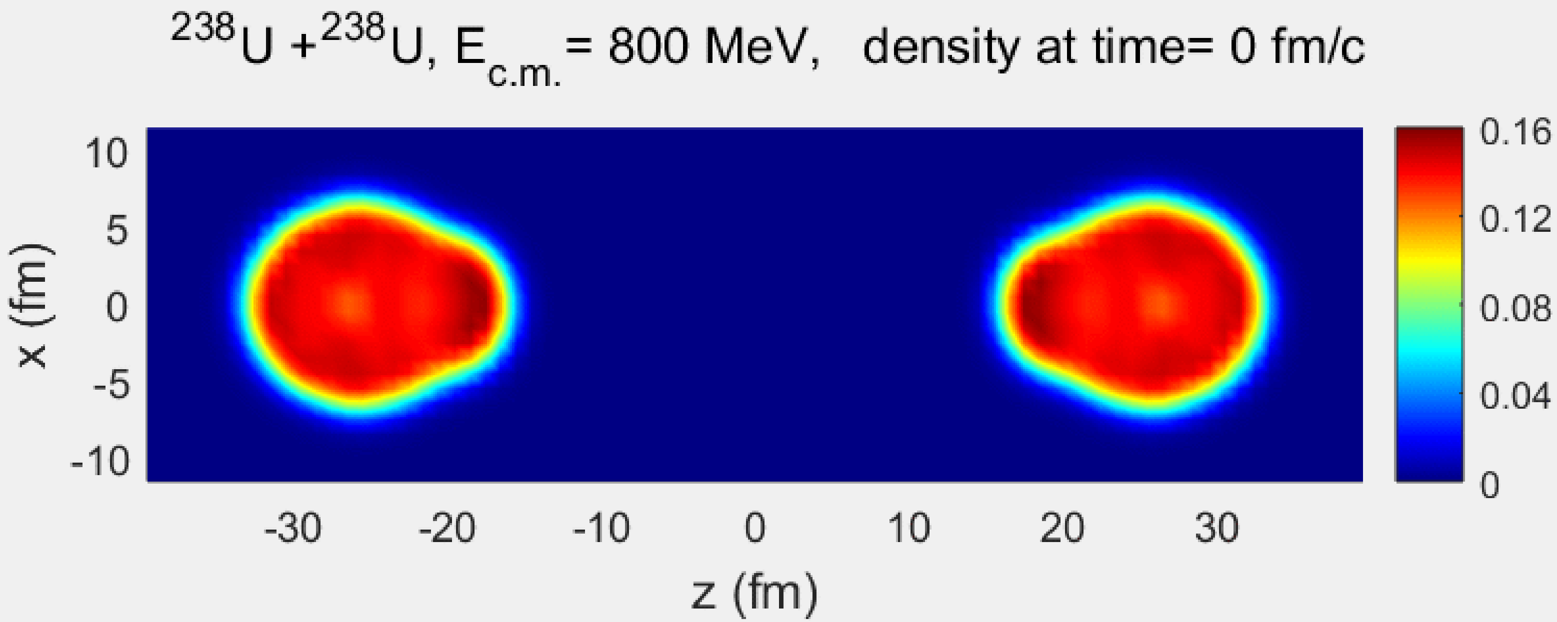


Collisions of actinide nuclei

Collisions of actinide nuclei

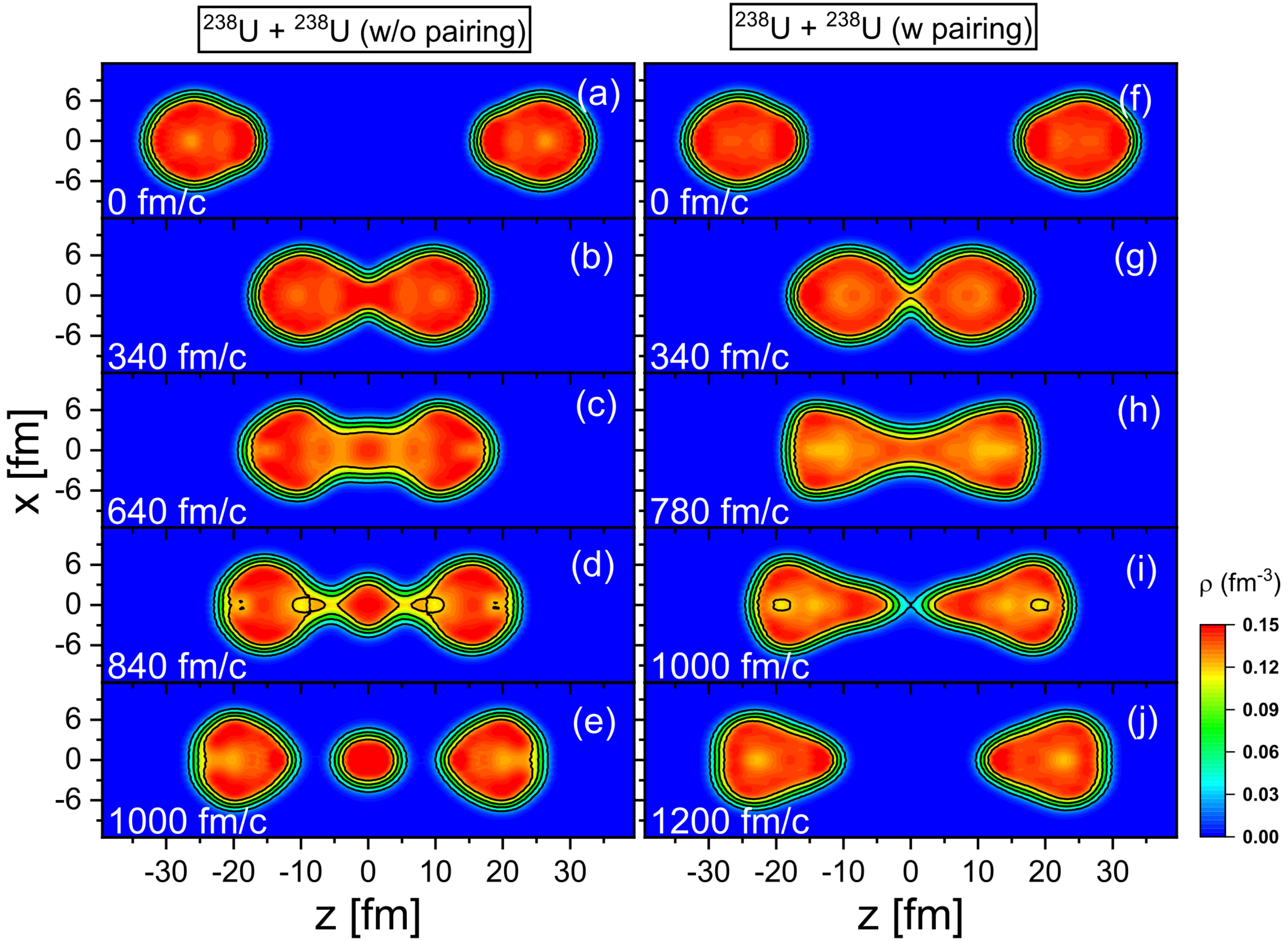
Octupole deformation effects: central collision of $^{238}U + ^{238}U$ at energy $E_{c.m.}$ = 900 MeV.

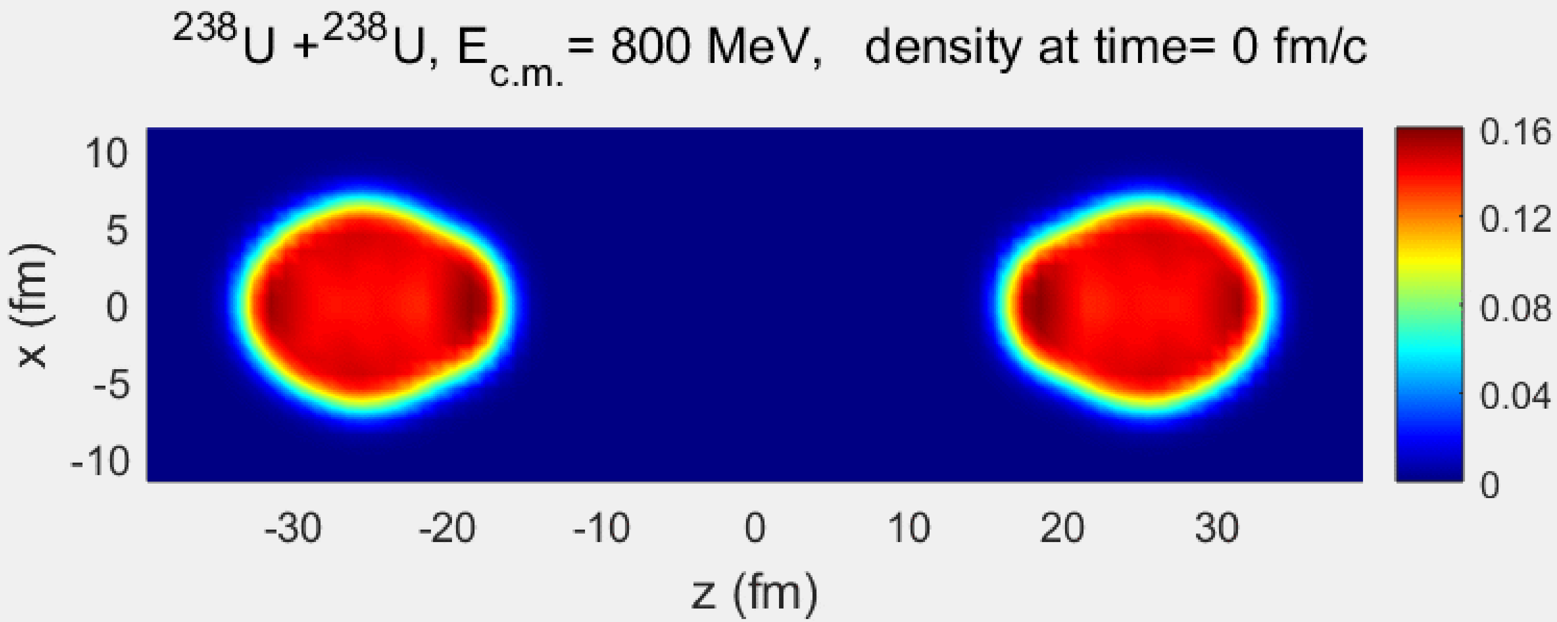




$^{238}U + ^{238}U$, $E_{c.m.}$ = 800 MeV, density at time= 0 fm/c
10
5
0
-5
-10
x (fm)
-30
-20
-10
0
10
20
30
z (fm)
0.16
0.12
0.08
0.04
0

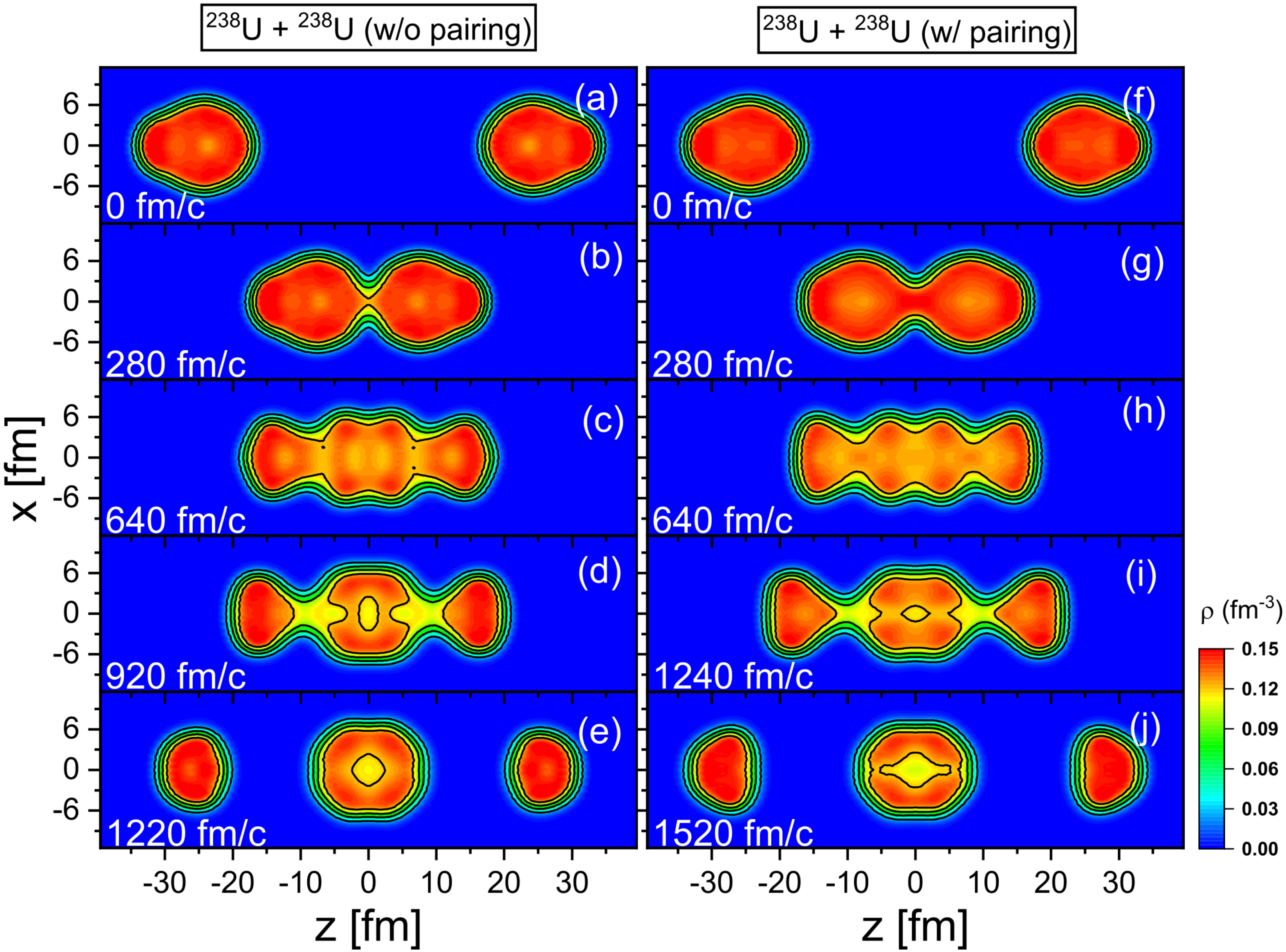
Pairing effects: central collision of $^{238}U + ^{238}U$ at energy $E_{c.m.}$ = 900 MeV.

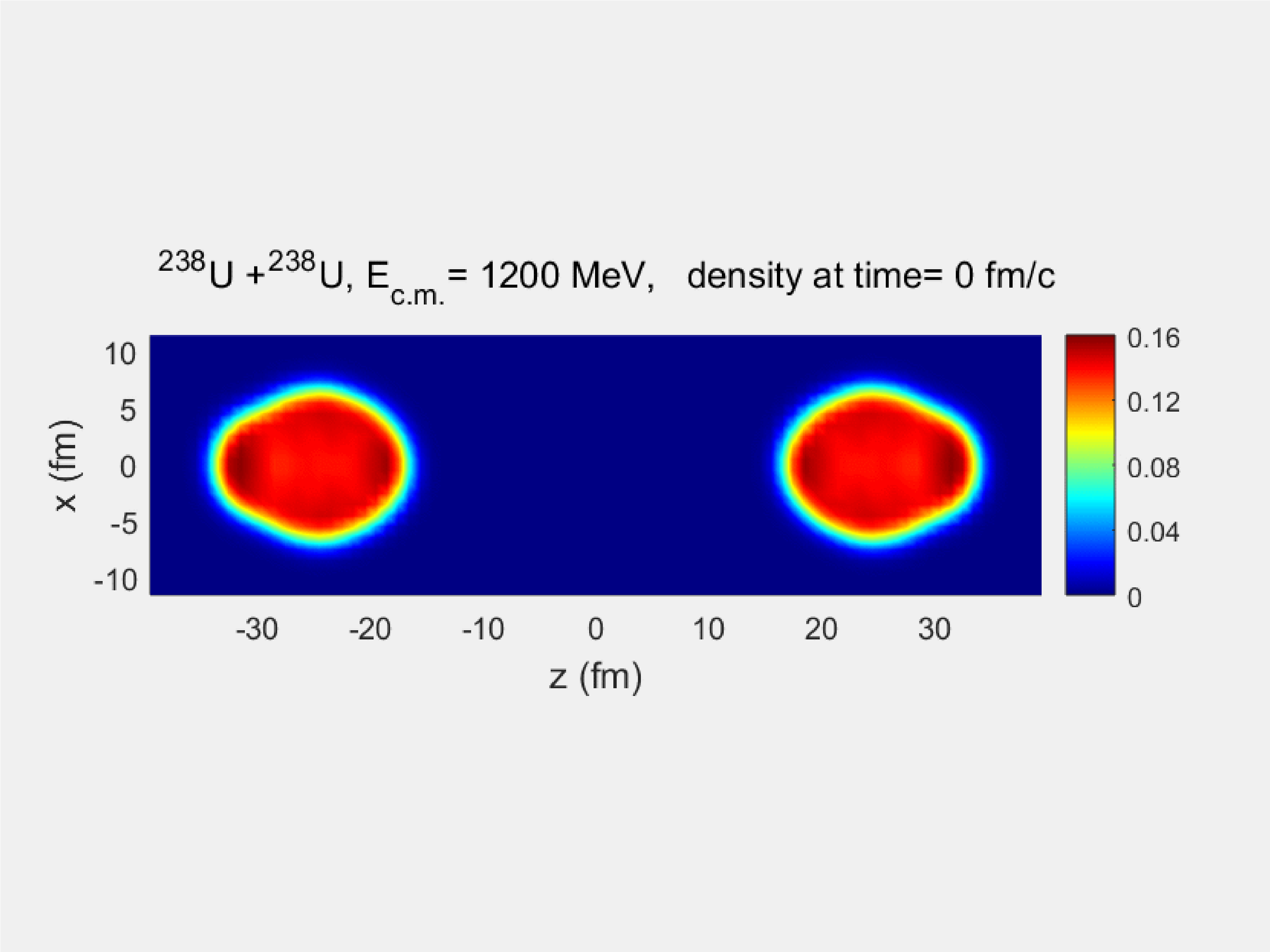




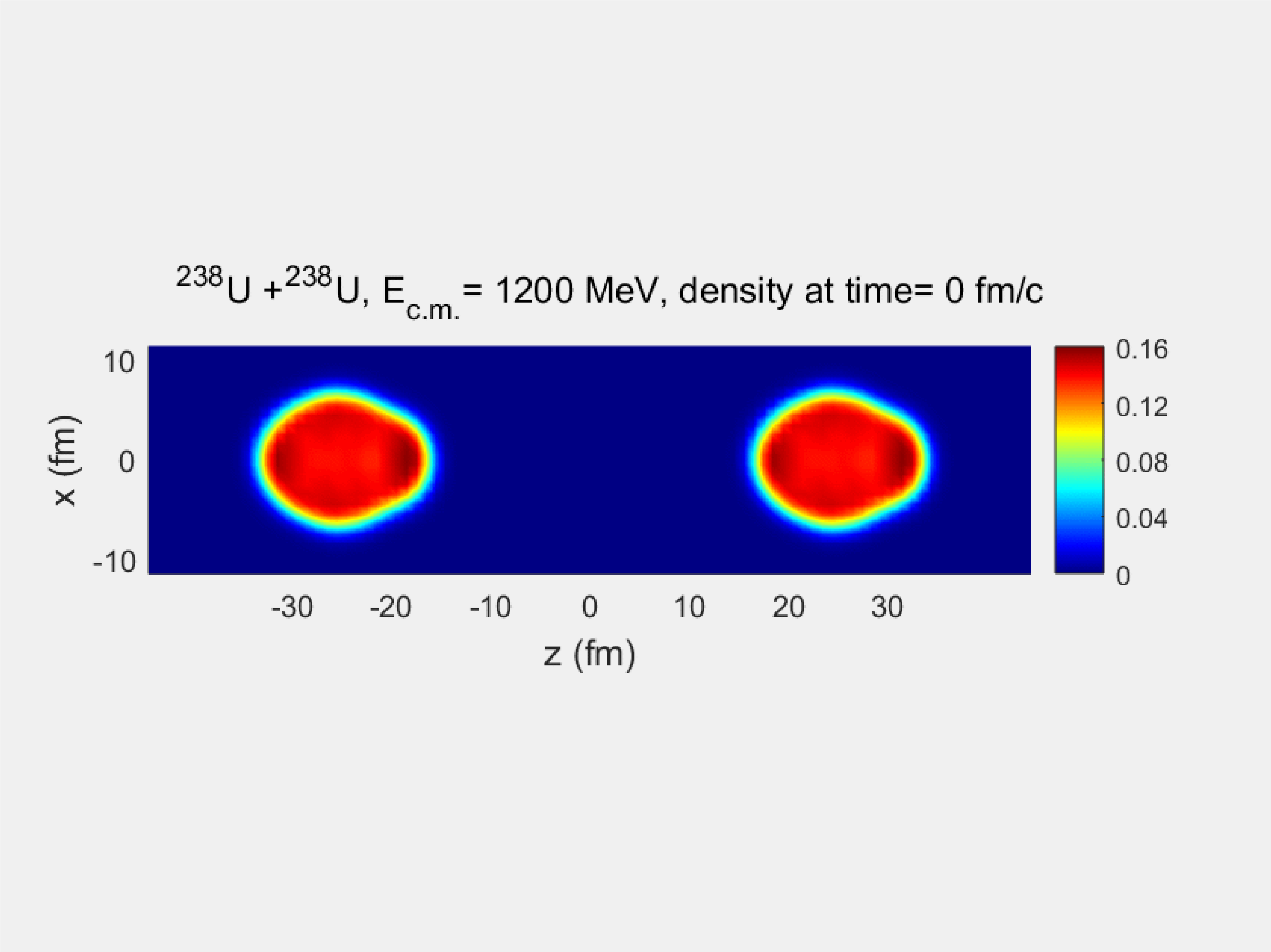
$^{238}U + ^{238}U$, $E_{c.m.}$ = 800 MeV, density at time= 0 fm/c
x (fm)
10
5
0
-5
-10
-30
-20
-10
0
10
20
30
z (fm)
0.16
0.12
0.08
0.04
0

Orientation effects: central (tail-to-tail) collision of $^{238}U + ^{238}U$ at energy $E_{c.m.}$ = 1200 MeV.



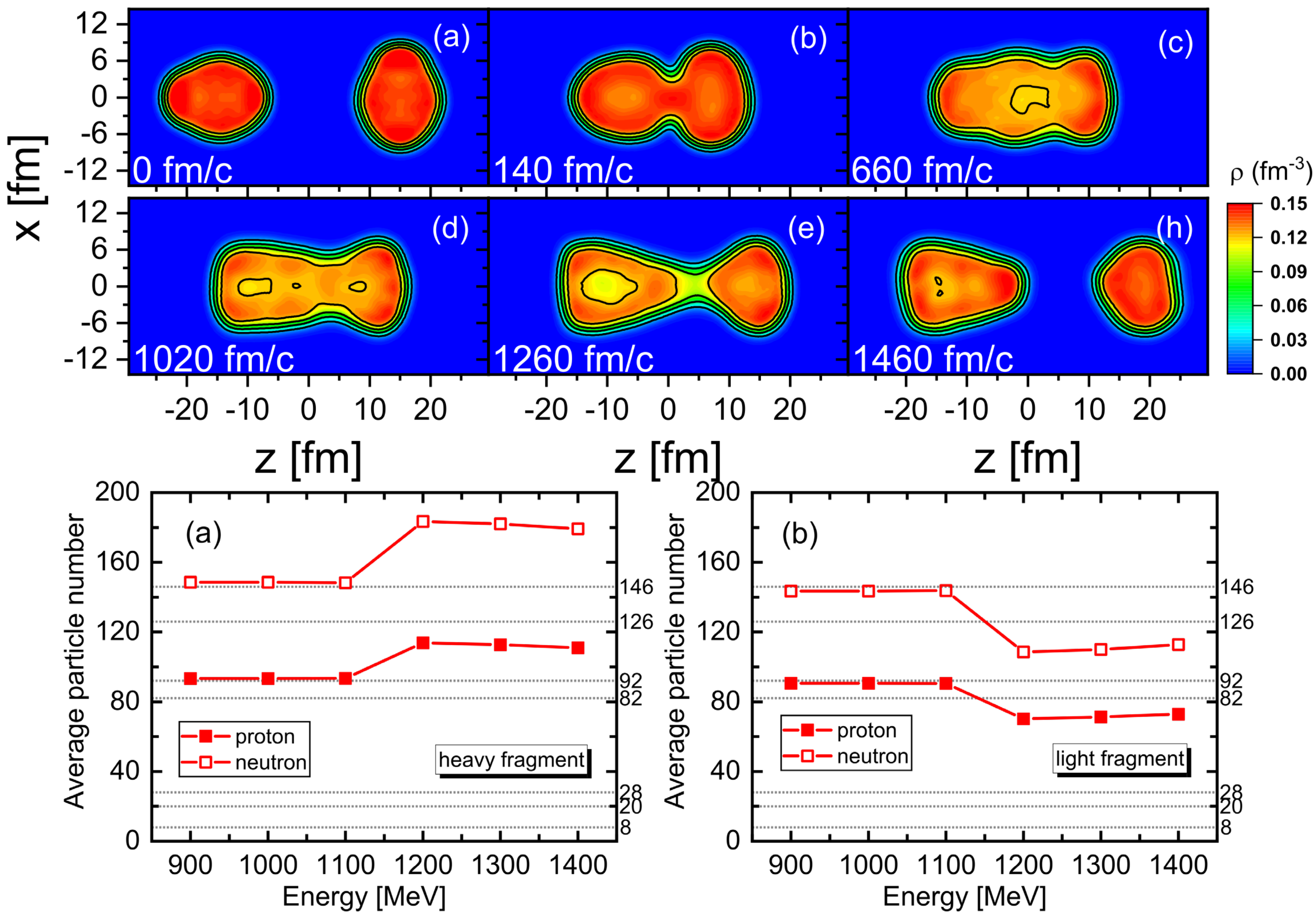


238U +238U, Ec.m.= 1200 MeV, density at time= 0 fm/c
x (fm)
10
5
0
-5
-10
-30
-20
-10
0
10
20
30
z (fm)
0.16
0.12
0.08
0.04
0



$^{238}U + ^{238}U$, $E_{c.m.}$= 1200 MeV, density at time= 0 fm/c
x (fm)
10
0
-10
-30
-20
-10
0
10
20
30
z (fm)
0.16
0.12
0.08
0.04
0

Orientation effects: central (tail-to-side) collision of $^{238}U + ^{238}U$ at energy $E_{c.m.}$ = 1300 MeV.



The average number of protons and neutrons in the heavy, and light fragments as functions of the c.m.s. energy for the central $^{238}U + ^{238}U$ collision, with tail-to-side initial orientation.